COSMIC STRINGS II : OBSERVATIONS

I. Recap : Network of Cosmic Strings

II. Spacetime around a Long Infinite String

III. Observations :
- Gravitional Lensing ⇒ Double Images
- Doupler shift ⇒ Cosmic Microwave Background (CMB) Signature
  Kaiser-Stebbins-Gott (KSG) Effect
- Cosmic String Wake
- Non-Gravitational Radiation : Cosmic Rays, Radio Bursts

IV. Strings with Small-scale Structures : Wiggle strings, Cusps, Kinks

V. Gravitational Radiation
- Stochastic Gravitational Wave Signals from Cosmic String Network
- Variety of Cosmic-String Models

Literatures :

1) Vilenkin, Shellard : Cosmic Strings and other topological defects

2) Maggoire : Gravitational Waves Vol. 2 : Astrophysics and Cosmology

3) Bailin, Love : Cosmology in Gauge Field Theory and String Theory

4) Hindmarsh, Kibble : Cosmic Strings 9411342

5) Vachaspati, Pogosian, Steer : Cosmic Strings 1506.04039v2

6) Polchinski, Cosmic String Loops and Gravitational Radiation
0707.0888v2

7) Polchinski, Introduction to Cosmic F- and D-strings 0412244
(For cosmic strings in context of string theory)

2 § Recap: Cosmic String Network

Cosmic scale is large. We can use the effective description. (Zero-thickness Limit)

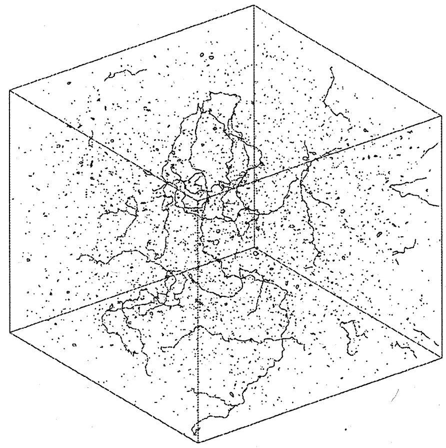
- String-liked solution from field theory
- Superstrings

} Nambu-Goto (NG) Action

Note: For global strings or superconducting strings ⇒ Kalb-Ramond Action

Relevant macroscopic parameters of string network

1) Tension μ
2) Intercommutation probability P
3) Interactions : gravitational, particle production



NG - Simulation
(Allen & Shellard, 1990)

<u>Traditional assumption for local string network</u>

- Cosmic strings were formed randomly by Kibble mech.
  with mass per length $G\mu \sim \left(\frac{\eta}{m_{Pl}}\right)^2$ ← SSB scale
- Only consider gravitational radiation/interaction
- Intercommutation probability : $P = 1$ Note: for superstring $P < 1$ since they can avoid collisions in extra-dimensions.
  └ long strings fragmented into loops
- Network maintained energy density as $\rho \sim \frac{\mu}{L^2}$ (Scaling regime)
  by losing energy through loop production and decay. ← L: correlation length scale
  - loops of length ℓ have lifetime : $\tau(\ell) \sim \frac{\ell}{\Gamma G\mu}$
  - a loop decay generate GW power : $P_{GW} \sim \Gamma G\mu^2$ ⇜ Not from quadrupole since loops oscillated relativistically. $T \sim L/2$

§ Gravity around Strings :

For static matter : $T^{\mu}{}_{\nu} = \mathrm{diag}(\rho, -P_1, -P_2, -P_3) \Rightarrow$

Einstein Eq. in Newtonian limit

$$\nabla^2\Phi = 4\pi G(\rho + P_1 + P_2 + P_3)$$

Potential

$\Rightarrow$ Non-rela. matter : $\nabla^2\Phi = 4\pi G\rho$

$\rho \gg P_i$

$\Downarrow$

For static, straight, featureless strings along z-axis

$P_3 = -\rho$, $P_1 = P_2 = 0$ $\leftarrow$ From average over string cross-section.

$\Rightarrow$ $T^{\mu}{}_{\nu} = \rho\,\mathrm{diag}(1,0,0,1) \Rightarrow \nabla^2\Phi = 0$ No grav. force on the surroundings.

$\Rightarrow$ Show that spacetime around a static, straight strings is locally flat in a full-rela. context

Consider a weak gravitational field $\Rightarrow$ Linearized theory $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$

EoM : $\Box h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^{\sigma}{}_{\sigma}\right)$ with harmonic gauge

$$\partial_{\nu}\left(h^{\nu}{}_{\mu} - \frac{1}{2}\delta^{\nu}{}_{\mu}h^{\sigma}{}_{\sigma}\right) = 0$$

For NG-string along z-axis

$$T^{\mu}{}_{\nu} = \underbrace{\mu\,\delta(x)\,\delta(y)}_{\text{due to zero-thickness limit!}}\,\mathrm{diag}(1,0,0,1)$$

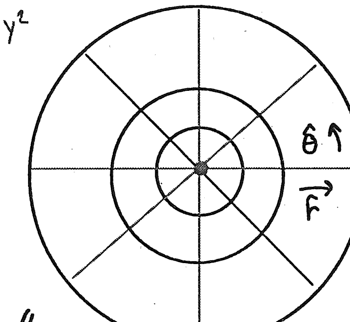
$r = \sqrt{x^2+y^2}$

$\Rightarrow$ $\underset{\text{By symm.}}{h_{00} = h_{33} = 0}$, $h_{11} = h_{22} = h = 8\pi G\mu \ln\left(\frac{r}{r_0}\right)$

r_0: integration constant

$\Rightarrow$ Metric around a long string :

$$ds^2 = dt^2 - dz^2 - (1-h)(dr^2 + r^2 d\theta^2)$$

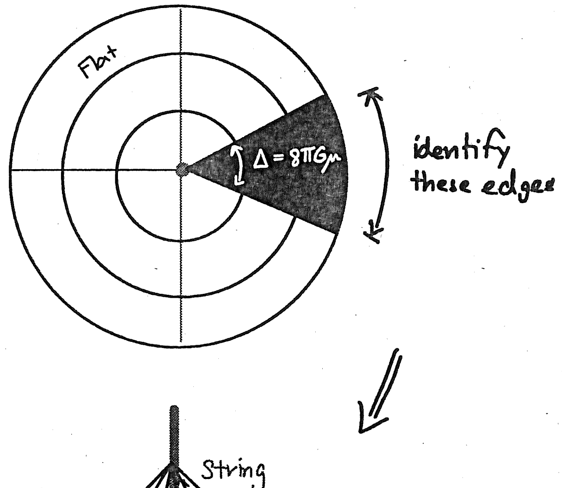


- h diverges as $r \to \infty$ due to a bad coord. choice.

Introduce a new coordinates : r', θ'

$G\mu \ll$ $\Downarrow$

$$(1-h)r^2 = (1-8G\mu)r'^2$$
$$\theta' = (1-4G\mu)\theta$$

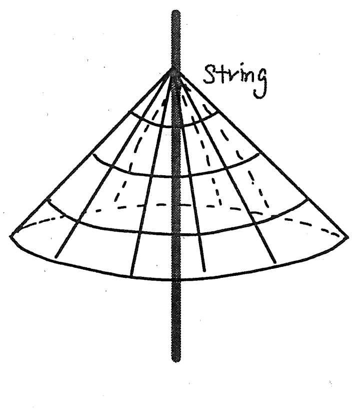


$\Rightarrow$ A locally flat spacetime around a string

$$\boxed{\begin{array}{l} ds^2 = dt^2 - dz^2 - dr'^2 - r'^2 d\theta^2 \\ \text{for } \quad 0 \le \theta' < 2\pi(1-4G\mu) \end{array}}$$

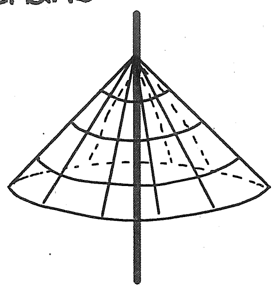
Globally "conical" spacetime with

deficit angle : $\Delta = 8\pi G\mu$
(azimuthal)

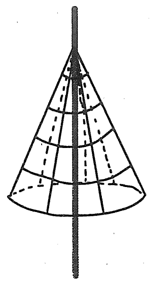


Remarks:

1)

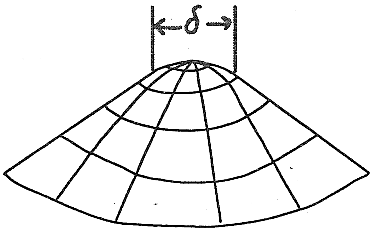


$G\mu_1 < G\mu_2$



2) The singularity gets resolved by the micro-structure i.e. local string thickness.

beyond linearized theory $\Rightarrow$ Asymthotically conical spacetime.



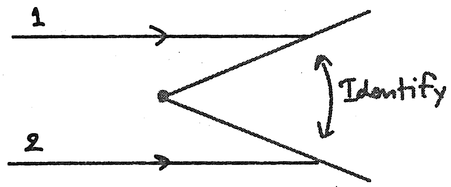
3) For global string, $T_{\mu\nu}$ is not vanishing outside the string's core due to Goldstones.

I) Deficit angle : $\Delta(r) = 8\pi G\mu(r)$, $\mu(r) \simeq 2\pi\eta^2 \ln\left(\frac{r}{\delta}\right)$

II) "Repulsive gravitational potential"

<u>Effects on test particles around a local string</u>

- Flat spacetime with a wedge of angle Δ $\Rightarrow$ Geodesic is straight line.
  $\Rightarrow$ Particle at rest remains at rest
- From global view, the moving particle gets deflected.

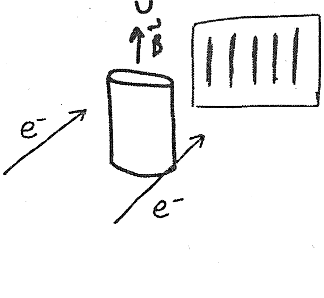


Two paths will cross at some points.

$\Downarrow$

"Felt" the existence of strings
even the spacetime curvature is confined in string's core

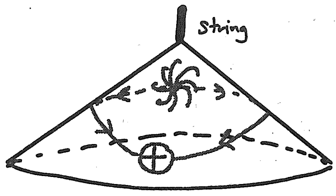
Analogous to Aharonov-Bohm effect in electrodynamics

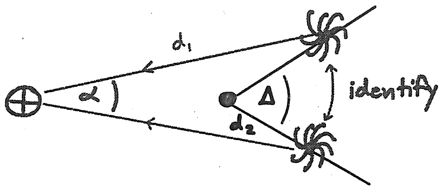


Interference-pattern change with $\vec{B}$
even $\vec{B}$ vanished outside solenoid.
$\Rightarrow$ Travelling e^- gained phase-shift.

§ Observations :

1) Lensing $\Rightarrow$ Double images of galaxies behind a string in our line of sight.





$$d_1 \alpha = d_2 \Delta$$

$\Rightarrow$ Double image of a galaxy separated by angle

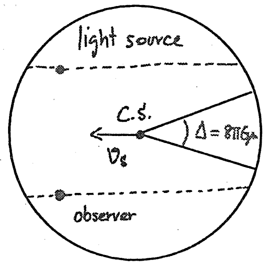
$$\boxed{\alpha = \left(\frac{d_2}{d_1}\right) \Delta}$$

2) CMB : Cosmic strings as a source of temperature fluctuation $\frac{\delta T}{T} \simeq 10^{-5}$

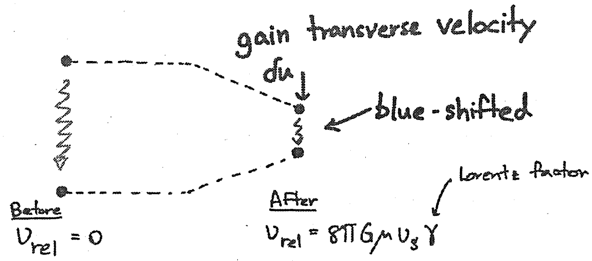
Possible effects : 1) Doppler shift of CMB signals (Kaiser - Stebbins - Gott eff.)

2) Curvature & Tensor perturbations emitted by Strings

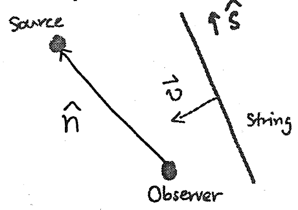
1) KSG effects : A light soure is at rest w.r.t. an observer.
A string moving with velocity v_s perpendicular to a line of sight.



$\Rightarrow$



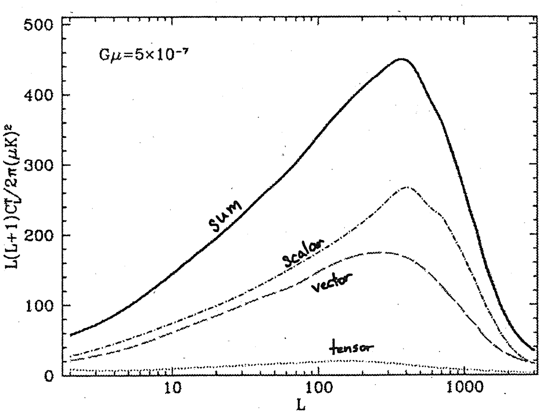
In arbitrary config. : Moving strings distorting CMB signals $\Rightarrow$ T-fluctuation



$$\boxed{\frac{\delta T}{T} = \delta u = 8\pi G\mu \gamma \, \hat{n} \cdot (\vec{v} \times \hat{s})}$$

2) Curvature & Tensor perturbations : Cosmic strings sourced CMB anisotropy before the time of recombination.

Result from Simulations



[Vachaspati & Pogosian & Steer , 2015]

No acoustic peak !

$\Downarrow$

Reason : strings randomly distributed from Kibble mechanism

$\Downarrow$

Decoherence perturbation

(Acoustic peak $\Leftarrow$ Inflation)

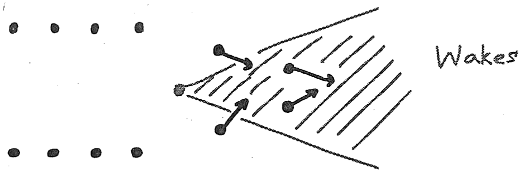
Constraints for featureless strings (smooth, NG strings with eff. string tension $\tilde{\mu}$)

Direct searches (line-liked features) : $G\tilde{\mu} \lesssim 10^{-6}$

Statistics : CMB anisotropy : $G\tilde{\mu} \lesssim 10^{-7}$ (Planck)
(Not the main source)

3) **Wakes** : particles got velocities from a moving string.

$\Rightarrow$ Wakes of matter flow behind the strings.



Observations : - 21 - cm radiation
- LSS

4) **Non - Grav. radiations** : for strings coupled to other forces (More model-dependent)

I) Cosmic rays : particles coupled to strings

Particle fluxes from kinks & cusps $\propto (G\mu)^{-1}$ [Bhattacharjee & Sigl , '99]

- strings with less tension fragmented into more loops $\Rightarrow$ More P. production $P_{pp} \propto L^{-1}$
- Less gravitational radiation $P_{GW} \sim \Gamma G\mu^2$ $\Rightarrow$ More loops @ given time

More loops @ a given time $\Rightarrow$ More particles fluxes from cusps/kinks

$\Rightarrow$ Lower bound on $G\mu$

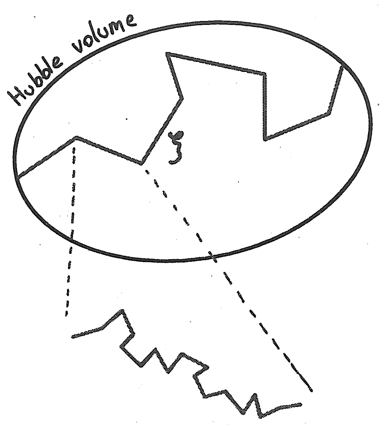
[Detailed calculation : peak flux is @ $G\mu \sim 10^{-15}$. [Bhattacharjee, '89]
of loop-production But too small to exceed the observation.

Below this $G\mu$, loop decay dominated by P.P.
(rate P.P. > rate G.radiation)
$\Rightarrow$ less loop density @ given time $\Rightarrow$ less P. flux. !]

II. Radio bursts : photons coupled to strings

As beams from cusps, kinks - emission.

§ Microstructures on Cosmic strings



At formation time

Characteristic scale : "correlation length" ξ

determined by Kibble mechanism.

After strings' evolution + intercommutation

$\Rightarrow$ Cusps, Kinks, Wiggles developed

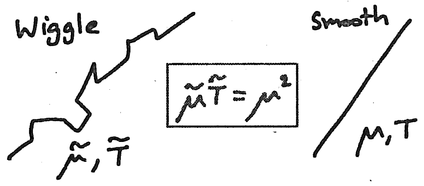
Normally, these structures are unable to resolve. An observer sees smooth strings, except the direct effects from them e.g. cusp / kink burst $\Rightarrow$ GW, cosmic rays
small loops production
* gravitational radiation of an infinite string

1) <u>Wiggle Strings</u> : After some times, smooth strings become more wiggle.

For observers, strings appear smooth with

1) Effective mass per length $\tilde{\mu} > \mu$

2) Effective tension $\tilde{T} < T$



Effects from $\tilde{\mu}$:

1) Gravitation potential : $T_{\mu\nu} = \delta(x)\delta(y)\,\mathrm{diag}(\tilde{\mu}, 0, 0, \tilde{T}) \Rightarrow \Phi = 2G(\tilde{\mu} - \tilde{T}) \ln(r/r_0)$

2) Lensing : deficit angle $\tilde{\Delta} = 8\pi G\tilde{\mu}$

3) Gravitational radiation : $\dfrac{dP^{\infty}_{GW}}{dl} \sim 2\pi G(\tilde{\mu} - \mu)^2 \bar{\omega}$ average characteristic radiation frequency

$\Downarrow$

$$\frac{\Omega^{\infty}_{GW}}{\Omega^{loop}_{GW}} \sim \frac{4\pi^2}{\Gamma}\left(\frac{\Gamma G\mu}{\alpha}\right)^{3/2}$$

scale of loops and structure of wiggles

$\Rightarrow$ Two sources are comparable for very small microstructu (small α)

Note : 1) For strings with no structure, $\tilde{\mu} = \mu \Rightarrow$ No gravitational radiation.

2) For small α, infinite strings act as many small loops.

3) For large α, amplitudes of oscillation is maximal for loops.

String Dynamics in Minkowski spacetime

Coordinates of strings : $x^0 = t$, $\vec{X}(\sigma, t)$ ← String worldsheet coordinates

EoM : $\ddot{\vec{X}} - \vec{X}'' = 0$ ⟵ $\ddot{\vec{X}} \sim R^{-1}$ (curvature)$^{-1}$ ⇒ String tends to smoothen.

Transverse Gauge : (I) $\dot{\vec{X}} \cdot \vec{X}' = 0$, (II) $\dot{\vec{X}}^2 + \vec{X}'^2 = 1$

Velocity of string $\dot{\vec{X}}$ is perpendicular to string

To have $d\sigma \propto dE$,

$\vec{x}' = (1 - \dot{\vec{x}}^2)^{\frac{1}{2}} \Rightarrow d\sigma = (1 - \dot{\vec{x}}^2)^{-\frac{1}{2}} |d\vec{x}| = \frac{1}{\mu} dE$

where $E = \mu \int (1 - \dot{\vec{x}}^2)^{-\frac{1}{2}} |d\vec{x}|$

Solution : $$\boxed{\begin{gathered} \vec{X}(\sigma, t) = \frac{1}{2}\left[\vec{a}(\sigma - t) + \vec{b}(\sigma + t)\right] \\ \text{with } \vec{a}'^2 = \vec{b}'^2 = 1 \end{gathered}}$$

Loop Oscillation : $0 \leq \sigma < L$ ← length $L = \frac{E}{\mu}$

Periodic condition : $\vec{X}(\sigma + L, t) = \vec{X}(\sigma, t)$

$$\Rightarrow -\vec{a}(\sigma - t + L) + \vec{a}(\sigma - t) = \vec{b}(\sigma + t + L) - \vec{b}(\sigma + t) = \Delta \quad \text{(some constant vector)}$$

can set since $\vec{b}, \vec{a}$ are ind.

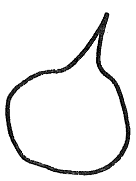
Consider the momentum of string $P = \mu \int d\sigma\, \dot{\vec{X}}(\sigma, t) = \frac{\mu}{2} \int_0^L d\sigma\, (b' - a') = \mu \Delta$

In CM Frame, $P = 0 \Rightarrow \Delta = 0 \Rightarrow$ $\vec{a}(\sigma - t + L) = \vec{a}(\sigma - t)$
$\vec{b}(\sigma + t + L) = \vec{b}(\sigma + t)$

⇨ $\vec{X}(\sigma + \frac{L}{2}, t + \frac{L}{2}) = \vec{X}(\sigma, t)$ ⇒ $\boxed{\text{Period of oscillation } T = \frac{L}{2}}$ ⇒ $f_n = \frac{2n}{L}$

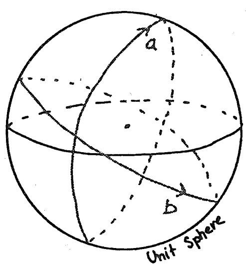
$T \sim L$ ⇒ String's motion is Relativistic !

2) **Cusp** : a point where string moved at light speed. (Generally, it developed in loops.)



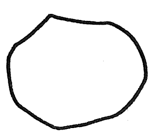
From $\dot{\vec{X}}^2(\sigma, t) = \frac{1}{4}\left[\vec{a}'(\sigma - t) - \vec{b}'(\sigma + t)\right]^2 = \frac{1}{4}\left[\vec{a}'^2 + \vec{b}'^2 - 2\vec{a}' \cdot \vec{b}'\right] = 1$

$\vec{a}'^2 = \vec{b}'^2 = 1$
$a(\sigma - t + L) = a(\sigma - t)$
$b(\sigma + t + L) = b(\sigma + t)$



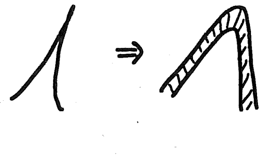
⇒ Generally, these paths intersect.
$\vec{a} = \vec{b}$

3) **Kinks** : a point where $\vec{a}'$, $\vec{b}'$ are discontinuous.



! loops with kinks can avoid the intersection between $\vec{a}', \vec{b}'$.
↑
Not likely to develop cusps.

Note : • These singular structures can be resolved and smoothened by the underlying field description.



§. Gravitational Radiation

The energy density spectrum : $\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df}\Big|_{t_0}$

In case of GW from cosmic-string loops : $\frac{d\rho_{GW}}{d\tilde{f}} = \int_{t_F}^{t_0} d\tilde{t}\, P_{GW} \frac{dN(\tilde{f},\tilde{t})}{d\tilde{f}}$

Number of loops density radiated at time $\tilde{t}$ with frequency f (Model dependent)

Radiation power per loop : $P_{GW} = \Gamma G\mu^2$

All GW that radiated since network formation at t_F until today t_0

Consider red-shift

1) loops created at time t_i ⇒ Decay into GW at time $\tilde{t}$

$$N(\tilde{f}, t_i) \longrightarrow N(\tilde{f},\tilde{t}) = \underline{N(\tilde{f},t_i)} \left[\frac{a(t_i)}{a(\tilde{t})}\right]^3$$

2) GW detected today with frequency: $f = \tilde{f}\left[\frac{a(\tilde{t})}{a(t_0)}\right]$

3) GW energy density : $\rho_{GW} \propto a^{-4}$ (as radiation)

Example : Velocity-dependent One-Scale (VOS) Model + NG simulation

For loop created at time t_i had size $l \sim \alpha t_i$

⇒ loop shrinked by G. radiation $l(t) = \alpha t_i - \Gamma G\mu (t - t_i)$ and radiated with freq. $\tilde{f} = \frac{2k}{l}$

Loops number density at time t_i : $\frac{dn}{dt_i} = (0.1)\frac{C_{eff}(t_i)}{\alpha t_i^4}$ loop production efficiency

$$\Rightarrow \quad n(l,t_i) = \frac{dn}{dt_i}\cdot\frac{dt_i}{dl} = (0.1)\frac{C_{eff}(t_i)}{t_i^4}\cdot\frac{1}{\alpha(\alpha+\Gamma G\mu)}$$

$$\Rightarrow \quad N(\tilde{f},t_i) = \sum_{k=\text{mode}} n(l,t_i)\, dl(\tilde{f},t_i)$$

$$\Rightarrow \quad \frac{dN(\tilde{f},t_i)}{d\tilde{f}} = \sum_k n(l,t_i)\left(\frac{2k}{\tilde{f}^2}\right) \Rightarrow \frac{dN(\tilde{f},t_i)}{d\tilde{f}} = \frac{dN(\tilde{f},t_i)}{d\tilde{f}}\cdot\left[\frac{a(t_i)}{a(\tilde{t})}\right]^3$$

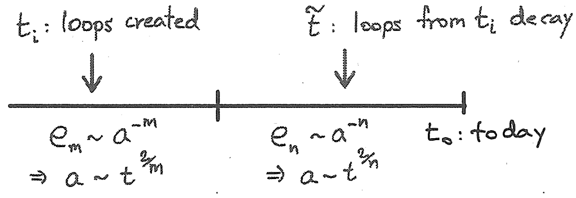
⇨ GW energy density spectrum from cosmic strings generated at time t_F

$$\boxed{\Omega_{GW}(f) = \sum_k \frac{1}{\rho_c}\cdot\frac{2k}{f}\cdot\frac{(0.1)\Gamma^{(k)}G\mu^2}{\alpha(\alpha+\Gamma G\mu)}\int_{t_F}^{t_0} d\tilde{t}\,\frac{C_{eff}(t_i)}{t_i^4}\left[\frac{a(\tilde{t})}{a(t_0)}\right]^5\left[\frac{a(t_i)}{a(\tilde{t})}\right]^3\Theta(t_i - t_F)}$$

where

$$\Gamma^{(k)} = \begin{cases} \dfrac{\Gamma k^{-4/3}}{\sum_{m=1}^{\infty} m^{-4/3}} & \text{for cusp loops} \\[2ex] \dfrac{\Gamma k^{-5/3}}{\sum_{m=1}^{\infty} m^{-5/3}} & \text{for kink loops} \end{cases}$$

Behavior of GW energy density spectrum



$$\Omega_{GW}(f) \propto \frac{1}{f}\int_{t_i}^{t_0} d\tilde{t}\, \frac{1}{t_i^4}\, a(\tilde{t})^5 \left[\frac{a(t_i)}{a(\tilde{t})}\right]^3$$

$a_n \sim t^{2/n}$, $a_m \sim t^{2/m}$

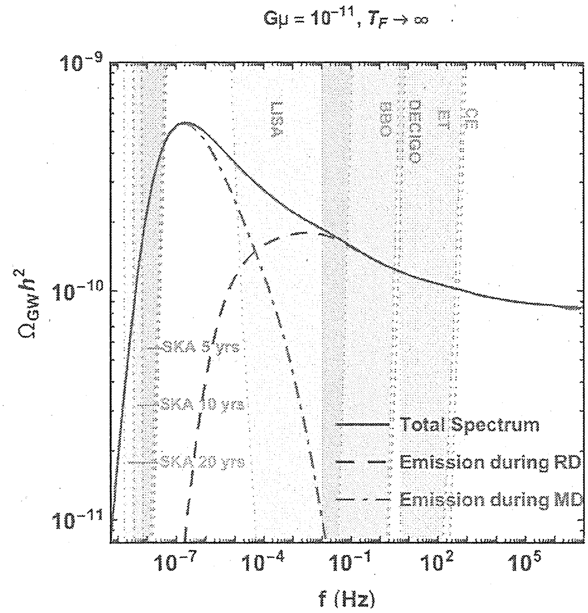
$$\propto \frac{1}{f}\int_{t_i}^{t_0} d\tilde{t} \cdot \tilde{t}^{4/n}\, t_i^{(6/m - 4)}$$

$t_i \sim \frac{a(\tilde{t})}{f}$

$$\propto f^{3-\frac{6}{m}} \int_{t_i}^{t_0} d\tilde{t}\ \tilde{t}^{\left(\frac{12}{mn} - \frac{4}{n}\right)}$$

$\tilde{t} \sim \frac{a(\tilde{t})}{f} \Rightarrow \tilde{t} \sim f^{\frac{n}{2-n}}$

$$\boxed{\Omega_{GW}(f) \propto f^{\frac{2m(n-1)-6n}{m(n-2)}}}$$



For loops created in RD and decayed in RD

$\Rightarrow (m,n) = (4,4) \Rightarrow \Omega_{GW}(f) \sim f^0$ (flat !)

For loops created in RD and decayed in MD

$\Rightarrow (m,n) = (4,3) \Rightarrow \Omega_{GW}(f) \sim f^{-\frac{1}{2}}$

Note : GW spectrum depends on history of the dominating energy density $e_n \sim a^{-n}$.

The future GW experiments $\Rightarrow$ probe the history of our Universe!

Cosmic Strings Models

I) Effective Strings : NG action: zero-thickness + Only gravitational decays

- No microstructure : corr. length scale $\xi \Rightarrow$ loops produced with length $\ell \sim \alpha t \sim (0.1)t$

  e.g. one-scale model [Kibble], Velocity-dep. one-scale (VOS) model [Villenkin, Shellard]

- included microstructures : loops with many sizes $\gamma_b \lesssim \ell < \alpha t$

  Gravitational backreaction scale

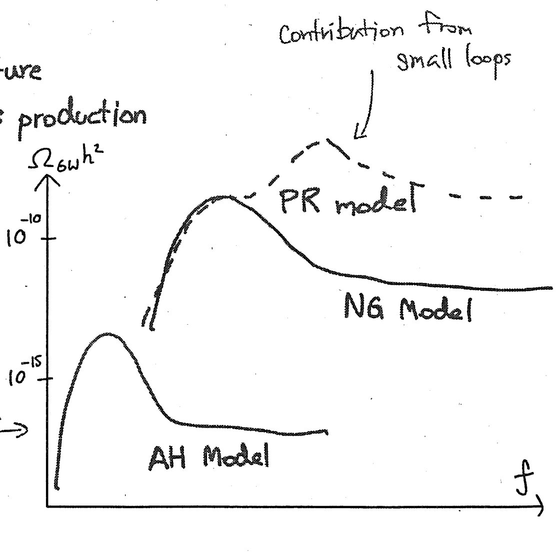
  e.g. Polchinski - Rocha model [Polchinski, Rocha, Ringeva]

II) Strings from fields

Resolved string thickness $\Rightarrow$ Included all microstructure
Included other channels of string interaction : particles production

e.g. Abelian - Higgs Model [Hindmarsh, Daverio]

Global $O(N) \to O(N-1)$ defect [Durrer, Figueroa]



GW amplitude suppressed since most of loops decay into "particles".