# Parton branching TMD distributions from fits to DIS precision data

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# Outline

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   - What are TMDs?
   - Why TMDs?
2. New developments
   - How to solve DGLAP evolution with PB method?
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3. Determination of TMD densities at LO from HERA DIS data
   - PDFs from PB method: fit to HERA data
   - Improved ordering dynamic $z_M$
   - Extension to include small-$x$ processes

# What are TMDs?

**TMDs (Transverse Momentum Dependent parton distribution)**

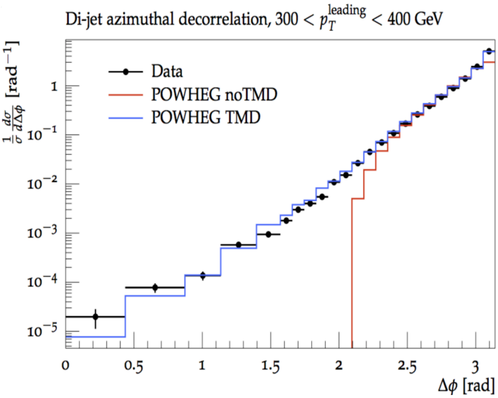
- $\rightarrow$ at very small $k_t$
- typically for small $k_t$ in DY production, or semi-inclusive DIS (CS, CSS, ...)

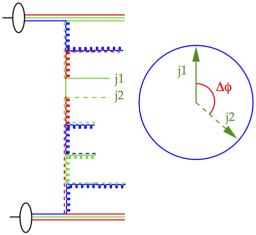
- $\rightarrow$ at very small $x$
- essentially only gluon densities (CCFM, BFKL, ...)

new approach: **Parton Branching Method**

- $\rightarrow$ cover all transverse momenta from small $k_t$ to large $k_t$ as well as large range in $x$ and $\mu^2$.

# Why TMDs?




- NLO-dijet with collinear POWHEG cannot describe small $\Delta\phi$
- NLO-dijet with TMD POWHEG describes spectrum at small and large $\Delta\phi$
- Region of higher order emissions described by TMDs. This approach gives further insights and improves the description of jet production.

# How to determine TMDs?

Transverse momentum effects are naturally coming from

- intrinsic $k_t$
- parton shower

Determine integrated PDFs form PB solution of evolution equation:

- at LO, NLO and NNLO
- advantages of PB method

Determine TMD:

- since each branching is generated explicitly, energy-momentum conservation is fulfilled and transverse momentum distributions can be obtained.

# How to solve DGLAP evolution with PB method?

- differential form of the DGLAP:

$$\mu^2 \frac{\partial}{\partial \mu^2} f(x,\mu^2) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \mathcal{P}_+(z) \, f(\frac{x}{z},\mu^2)$$

- sudakov FF

$$\Delta_s(\mu^2) = exp\left( - \int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} \mathcal{P}(z) \right)$$

- DGLAP eq. in the form including the $\Delta_s$

$$\mu^2 \frac{\partial}{\partial \mu^2} \frac{f(x,\mu^2)}{\Delta_s(\mu^2)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{\mathcal{P}(z)}{\Delta_s(\mu^2)} f(\frac{x}{z},\mu^2)$$

# Parton Branching method

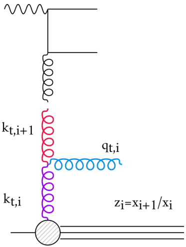
- integral form with a very simple physical interpretation:

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int \frac{dz}{z}\frac{d\mu'^2}{\mu'^2}.\frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)}P^R(z)f(\frac{x}{z},\mu'^2)$$

- solve integral equation via iteration:

$$f_0(x,\mu^2) = f(x,\mu_0^2)\ \Delta_s(\mu^2)$$

$$f_1(x,\mu^2) = f(x,\mu_0^2)\ \Delta_s(\mu^2) + \int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)}\int\frac{dz}{z}P^R(z)f(x/z,\mu_0^2)\Delta(\mu'^2)$$




iterate with second branching and so on to get the full solution

# Evolution equation and parton branching method

- use momentum weighted PDFs with real emission probability

$$
\begin{aligned}
xf_a(x,\mu^2) &= \Delta_a(\mu^2) xf_a(x,\mu_0^2) \\
&+ \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} \int_x^{z_M} dz\, P_{ab}^R(\alpha_s(t'),z)\, \frac{x}{z}\, f_b(x/z,\mu^2)
\end{aligned}
$$

- $z_M$ introduced to separate real from virtual and non-resolvable branching
- reproduces DGLAP up to $\mathcal{O}(1-z_M)$

- associate the evolution scale with some physical interpretation
  - set1 $\to \alpha_s(\mu_i^2)$
  - set2 $\to \alpha_s((1-z_i)^2\mu_i^2)$
- check what the dependence of this different choice is

A. Bermudez Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu and R. Žlebčík, arXiv:1804.11152 [hep-ph].

# PDFs from PB method: fit to HERA data

Convolution of kernel with starting distribution

$$
\begin{aligned}
xf_a(x,\mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x')\, \tilde{\mathcal{A}}_a^b(x'',\mu^2)\, \delta(x'x''-x) \\
&= \int dx'\, \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'}\, \tilde{\mathcal{A}}_a^b(\frac{x}{x'},\mu^2)
\end{aligned}
$$

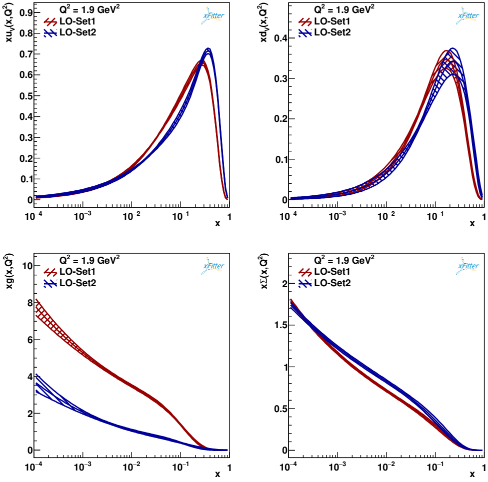
Fit performed using xFitter frame in **LO** to investigate the small-x corrections to splitting functions (CCFM)

- Full coupled-evolution with all flavors
- using full HERA 1+2 inclusive DIS (neutral current, charged current) data
- in total 1145 data points
- $3.5 < Q^2 < 50000$ GeV$^2$ & $4.10^{-5} < x < 0.65$

Can be easily extended to include any other measurement for fit.

A. Bermudez Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu and R. Žlebčík, arXiv:1804.11152 [hep-ph].

# Standard LO full fit with different scale in $\alpha_s$

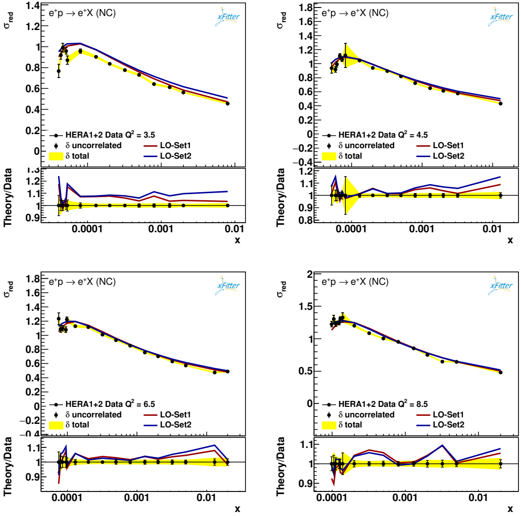



- Set1 with $\alpha_s(\mu_i^2)$
  $\chi^2/dof = 1.239$
- Set2 with $\alpha_s((1-z_i)^2\mu_i^2)$
  $\chi^2/dof = 1.262$
- $\chi^2/dof$ is comparable to NLO fit.

- Very different gluon distribution obtained at small $Q^2$
- The differences are wash out at higher $Q^2$

# Fit to DIS x-section at LO: $F_2$

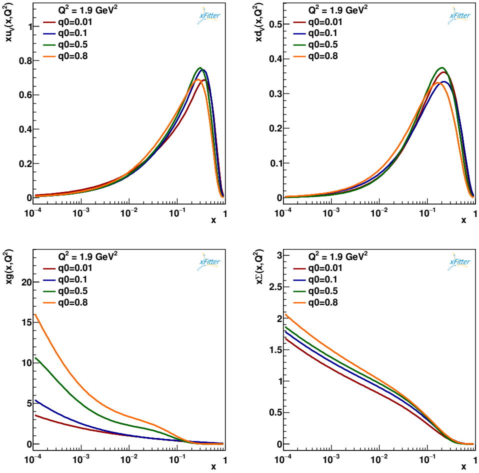
How well can we describe $F_2$ with the two sets at LO?



# Improved ordering dynamic $z_M$

- At the moment all our fits are with $z_M = 1 - \frac{q_0}{q}$ and $q_0$ is chosen to be $q_0 = 0.01$ GeV$^2$.
- Soft gluon resolution scale is chosen to be very small to make sure that the collinear evolution coincides with the standard DGLAP evolution.
- We study the $z_M$ dependence with $q_0$ to suppress the soft gluons
- Generate LO kernels at larger $q_0$ and perform LO fits.

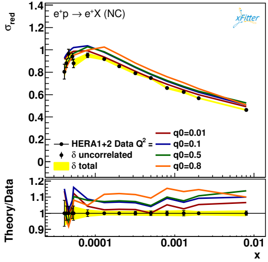
# Studying the effect of different $q_0$ on PDFs



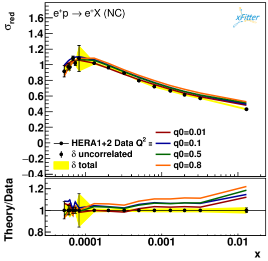

- Perform LO fit to all HERA data
- Full evolution with all flavors
- Change $q_0$
- At large $q_0$, we remove quite a lot of soft gluons so we do not have appropriate fit.

$$q_0 = 0.01 \rightarrow \chi^2/dof = 1.263$$
$$q_0 = 0.1 \rightarrow \chi^2/dof = 1.405$$
$$q_0 = 0.5 \rightarrow \chi^2/dof = 1.538$$
$$q_0 = 0.8 \rightarrow \chi^2/dof = 1.824$$

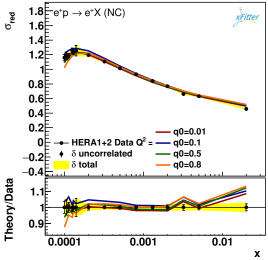
# Different $q_0$ in fit



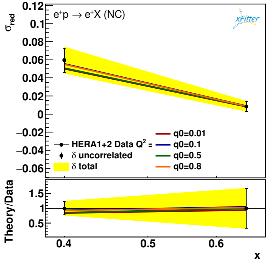

$Q^2 = 3.5 GeV^2$




$Q^2 = 4.5 GeV^2$



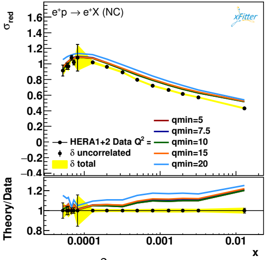

$Q^2 = 8.5 GeV^2$



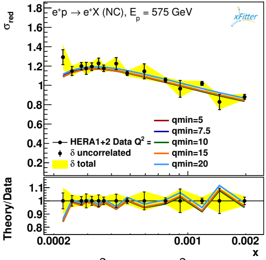

$Q^2 = 30000 GeV^2$

# Checking $q_{min}$ dependence

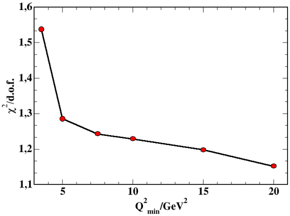
- Fits performed for different $q_{min}$ while $q_0 = 0.5$ GeV$^2$
- The larger $q_{min}$, the smaller $\chi^2/dof$.
- If we go to higher $q^2$, we get again a reasonable fit. We just sum up the large enough number of soft gluons



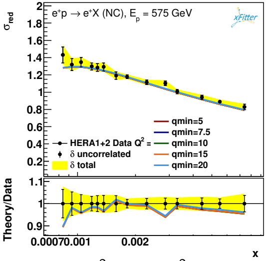

$Q^2 = 4.5 GeV$



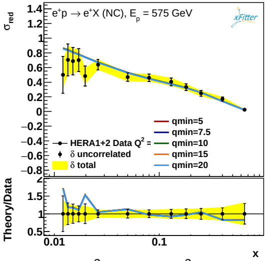

$Q^2 = 12 GeV^2$




*The dependence of $\chi^2/dof$ on $q^2_{min}$ of LO fit for $q_0 = 0.5$ $GeV^2$*




$Q^2 = 45 GeV^2$




$Q^2 = 800 GeV^2$

# Extension to include small-$x$ processes

- Enlarging the phase space to include full angular ordering
  - DGLAP ordering: $q_i > q_{i-1}$
  - Full angular ordering: $q_i > z_{i-1} q_{i-1}$
- Opening up the phase space in DGLAP scenario ($k_t < q$), we get larger $k_t$ emission ($k_t > q$). Then, we get an enhancement at small-x.
- We need the additional non-sudakov FF which suppresses the growth in small-x region.
- CCFM splitting functions at LO including the non-sudakov FF

$$P_{gg}^{(0)} = 6\,(\frac{\alpha_s}{2\pi})\,(\frac{1}{\mathbf{z}}\widetilde{\boldsymbol{\Delta}}_{\mathbf{ns}} + \frac{1}{1-z} + \ldots)\,\widetilde{\Delta}_s$$

$$P_{gq}^{(0)} = \frac{4}{3}\,(\frac{\alpha_s}{2\pi})\,(z - 2 + \frac{2}{\mathbf{z}}\widetilde{\boldsymbol{\Delta}}_{\mathbf{ns}})\,\widetilde{\Delta}_s$$

$$P_{qg}^{(0)} = \frac{1}{2}\,(\frac{\alpha_s}{2\pi})\,(z^2 + (1-z)^2)\,\widetilde{\Delta}_s$$

$$P_{q_i q_i}^{(0)} = \frac{4}{3}\,(\frac{\alpha_s}{2\pi})\,(\frac{1+z^2}{1-z})\,\widetilde{\Delta}_s$$

$$\widetilde{\Delta}_s = exp\left(-\int_{\mathbf{z_{i-1}q_{i-1}}}^{\mathbf{q_i}} \frac{dq'^2}{q'^2}\int^{z_M} dz\,\frac{1}{1-z}\right)$$

$$\widetilde{\boldsymbol{\Delta}}_{\mathbf{ns}} = exp\left(-\int_{\mathbf{z_{i-1}q_{i-1}}}^{\mathbf{k_t}} \frac{dq'^2}{q'^2}\int^{z_M} dz\,\frac{1}{z}\right)$$

S. Catani, F. Fiorani and G. Marchesini, Nucl. Phys. B **336**, 18 (1990).

# Small-x corrections to splitting functions (CCFM)

- Original CCFM includes only gluon splitting function with the singular term.
- We re-write CCFM with the full splitting function into the sudakov as we do for DGLAP.
- $\Delta_{ns}$ acts only if $k_t > q_i$ because the other region $k_t < q_i$ is already covered by the sudakov in PB approach.
- Only for $P_{gg}$ and $P_{gq}$ splitting function.

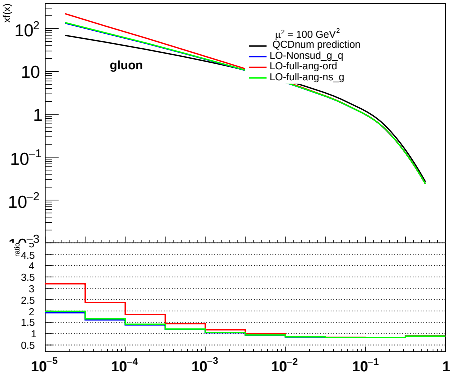
$$\widetilde{\Delta}_s \rightarrow \Delta_s = exp\left(-\int_{\mathbf{z_{i-1}q_{i-1}}}^{\mathbf{q_i}} \frac{dq'^2}{q'^2} \int^{z_M} dz\,(\frac{1}{z} + \frac{1}{1-z} + ...)\right)$$

$$\widetilde{\Delta}_{ns} \rightarrow \Delta_{ns} = exp\left(-\int_{\mathbf{q_i}}^{\mathbf{k_t}} \frac{dq'^2}{q'^2} \int^{z_M} dz\,\frac{1}{z}\right) \quad \text{for } k_t > q_i$$

$$\widetilde{\Delta}_{ns} \rightarrow \Delta_{ns} = 1 \quad \text{for } k_t < q_i$$

- How much do the PDFs would change in small-x region by including the $\Delta_{ns}$ for $k_t > q_i$?

- This is not a fit!
- QCDNUM agrees with PB method in the DGLAP limit.
- Red curve: opening up the phase space to include angular ordering. enhancement at small $x$, essentially below $10^{-3}$.
- With angular ordering we need some extra piece which covers the no branching probability from $q_i$ to $k_t$ which is not covered in the DGLAP.



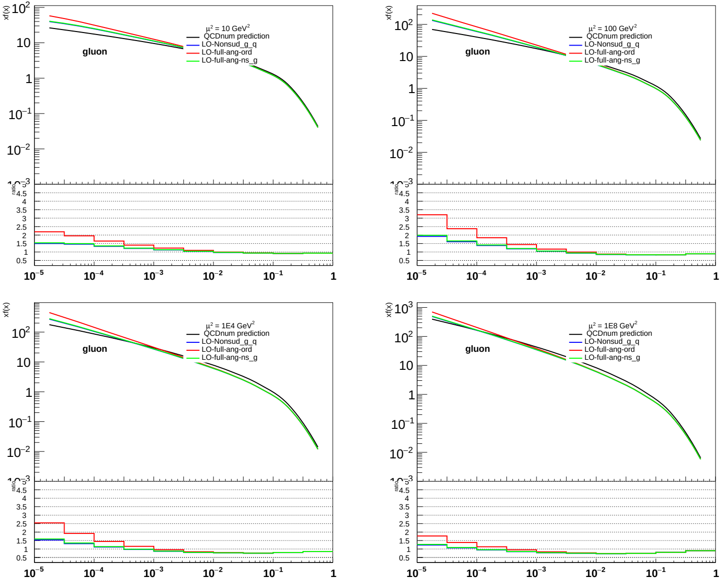
# Conclusion

- PB method to solve DGLAP equation at LO, NLO, NNLO
- Advantages of PB method
- LO fit with coupled CCFM evolution equation based on PB solution for all flavors
- Reasonable $\chi^2/dof$
- The effect of suppressing soft gluons with different $z_M$ is studied
- Small-x corrections included to the DGLAP splitting function (CCFM)

Thank you for your attention

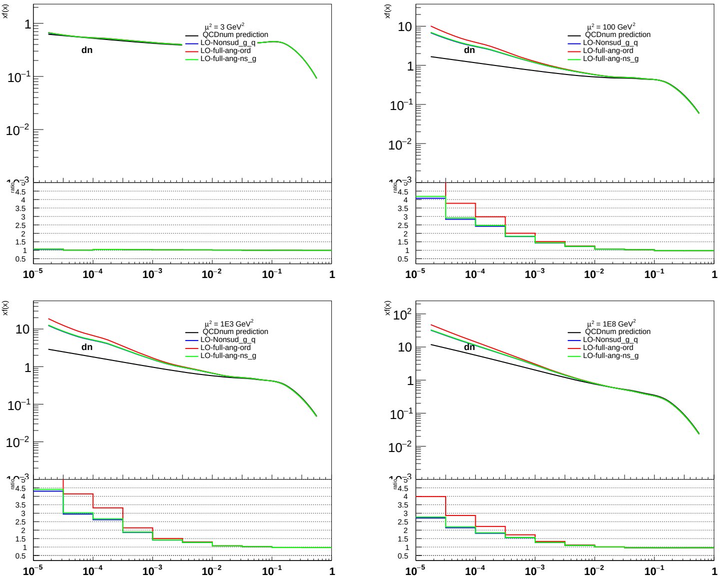
# Backup

# Gluon



gluon
μ² = 10 GeV²
μ² = 100 GeV²
μ² = 1E4 GeV²
μ² = 1E8 GeV²
QCDnum prediction
LO-Nonsud_g_q
LO-full-ang-ord
LO-full-ang-ns_g
xf(x)
ratio

# Quark



$\mu^2$ = 3 GeV$^2$
QCDnum prediction
LO-Nonsud_g_q
LO-full-ang-ord
LO-full-ang-ns_g
dn
$\mu^2$ = 100 GeV$^2$
$\mu^2$ = 1E3 GeV$^2$
$\mu^2$ = 1E8 GeV$^2$
xf(x)
ratio