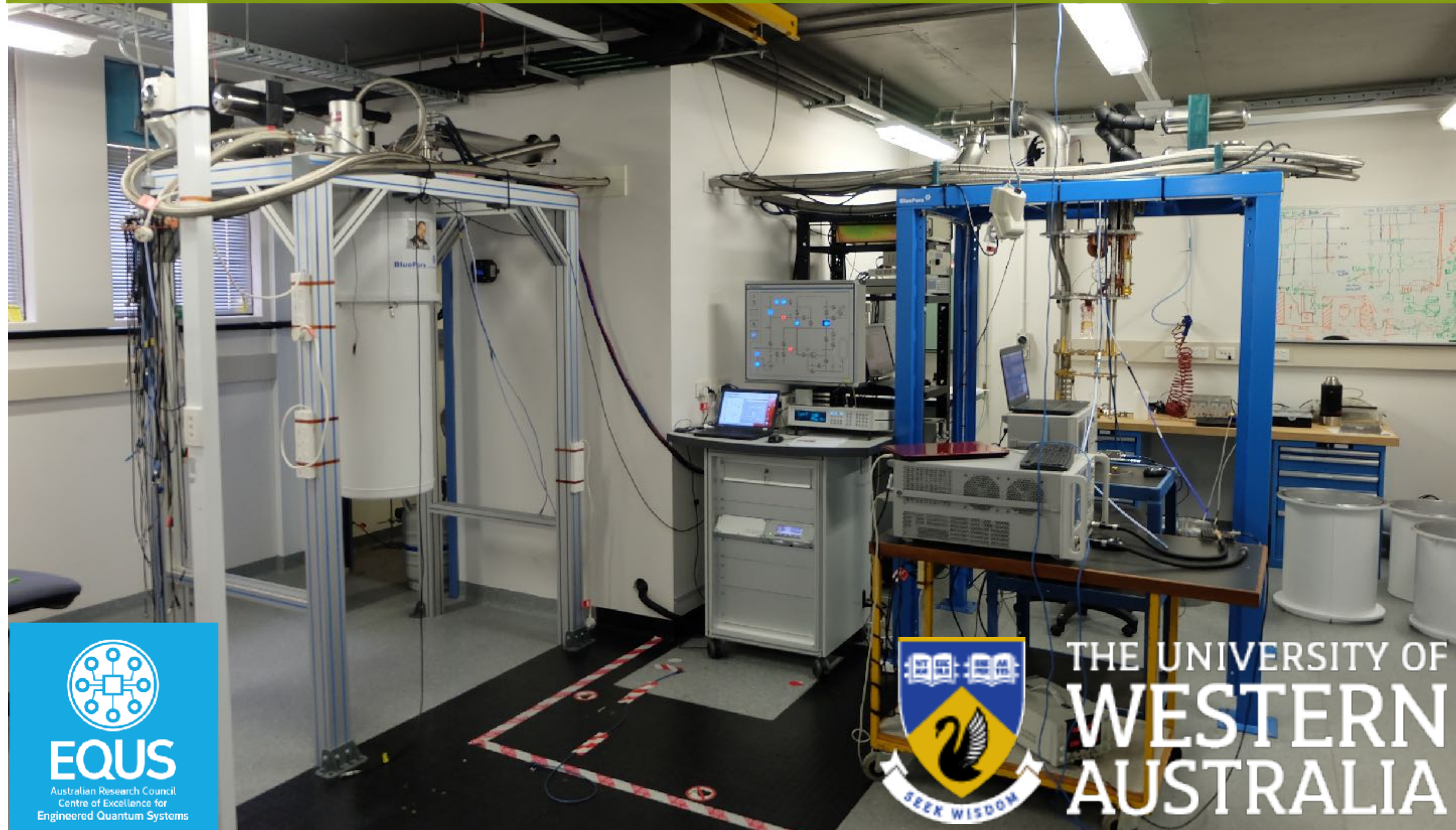


Axion Haloscope Experiments at the University of Western Australia

Michael Tobar

https://arxiv.org/a/tobar_m_1.html

FACULTY OF SCIENCE



EQUS

Australian Research Council
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Engineered Quantum Systems



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AUSTRALIA**

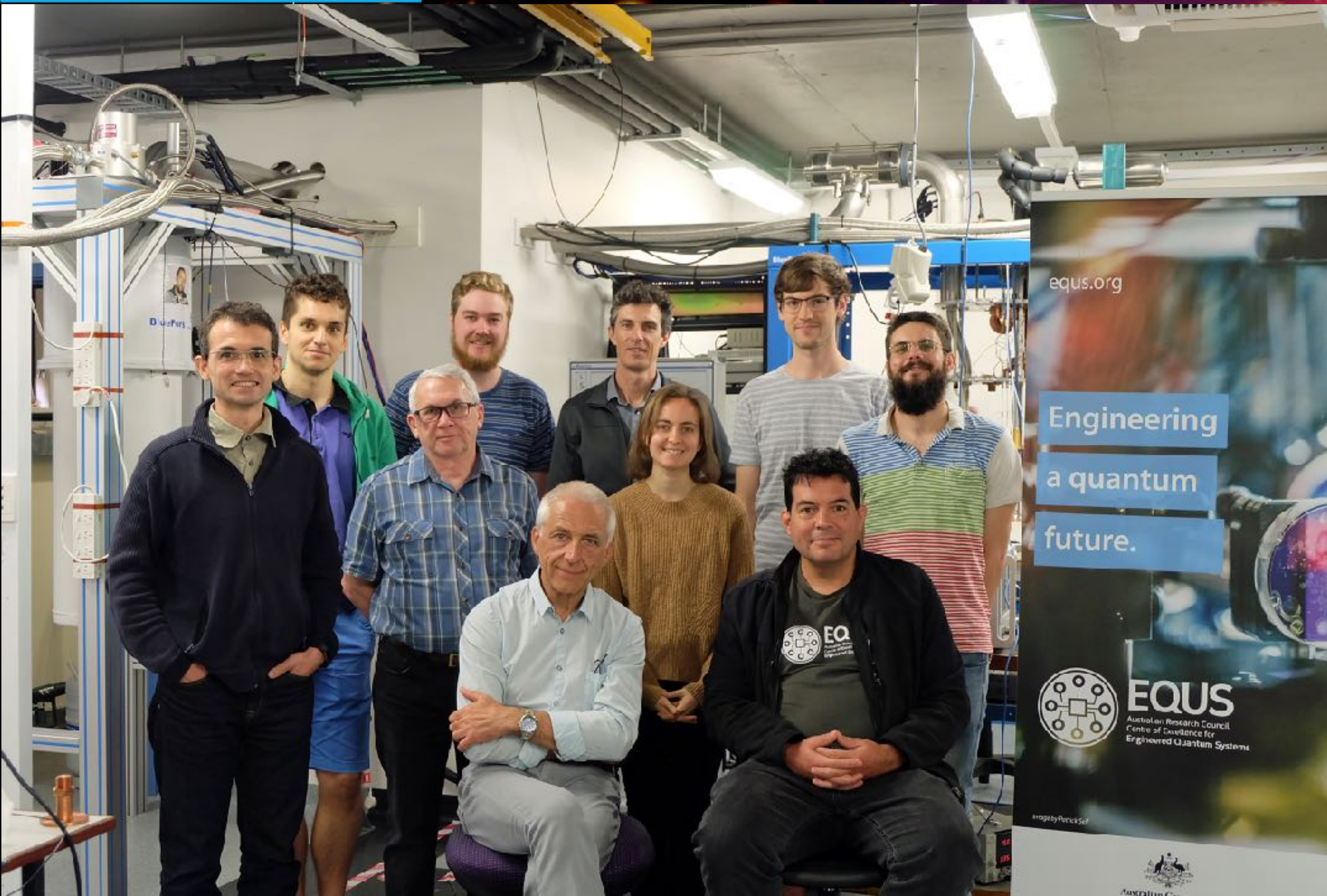


Australian Research Council
Centre of Excellence for
Engineered Quantum Systems

Frequency and Quantum Metrology Research Group at UWA



THE UNIVERSITY OF
**WESTERN
AUSTRALIA**



ACADEMIC STAFF

Michael Tobar

Eugene Ivanov

John McFerran

Alexey Veryaskin

POSTDOCS

Sascha Schediwy

Maxim Goryachev

Ben Kaebe

Ben McAllister

STUDENTS

Graeme Flower

Ben Dix-Mathews

Catriona Thomas

Jacob Ma

William Campbell

Aaron Quiskamp

Elrina Hartman

Rayman Watson

Brett Leask

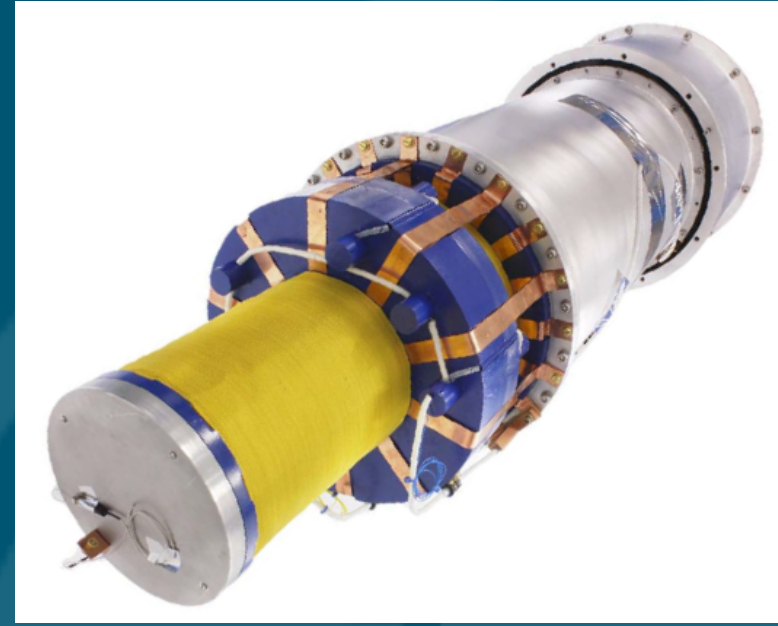
Mushfiq Shah

Naijiao Jin

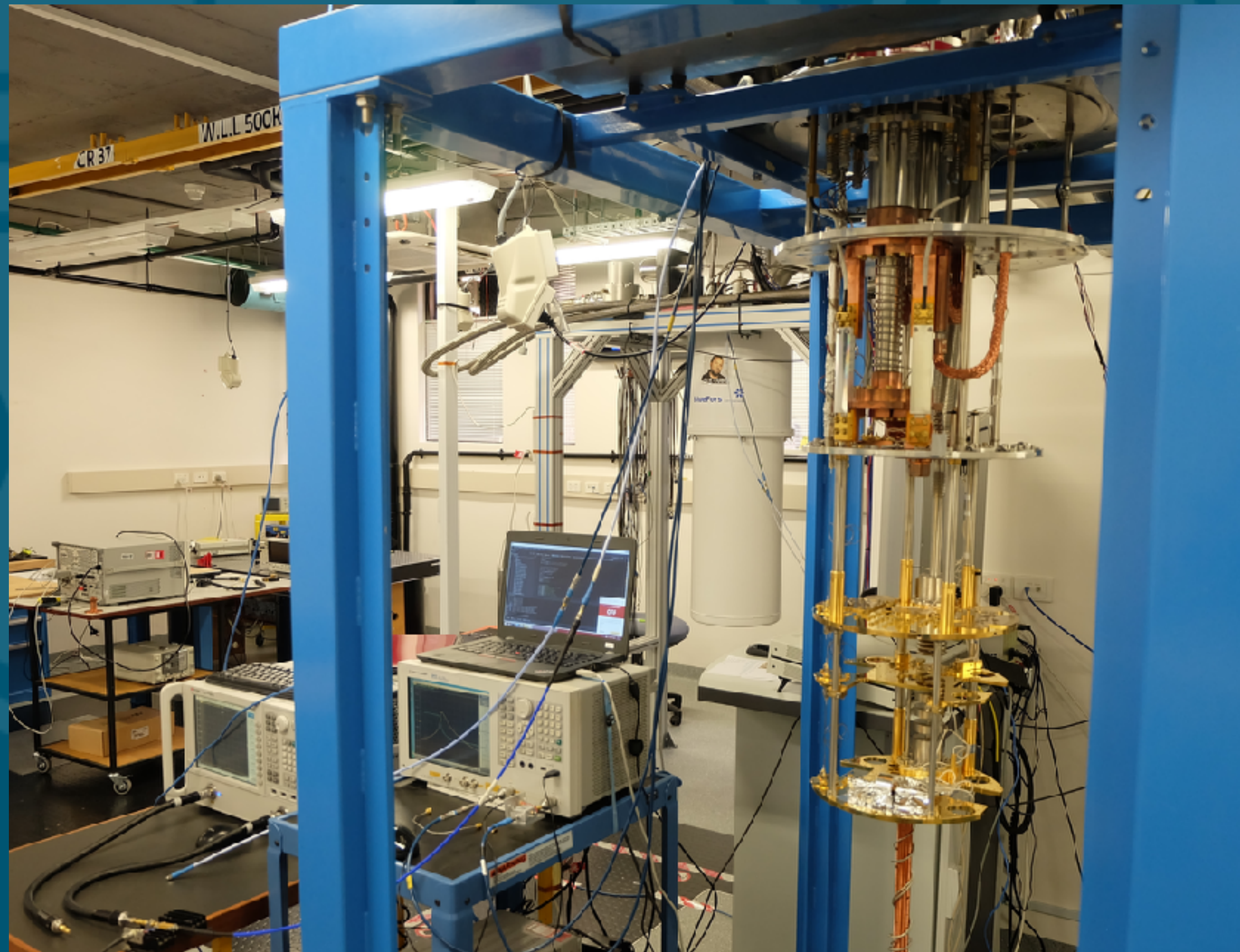
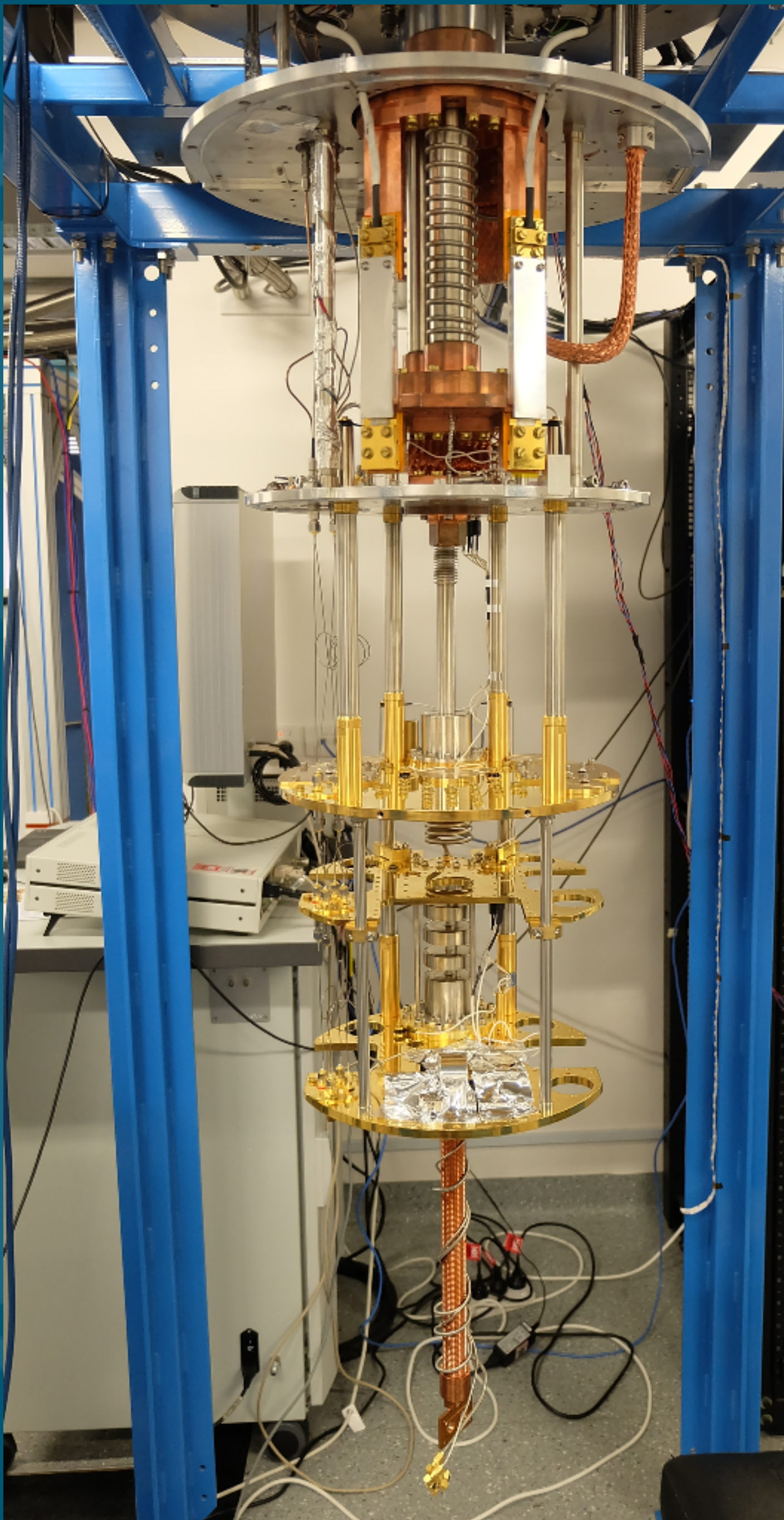
Ignasius Setiাপুত্রা

Dark Matter Experiments at UWA

Dilution Fridge Lab



7 T Magnet (10 cm bore)



Cavity-Magnon Polariton Axion Detection Experiment

arXiv:1811.09348v3 [physics.ins-det]

$g_{aee} > 3.7 \times 10^{-9}$ in the range $33.79 \mu\text{eV} < m_a < 33.94 \mu\text{eV}$ with 95% confidence

$$\frac{H}{\hbar} = \omega_c a^\dagger a + \omega_m b^\dagger b + g_{cm}(a^\dagger + a)(b^\dagger + b)$$

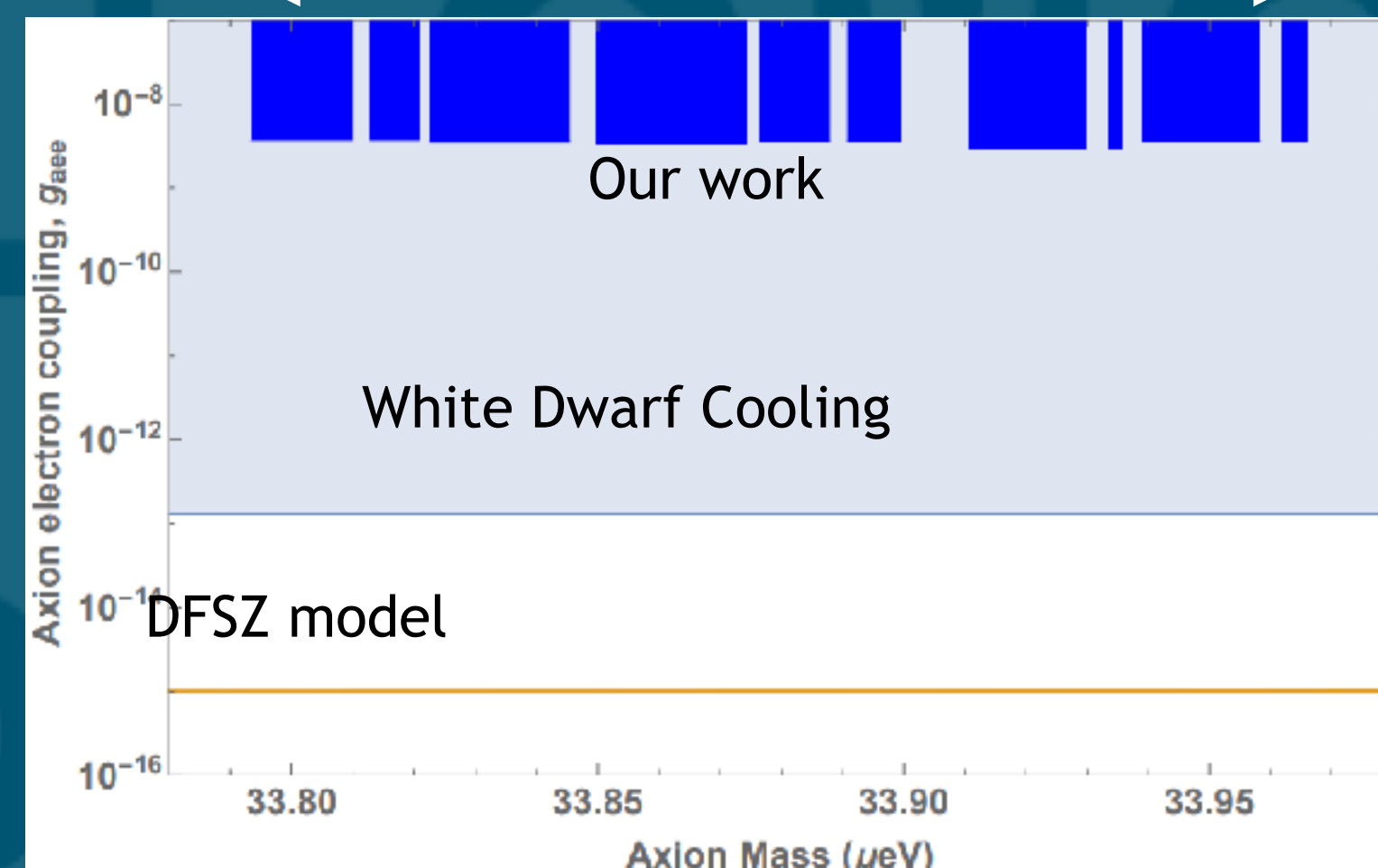
Photons

Magnons

Interaction

36MHz in 6 MHz blocks
from 8hrs of averages

Centred at
8.2GHz



Contents lists available at ScienceDirect

Physics of the Dark Universe

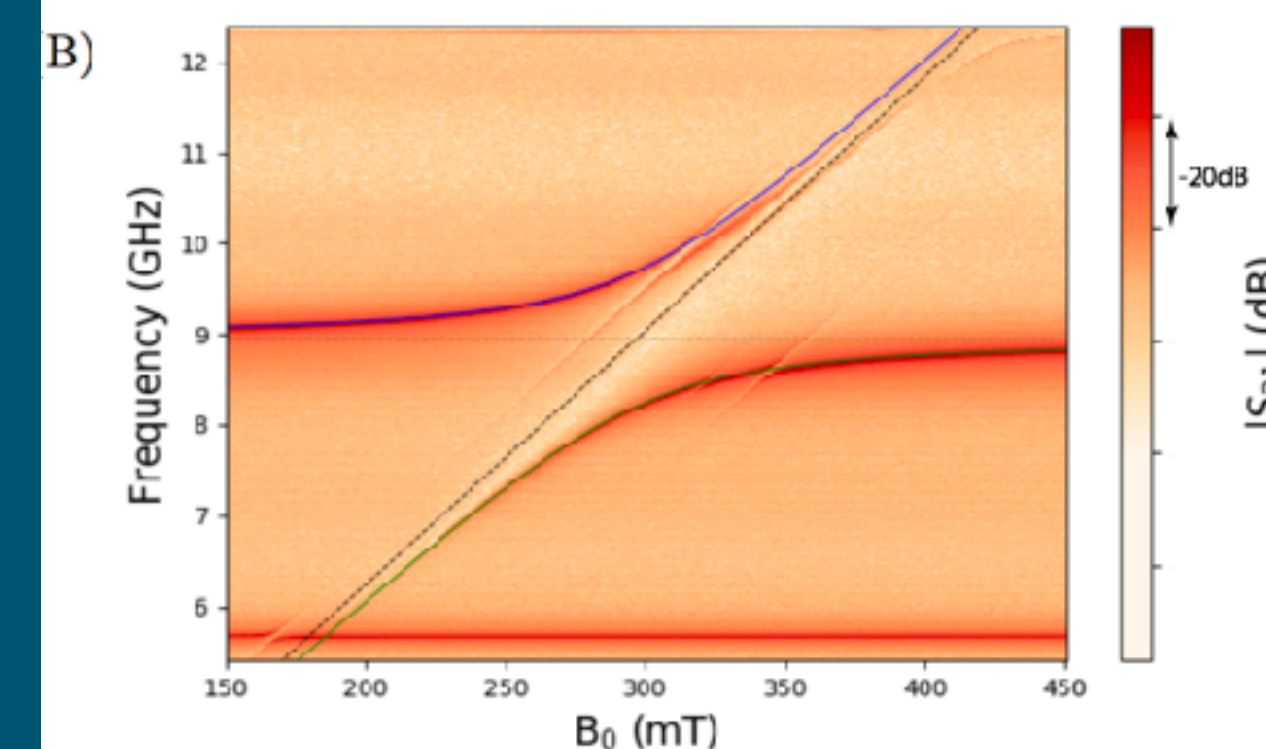
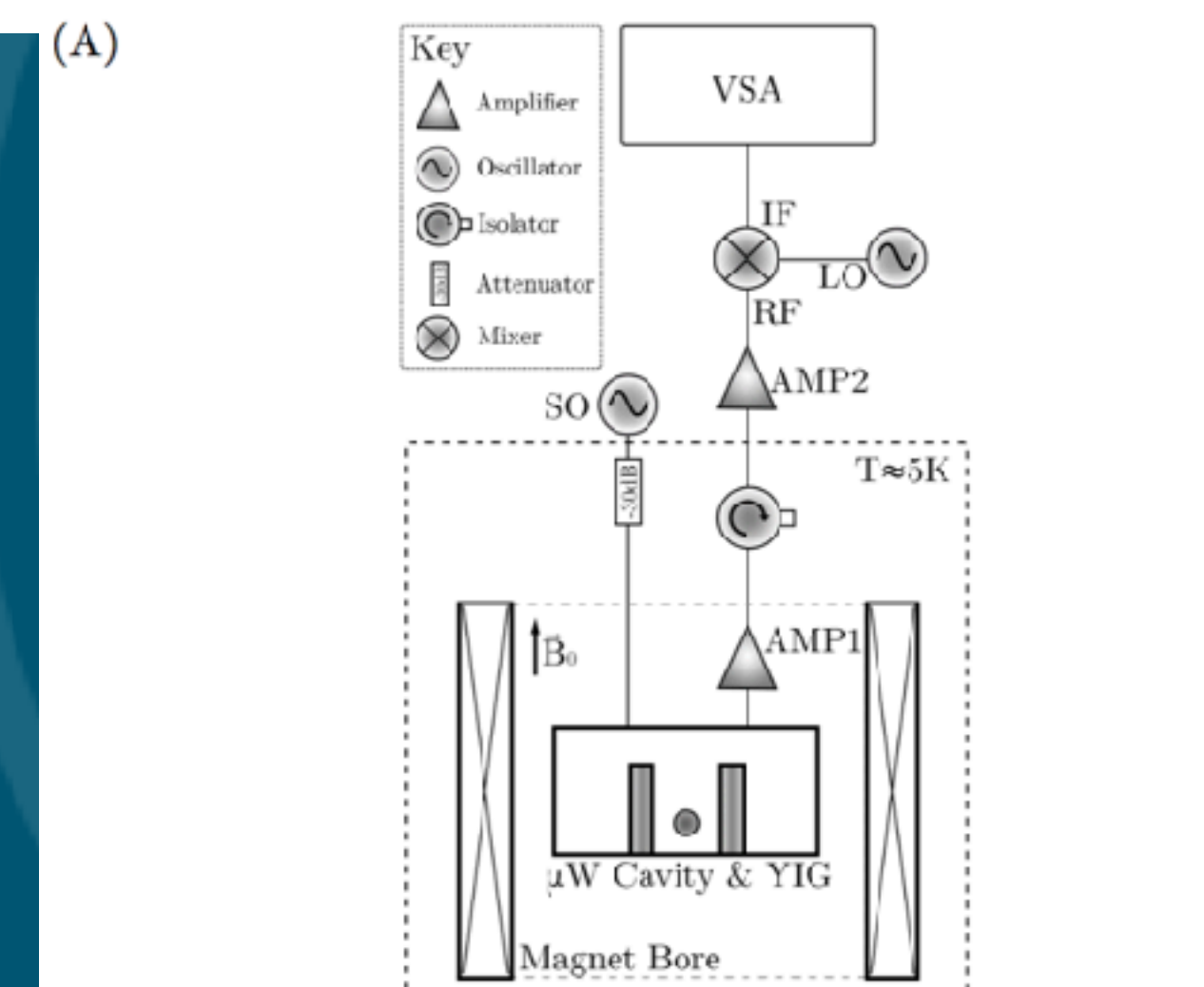
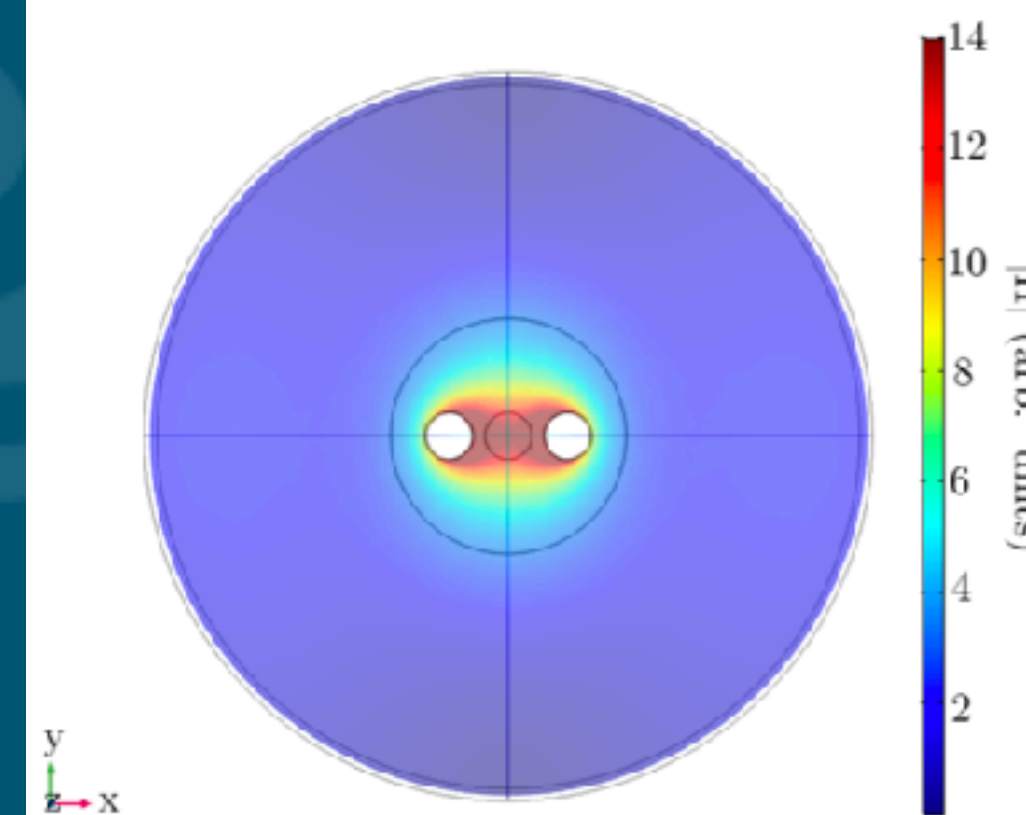
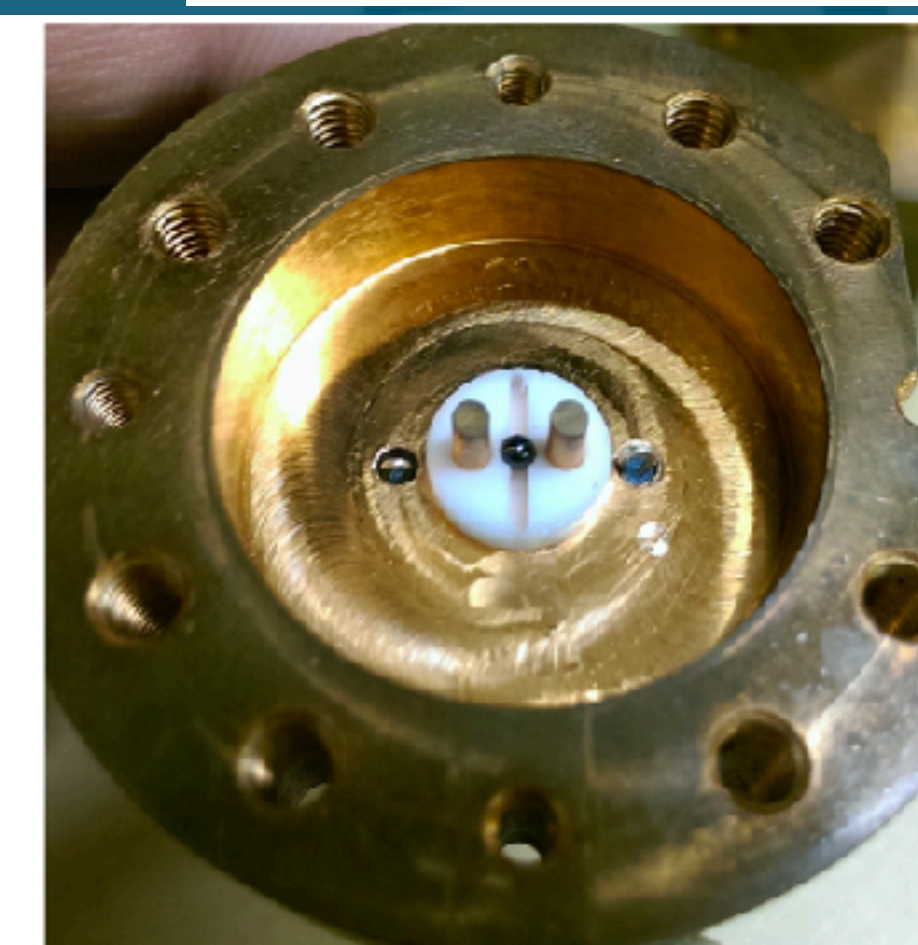
journal homepage: www.elsevier.com/locate/dark



Broadening frequency range of a ferromagnetic axion haloscope with strongly coupled cavity-magnon polaritons

Graeme Flower*, Jeremy Bourhill, Maxim Goryachev, Michael E. Tobar

ARC Centre of Excellence for Engineered Quantum Systems, Department of Physics, University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia



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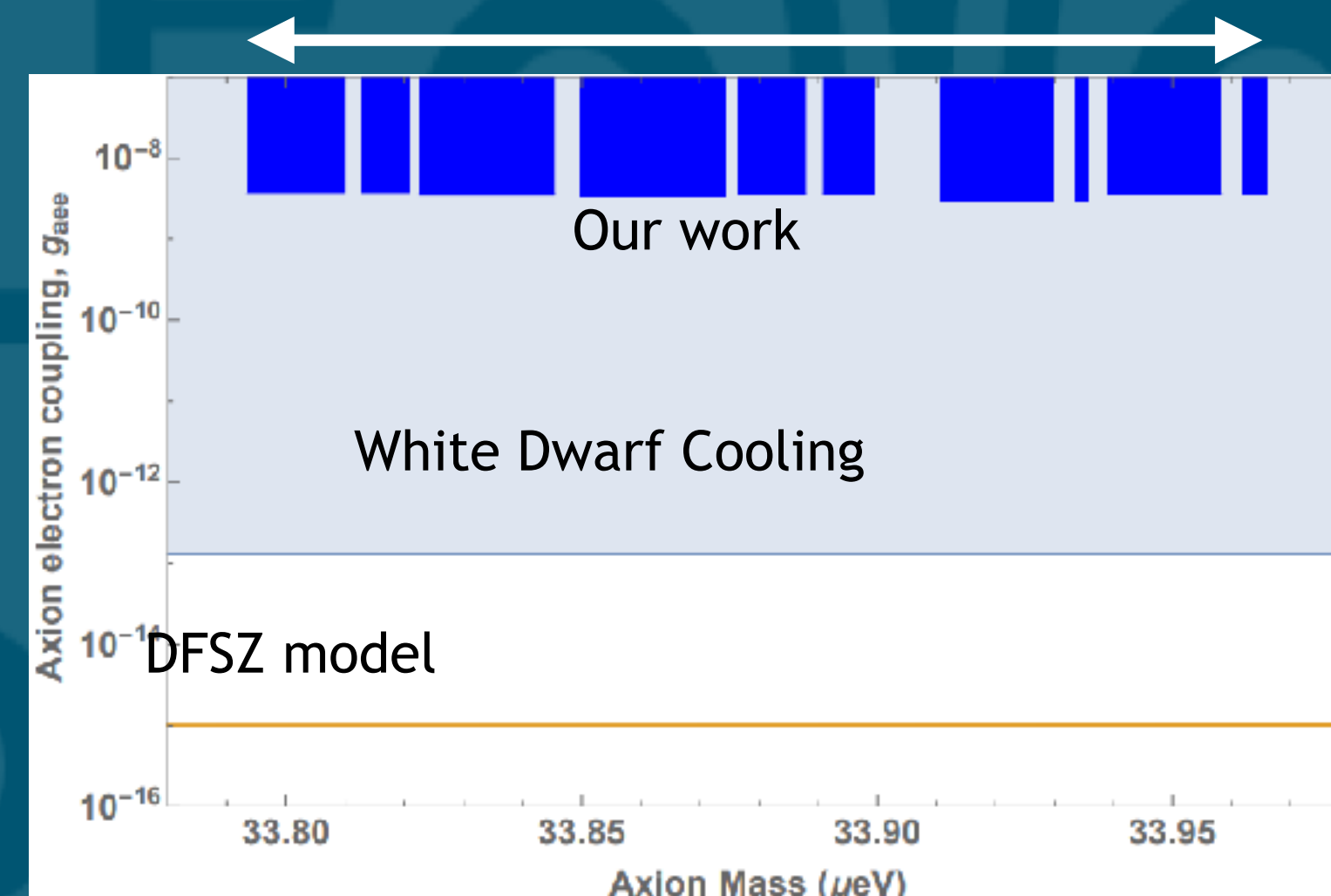
Photons

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36MHz in 6 MHz blocks
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Centred at
8.2GHz



Poster Session1 Tuesday, June 4, 14:40:
Broadening Frequency Range of a Ferromagnetic
Axion Haloscope with Strongly Coupled Cavity-
Magnon Polaritons: **Graeme Flower**



Contents lists available at ScienceDirect

Physics of the Dark Universe

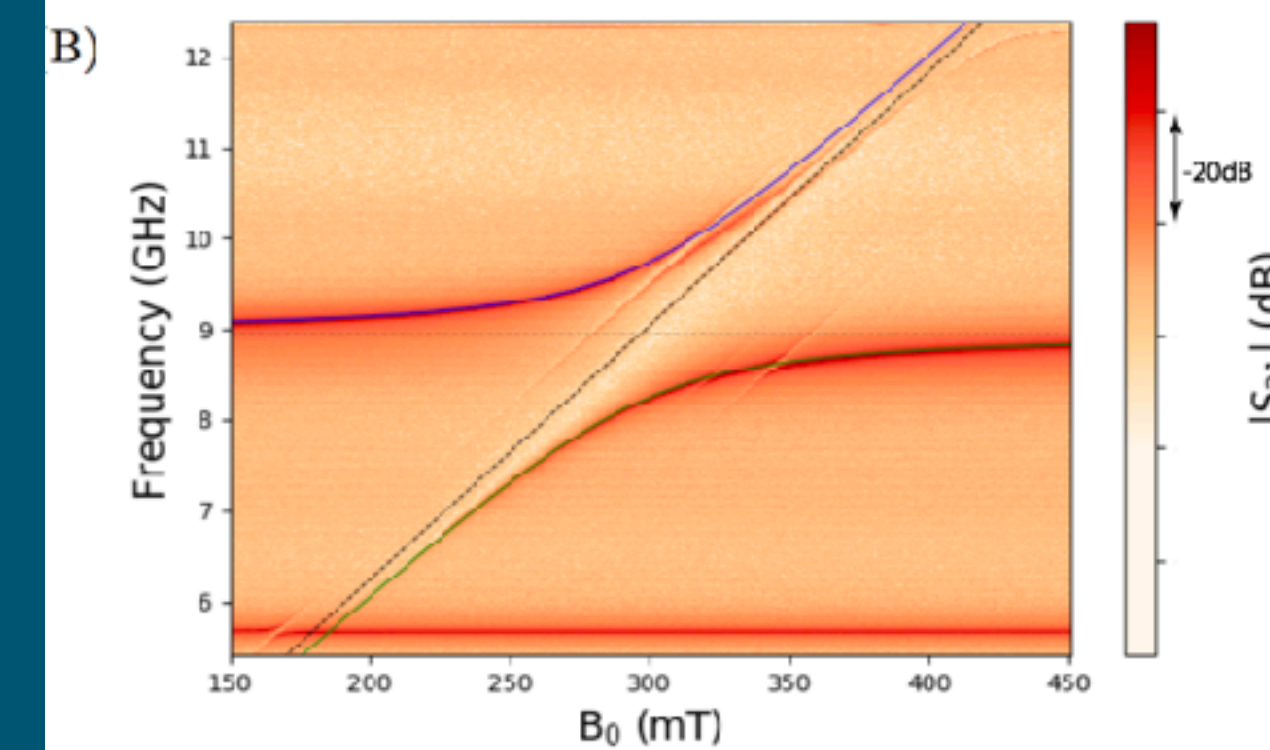
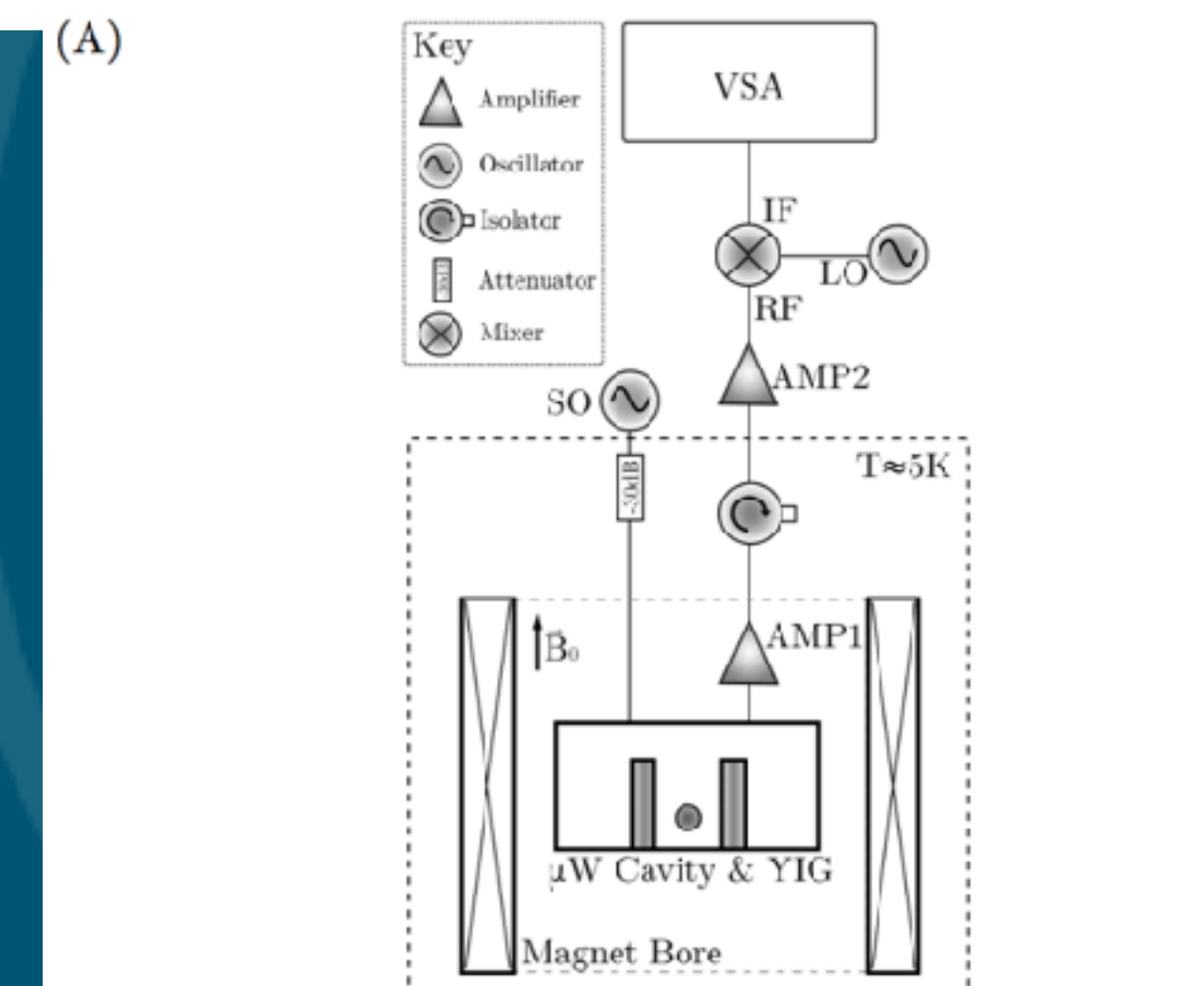
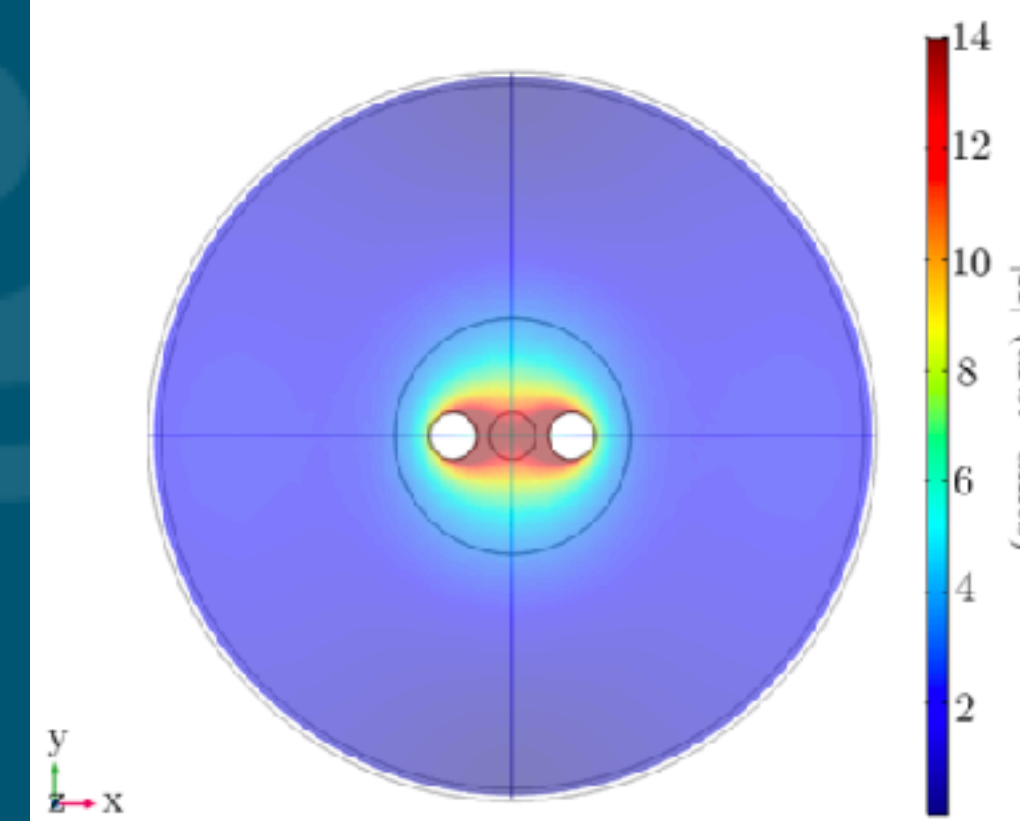
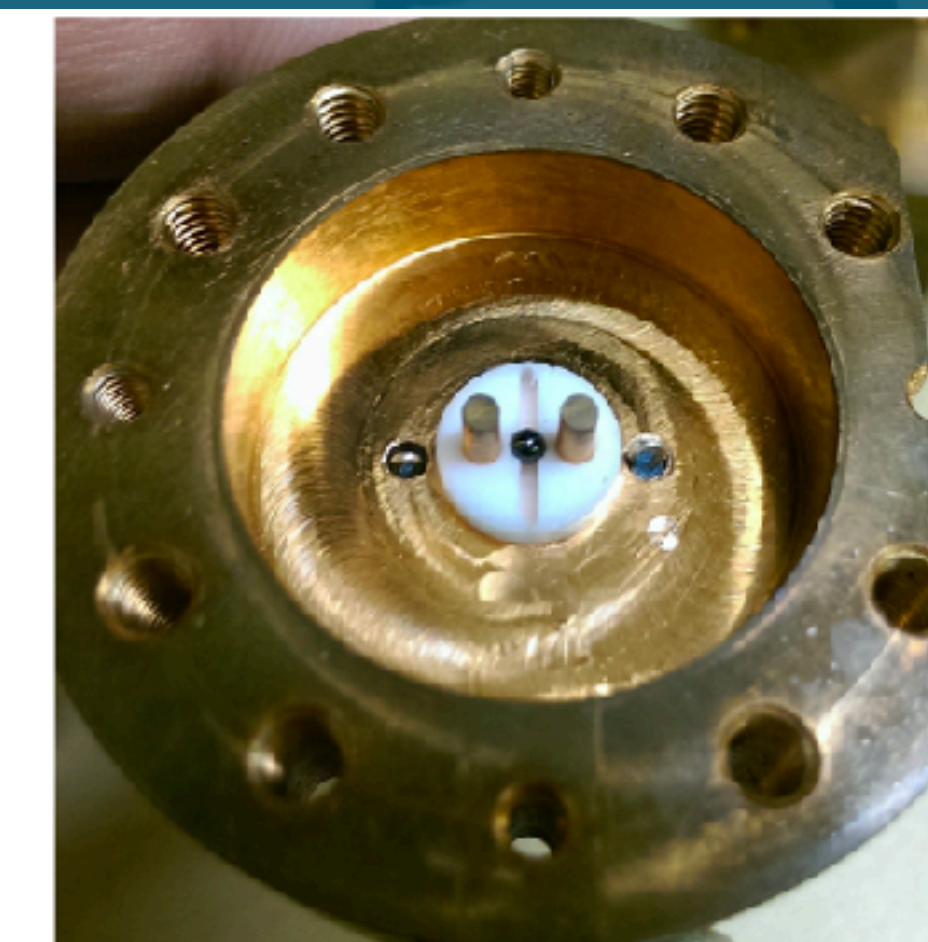
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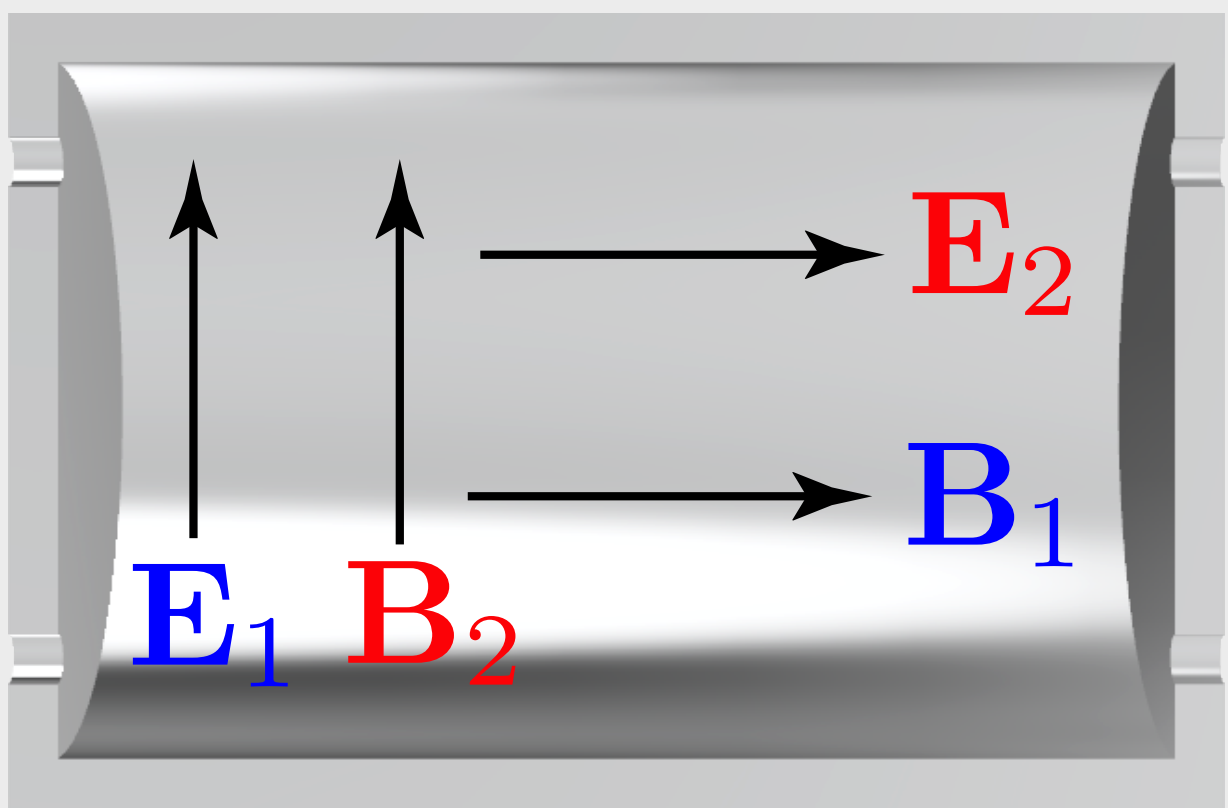
ARC Centre of Excellence for Engineered Quantum Systems, Department of Physics, University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia



Axion Detection with Precision Frequency Metrology

arXiv:1806.07141

photonic cavity with two mutually orthogonal modes



Axion UpConversion

$$\omega_a = \omega_2 - \omega_1$$

$$H_U = i\hbar g_{\text{eff}} \xi_- (a^* c_1 c_2^\dagger - a c_1^\dagger c_2)$$

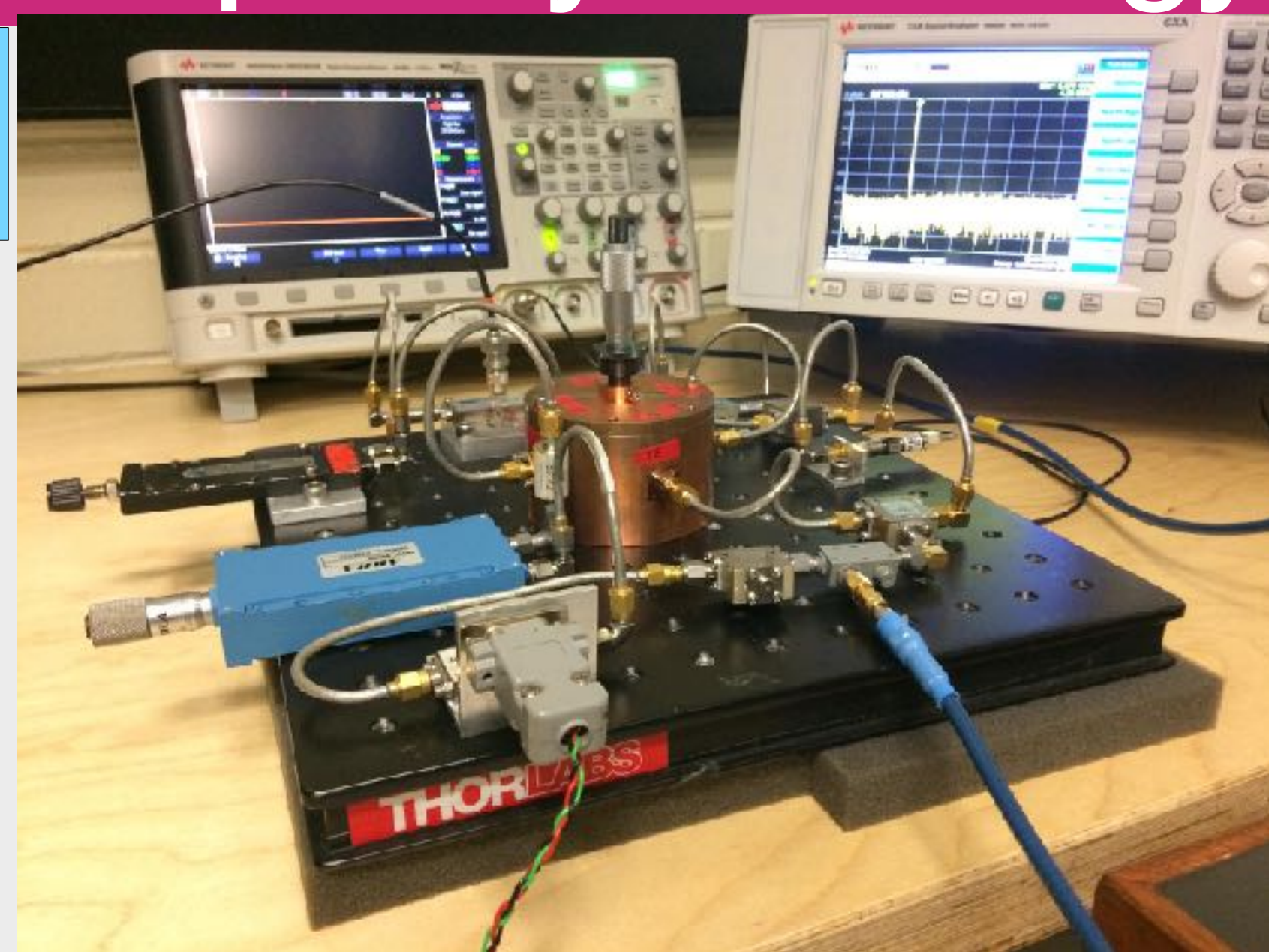
beam splitter

Axion DownConversion

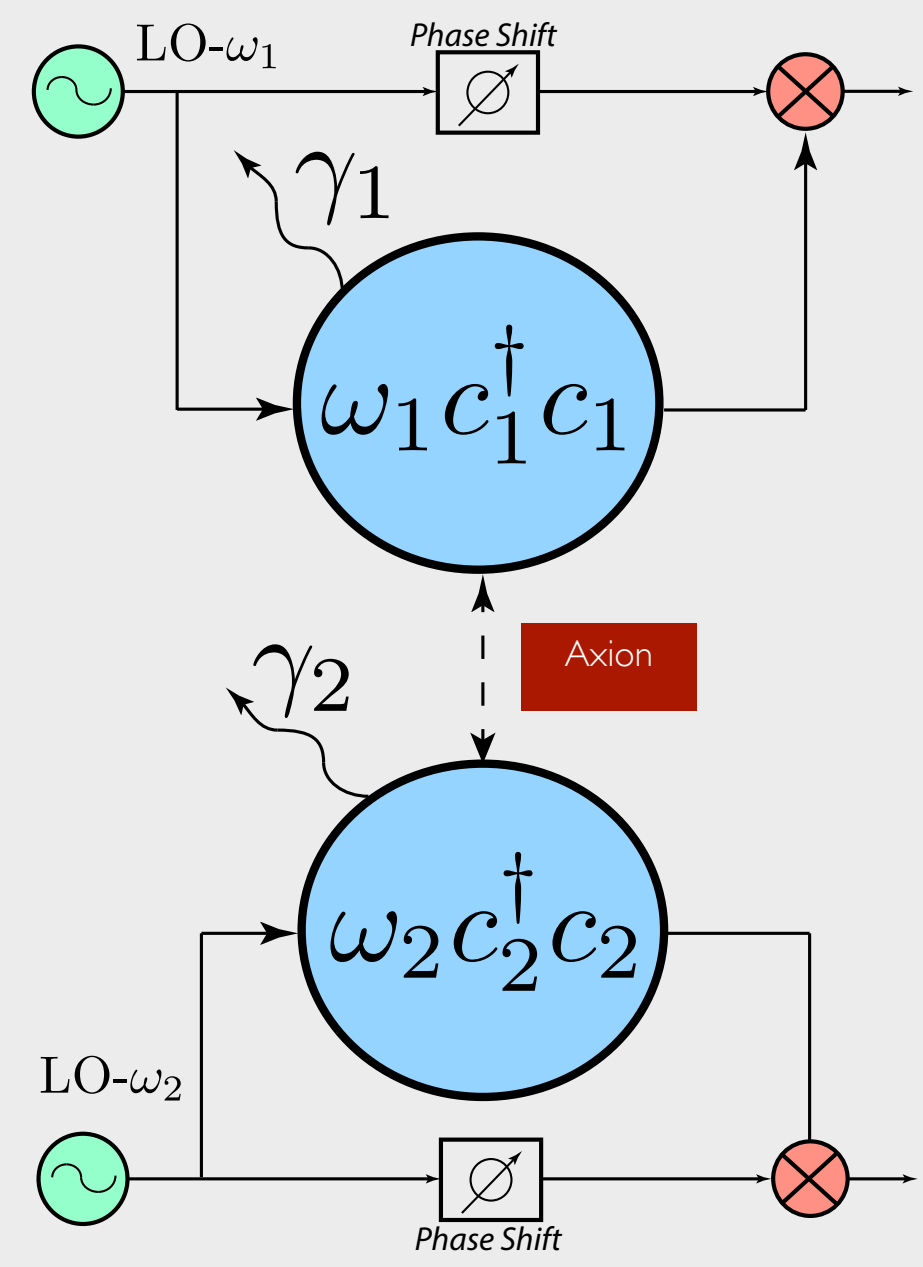
$$\omega_a = \omega_2 + \omega_1$$

$$H_D = i\hbar g_{\text{eff}} \xi_+ (a c_1^\dagger c_2^\dagger - a^* c_1 c_2)$$

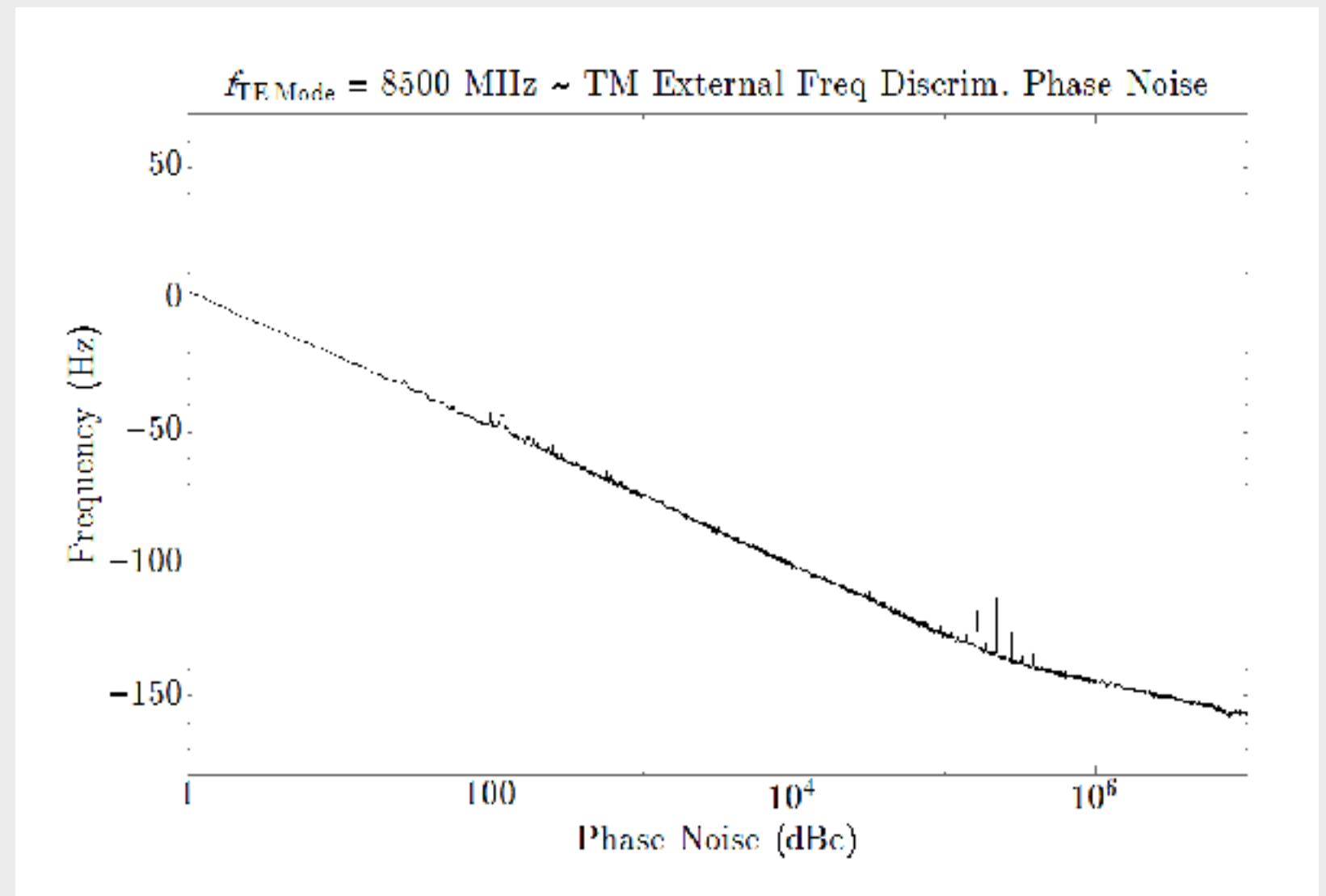
parametric amplification



Eigenfrequency Shift



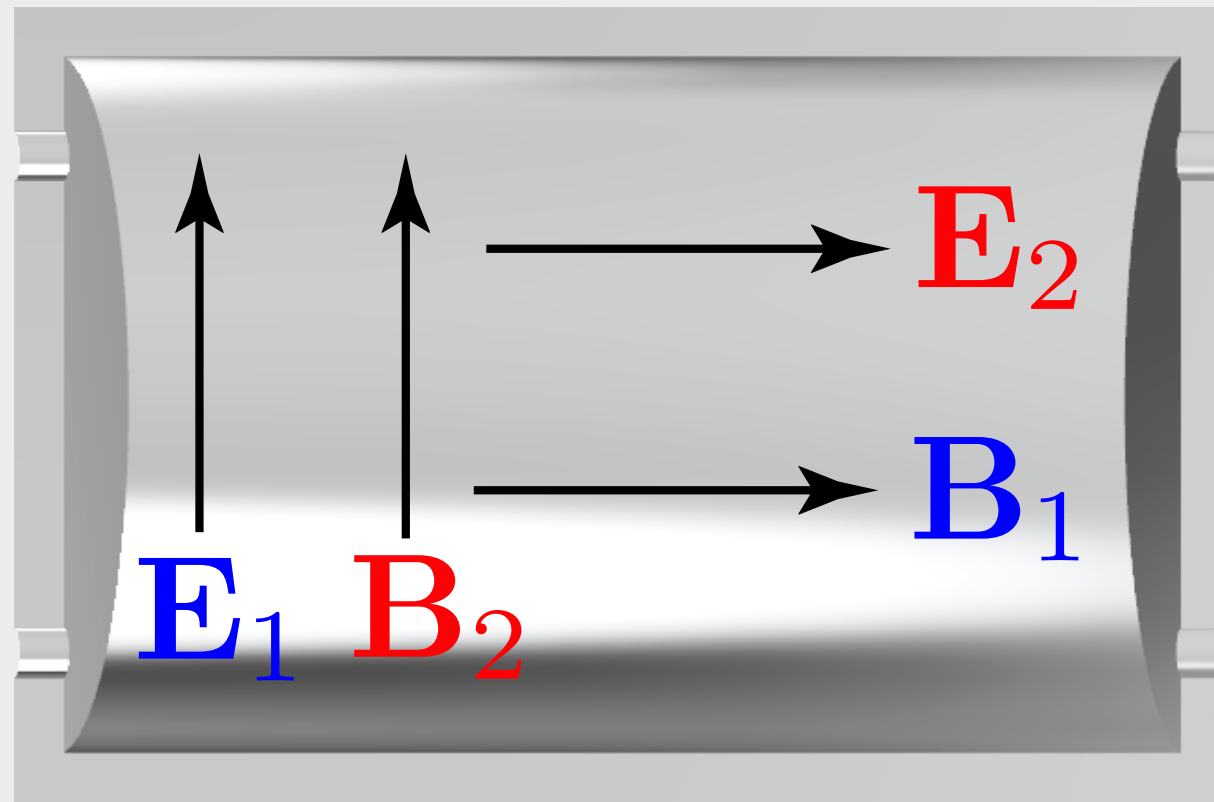
Proto-type built and taking data with first limits
Graduate Student: Catriona Thomas



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photonic cavity with two mutually
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beam splitter

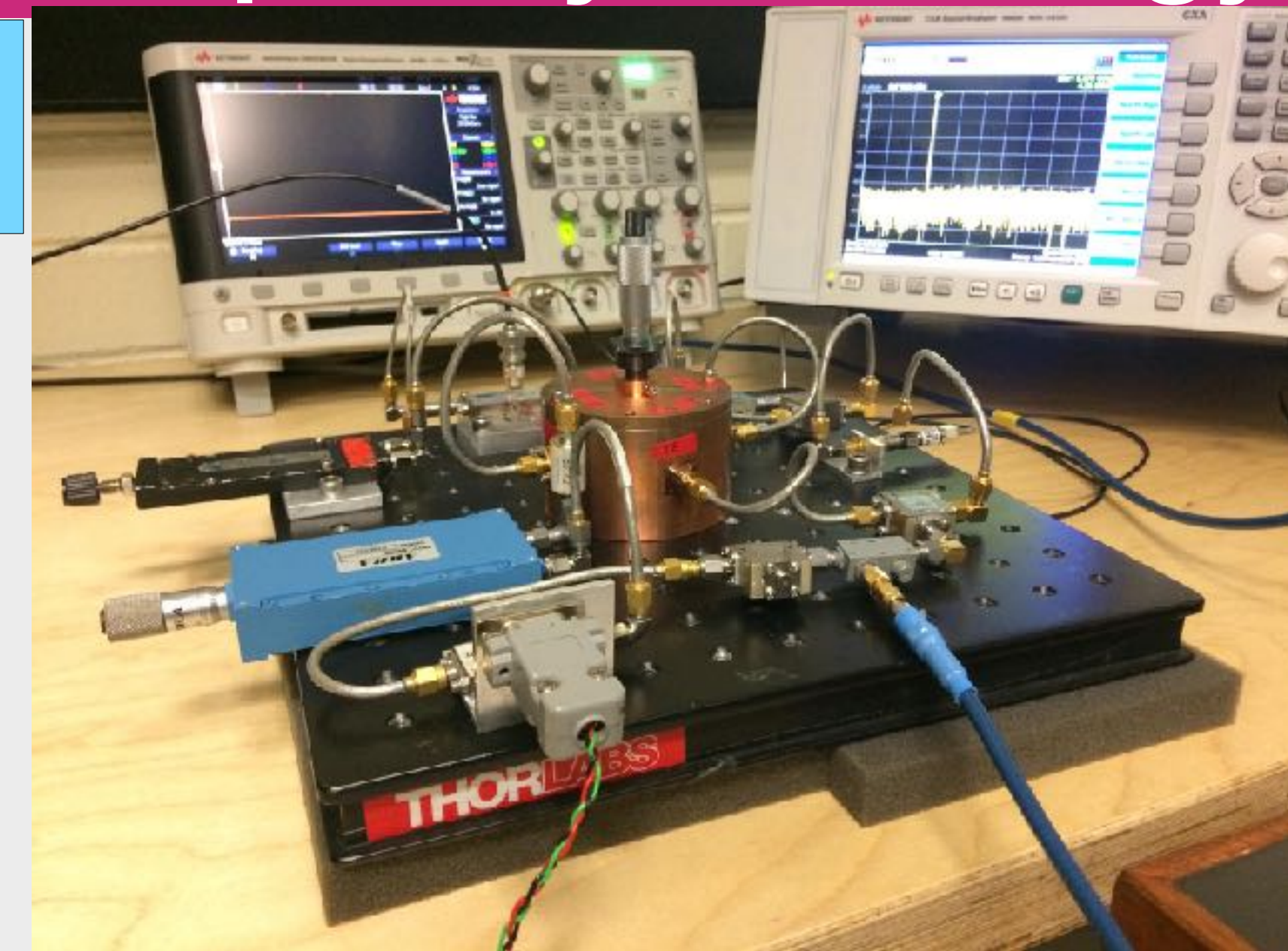
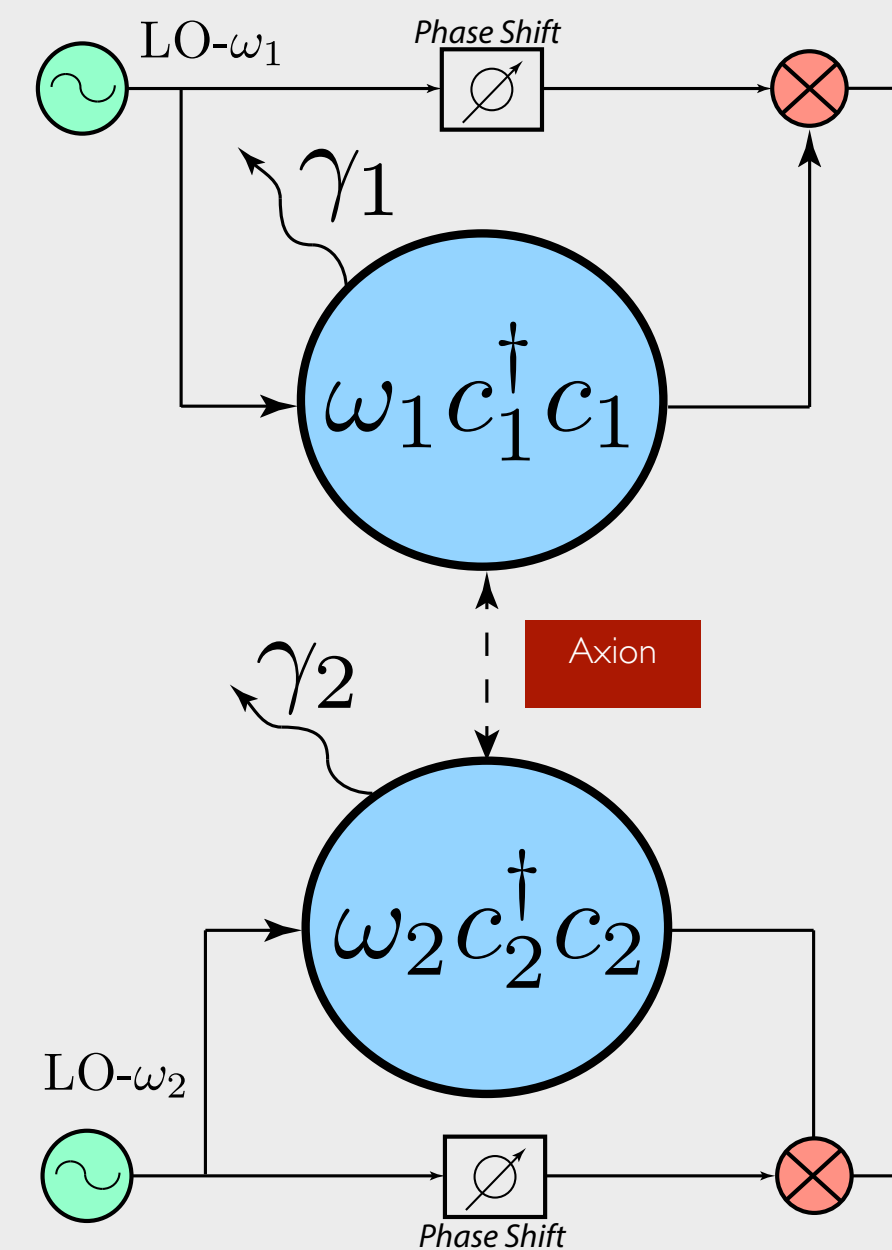
Axion DownConversion

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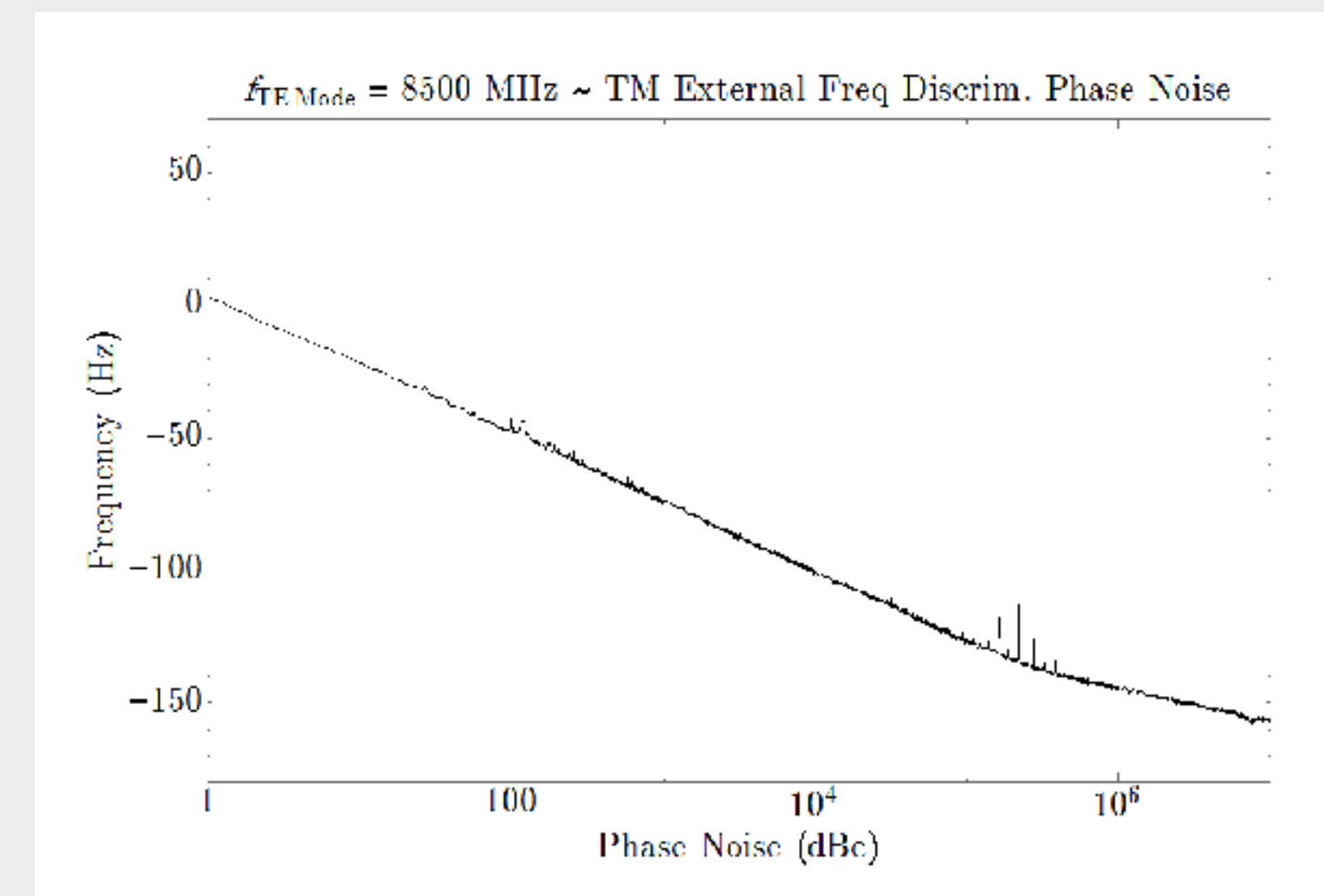
$$H_D = i\hbar g_{\text{eff}} \xi_+ (a c_1^\dagger c_2^\dagger - a^* c_1 c_2)$$

parametric amplification

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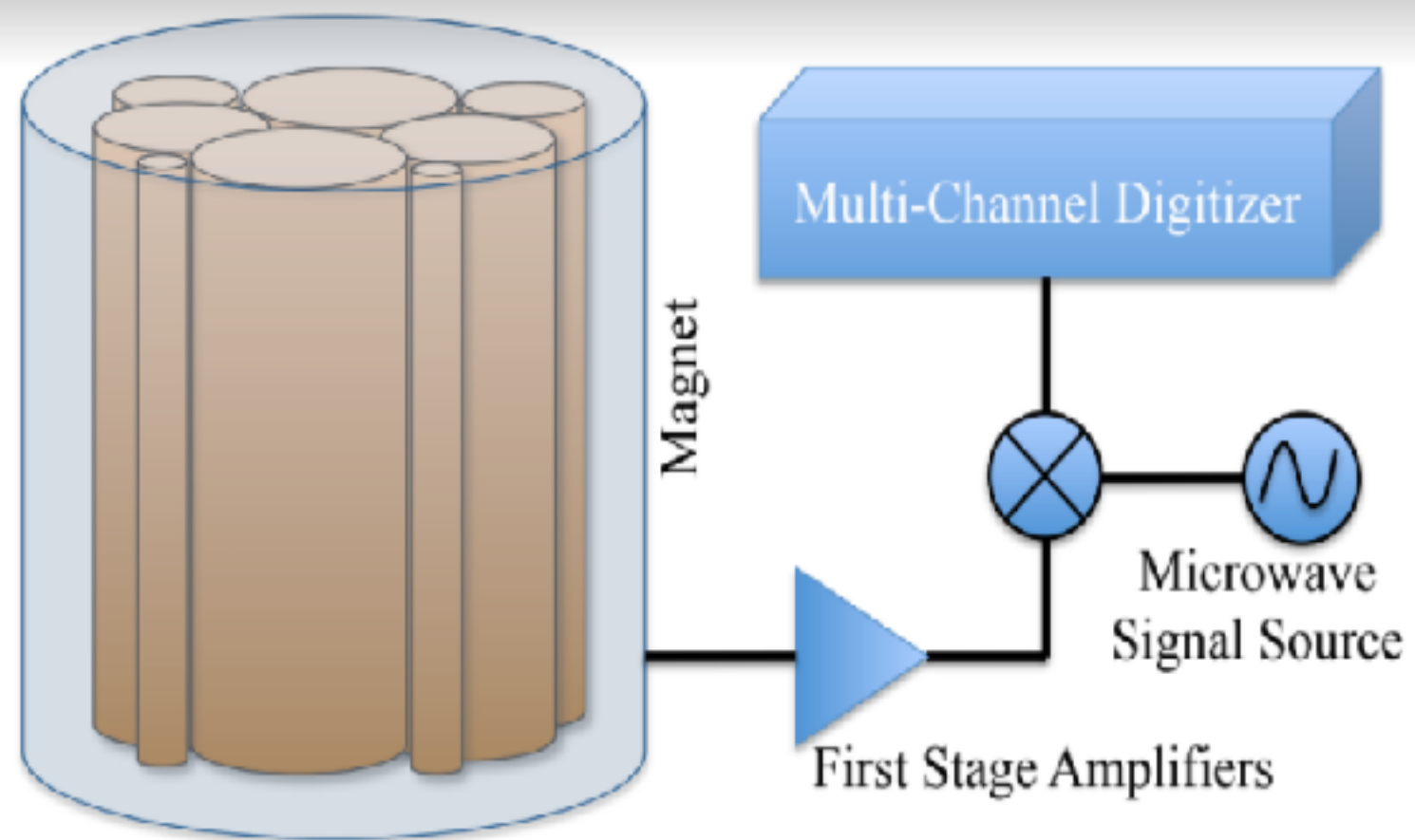
Proto-type built and taking data with first limits
Graduate Student: Catriona Thomas



Maxim Goryachev
Tuesday, June 4 11:05 AM
Employing Precision
Frequency Metrology for
Axion Detection

The ORGAN Experiment:

26 GHz
Haloscope



McGillivray Organ at UWA

26 GHz
Haloscope

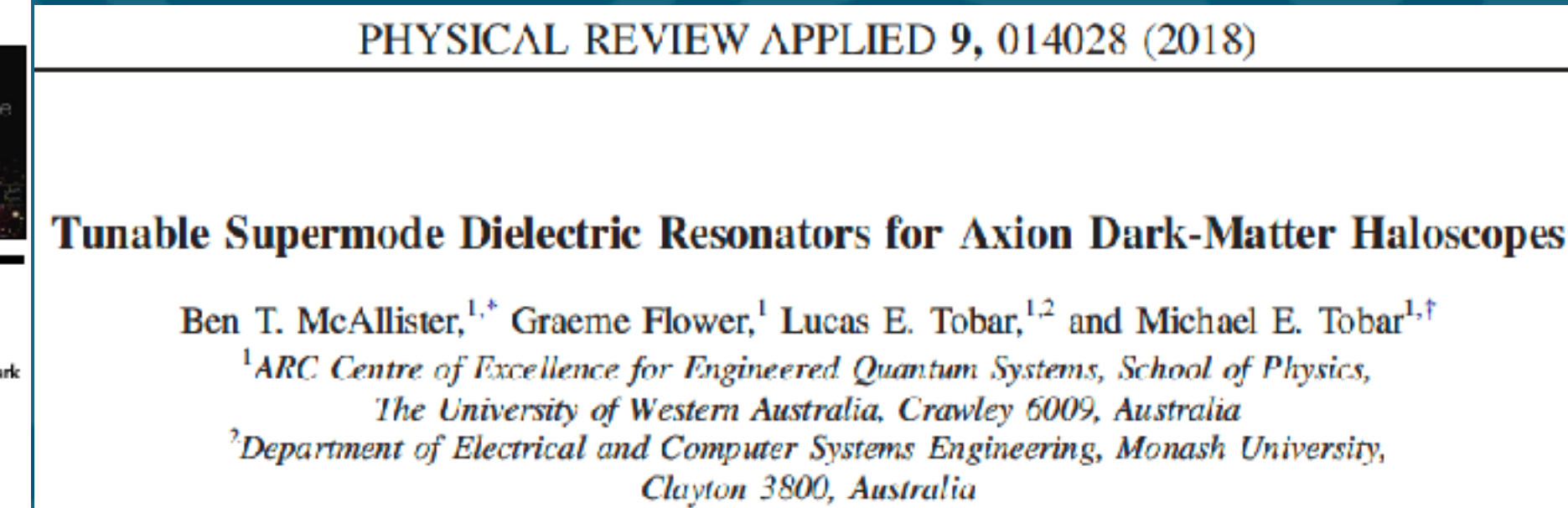
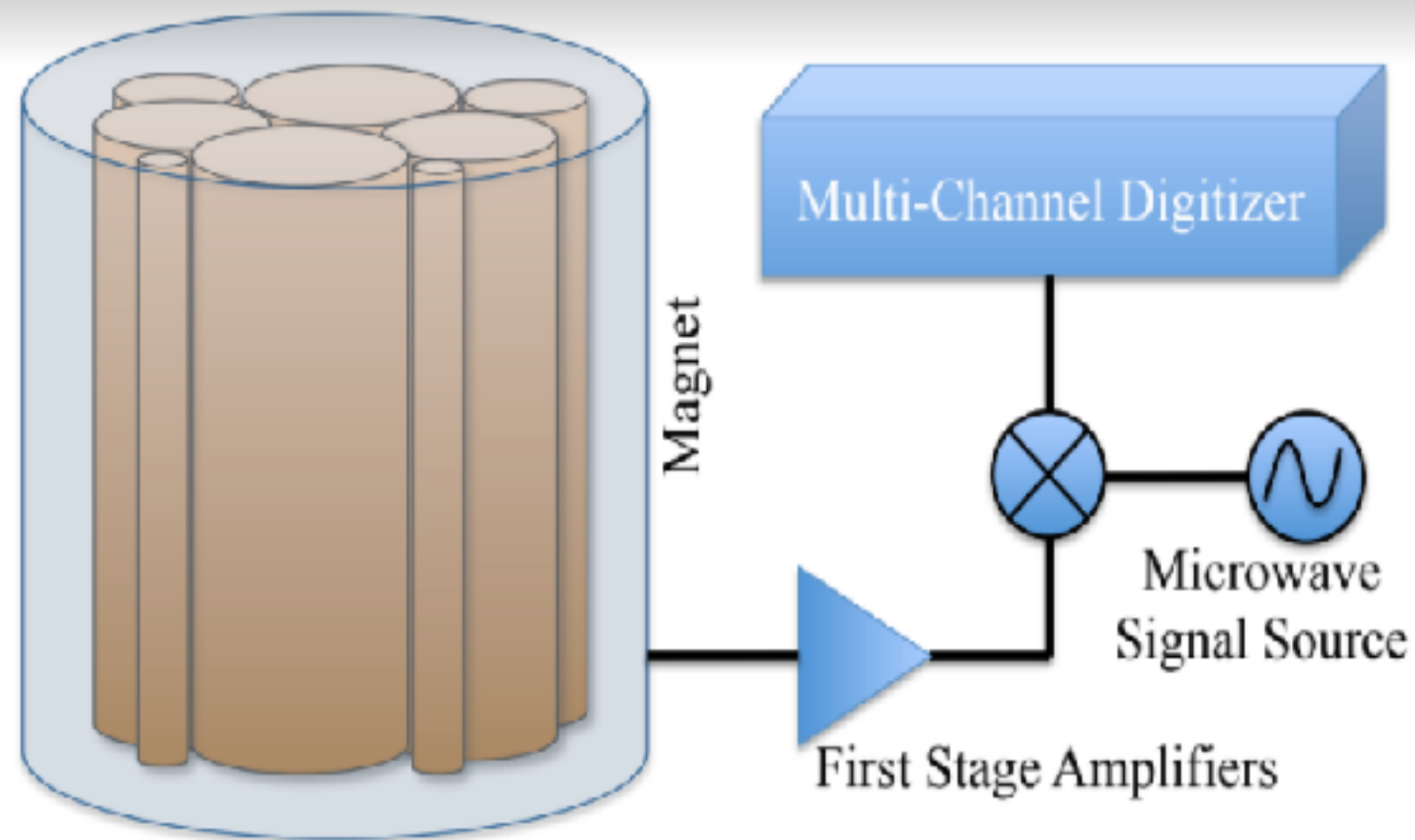


The ORGAN Experiment:

STATUS: First Stage Run Completed

Second Stage: Dil Fridge +14 T mag Lab upgrade: by 10/19

New Research Grant; includes Joining ADMX 2019-21



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AUSTRALIA**



26 GHz
Haloscope

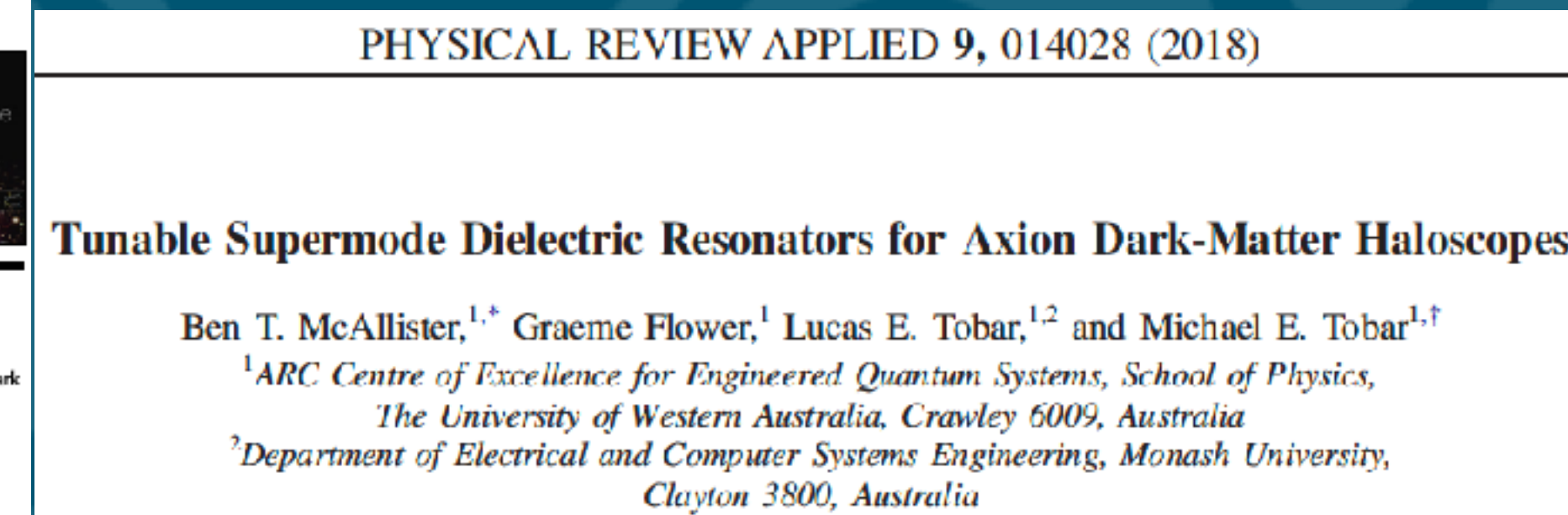
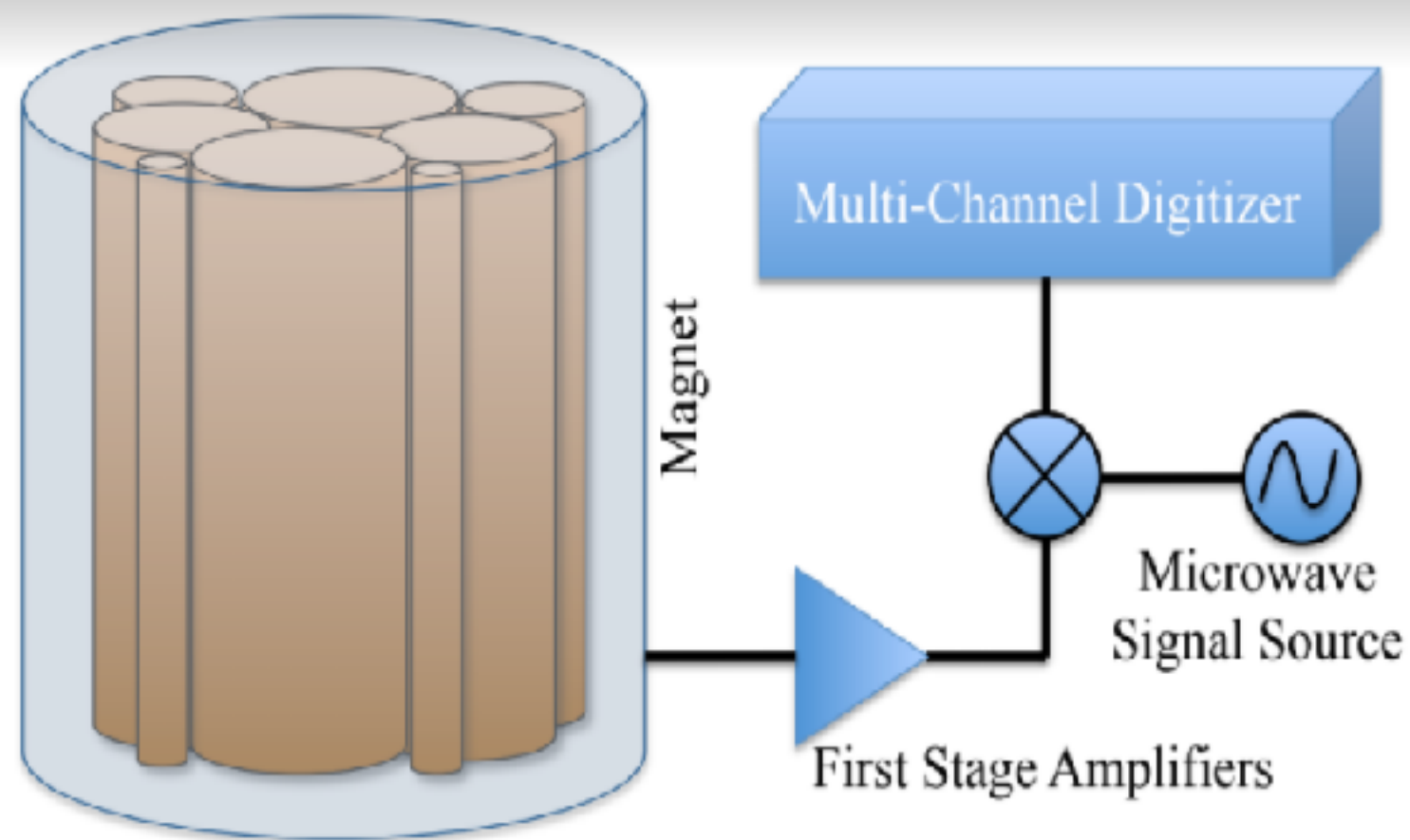


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Developing Tuneable Cavities:



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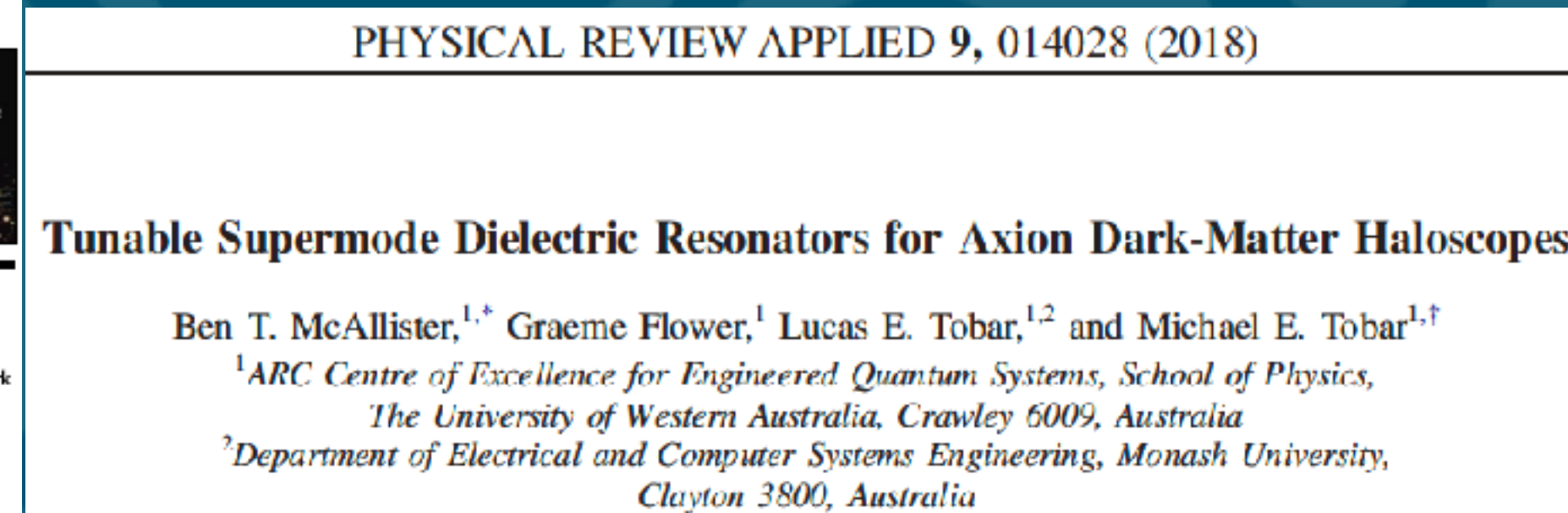
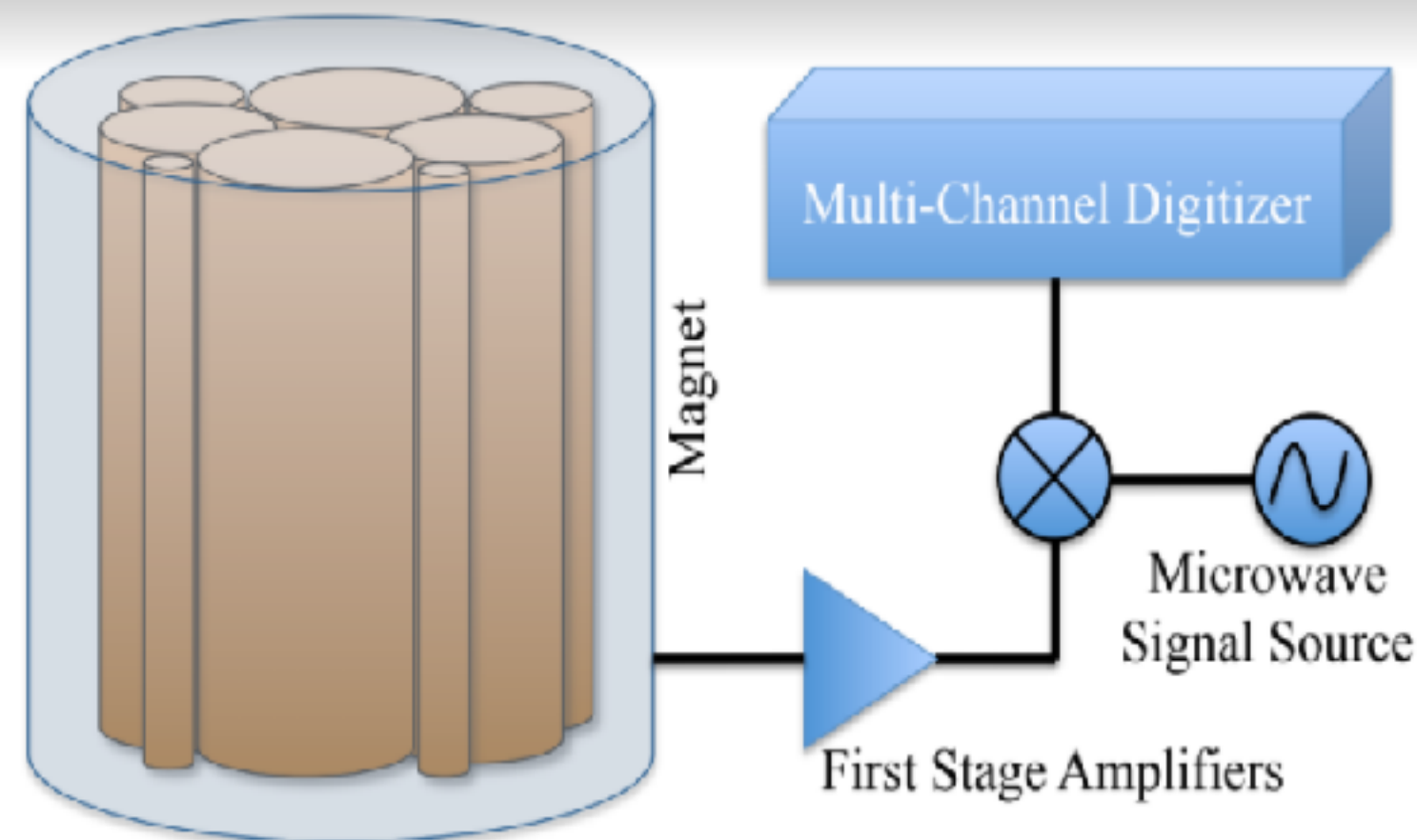
26 GHz
Haloscope



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Developing Tuneable Cavities:

1) Collaboration with ADMX on New Haloscopes for ORGAN and
ADMX Extended Frequency Range 2-4 GHz (No updates)



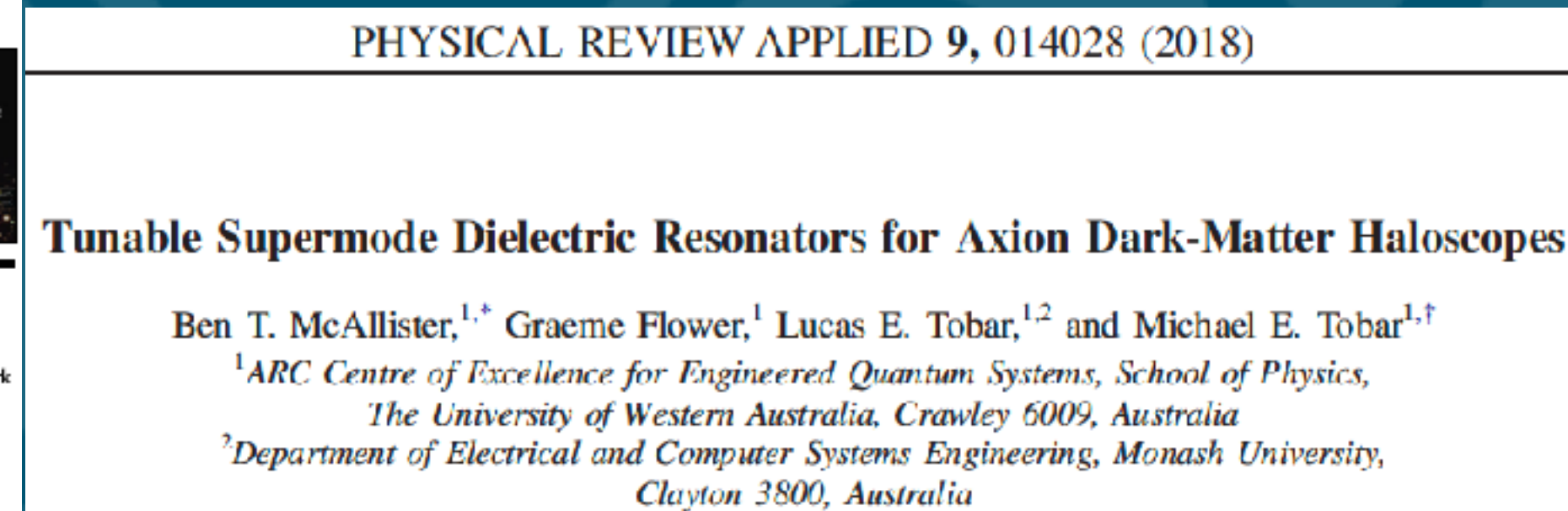
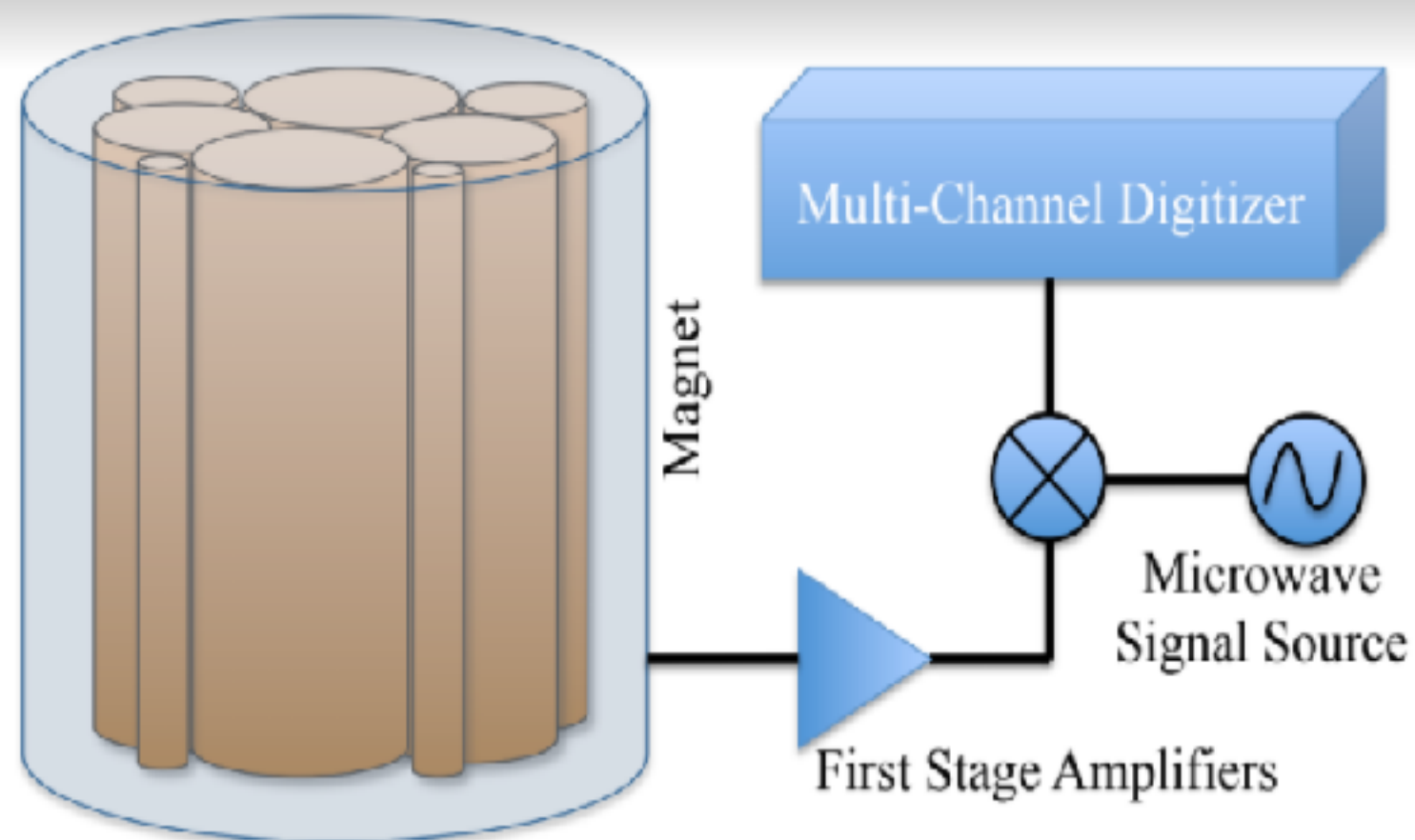
26 GHz
Haloscope



The ORGAN Experiment:

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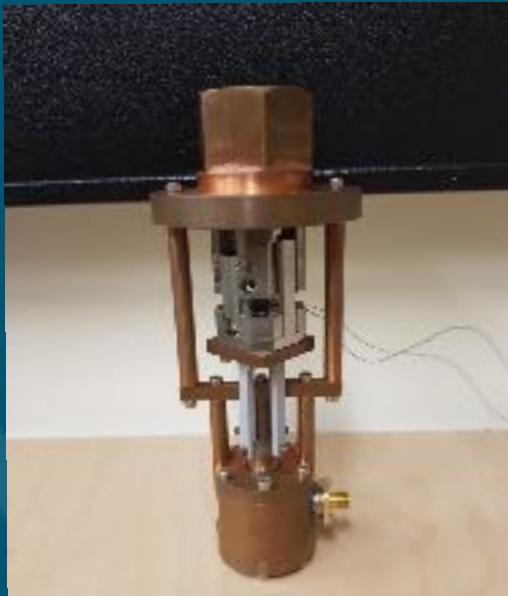
Developing Tuneable Cavities:

- 1) Collaboration with ADMX on New Haloscopes for ORGAN and ADMX Extended Frequency Range 2-4 GHz (No updates)
- 2) Developing Quantum Technologies for low noise readout:



McGillivray Organ at UWA

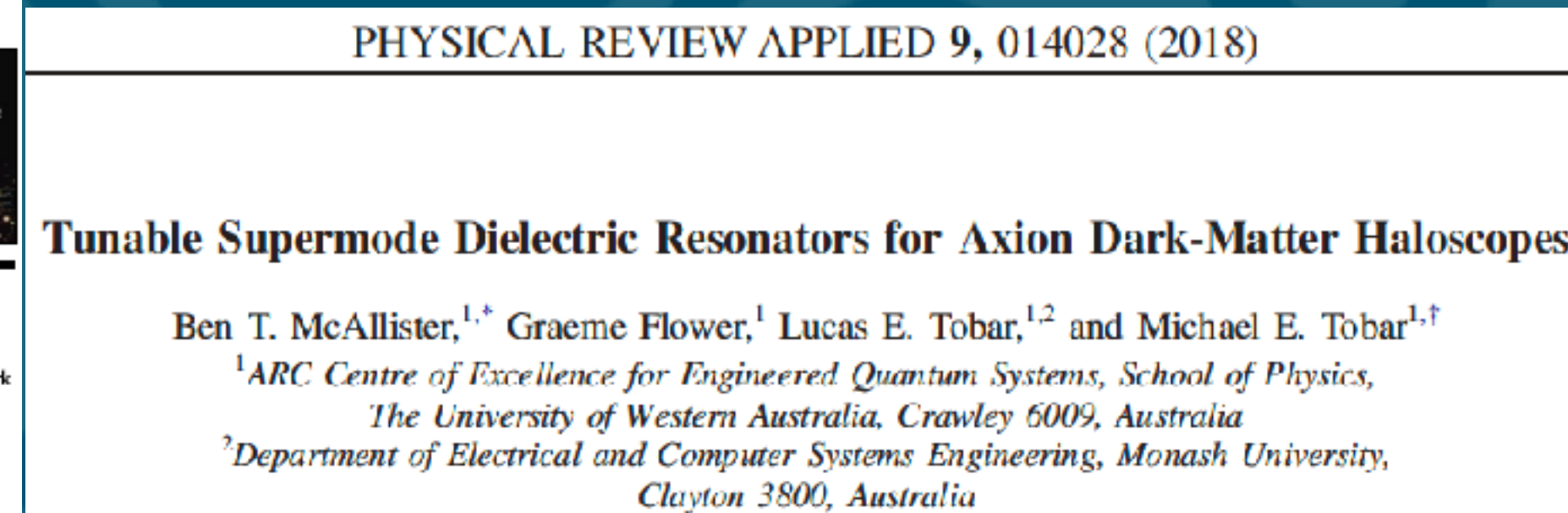
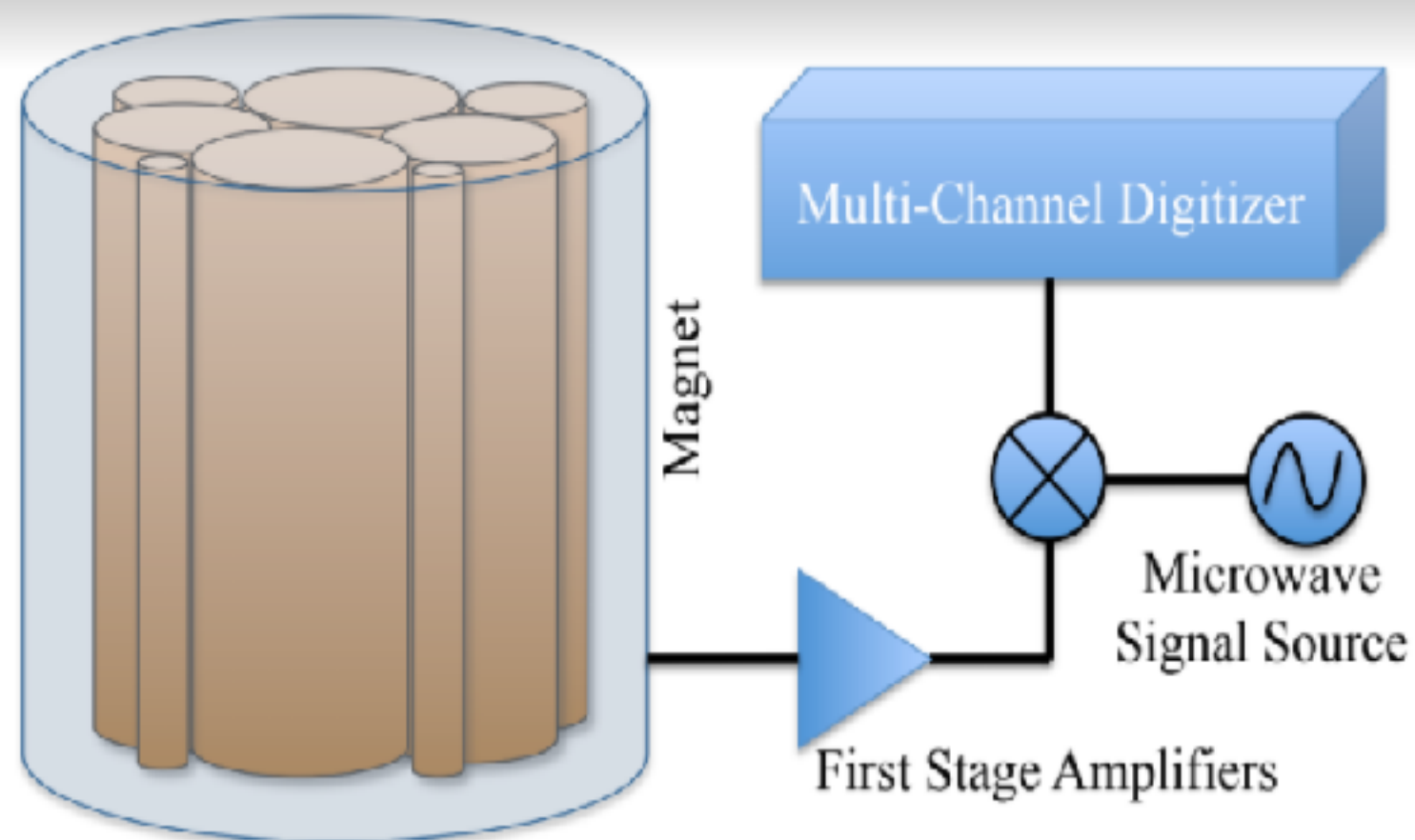
26 GHz
Haloscope



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Developing Tuneable Cavities:

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 - a) EQUS; Parametric Amplifiers at 26 GHz



McGillivray Organ at UWA

26 GHz
Haloscope

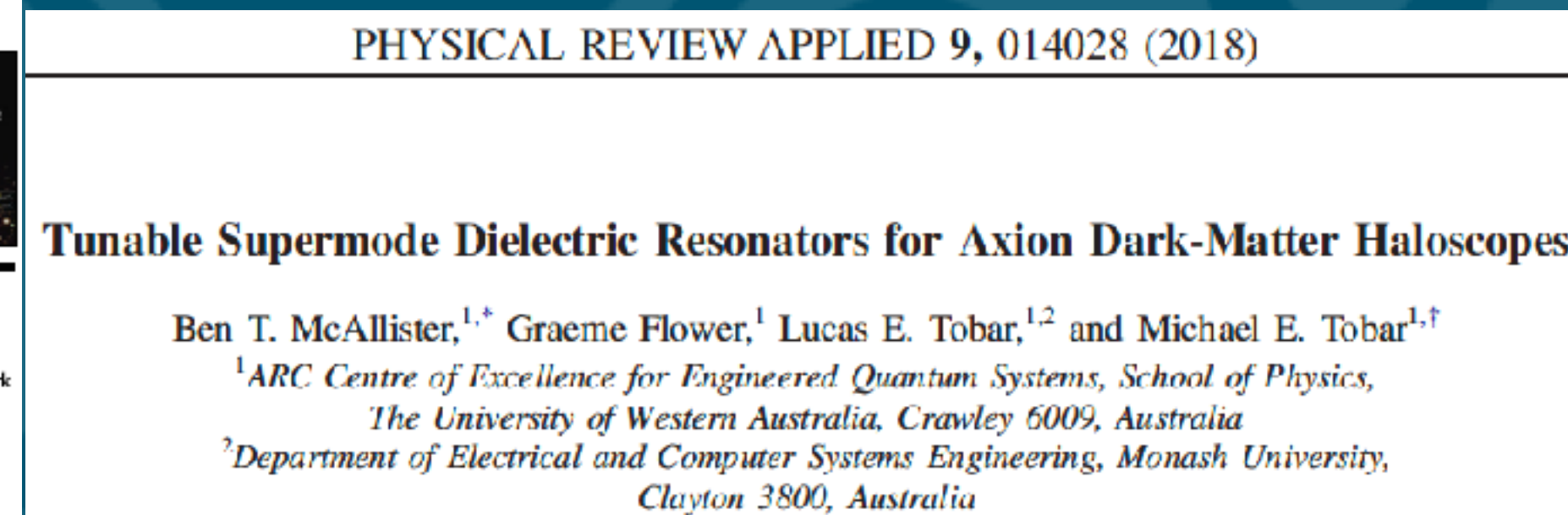
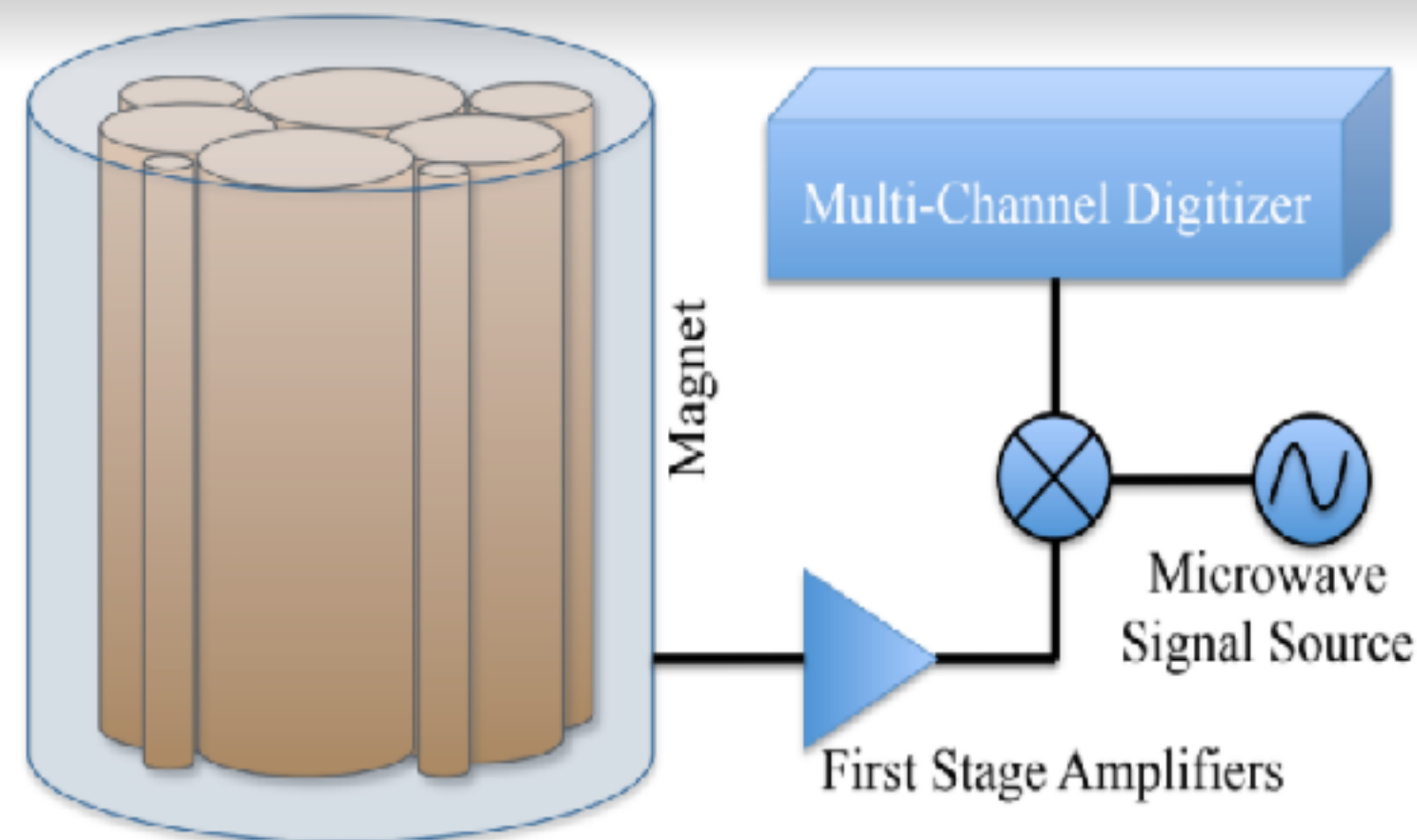


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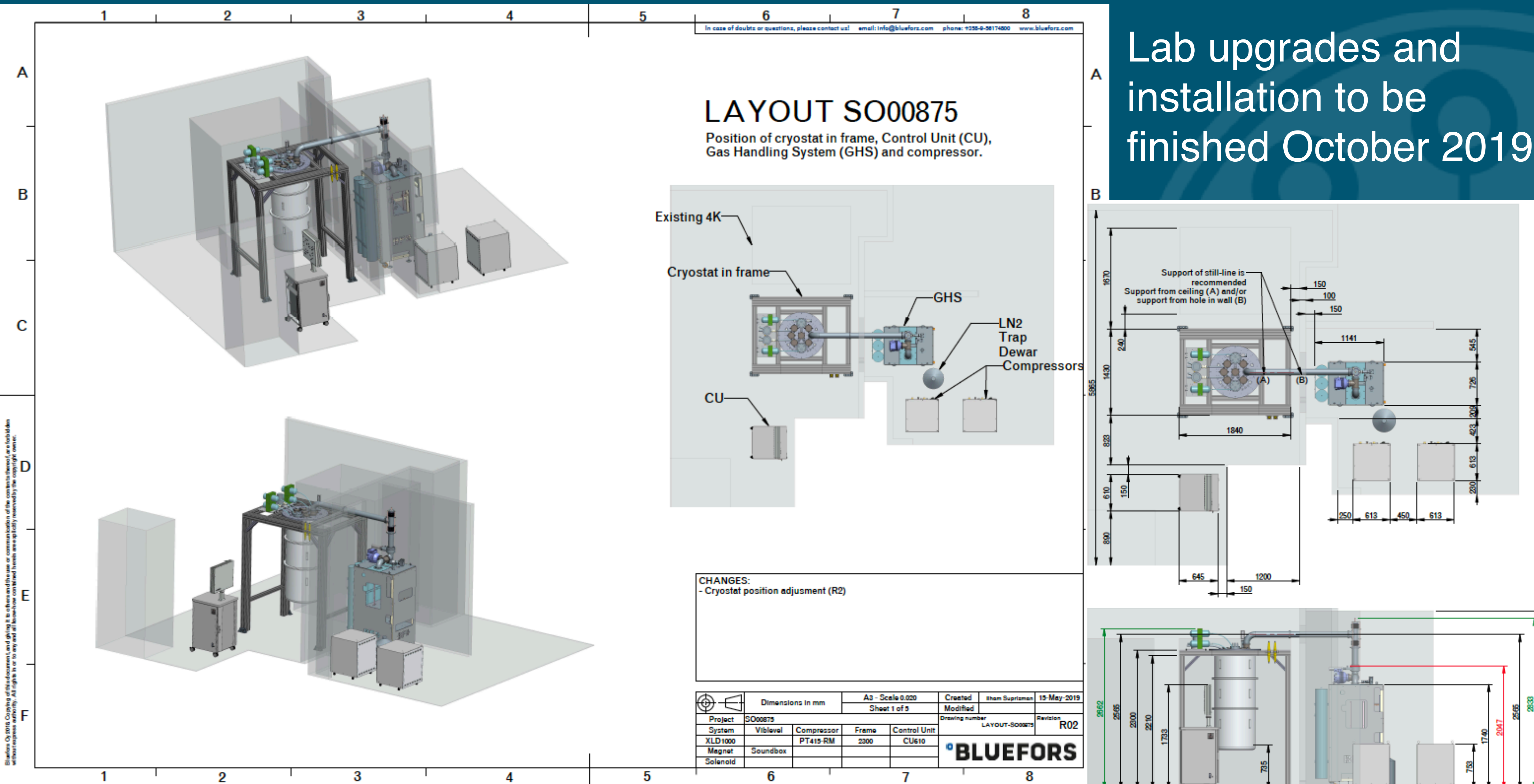
Developing Tuneable Cavities:

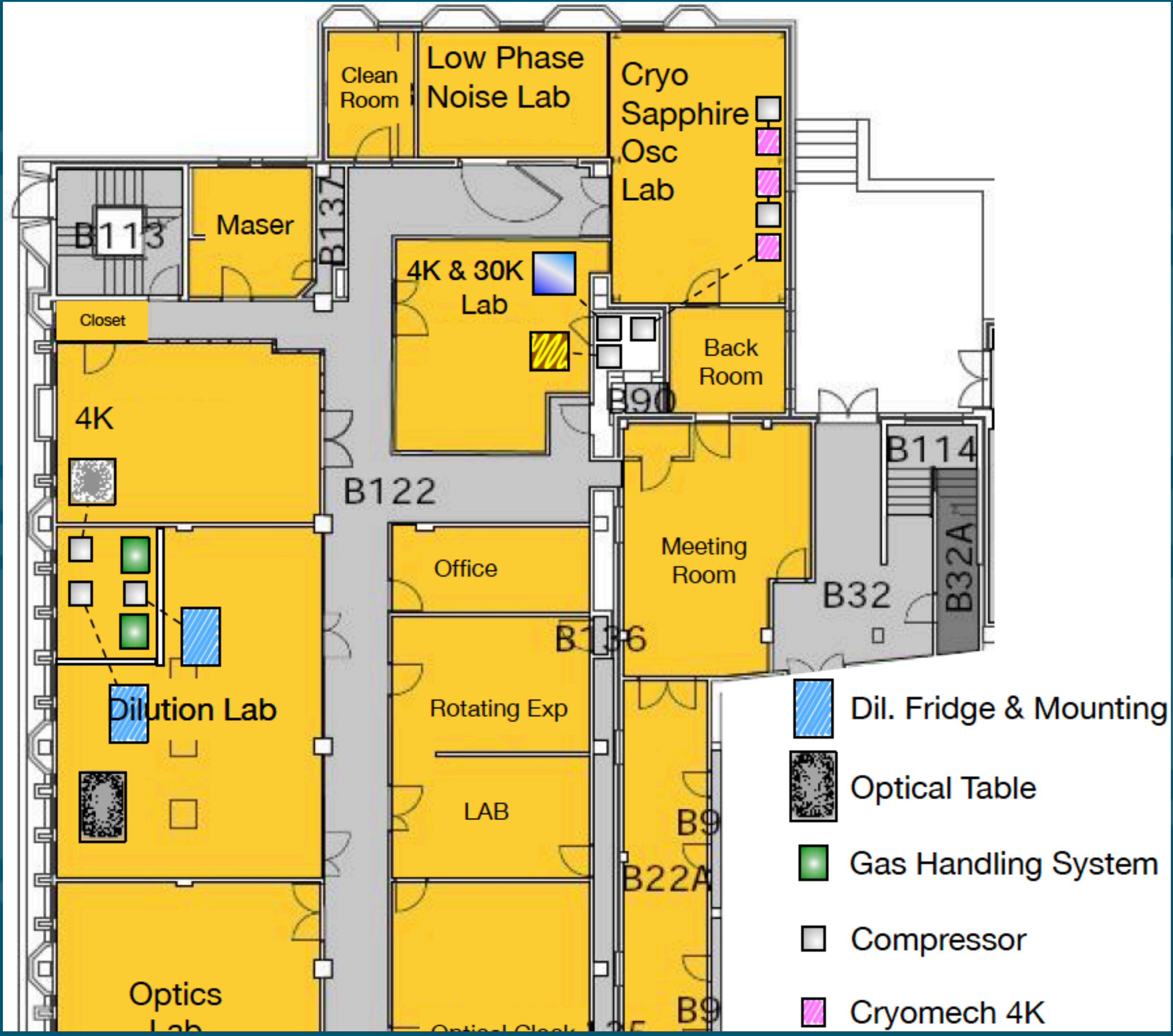
- 1) Collaboration with ADMX on New Haloscopes for ORGAN and ADMX Extended Frequency Range 2-4 GHz (No updates)
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 - a) EQUS; Parametric Amplifiers at 26 GHz
 - b) **Leonid Kuzmin, Thursday, June 6:** Single Photon Counter at 14 and **26 GHz** for searching Galactic Axions within the QUAX and **ORGAN** projects

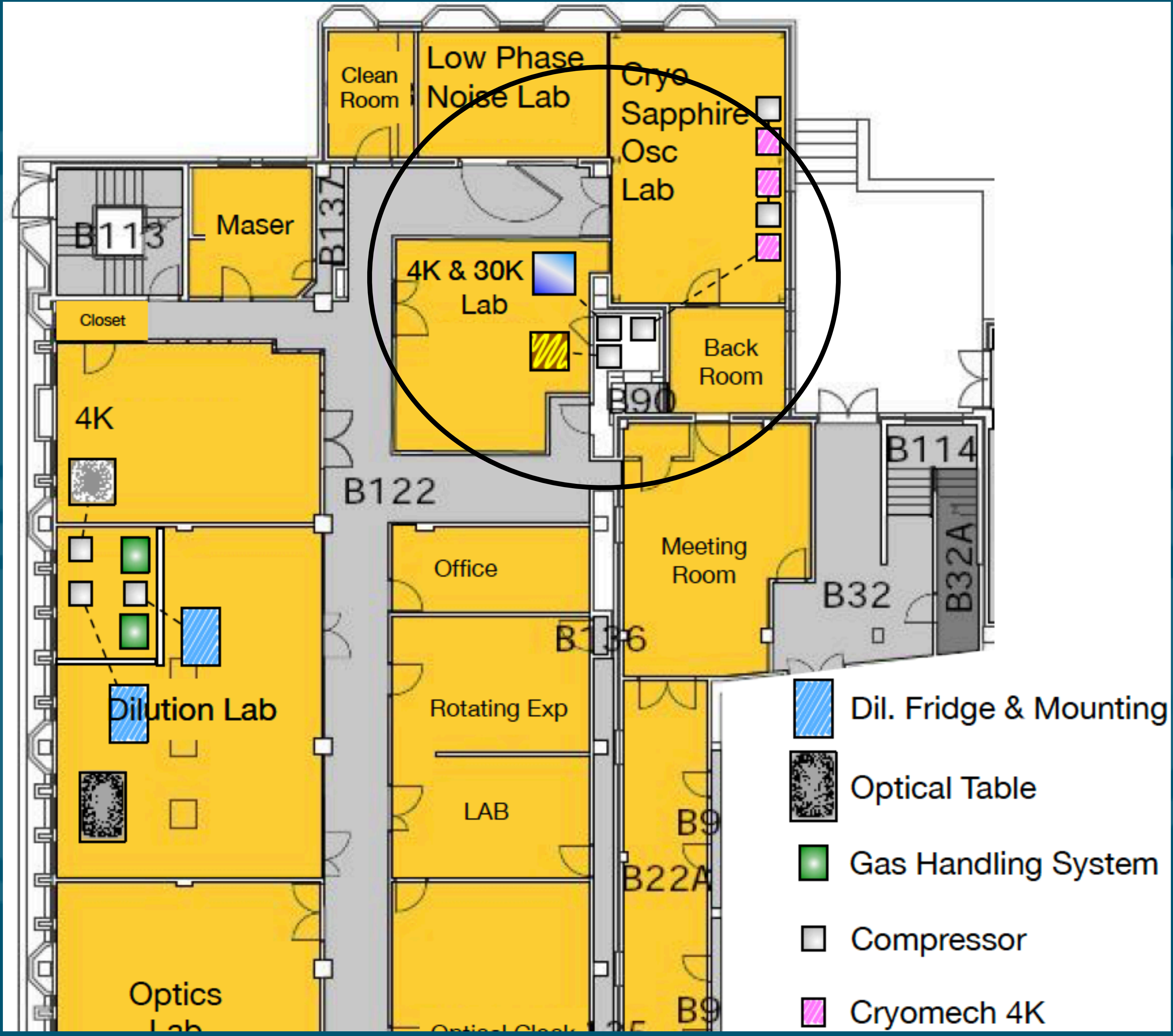


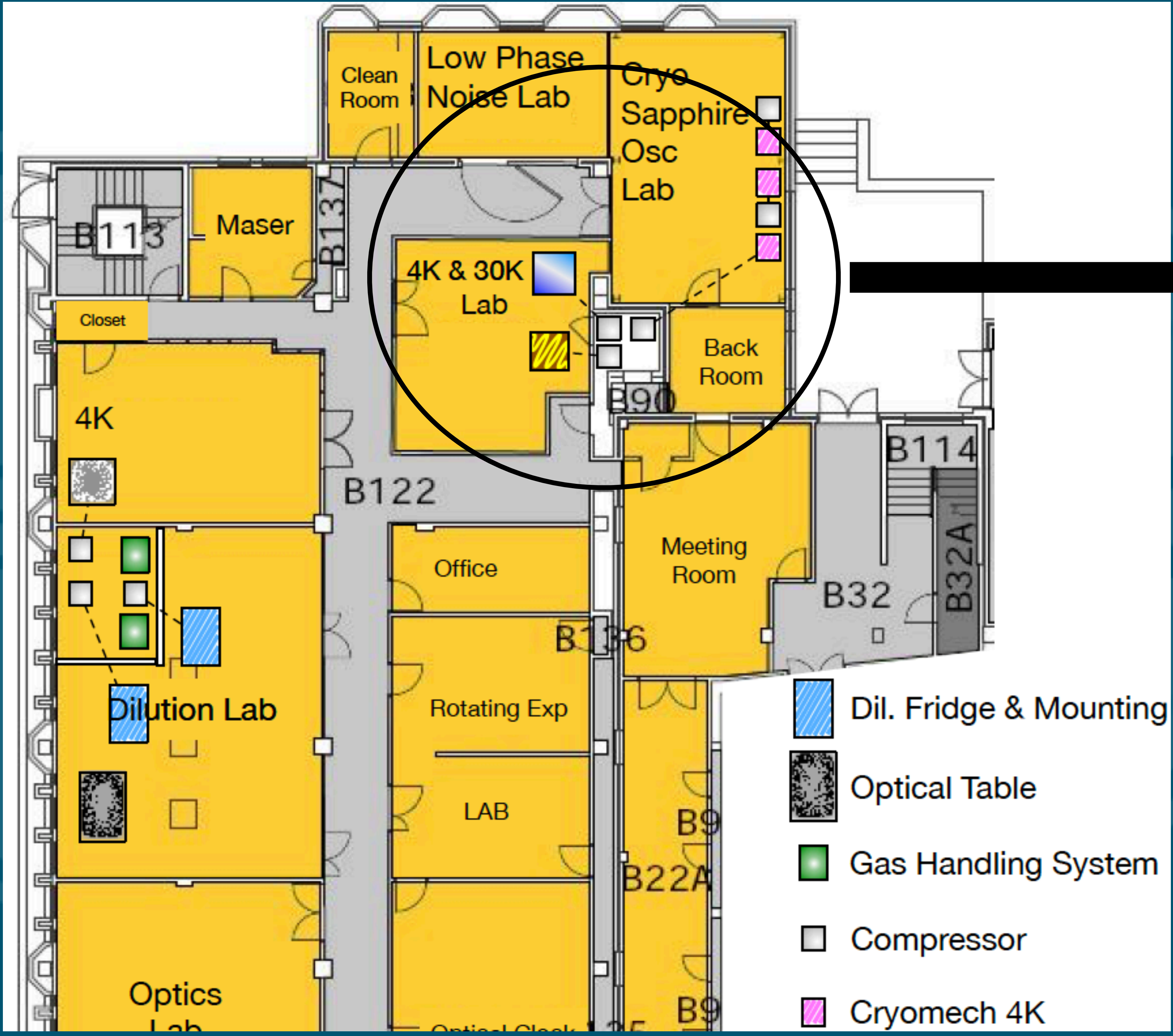
BF-XLD1000 with 14T-65mm Cryogen-Free Solenoid Magnet

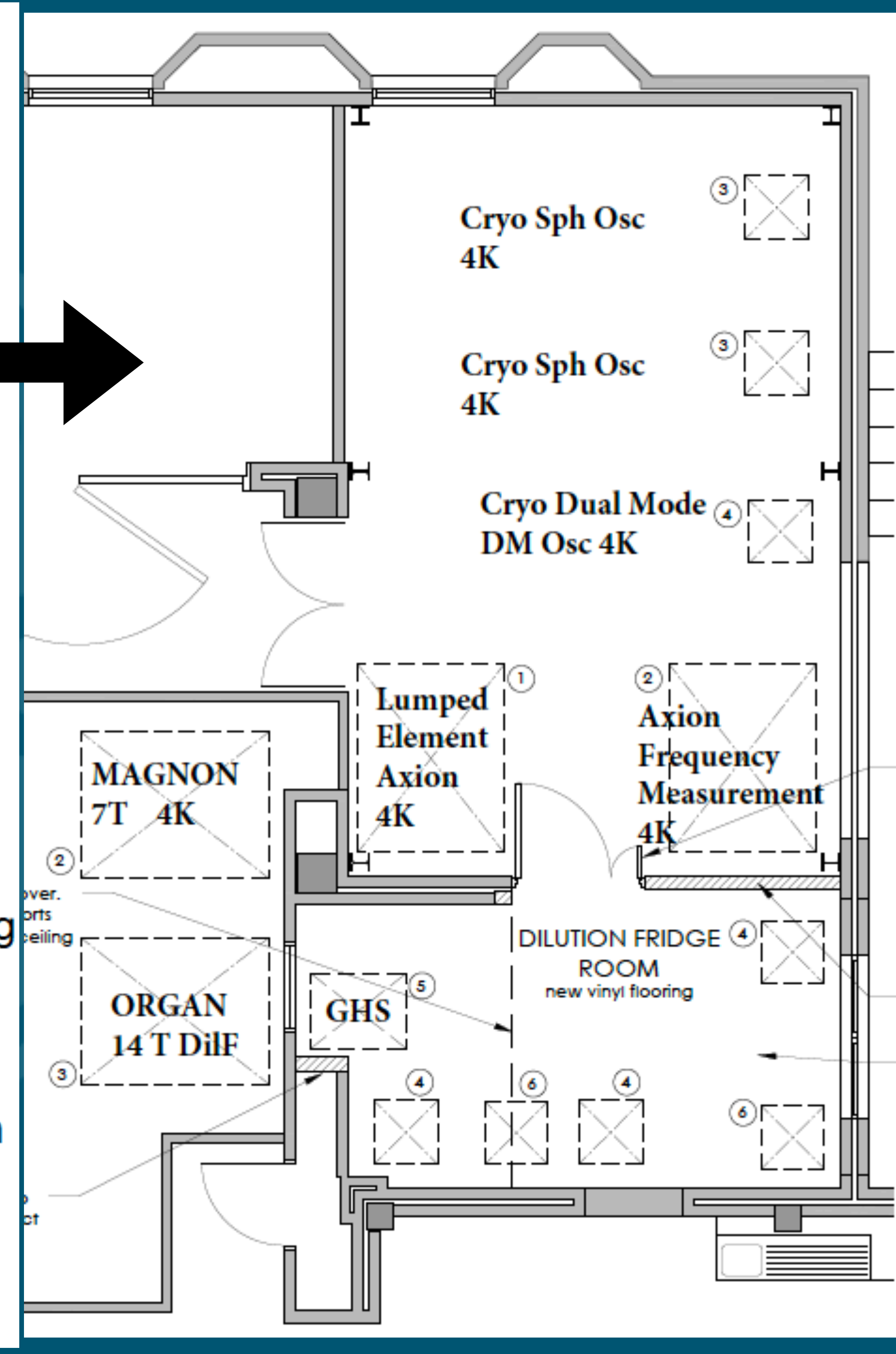
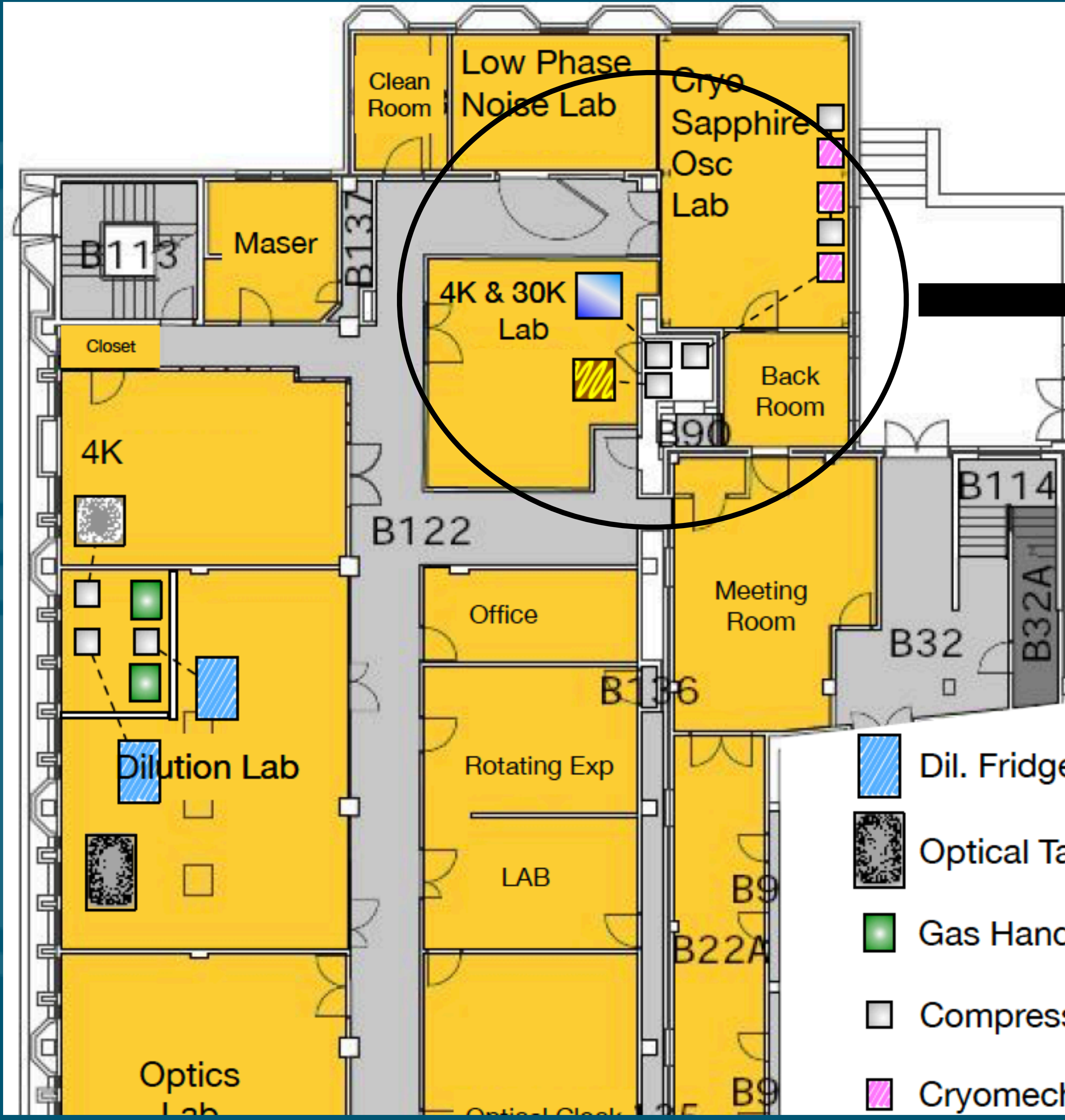
Lab upgrades and installation to be finished October 2019







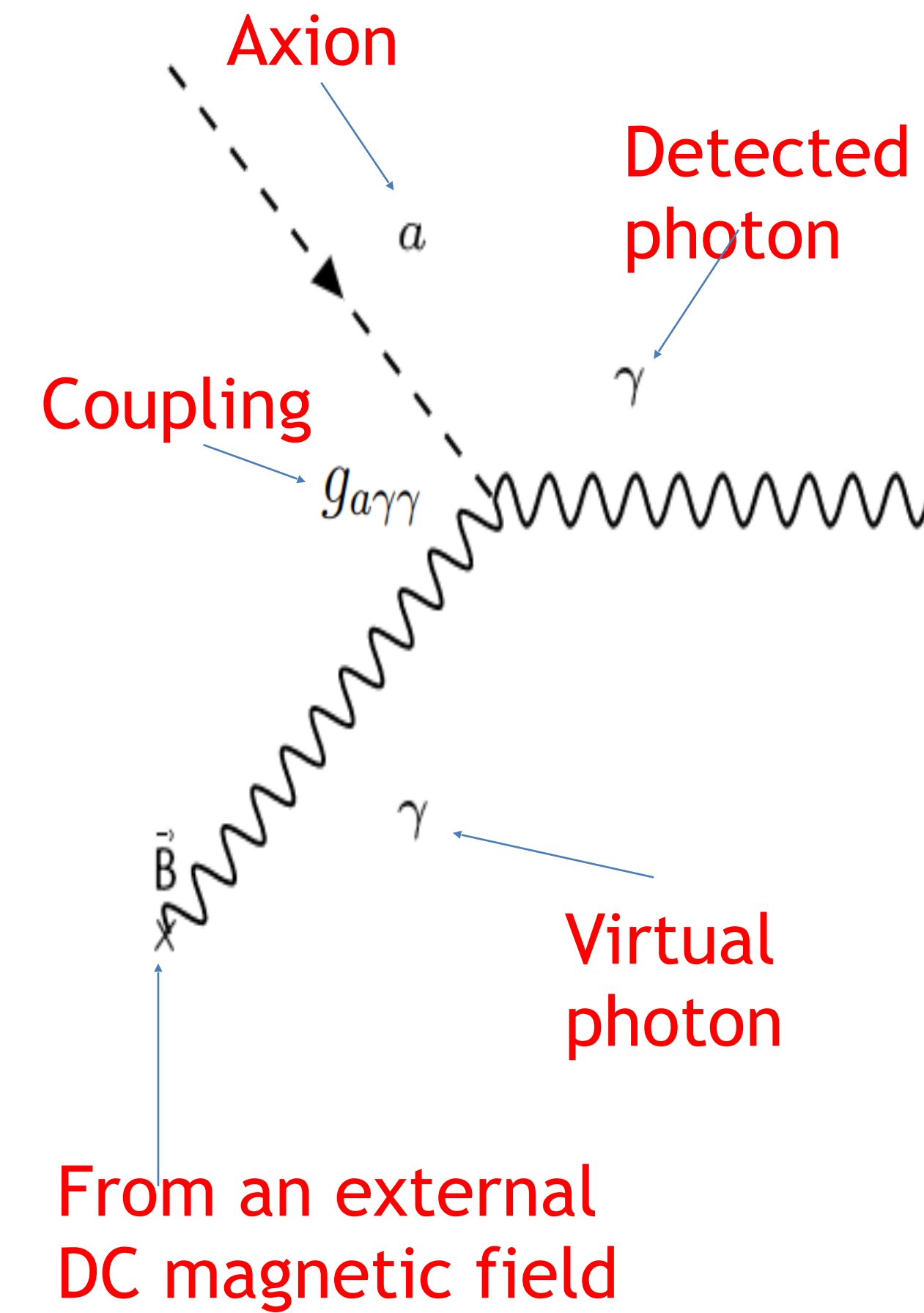




The Axion Haloscope Technique



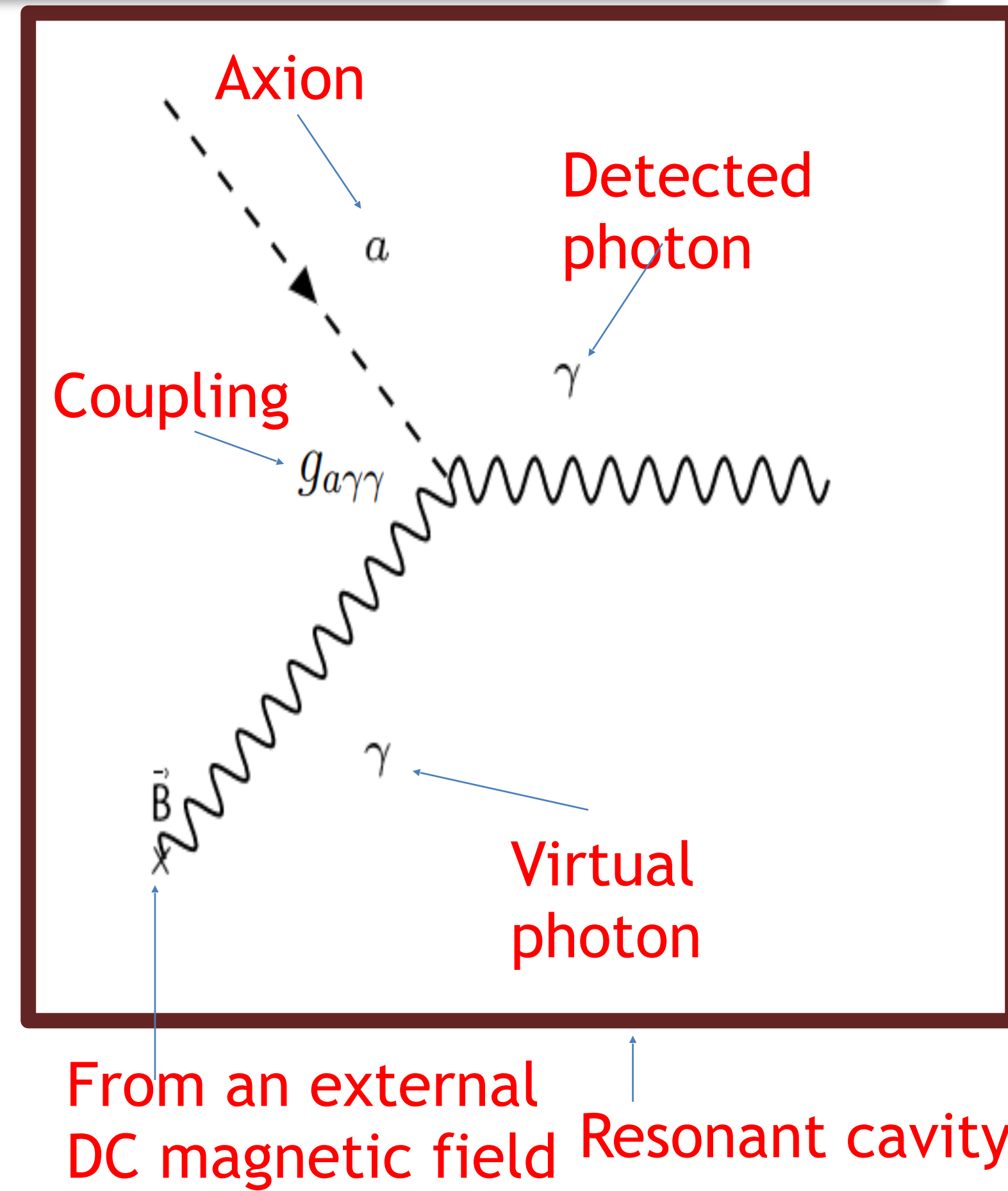
Pierre Sikivie



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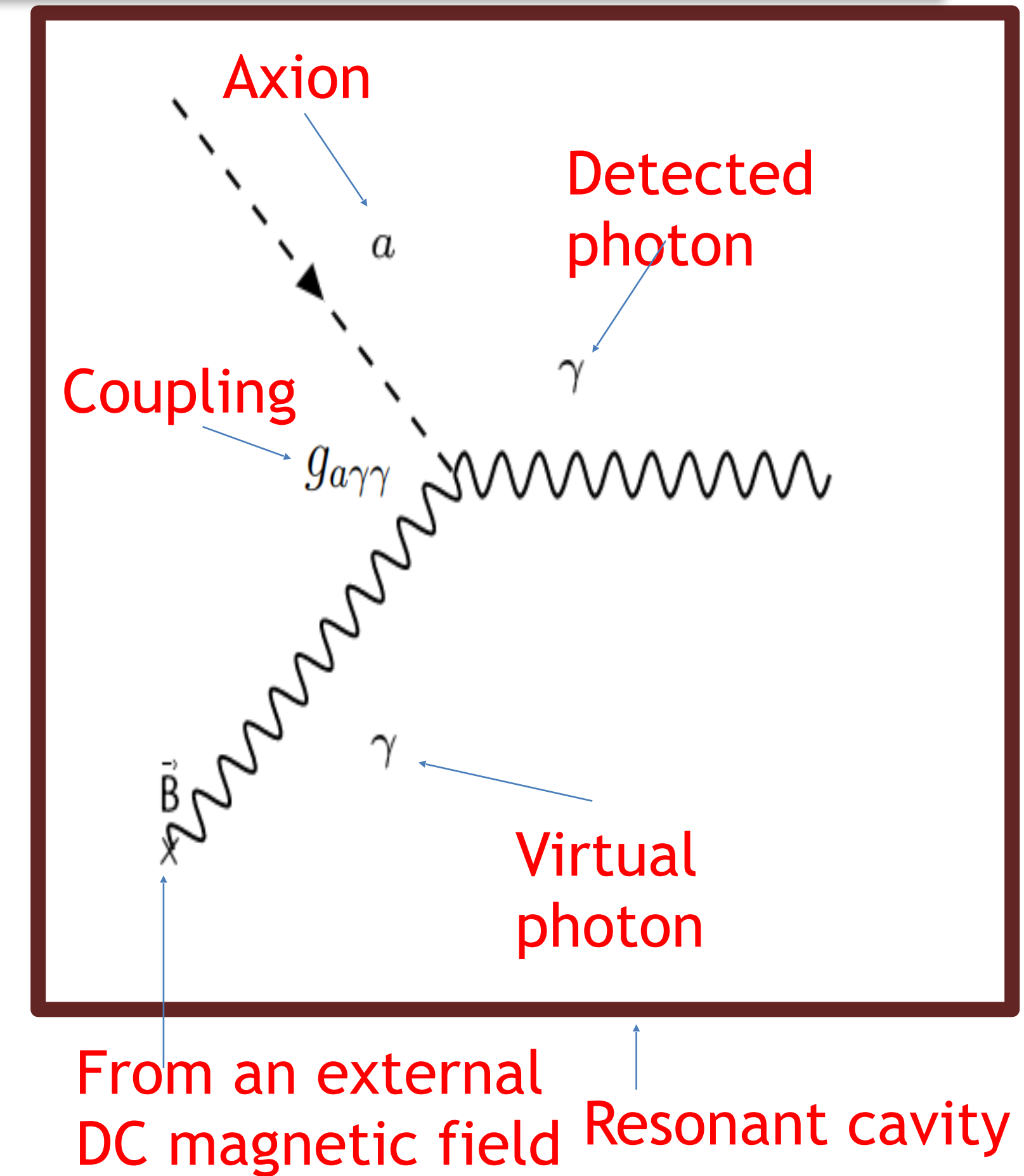
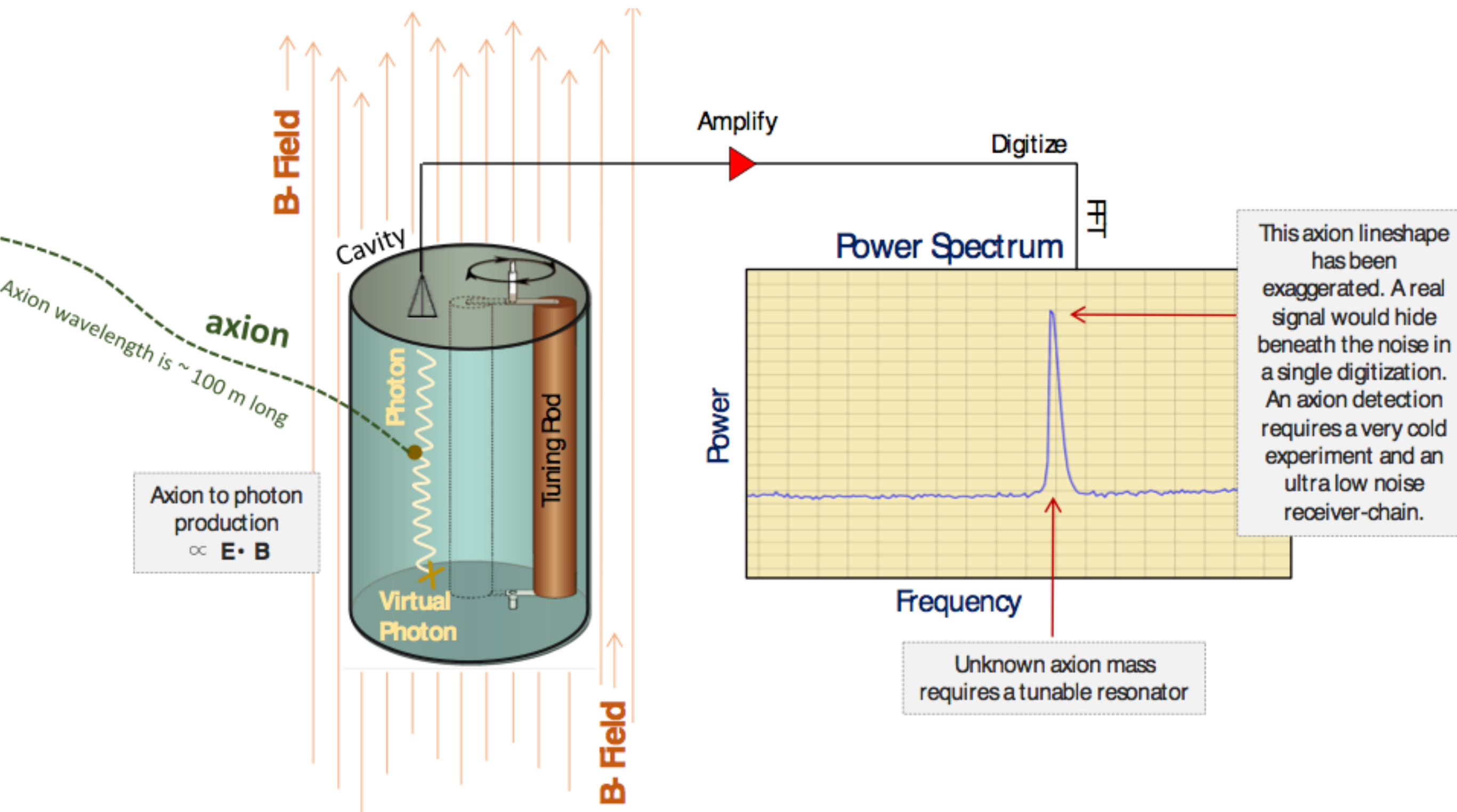
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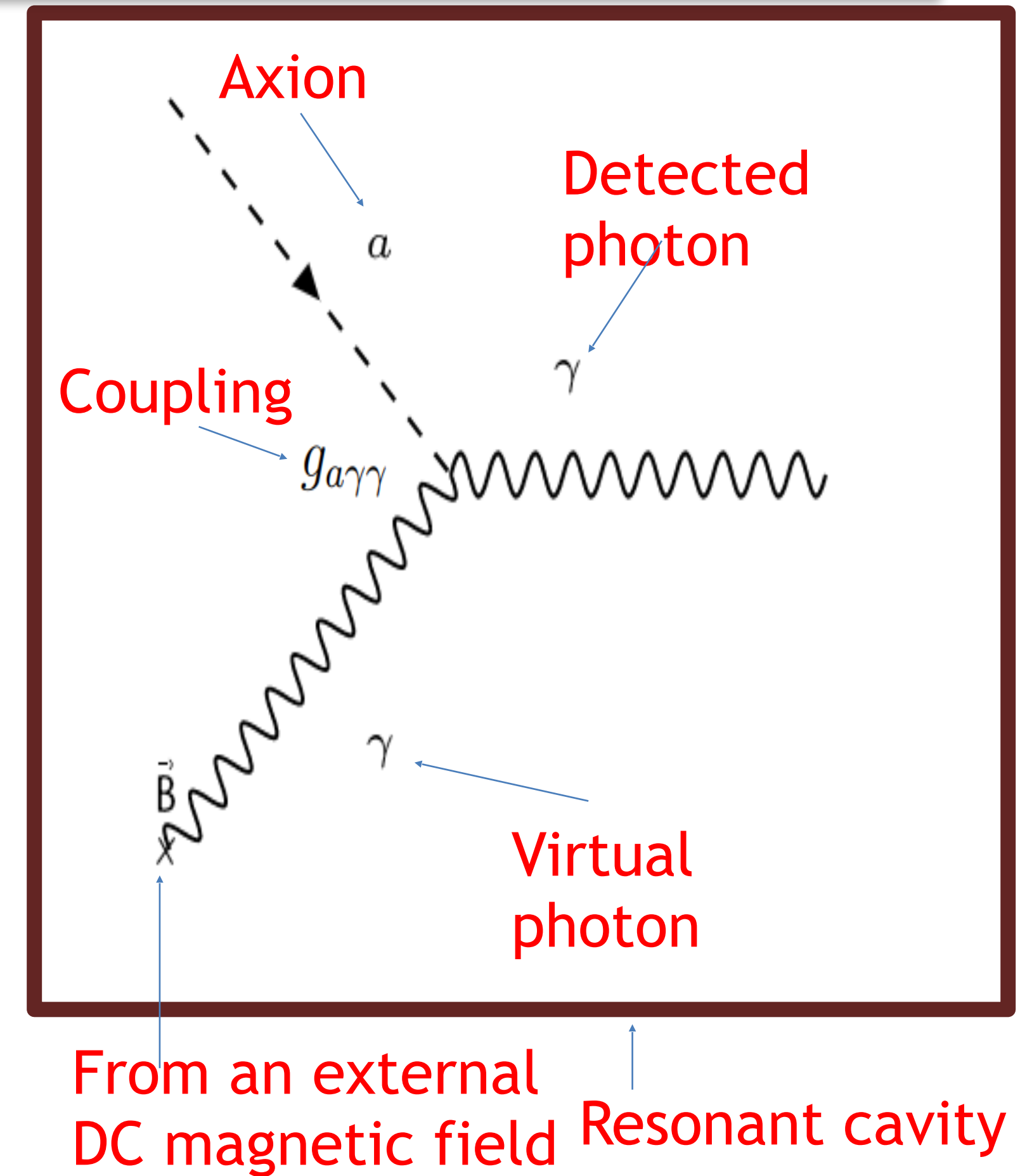
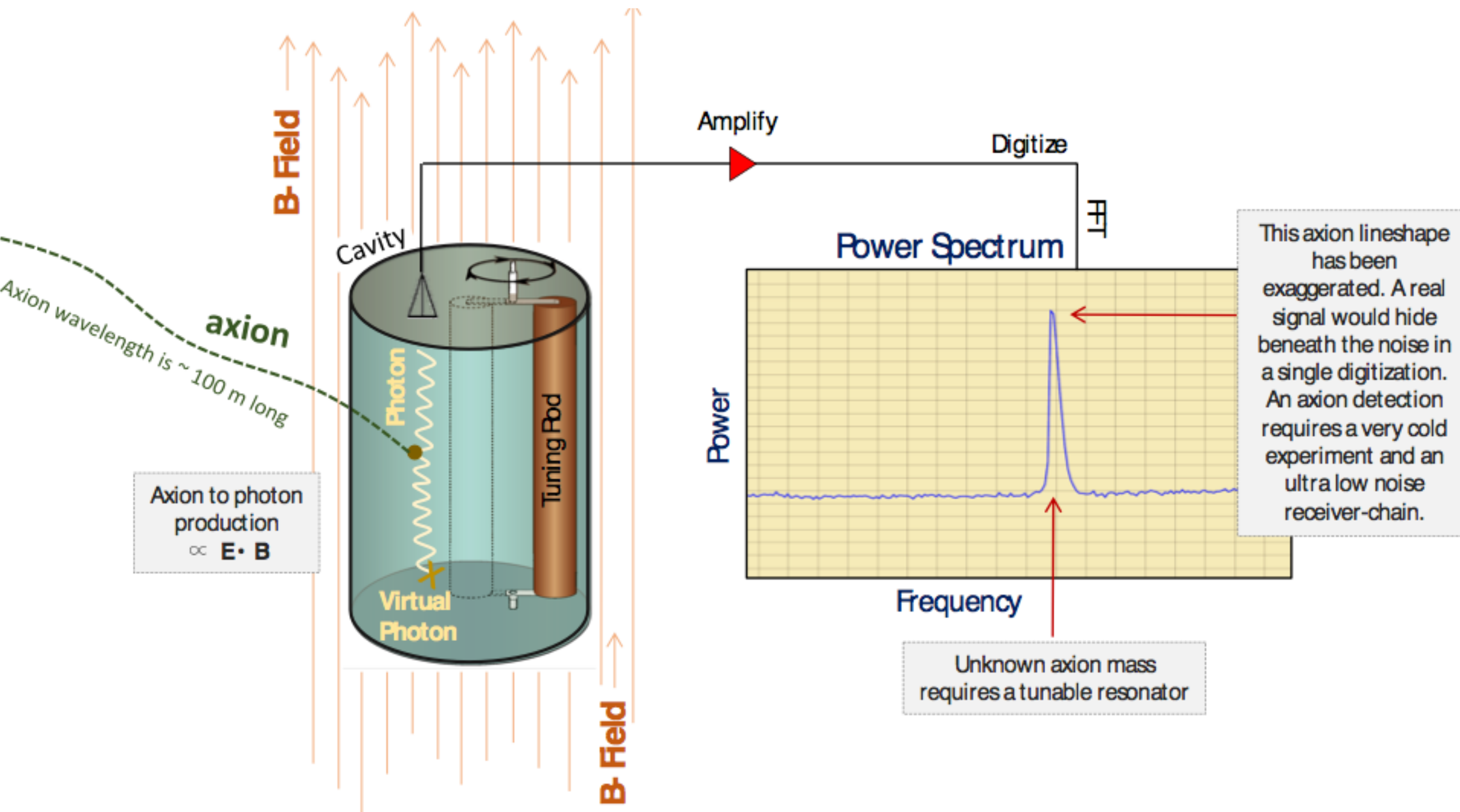
Pierre Sikivie



The Axion Haloscope Technique



Pierre Sikivie



$$\mathcal{L} \propto ag_{a\gamma\gamma} \vec{E}_{cavity} \cdot \vec{B}_{ext}$$

Lagrangian gives effective strength

Low-Mass Axion Detection with Lumped Elements; Implications of Modified Electrodynamics



PHYSICAL REVIEW D 94, 042001 (2016)

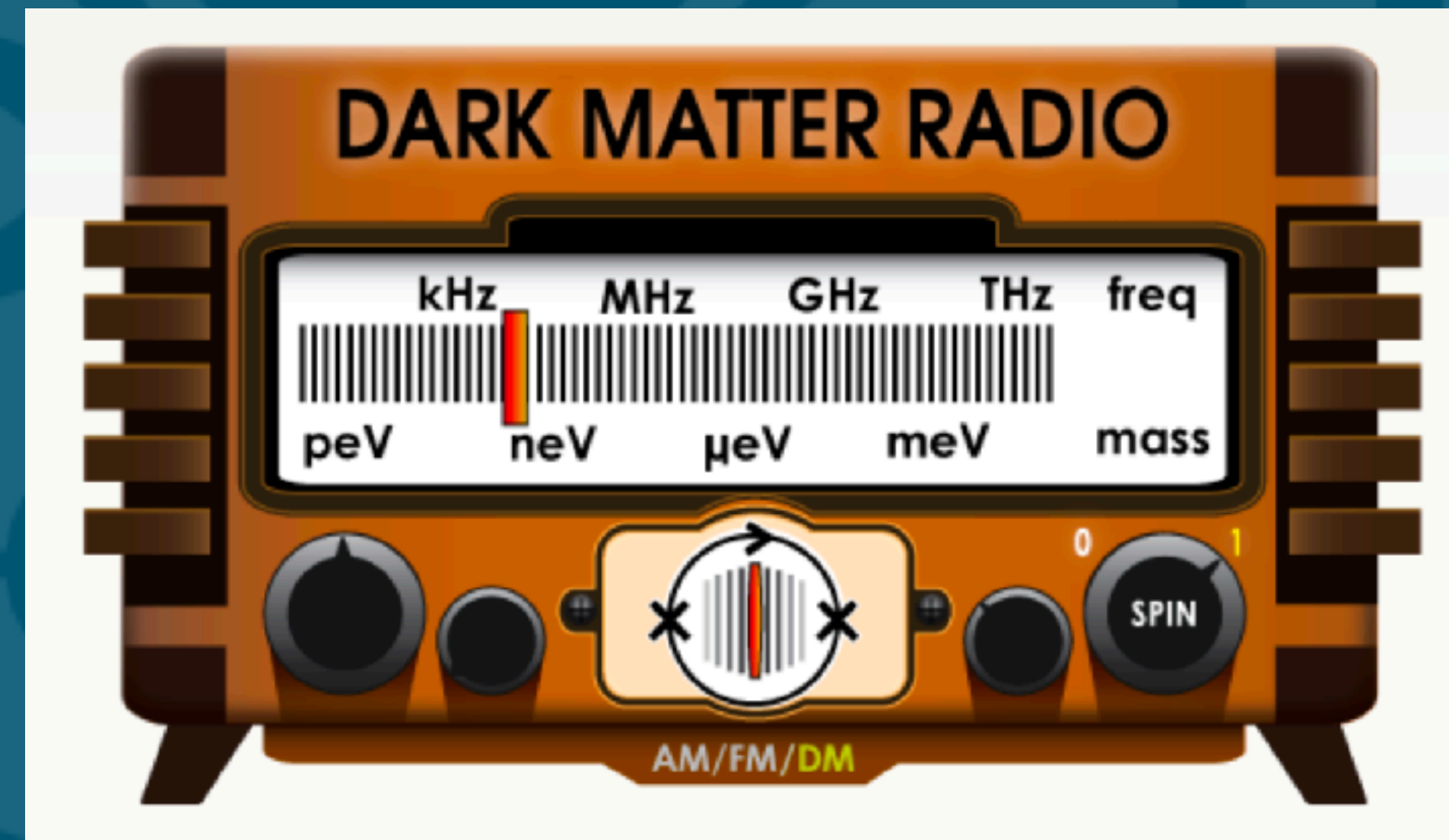
3D lumped LC resonators as low mass axion haloscopes

Ben T. McAllister,^{*} Stephen R. Parker, and Michael E. Tobar[†]

ARC Centre of Excellence for Engineered Quantum Systems, School of Physics,

The University of Western Australia, 35 Stirling Highway, Crawley 6009, Western Australia, Australia

(Received 18 May 2016; published 11 August 2016)



PRL 112, 131301 (2014)

PHYSICAL REVIEW LETTERS

week ending
4 APRIL 2014

Proposal for Axion Dark Matter Detection Using an *LC* Circuit

P. Sikivie, N. Sullivan, and D. B. Tanner

Department of Physics, University of Florida, Gainesville, Florida 32611, USA

(Received 31 October 2013; revised manuscript received 22 January 2014; published 31 March 2014)

Controversy at Low Mass; Compton Wavelength of Axion is Large

[1] [arXiv:1904.05774](#) [[pdf](#), [other](#)]

Electrodynamics of Impressed Bound and Free Charge Voltage Sources

[Michael E. Tobar](#), [Ben T. McAllister](#), [Maxim Goryachev](#)

Subjects: **Classical Physics** ([physics.class-ph](#))

[4] [arXiv:1809.01654](#) [[pdf](#), [other](#)]

Modified Axion Electrodynamics as Impressed Electromagnetic Sources Through Oscillating Background Polarization and Magnetization

[Michael Edmund Tobar](#), [Ben T. McAllister](#), [Maxim Goryachev](#)

Comments: This version shows that axion electrodynamics under DC magnetic field is driven by an electric vector potential

Subjects: **High Energy Physics – Phenomenology** ([hep-ph](#)); Astrophysics of Galaxies ([astro-ph.GA](#)); General Relativity and Quantum Cosmology ([gr-qc](#)); Instrumentation and Detectors ([physics.ins-det](#))

[13] [arXiv:1803.07755](#) [[pdf](#), [other](#)]

Broadband Axion Dark Matter Haloscopes via Electric Sensing

[Ben T. McAllister](#), [Maxim Goryachev](#), [Jeremy Bourhill](#), [Eugene N. Ivanov](#), [Michael E. Tobar](#)

Comments: 6 pages, 4 figures. V4: Updated figures/text/appendices

Subjects: **Instrumentation and Detectors** ([physics.ins-det](#)); General Relativity and Quantum Cosmology ([gr-qc](#)); High Energy Physics – Experiment ([hep-ex](#)); High Energy Physics – Phenomenology ([hep-ph](#))

Axion Electrodynamics

$$\theta \frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \equiv \theta \frac{e^2}{32\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu} \sim \vec{E} \cdot \vec{B}$$

Axion Electrodynamics

$$\theta \frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \equiv \theta \frac{e^2}{32\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu} \sim \vec{E} \cdot \vec{B}$$

Parity or Time-reversal: $\vec{E} \cdot \vec{B} \rightarrow -\vec{E} \cdot \vec{B}$ CP odd interaction

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Total Derivative $\varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} = \partial_\mu \left(\varepsilon^{\alpha\beta\mu\nu} A_\nu F_{\alpha\beta} \right)$ Does not change Equation of motion but adds CP odd interaction

Axion Electrodynamics

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QCD

$\varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} = \partial_\mu \varepsilon^{\alpha\beta\mu\nu} \left(A_\nu F_{\alpha\beta} + A_\alpha A_\beta A_\nu \right)$ Instanton \rightarrow can not ignore the surface terms!!!!

Axion Electrodynamics

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$$\int d^4x \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \neq 0$$

Axion Electrodynamics

$$\theta \frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} \equiv \theta \frac{e^2}{32\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu} \sim \vec{E} \cdot \vec{B}$$

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Total Derivative $\varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} = \partial_\mu \left(\varepsilon^{\alpha\beta\mu\nu} A_\nu F_{\alpha\beta} \right)$ Does not change Equation of motion but adds CP odd interaction

QCD

$\varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} = \partial_\mu \varepsilon^{\alpha\beta\mu\nu} \left(A_\nu F_{\alpha\beta} + A_\alpha A_\beta A_\nu \right)$ Instanton -> can not ignore the surface terms!!!!

$$\int d^4x \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \neq 0$$

Electrodynamics -> surface term due to impressed large DC current -> creates the DC magnetic field -> Excitation (or forcing) term (excites the system but does not change equation of motion)

Axion Electrodynamics

$$\theta \frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} \equiv \theta \frac{e^2}{32\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu} \sim \vec{E} \cdot \vec{B}$$

Parity or Time-reversal: $\vec{E} \cdot \vec{B} \rightarrow -\vec{E} \cdot \vec{B}$ CP odd interaction

Total Derivative $\varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} = \partial_\mu \left(\varepsilon^{\alpha\beta\mu\nu} A_\nu F_{\alpha\beta} \right)$ Does not change Equation of motion but adds CP odd interaction

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Modified Axion Electrodynamics as Impressed Electromagnetic Sources Through Oscillating Background Polarization and Magnetization

$$\begin{aligned}\vec{\nabla} \cdot \vec{D} &= \rho_f + g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{B} \cdot \vec{\nabla} a \\ \vec{\nabla} \times \vec{H} &= \vec{J}_f + \frac{\partial \vec{D}}{\partial t} - g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} \left(\vec{B} \frac{\partial a}{\partial t} + \vec{\nabla} a \times \vec{E} \right) \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= - \frac{\partial \vec{B}}{\partial t} \\ \vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \vec{H} &= \frac{\vec{B}}{\mu_0} - \vec{M}\end{aligned}$$



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Modified Axion Electrodynamics as Impressed Electromagnetic Sources Through Oscillating Background Polarization and Magnetization

$$\vec{\nabla} \cdot \vec{D} = \rho_f + g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{B} \cdot \vec{\nabla} a$$

Apply DC B-field

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} - g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} \left(\vec{B} \frac{\partial a}{\partial t} + \vec{\nabla} a \times \vec{E} \right)$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Usual lumped element
experiments assume

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} a = 0$$

Fine, but approximation made
too early

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

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$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} + \vec{J}_{aB} + \vec{\nabla} a \times \vec{E}$$

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Usually it is assumed the axion current is non-interacting
NOT TRUE ignores surface term

Rearrange Axion Electrodynamics to reflect

$$\mathcal{L}'_{a\gamma} = -g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{E}_T \cdot (a \vec{B}_0)$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

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$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

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Vector Identities

$$\vec{B} \cdot (\vec{\nabla} a) = \vec{\nabla} \cdot (a \vec{B}) + a(\vec{\nabla} \cdot \vec{B})$$

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Modified Gauss' Law and Ampere's Law

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Reformulate Modified Electrodynamics

$$\begin{aligned}\vec{\nabla} \cdot \vec{D}_T &= \rho_f \\ \vec{\nabla} \times \vec{H}_T &= \vec{J}_f + \frac{\partial \vec{D}_T}{\partial t} \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t}\end{aligned}$$

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Similar to Standard
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Signals for Lorentz violation in electrodynamics

V. Alan Kostelecký and Matthew Mewes

Physics Department, Indiana University, Bloomington, Indiana 47405

(Received 20 May 2002; published 23 September 2002)

$$\begin{aligned}
\vec{\nabla} \times \vec{H} - \partial_0 \vec{D} &= 0, & \vec{\nabla} \cdot \vec{D} &= 0, & \begin{pmatrix} \vec{D} \\ \vec{H} \end{pmatrix} &= \begin{pmatrix} 1 + \kappa_{DE} & \kappa_{DB} \\ \kappa_{HE} & 1 + \kappa_{HB} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} \\
\vec{\nabla} \times \vec{E} + \partial_0 \vec{B} &= 0, & \vec{\nabla} \cdot \vec{B} &= 0.
\end{aligned}$$

New methods of testing Lorentz violation in electrodynamicsMichael Edmund Tobar,^{1,*} Peter Wolf,^{2,3} Alison Fowler,¹ and John Gideon Hartnett¹¹*University of Western Australia, School of Physics, M013, 35 Stirling Highway, Crawley 6009 WA, Australia*²*Bureau International des Poids et Mesures, Pavillon de Breteuil, 92312 Sèvres Cedex, France*³*BNM-SYRTE, Observatoire de Paris, 61 Avenue de l'Observatoire, 75014 Paris, France*

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$$g_{a\gamma\gamma}a \sim \kappa_{DB} \kappa_{HE}$$

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**Axion Interaction similar to odd
parity Lorentz Invariance Violation**

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Axion Interaction similar to odd parity Lorentz Invariance Violation

Axion Induced Background Bound Charges and Currents

Background Bound Charge

$$\rho_{aB} = g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{\nabla} \cdot (a \vec{B})$$

Background Polarization Current (or Displacement Current)

$$\vec{P}_{aE} = -g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} (a \vec{B}) \quad \vec{J}_{aB} = -g_{a\gamma\gamma} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{\partial(a \vec{B})}{\partial t} = \frac{\partial \vec{P}_{aB}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{J}_{aB} = -\frac{\partial \rho_{aB}}{\partial t}$$

Satisfies the Continuity Equation

Axion Modifications as Impressed Sources (Harrington)

$$\vec{E}_T = \frac{1}{\epsilon_r \epsilon_0} \vec{D}_T = \vec{E} + \vec{E}_{aB}, \text{ where } \vec{E}_{aB} = -g_{a\gamma\gamma} \frac{c}{\epsilon_r} (a \vec{B})$$

$$\vec{B}_T = \mu_r \mu_0 \vec{H}_T = \vec{B} + \vec{B}_{aE}, \text{ where } \vec{B}_{aE} = g_{a\gamma\gamma} \frac{\mu_r}{c} (a \vec{E})$$

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curl free



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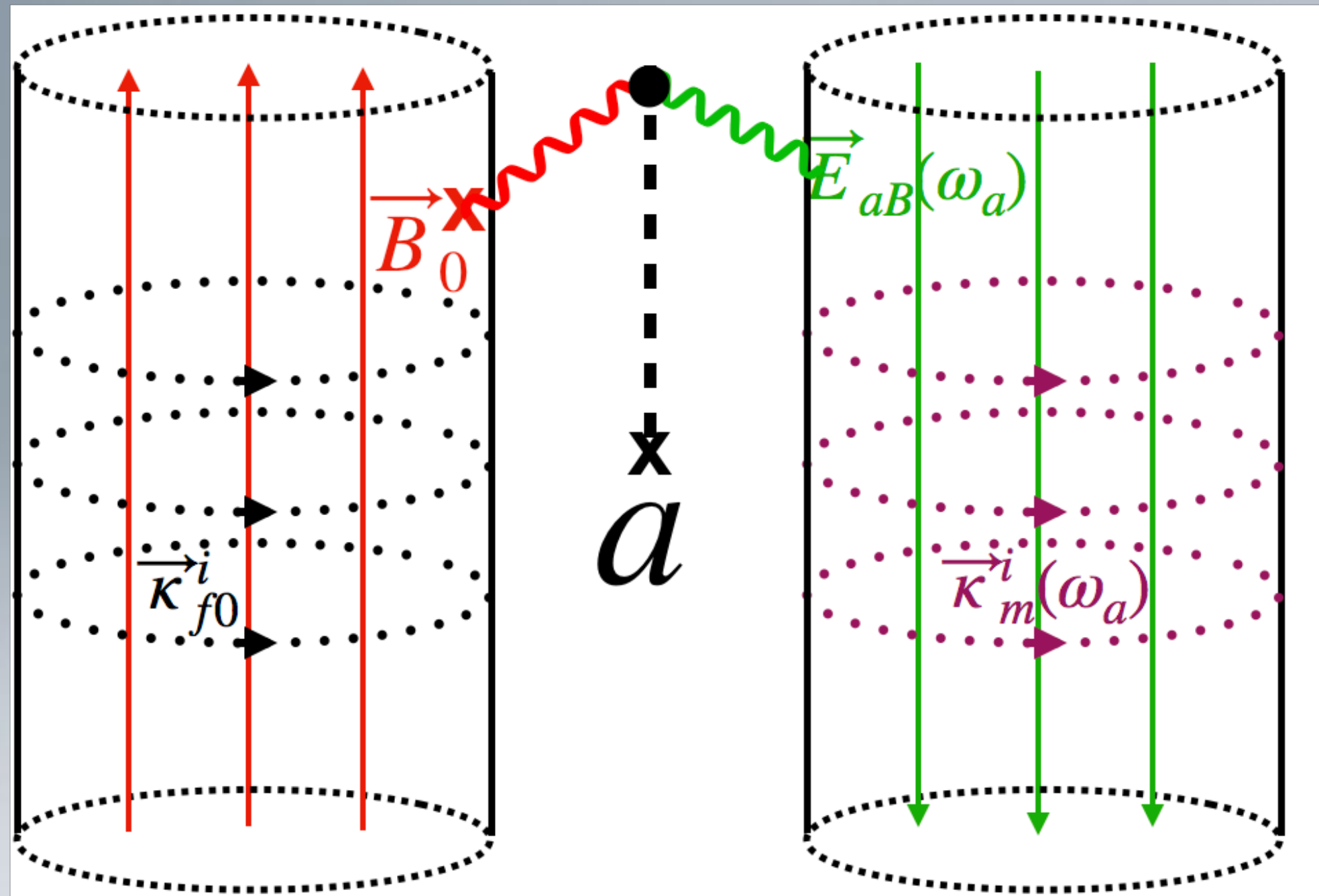
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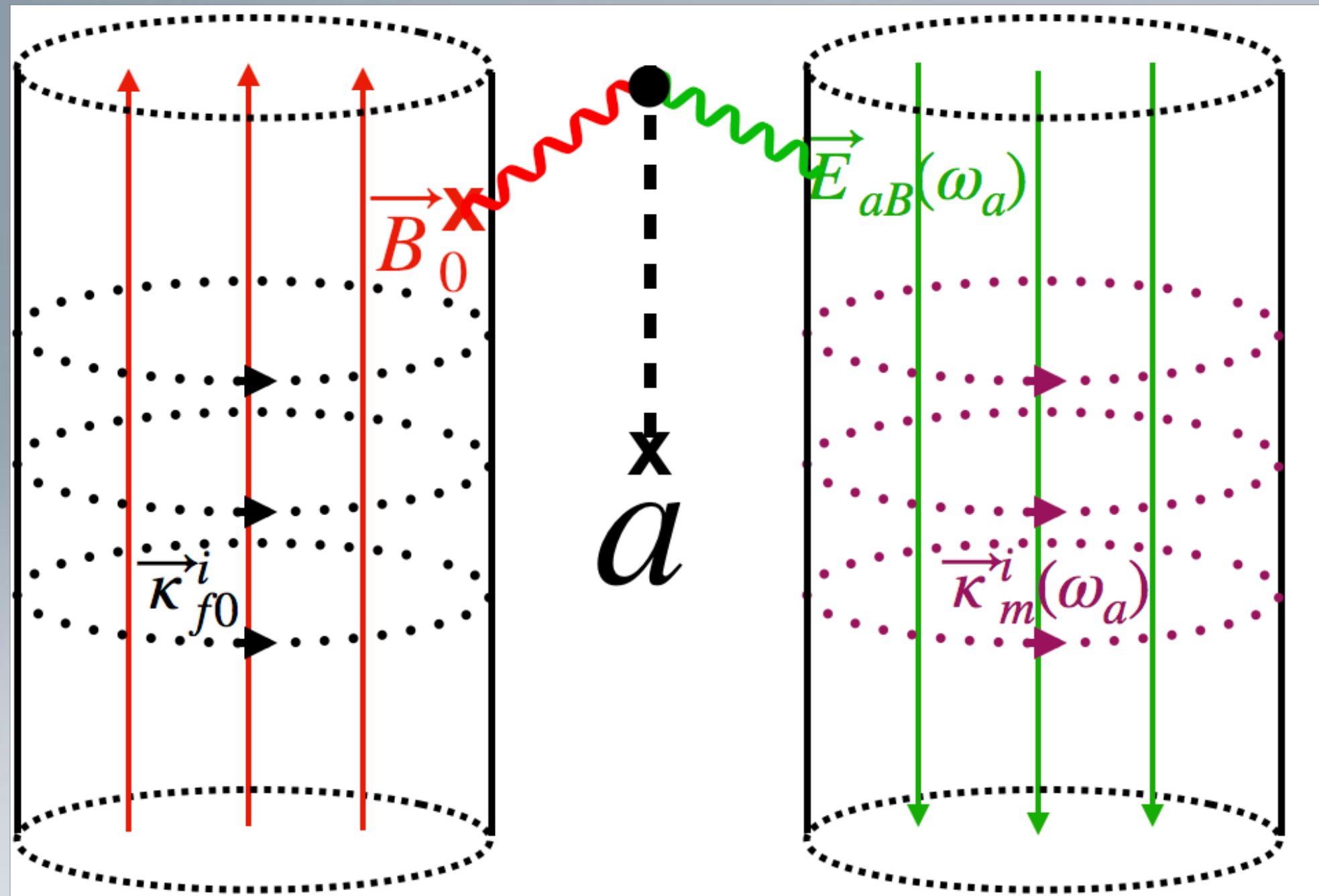
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Axion Induced Oscillating Sources and Fields under a DC Magnetic field



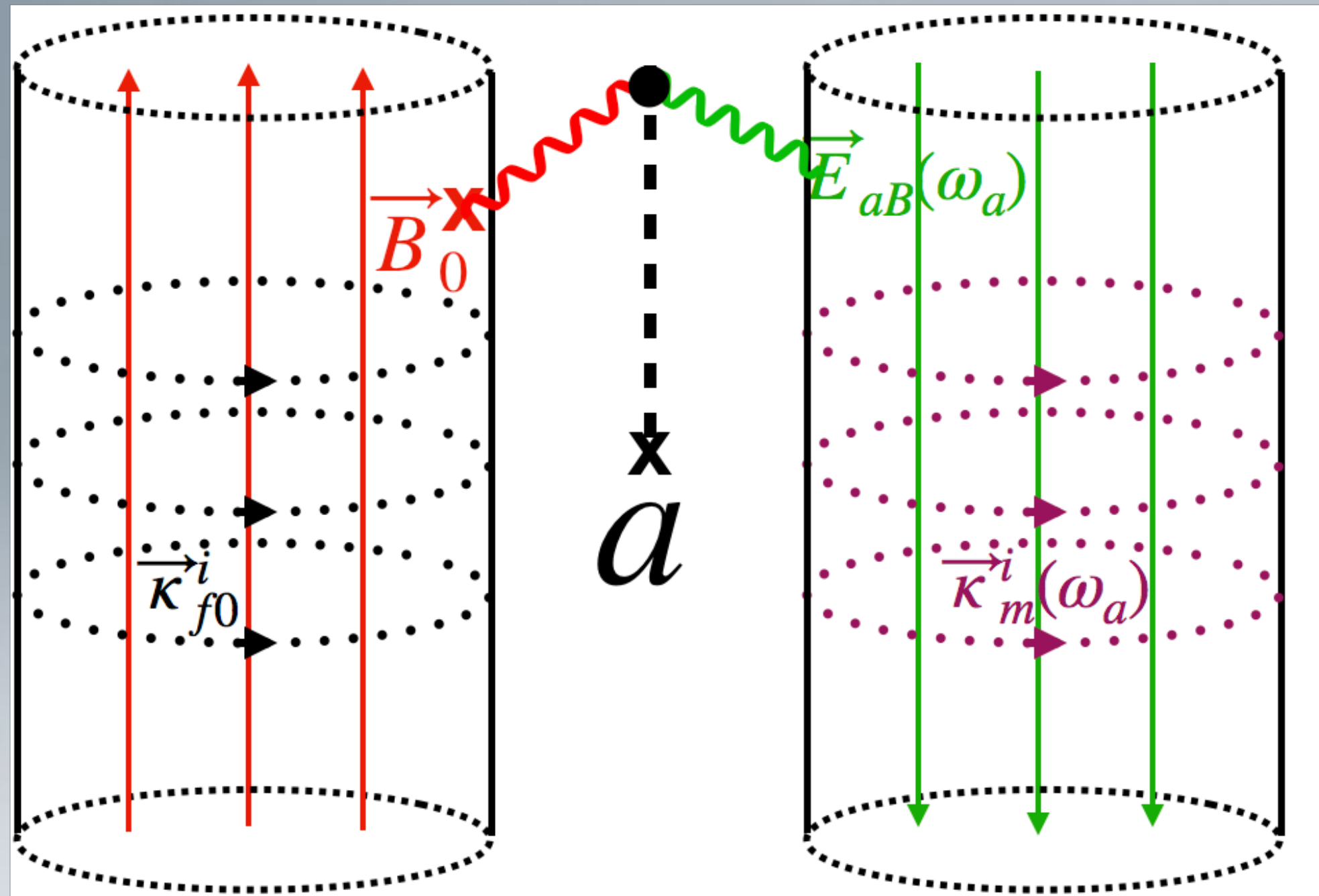
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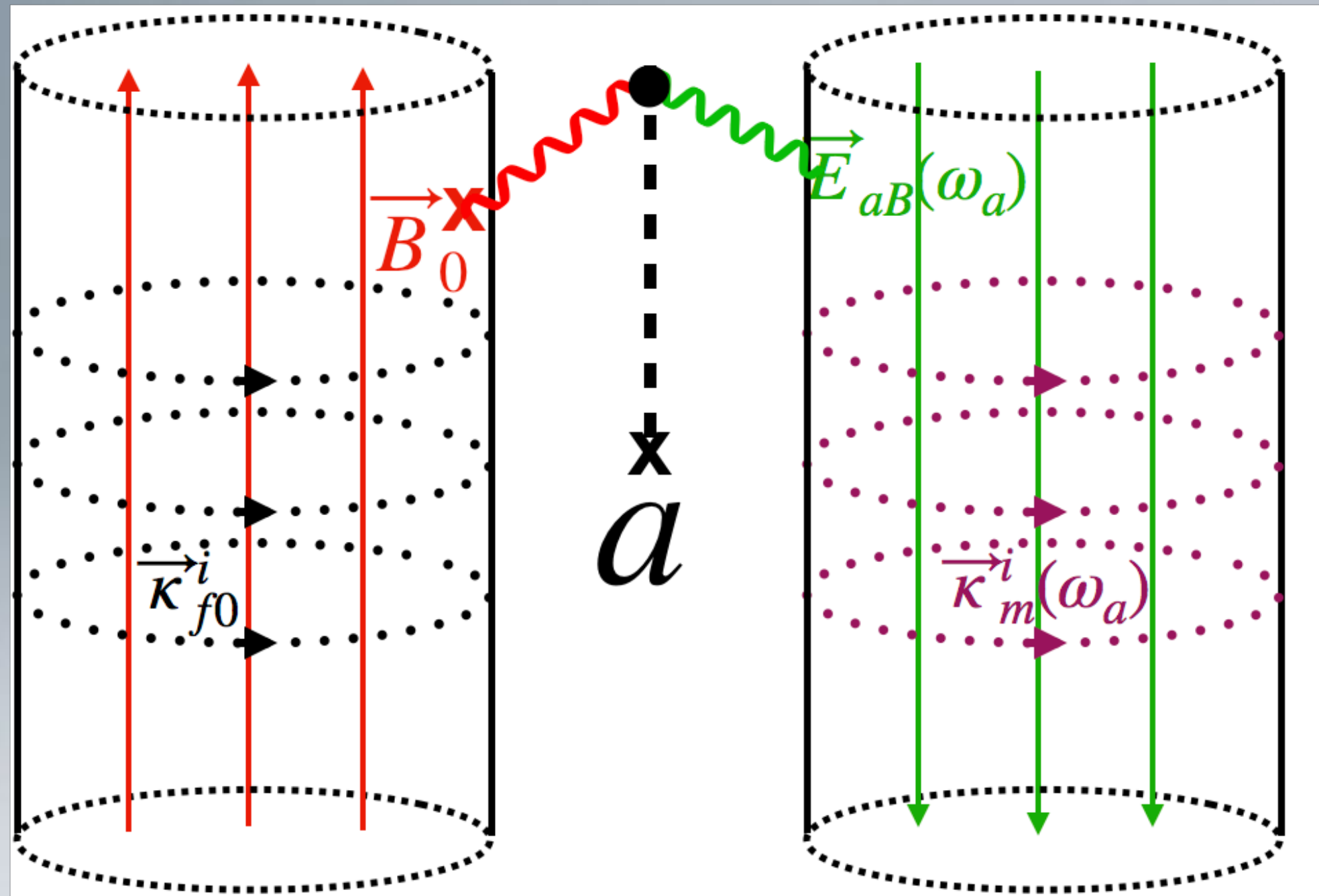


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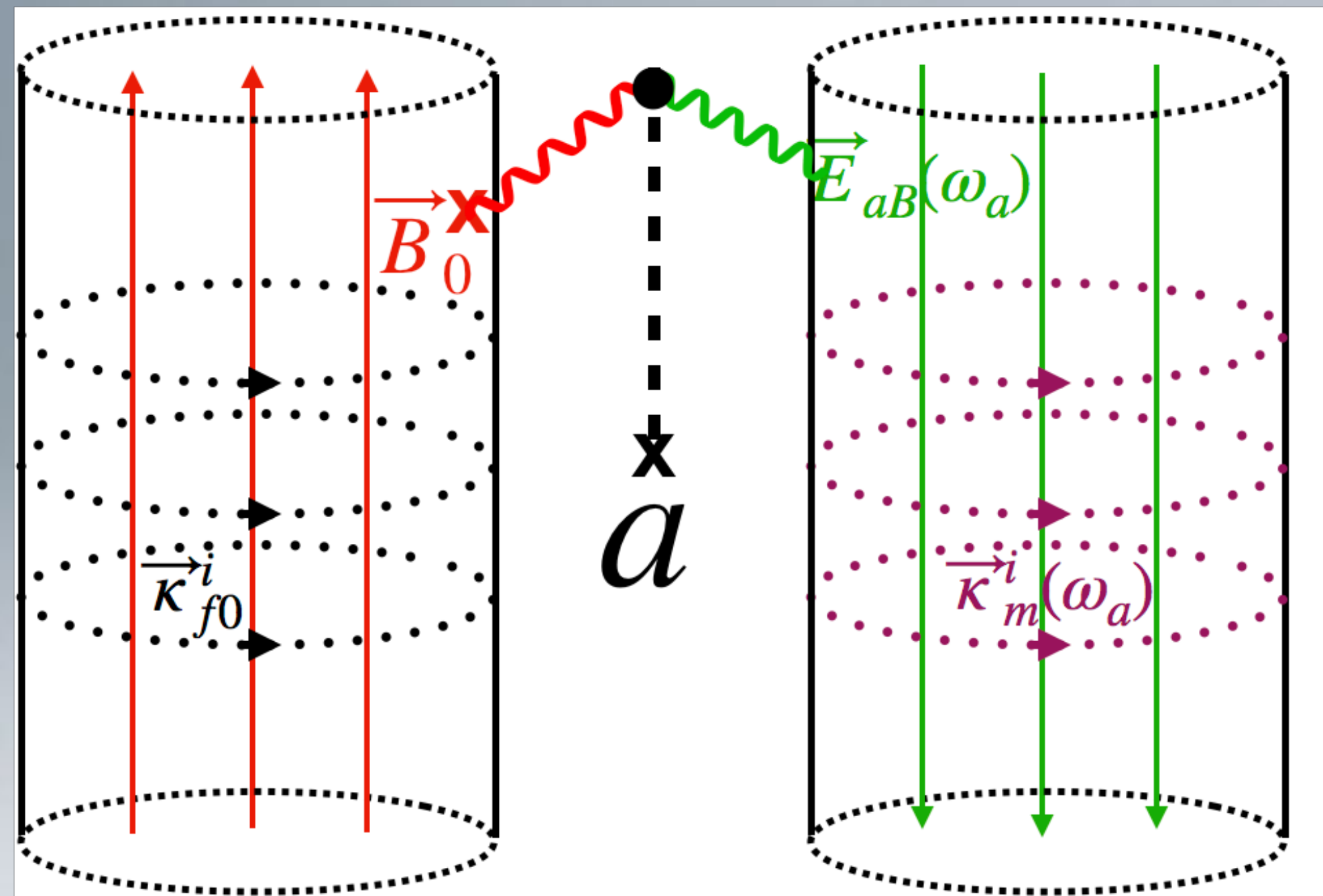
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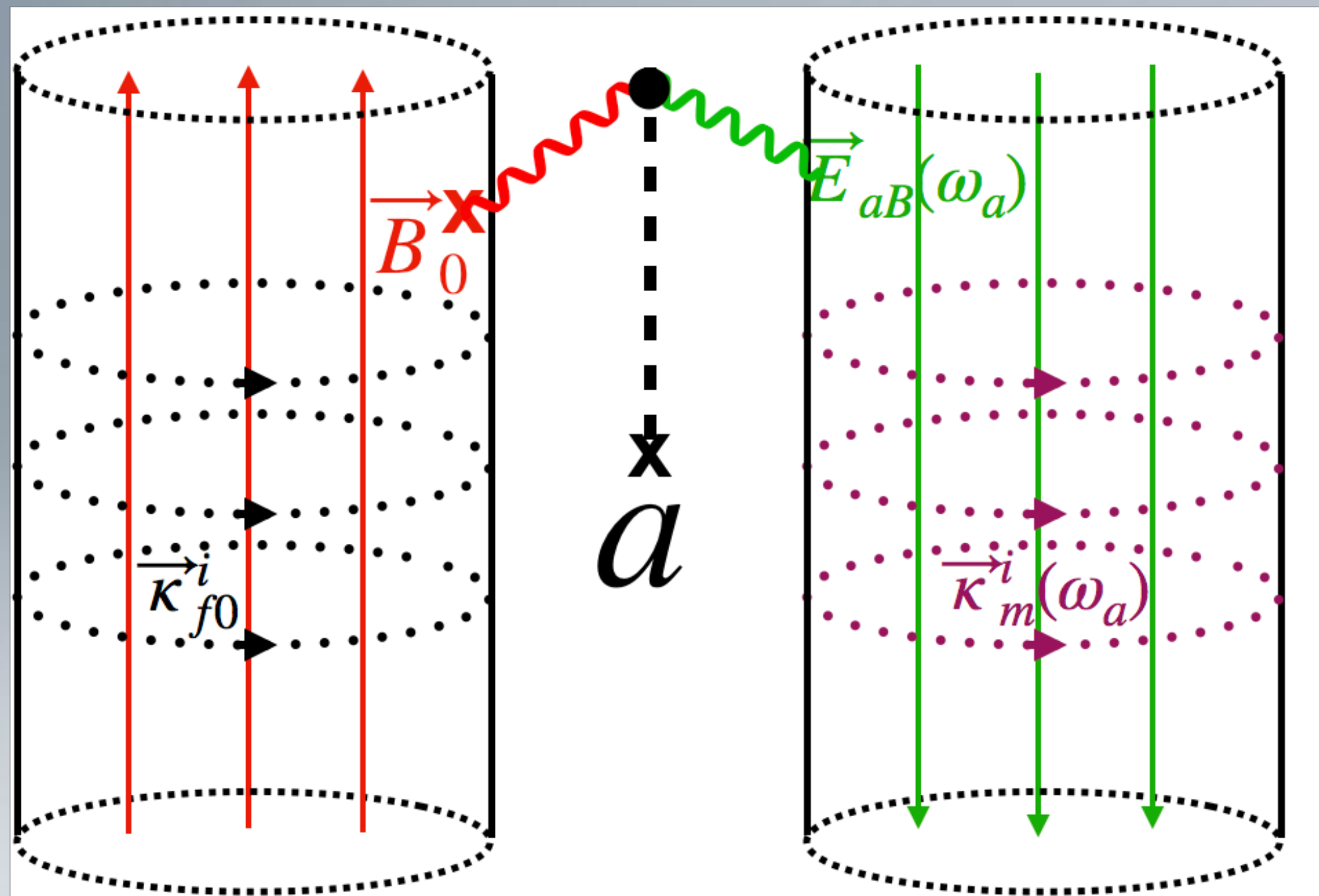
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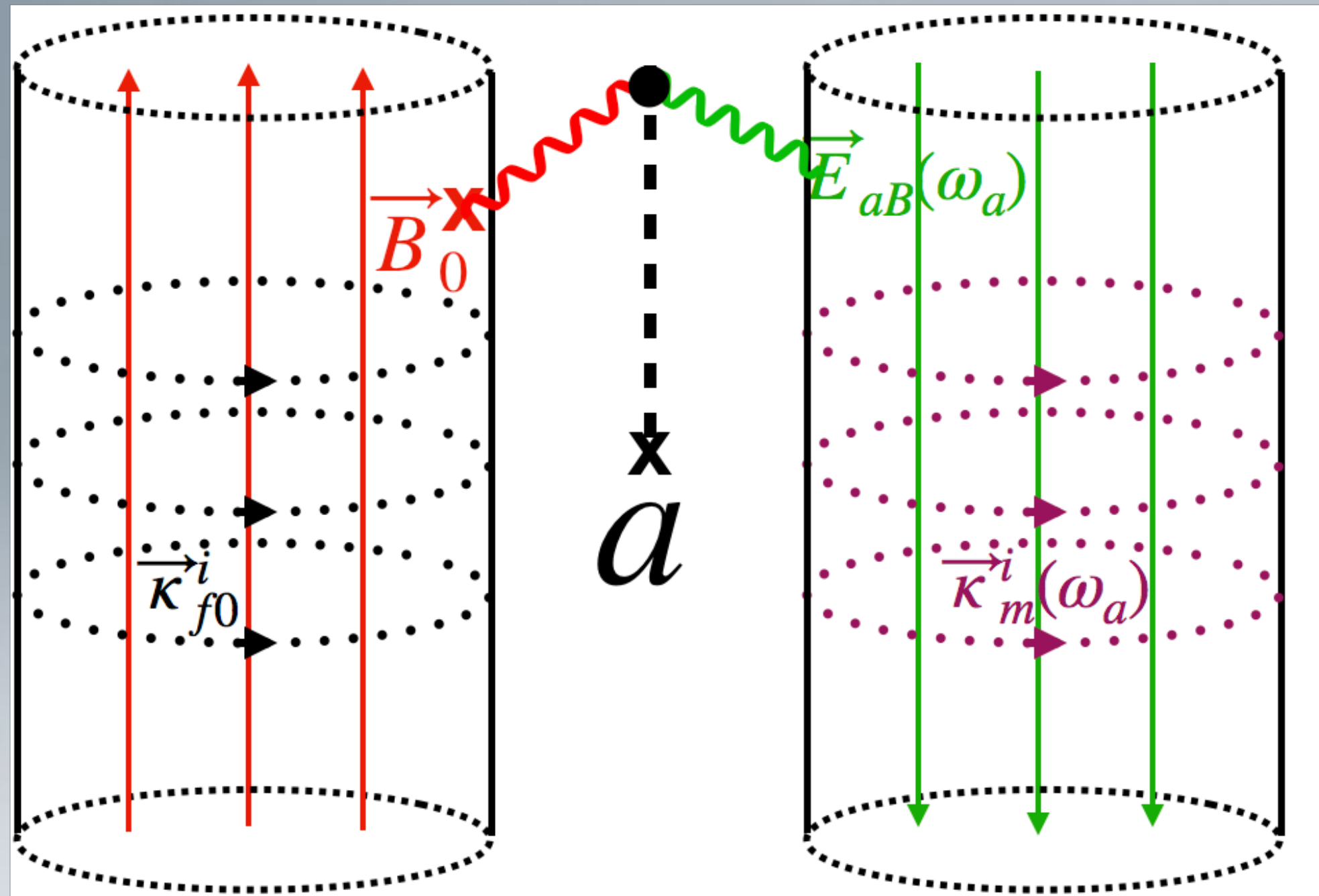
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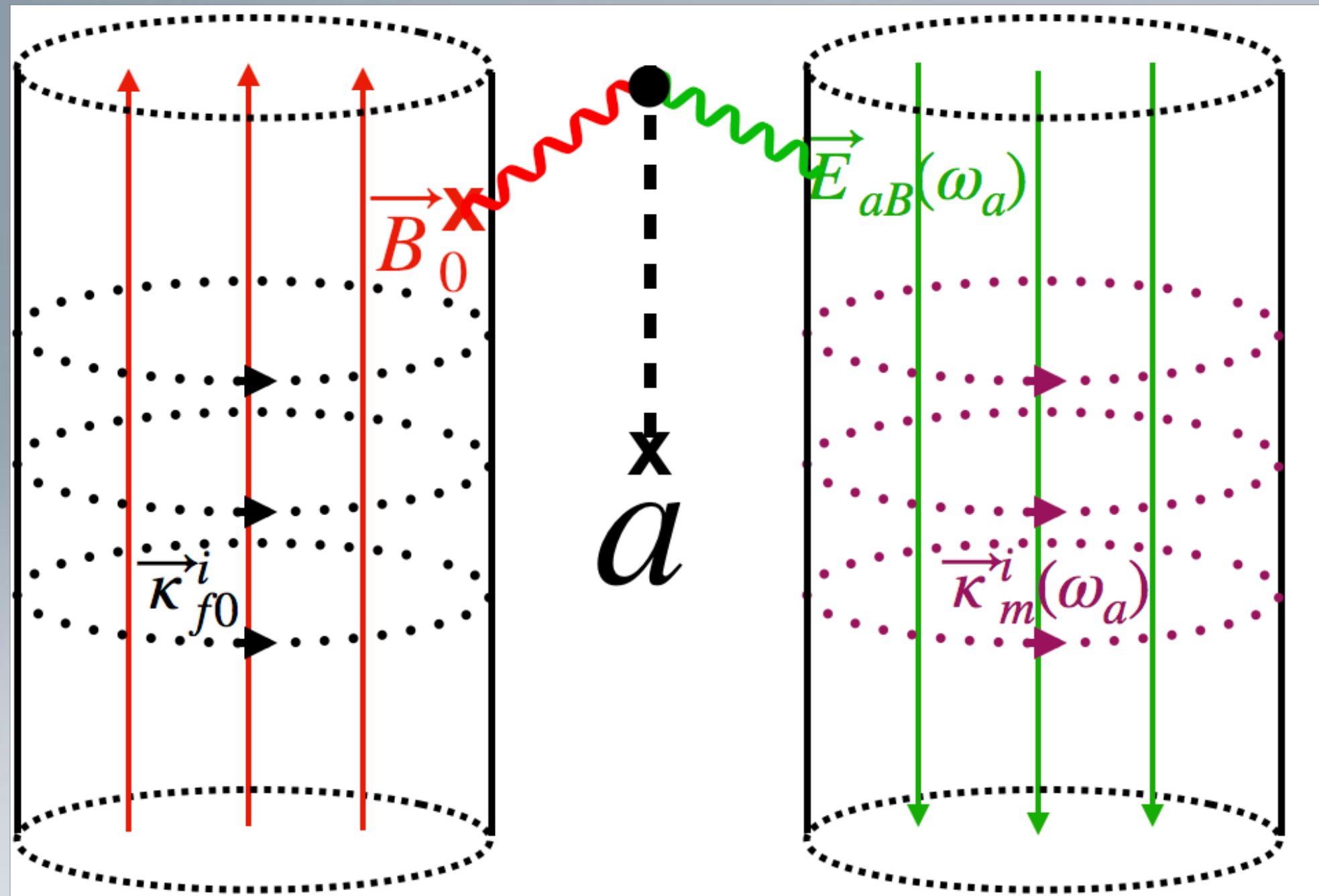
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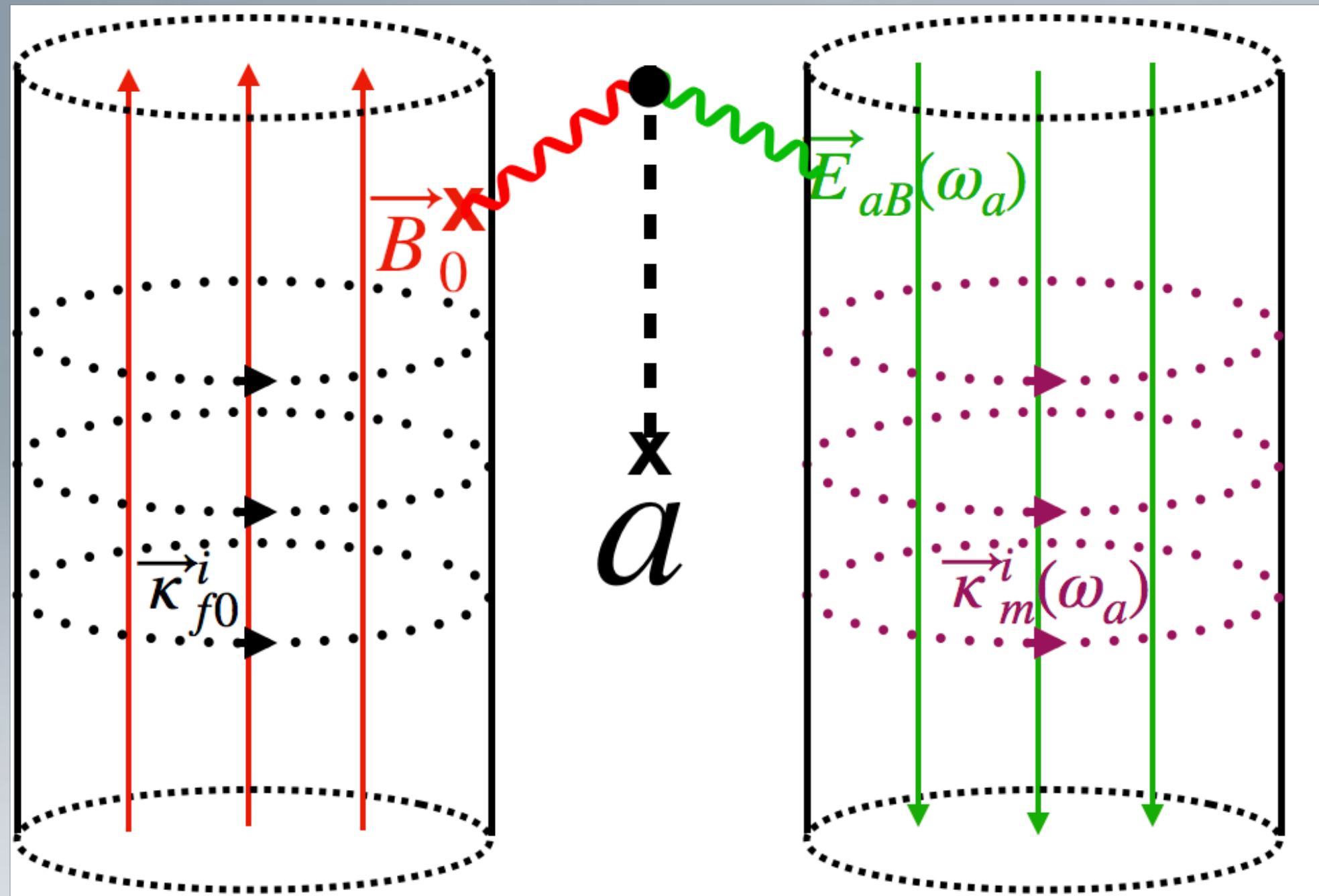
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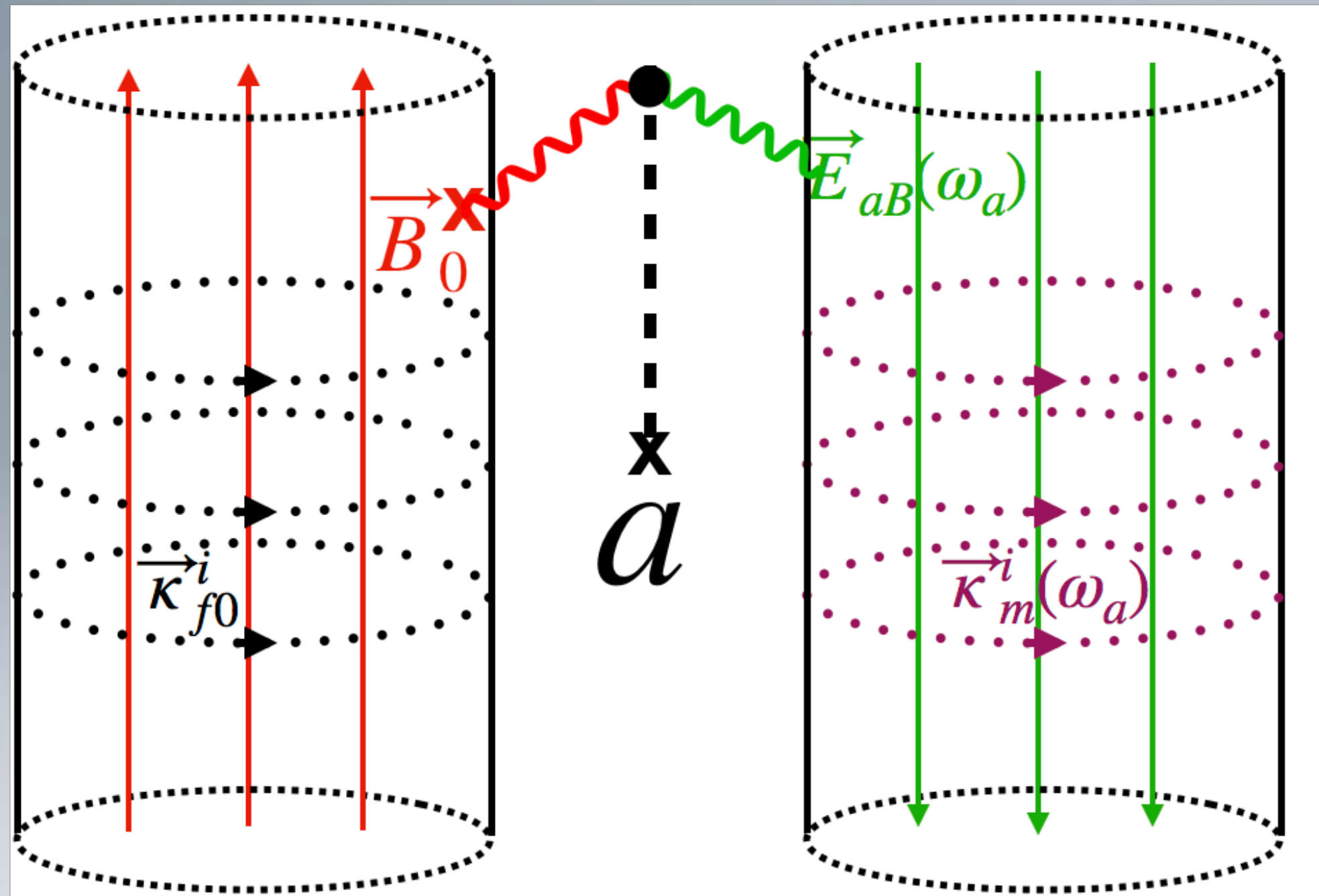
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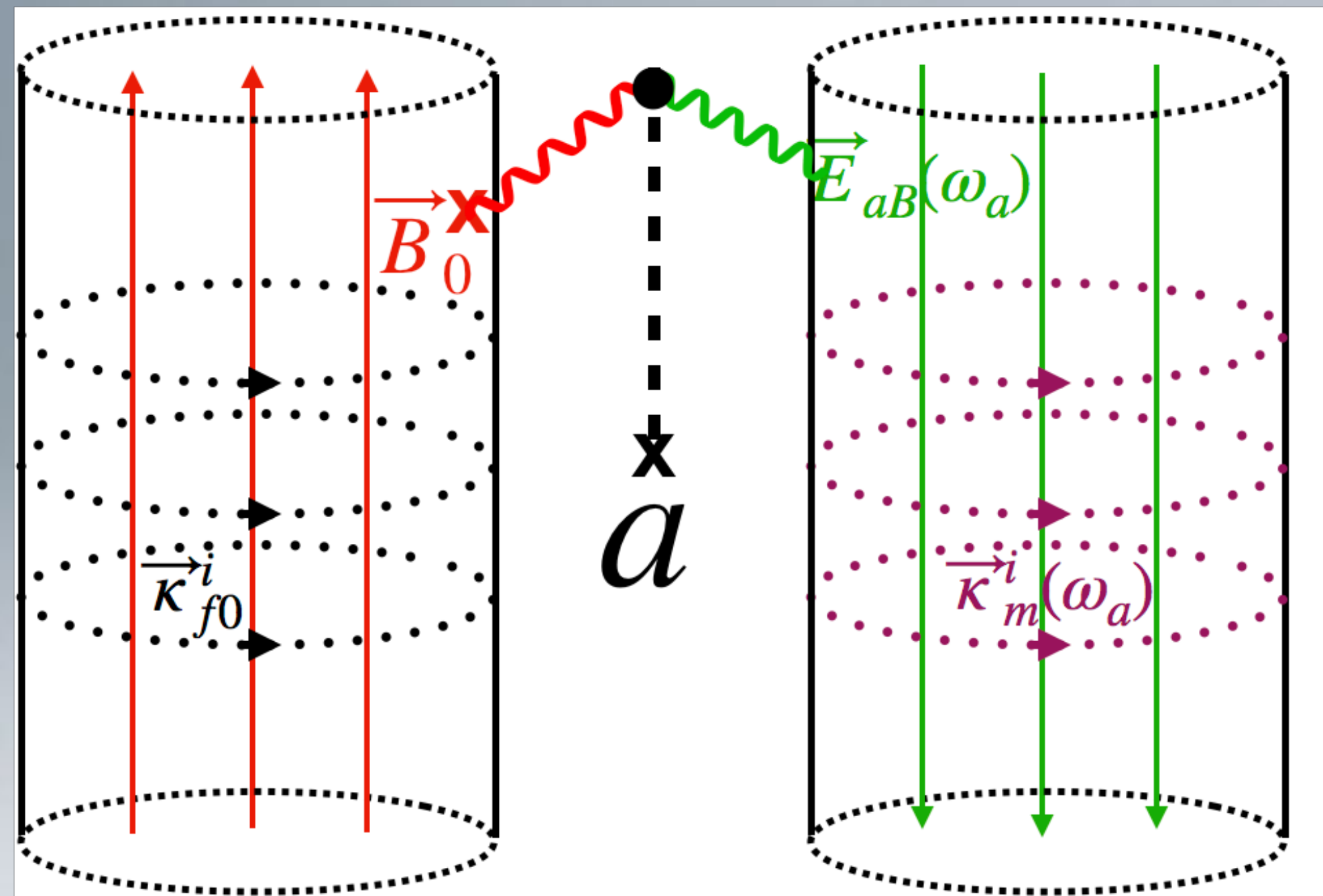
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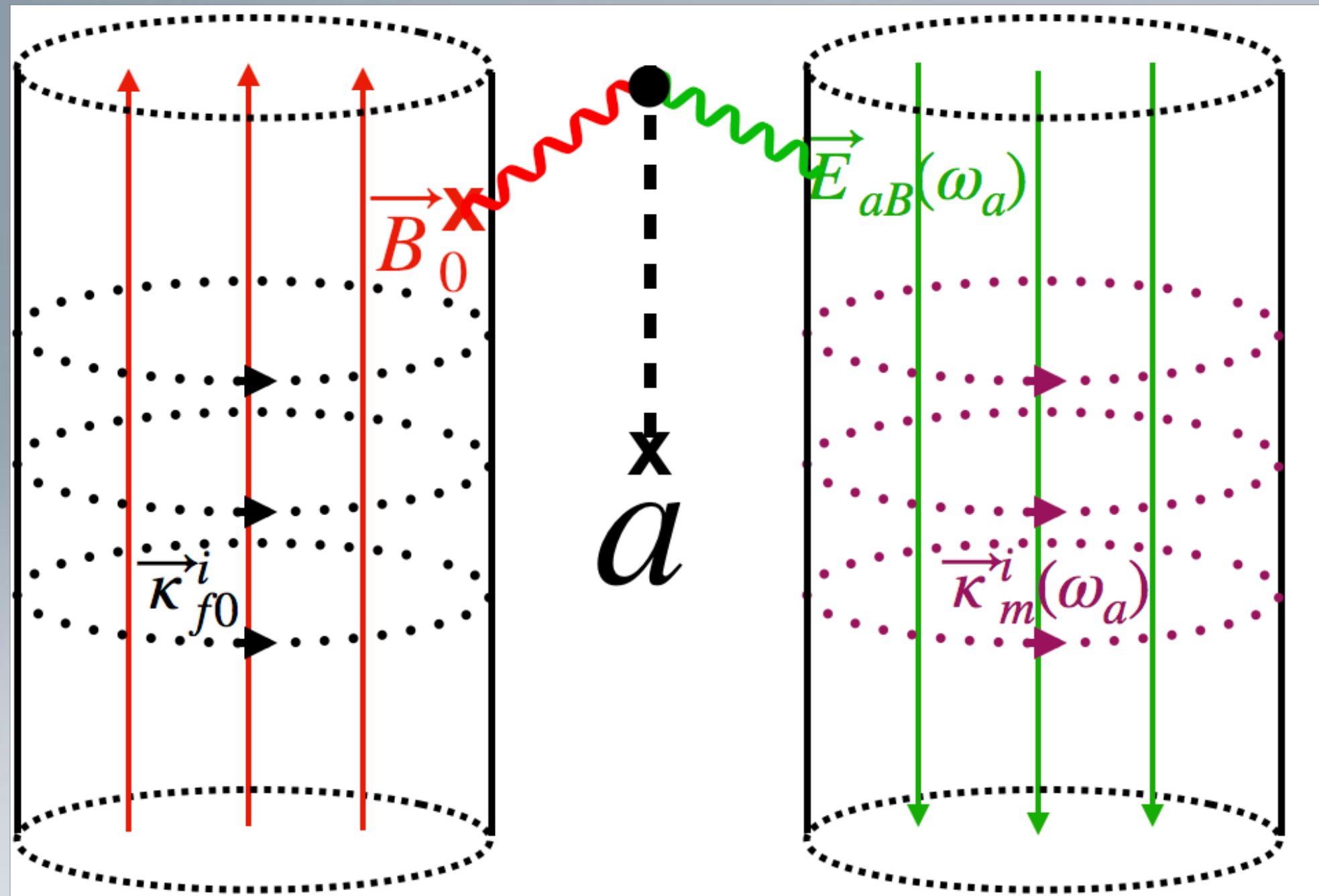
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The impressed DC current density induces an impressed magnetic current (or voltage source) oscillating at the axion Compton frequency through the inverse Primakoff effect

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PHYSICAL REVIEW D **99**, 055010 (2019)

Solutions to axion electrodynamics in various geometries

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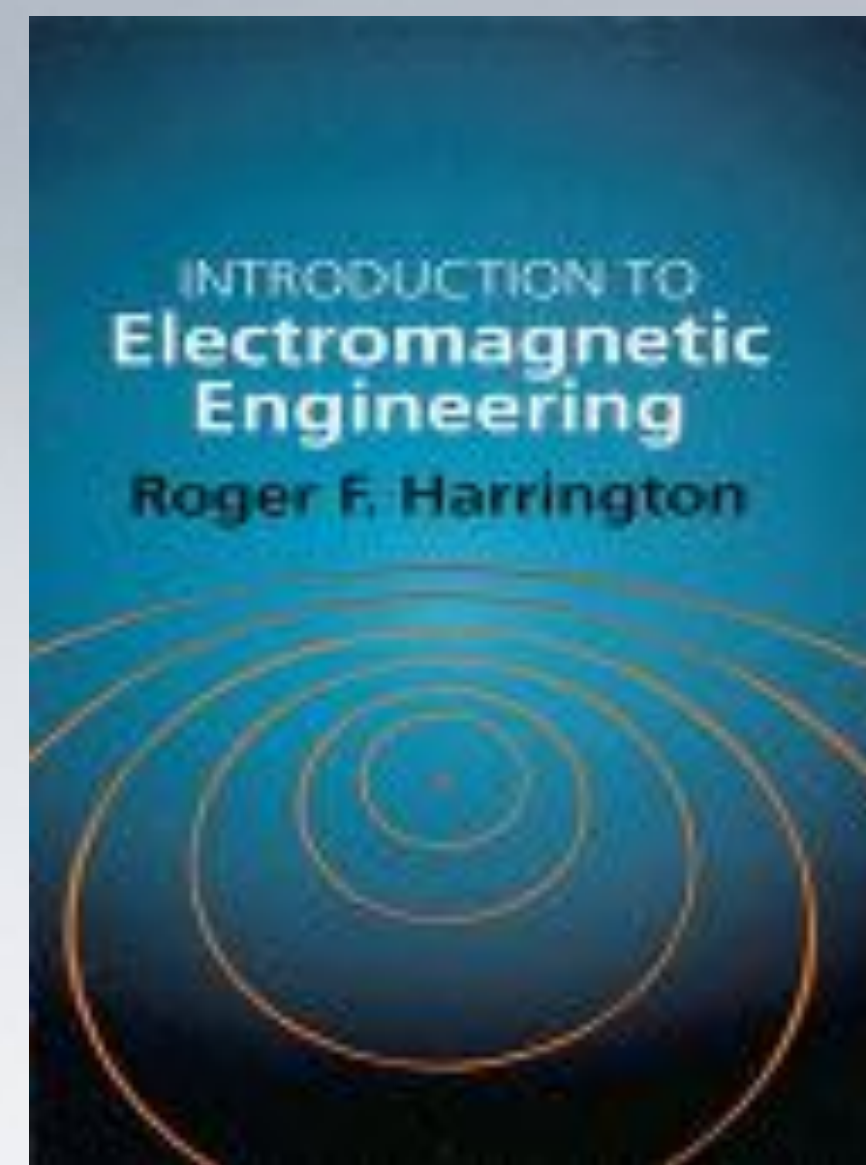
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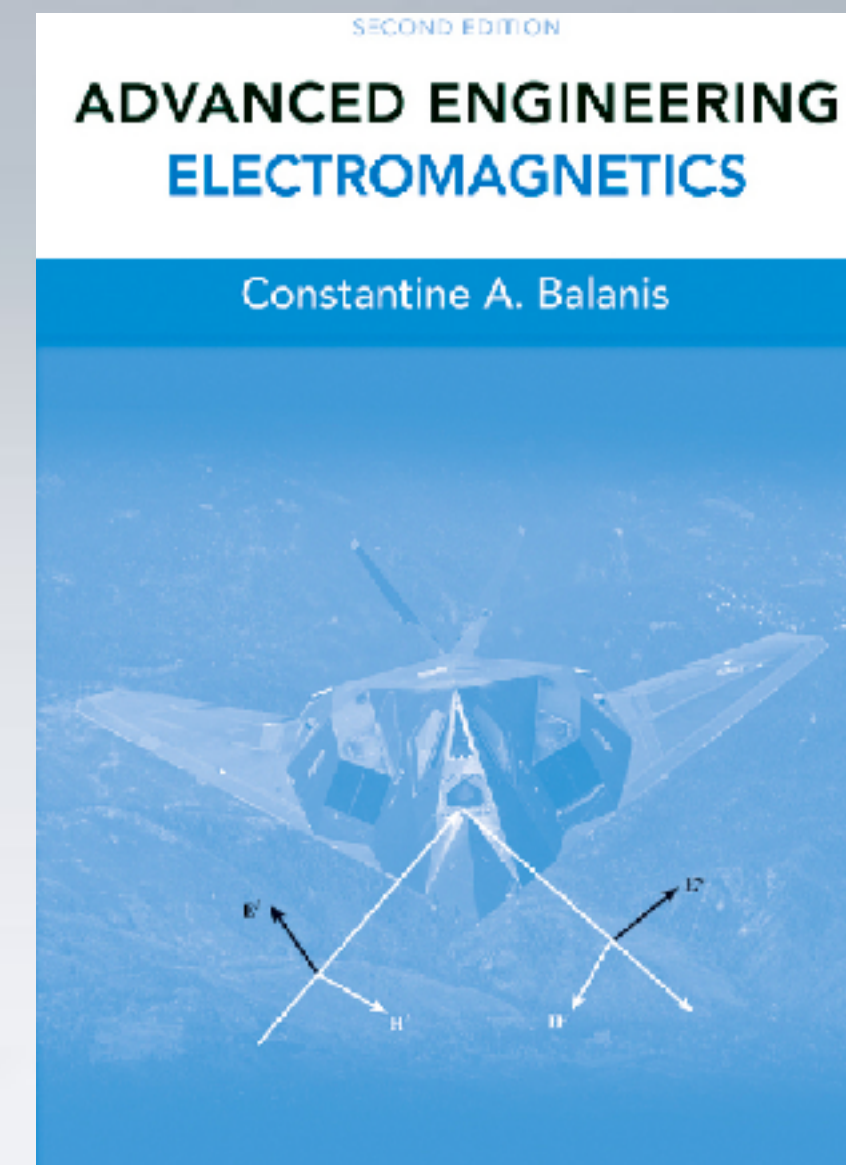
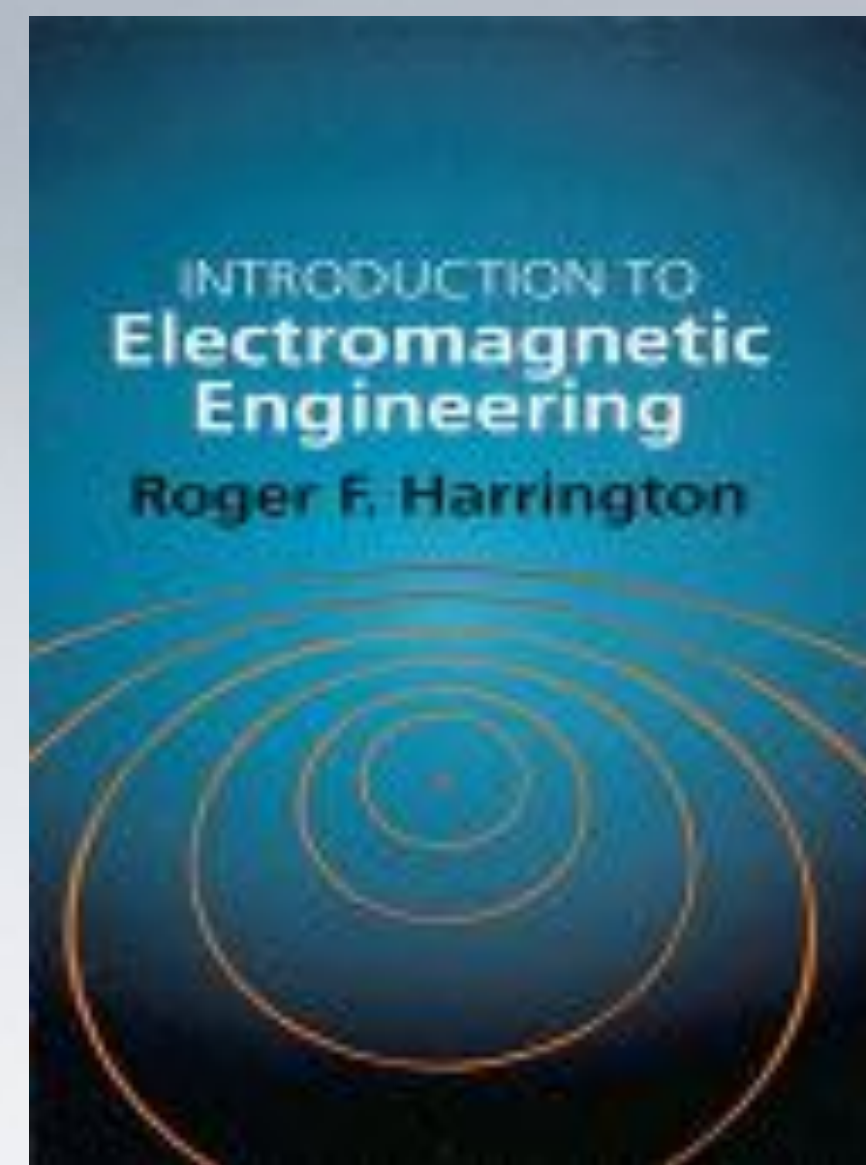
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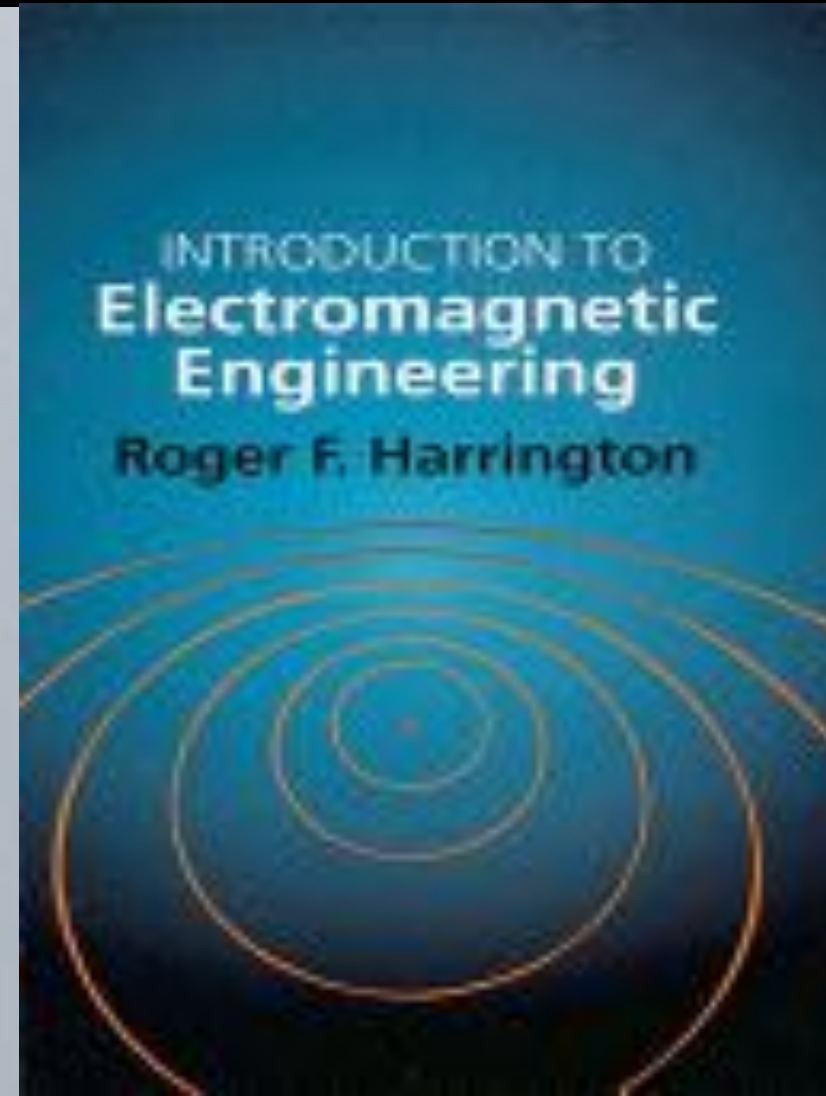


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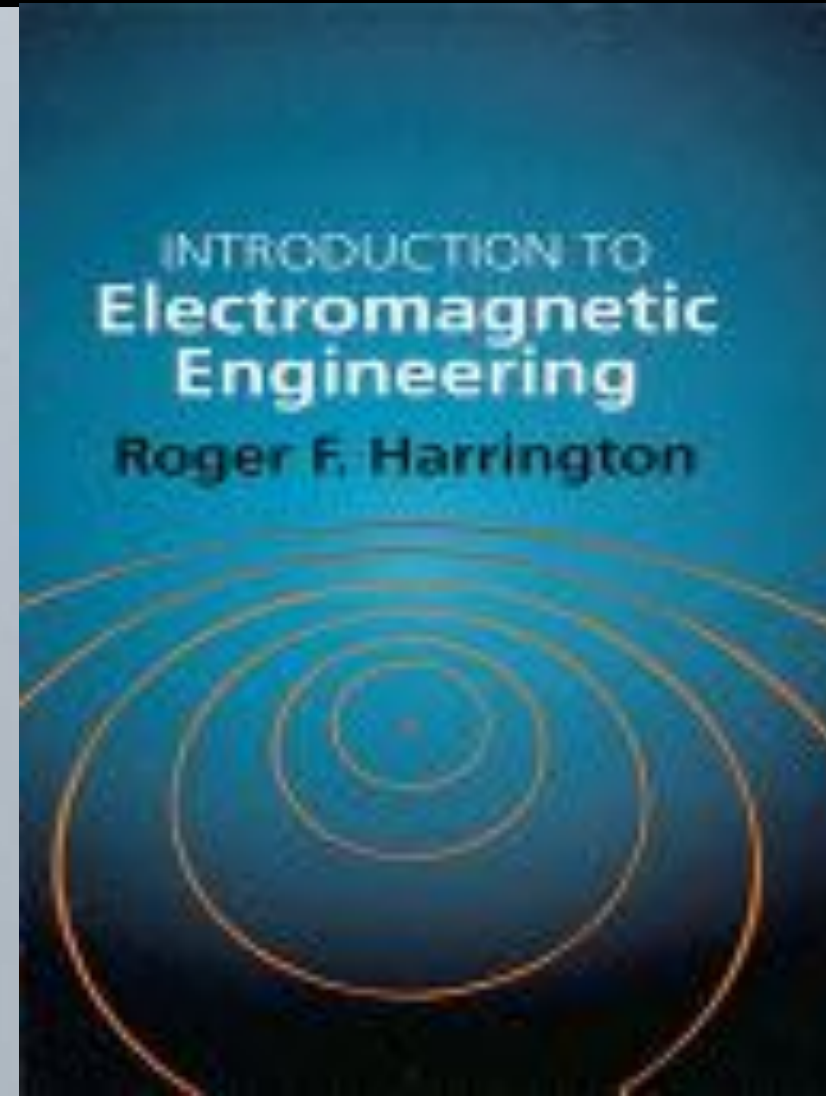


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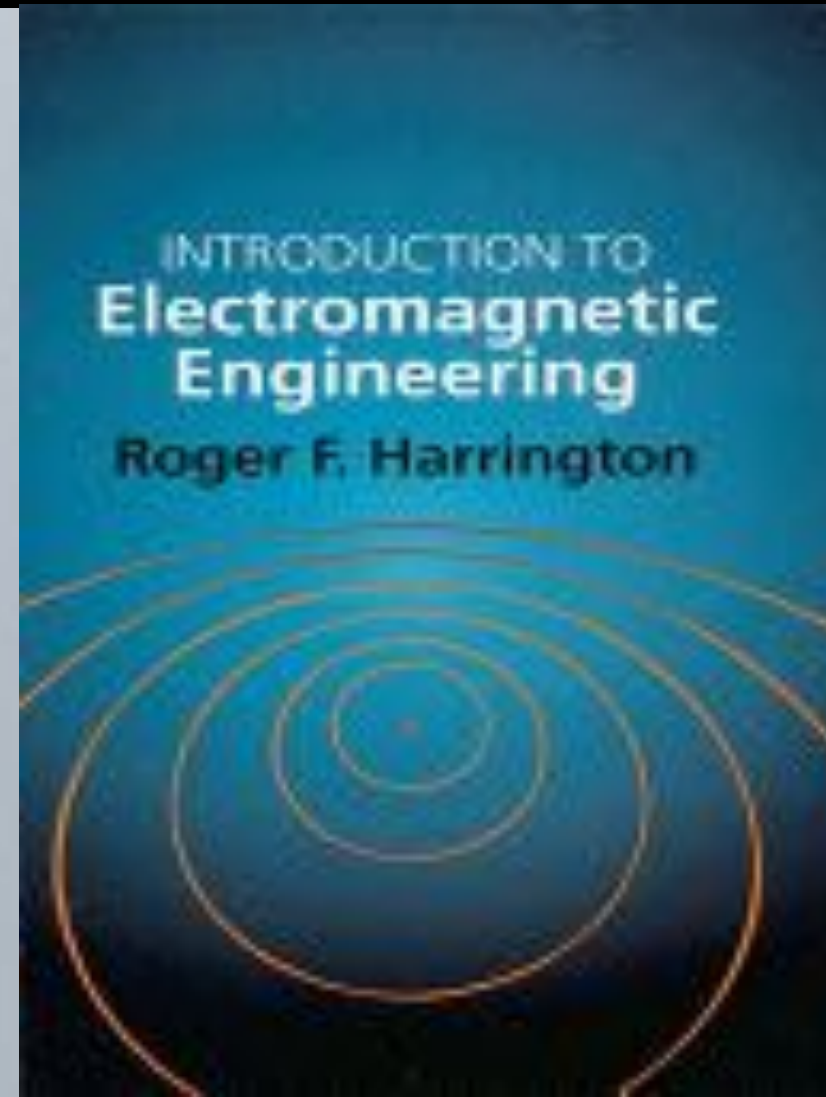


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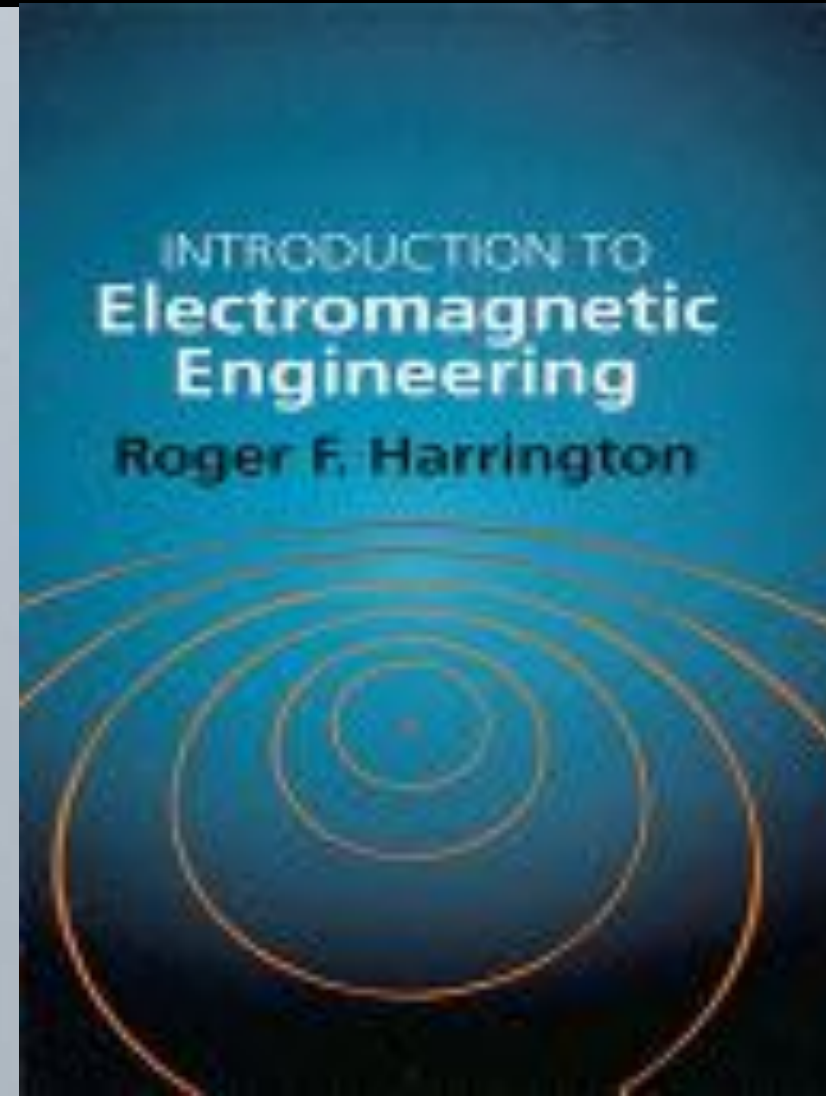
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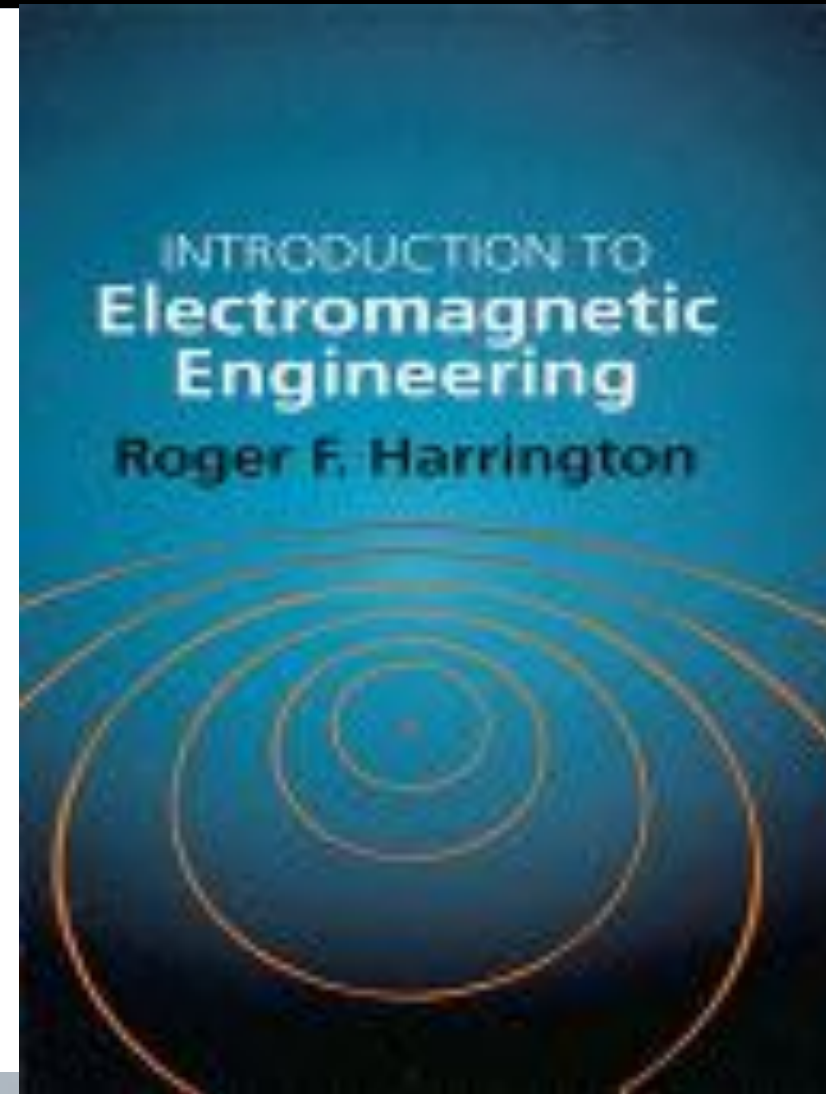
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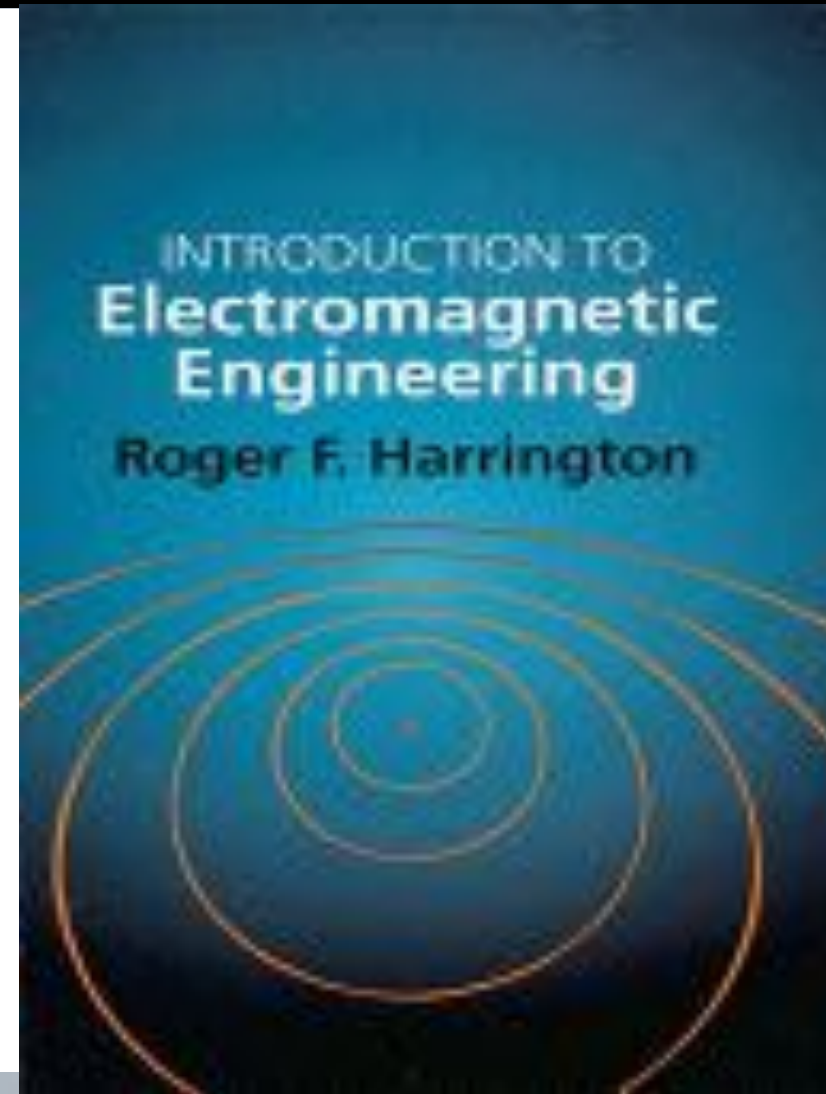
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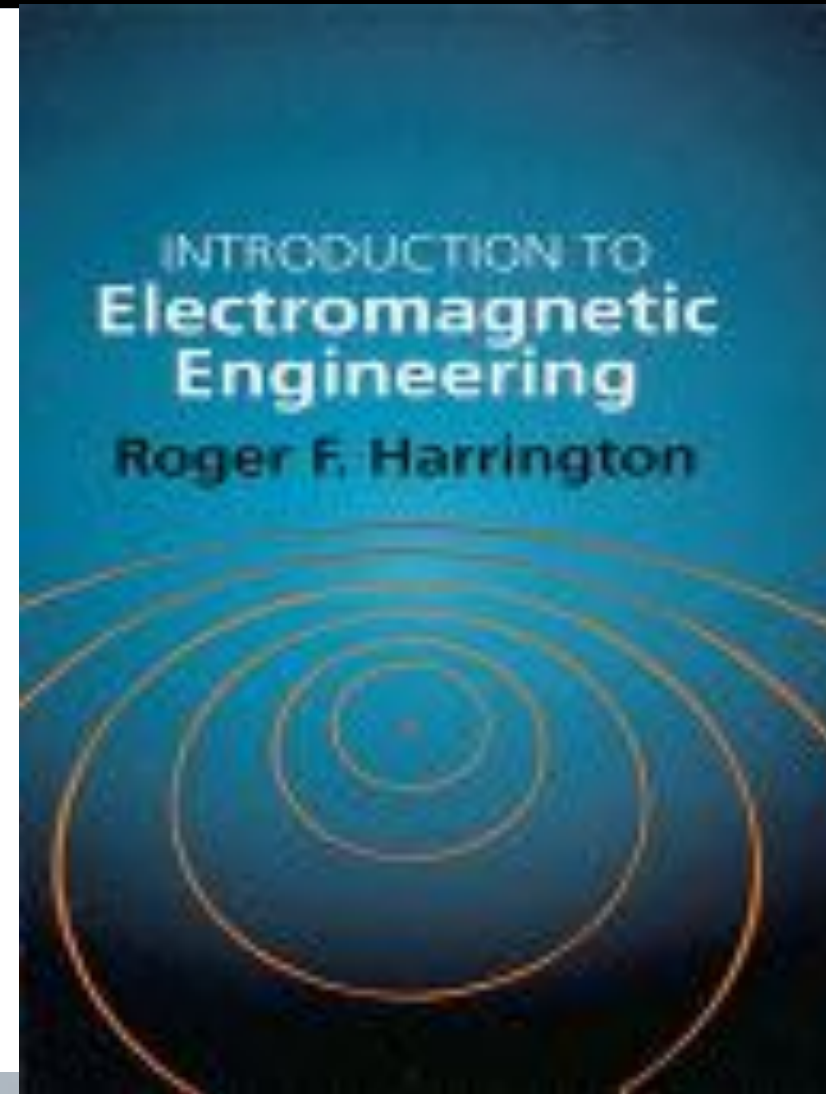
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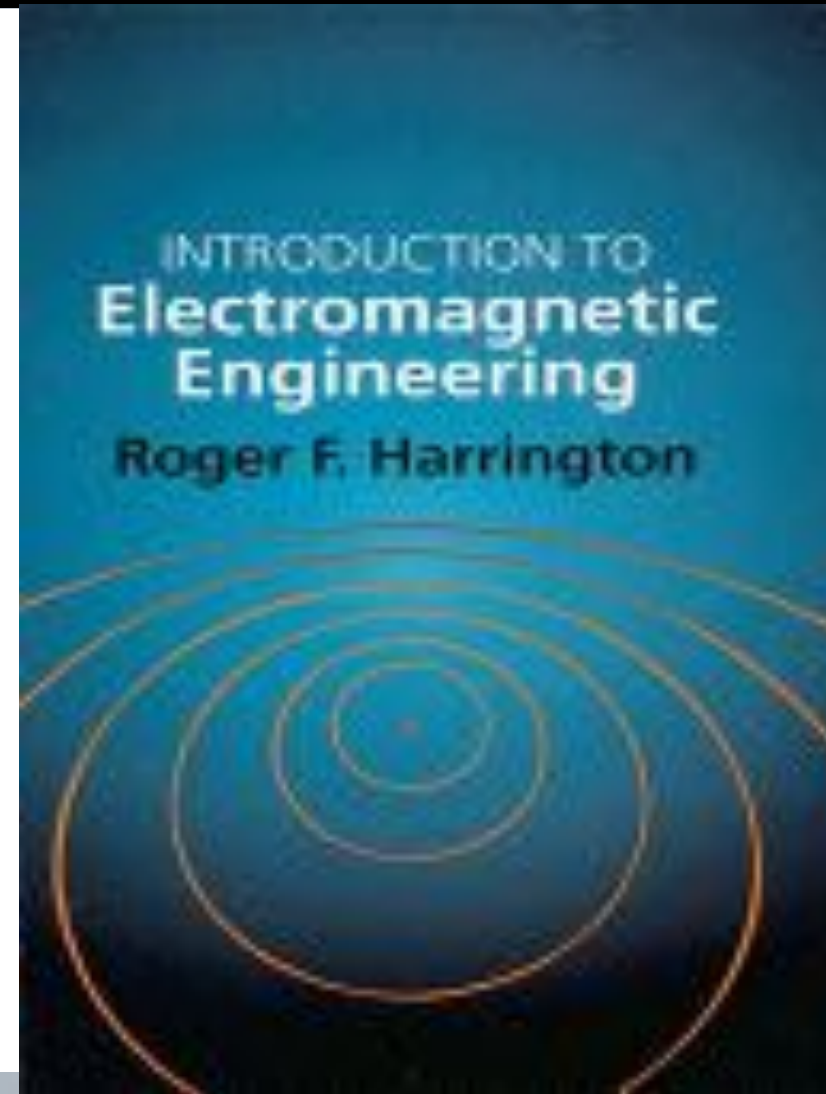
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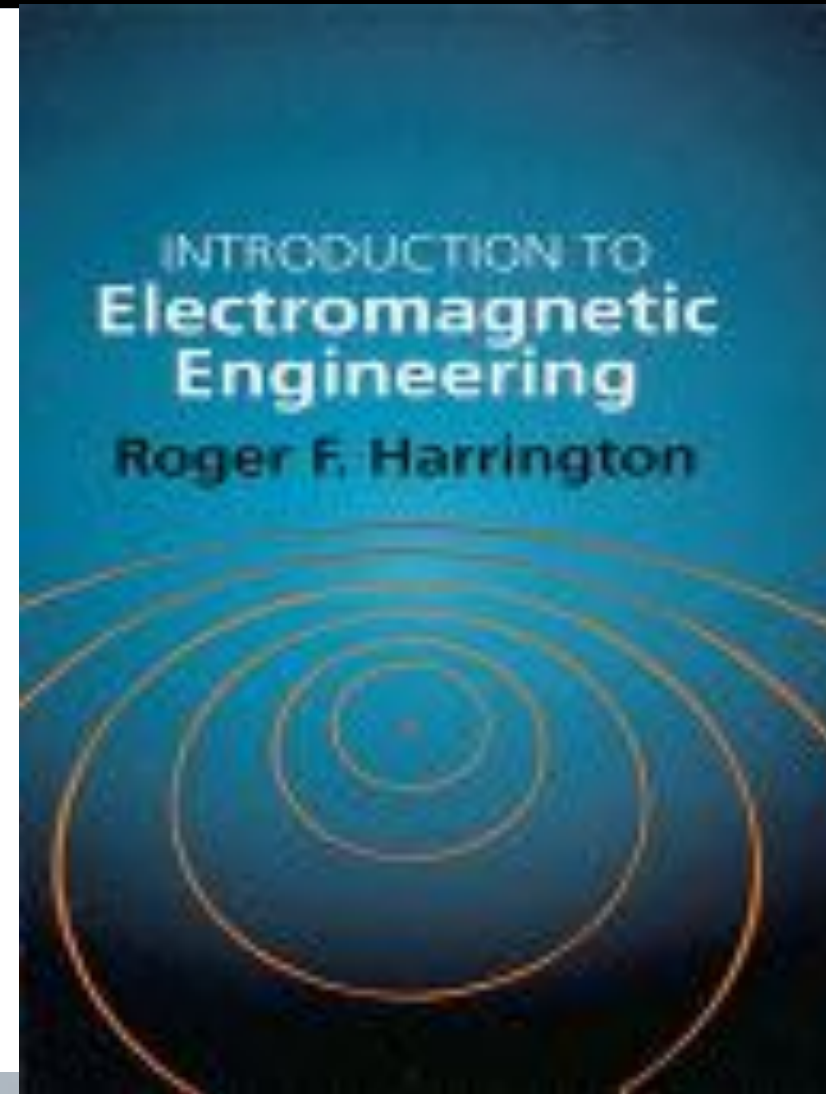
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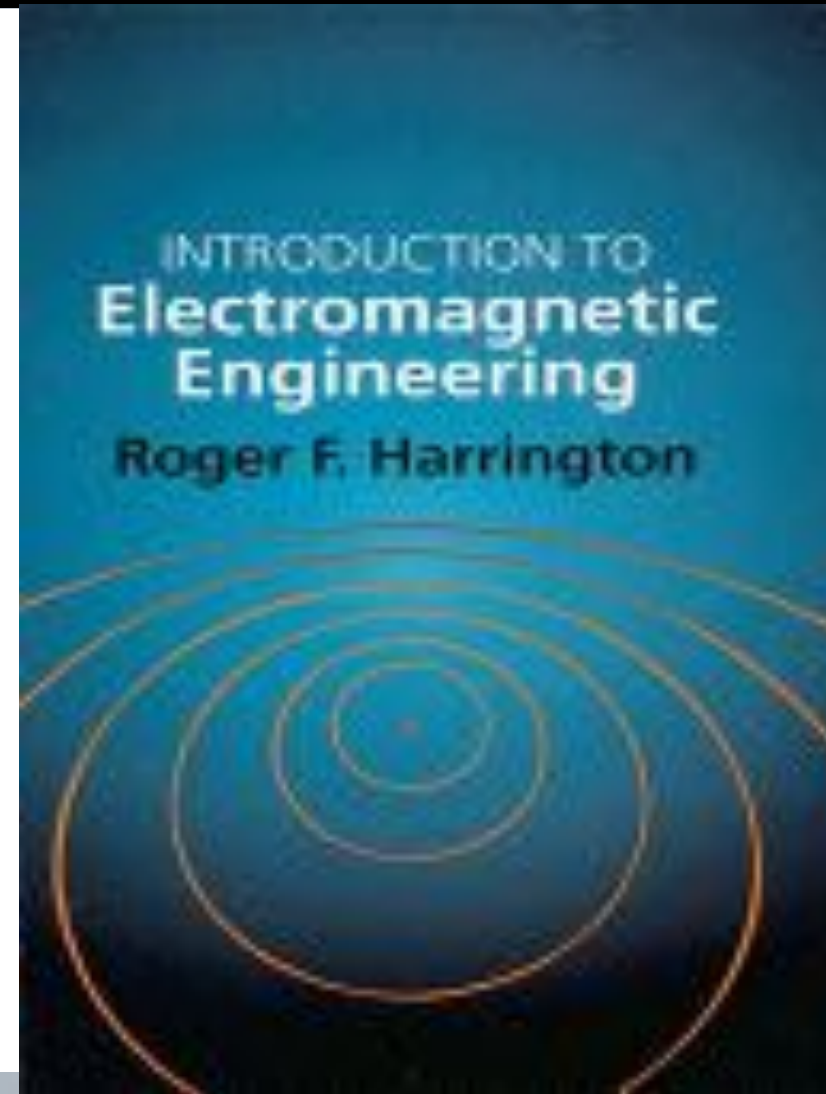
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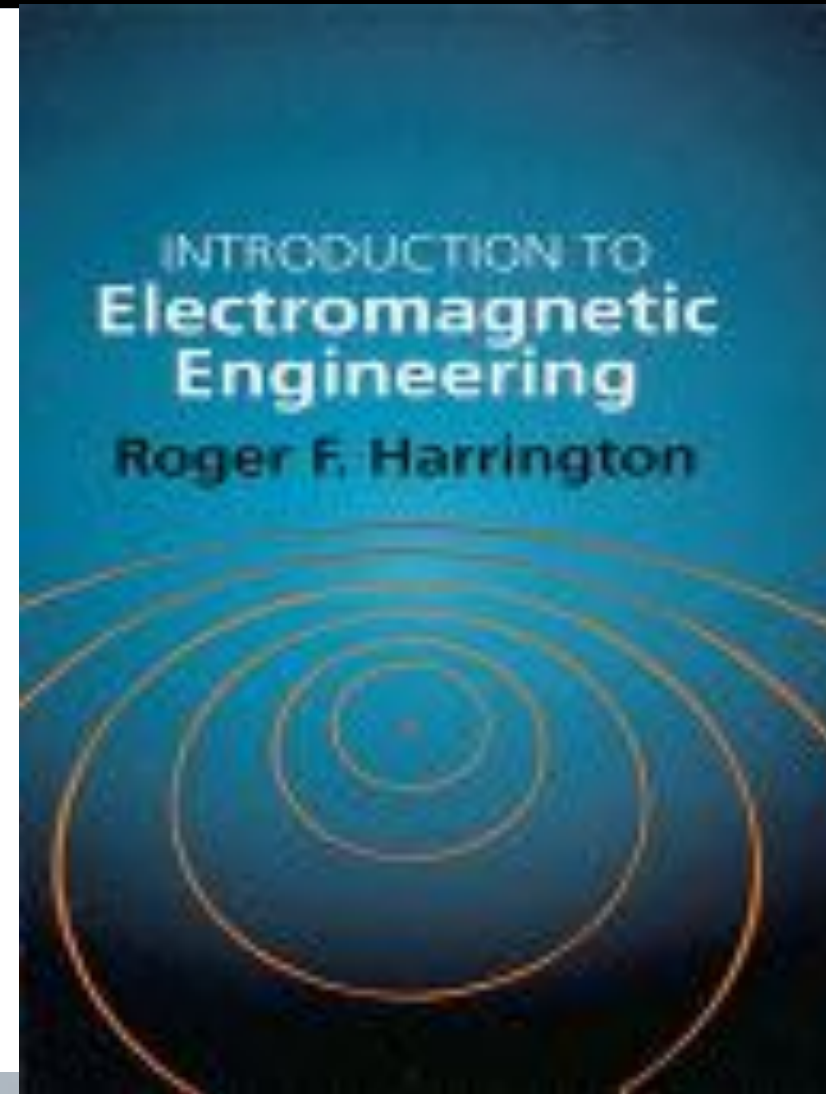
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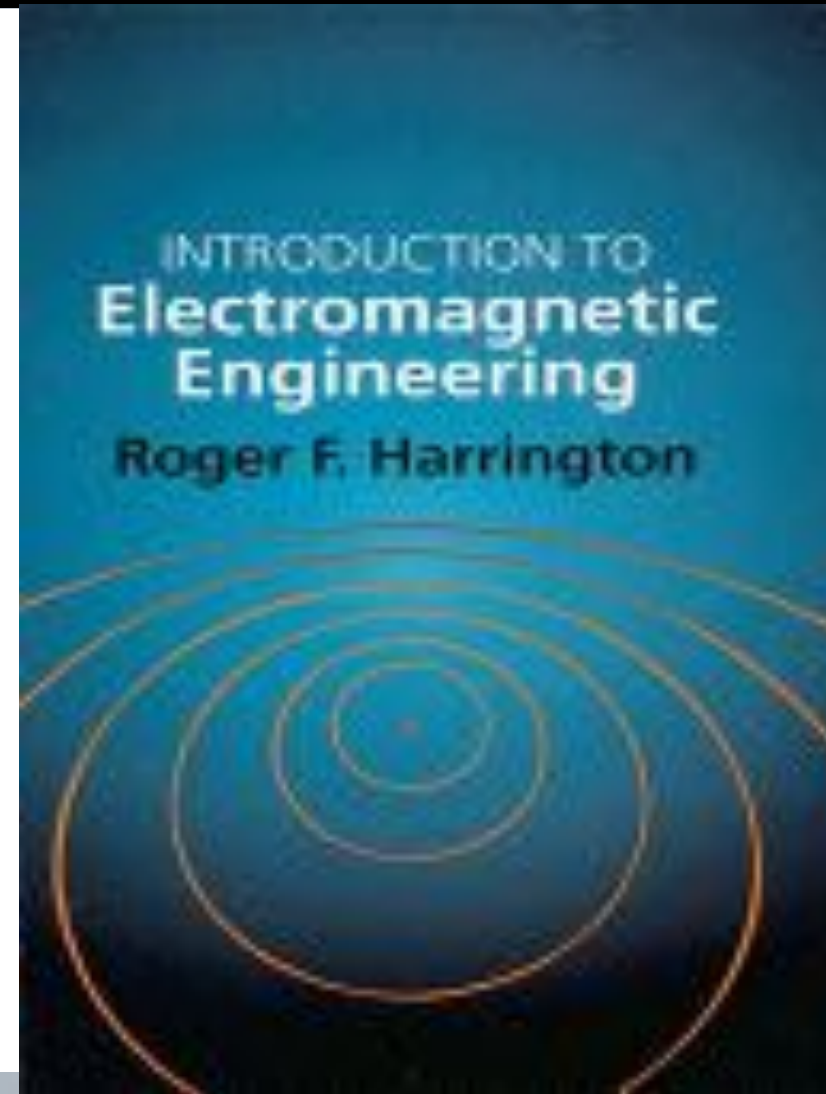
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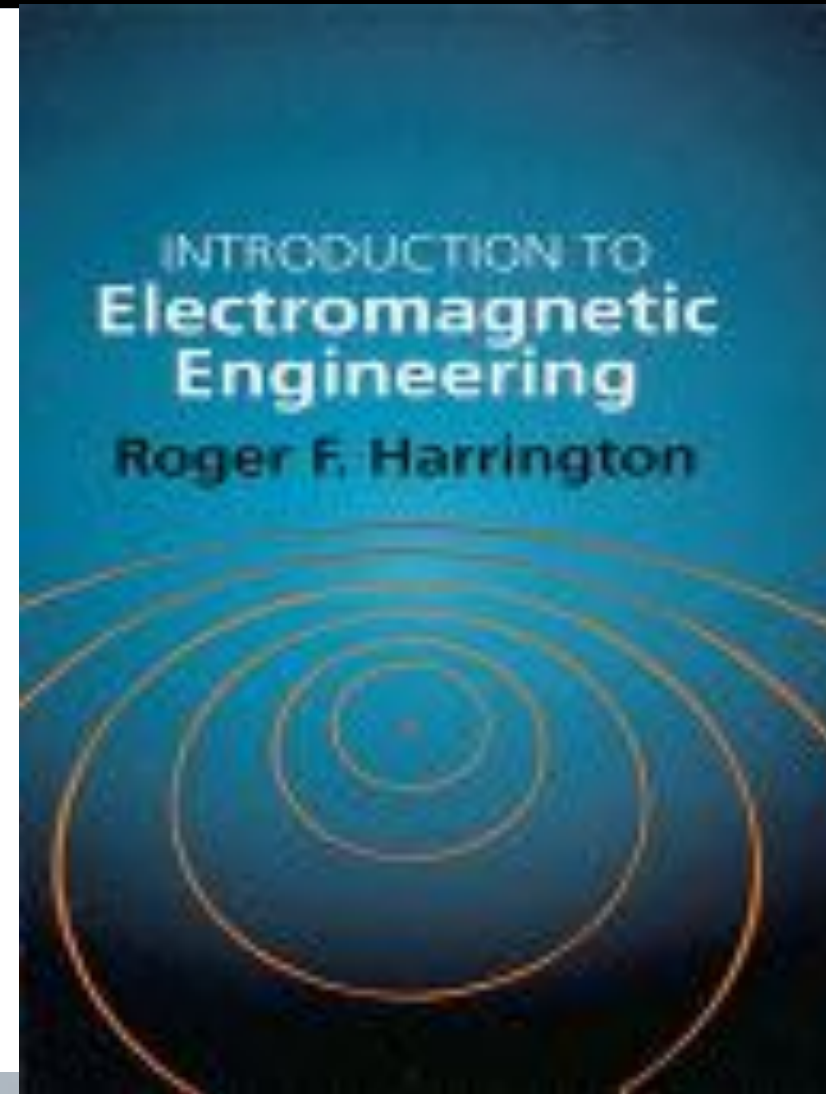
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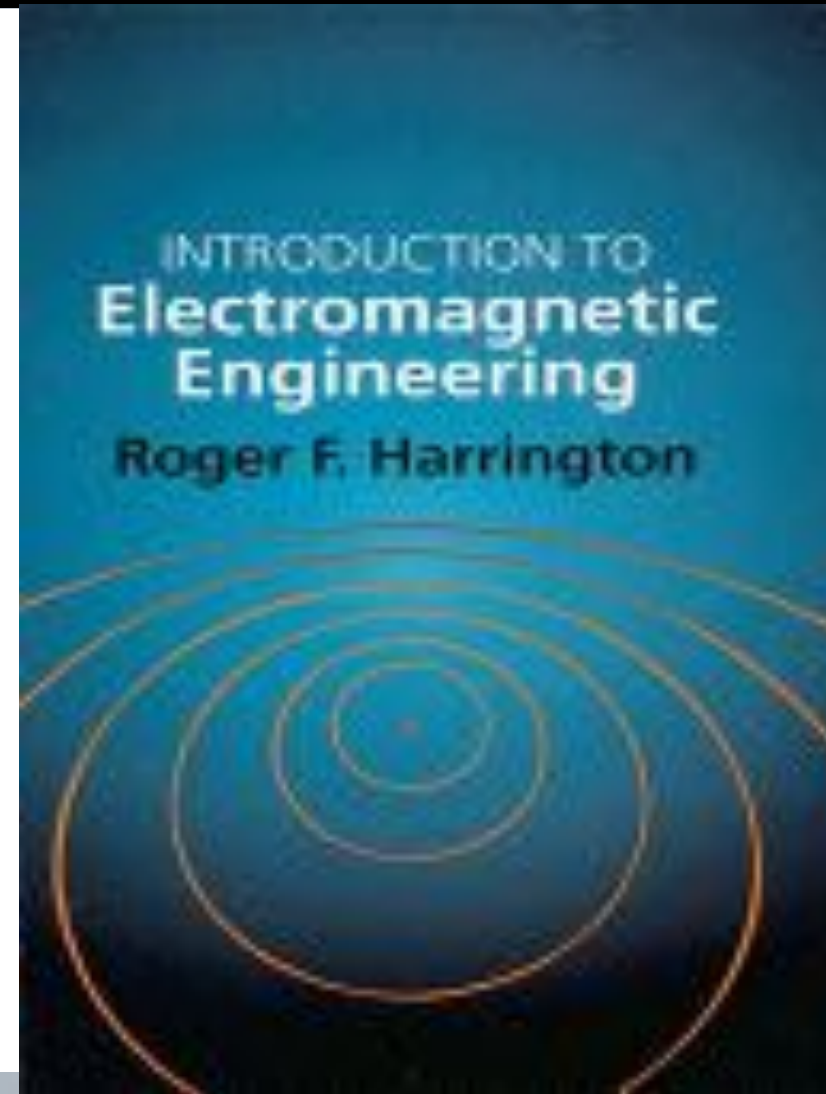
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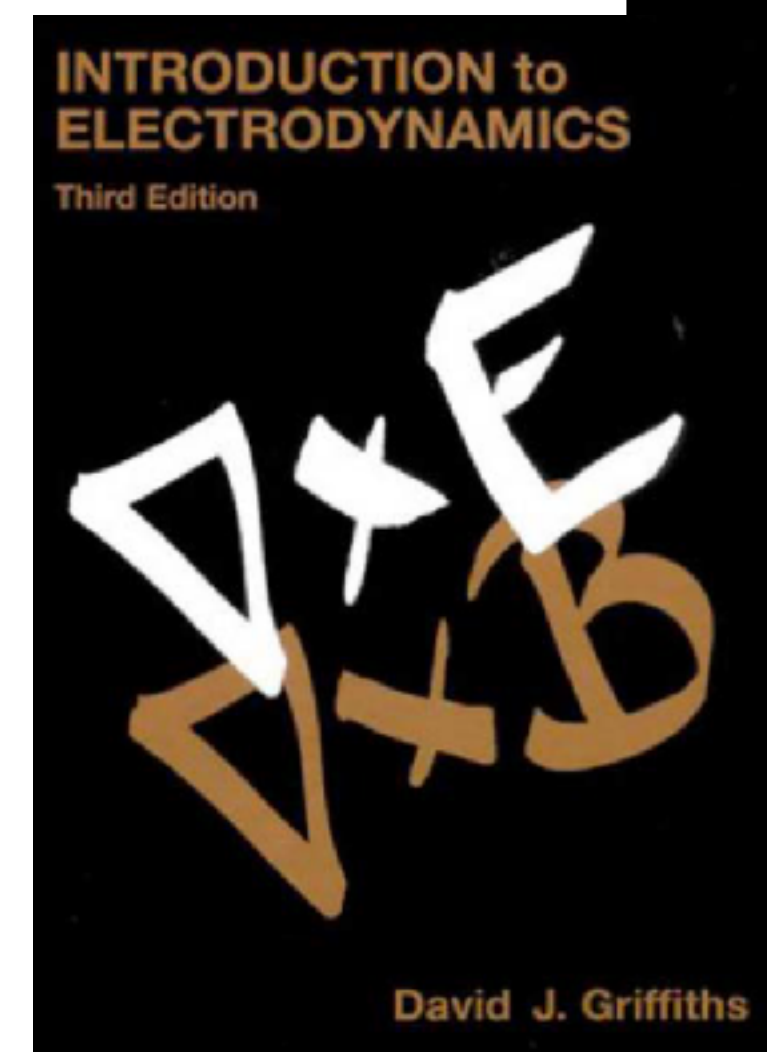
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IMPRESSED VOLTAGE SOURCE TERMS



Chapter 7

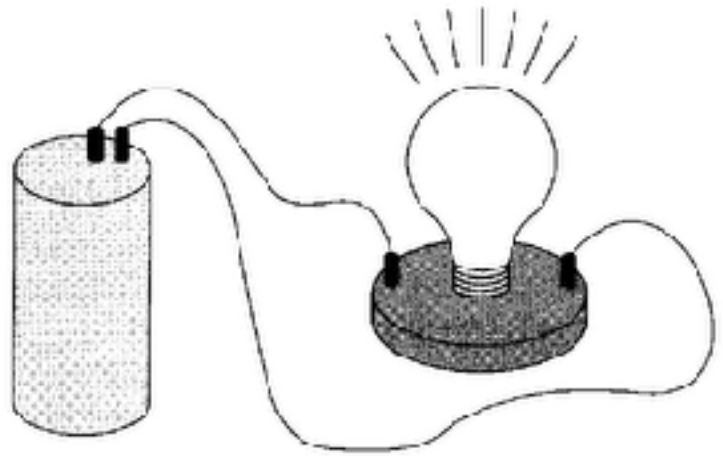
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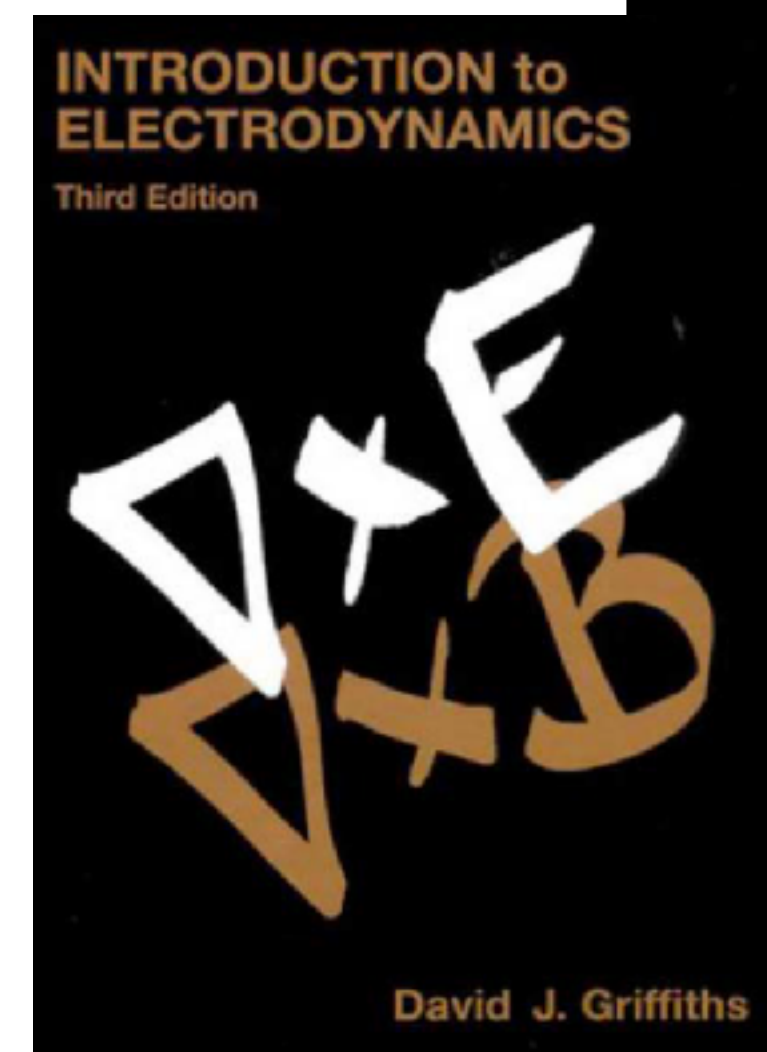
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Chapter 7

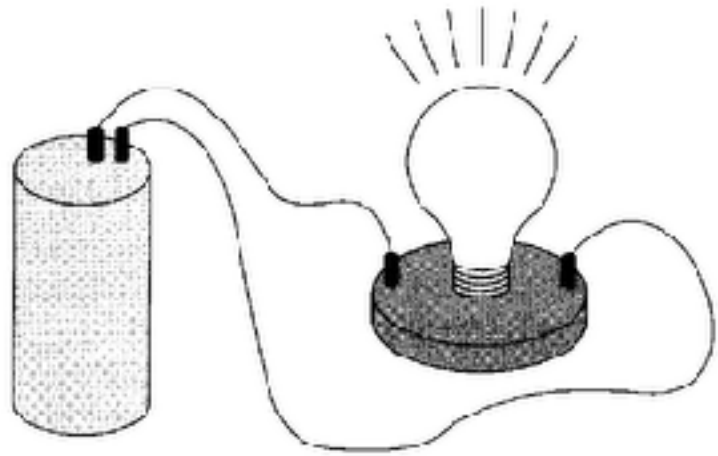
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IMPRESSED VOLTAGE SOURCE TERMS

Chapter 7

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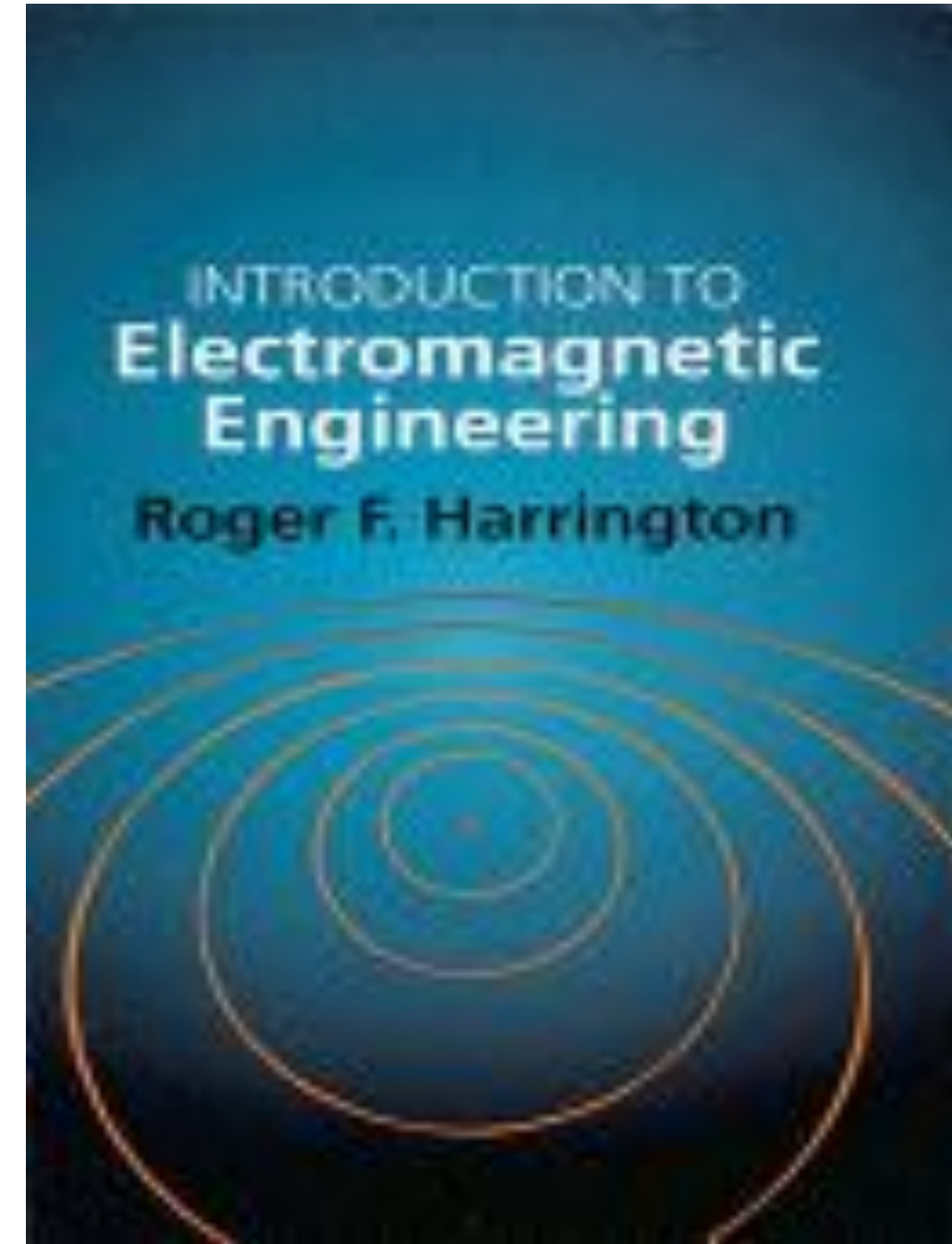
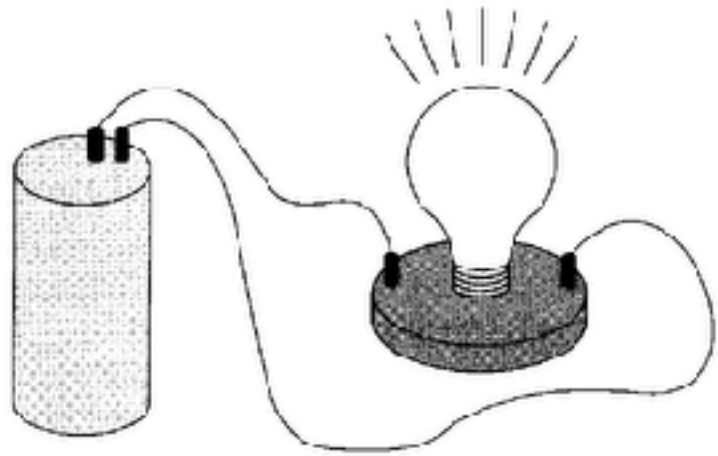
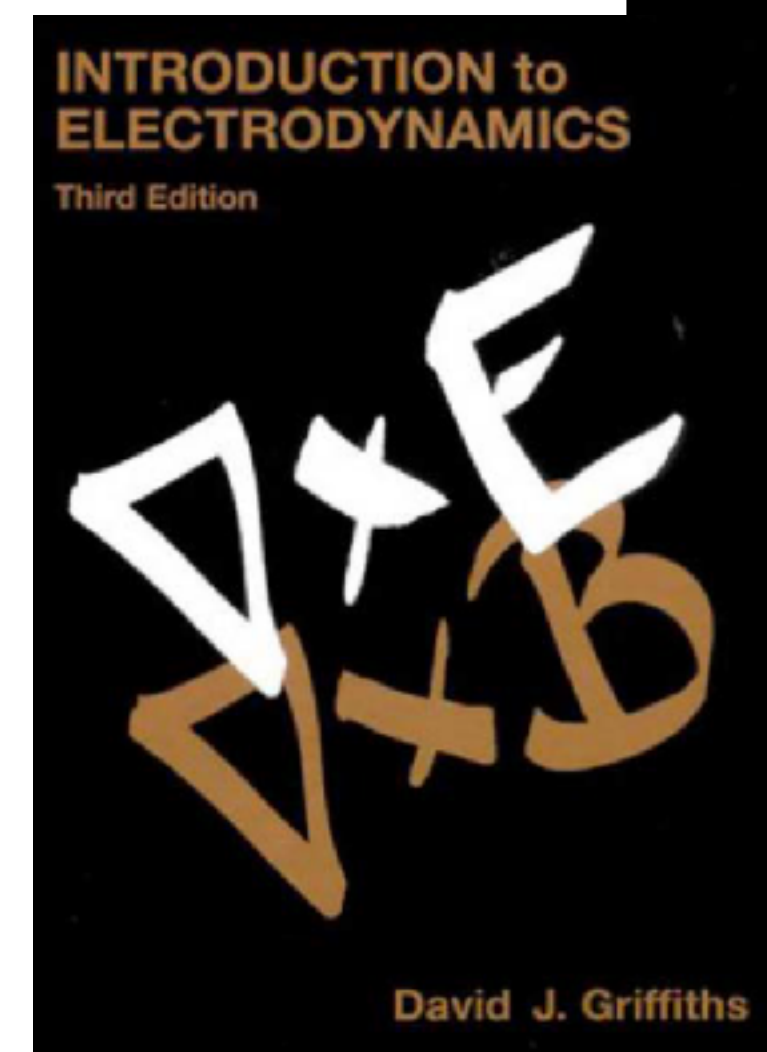
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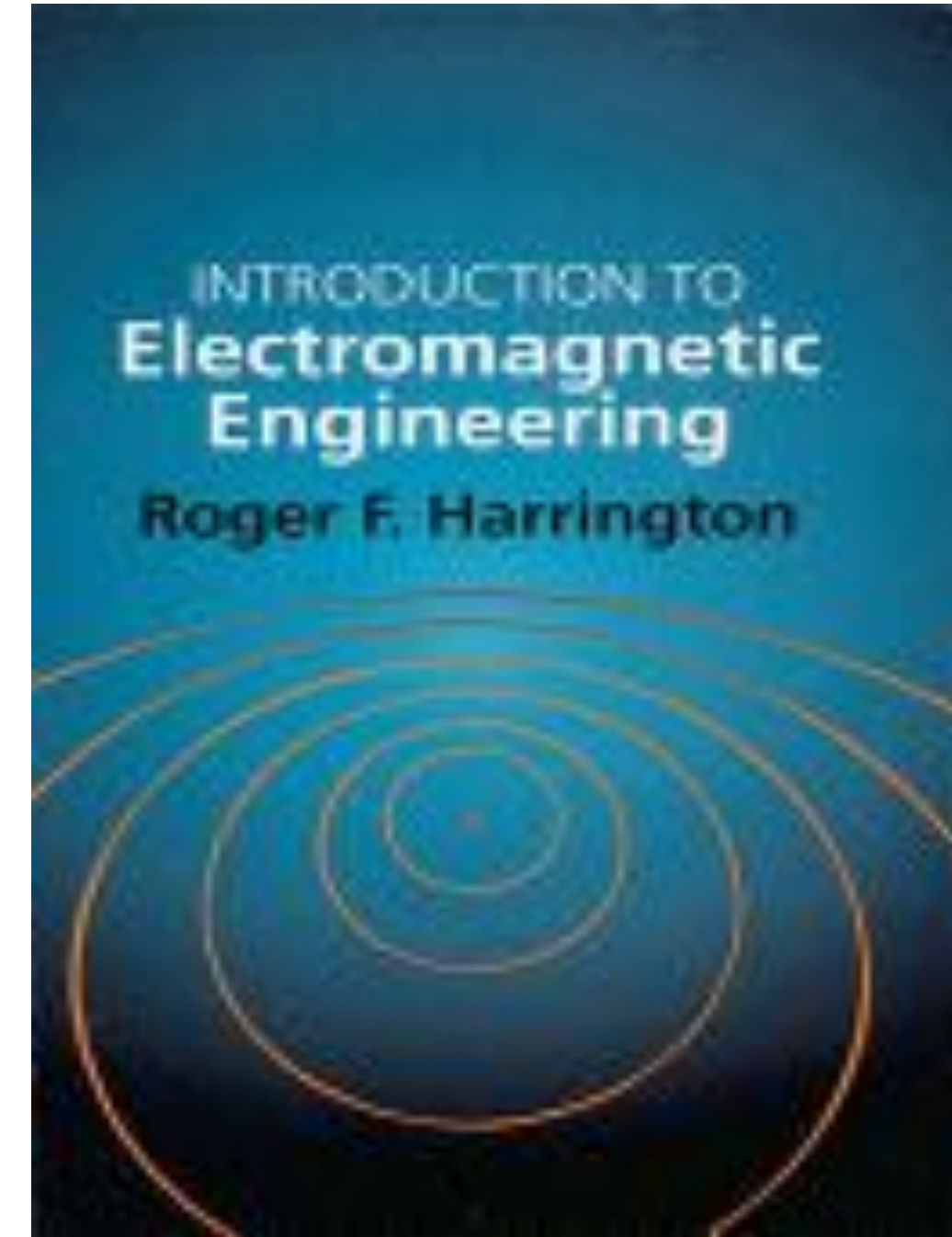
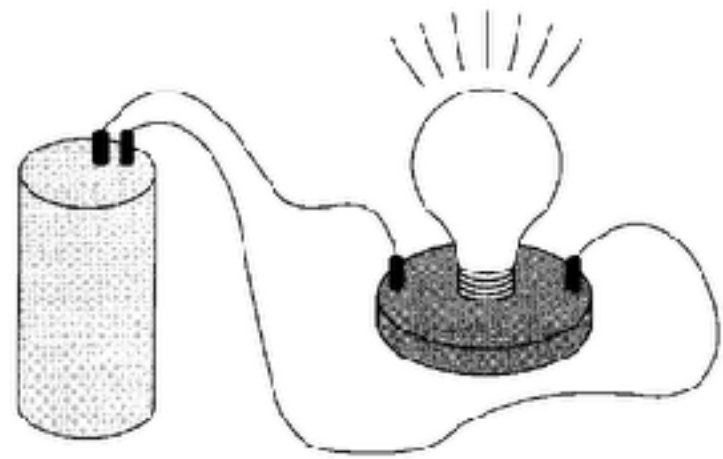
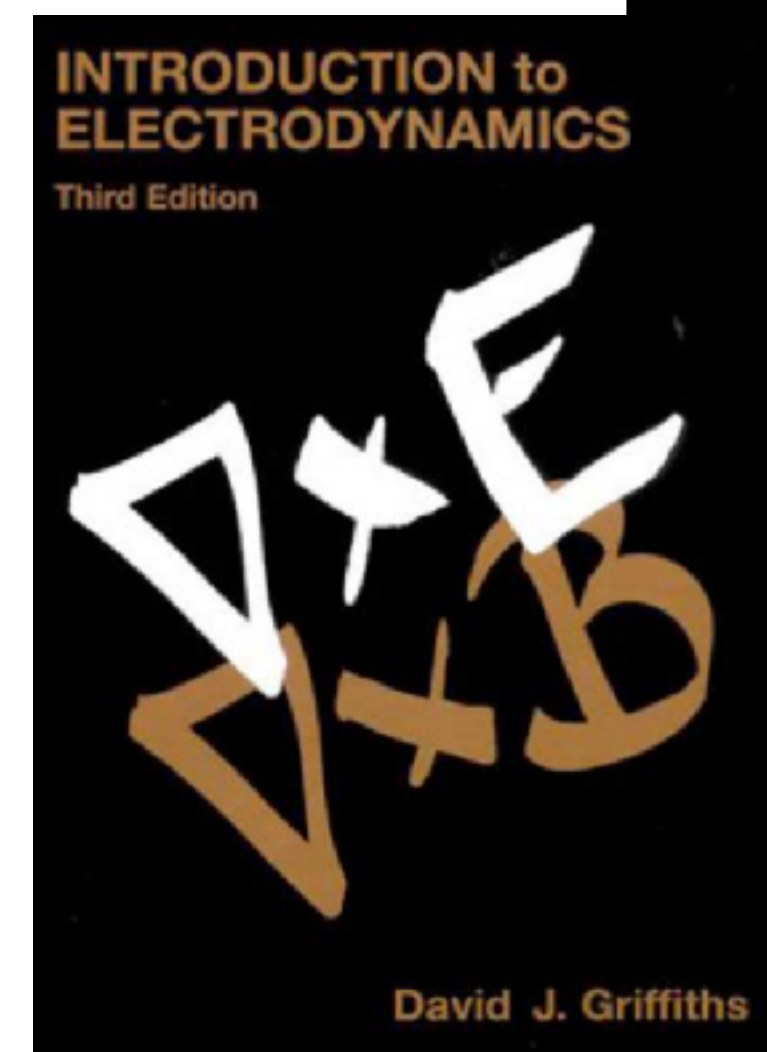
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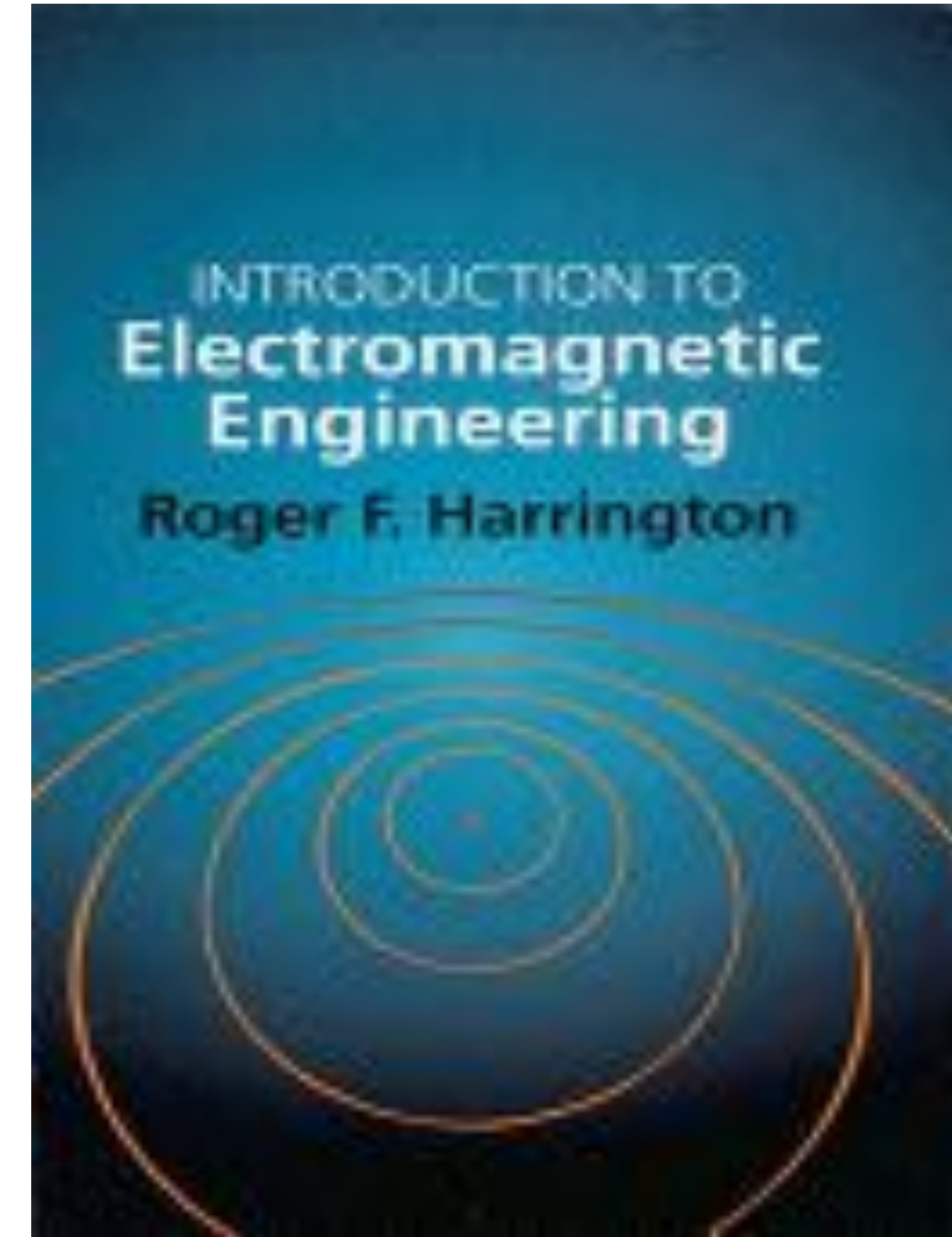
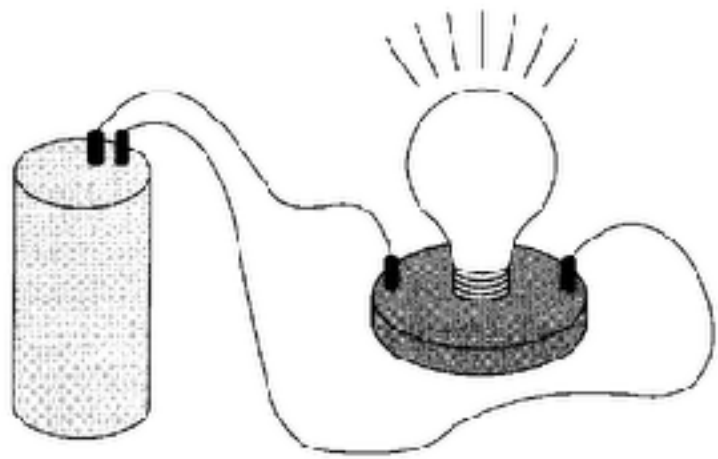
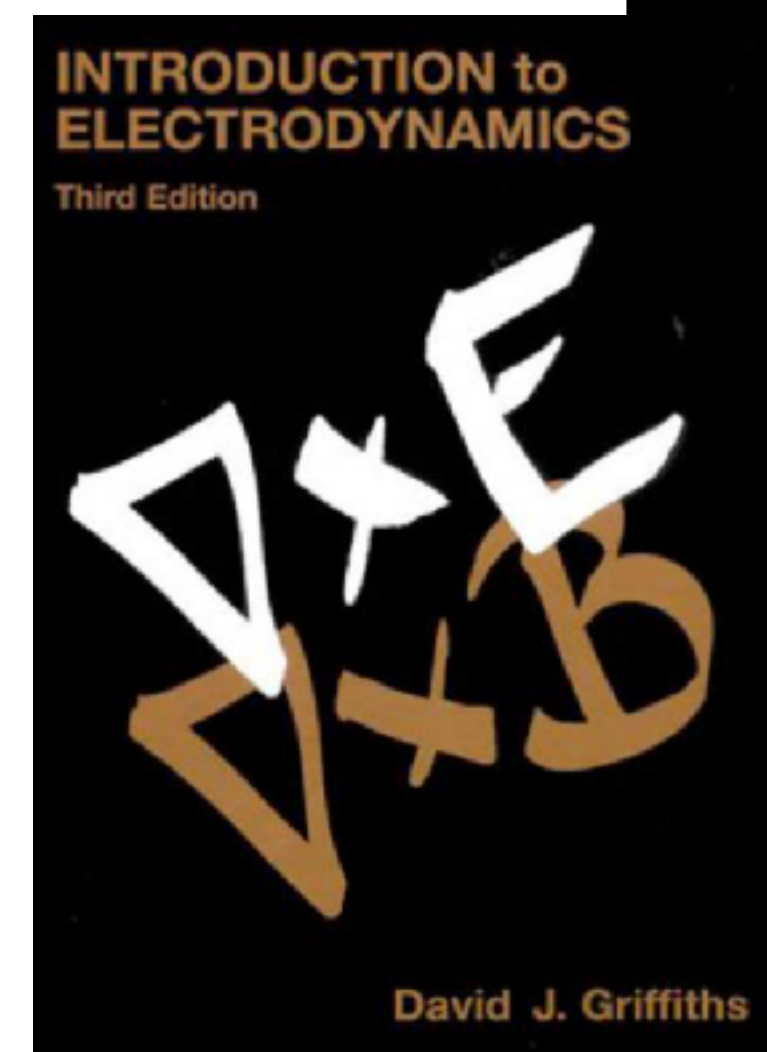
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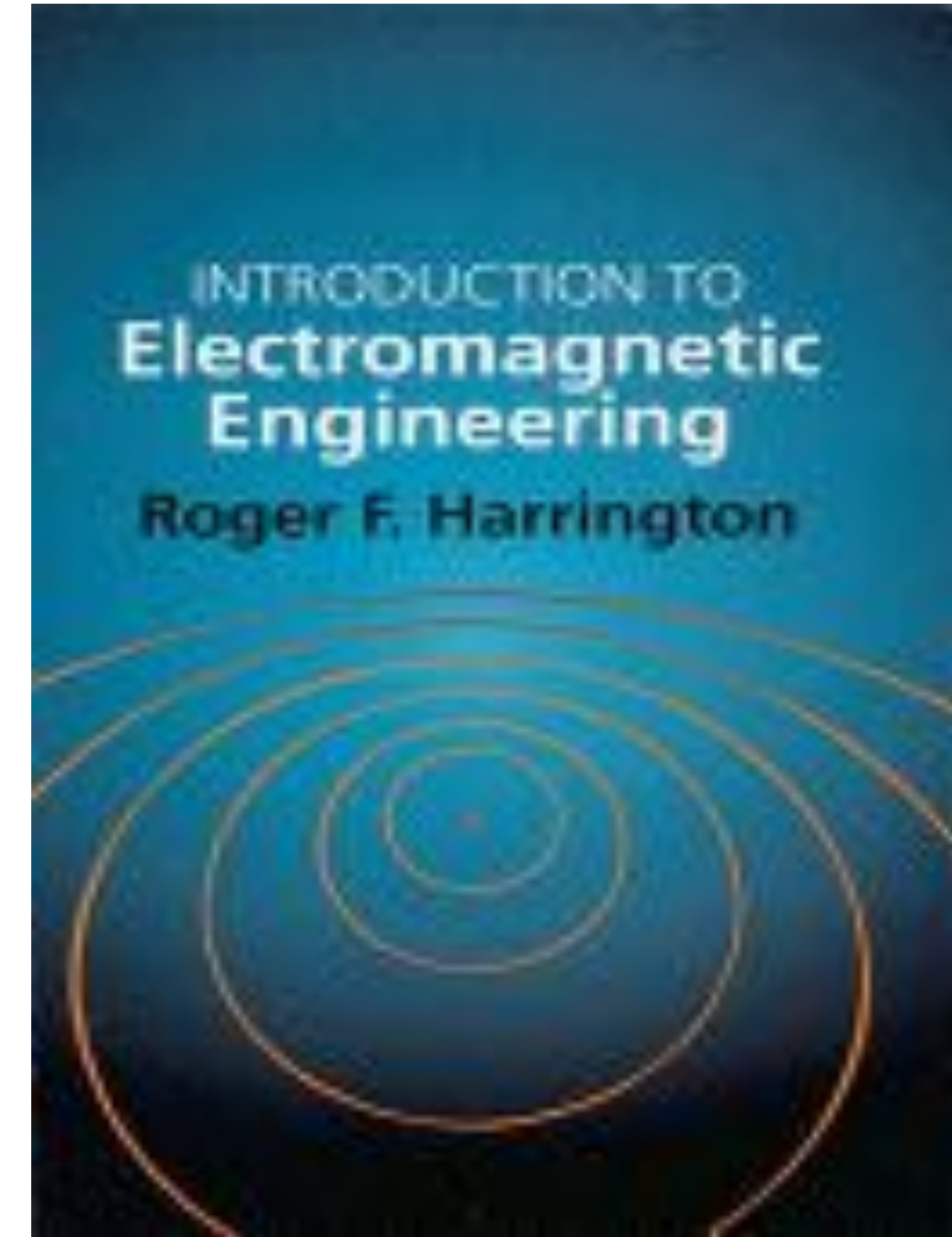
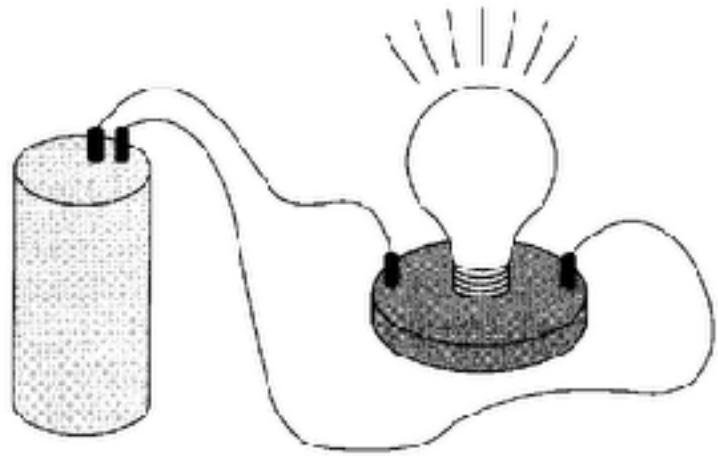
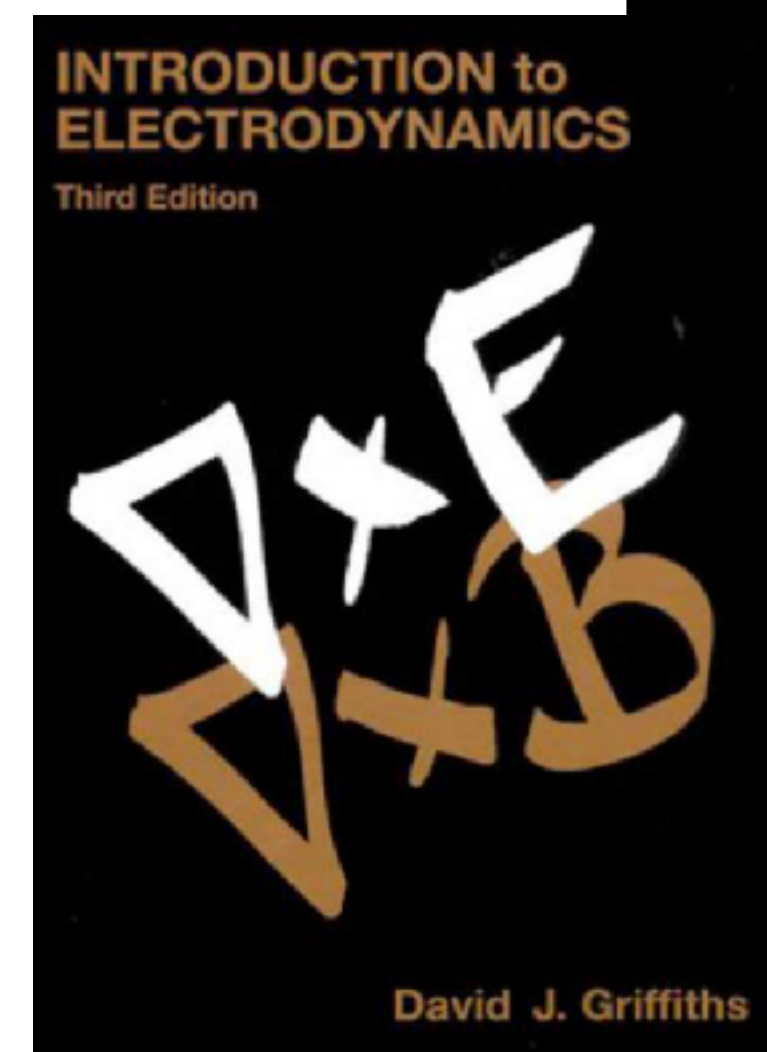
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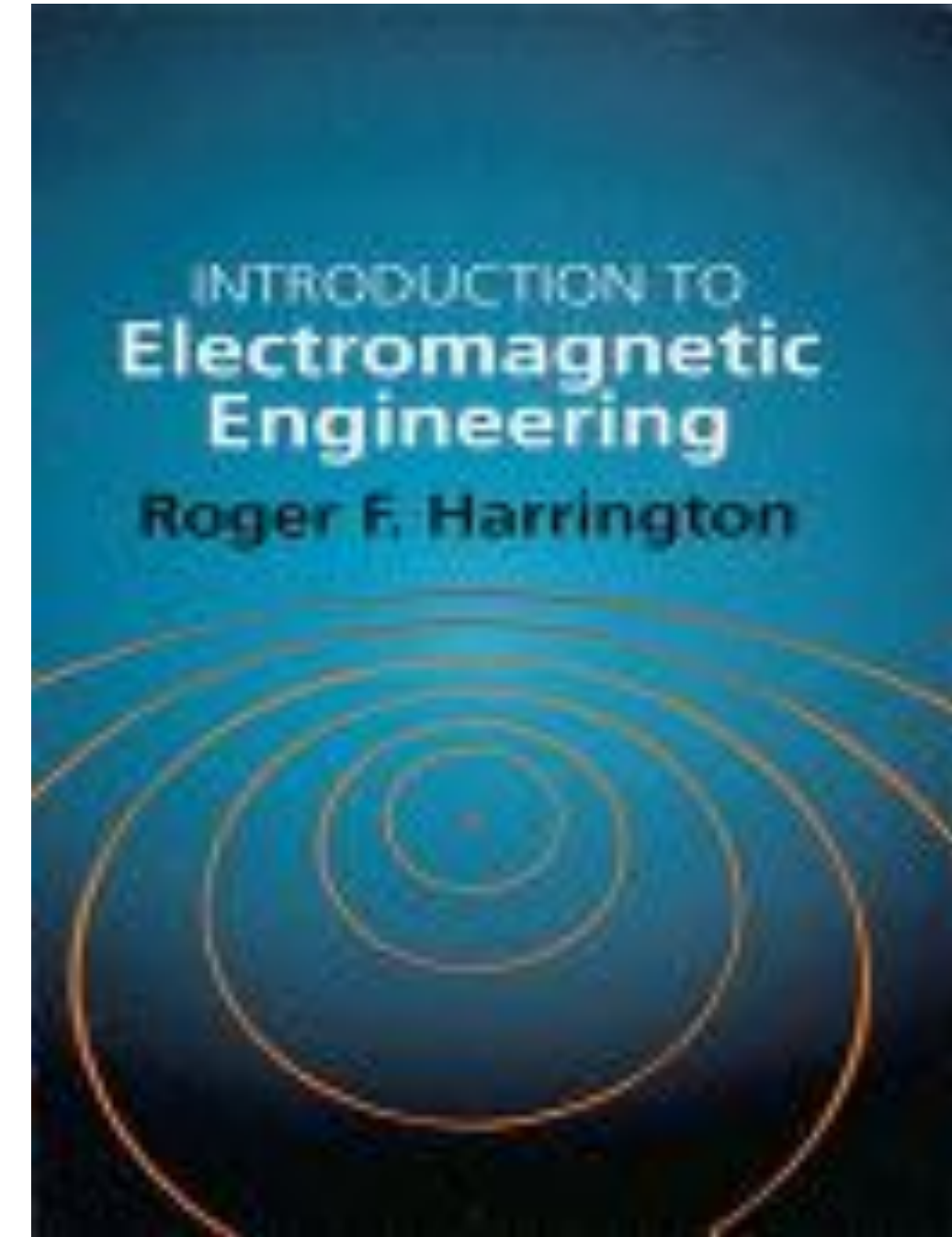
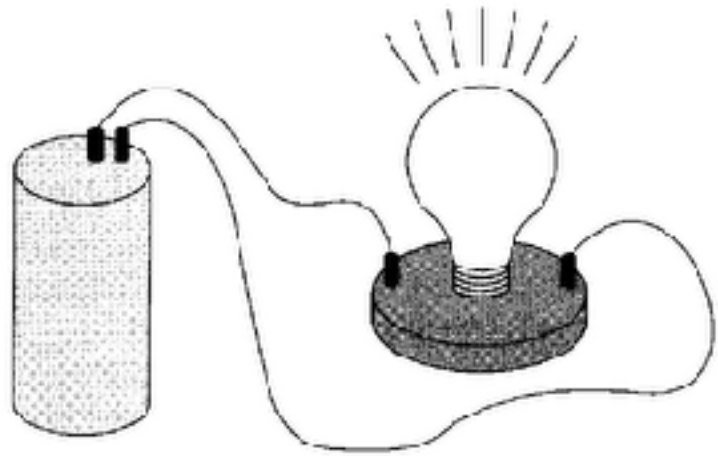
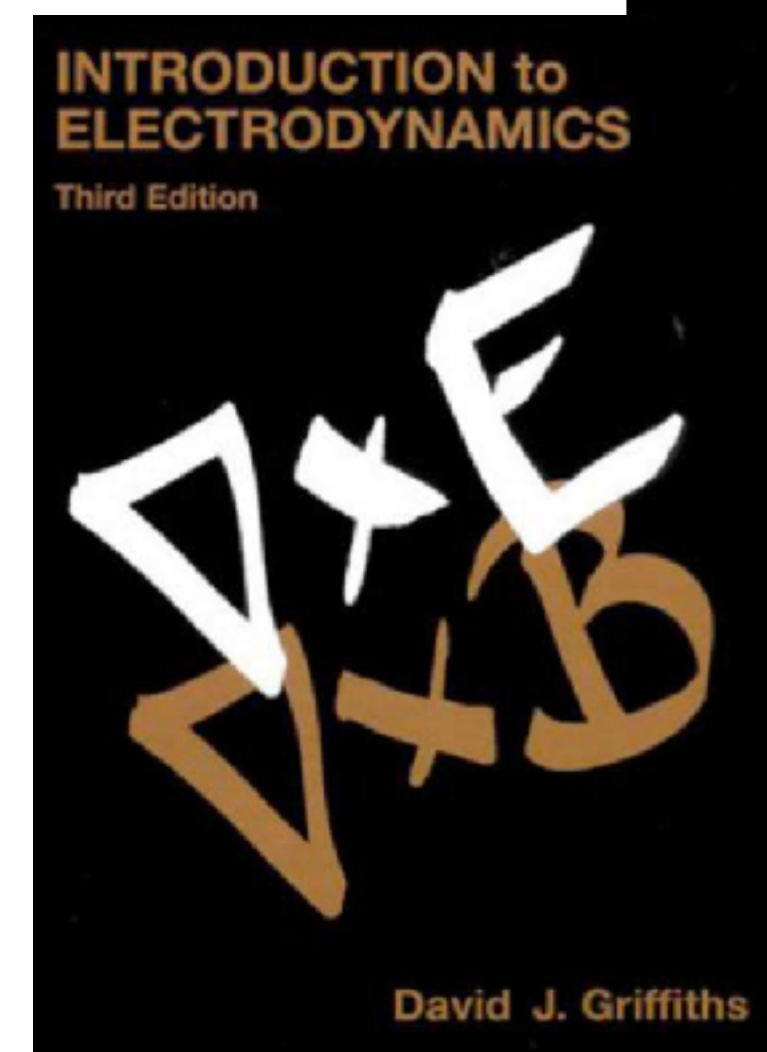
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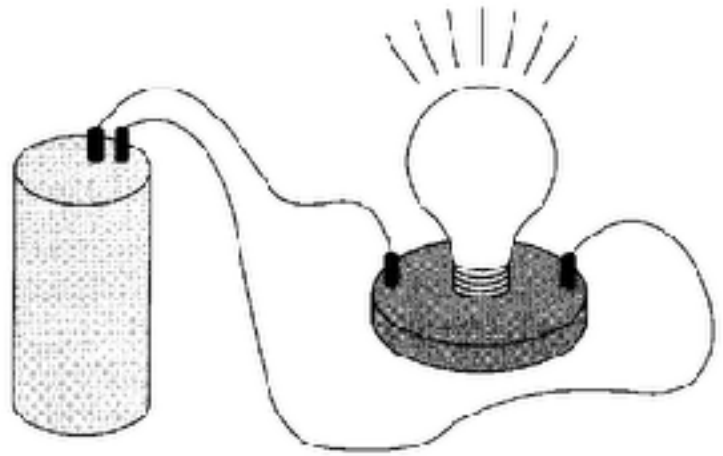
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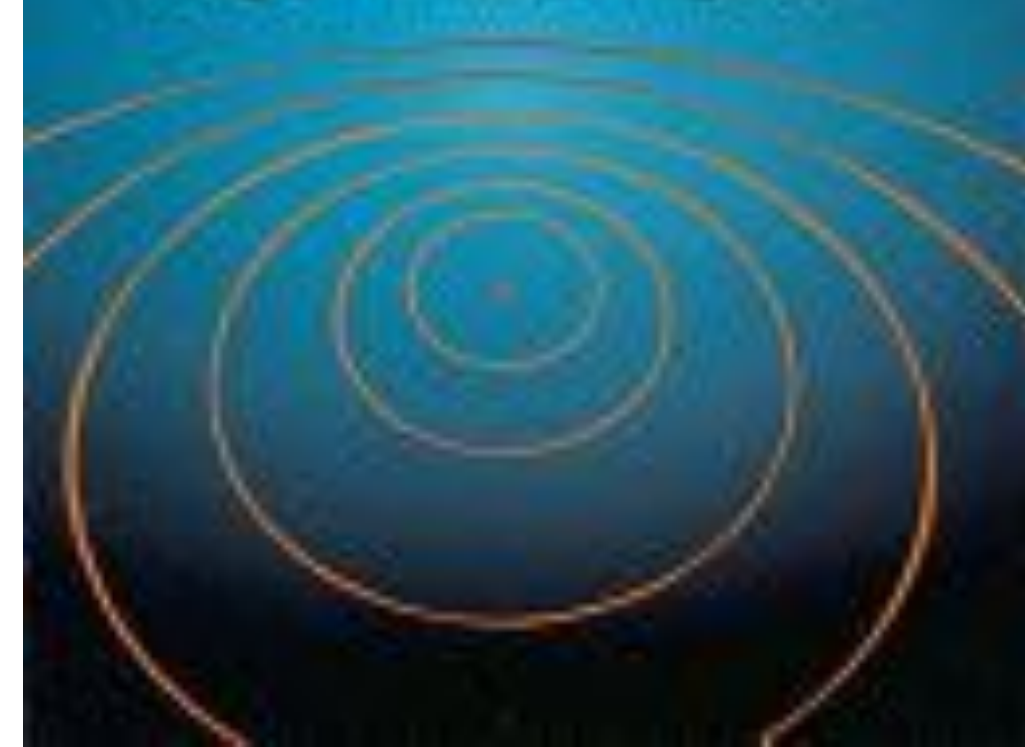


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$$\vec{E}_T = \vec{E}_f^i + \vec{E}$$

$$\vec{\nabla} \times \vec{E}_T = -\left(\frac{\partial \vec{B}}{\partial t} + \vec{J}_{mf}^i\right)$$

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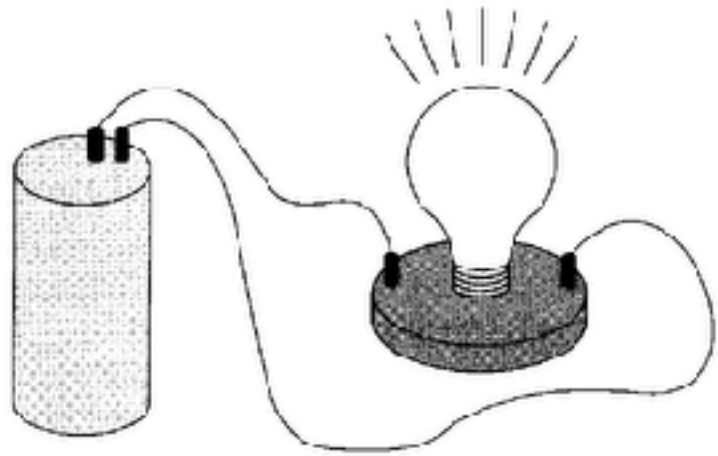
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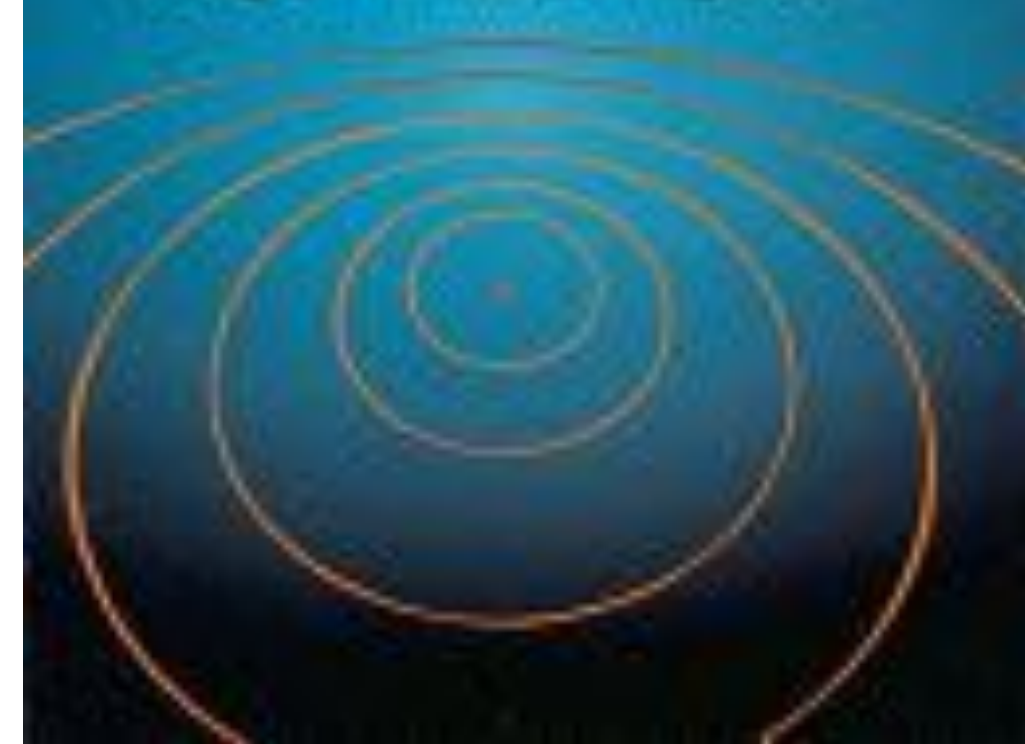
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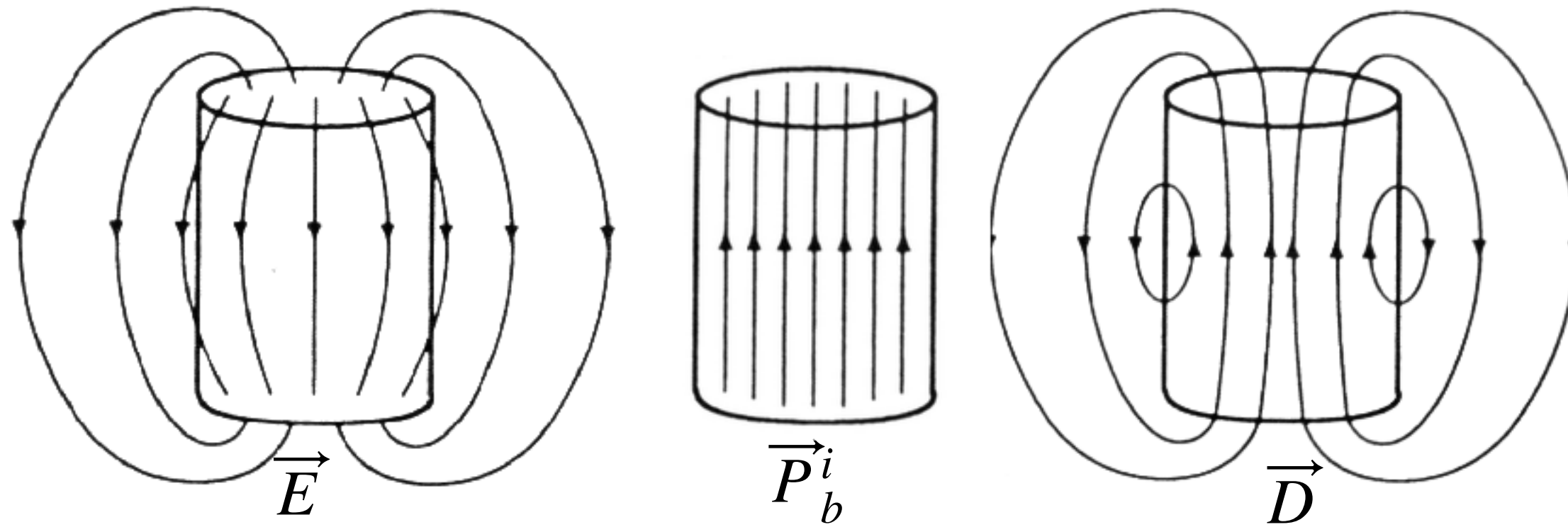
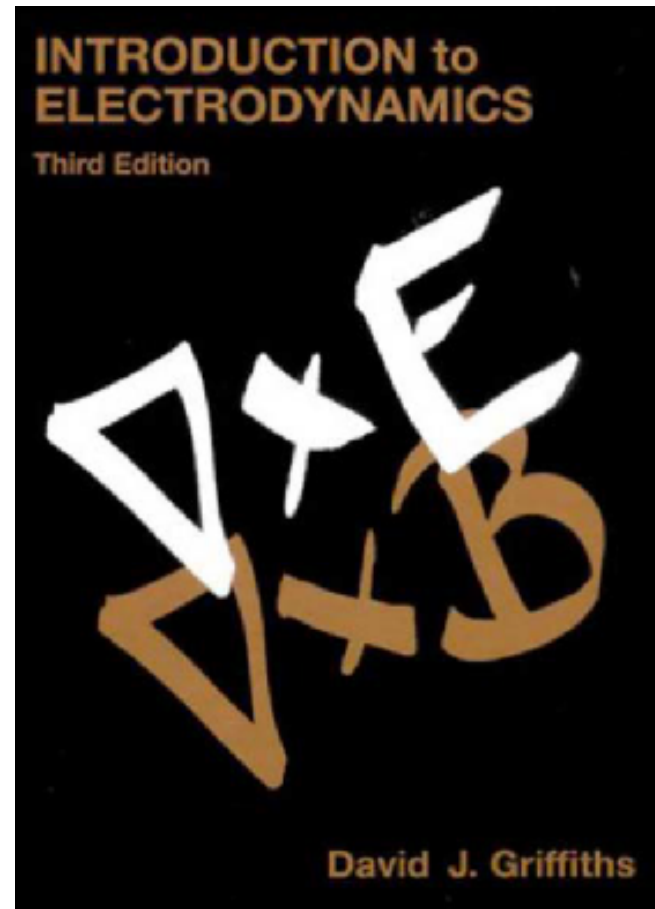
$$\vec{\nabla} \times \vec{E}_T = -\left(\frac{\partial \vec{B}}{\partial t} + \vec{J}_{mf}^i\right)$$

Define an impressed free charge polarization or impressed electric field vector

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TIME VARYING BOUND CHARGE VOLTAGE SOURCE: BAR ELECTRET



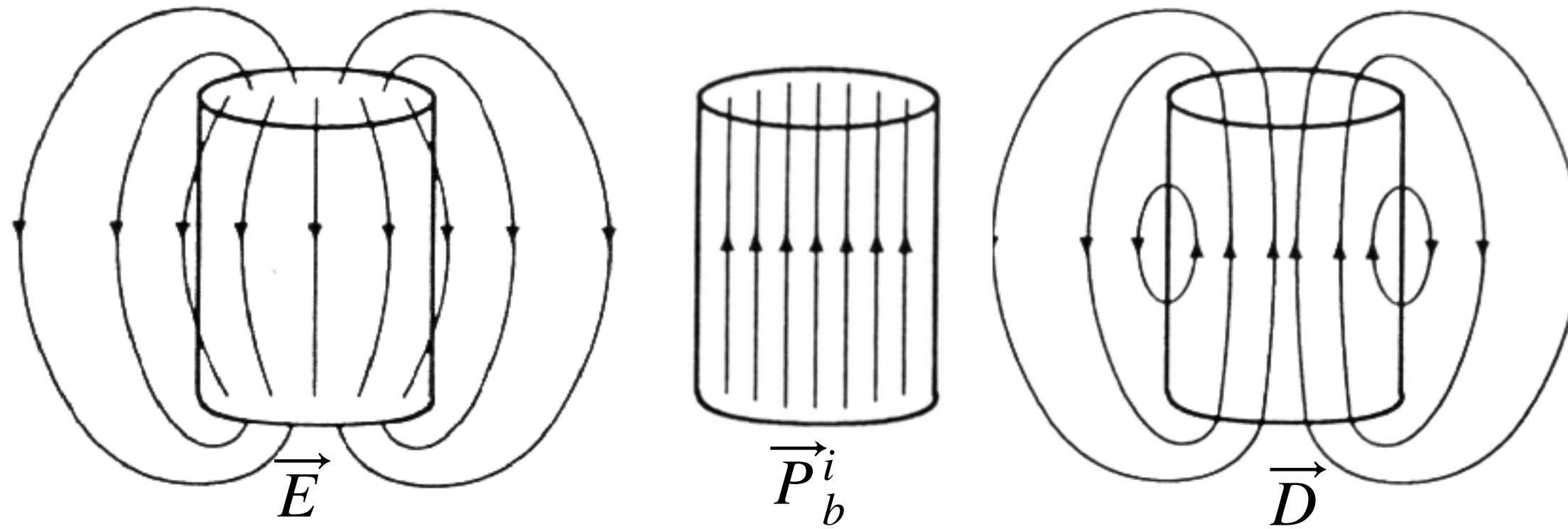
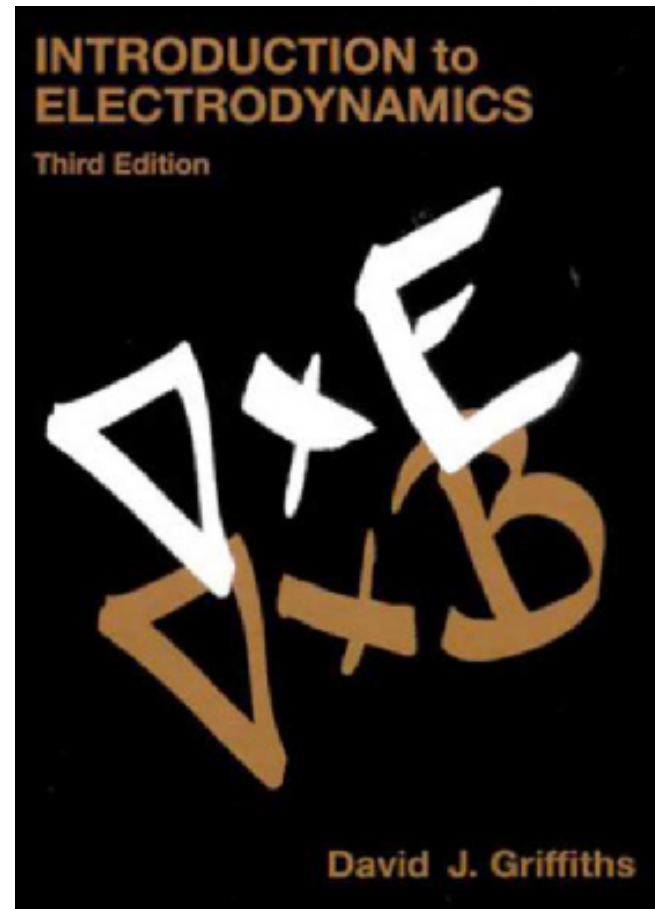
$$\vec{\nabla} \cdot \vec{D} = 0,$$

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TIME VARYING BOUND CHARGE VOLTAGE SOURCE: BAR ELECTRET



$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

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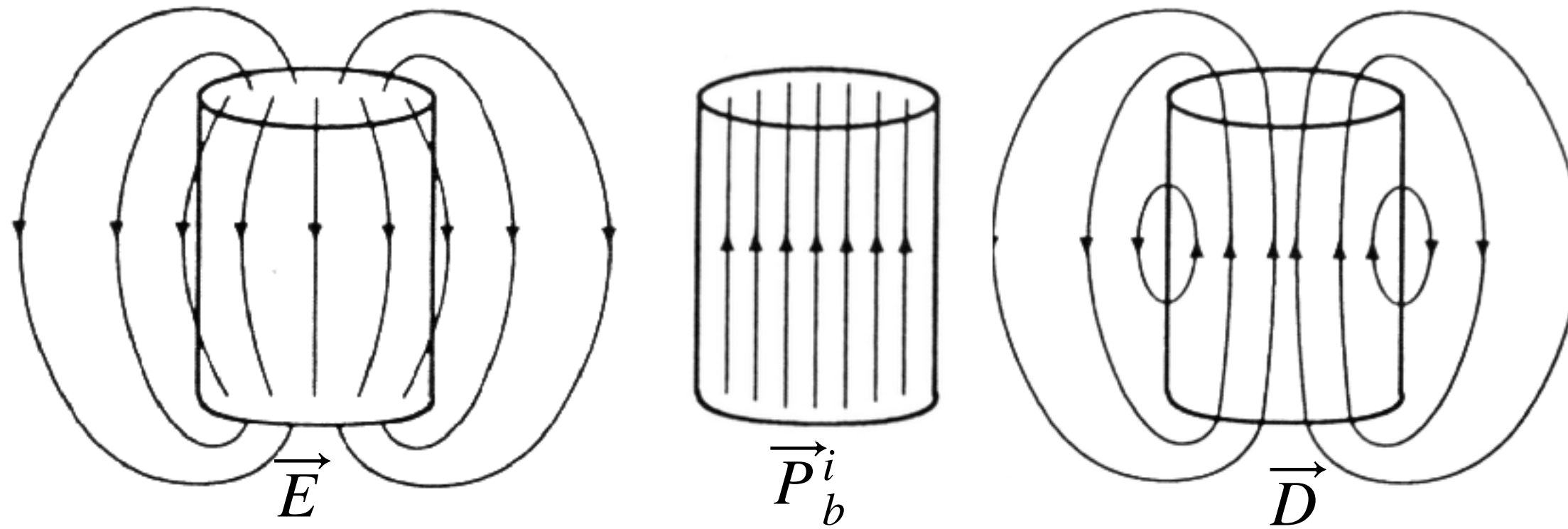
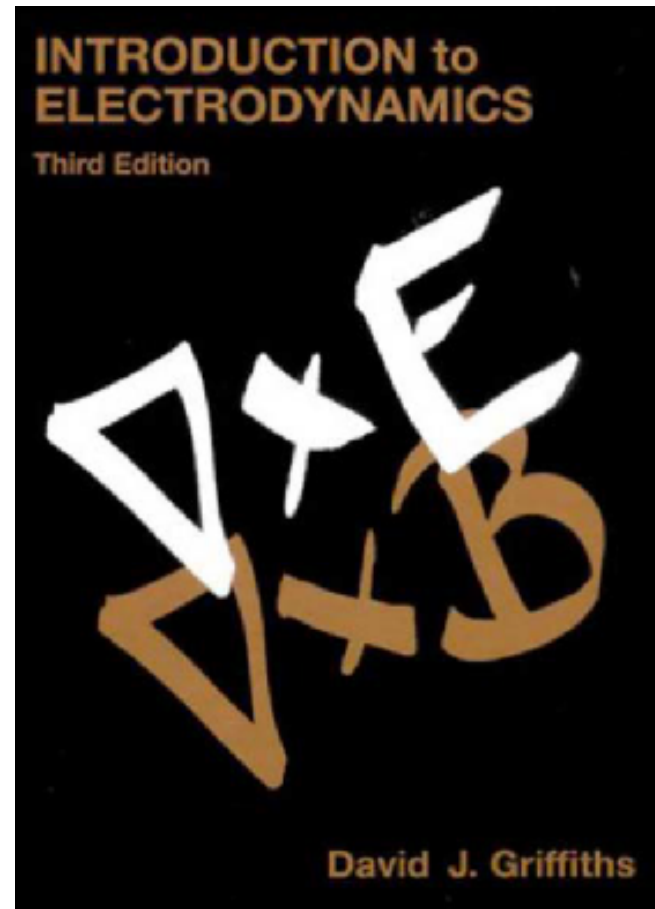
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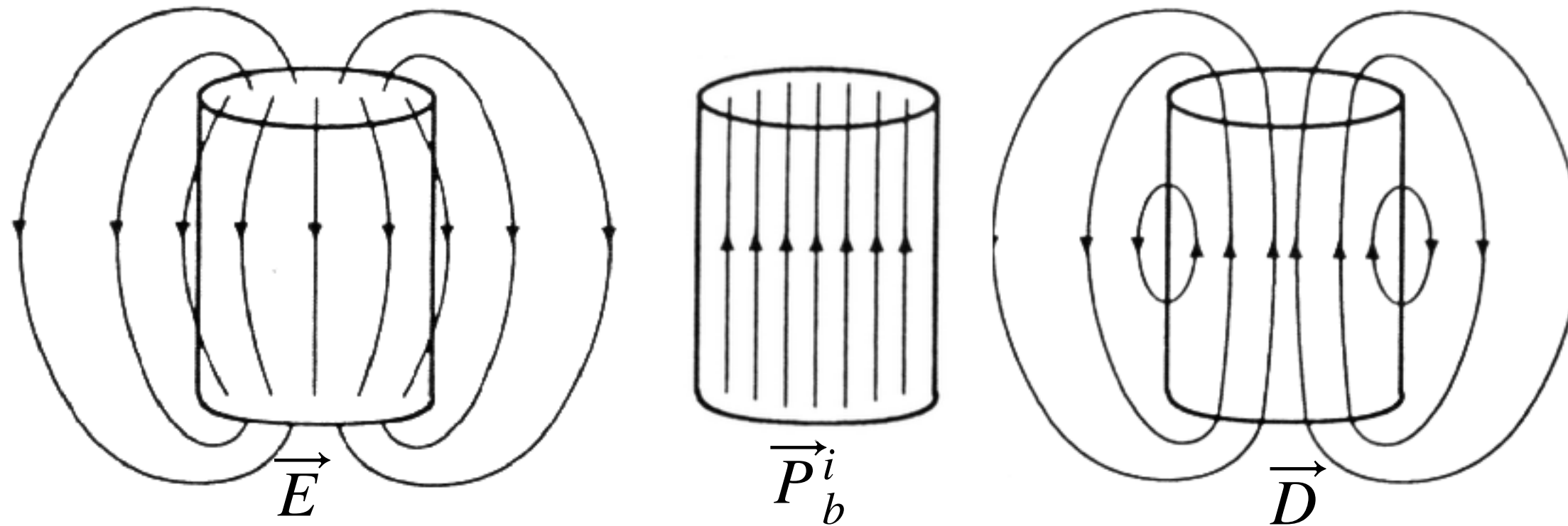
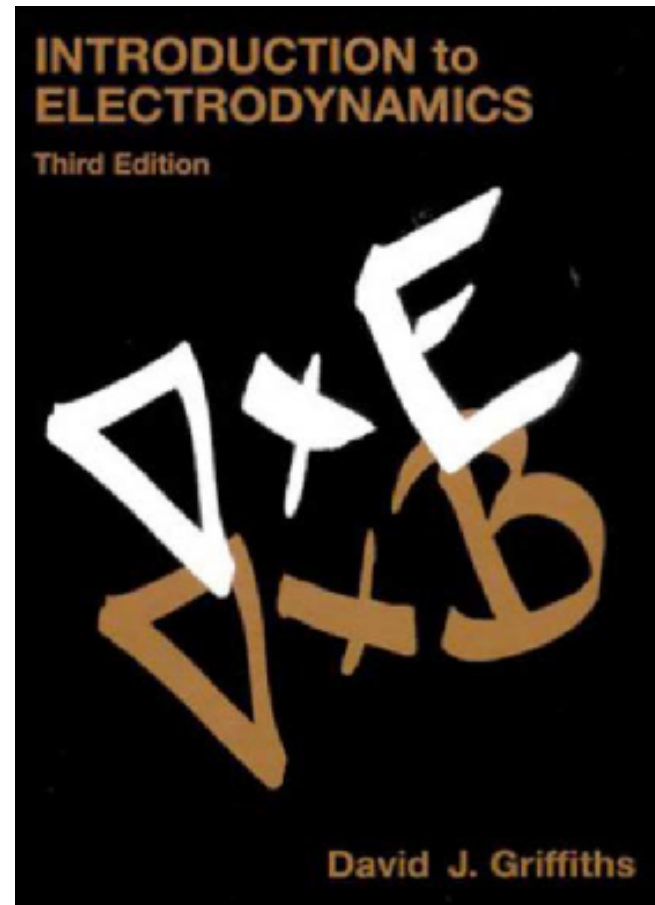
$$\vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t},$$

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TIME VARYING BOUND CHARGE VOLTAGE SOURCE: BAR ELECTRET



$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \chi_e \epsilon_0 \vec{E} + \vec{P}_b^i$$

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[1] [arXiv:1904.05774](#) [pdf, other]

Electrodynamics of Impressed Bound and Free Charge Voltage Sources

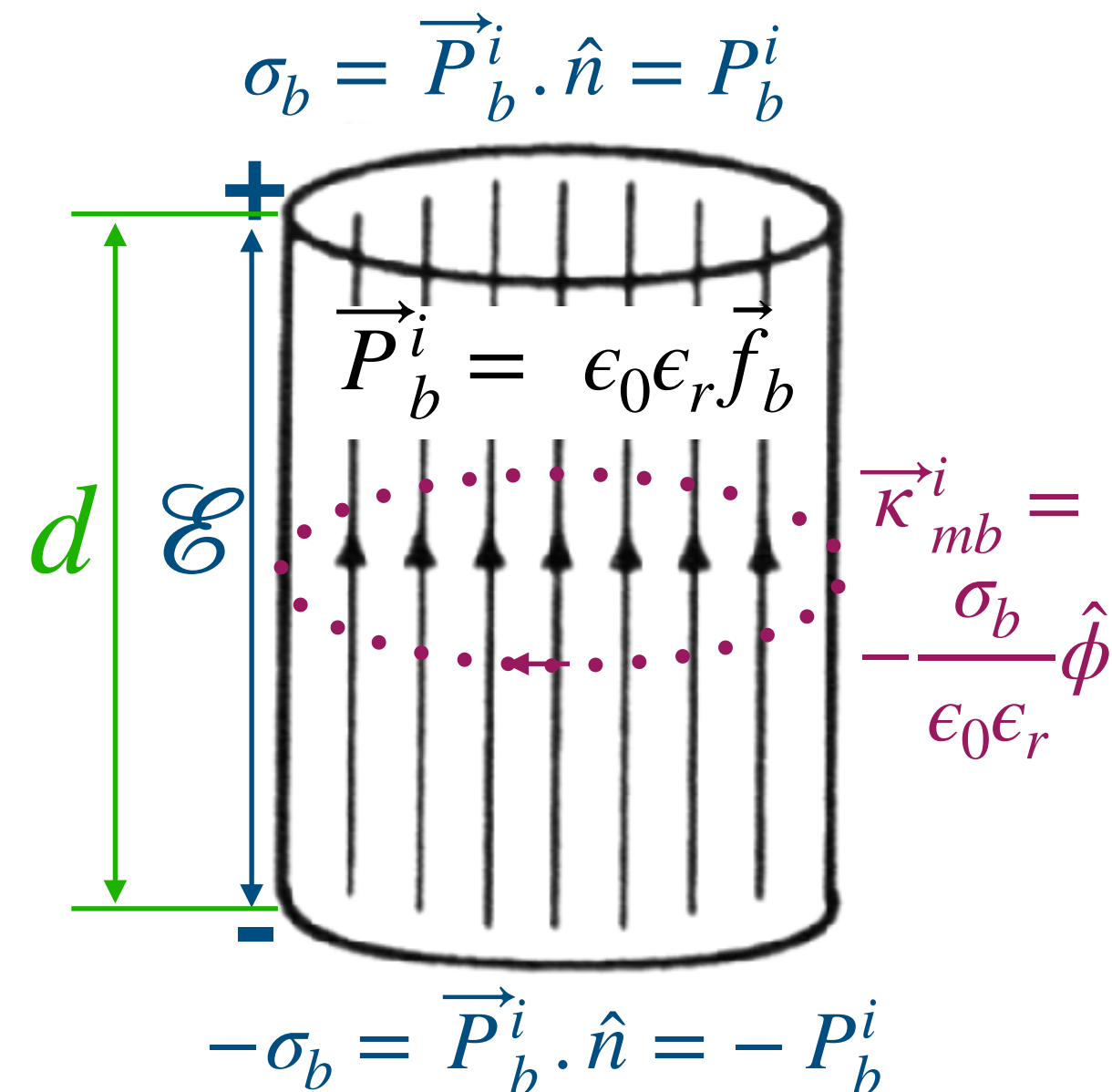
Michael E. Tobar, Ben T. McAllister, Maxim Goryachev

Subjects: Classical Physics (physics.class-ph)

Two-Potential Formulation of Electrodynamics

$$\vec{E}_T = \vec{f}_T = \vec{D}/(\epsilon_0\epsilon_r) = \vec{E} + \vec{E}^i \text{ where } \vec{E}^i = \vec{P}_b^i/(\epsilon_0\epsilon_r)$$

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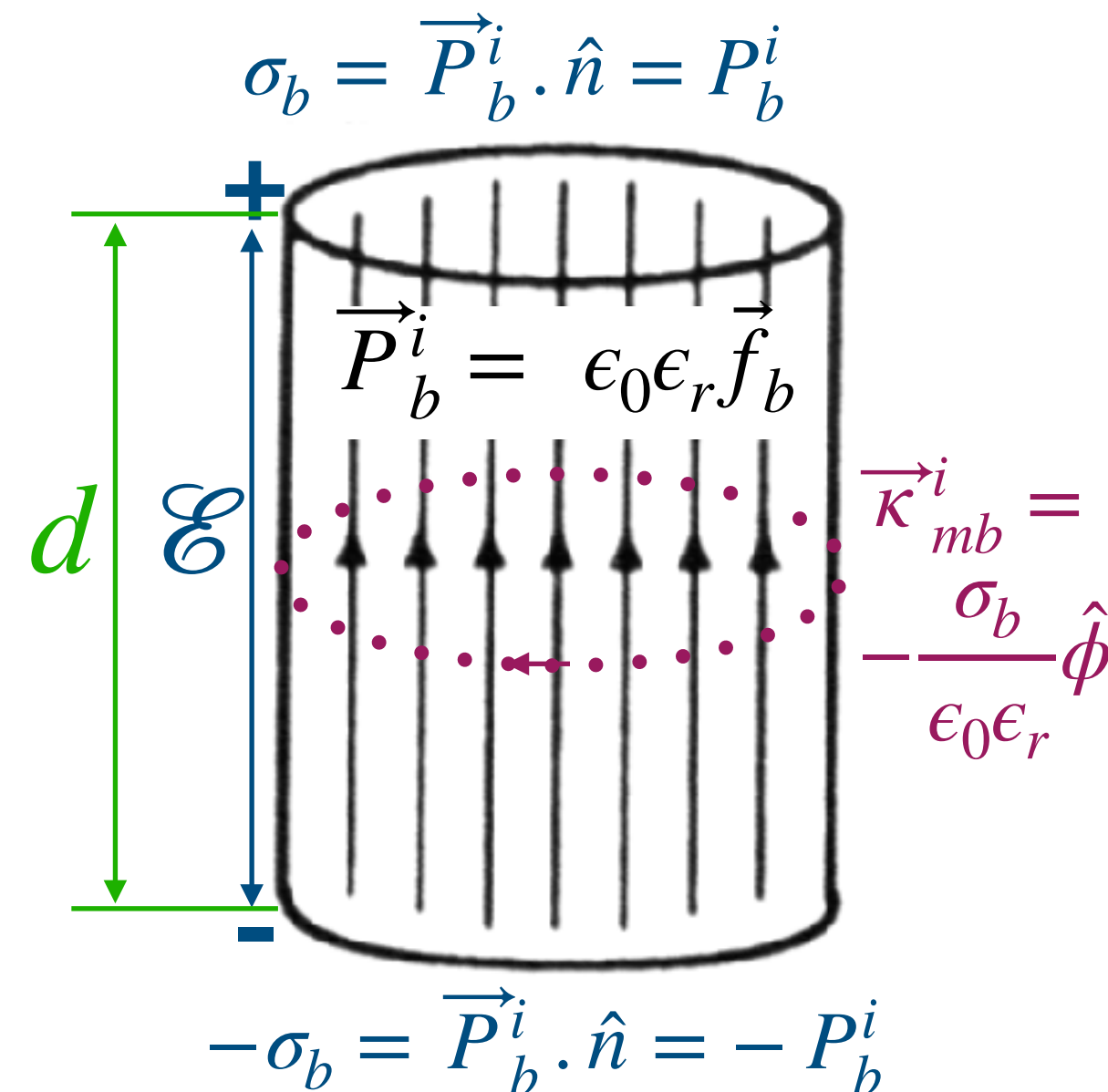


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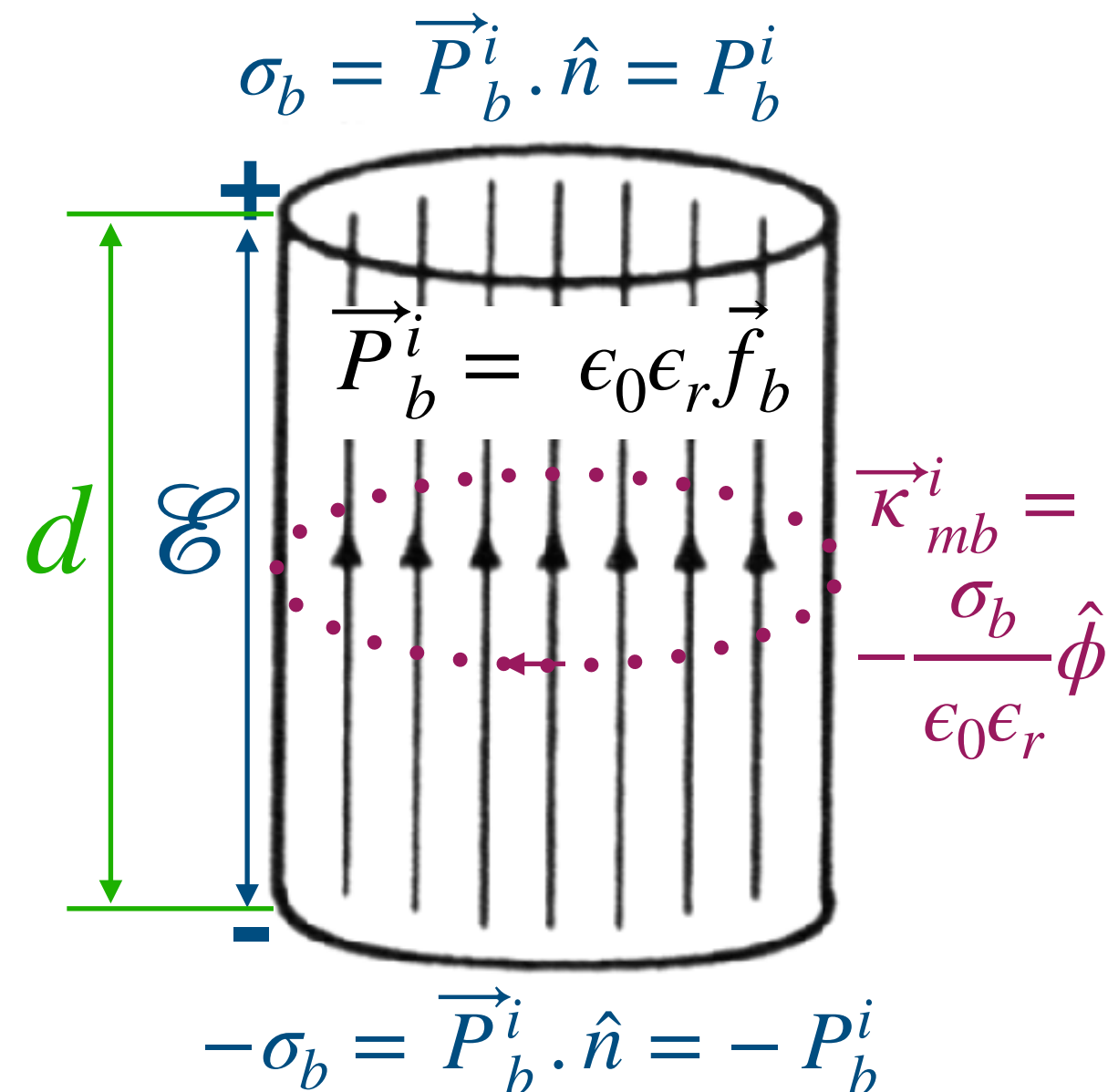
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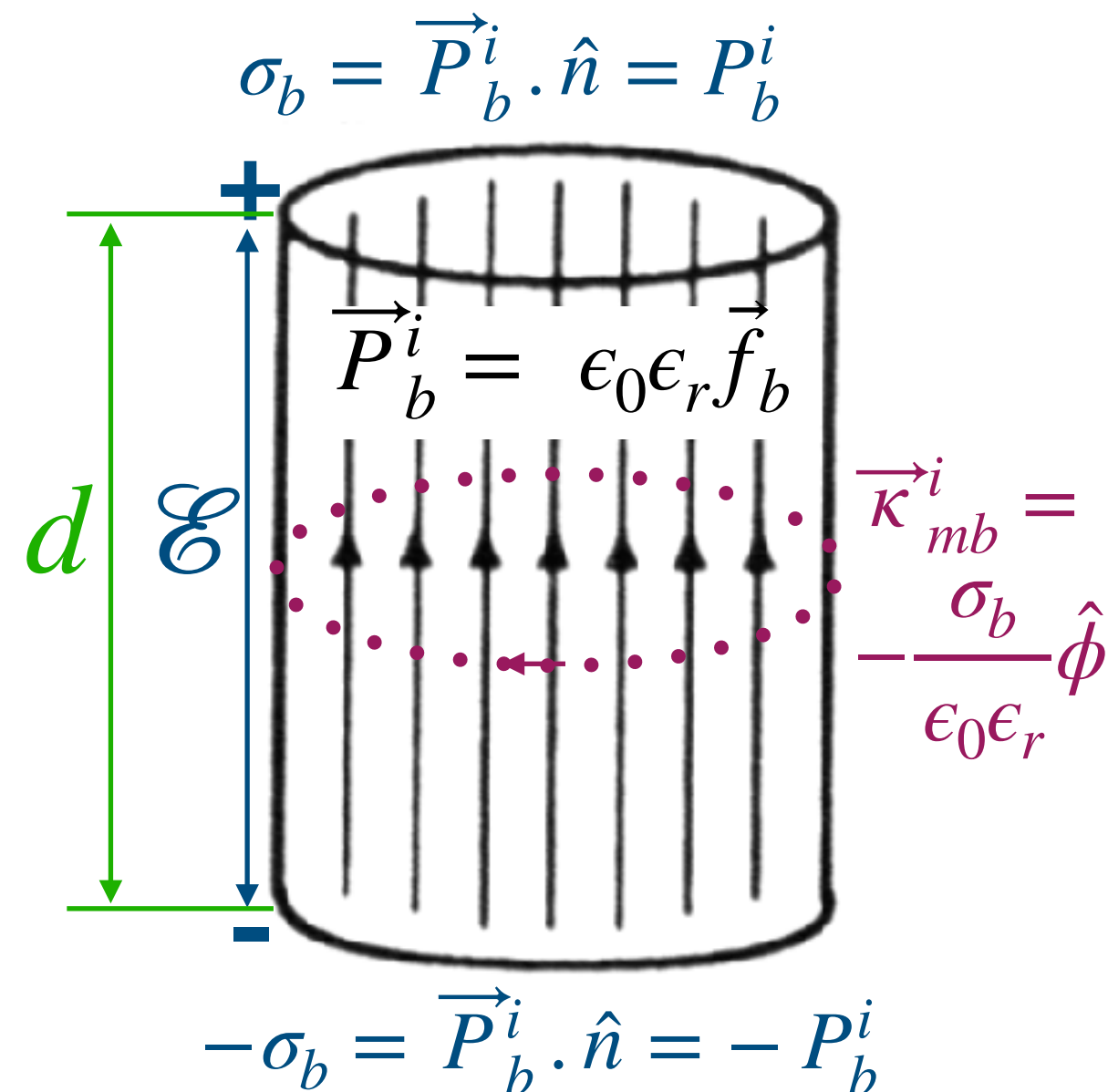
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Two-Potential Formulation of Electrodynamics

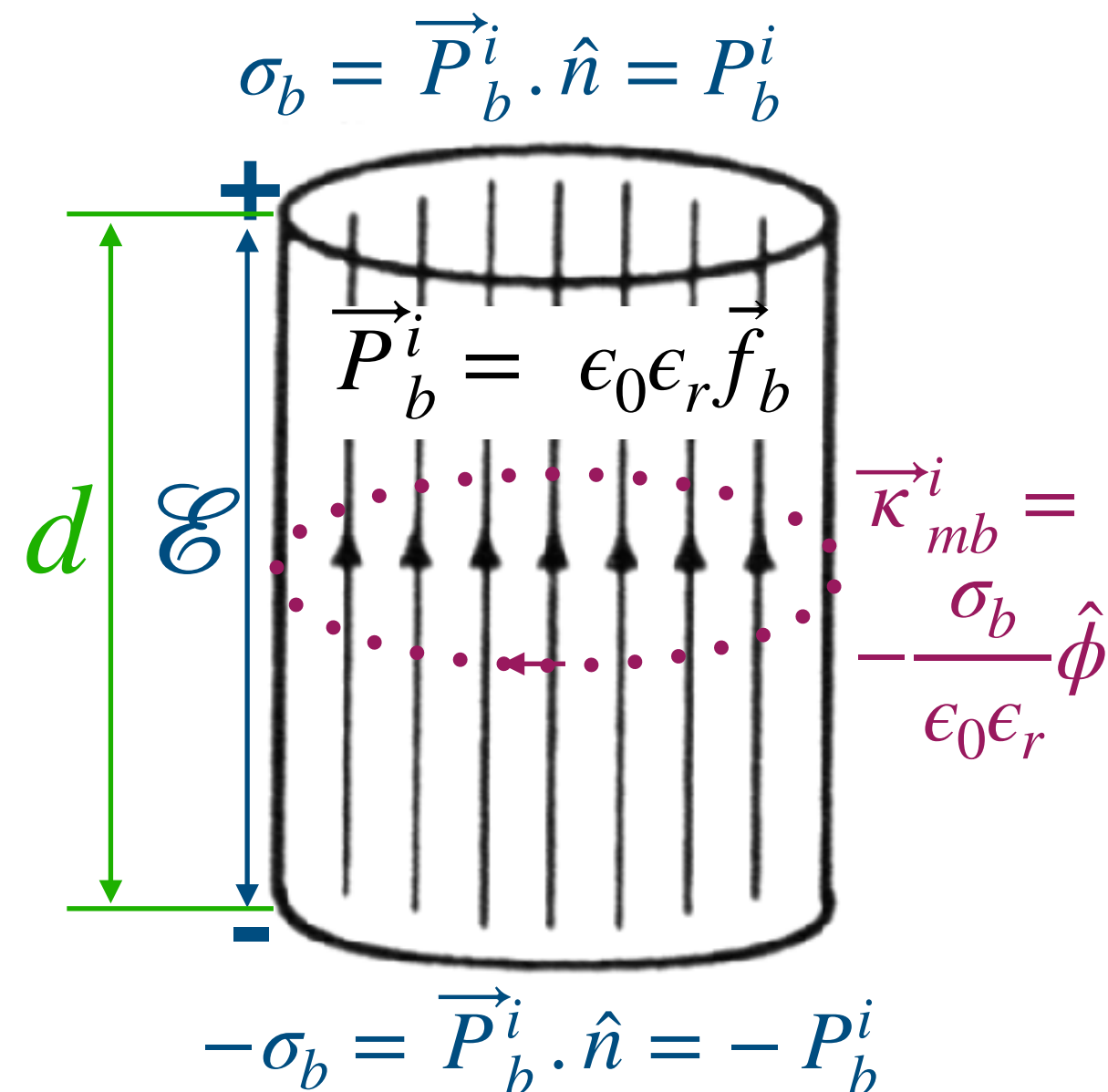
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Apply Superposition(See Harrington and Balanis)



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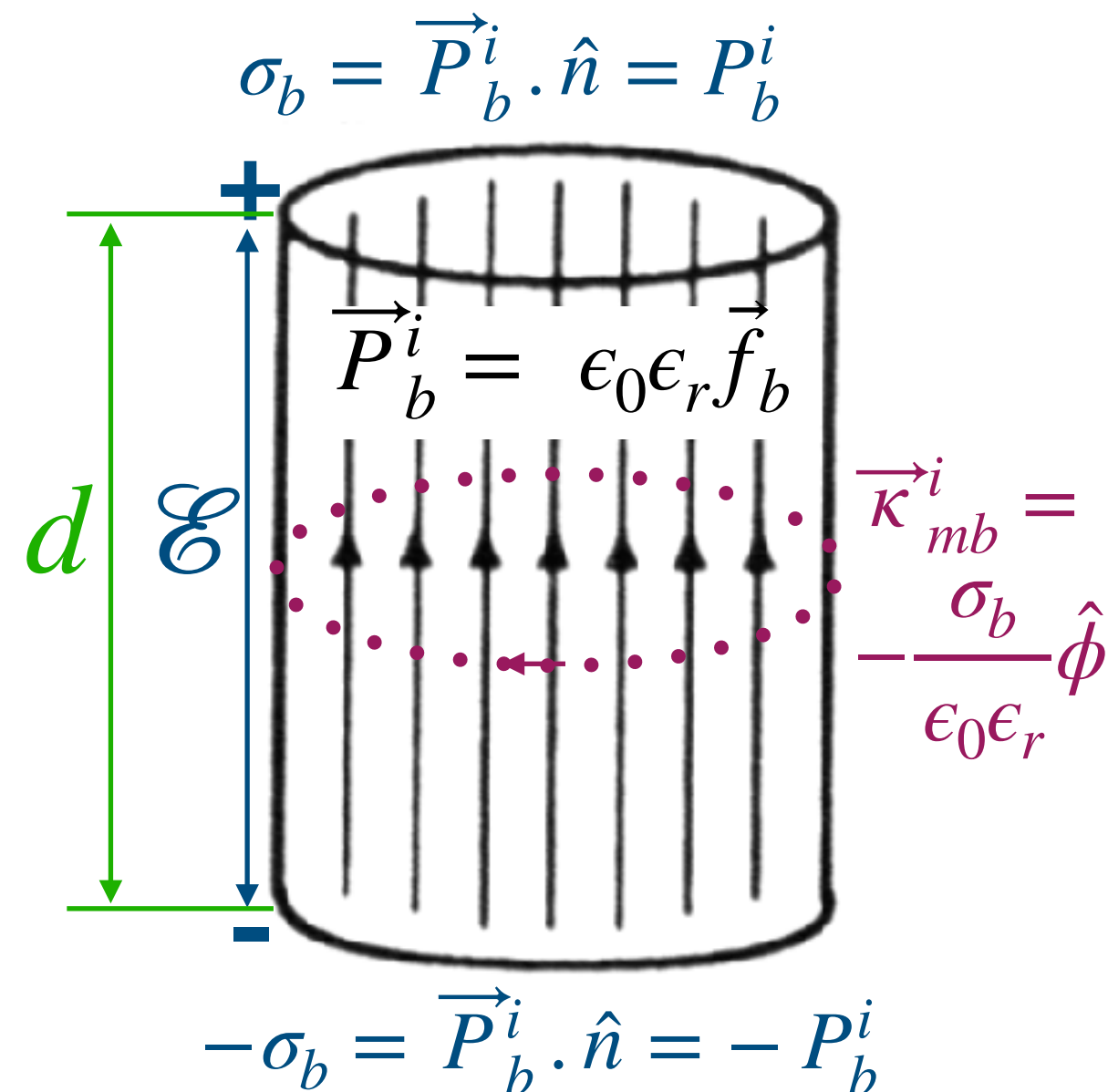
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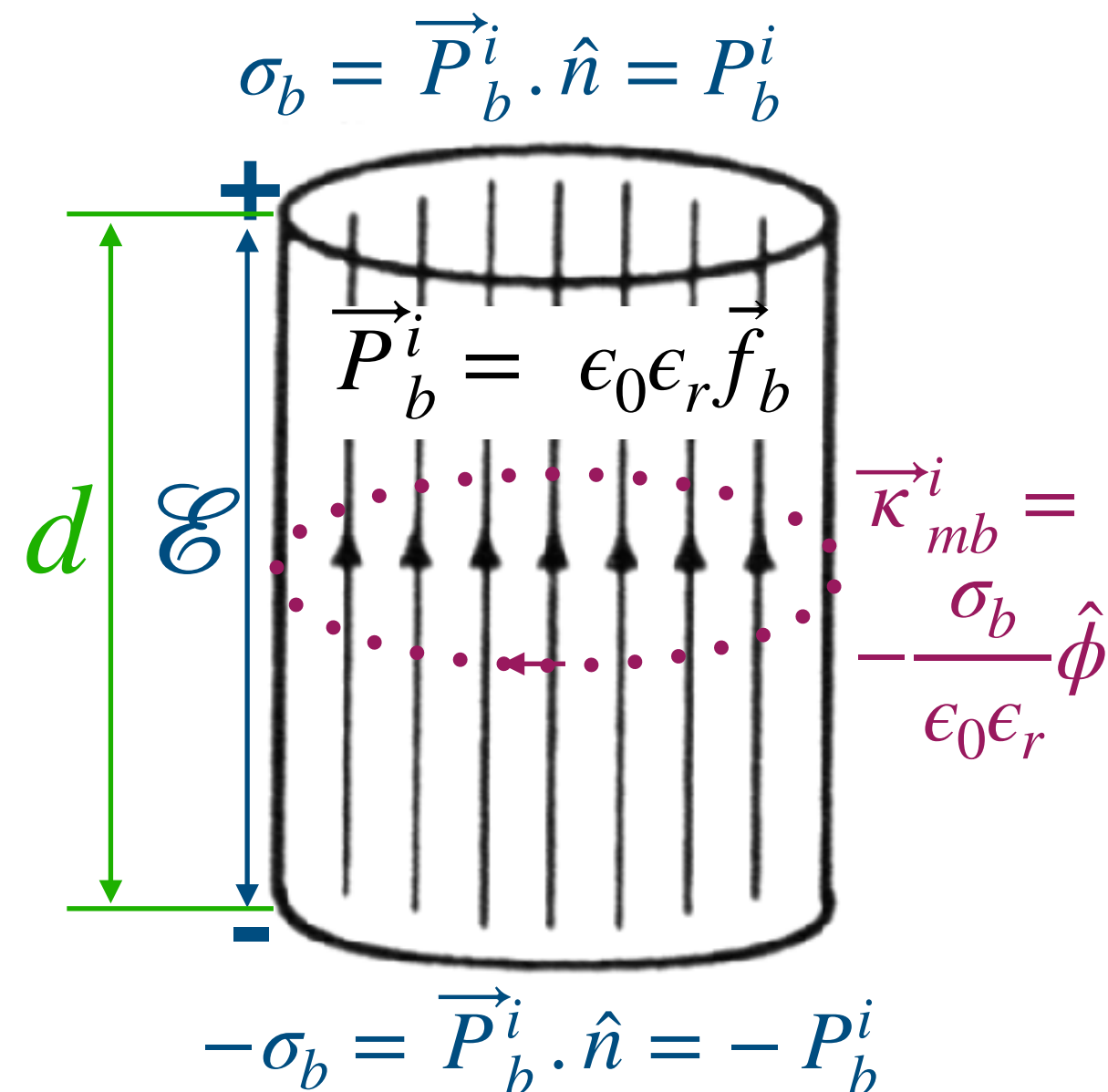
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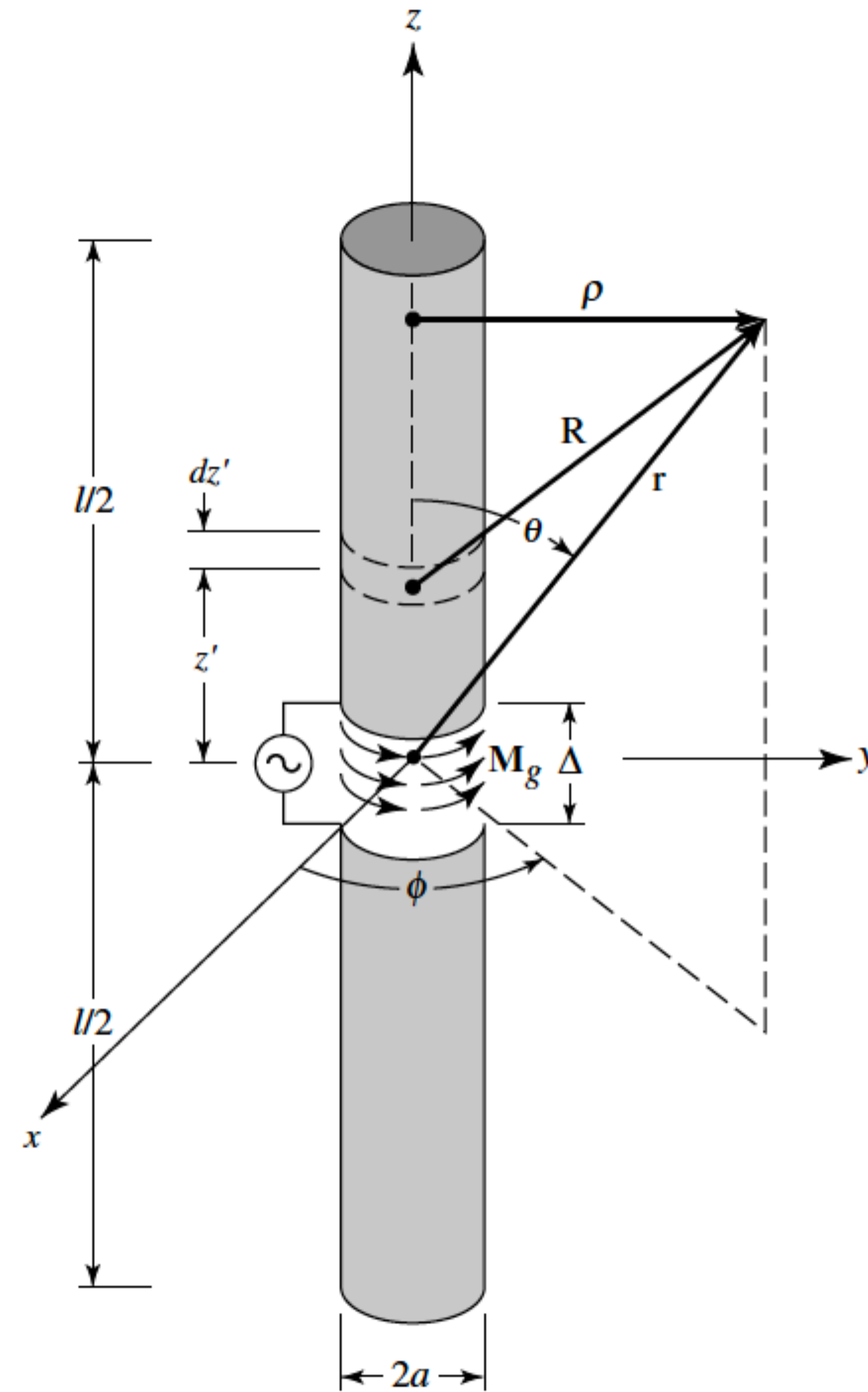


$$\vec{E}_T = \vec{E}_A + \vec{E}_C = -\frac{\partial \vec{A}}{\partial t} - \nabla V - \frac{1}{\epsilon_0\epsilon_r}\nabla \times \vec{C}$$

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Another example is modelling antenna near and far fields driven by a voltage source -> Modelled by a magnetic current and Two-Potential formulation.



(a) Dipole

Solutions to axion electrodynamics in various geometries

Jonathan Ouellet^{*} and Zachary Bogorad

*Laboratory for Nuclear Science, Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139, USA*



(Received 27 September 2018; published 12 March 2019)

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For axion at low-mass, only vector potential exists, scalar potential suppressed

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$$\begin{aligned} \mathcal{E} = & \oint_C [\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}] \cdot d\boldsymbol{\ell} \\ & + \frac{1}{q} \oint_C \text{Effective chemical forces} \cdot d\boldsymbol{\ell} \\ & + \frac{1}{q} \oint_C \text{Effective thermal forces} \cdot d\boldsymbol{\ell} , \\ & + \frac{1}{q} \oint_C \text{Effective axion forces} \cdot d\boldsymbol{\ell} \end{aligned}$$

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[arXiv:1809.01654](https://arxiv.org/abs/1809.01654) [hep-ph]

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Axion Induced Effective Magnetic Current under DC Magnetic Field: an Impressed Voltage Source

Modified Electrodynamics

$$\vec{\nabla} \cdot \vec{E}_T = 0, \quad (29)$$

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VECTOR POTENTIAL ACTS ON CHARGES

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VECTOR POTENTIAL ACTS ON CHARGES

$$\vec{C}(\vec{r}, t) = \frac{\epsilon_0 \epsilon_r}{4\pi} \int_{\Omega} \frac{\vec{J}_{ma}^i(\vec{r}', t')}{|\vec{r} - \vec{r}'|} d^3 \vec{r}' = \frac{g_{a\gamma\gamma} a}{4\pi c} \int_{\Omega} \frac{\vec{J}_{f_0}^i(\vec{r}', t')}{|\vec{r} - \vec{r}'|} d^3 \vec{r}'$$

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QUANTUM FIELD CALCULATION

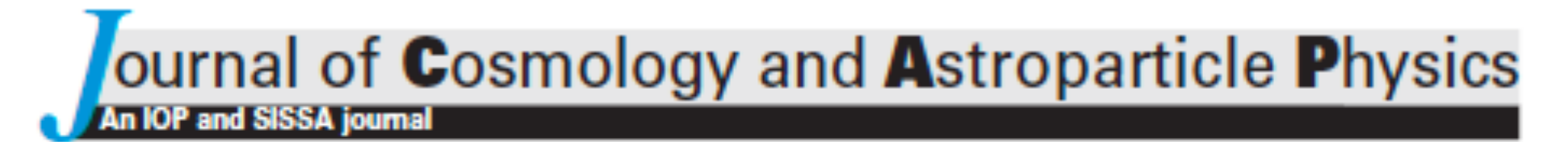
1. [arXiv:1812.05487](#) [[pdf](#), [other](#)]

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Marc Beutter, Andreas Pargner, Thomas Schwetz, Elisa Todarello

Comments: 16 pages, 1 figure. Minor corrections. Matches published version in JCAP

Subjects: **High Energy Physics – Phenomenology** (hep-ph)



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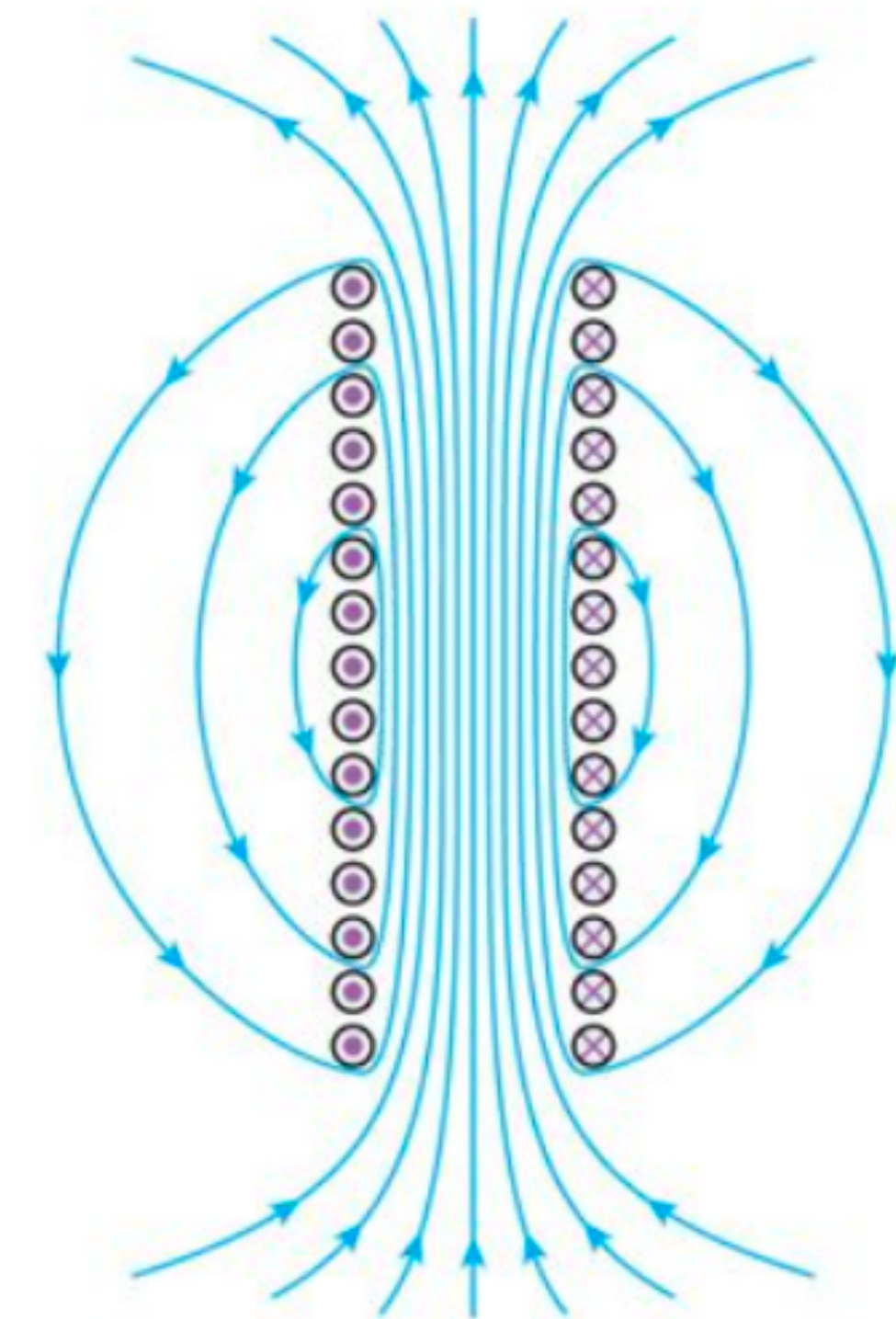
IGNORES THE ELECTRIC VECTOR POTENTIAL

Effective Approximation of Electromagnetism for Axion Haloscope Searches

Younggeun Kim, Dongok Kim, Junu Jung, Jinsu Kim, Yun Chang Shin, Yannis K. Semertzidis

(Submitted on 4 Oct 2018 (v1), last revised 1 Feb 2019 (this version, v4))

We applied an effective approximation into Maxwell's equations with an axion interaction for haloscope searches. A set of Maxwell's equations acquired from this approximation describes just the reacted fields generated by the anomalous interaction. Unlike other approaches, this set of Maxwell's equations inherently satisfies the boundary conditions for haloscope searches. The electromagnetic field solutions from the Maxwell's equations were evaluated for both cylindrical and toroidal cavity geometries including when the axion mass becomes ultra-light (sub-meV). A small but non-zero difference between the electric and magnetic stored energies appeared in both cases. The difference may come from an anomalous non-dissipating current induced by oscillating axions.



$$\nabla \cdot (\mathbf{E} - cg_{a\gamma\gamma}a\mathbf{B}) = \frac{\rho_e}{\varepsilon},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times (c\mathbf{B} + g_{a\gamma\gamma}a\mathbf{E}) = \frac{1}{c} \frac{\partial}{\partial t} (\mathbf{E} - cg_{a\gamma\gamma}a\mathbf{B}) + c\mu\mathbf{J}_e.$$

• zero current : $\mathbf{J}_e = 0$,

• zero charge density : $\rho_e = 0$,

• zero external electric field : $\mathbf{E}_{\text{ext}} = 0$,

• nonzero external magnetic field : $\mathbf{B} = \mathbf{B}_{\text{ext}}$,

• curlless magnetic field : $\nabla \times \mathbf{B} = \nabla \times \mathbf{B}_{\text{ext}} = 0$,

• time independent magnetic field : $\dot{\mathbf{B}} = 0$.

Boundary Conditions

$$\oiint_S \vec{E}_T \cdot d\vec{a} = 0,$$

$$\oint_P \vec{B} \cdot d\vec{l} = \mu_0 I_{f_0 enc}^i + \mu_0 \epsilon_r \epsilon_0 \frac{d}{dt} \int_S \vec{E}_T \cdot d\vec{a}$$

$$\oiint_S \vec{B} \cdot d\vec{a} = 0,$$

$$\oint_P \vec{E}_T \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{a} - g_{a\gamma\gamma} a \frac{c}{\epsilon_r} \mu_0 I_{f_0 enc}^i$$

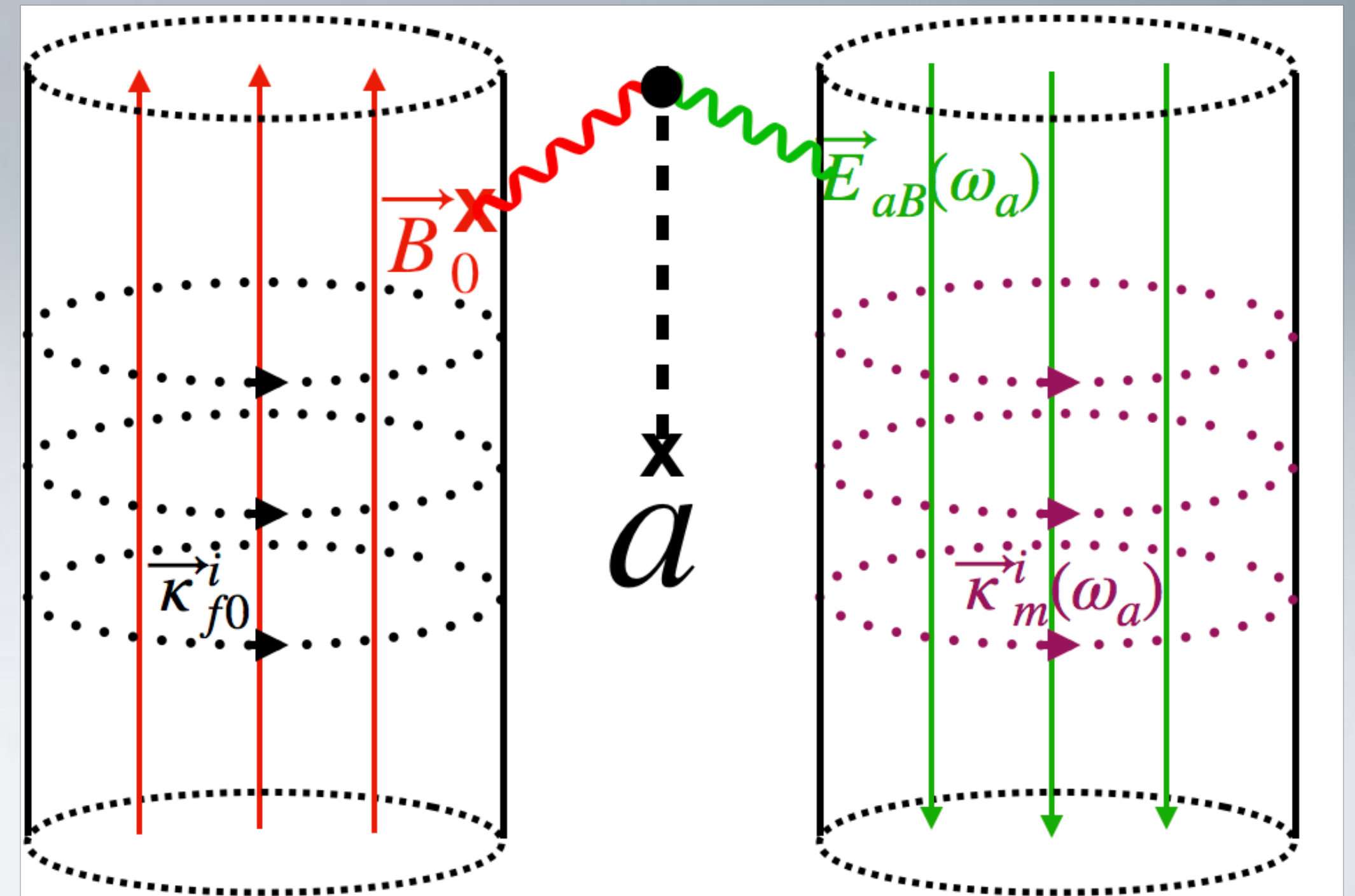
Here, $I_{f_0 enc}^i = \int_S \vec{J}_{f_0}^i \cdot d\vec{a}$.

$$\vec{E}_{T1}^{\parallel} - \vec{E}_{T2}^{\parallel} = -g_{a\gamma\gamma} a \frac{c}{\epsilon_r} \mu_0 \vec{\kappa}_{f_0}^i \times \hat{n} = -\vec{\kappa}_{f_m}^i \times \hat{n} = \vec{E}_{aB}$$

$$\vec{E}_{T1}^{\perp} = \vec{E}_{T2}^{\perp},$$

$$\vec{B}_1^{\parallel} - \vec{B}_2^{\parallel} = \mu_0 \vec{\kappa}_{f_0}^i \times \hat{n},$$

$$\vec{B}_1^{\perp} = \vec{B}_2^{\perp}.$$



THE END: THANK YOU

QUESTIONS?

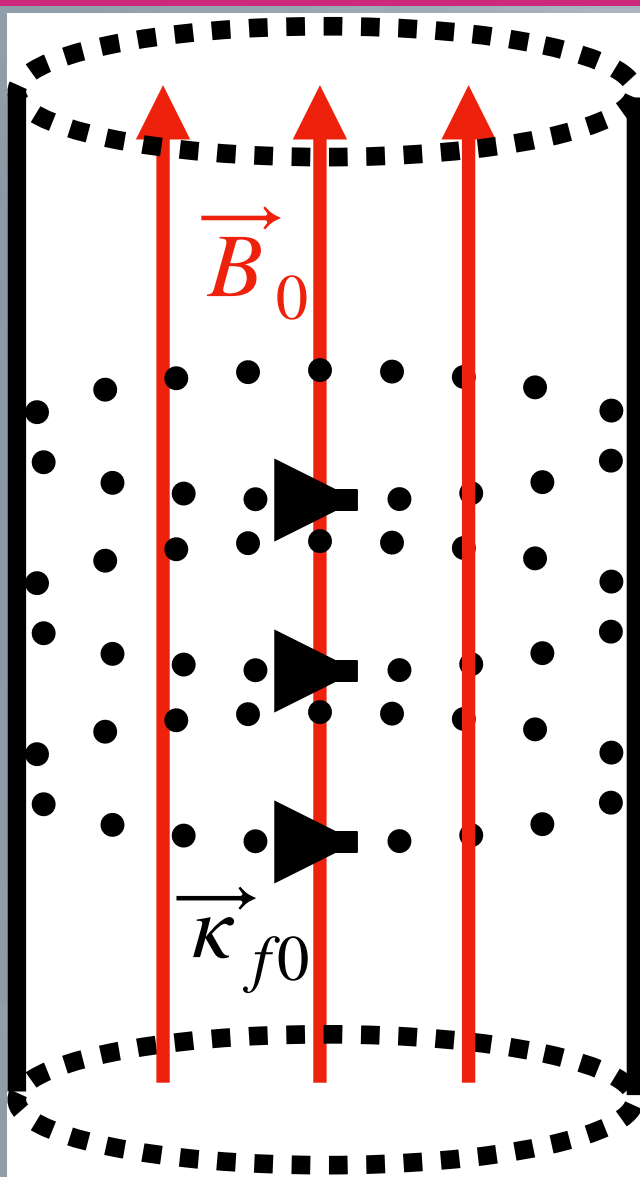
What is the Orthogonality of E and B in the Low-Mass Limit under DC Magnetic Field?

$$\mathcal{L}_{a\gamma} = \frac{g_{a\gamma}}{4} a F \tilde{F} = -g_{a\gamma\gamma} a \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{E} \cdot \vec{B}$$

Axion Two Photon Coupling

$$a(t) \approx a_0 \cos(\omega_a t)$$

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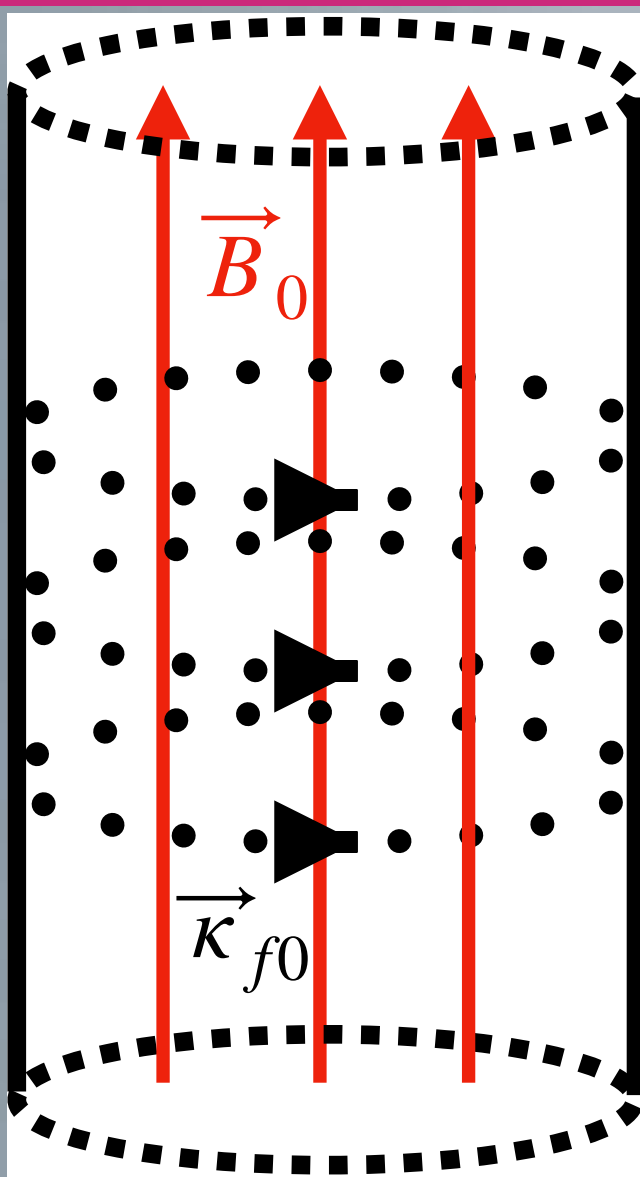
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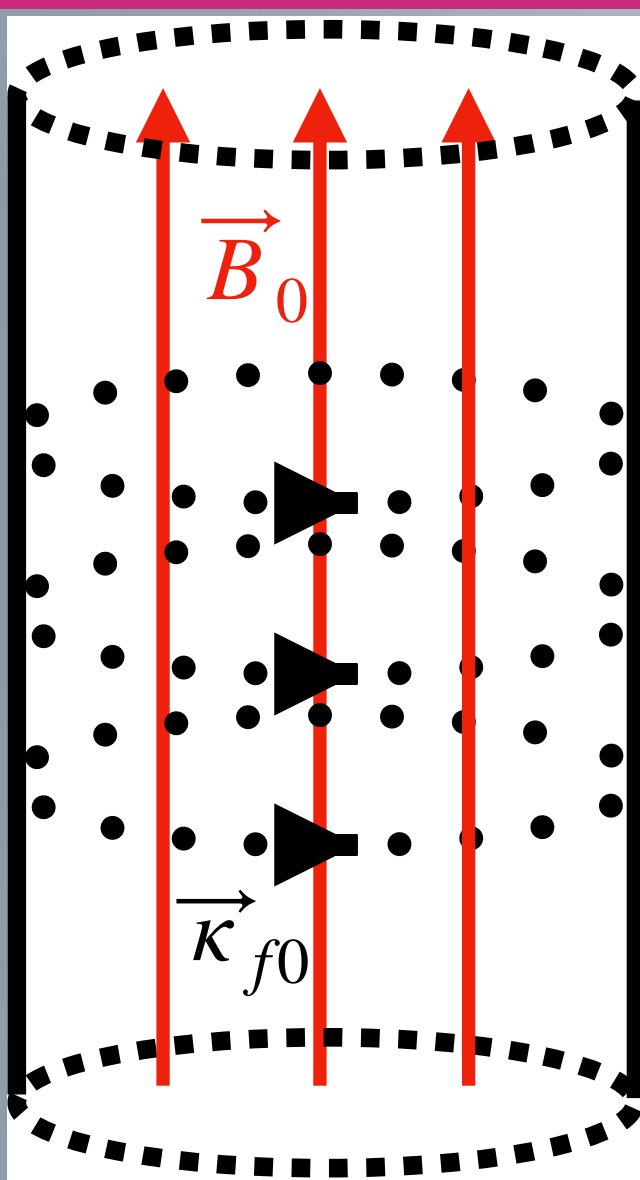
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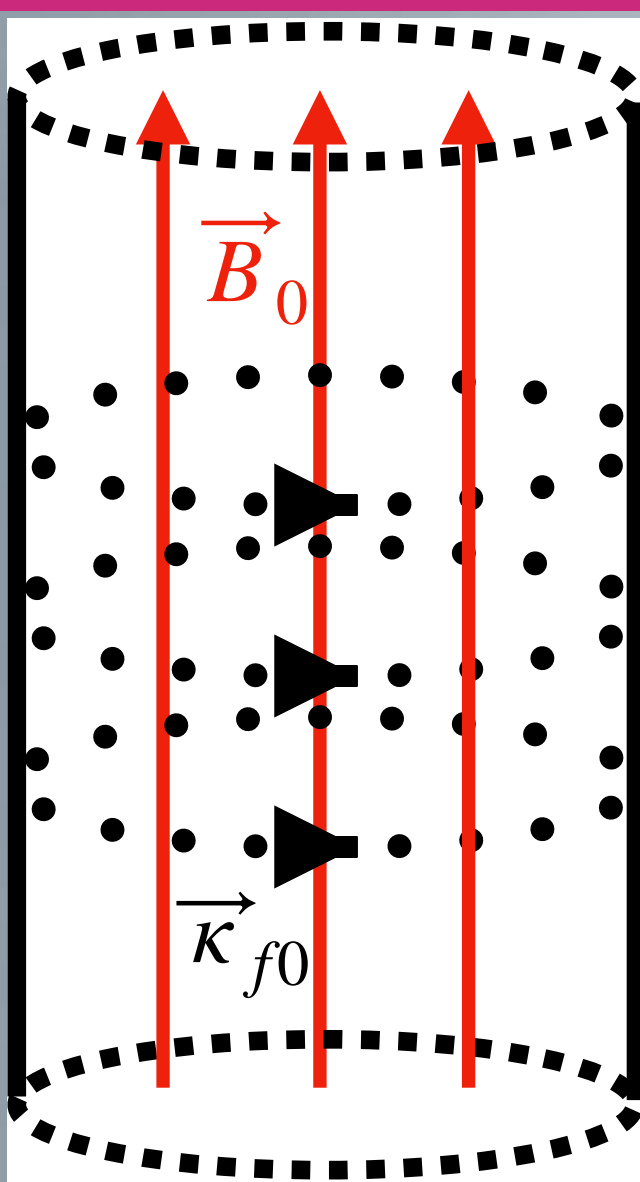
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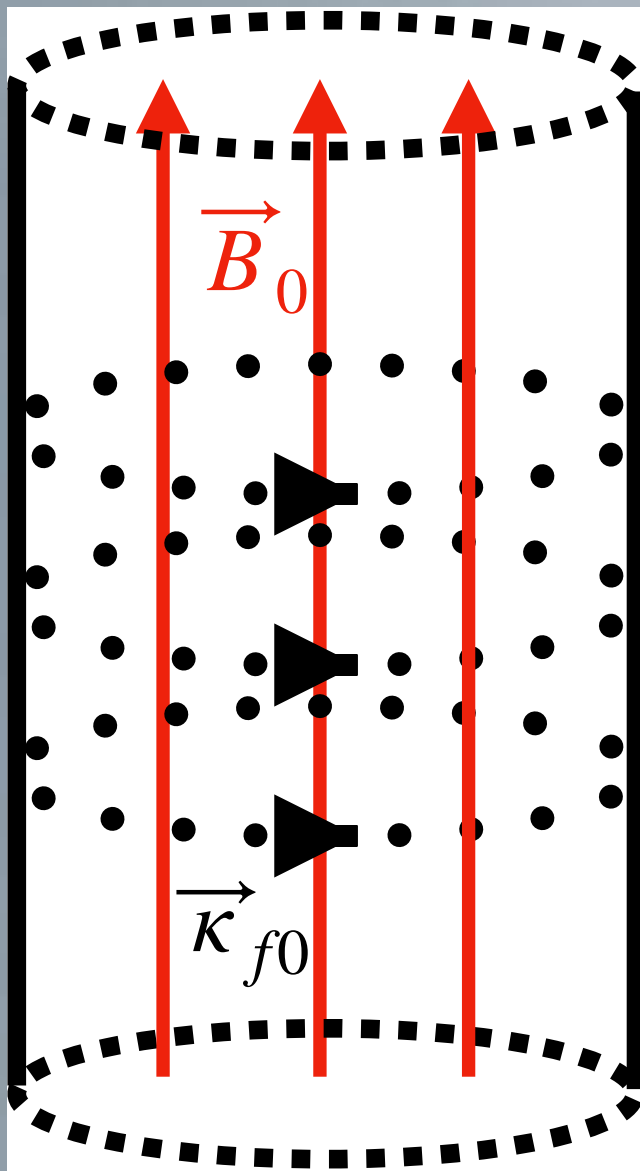
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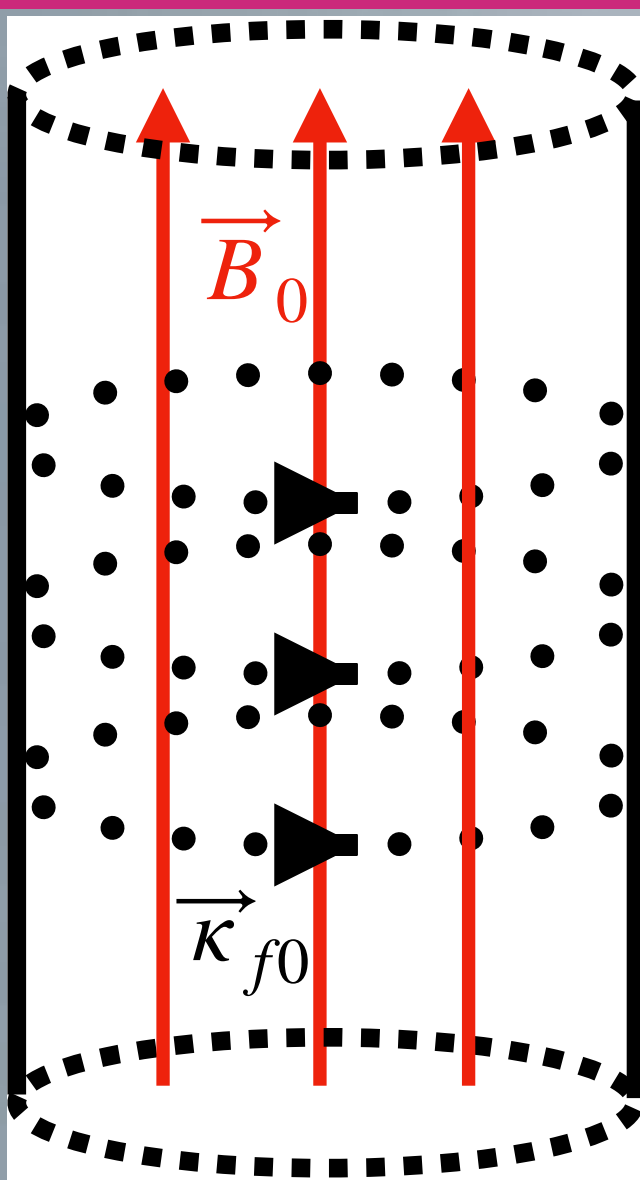
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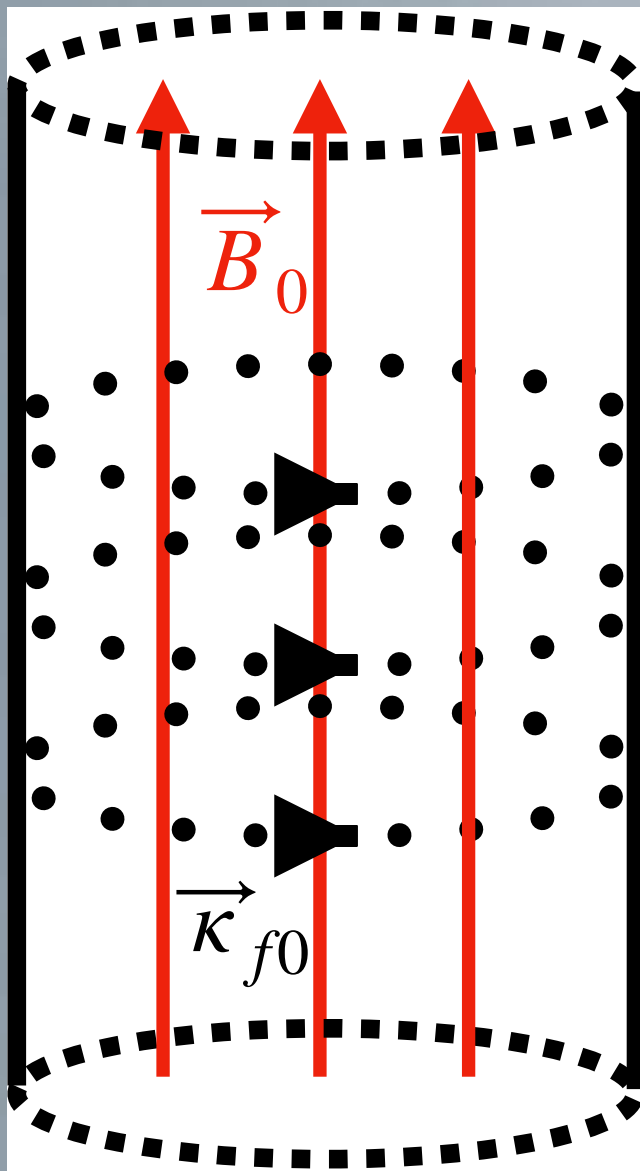
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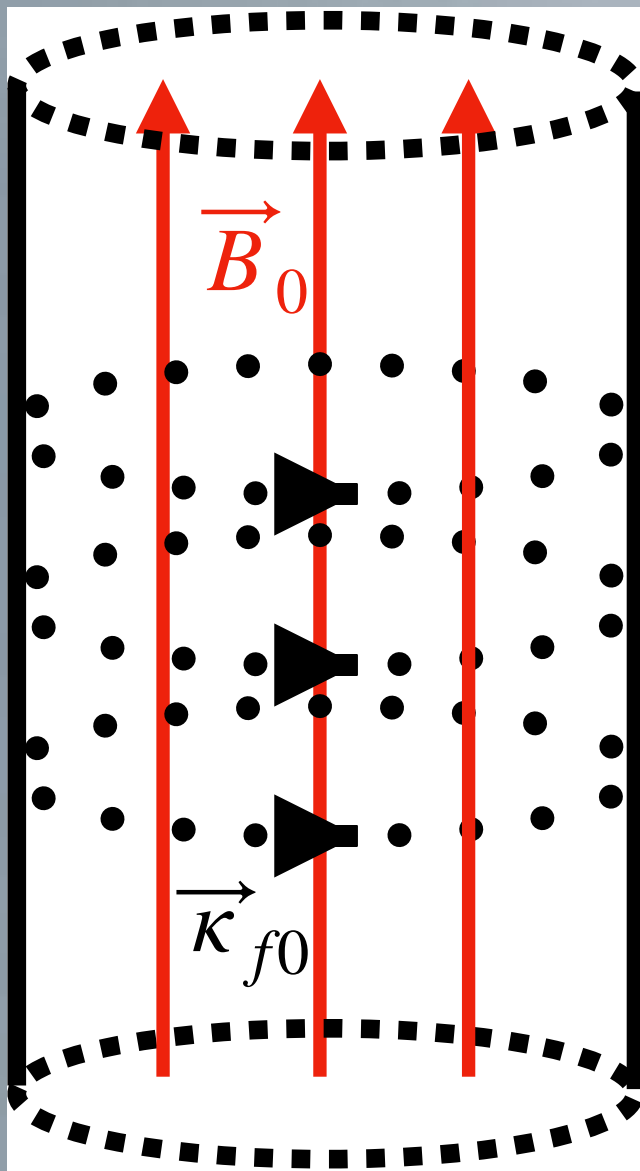
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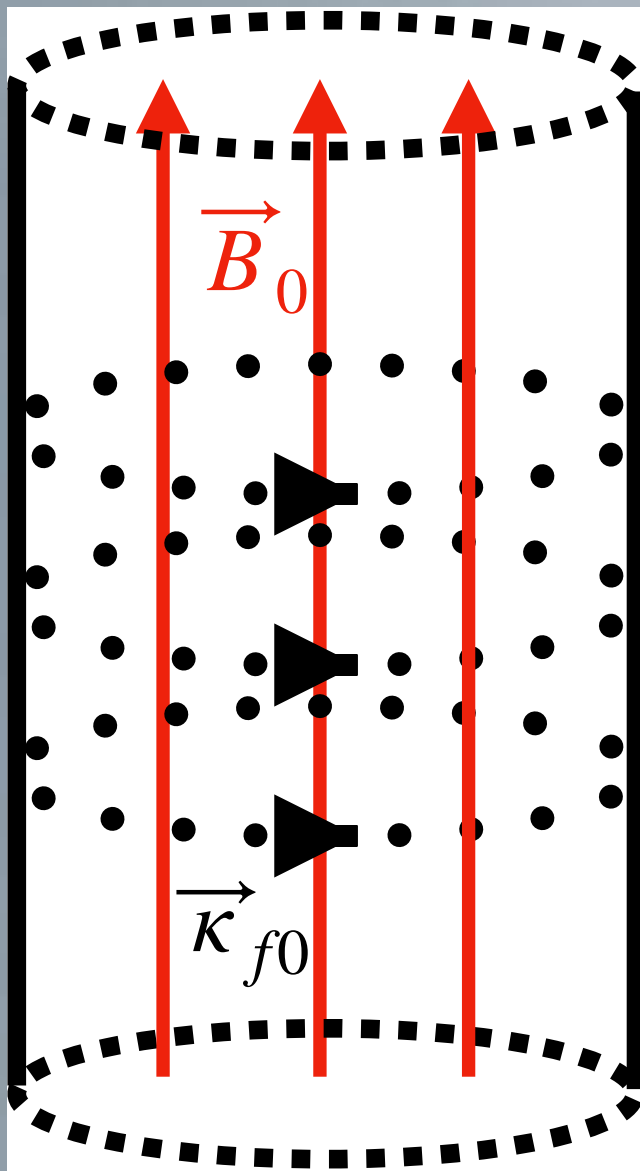
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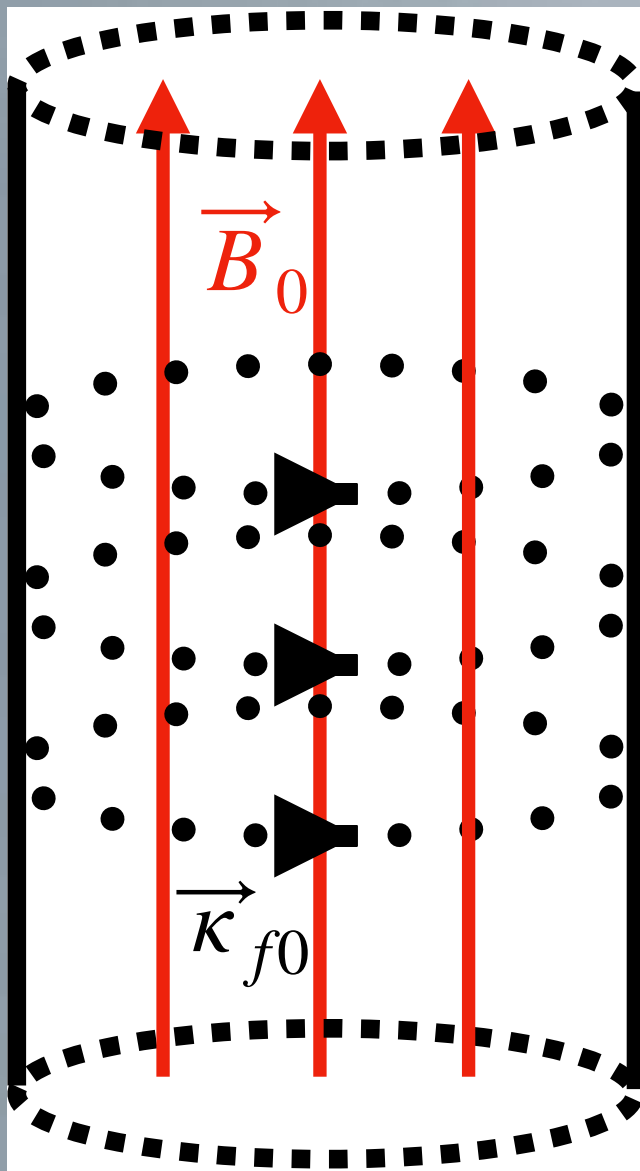
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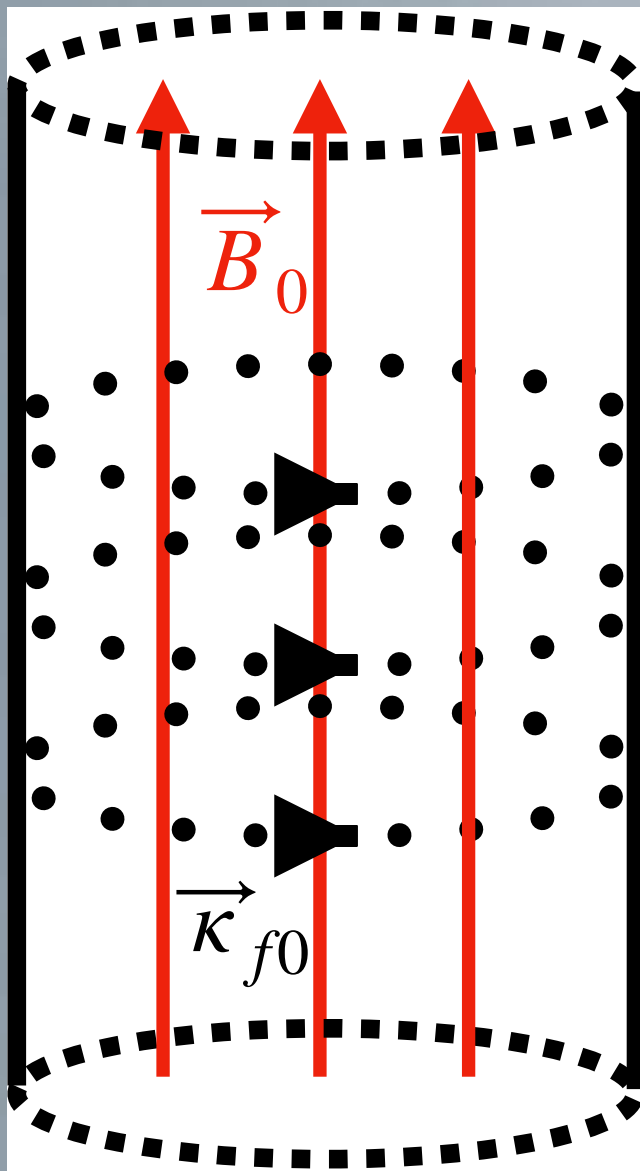
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