

Axion superradiance in rotating neutron stars

Francesca Day

DAMTP, University of Cambridge

PATRAS

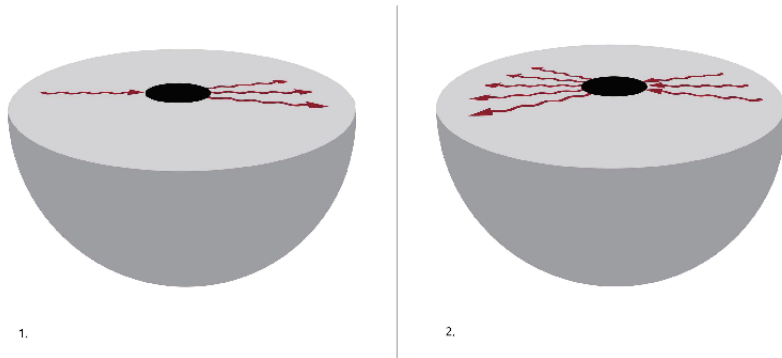
ALU Freiburg June 2019

1904.08341: FD and Jamie McDonald

Outline

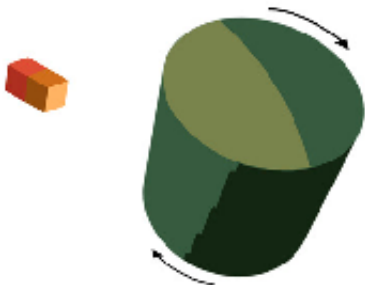
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- 2 Axion-photon superradiance
- 3 Axion superradiance in neutron stars

Black Hole Superradiance

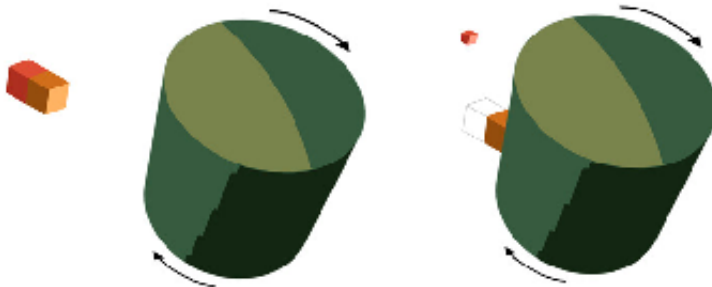


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Carrousel Analogy



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Axion Black Hole Superradiance

- Axions build up around Kerr black hole from an initial quantum fluctuation.
- ‘Bosenova’ explosion as axion cloud collapses (Arvanitaki & Dubovsky, 1004.3558)
- Depletion of black hole spin
- Stellar mass BH spin measurements exclude $6 \times 10^{-13} \text{ eV} < m_a < 2 \times 10^{-11} \text{ eV}$ for $f_a \gtrsim 10^{13} \text{ GeV}$. (Arvanitaki, Baryakhtar & Huang, 1411.2263)
- Advanced Ligo will be sensitive to $m_a \lesssim 10^{-10} \text{ eV}$. (Arvanitaki *et al*, 1604.03958).
- Constraints on the string axiverse mass spectrum (Stott & Marsh, 1805.02016)

Rotational Superradiance in Stars

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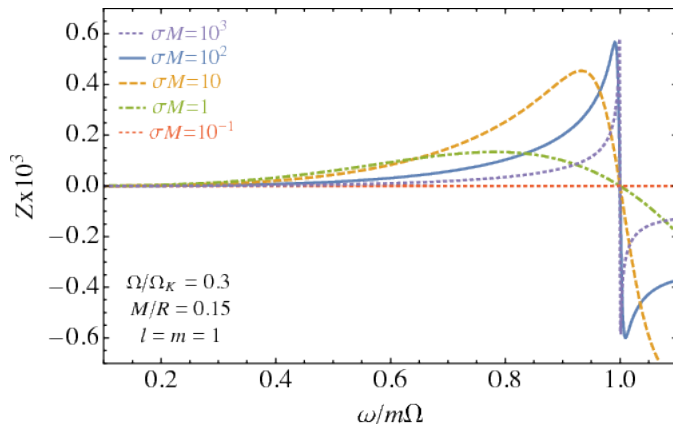
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- The effective damping parameter $\alpha\omega'$ becomes negative.
- Superradiance corresponds to a change in sign of the imaginary part of the eigenfrequency $\text{Im}[\omega]$.

Superradiance

For superradiance we need:

- Rotation
- Bound states - i.e. massive particle for gravitational bound state
- Dissipation

Example: Dark photons in neutron stars



$$Z := \frac{|A_{\text{out}}|^2}{|A_{\text{in}}|^2} - 1.$$

Reproduced from V Cardoso, P Pani and T Yu, 1704.06151.

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Can the massive particle and the dissipation be in different sectors that talk to eachother?

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- *Photons* dissipate energy into the neutron star magnetosphere via the magnetosphere's bulk conductivity.
- Axions and photons mix, so these effects together lead to a superradiant instability in the neutron star magnetosphere.

The axion-photon system

$$\mathcal{L} \supset \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\mu^2}{2} \phi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{g_{a\gamma\gamma}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} - A_\mu j^\mu \right],$$

$$j^\mu = \sigma F^{\mu\nu} u_\nu + \rho u^\mu,$$

giving the linearised equations of motion:

$$\square \phi + \mu^2 \phi = -\frac{g_{a\gamma\gamma}}{2} F_{\mu\nu} \tilde{F}_B^{\mu\nu},$$

$$\partial_\mu F^{\mu\nu} - \sigma F^{\nu\mu} u_\mu = -g_{a\gamma\gamma} (\partial_\mu \phi) \tilde{F}_B^{\mu\nu}.$$

The axion-photon system

In Lorenz gauge:

$$\square\phi + \mu^2\phi = -g_{a\gamma\gamma} \left[\nabla A^0 + \dot{\mathbf{A}} \right] \cdot \mathbf{B}$$

$$\square A^0 = -g_{a\gamma\gamma} \nabla\phi \cdot \mathbf{B} - \sigma \mathbf{u} \cdot \left[\nabla A^0 + \dot{\mathbf{A}} \right]$$

$$\square \mathbf{A} = g_{a\gamma} \dot{\phi} \mathbf{B} - \sigma \left[\nabla A^0 + \dot{\mathbf{A}} \right] + \sigma \mathbf{u} \times [\nabla \times \mathbf{A}]$$

For a neutron star magnetosphere, $\mathbf{B} \sim 10^{10}$ G, $0.01\Omega \lesssim \sigma \lesssim 100\Omega$.

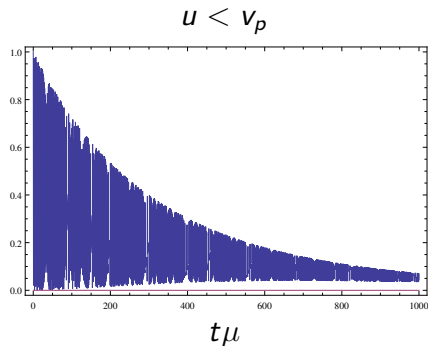
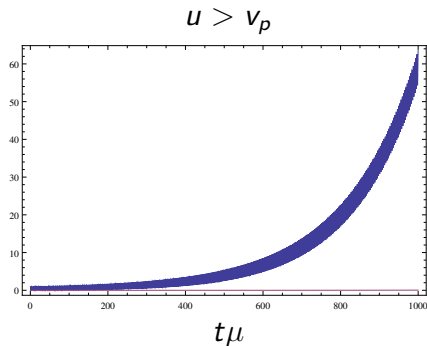
Warm up: 1D Example

- Consider a 1D fluid flow with $\mathbf{A} \cdot \mathbf{u} = 0$, $\mathbf{B} \cdot \mathbf{k} = 0$.
- Introduce a sound speed: $\square\phi \rightarrow \ddot{\phi} - c_s^2 \nabla^2 \phi$.
- The unperturbed axion sound speed is $v_p(k)^2 = (c_s^2 + \mu^2/k^2)$.
- We find:

$$\ddot{\phi} + k^2 c_s^2 \phi + \mu^2 \phi = -g_{a\gamma\gamma} \dot{A}_{\parallel} B$$

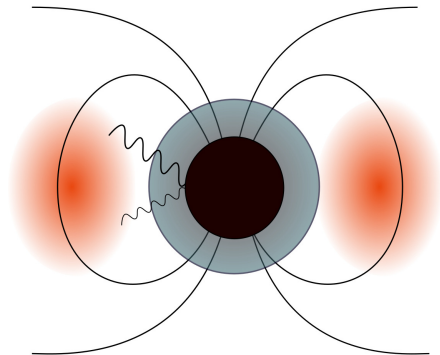
$$\ddot{A}_{\parallel} + k^2 A_{\parallel} + \sigma u^0 \dot{A}_{\parallel} + i\sigma(\mathbf{k} \cdot \mathbf{u}) A_{\parallel} = g_{a\gamma\gamma} \dot{\phi} B$$

Warm up: 1D Example



The axion (blue) and photon component (red) (arbitrary units) for the mode $|\mathbf{k}| = 3\mu$. The left- and right-hand plots correspond to sound speeds $c_s = 0.01$ and $c_s = 0.7$, respectively. The other parameter values are $Bg_{a\gamma\gamma}/\mu = 0.3$, $u = 0.5$ and $\sigma/\mu = 3$.

Axion superradiance in neutron stars



Schematic illustration of the instability. The axion boundstate (orange) mixes with a photon mode which is then amplified by scattering off the rotating magnetosphere (grey). The photon energy is then deposited back into the axion sector.

3D Case

$$\square\phi + \mu^2\phi = -g_{a\gamma\gamma} \left[\nabla A^0 + \dot{\mathbf{A}} \right] \cdot \mathbf{B}$$

$$\square A^0 = -g_{a\gamma\gamma} \nabla\phi \cdot \mathbf{B} - \sigma \mathbf{u} \cdot \left[\nabla A^0 + \dot{\mathbf{A}} \right]$$

$$\square \mathbf{A} = g_{a\gamma} \dot{\phi} \mathbf{B} - \sigma \left[\nabla A^0 + \dot{\mathbf{A}} \right] + \sigma \mathbf{u} \times [\nabla \times \mathbf{A}]$$

- 3D geometry
- Bound states
- Rotation of star removes need for sound speed

Perturbation theory

We re-write our equations of motion as a time-independent Schrodinger equation:

$$[H(\sigma_*) + V(\sigma_M, g_{a\gamma\gamma})] \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix} = \omega^2 \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix}$$

$$H = \begin{pmatrix} -\frac{d^2}{dr_*^2} + U(r), & 0 & 0 \\ 0 & -\nabla^2 & 0 \\ 0 & 0 & -\nabla^2 \end{pmatrix}$$

Perturbation theory

$$V = V_A + V_{a\gamma\gamma}$$

$$V_{a\gamma\gamma} = ig_{a\gamma\gamma} \begin{pmatrix} 0 & \mathbf{B}(\hat{x}) \cdot \hat{p} & -\omega \mathbf{B}(\hat{x}) \\ \mathbf{B}(\hat{x}) \cdot \hat{p} & 0 & 0 \\ \omega \mathbf{B}(\hat{x}) & 0 & 0 \end{pmatrix}$$

$$V_A = i\sigma_M(\hat{x}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{u}(\hat{x}) \cdot \hat{p} & -\omega \mathbf{u}(\hat{x}) \\ 0 & \hat{p} & -\omega - \mathbf{u}(\hat{x}) \times \hat{p} \end{pmatrix}$$

Perturbation theory

Use quantum mechanical perturbation theory to find the eigen-frequencies of the axion-photon system as a double expansion in σ and $g_{a\gamma\gamma}$:

$$\omega = \omega_{\ell n} + \omega^{(1)} + \omega^{(2)} + \omega^{(3)} + \dots$$

Axion superradiance in neutron stars

$$\text{Im} [\omega_{\ell mn}] \simeq \pi g_{a\gamma\gamma}^2 B^2 \sigma_M \left((m\Omega - \omega) - \omega S[\ell, \ell + 1, m]^2 \right) \frac{(R_{\text{LC}}\omega)^{2\ell+3} - (R\omega)^{2\ell+3}}{32\omega^3} \alpha^{2\ell+5} \mathcal{F}_{\ell n}$$

$$V_A = i\sigma_M(\hat{x}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{u}(\hat{x}) \cdot \hat{p} & -\omega \mathbf{u}(\hat{x}) \\ 0 & \hat{p} & -\omega - \mathbf{u}(\hat{x}) \times \hat{p} \end{pmatrix}$$

Axion superradiance in neutron stars

- We typically find superradiant timescales $\tau = \frac{1}{\text{Im}[\omega_{\ell mn}]}$ a few orders of magnitude higher than the neutron star spin down time.
- Therefore, we do not expect this process to be observable.
- Our result is an example of a more general phenomenon which can arise when there is an instability in the plasma sector.
- For axion modes which couple to an unstable mode of the neutron star, one could in principle find similar instabilities.

Perturbation theory

$$\omega = \omega_{\ell n} + \omega^{(1)} + \omega^{(2)} + \omega^{(3)} + \dots$$

$$\omega^{(1)} = \frac{1}{2\omega_{\ell n}} \langle \phi_{\ell mn} | V | \phi_{\ell mn} \rangle = 0$$

$$\omega^{(2)} = \frac{1}{2\omega_{\ell n}} \sum_{\ell', m', i} \int d[\omega^2] \frac{\langle \phi_{\ell mn} | V_{a\gamma\gamma}(\omega_{\ell n}) | A_{\ell' m'}^{(i)}(\omega) \rangle \langle A_{\ell' m'}^{(i)}(\omega) | V_{a\gamma\gamma}(\omega_{\ell n}) | \phi_{\ell mn} \rangle}{\omega_{\ell n}^2 - \omega^2} \in \mathbb{R}$$

Perturbation theory

$$\delta\omega_{\ell mn}^{(3)} = \frac{1}{2\omega_{\ell n}} \sum_{\ell_{1,2}, m_{1,2}, i, j} \int d[\omega_{1,2}^2] \frac{\langle \phi_{\ell mn} | V | A_{\ell_1 m_1}^{(i)}(\omega_1) \rangle \langle A_{\ell_1 m_1}^{(i)}(\omega_1) | V | A_{\ell_2 m_2}^{(j)}(\omega_2) \rangle \langle A_{\ell_2 m_2}^{(j)} | V | \phi_{\ell mn} \rangle}{(\omega_{\ell n}^2 - \omega_1^2)(\omega_{\ell n}^2 - \omega_2^2)}$$

$$\delta\omega_{\ell mn}^{(3)} = \frac{\pi^2}{8\omega_{\ell n}} \sum_{\ell_{1,2}, m_{1,2}} \sum_{i, j} \langle \phi_{\ell mn} | V_{a\gamma\gamma} | A_{\ell_1 m_1}^{(i)}(\omega_{\ell n}) \rangle \langle A_{\ell_1 m_1}^{(i)} | V_A | A_{\ell_2 m_2}^{(j)}(\omega_{\ell_2}) \rangle \langle A_{\ell_2 m_2}^{(j)} | V_{a\gamma\gamma} | \phi_{\ell mn} \rangle$$

Perturbation Theory

$\phi_{\ell mn} \sim \text{Laguerre polynomial} \times \text{spherical harmonic}$

$A_{\ell m}(\omega) \sim \text{Bessel function}(r\omega) \times \text{spherical harmonic}$

$$\text{Im} [\omega_{\ell mn}] \simeq$$

$$\pi g_{a\gamma\gamma}^2 B^2 \sigma_M \left((m\Omega - \omega) - \omega S[\ell, \ell + 1, m]^2 \right) \frac{(R_{\text{LC}}\omega)^{2\ell+3} - (R\omega)^{2\ell+3}}{32\omega^3} \alpha^{2\ell+5} \mathcal{F}_{\ell n}$$