Gamin Gouttenoire
28/05/19

Cosmic String I

Literature:

↳ Vilenkin & Shellard — cosmic strings and other topological defects 1994

↳ Hindmarsh & Kibble — cosmic strings - 1994

plan:

I) String field theory
1) Classification of topological defects
2) Local string
3) Global string
4) Nambu-Goto string

II) String formation and evolution
1) Kibble mecanism
2) Damped epoch evolution
3) Scaling evolution

III) Gravitational & particle production
1) Grav. prod.
2) part. prod.
3) Application to axion string network

I] String field theory

1) Topological classification of defects

↳ <u>Homotopy group</u> $\pi_n(\mathcal{M})$: set of equivalent classes of map

$$f : S^n \longrightarrow \mathcal{M}$$

Ex : $\pi_1(\text{Torus}) = \mathbb{Z}^2$ $\qquad \pi_n(S^n) = \mathbb{Z}$

$\pi_1(S^2) = 0$

↳ Spontaneous sym breaking : $\underset{\substack{\uparrow \\ \text{vacuum manifold}}}{\mathcal{M}} = G/H$

use <u>fondamental theorem</u> :

$$\pi_n(G/H) \cong \pi_{n-1}(H)$$

topol. defect	Dim	classification	example
Domain walls	2	$\pi_0(\mathcal{M})$	$\mathbb{Z}_N \to 1$ } axiomic defects
Strings	1	$\pi_1(\mathcal{M})$	$U(1) \to 1$
Monopoles	0	$\pi_2(\mathcal{M})$	$SU(5) \xrightarrow{GUT} SU(3) \times SU(2) \times U(1)$
Textures	—	$\pi_3(\mathcal{M})$	$SO(4) \to SO(3)$ $\quad \uparrow \nearrow \rightarrow \downarrow \leftarrow \nwarrow \uparrow$

↳ <u>String examples:</u>

↳ <u>PQ</u>: $U(1) \to 1$ $\quad U(1) = \mathbb{R}/\mathbb{Z} \quad \pi_1(U(1)) = \pi_0(\mathbb{Z}) = \mathbb{Z}$

↳ <u>GUT</u>: $\underset{\substack{\uparrow \\ \text{double covering of } SO(10)}}{Spin(10)} \longrightarrow SU(5) \times \mathbb{Z}_2$

$$\pi_1\left(Spin(10)/SU(5)\times\mathbb{Z}_2 \right) = \pi_0(\mathbb{Z}_2) = \mathbb{Z}_2$$

↳ <u>SM</u>: $SU(2)_\ell \times U(1)_\ell \longrightarrow U(1)_\ell$

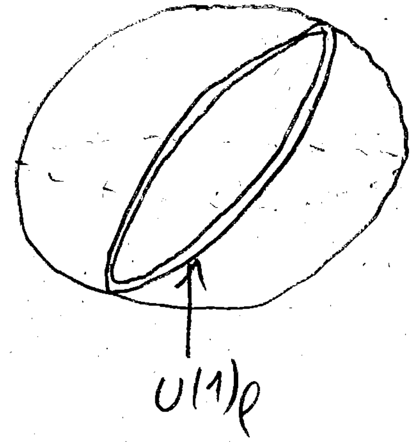
$$\pi_1\left(SU(2)\times U(1)/U(1) \right) = \pi_0(U(1)) = 1$$

$\longrightarrow$ no topological strings in SM

↳ <u>Semi-local string</u>: $\underset{\substack{\uparrow \\ \text{global}}}{SU(2)_g} \times \underset{\substack{\uparrow \\ \text{local (gauge)}}}{U(1)_\ell} \longrightarrow U(1)_g$

⊕ 2 complex scalars

$$\frac{SU(2)_g \times U(1)_\ell}{U(1)_g} :$$



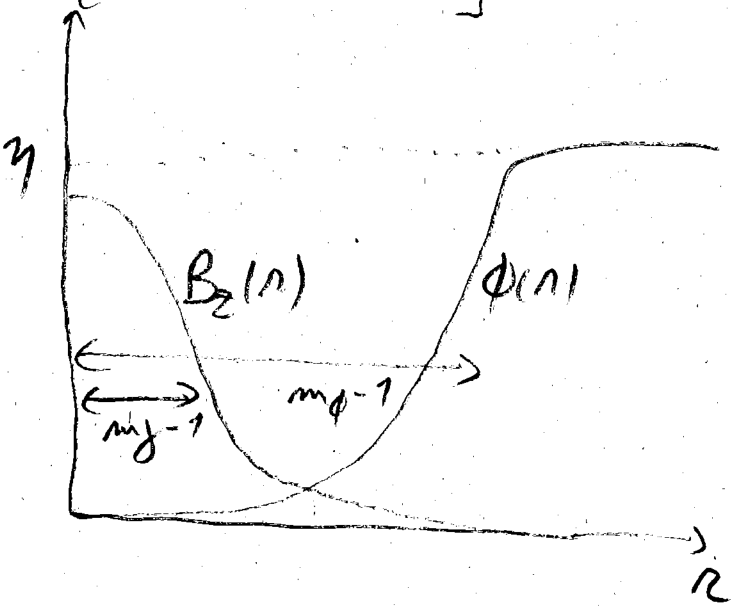
If You want to move the vacuum state out of the $U(1)_\ell$ circle, you have to overcome a large (divergent) gradient energy barrier.

$$E_g = \int dr^2 (\partial\phi)^2 = 2\pi\eta^2 \ln\left(\frac{\overset{\text{cut-off}}{\Lambda}}{m_\phi^{-1}}\right) \gg E_\ell = \int dr^2 (D\phi)^2 = 2\pi m \eta^2$$

2) Local string (abelian-Higgs model)

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\lambda\left(|\phi|^2 - \eta^2\right)^2$$

$$E = \int dr^2 \left[|D\phi|^2 + \frac{1}{2}(E^2+B^2) + V(\phi) \right]$$



Properties:

↳ energy confinement

↳ Higgs screening

$$E_{global} = \int dr^2 (\partial\phi)^2 = 2\pi\eta^2 \ln\left(\frac{\Lambda}{m_\phi^{-1}}\right)$$

Λ: cut-off

(because $\phi = \eta e^{i\varphi} \rightarrow \vec{\partial}\phi = \frac{i}{r}\phi \rightarrow |\vec{\partial}\phi|^2 = \frac{\eta^2}{r^2}$)

$$E_{local} = \int dr^2 (D\phi)^2 = 2\pi\eta^2 n$$

↳ $n = \frac{e}{2\pi}\oint \vec{A}\, d\vec{l}$

↳ $\epsilon_{int} = \underbrace{\epsilon_I}_{<0} e^{-m_\phi d} + \underbrace{\epsilon_{II}}_{>0} e^{-m_\gamma d}$

$\beta = \frac{m_\phi}{m_\gamma}$

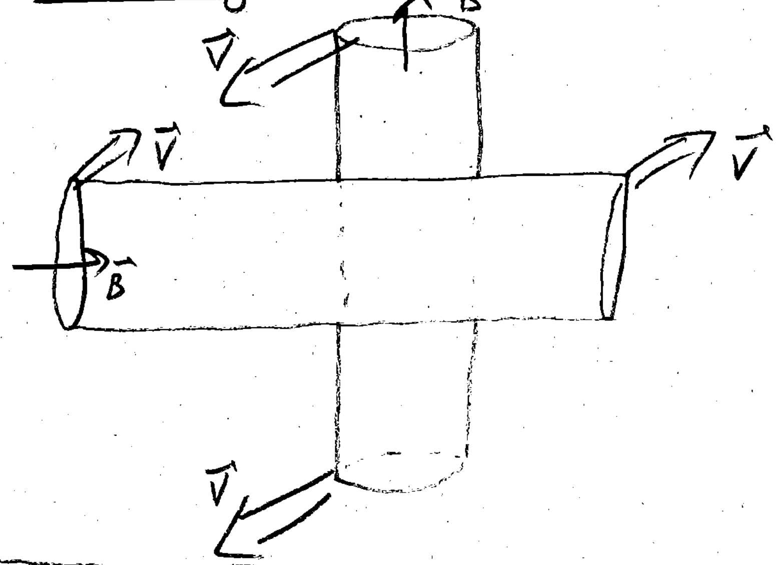
attractive — repulsive / $\overbrace{\text{attractive}}^{\text{vortex-anti-vortex}}$

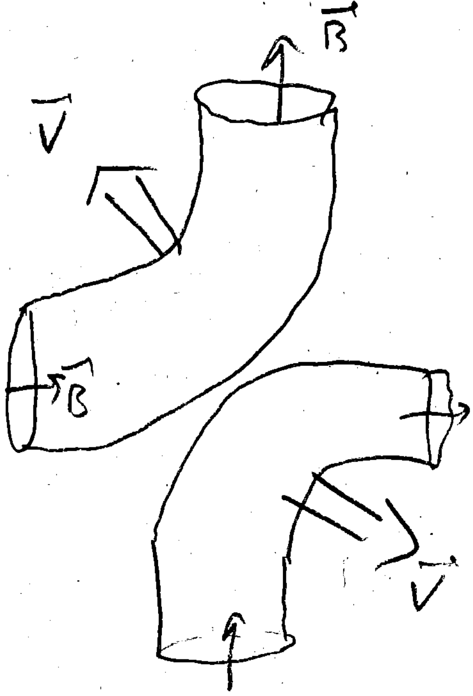
Vortex core-overlap — circulation of $\vec{B}$ field outside the vortex core

$\beta < 1$ - type I — $\beta > 1$ - type II

↳ String interactions

(1)

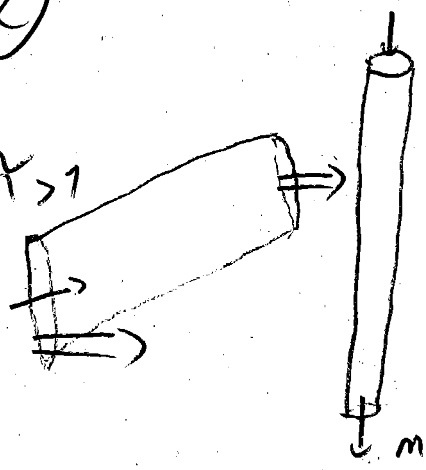


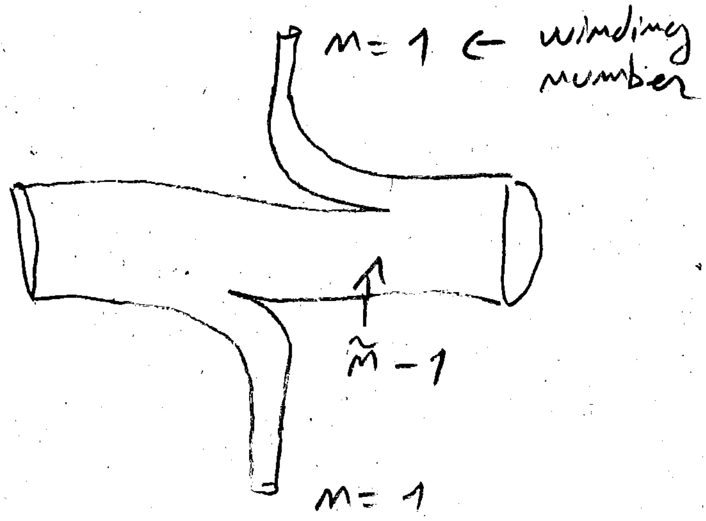


$\boxed{P_{\text{intercommutation}} = 1}$ (except if $\frac{1}{\sqrt{1-v^2}} \gg 1$)

For both type I and II

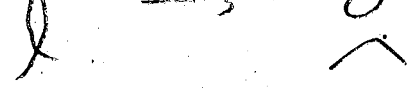
(2)





csq of <u>Peeling</u>: interact network with high winding number string rapidly break down to $n = 1$ strings

(3) <u>loop formation</u>



3) Global string

↳ no gauge field → $E = \int d^2 r \, |\partial\phi|^2 = 2\pi\eta^2 \ln\left(\frac{\Lambda}{m_\phi^{-1}}\right)$

↳ energy diverges due to existence of the massless goldstone which create a long range force.

↳ cut-off $\Lambda = H^{-1}$: causal horizon

$$E = 2\pi\eta^2 \ln\left(\frac{m_{radial}}{H}\right)$$

Ex : axion – $m_{radial} = \lambda f_a$ $\quad \lambda \sim O(1)$

$m_{axion} = \frac{\Lambda_{QCD}^2}{f_a}$

$$\phi(x) = \frac{r(x) + f_a}{\sqrt{2}} e^{i\frac{a(x)}{f_a}}$$

radial mode getting vev η whose gradient energy is confined in the vortex core

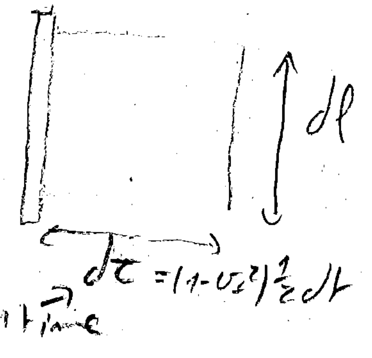
axion (goldstone) whose gradient energy spread out of the core

4) Nambu_Goto string

↳ Action of relativistic $\vec{B}$ flux line $\propto$ area of the worldsheet swept out by the motion

↳ $S = -\mu \int dt \, dl \, (1 - v_\perp^2)^{\frac{1}{2}}$

covariant form $= -\mu \int d^2\xi \sqrt{-\gamma}$



↳ $S = -\mu \int d^2\sigma \sqrt{-\gamma}$ ← classical object

$\gamma \equiv \det \gamma_{ab}$

$\gamma_{ab} \equiv g_{\mu\nu} \frac{\partial x^\mu}{\partial \sigma^a} \frac{\partial x^\nu}{\partial \sigma^b}$: induced metric

↳ String tension : μ = Energy/length

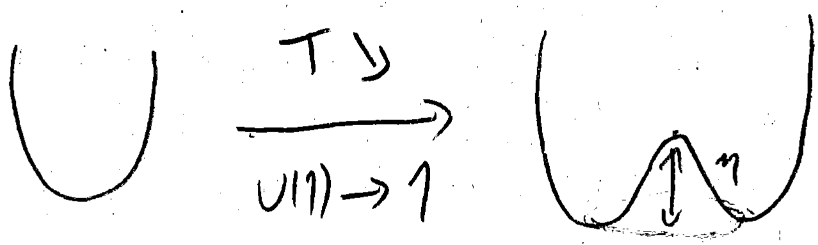
$= \int dr^2 \left(|D\phi|^2 + \dots \right)$

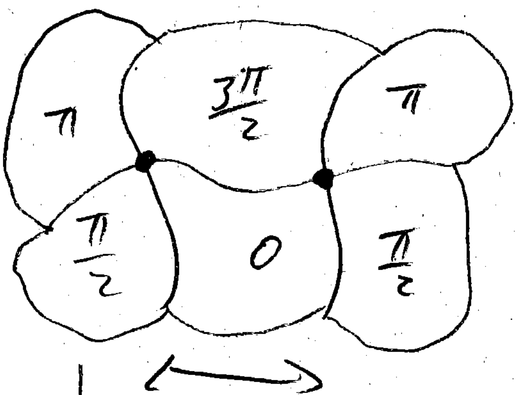
local : $\mu = 2\pi\eta^2$ ($n=1$ due to peeling)

global : $\mu = 2\pi\eta^2 \ln\left(\frac{m_\phi}{H}\right)$

II String formation and evolution

1) Kibble mecanism





ξ ≡ correlation length

↳ Upper bound from causality

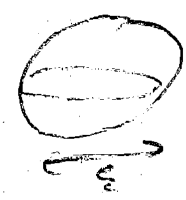
$$\xi(t) < t = H^{-1}$$

↳ At the phase transition $T_\delta \sim \eta$

$$\xi(T_\delta) \sim m_\phi^{-1} \sim (\lambda\eta)^{-1}$$

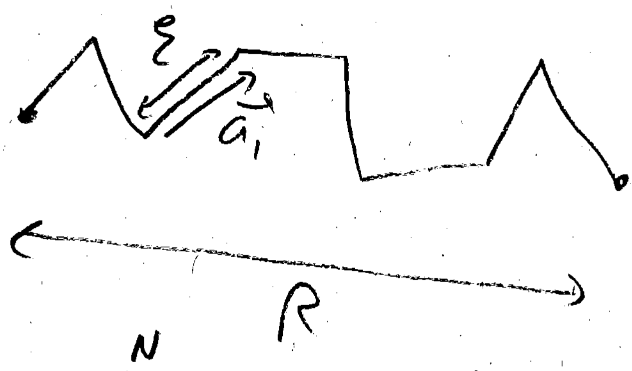
$$\xi / H^{-1} \sim \frac{1}{\lambda\eta} \frac{\eta^2}{M_{Pl}} \sim \frac{\eta}{\lambda M_{P}} \ll 1$$

For a 1st order PT



bubble size at nucleation

↳ Random walk



String length L

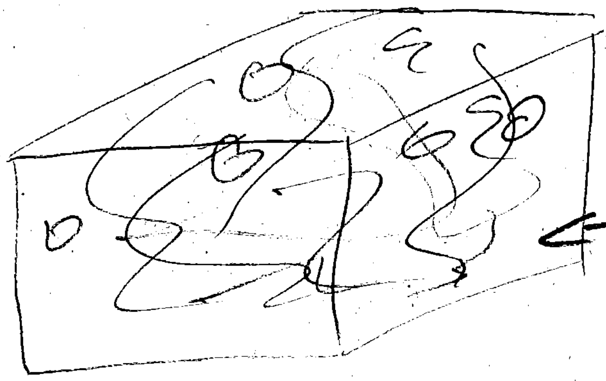
$$R = \sum_{i=1}^{N} \vec{a}_i \longrightarrow R^2 = \sum_{i=1}^{N} |\vec{a}_i|^2 + \underbrace{\sum_{i \neq j}^{N} \cancel{\vec{a}_i \cdot \vec{a}_j}}_{0 \text{ randomness}}$$

$$= N \xi^2$$

$$= L \xi$$

$$\longrightarrow L = R^2 / \xi$$

checked by simulation Vachaspati & Vilenkin 1984



← How does this evolve?

2) Damped epoch evolution

Interaction with plasma: $F_d = -\beta T^3 \vec{v}$ $\beta = O(1)$

Note: If $T < m_\phi \exp\left[\frac{1}{\kappa}\right] \rightarrow$ interaction indpt of coupling constant κ

Kinetic energy per unit length: $\mathcal{E} \sim \mu v^2$

is dissipated with rate $\dot{\mathcal{E}} = F_d v \sim T^3 v^2$

↳ characteristic damping time: $t_d \sim \frac{\mathcal{E}}{\dot{\mathcal{E}}} = \frac{\mu}{T^3}$

$\hookrightarrow$ damping stops when $t_{osc} \sim H^{-1} \sim \frac{M_{pl}}{T^2}$
oscillation starts

$\rightarrow T_{osc} \sim \frac{\mu}{M_{pl}}$

$\rightarrow t_{osc} \sim (G\mu)^{-2} t_p \sim (G\mu)^{-1} t_g$

t_p: Planck time; t_g: formation time, $T_g \sim \eta$

$G\mu$	$T_g \sim \eta \sim \sqrt{\mu}$	$T_{osc} \sim \frac{\mu}{M_p}$
10^{-6}	10^{15} GeV	10^{12} GeV
10^{-10}	10^{13} GeV	10^{8} GeV
10^{-18}	10^{9} GeV	1 GeV
10^{-24}	10^{6} GeV	1 eV

3) Scaling evolution

Q: what happens after t_{osc} ?

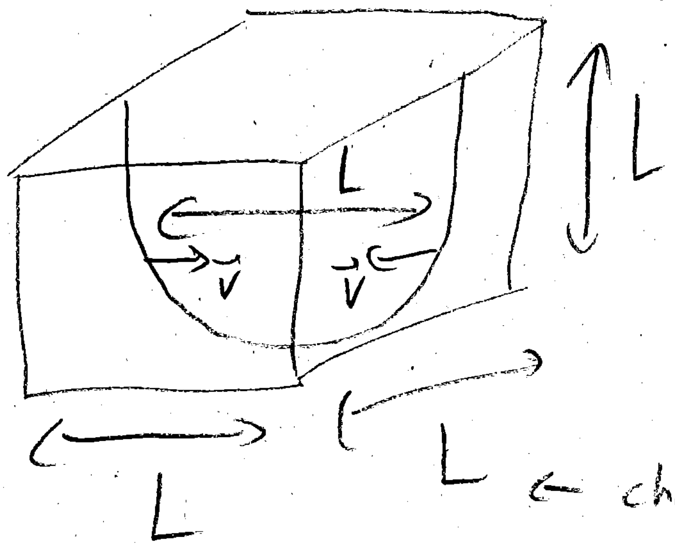
Energy of Network: $E = \frac{\mu V}{L^2}$ $\quad \rho = \frac{\mu}{L^2}$ (L: correlation scale (ξ))

$\hookrightarrow$ 2 competing dynamics

(1) Hubble expansion: $L \propto a \rightarrow \epsilon \propto a$
whereas $\epsilon_{rad} \propto a^{-1}$

(2) $\underbrace{\text{Fragment into tiny}}_{P_{intercommutation} = 1}$ loops and disappear (by gravitational or particle production)

chopping rate: segment of length L must travel L before encountering other segment in volume L^3



$$\# \text{ collisions}/\text{volume} = \frac{v\Delta t}{L} \cdot \frac{1}{L^3}$$

$\uparrow$ proba. collision

$L \leftarrow$ characteristic length (correlation length)

↳ loop of size L is formed

↳ the network loses energy μL with rate $\frac{1}{L^4} \cdot \mu L \cdot V$

↳ $$\boxed{\dot{E} = \frac{\dot{a}}{a} E - \frac{1}{L^4} \cdot \mu L \cdot V}$$

$$\rho = \frac{E}{V} \rightarrow \dot{\rho} = \frac{\dot{E}}{V} - 3\frac{\dot{a}}{a} E$$

$$\dot{\rho} = -2\frac{\dot{a}}{a} \rho - \frac{\rho}{L}$$

↳ Try $L = \gamma t$

$$\boxed{\dot{\gamma} = \frac{1}{2t}(1 - \gamma)}$$

↳ Clearly $\gamma = 1$ is a fix point

↳ if high density of string $\gamma \lesssim 1$

↳ $\dot{\gamma} > 0$: loop production becomes efficient

↳ if low density of string $\gamma \gtrsim 1$

↳ $\dot{\gamma} < 0$: loop production becomes deficient

↳ attractor : scaling evolution

$$\boxed{L \sim t}$$

↳ $\rho \propto \frac{\mu}{t^2} \rightarrow \frac{\rho H^{-3}}{\mu L} \simeq cst$

$$\frac{\rho H^{-3}}{\mu L} \equiv \# \text{ strings per Hubble patch}$$

$$\boxed{\# \text{ strings per Hubble patch is conserved}}$$

III Gravitational and particle production

1) Gravitational production (loops)

↳ $\dot{E} \sim G \left(\frac{d^3 Q}{dt^3} \right)^2 \sim G M^2 L^4 \omega^6$

$Q \sim ML^2$: quadrupole moment

$M \sim \mu L$: loop mass

$\omega \sim L^{-1}$: characteristic frequency

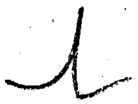
$\dot{E} \sim \Gamma G \mu^2 \qquad \Gamma \sim 50$

↳ Lifetime : $\tau \sim \frac{M}{\dot{E}} \sim \frac{L}{\Gamma G \mu}$

↳ $(G\mu)^{-1}$ oscillations !

↳ Gravitational production from infinite long string is suppressed

↳ high frequency modes come from

cusp  $P_n \propto n^{-\frac{4}{3}}$ $\quad n \equiv$ mode number

kink  $P_n \propto n^{-2}$ $\quad \omega_n = \frac{2n}{L}$

2) particle production (global loops)

↳ $\dot{E} = \Gamma \mu$ $\qquad \Gamma \sim 100$

↳ lifetime: $\tau \sim \frac{E}{\dot{E}} = \frac{2\pi L \mu \ln(L/\delta)}{\Gamma \mu}$

↳ $\tau \sim k L$ $\qquad k = 10$

$\ln \frac{L}{\delta} = 100$

↳ 20 osc before death

$\ll (G\mu)^{-1}$ oscillations for local string

Note: particle production only possible for global string loops because of existence of massless goldstone

Application | 1806.04677 Gorghetto, Hardy, Villadoro

Axion production from string network

↳ loops radiate goldstone modes with power $P \simeq \Gamma \mu$

with $\mu = 2\pi \eta^2 \ln\left(\frac{m_{radial}}{H}\right)$

$$\phi(x) = \frac{r(x) + f_a}{\sqrt{2}} e^{i\frac{a(x)}{f_a}}$$

radial mode getting very heavy, whose gradient energy is confined in the vortex core

$m_{radial} = \lambda f_a$

axion (goldstone) whose gradient energy spreads out of the core

$m_{axion} = \frac{\Lambda_{QCD}}{f_a}$

↳ N_{axion} from string network decay $\simeq \frac{E_{network}}{\langle E_{axion} \rangle} \simeq \frac{\xi \mu H^2}{\langle k_{axion} \rangle}$

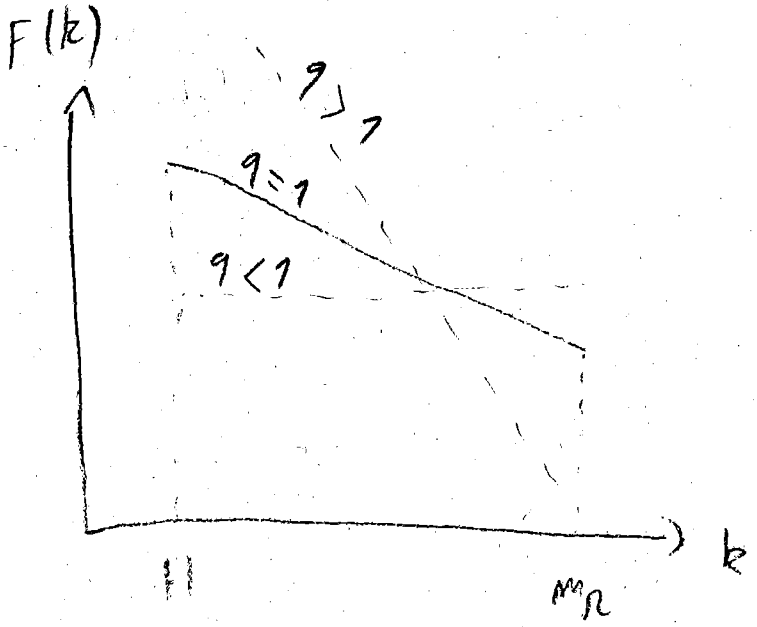
where ξ = # string per Hubble patch

↳ Scaling violation $\xi \neq$ cst

$$= 0.2 \ln\Big(\underbrace{\frac{m_{rad}}{H}}_{70 \to \text{boost factor}}\Big) + 0.1$$

$\hookrightarrow \langle k_{axion} \rangle = \int dk \; k \; F(k)$

$\uparrow$ axion spectrum



$F(k) \propto \frac{1}{k^q}$

$q \gg 1$ (soft) $\longrightarrow \langle k_{axion} \rangle \simeq H$

$q \simeq 1$ (medium) $\longrightarrow \langle k_{axion} \rangle \simeq H \ln\left(\frac{m_r}{H}\right)$

$q \lesssim$ (hard) $\longrightarrow \langle k_{axion} \rangle \ll H$