The Sphaleron Awakens

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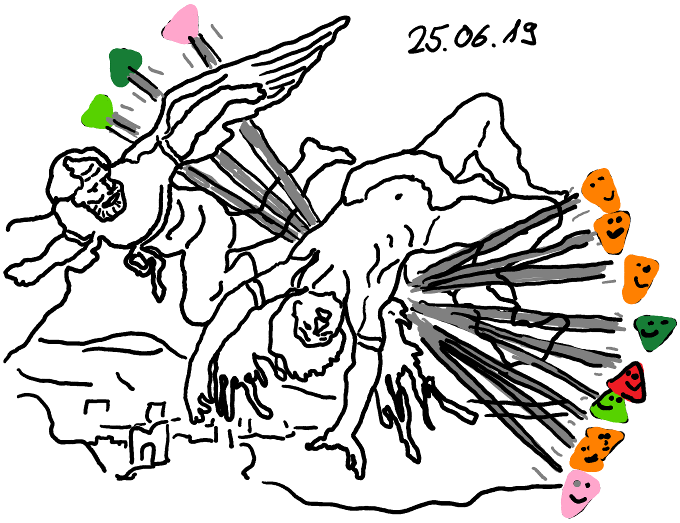
25.06.19



Prélude

Sphaleron

greek "ready to fall"

coined by Manton, Klinkhamer 1984

Instanton vs. Sphaleron

Last time we learned:

- Instantons are of use when calculating tunneling amplitudes (between topologically distict vacua, e.g θ vacuum) (at zero energy & temperature)

- Relation between anomalous Non-conservation of Baryon number B and change of topological charge Q

$$\Delta B = \Delta Q$$

- At zero temperatures tunneling amplitude supressed:

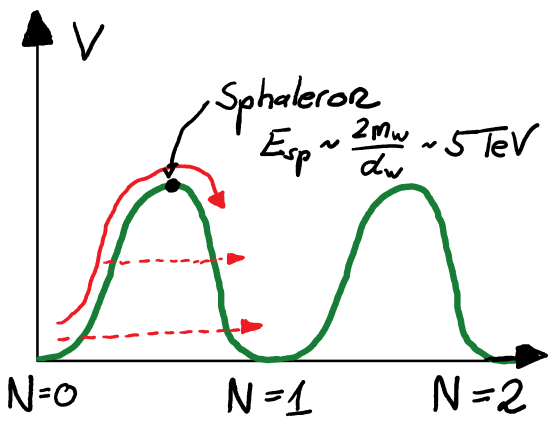
$$\Gamma_{inst} \propto e^{-S_{inst}} = e^{-\frac{4\pi}{\alpha_w}} \sim 10^{-170}$$

Instanton:

Lowest Euclidean action configuration with $\Delta B = 1$

Sphaleron:

Lowest static energy configuration with $\Delta B = \frac{1}{2}$.



Literature Roadmap

Numerical

R.F. Dashen, B. Hasslacher, A. Neveu
Nonperturbative Methods and extended hadron models in field theory. III. Four-dimensional non-Abelian models
Phys. Rev. D 10 (1974)

Topological

C.H. Taubes
The Existence of a non-minimal solution to the SU(2) Yang-Mills-Higgs equations on $\mathbb{R}^3$
Commun. Math. Phys. (1982) 86: 257

N. S. Manton
Topology in the Weinberg-Salam theory
Phys. Rev. D 28, 2019 (1983)

P. Forgács, Z. Horváth
Topology and saddle points in field theories
Phys. Lett. 138B (1984) 397

W boson
1983

[1]
F. R. Klinkhamer, N. S. Manton
A saddle-point solution in the Weinberg-Salam Theory
Phys. Rev. D 30, 2212 (1984)

Early universe

V. A. Kuzmin, V. A. Rubakov, M. E. Shaposhnikov
On anomalous electroweak baryon-number non-conservation in the early universe
Phys. Lett. 155B (1985) 36

V. A. Rubakov, M. E. Shaposhnikov
Electroweak Baryon Number Non-Conservation in the early universe and in high energy collisions
hep-ph/9603208

Pedagogical papers

[2] P. B. Arnold, L. D. McFerran
The Sphaleron strikes back
Phys. Rev. D37 (1988)

[3] P. B. Arnold
An introduction to Baryon Violation in the Standard Electroweak theory (1990)

Historical questions: **A.** In order to overcome potential barrier one needs gauge fields with energies $E_0 \sim \frac{M_W}{\alpha_W}$.

- Tunneling is highly supressed ('t Hooft 76) $\frac{S}{\hbar} \sim \frac{1}{\alpha_W}$
  also at finite temperature (Gross et al '81)
- Finite temperature: Go over barrier classically $\rightarrow \exp(-\beta E_0)$

$\longrightarrow$ Are both claims consistent?

B. What about the High energy limit?

$\longrightarrow$ Still some controversy (Tye, Wong 2015)

Contents [1,2,3]

I. Baryon number & instantons

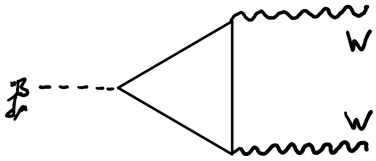
SM: Baryon number conservation at tree level

$$Q_i = e^{i\frac{\alpha}{3}} Q_i \qquad (u_i^c, d_i^c) = e^{-i\frac{\alpha}{3}}(u_i^c, d_i^c)$$

$$j_B^\mu = \frac{1}{3}\left[\sum_i \overline{Q}\,\frac{1}{2}(1-\gamma^5)\gamma^\mu Q - \overline{u}_i^c\,\frac{1}{2}(1-\gamma_5)\,u_i^c\right]$$

- Anomalous non-conservation due to triangle anomaly

$\langle WW|\partial^\mu j_\mu^B|0\rangle$



$$\partial_\mu j_B^\mu = \frac{g^2}{32\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,F^{\mu\nu}F^{\rho\sigma}$$

$$= \frac{g^2}{16\pi^2}\,F^{\mu\nu}\tilde{F}^{\rho\sigma}$$

$F\tilde{F}$ is a total divergence $\qquad F\tilde{F} = \partial_\mu K^\mu$

$$K^\mu = \epsilon_{\mu\nu\rho\sigma} \, \mathrm{Tr}\left[A_\nu \partial_\rho A_\sigma + \tfrac{2}{3} A_\nu A_\rho A_\sigma \right]$$

$$\tilde{j}_5^\mu = j_5^\mu - \frac{g^2}{16\pi^2} K^\mu$$ conserved but not gaugeinvariant

Applies equally to baryon number & lepton number

$$N[A] = \frac{1}{32\pi^2} \int d^4x \, \mathrm{Tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu} \qquad \Delta N_f = N[A]$$

"Selection rule"

The number of fermions changes by the same amount for every species $\Delta N_e = \Delta N_\mu = \Delta N_\tau = N[A]$

$$\Delta B = \frac{1}{3} \cdot 3 \cdot 3 \cdot N[A]$$

$B - L$ conserved $\qquad B + L$ violated

Bottomline : Baryon number violation in the SM is due to winding of weak gauge fields relating to the anomaly. Change in winding number is mediated by instanton transitions.

Comment on gauge choice

$A_0 = 0$ $\qquad$ In this gauge $\exists$ discrete set of classical vacua

i.e. pure gauge transformations

$$A_i = \omega \partial_i \omega^{-1} \qquad \phi = \omega \phi_0 \qquad \phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$ In trivial vacuum

$$\omega = \omega(\vec{x})$$

$$n[\omega] = -\frac{1}{24\pi^3}\int d^3x\ \epsilon^{ijk}\ \mathrm{Tr}\left(\omega\,\partial_i\omega^{-1}\cdot\omega\,\partial_j\omega^{-1}\cdot\omega\,\partial_k\omega^{-1}\right)$$

$$N[A] = \int d^3x\, dt\ \partial_\mu K_\mu = \left[\, d^3x\ K_0\right]_{t=-\infty}^{t=\infty} = n\left[\omega_{t=+\infty}\right] - n\left[\omega_{t=-\infty}\right]$$

Processes that mediate baryon number violation

1. $T=0,\ E=0$ Instantons $N[A]=1$

$$\text{rate}\quad \Gamma \sim \max\left|e^{-S_E}\right|^2$$

S_E is minimized by eucl. e.o.m e.g.

$$A^a_\mu = \frac{2}{g}\,\frac{\eta^a_{\mu\nu}\,x^\nu}{(x^2+R^2)}$$

$$S_E = \frac{2\pi}{\alpha_W} \qquad \frac{g^2}{16\pi^2}\int d^4x\ F\tilde{F} = 1$$

$$\Gamma_{inst} \propto e^{-S_{inst}} = e^{-\frac{4\pi}{\alpha_W}} \sim 10^{-170}$$

2. Calorons (finite temperature instantons)

[Gross, Pisarski, Yaffe, Rev. Mod. Phys 53 (1981) 43]

Zero temperature path integral rotated by $i \to -i\tau$

$$\int [\mathcal{D}\phi]\ \exp\left[-\int_{-\infty}^{\infty} d\tau\ \mathcal{L}_E(\phi)\right]$$

The equivalent at finite temperature is the partition function Z

$$Z = \mathrm{Tr}\, e^{-\beta H} = \sum_i \langle i | e^{-\beta H} | i \rangle$$

Same as evolution operator but evolves for imaginary time $i\beta$

$$Z = \int [\mathcal{D}\phi]_\beta \exp\left[i \int_0^{i\beta} dt\, L(\phi) \right] = \int [\mathcal{D}\phi]_\beta \exp\left[- \int_0^\beta d\tau\, L_E(\phi) \right]$$

Path integral evaluated over periodic paths : $\phi(\vec{x}, \tau=\beta) = \phi(\vec{x}, \tau=0)$

Periodicity constraint implements the trace constraint.

$$\Delta N = \frac{g^2}{16\pi^2} \int_0^\beta d\tau \int d^3x\, \mathrm{tr}\, F\tilde{F} = 1 \qquad S_E = \int_0^\beta d\tau\, L_E = \frac{2\pi}{\alpha}$$

S_E becomes singular in High-T limit.

3. Real time evolution $\to$ Sphaleron! In Euclidean time the sphaleron does not wind

II. The $SU(2) \times U(1)$ Sphaleron

Energy functional for EW theory

$$E = \int \left[\frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{4} f_{ij} f_{ij} + (D_i \varphi)^\dagger (D_i \varphi) + V(\varphi) \right] dx^3$$

$$F_{ij} = \partial_i W_j^a - \partial_j W_i^a + g \, \epsilon^{abc} W_i^b W_j^c$$

$$f_{ij} = \partial_i a_j - \partial_j a_i$$

$$D_i \varphi = \partial_i \varphi - \frac{1}{2} i g \, \sigma^a W_i^a \varphi - \frac{1}{2} i g' a_i \varphi$$

$$V(\varphi) = \lambda \left(\varphi^\dagger \varphi - \frac{v^2}{2} \right)^2$$

Weak mixing angle

$$\tan \Theta_W = \frac{g'}{g} \qquad \left. \begin{matrix} g' = 0.35 \\ g = 0.65 \end{matrix} \right\} \; \Theta_W = 0.51 \sim 30^\circ$$

$$M_W = \frac{1}{2} g v \qquad M_Z = \frac{g+g'}{2} v \qquad M_H = \sqrt{2 \lambda v^2}$$

Equations of motion:

$$(D_j F_{ij})^a = -\frac{1}{2} i g \left[\varphi^\dagger \sigma^a D_i \varphi - (D_i \varphi)^\dagger \sigma^a \varphi \right] \qquad (1)$$

$$\partial_j f_{ij} = -\frac{1}{2} i g' \left[\varphi^\dagger D_i \varphi - (D_i \varphi)^\dagger \varphi \right] \qquad (2)$$

$$D_i D_i \varphi = 2 \lambda \left(\varphi^\dagger \varphi - \frac{v^2}{2} \right) \varphi \qquad (3)$$

with $(D_j F_{ij})^a = \partial_j F_{ij}^a + g \, \epsilon^{abc} W_j^b F_{ij}^c$

Idea: Solve $SU(2) \times U(1)$ in orders of θ_W

Go to limit $g' \to 0$ $SU(2)$ only! $\boxed{\theta_W = 0}$

$$W_i^a \sigma^a dx^i = -\frac{2i}{g} f(gvr)\, dU^\infty (U^\infty)^{-1} \qquad \xi \equiv gvr$$

$$\varphi = \frac{v}{\sqrt{2}} h(gvr)\, U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U^\infty = \frac{1}{r} \begin{pmatrix} z & x+iy \\ -x+iy & z \end{pmatrix} \quad \text{or} \quad (U^\infty)' = U_L U^\infty U_R$$

$$U_{L,R} \in \text{Mat}(SU(2))$$

A N S A T Z

$\longrightarrow$ Energy density spherically symmetric

New field equations $\qquad \frac{4\pi v}{g} \simeq 5\ \text{TeV}$

$$E = \frac{4\pi v}{g} \int_0^\infty d\xi \left(4 \frac{df}{d\xi} + \frac{8}{\xi^2} \left[f(1-f) \right]^2 + \frac{\xi^2}{2} \left[\frac{dh}{d\xi} \right]^2 + \right.$$

$$\left. + \left[h(1-f) \right]^2 + \frac{1}{4} \left(\frac{\lambda}{g^2} \right) \xi^2 (h^2-1)^2 \right)$$

with boundaries

$$\xi : 0 \qquad f = \alpha \xi^2 \qquad h = \beta \xi^2$$

$$\xi : \infty \qquad f = 1 - \gamma e^{-\frac{\xi}{2}} \qquad h = 1 - \left(\frac{\delta}{\xi} \right) e^{-\sqrt{\frac{2\lambda}{g^2}}\, \xi}$$

$\alpha, \beta, \gamma, \delta$ $\quad \mathcal{O}(1)$ coefficients

$\boxed{\theta_W \neq 0}$

a_i cannot be zero because of $U(1)$ current in the case $\theta_W \neq 0$. Assume W, φ unchanged by presence of current, look at what a-field is induced.

Energy shift by terms containing a

$$\Delta E = \int d^3x \left(\frac{1}{4} f_{ij} f_{ij} - a_i j_i \right) d^3x \overset{(2)}{=} - \int d^3x \, \frac{1}{2} a_i j_i$$

$$j_i = - \frac{1}{2} i g' \left[\varphi^\dagger D_i \varphi - (D_i \varphi)^\dagger \varphi \right]$$

Ansatz $\rightarrow$ $\vec{j} = \frac{1}{2} g v^2 \frac{h^2(\xi)(1-f(\xi))}{r^2} \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$ Axial symmetry

Parametrize a by $\vec{a} = \frac{1}{2} g' v^2 p(\xi) \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$

$$-\nabla^2 \vec{a} = \vec{j} \longrightarrow \xi^2 \frac{d^2 p(\xi)}{d\xi^2} + 4 \xi \frac{dp}{d\xi} = - h^2 (1-f)$$

Boundary conditions $\xi \to 0$ $\quad \xi^2 p(\xi) = 0$

$\xi \to \infty$ $\quad p(\xi) = 0$

$$p(\xi) = \frac{1}{3\xi^2} \int_0^\xi d\eta \, \eta^2 h^2(\eta) \left[1 - f(\eta) \right] + \int_\xi^\infty d\eta \, \frac{1}{3\eta} h^2(\eta) \left[1 - f(\eta) \right]$$

Magnetic Dipole moment of the sphaleron

$$\vec{a} = \frac{\vec{\mu} \times \vec{x}}{4\pi r^3} \qquad \mu = \frac{2\pi}{3} \frac{g'}{g^3 v} \int_0^\infty \xi^2 h^2(\xi) \left[1 - f(\xi) \right] d\xi$$

$$\Delta E = -\frac{\pi}{3}\frac{g'^2 v}{g^3}\int_0^\infty \xi^2 h^2(\xi)\left[1-f(\xi)\right] p(\xi)\, d\xi$$

Numerical solution

[1] $\nearrow$ Ansatz a) Similar to DHN '74
[1] $\searrow$ Ansatz b) Given here:

$$f^b(\xi) = \frac{\xi^2}{\Xi(\Xi+4)} \quad \xi \le \Xi \;, \quad 1-\frac{4}{\Xi+4}\exp\frac{\Xi-\xi}{2} \;, \quad \xi \ge \Xi$$

$$h^b(\xi) = \frac{\sigma\Omega+1}{\sigma\Omega+2}\frac{\xi}{\Omega} \;, \quad \xi \le \Omega \;, \quad 1-\frac{\Omega}{\sigma\Omega+2}\frac{1}{\xi}\exp\sigma(\Omega-\xi) \;, \quad \xi \ge \Omega$$

$\rightarrow$ $E_{sph} = 8 - 14$ TeV depending on $\frac{\lambda}{g^2}$ $\quad \lambda \in (0,\infty)$

$\rightarrow$ $\frac{\mu}{\mu_W} = 83 \quad (\lambda = 0) \qquad \mu_W = \frac{e}{M_W}$

Further Klinkhamer & Manson also calculate the charge of the sphaleron by using the sphaleron Ansatz above

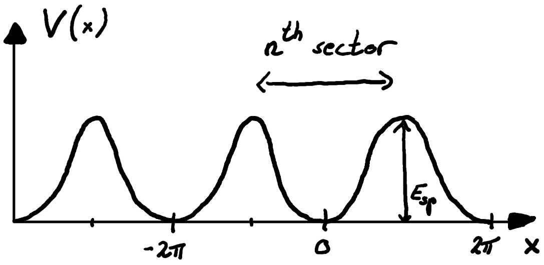
$\rightarrow$ $Q_B(\text{sphaleron}) = \frac{1}{2}$

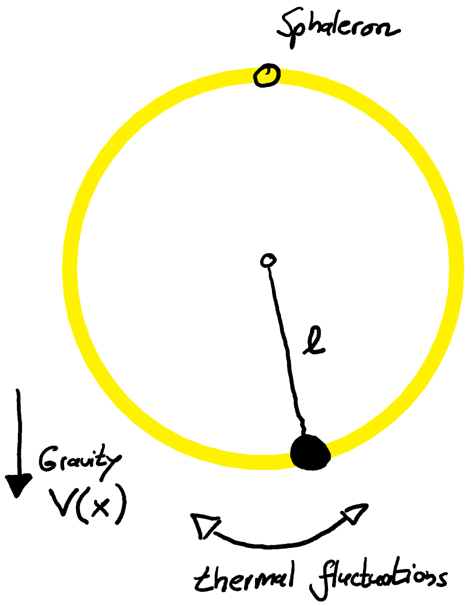
Summary:

Constructed approximate solution to static field equations of EW theory which is unique up to choice of axis of axial symmetry and translations. Its topological charge is $\frac{1}{2}$. It further has a magnetic moment which is a semi-quantitative measure of its coherence

II. QM toy model

To show: It is possible to have winding in t-dependent correlations of B-number supressed in Euclidean time but unsupressed when extended to real time.





Rigid pendulum (toy model)

$T=0 \rightarrow$ Instanton potential

$T \neq 0 \rightarrow$ finite Temperature ensemble $\sim e^{-\beta E}$

$-\infty < x < \infty \qquad x_{sp}(t) = \pi$

Assume Temperature high enough such that gravity can be neglected

$$S_E = \int_0^{\hbar\beta} d\tau \, \frac{\dot{x}^2}{2}$$

Units where moment of inertia is 1

each term integrate for paths fixed winding number

$$n = \frac{1}{2\pi} \int_0^{\hbar\beta} d\tau \, \dot{x} \quad \text{gives} \quad Z = \sum_n \int [\mathcal{D}x]_n \exp\left(-\frac{S_E}{\hbar}\right)$$

$$x_n(\tau) = \frac{2\pi n \tau}{\hbar\beta} \qquad S_n = \frac{2\pi^2 n^2}{\hbar\beta}$$

Adding θ term coupled to winding number.

$$S_E = \int_0^{\hbar\beta} d\tau \left[\frac{\dot{x}^2}{2} + i\hbar \frac{\theta}{2\pi} \dot{x} \right]$$

Measure of real time rate at which pendulum wraps around the circle

$$A(t) \equiv \langle \mathcal{T} : \left(\frac{x(t)-x(0)}{2\pi}\right)^2 \rangle$$

Classically: $A(t) = \frac{t^2 T}{4\pi} + \mathcal{O}(\hbar)$ $\left(\text{Using } \frac{\overline{v}^2}{2} = \frac{T}{2}\right)$

Free energy: $\frac{\partial^2 F}{\partial \theta^2} = \langle n^2 \rangle = A(-i\hbar\beta)$

Exponentially small quantity. See this in each sector n by $x(\tau) \to x_n(\tau) + \delta x(\tau)$

$$Z = \sum_n \exp\left(\frac{-2\pi^2 n^2}{\hbar^2\beta}\right) \exp(in\theta) \times \int [\mathcal{D}\delta x] \exp\left(-\frac{1}{\hbar}\int_0^{\hbar\beta} d\tau \frac{(\delta\dot{x})^2}{2}\right)$$

$$\langle n^2 \rangle = \frac{\sum n^2 \exp\left(\frac{-2\pi^2 n^2}{\hbar^2\beta}\right)\exp(in\theta)}{\sum \exp\left(-\frac{2\pi^2 n^2}{\hbar\beta}\right)\exp(in\theta)} = 2\exp\left(\frac{-2\pi^2}{\hbar\beta}\right) + \mathcal{O}\left(e^{-\frac{4\pi^2}{\hbar^2\beta}}\right)$$

How can these two results be consistent?

Calculate $A(t)$ for arbitrary t

$n \neq 0$ exponentially supressed

$n = 0$:

$$A(t) = \langle \mathcal{T} : \left(\frac{\delta x(t) - \delta x(0)}{2\pi}\right)^2 \rangle + \mathcal{O}\left(e^{-\frac{2\pi^2}{\hbar^2\beta}}\right)$$

finite T propagator:

$$\langle \mathcal{T}\, \delta x \delta x \rangle = \frac{i}{k^2 - \frac{m^2}{\hbar^2} + i\epsilon} + \frac{1}{\exp(\hbar\beta|k_0|) - 1}\, \delta\left(k^2 - \frac{m^2}{\hbar^2}\right)$$

Same in configuration space:

$$\langle \delta x(t_1)\,\delta x(t_2)\rangle = \frac{\hbar}{2m}\left(\frac{\exp\left(+\frac{im\,|t_1-t_2|}{\hbar}\right)}{\exp(+\beta m)-1} - \frac{\exp\left(-\frac{im\,|t_1-t_2|}{\hbar}\right)}{\exp(-\beta m)-1} \right)$$

$\longrightarrow$

$$A(t) = \frac{\hbar}{4\pi^2 m}\left(\frac{\exp\left(-i\frac{m}{\hbar}|t|\right)-1}{\exp(-\beta m)-1} - \frac{\exp\left(i\frac{m}{\hbar}|t|\right)-1}{\exp(\beta m)-1} \right)$$

$$+\,\mathcal{O}\left(e^{-\frac{2\pi^2}{\hbar^2\beta}}\right)$$

$\longrightarrow$ Key result:

$$\boxed{A(t) = \frac{1}{4\pi^2}\left(\frac{t^2}{\beta} + i\hbar t\right) + \mathcal{O}\left(e^{-\frac{2\pi^2}{\hbar^2\beta}}\right)}$$

Classical limit: $\quad A(t) \approx \dfrac{t^2 T}{4\pi}$

but also: $\quad A(-i\hbar\beta) = \mathcal{O}\left(e^{-\frac{2\pi^2}{\hbar^2\beta}}\right)$

Large real time winding comes from $n=0$ sector. Fluctuations in Euclidean time must be small since otherwise supression due to large S_E

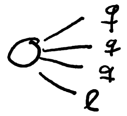
$$|A(-i\tau)| \leq \frac{\hbar^2\beta}{4\pi^2} \quad \text{for} \quad 0 \leq \tau \leq \hbar\beta$$

When analytically continued to real time fluctuations become large.

IV. Sphaleron as coherent state

Set number of generations to 1.

Baryon number violation requiring Euclidean winding

$\Delta N = 1 \longrightarrow$ 

$$\int [\mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}A]\, e^{-S_E}\, qqq\ell = \int [\mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}A]\, e^{-S_E}\, qqq\ell \;\delta\!\left(\frac{g^2}{16\pi^2}\int F\tilde{F} - 1\right)$$

Only configurations that tunnel are those with Euclidean winding number $N = 1$

S_E is minimized by (finite temperature) instantons $\langle qqq\ell \rangle \sim e^{-\frac{2\pi}{\alpha_W}}$

Classical sphaleron do not relate to $\langle qqq\ell \rangle$ but to

$$\langle qqq\ell \; A_{in}^{\frac{1}{\alpha_W}} A_{out}^{\frac{1}{\alpha_W}} \rangle$$

These are Green functions with non-perturbative number of legs

Simple example from calculus:

$$I_n \equiv \int_{-\infty}^{\infty} dx\; e^{-S(x)}\, x^{2n} \quad \text{where } S(x) = g^{-2}\left[1 + (x - g^{-1})^2\right]$$

The minimum of S is $S_{min} = \frac{1}{g^2}$ at $x_0 = \frac{1}{g}$

$I_0 \leq \exp(-S_{min}) \longrightarrow 0$ for $g \to 0$

Well behaved in weak coupling limit

Large n limit $n = \frac{1}{g^2}$

$$I_{\frac{1}{g^2}} = \int_{-\infty}^{\infty} dx \, \exp\left(-S(x) + g^{-2} \ln x^2\right) \gtrsim \left(\frac{1}{g}\right)!$$

At x_0 integrand $e^{-\frac{1}{g^2}} \left(\frac{1}{g}\right)^{\frac{2}{g^2}} \sim \left(\frac{1}{g}\right)!$!

$I_{\frac{1}{g^2}}$ does not vanish in the small g limit

Instanton methods are not effective for amplitudes involving many quanta.

Sphalerons become supressed in the limit of few quanta.

# of quanta in a classical coherent state are Poisson distributed

$$P_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$\langle n \rangle \sim \frac{1}{\alpha_w}$ one finds supression similar to the instanton one. To see this look at overlap between n-particle & coherent state.

Coherent state:

$$|out\rangle = e^{-\frac{\langle n \rangle}{2}} \exp\left[\int \frac{d^3k}{(2\pi)^3 \sqrt{2E}} \Phi_d(k) \, a^\dagger(k)\right] |0\rangle$$

E energy of quanta created by $a^{t}(k)$

$$\langle n \rangle = \int \frac{d^3k}{(2\pi)^3 2E} \, |\Phi(k)|^2$$

work in frame where average of spatial momentum vanishes and energy is given by

$$E_{sp} = \int \frac{d^3k}{(2\pi)^3 2E} \, E \, |\Phi(k)|^2$$

$$\left| \langle out \mid \overbrace{k_1 \ldots\ldots k_n}^{n \text{ particle state}} \rangle \right|^2 = \left| \langle out \mid a^{\dagger}(k_1) \ldots\ldots a^{\dagger}(k_n) \mid 0 \rangle \right|^2$$

$$= e^{-\langle n \rangle} \frac{|\Phi(k_1)|^2}{(2\pi)^3 2E_1} \cdots \frac{|\Phi(k_n)|^2}{(2\pi)^3 2E_n} \qquad |\Phi| \sim \frac{1}{\alpha_w}$$

$\frac{1}{n!}$ for identical particles, integrating over 3-momenta gives Poisson distribution

$\langle n \rangle$ large   Supression of sphaleron decays into small # of quanta.

P.B. Arnold's depiction of how a Sphaleron presumably looks like [3]

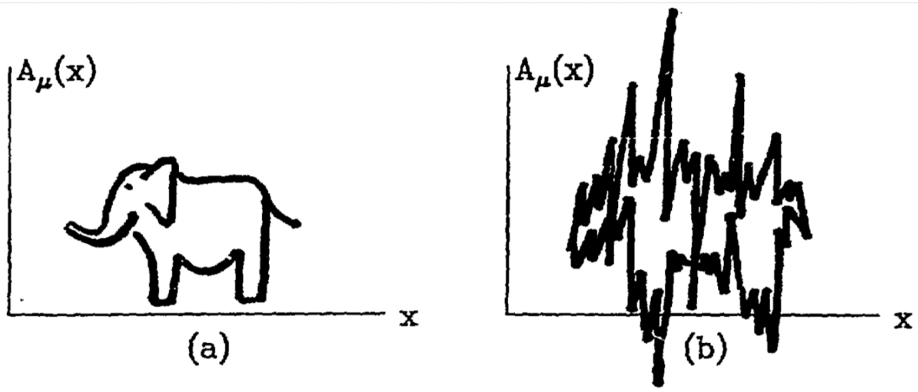


FIG. 7. (a) the sphaleron; (b) the sphaleron distorted by high-frequency thermal noise.

V. Winding in real time (Abelian Higgs 1+1)

Classical evolution of the Sphaleron in EW theory to see what it does to Baryon number

Classical e.o.m's hard to solve for Weinberg-Salam

$\longrightarrow$ Abelian Higgs model 1+1 (Instantons ✓)

Problem : Classical physics misleading in 1+1
(Quantum fluctuations dominate the IR)

$\longrightarrow$ Hope that here there is no problem

$$S = \int d^2x \left[-\frac{1}{4} F_{\mu\nu} + (D_\mu \Phi)^2 - V(\Phi) + \bar{\Psi} \gamma_\mu (i\partial^\mu + g\gamma_5 A^\mu) \Psi \right.$$

$$V(\phi) = \lambda \left(|\phi|^2 - \frac{v^2}{2} \right)^2$$

finite & periodic in spatial direction L

Axial coupling with anomalous fermion number current

spatial Vector potential

$$\partial_\mu \bar{\Psi} \gamma^\mu \Psi = -\frac{g}{4\pi} \epsilon_{\mu\nu} F^{\mu\nu} \qquad Q_t = -\frac{g}{4\pi} \int dx\, A_1(x)$$

$$\frac{d}{dt} \left(Q_F - Q_t \right) = 0$$

Exact sphaleron solution $\qquad M_H = 2\lambda v^2$

gauge fixing

$$\Phi = i e^{-i\pi \frac{x}{L}} \frac{v}{\sqrt{2}} \tanh \frac{M_H x}{2} \qquad A = \begin{pmatrix} 0 \\ \frac{\pi}{gL} \end{pmatrix}$$

$$Q_t \,(\text{sphaleron}) = \frac{1}{2}$$

Decay in absence of fermions (Numerical solution from [2])

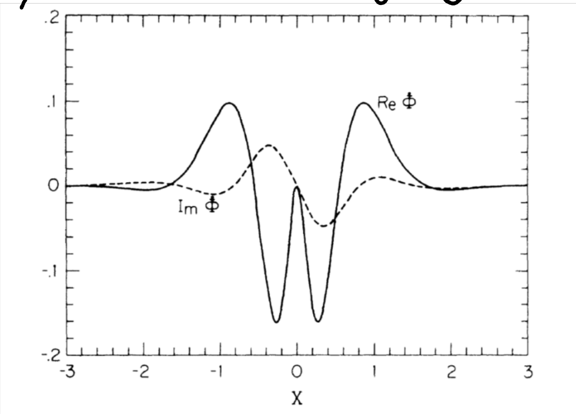


FIG. 5. The initial values of ReΦ (solid line) and ImΦ (dashed line) used in our simulation.

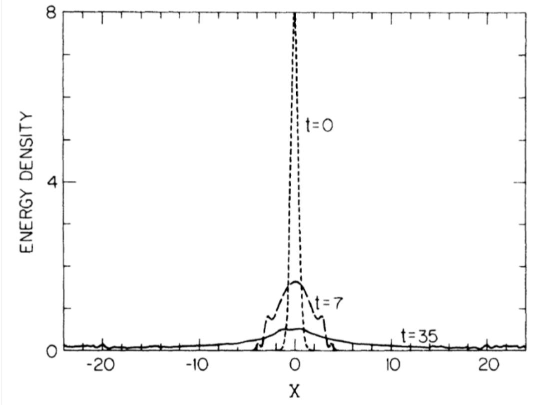


FIG. 6. The energy densities at $t = 0$, 7, and 35.

Q should change by one unit

Fourier decomposition of plane waves:

$$A_\mu(x) = \int \frac{dk}{2\pi}\, f_\mu(k)\, \cos\left(kx + \phi(k)\right)$$

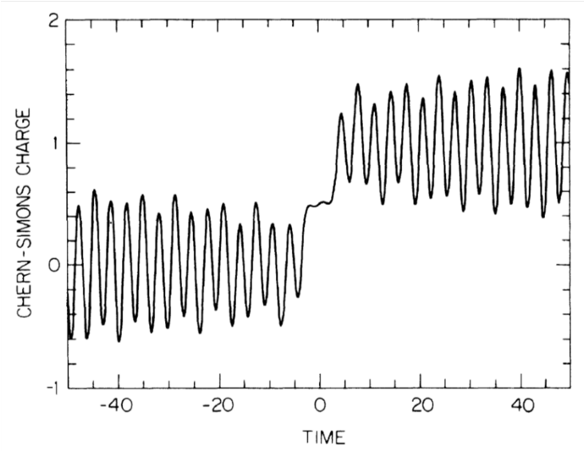


FIG. 7. The topological charge as a function of time.

$$\dot{Q}^{1D} = g \int dx\, \tilde{F} \sim g\, M_W\, f(0)\, \cos\left(M_W t + \phi_0\right)$$

$$\dot{Q}^{3D} = g^2 \int d^3x\, F\tilde{F} \sim g^2 \int d^3k\, \epsilon^{\mu\nu\rho\sigma}\, k_\mu f^a_\nu(k)\, \tilde{k}_\rho\, f^a_\sigma(-k)$$

$$\times \cos\left[2\omega_k t + \phi(\vec{k}) + \phi(-k)\right]$$

Strongly oscillating cosine will set $\dot{Q} \to 0$ (for smooth f)