Theory Group

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Semiclassical objects in QFT

Q-balls

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Contents

1 What is a Q-ball?

During this seminar series, we have learned mainly about Topological Solitons (TS), i. e. extended objects which are a solution to the Equations of Motion (EOMs) of a Quantum Field Theory (QFT) and are stable due to the conservation of some topological quantity. However, the seminar swansong will be dedicated to a remarkable kind of Non-topological Solitons (NTS): the Q-ball.

Before proceeding with today's main topic, it is convenient to clarify the general properties of a NTS. These are non-dissipative, extended in space solutions to the classical field equations that arise in field theories with an unbroken global symmetry, which has an associated set of conserved Noether charges Q[1]. A NTS is, for a fixed charge Q, the field configuration with the lowest energy and is stable as long as the global symmetry remains unbroken [1]. And here is the main difference with a TS: in a NTS, the conserved charge associated with the global symmetry is the responsible of the stability of the solution. On the other hand, in a TS, that role is played by a topological conserved quantity or winding number.

After such a long prologue, let us finally tackle the first big question: what is a Q-ball? A Q-ball is a spherically symmetric[2] NTS composed by scalar fields in a theory with a continuous unbroken global symmetry. These objects behave in many ways like a lump of matter with fixed Q charge density. The seminal paper on this subject was published by Sidney Coleman in 1985 [2], although there were a few previous works which found similar solutions but did not address some fundamental issues like stability nor did such a comprehensive study. Since then, Q-balls have been studied in different Beyond Standard Model (BSM) theories. Their main phenomenological features are that they could have been produced in the early Universe and they could even be Dark Matter (DM) candidates.

2 Simplest Q-balls

As usually happens in theoretical Physics, Q-balls were discovered in a very simple theory. Coleman studied comprehensively and rigorously these solutions in such theory [2], but only in the case of large Q-balls. In this section, you will be talked through the most important parts of such work from a practical and not so formal point of view.

2.1 Model setup

The simplest theory with Q-balls is a SO(2) symmetric theory of real scalars:

$$\mathcal{L} = \sum_{i=1}^{2} \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_i - U(\phi), \tag{1}$$

where:

$$\phi = \sqrt{\sum_{i=1}^{2} \phi_i^2}, \tag{2}$$

and $U(\phi)$ is a general scalar potential, which might even be non-renormalisable. We must ask the global minimum of the potential to be at $\phi = 0$ to keep the global symmetry unbroken. By convention, we will choose $U(0) = 0$. The spectrum consists of scalar bosons with $Q = \pm 1$ and mass μ, defined like:

$$\mu^2 = \frac{d^2 U}{d\phi^2}(0) = \left. \frac{2U(\phi)}{\phi^2} \right|_{\phi=0}. \tag{3}$$

[1] Can you see now the origin of the Q in Q-ball?

[2] Now the name is completely justified. For the sake of rigour, I will show later that these objects must be spherical.

The conserved currents and charges of this theory are:

$$j_\mu = \phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1 \tag{4}$$

$$Q = \int j_0 \, d^3\mathbf{x}. \tag{5}$$

From now on, we will do mostly classical field theory[3].

2.2 Introducing the Q-ball

If the minimum of U/ϕ^2 is at some point $\phi_0 \neq 0$, the spectrum changes and new objects with arbitrary charge Q appear. These will be stable if the following condition is satisfied:

$$\min\left[\frac{2U(\phi)}{\phi^2}\right] = \frac{2U(\phi_0)}{\phi_0^2} = \omega_0^2 < \mu^2, \tag{6}$$

where we use such equation to define the frequency ω_0 for future convenience. These new states in the spectrum are, for large enough Q, non-dissipative solutions of the classical field equations that are absolute minima of the energy for fixed Q. For a suitable choice of the reference frame, those solutions are:

$$\phi_1 = \phi(r)\cos(\omega t), \tag{7}$$

$$\phi_2 = \phi(r)\sin(\omega t), \tag{8}$$

where $\phi(r)$ is a monotonically decreasing function of the distance from the origin, going to zero at infinity. ω is a real constant which must obey the inequalities:

$$\omega_0^2 < \omega^2 < \mu^2. \tag{9}$$

We have found the Q-balls. The scalar field inside a Q-ball has a non-vanishing absolute value but it rotates steadily with frequency ω in its internal space.

As the charge Q goes to infinity, the frequency ω approaches ω_0. In such limit, the spatial distribution of the field resembles a smoothed-out step function. Let's call R to the radius where the field goes from ϕ_0 inside to 0 outside. Both regions are connected by a smooth transition zone with thickness $1/\mu$. A sketch of the field profile for a Q-ball is showed in Fig. 1.

2.3 Building Q-balls

Let us assume the existence of a solution like the one discussed before and minimize the energy at fixed Q. The field will be given by Eq. 8 and we will start with the roughest and crudest approximation for the spatial component. We will assume that the field is a step function, with some constant non-zero value around the origin of coordinates:

$$\phi(\vec{r}) = \begin{cases} \phi & \text{if } \vec{r} \in \mathcal{V} \\ 0 & \text{if } \vec{r} \notin \mathcal{V} \end{cases}, \tag{10}$$

where $\mathcal{V}$ is a simply connected region around the origin with volume V. Monotonically decreasing and approaching zero at infinity, as we wanted, it is for sure. Indeed, it is a good approximation for a very large Q-ball [2]. We will study the transition zone, dropping the assumption of a step function, later on.

[3] No, Q-ball does not mean Quantum ball.

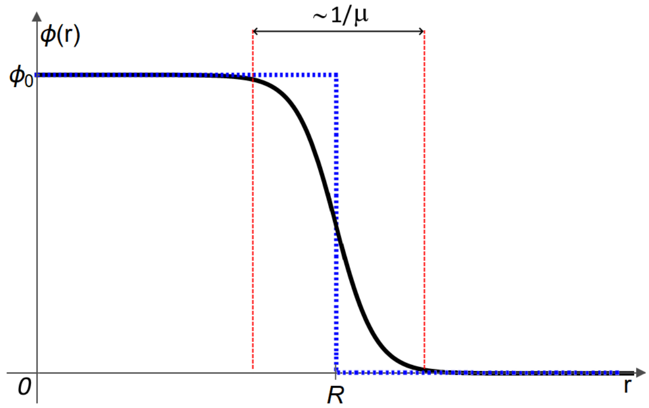

Figure 1: Sketch of the field profile of a large Q-ball. In dashed blue lines, we plot the step function approximation for such profile.

The energy of this field configuration is simply:

$$E = \int \left[\sum_{i=1}^{2} \frac{1}{2} \dot{\phi}_i^2 + \sum_{i=1}^{2} \frac{1}{2} \left(\nabla \phi_{\mathrm{i}} \right)^2 + U\left(\phi \right) \right] d^3x$$
$$= \frac{1}{2} \omega^2 \phi^2 V + U\, V, \tag{11}$$

and the charge:

$$Q = \omega \phi^2 V. \tag{12}$$

Therefore, the energy can be rewritten trading ω for Q:

$$E = \frac{1}{2} \frac{Q^2}{\phi^2 V} + U\, V. \tag{13}$$

For a fixed Q and as a function of V, the energy is minimum for:

$$V = \frac{Q}{\sqrt{2\phi^2 U}}. \tag{14}$$

Replacing:

$$E = Q \sqrt{\frac{2U}{\phi^2}}. \tag{15}$$

Minimizing this as a function of ϕ, we find the definition of $\phi = \phi_0$, where ϕ_0 was defined in Eq. 6.

This is a good moment to stress the meaning of the condition in Eq. 6. If $\omega_0^2 \geqslant \mu^2$, it is energetically favourable to have a bunch of free scalar bosons with total charge Q and then the Q-ball does not exist. When the condition is fulfilled, the Q-ball solution is not only allowed but its decay via emission of charged scalars (with charge ± 1 and mass μ) is completely forbidden. Why? Because the energy per unit of charge $\frac{E}{Q} = \sqrt{\frac{2U_0}{\phi_0^2}}$ is forced to be smaller than the one of a scalar, μ.

Another important consequence is that the value of the field inside the Q-ball is determined by the potential. We did not ask ϕ to be independent of the charge, so it is remarkable that turned out to be so! This implies that the energy (see Eq. 15) and the volume (see Eq. 14) are proportional to the charge. In other words, the energy and charge density inside the Q-ball are independent of the total charge. Later, we will see that the frequency ω has a similar property. Therefore, we are entitled to think these solutions as a lump of Q-matter, with constant local properties. But we have to stress the validity of these claims only for large enough Q-balls.

But, is this a Q-ball or just a Q-thing with undefined shape? Because we never specified the shape of $\mathcal{V}$... To elucidate this question, we have to consider the contribution to the energy from the transition zone at the surface of $\mathcal{V}$. This contribution is expected to be positive and proportional to the surface area. Hence, in order to minimize the energy, we have to pick the shape with minimal surface area for a given volume, i. e. the sphere. In conclusion, yes, $\mathcal{V}$ is a sphere and we really have Q-balls.

The minimization of the surface also forbids the decay via quantum tunneling (the appearance of a vacuum bubble inside the Q-ball). Quantum tunneling preserves Q and E and the previous arguments say that the only state with the energy and charge of a Q-ball is a spherical one, not one with a cavity inside.

Let us make a small detour to think a little more about the name of these objects. A bunch of ordinary matter is stable if the number of particles in it is so. And if that bunch has spherical symmetry, I would say that we have a ball of particles or a particle ball. In the case of a Q-ball, the radius of the ball is constant if the charge is preserved. So the charge Q plays the role of the number of particles in ordinary matter. The name "Q-ball" is more than well justified.

2.4 Newton's lessons

Replacing the Q-ball solution (Eq. 8) in the EOM of the field, we obtain:

$$\frac{d^2\phi}{dr^2} = -\frac{2}{r}\frac{d\phi}{dr} - \omega^2\phi + U'(\phi). \tag{16}$$

Through a analogy with Newton mechanics, it is possible to show that a Q-ball is a solution to the EOMs. You can find the details in the original paper [2].

Here, we will summarise what such analogy teaches us about the Q-ball. First, it justifies the condition over ω in Eq. 9. Second, it tells us that for very large Q-balls, i.e. $\phi(r) \cong \phi_0$ even far from the origin, $\omega \to \omega_0^+$. Indeed, it is useful to take $\omega \cong \omega_0$.

Third, it allows to characterize the surface of the Q-ball. If we define sort of an "effective potential" $\hat{U}$ as:

$$\hat{U} = U - \frac{\omega_0^2}{2}\phi^2, \tag{17}$$

we find the relation:

$$R - r = \int_{\bar{\phi}}^{\phi(r)} \frac{1}{\sqrt{2\hat{U}}} d\phi, \tag{18}$$

where R is defined such that $\phi(R) = \bar{\phi}$. And $\bar{\phi}$ can be chosen anywhere in the fuzzy transition region. It is convenient to choose it such that:

$$\int \phi^2 d^3x = \frac{4\pi}{3} R^3 \phi_0^2. \tag{19}$$

It can be shown that such choice gives a $\bar{\phi}$ independent of R for large R [2]. Now we can compute the charge, the volume energy and the surface energy of the Q-ball as a function of R and using $\omega \cong \omega_0$:

$$Q = \frac{4\pi}{3} R^3 \phi_0^2 \omega_0, \tag{20}$$

$$E_{vol} = \int \omega_0^2 \phi^2 d^3x = \frac{4\pi}{3} R^3 \phi_0^2 \omega_0^2 = \frac{8\pi}{3} R^3 U(\phi_0), \tag{21}$$

and the surface energy:

$$E_{surf} = \int \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + \hat{U} \right] d^3x \tag{22}$$

$$= 4\pi R^2 \int \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + \hat{U} \right] dr \tag{23}$$

$$= 4\pi R^2 \int_0^{\phi_0} \sqrt{2\hat{U}} d\phi. \tag{24}$$

And the integral of the last line is the surface tension coefficient. Finally, it is worth noting too that finding the Q-ball solution for a fixed ω is identical to the problem of finding a bounce solution for tunneling in 3 Euclidean dimensions with the potential $\hat{U}$.

2.5 Existence and stability of Q-balls

Coleman proves that Q-balls exist and are absolutely stable through a lemma and a theorem. What do we learn from them? We can extract two useful definitions.

Q-ball type set of initial-value data

A set of initial-value data is of Q-ball type if:

- $\phi_1 = \phi(r) \wedge \phi_2 = 0 \wedge \dot{\phi}_1 = 0 \wedge \dot{\phi}_2 = \omega\phi(r)$,
- ω is a constant and,
- $\phi(r)$ is a positive function monotonically decreasing to zero as r goes to ∞.

Acceptable potential

A potential will be called "acceptable", i. e. that gives rise to Q-balls, if:

- $U(\phi) \geqslant 0$
- $U(\phi) = 0$ if and only if $\phi = 0$.
- U is twice continuously differentiable in all its domain.
- $U'(0) = 0$ and $U''(0) = \mu^2 > 0$. So far, we are just asking the global symmetry to be unbroken and the scalars to be massive.
- The minimum of U/ϕ^2 is attained at some $\phi_0 \neq 0$. This is just the old condition in Eq. 6 written in words. It means that U must, somewhere, dip below $\frac{\mu^2}{2}\phi^2$.
- There must exist three positive numbers a, b and c, with $c > 2$, such that:

$$\frac{\mu^2}{2}\phi^2 - U \leqslant \min\{a, b\phi^c\}, \tag{25}$$

  which means that U should not dip below $\frac{\mu^2}{2}\phi^2$ too far.

For the sake of completeness, let us state the theorem:

If U is an acceptable interaction, there exists $Q_{min} \leqslant 0$, such that for any $Q > Q_{min}$, there is initial-value data of Q-ball type that minimizes the energy for that value of Q. Furthermore, this is the initial-value data for a Q-ball solution of the EOM (Eq. 16).

It is worth stressing that Coleman does not give any formula to compute Q_{min}, he just proves its existence. But the proof was done always assuming a large Q-ball. 12 years later, a much more careful study of small Q-balls performed by Kusenko found out that there is no classical lower bound for the charge of a Q-ball [3]. Q_{min} is just a mirage produced by Coleman's techniques.

2.6 A digression on the potential

So far, we have worked in a very potential independent way, just making assumptions on how it should behave and on their global and local minima. Let us try to characterize it a bit better.

At some moment, it was pointed that it could be non-renormalizable. Indeed, the only renormalizable potential allowed for the theory in Eq. 1 is:

$$U(\phi) = \frac{\mu^2}{2}\phi^2 + \lambda\phi^4, \tag{26}$$

which can not fulfil the condition in Eq. 6 and then does not give rise to Q-ball solutions.

Is this a problem? No. First, because U can be an effective potential and the difference would be only noticeable when computing loop induced corrections, which will be out of this discussion. However, it should be mentioned that it was claimed that quantum corrections can stabilise a classically unstable Q-ball [4]. Second, because just a slight refinement of the theory would allow a renormalizable potential able to produce Q-balls, like changing SO(2) for SO(3) and putting ϕ in the adjoint. And we will do this a bit later.

Most phenomenological applications of Q-balls are related to their possible production in the early universe. This is because they appear whenever we are near or at a first-order symmetry-breaking phase transition. We can see an example in Fig. 2, where the point ϕ_+ is the candidate to satisfy the condition Eq. 6. If U_+ is low enough, such condition will be satisfied and Q-balls will appear. When the parameters are deformed in such a way that the transition happens, i. e. U_+ goes below 0 and ϕ_+ is the new minimum, the Q-ball becomes adiabatically the new asymmetric vacuum ($\omega_0 \to 0$).

2.7 Exciting Q-balls

Let us find those small oscilations of Q-balls whose frequencies go to zero as $R \to \infty$. There are two kinds of them.

2.7.1 Bulk sound waves

Pretend to be in the center of a Q-ball and send R to infinity. Now, you are inside an infinite volume filled with Q-matter. There is a zero mode which consists in doing infinitesimal Q-rotations. This turns out to be the zero wave-vector limit of a sound wave which has a frequency proportional to $|\vec{k}|$, for small wave-vector $\vec{k}$ [2]. The proportionality constant is the sound speed inside the Q-ball, v_S:

$$\omega_{Bulk}^2 = \left(k^0\right)^2 = v_S^2 \left|\vec{k}\right|^2. \tag{27}$$

For a Q-ball of radius R, the wave vector will be of order $\frac{1}{R}$, so the energy of these sound waves will be $k^0 \sim \frac{1}{R}$.

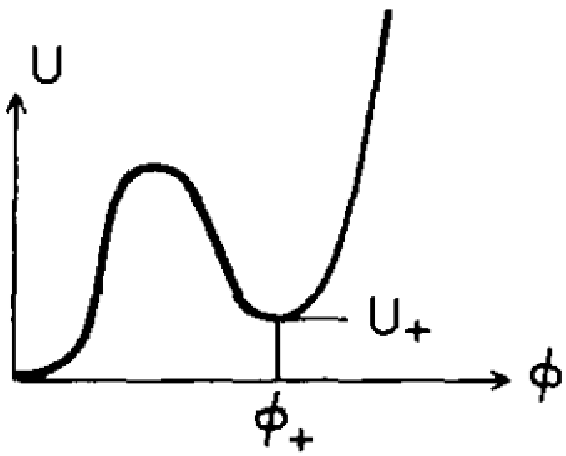

Figure 2: A potential near a phase transition. The local minimum at ϕ_+ is an excellent candidate to give rise to a Q-ball. Plot extracted from [2].

It is possible to compute the sound speed of such wave through 2 methods: solving the EOM in a small perturbations limit or using classical relativistic fluid dynamics and taking advantage of the local conservation of energy, momentum and Q [2]. Either way, the result is:

$$v_S^2 = \frac{U''(\phi_0) - \omega_0^2}{U''(\phi_0) + 3\omega_0^2}. \tag{28}$$

Through the fluid dynamics way, we also learn that the local energy and pressure density in the Q-matter are[4] [2]:

$$e = \frac{\omega^2}{2}\phi^2 + U = \frac{1}{2}\phi U' + U, \tag{29}$$

$$p = \frac{\omega^2}{2}\phi^2 - U = \frac{1}{2}\phi U' - U. \tag{30}$$

The pressure density tells us that in the ground state, ϕ_0 with ω_0, the pressure is zero (replace the definition of ω_0 given in Eq. 6). This means that the ground state does not want to contract nor expand.

2.7.2 Surface waves

What happens if we stand at the surface of a Q-ball? In that case, the $R \to \infty$ limit is a half-space filled with Q-matter, separated from the vacuum by a flat boundary. The most essential zero-mode one can think of consists of translations of the surface normal to itself.

One can compute the dispersion relation for these surface waves through fluid dynamics techniques and the result is:

$$\left(k^0\right)^2 = \frac{\alpha}{e_0}\left|\vec{k}_\perp\right|^3, \tag{31}$$

where α is the surface tension coefficient found in Eq. 24:

$$\alpha = \int_0^{\phi_0} \sqrt{2\hat{U}}d\phi = \int_0^{\phi_0} \sqrt{2U - \omega_0^2\phi^2 d\phi}, \tag{32}$$

[4]These equalities are valid assuming a Q-ball field configuration like the one in Eq. 8.

e_0 is the energy density of Eq. 29 evaluated in the ground state and $\vec{k}_\perp$ is the projection of the wave vector on the surface of the Q-ball.

The previous result is valid for a Q-ball of infinite radius. For a finite radius, the waves would take the shape of spherical harmonics $Y_{lm}(\theta, \varphi)$ and the dispersion equation would be:

$$\left(k^0\right)^2 = \frac{\alpha}{e_0 R^3} l\,(l+2)\,(l-1)\,, \tag{33}$$

which makes clear that the energy of these waves is expected to be $k^0 \sim \frac{1}{R^{3/2}}$. What does this mean? It means that for a large Q-ball, the surface waves have much smaller typical energies than the bulk waves.

3 Non-abelian Q-balls

Shortly after the discovery of Q-balls, Coleman and his collaborators extended the study to the case with a non-abelian global symmetry [5]. They studied models with only scalar fields and the chosen global symmetry groups were SU(3) and SO(3).

3.1 General setup of the models

In the SU(3) case, the fields transform in the adjoint representation of the group (**8**) and therefore, they can be arranged in a 3x3 traceless hermitian matrix, ϕ, and the Lagrangian density is:

$$\mathcal{L} = \mathrm{Tr}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4\right]. \tag{34}$$

In order to have the global minimum of the potential at $\phi = 0$, the coefficients must satisfy [5]:

$$\mu^2 > 0,\ \ \lambda > 0,\ \ 9\lambda\mu^2 > g^2. \tag{35}$$

The model with SO(3) symmetry has scalar fields in the spin-2 representation (**5**). Hence, it can be defined with the same Lagrangian of Eq. 34 with the restriction that all the fields must be real.

The 8 or 3 conserved charges of each model can be arranged conveniently in a traceless hermitian matrix which can be computed as [5]:

$$Q = i\int \left[\dot{\phi}, \phi\right] d^3x. \tag{36}$$

In SU(3), every charge matrix is unitarily equivalent to a diagonal matrix with two eigenvalues [5]:

$$Q_{\mathrm{SU}(3)} \sim \mathrm{diag}\left[q_1, q_2, -(q_1+q_2)\right]. \tag{37}$$

Meanwhile, in the SO(3) case the charge matrix is always equivalent to [5]:

$$Q_{\mathrm{SO}(3)} \sim \mathrm{diag}\left[q, -q, 0\right]. \tag{38}$$

3.2 When a Q-ball is not a ball-Q

The Q-ball solutions in these models are qualitatively equal to those found in the abelian model [5]. For both considered symmetry groups, Q-balls appear as long as:

$$g^2 > \lambda\mu^2. \tag{39}$$

This is the first huge difference with the abelian case. Here, Q-balls appear already with a renormalizable potential. The reason is that in the abelian model, the symmetry forbids a cubic interaction. Additionally, every Q-ball of the SO(3) model is also a Q-ball of the SU(3) model, but the inverse is not valid[5].

Q-balls are constructed in the same way that in the abelian case. The field and its time derivative are non-zero only inside a spherical volume V and there, the field is in steady rotation in the internal space. So, the general form for the field is:

$$\phi(x,t) = e^{i\Omega t}\phi(x,0)\,e^{-i\Omega t}, \tag{40}$$

where Ω is a traceless hermitian 3x3 frequency matrix.

A very different feature of the non-abelian models is that Q-balls may decay. In particular, a bigger richness of solutions in the SU(3) model is expected due to the higher amount of charges. It turns out that, in the SU(3) model, only those Q-balls with one vanishing eigenvalue seem to be stable. This was proved with energy arguments without following a continuous path from the initial to the final state and neglecting surface energy [5]. Therefore, it is possible that those Q-balls are stable under small deformations and that small enough Q-balls are stabilized by their surface energy.

Let us go a bit further. Those SU(3) Q-balls with a vanishing charge eigenvalue (either q_1 or q_2) are, then, unitarily equivalent to a SO(3) Q-ball[6]. So, another way of stating the stability condition is that only those SU(3) Q-balls which are unitarily equivalent to a SO(3) Q-ball are stable. All the others are allowed to decay to two smaller Q-balls, each one of them unitarily equivalent to a SO(3) solution. This decay, at least for large Q-balls, is energetically favoured. This "fission" process was addressed with bigger detail in a later work [6] which also found that is in general energetically favourable. A special method to find the "minimal" Q-balls which have one vanishing charge eigenvalue was developed in the original paper [5].

Finally, what happens to the excitations of the Q-balls? Surface-wave spectrum is always the same regardless the global symmetry group of the theory because the infinite wavelength limit of a surface wave is an infinitesimal translation of the Q-ball normal to its surface, which has nothing to do with the internal space of fields. On the other hand, the spectrum of acoustic waves might be richer, but is not in the two models here studied. Again, there is a method to determine whether such spectrum is richer or not that can be found in the original paper [5].

4 Small, thick-walled and composite Q-balls

So far, we have used constantly the thin-wall approximation for Q-balls and worked mainly with big balls and huge charges. In 1997, Kusenko developed the right formalism required to work with small and thick-walled Q-balls [3]. It is useful to visualise the differences between the two limiting cases from the plot of $\hat{U}$, as can seen in Fig. 3. As we pointed out before, Kusenko found that there is no classical lower limit on the charge. However, for reasons of quantum stability[7], Q must be an integer and then $Q \geqslant 1$ [3].

Additionally, it was showed that these small Q-balls are stable with respect to small perturbations and classically stable. On the other hand, quantum corrections are small, and therefore the semiclassical treatment is suitable only for $Q^2 \gg 1$. For $Q \sim 1$, the quantum corrections to the mass of the soliton can be significant and then the stability of these solutions is not clear.

[5]If you do not see quickly the reason for this, think about it from a group theory point of view. Or look again at the charge matrices of the previous subsection.

[6]Two diagonal matrices with the same elements in the diagonal but in different order are unitarily equivalent. This is easy to prove, at least for 3x3 matrices, using Specht's theorem.

[7]Kusenko does not give further detail about this in his paper. The lecturer's guess is that is because the field quanta has charge ± 1, so you can not add non-integer amounts of charge to the Q-ball.

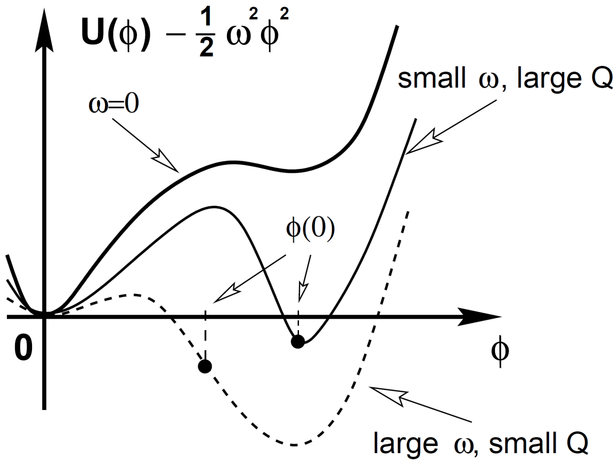

Figure 3: Modified potential $\hat{U}$ for the two limiting cases of small and big Q. Plot extracted from [3].

It is usually assumed that the scalar fields these solitons are made of are elementary, but the case where the bosons are composite scalars has also been considered[8] [7, 8]. More specifically, the bosons could be the Nambu-Goldstone bosons (NGBs) of a spontaneously broken global symmetry in a strongly interacting theory. In those cases, the dynamics of such bosons is described with the Coleman-Callan-Wess-Zumino (CCWZ) formalism [9, 10] and the best known case is the chiral lagrangian which describes the mesons in QCD. Indeed, the first study in this topic looked for Q-balls made of mesons in the strangeness conserving limit. In such case, the candidate mesons for shaping up the Q-ball are the kaons and hence, the authors decided to name that kind of Q-ball as K-ball.

However, a problem was quickly found: the leading order in derivatives CCWZ lagrangian does not allow the existence of Q-balls[9] [7, 8]. There are, at least, two ways out of this problem. The first one was used to show the possible existence of K-balls and it is simply adding to the CCWZ lagrangian the next-to-leading order terms. With these terms and some mild assumptions on the values of their coefficients, K-balls are allowed [7]. Do they exist in nature? That question could not be answered at that time because there was no experimental measurement of the next-to-leading order term coefficients and newer papers addressing that question have not been found [7]. It is worth mentioning that Isospin balls have also been studied [11].

Another way out is considering a conceptually rather different setup where the strongly interacting sector is a New Physics (NP) hidden sector at a scale of the TeV or above and the connection with the SM allows the presence of Q-balls. This approach was considered by Bishara et al [8] nearly 30 years later than the previous one . Let us be more precise about their model. The NP sector features the spontaneous breaking of a non-abelian global symmetry which is additionally explicitly broken, so the Nambu-Goldstone bosons (NGBs) turn out to be massive pseudo-Nambu-Goldstone bosons (pNGBs). Those pNGBs carry a global conserved U(1) charge. The NP sector communicates with the SM through a Higgs-portal interaction. Given the assumptions on the scale of the NP sector, the SM Higgs boson is lighter than the pNGBS and through its interaction "assists" in the formation of the Q-balls [8].

Analytical and numerical results show that in the described model there are long lived Q-balls with $Q \sim 10 - 10^4$ and they are of the thick-wall type [8]. It is even more remarkable that the Q-balls are composed of both the charged (under the global U(1)) pNGBs and the Higgs boson, stressing the importance of the latter in their existence. When the strongly interacting sector has quarks, the

[8]I am grateful to Fady Bishara for having read and checked and earlier version of this section, pointed out the reference [7] and clarified several doubts about Q-balls.

[9]The general proof of this general result can be found in an appendix of [8]

solutions are largely independent from the number of quarks. It is expected that the lighter the Higgs is with respect to the pNGBs, the larger the Q-balls are [8].

Before wrapping up this section, two general remarks about the influence of the strong dynamics on the Q-balls properties are in order. First, the underlying strong dynamics plays a small role in the analysis. Its main influence is that, if the pNGBs are composed of fermions, Fermi repulsion will make the Q-ball bigger and more weakly bound. Second, if the U(1) global symmetry has a small explicit breaking, the Q-balls will not be absolutely stable, but they might be very long-lived [8].

5 Meeting the fermions (or the evaporation of a Q-ball)

The effect of an interaction with massless chiral fermions, i.e. purely SM-like neutrinos, on the properties of a Q-ball was studied in an early paper by Coleman and collaborators [12]. They used the simplest possible model:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - U\left(|\phi|\right) + i\psi^\dagger \bar{\partial\!\!\!/} \psi - iy\phi\psi^\dagger\sigma_2\psi^* + h.c., \tag{41}$$

where ψ is the neutrino field, a Weyl spinor. The fermion has lepton number $L = 1$ and the scalar has $L = 2$. Only massless chiral fermions will be considered and given their similarity with the massless neutrinos of the SM, they will be usually called neutrinos throughout this text.

The interaction with the neutrinos makes the L-ball unstable. But there are some subtleties, because in the leading semiclassical approximation, the L-ball turns out to be stable, although it would not be so if the neutrinos were scalar bosons. What happens is that the exclusion principle and the consequent Fermi pressure prevents the formation of pairs inside the L-ball. Hence, neutrinos can only be produced at the surface of the L-ball and this "evaporates" [12]. It is important that the evaporation process is exclusive of the decay to fermions. If the L-ball decayed to scalar bosons, they could be produced anywhere in the L-ball.

It is possible to derive the following upper bound for the neutrino production rate per unit of area:

$$\frac{dN}{dtdA} \leqslant \frac{\omega_0^3}{192\pi^2}, \tag{42}$$

where N is the number of neutrinos and A, the area. Using that the lepton number density inside the L-ball[10] is $4\omega_0\phi_0^2$ and each neutrino pair has lepton number $2\hbar$[11], the previous bound can be turned into a bound for the contraction speed of the L-ball radius R:

$$\left|\frac{dR}{dt}\right| \leqslant \frac{\hbar\omega_0^2}{384\pi^2\phi_0^2}, \tag{43}$$

which is a non-trivial bound because in the semiclassical limit, i. e. $\hbar \ll 1$, this speed is far below the causality bound, the speed of light [12].

The exact neutrino production rate could not be computed analytically even in the thin-wall approximation, but it was checked that it vanishes when the Yukawa coupling goes to 0, as it should. That limit can be computed analytically in an universal case, not relying on any assumption about the properties of the L-ball. The result is [12]:

$$\frac{dN}{dtdA} \cong 3\pi \frac{y\,\phi_0}{\omega_0}\frac{\omega_0^3}{192\pi^2} + \mathcal{O}\left(y^2\right). \tag{44}$$

[10]Use Eq. 21 where now the charge is the lepton number. The factor of 4 comes from two sides: first, there is a 2 because the scalar field has lepton number 2, and there is another 2 coming from the normalization factor $1/\sqrt{2}$ of the complex scalar field with respect to the real scalar.

[11]Only for this computation, we are purposefully keeping $\hbar \neq 1$ while $c = 1$. To see that the lepton number of the pair carries a power of $\hbar$, analyse the dimension of the conserved charge of the fermion and find that is the same that for $\hbar$, energy times longitude.

6 Gauging Q-balls

What happens if we take the minimal setup for Q-balls and promote the U(1) symmetry to the status of gauge symmetry? This issue was addressed not long after Q-ball's discovery [13]. And the answer is that Q-balls also exist but their properties change [13]. To start with, inside the Q-ball, given the non-zero value of the scalar field, the gauge symmetry is spontaneously broken and the gauge field has mass $m_V \sim e\phi_0$.

In the limit of weak gauge coupling, the Q-ball is very large compared to the penetration length of the gauge field and the charge density is concentrated on the surface. What does this mean? It means that if the Q-ball is in an external magnetic field[12], the photon will not penetrate it and the magnetic field inside the Q-ball will vanish, i. e. it behaves like a perfect diamagnetic material. This is the Meissner effect, the fundamental property of a superconductor[13]. Therefore, the Q-ball interior becomes superconducting [13]. The localization of the charge density on the surface is a direct consequence of superconductivity[14] [14]. Even when the gauged Q-ball is not superconducting, there is a slight concentration of charge near the surface, contrastingly to the global case where the charge density is constant, as can be seen in Fig. 4 and 5.

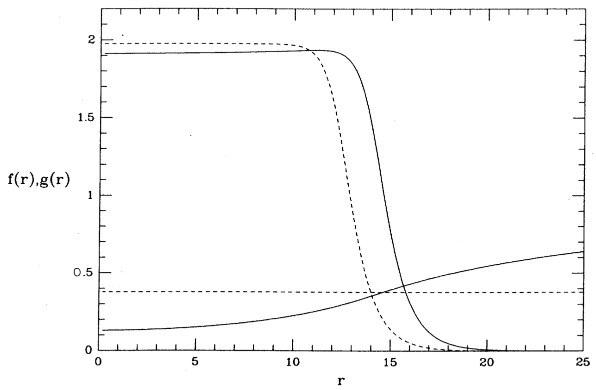

Figure 4: Plot of the scalar field $f(r)$ and the gauge field $g(r) = \omega - eA_0$ as a function of the distance to the center of the Q-ball, r. In solid lines, a gauged Q-ball with $e^2 = 0.01$ and $Q = 11119$ and in dashed lines, a global Q-ball ($e = 0$) with $Q = 10941$. Plot extracted from [13].

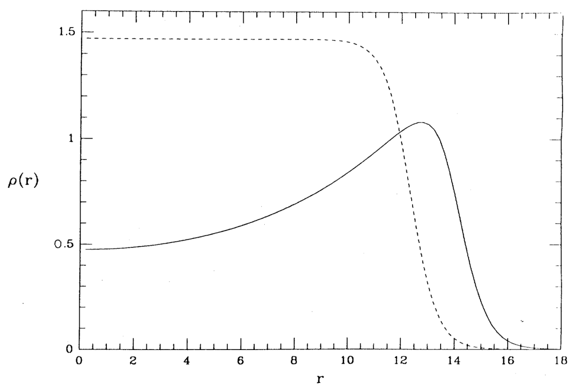

Figure 5: Charge density $\rho(r)$ as a function of the radial coordinate for a gauged Q-ball, in solid lines, and a global Q-ball, in dashed lines. In solid lines, a gauged Q-ball with $e^2 = 0.01$ and $Q = 11119$ and in dashed lines, a global Q-ball ($e = 0$) with $Q = 10941$. Plot extracted from [13].

As you might have expected, the gauge interaction also affects the conditions of existence and stability of the Q-ball. For small enough values of the charge Q and the gauge coupling e, the "electrostatic" energy of the Q-ball should be smaller than the rest of energies, the situation resembles to the global symmetry case and the Q-ball is expected to be stable. On the other hand, if e is large enough, there are no stable Q-balls. The reason is that the Coulomb repulsion would be stronger than all the other forces (scalar self interactions), preventing the accumulation of charged matter [13].

And for any fixed value of e, the same happens if Q is too large: the Coulomb repulsion[15] would also

[12] This is an abuse of terminology. Since now and for the following paragraphs, I will borrow all the terminology from the electromagnetism that we all know, such as magnetic field, diamagnetism, photon and superconductor, even though the gauged U(1) interaction in this theory might be a completely different and new interaction.

[13] Against the most naive belief, the Meissner effect is the fundamental property of a superconductor and no the vanishing resistivity.

[14] I am grateful to my friend Yasuo Oda for a very useful discussion on superconductivity which was fundamental for the writing of this paragraph.

[15] Which, remember, grows with the square of Q.

be too large and then the energy cost prevents the addition of charge to the Q-ball. In that situation, it will be energetically convenient to put the additional charge in the form of free-particles at infinity. So, there is a Q_{max}. The presence of a maximum charge is a stark difference with the global case [13]. Both the existence of Q_{max} and its dependence on e can be seen in the numerical results of Fig. 6.

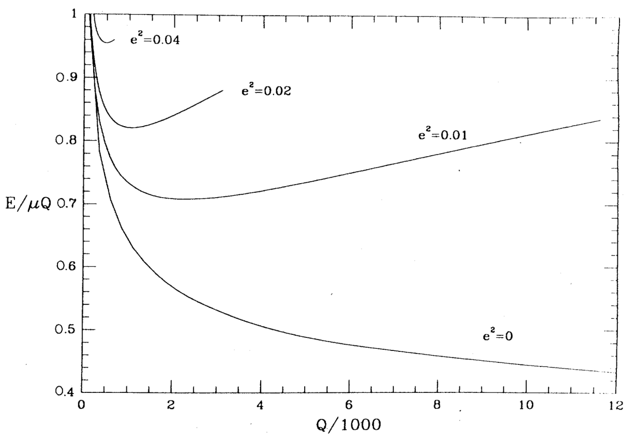

Figure 6: Plot of $E/(\mu Q)$ as a function of Q for a Q-ball with different values of the gauge coupling constant e, where μ is the mass of the free scalar in the theory. If $E/(\mu Q) > 1$, there is no Q-ball solution. Plot extracted from [13].

The same Coulomb repulsion has two more effects on Q-balls: a gauged Q-ball will have a higher energy and a larger radius than a global Q-ball with the same charge (see Fig. 4). In the thin-wall approximation, the authors find that the radius of a gauged Q-ball is [13]:

$$R = \left(\frac{3Q}{4\pi\phi\sqrt{2U(\phi)}}\right)^{1/3}\left(1 + \frac{e^2 Q^{2/3} C^{2/3}}{45}\right), \tag{45}$$

where $C = \frac{3\phi^2}{4\pi\sqrt{2U(\phi)}}$ and ϕ is the value of the scalar field inside the Q-ball. And the energy of the same Q-ball turns out to be:

$$E = Q\sqrt{\frac{2U}{\phi^2}} + \frac{3e^2Q^2}{20\pi R}. \tag{46}$$

And the effect of the Coulomb repulsion is evident in both.

The authors of the paper on gauged Q-balls claimed the existence of a Q_{min}, just like Coleman did [13]. At that time, Kusenko's result on Q_{min} were not known. Now, it seems clear (although the lecturer has not found any literature to support this) that $Q_{min} \geqslant 1$. And it seems feasible that, for e large enough, $Q_{max} < Q_{min}$ and then Q-balls do not exist due to Coulomb repulsion.

7 SUSY and Dark Matter, always there

7.1 A prologue: flavourful Q-balls

When Kusenko wanted to study the presence of Q-balls in supersymmetric (SUSY) extensions of the SM, he found out that there was no knowledge about the case where you have several types of charged scalars with different charges and masses [15]. Fortunately, the generalization to this case is rather trivial. Let us suppose a theory with N complex scalars φ_k, $k = 1, ..., N$, each of them with a charge q_k under a conserved U(1) global symmetry. As in the simplest case, let us assume that the scalar potential has its global minimum for $\vec{\varphi} = 0$ and that $U(0) = 0$.

Following the procedure first developed by Coleman to find large Q-balls, in this model one finds a few differences. First, each scalar field will be rotating with constant speed in their internal space inside the Q-ball, but their rotation speeds will be proportional to their charges [15]. More explicitly, the scalar fields in the Q-ball solution will be:

$$\varphi_k(x,t) = e^{iq_k\omega t}\varphi_k(x), \tag{47}$$

where ω is a real constant. Second, the existence condition for Q-balls is modified to [15]:

$$\min\left[\frac{2U(\vec{\varphi})}{\sum_{k=1}^{N} q_k|\varphi_k|^2}\right] \quad \text{for } \vec{\varphi} \neq \vec{0}. \tag{48}$$

If such minimum is a global minimum, the Q-ball is stable and if it is just a local minimum, the Q-ball is metastable. It is also worth pointing out that Kusenko and collaborators established the sufficient conditions for the existence of global Q-balls in a class of gauge theories [16], which we will not review here because they are not particularly enlightening.

7.2 Q-balls: just another signature of SUSY

In the Minimally Supersymmetric Standard Model (MSSM) and in any other SUSY extension of the SM, one finds in the lagrangian terms like the followings:

$$\mathcal{L} \supset y_{i,j,k} H_i \tilde{Q}_L^j \tilde{q}_R^k + \tilde{y}_{i,j,k} H_i \tilde{L}_L^j \tilde{l}_R^k, \tag{49}$$

where H_i is one of the 2 Higgs doublets, $\tilde{Q}_L^j$ is a squark associated to the left handed quark doublet, $\tilde{q}_R^k$ is a squark associated to the right handed quark singlet, $\tilde{L}_L^j$ is a squark associated to the left handed lepton doublet, $\tilde{l}_R^k$ is a squark associated to the right handed lepton singlet, $y_{i,j,k}$ and $\tilde{y}_{i,j,k}$ are Yukawa couplings and j and k are generation indices. These terms will come in general from both the SUSY superpotential and the soft-SUSY-breaking terms [15].

More importantly, the aforementioned terms are cubic interactions among scalar fields. And the squarks and sleptons carry baryon or lepton number and electric charge. So, under the assumption that lepton and baryon number are pretty much conserved (at least up to some very high scale), in these SUSY models there are both B-balls and L-balls which might also be electrically charged. Notice that the Higgs doublets do not carry lepton nor baryon number but some of the physical states are electrically charged. Hence, they can form part of the Q-balls too [15]. An intriguing possibility is that a Q-ball with both non-vanishing lepton and baryon numbers would interact as a leptoquark [15]. Shortly after, non-abelian Q-balls in SUSY models were also studied [17] and the influence of the abelian Q-balls on proton stability was addressed too [18].

7.3 Q-balls: just another DM candidate

Just after arguing that there were Q-balls in any SUSY extension of the SM, Kusenko and his collaborators studied their properties as DM candidates [19]. The first main concern was their production: none of the production mechanisms known at the time (pair production at high temperature, fusion like in nucleosynthesis and production in a phase transition) was able to generate Q-balls large enough to survive the evaporation in massless (or very light) fermions that we already discussed [12, 19]. They discovered that there is a production mechanism which could achieve such goal.

This mechanism uses the similarity between an infinite size Q-ball and a coherent scalar condensate with non-vanishing lepton or baryon numbers[16]. If in the early Universe, there was a condensate of

[16]This kind of condensate is the starting point in the already known Affleck-Dine scenario for baryogenesis.

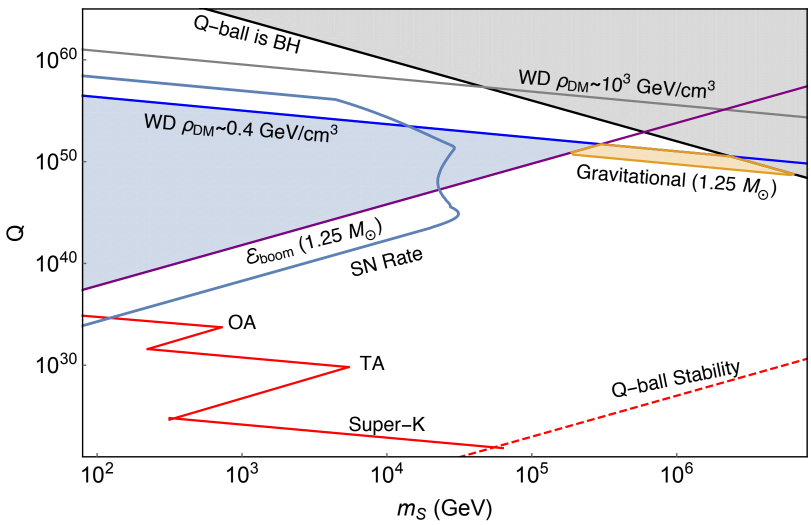

Figure 7: Constraints on the charge and the mass of Q-ball DM, where Q is its charge (baryon number) and M_S is a mass scale proportional to the Q-ball mass (see Eq. 128 in [20]). Plot extracted from [21]. Bounds come from demanding that the Q-ball interaction during a DM transit is capable of igniting White Dwarfs (WDs), occurring at a rate large enough to either ignite a single observed 1.25 $M_\odot$ WD in its lifetime (WD in local DM density is blue shaded) or exceed the measured Supernovas (SN) rate in our galaxy. The corresponding constraint from gravitational heating of WDs (orange shaded) and existing limits from terrestrial detectors (red) are also plotted [20], where TA and OA stand for Telescope Array and Owl-Airwatch telescope.

squarks or sleptons with a large VEV along some flat direction of the potential. Due to the large VEV, the baryon number can be strongly violated and then, the condensate acquires a non-zero baryon number. The subsequent evolution of the Universe leads to a vacuum where the baryon number is conserved and since then the scalar condensate is just Q-matter, what a Q-ball is made of, but without the right spatial distribution. But this condensate, due to small perturbations, might become unstable and it will decay to the lowest energy state: a Q-ball! (If its existence is allowed) [19].

The B-balls, i.e. a Q-ball with non-vanishing baryon number, generated through such mechanism, in order to be stable, should have a baryon number satisfying:

$$B \gtrsim \left(\frac{m_\varphi}{m_n}\right)^4 \gtrsim 10^{12}, \tag{50}$$

where m_φ is the typical mass of the squarks and $m_n \sim 1$ GeV is the mass of the lightest baryons. Given the LHC results, we expect $m_\varphi \gtrsim 10^3$ GeV. Let us remark that the lifetime of L-balls (with non-vanishing lepton number) will be much shorter due to the lightness of leptons (neutrinos especially). The authors point out that L-ball decays could distort the cosmic microwave background radiation (CMB) [19].

In a subsequent paper, Kusenko et al [22] analysed the experimental signatures of such B-balls in experiments such as Super-Kamiokande, Baikal Deep Underwater Neutrino Experiment and the MACRO search. The bound at that moment (1998) was $B \gtrsim 10^{21}$ [22]. Some updated bounds can be found in [20, 21]. In Fig. 7, we reproduce the plot shown in [21]. It is clear that the allowed parameter space is well constrained.

To wrap up this topic, let us take a quick look at some very recent works. Q-ball DM could be detected or constrained through White Dwarfs [21] or using IceCube [23]. But apparently several astrophysical constraints could be lifted if there were baryon violating operators [24]. 2 years ago a group claimed to have found a new type of Q-balls in gauge mediated SUSY breaking models [25]. And just a few weeks ago, another group studied Q-balls as DM candidates in the simple Higgs-portal dark matter model [26].

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A A soliton's story

As a conclusion to this Workshop Seminar series, I thought it would be nice to recall briefly the history of the concept soliton.

The concept soliton was born much before that particle physics and in a very different field: fluid dynamics. The first recorded observation and description of a soliton, then dubbed "wave of translation", was made by the Scottish engineer John Scott Russell (9 May 1808 - 8 June 1882) in the Union Canal in 1834[17]. Let us read his own description of the first observation of such phenomenon:

> *I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped – not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation.*

After his discovery, Russell spent a few years studying solitons in special water tanks. His studies were enough to discover that these waves are stable, can travel long distances, its speed depends on their size (the bigger, the quicker), its width depends on the depth and two other remarkable phenomena.

[17] All the information in this appendix was extracted from the English edition of Wikipedia.

First, that solitons do not merge and a big one will overtake smaller ones. Second, if a wave is too big for the depth of the water, it will split into two smaller solitons. At that time, experimental physics was ahead of theory because notable physicists of the time like Airy and Stokes could not explain solitons with their water wave theories. Just in 1870s Joseph Boussinesq and Lord Rayleigh provided the first theoretical descriptions of solitons. With the time, solitons spread around Physics and Science in general. Today, solitons are of interest in fiber optic physics, magnets and they may even appear in proteins and DNA!

But solitons' discoverer is also worth of a few words. Russell was a very accomplished civil engineer and naval architect who was also a Fellow of the Royal Society of Edinburgh and the Royal Society. He designed some early steam carriages in 1834, revolutionised ship hulls design in 1840s by studying the shape which would offer the least resistance to water and in 1848 made one of the first experimental observations of the Doppler effect, just 6 years after the publication of Christian Doppler's theory. He wrote articles for the British Encyclopaedia and was an accomplished lecturer and speaker. But he was very notable as a builder. He was responsible for the construction, although not the whole design, of the SS Great Eastern, the largest (by much) ship ever built at the time of her launch in 1858. Let me put some numbers to that ship: 211m long, surpassed in 1899; almost 19000 gross ton, surpassed in 1901; able to transport 4000 passengers from UK to Australia non-stop, surpassed just in 1913; two steam powered paddle wheels; one steam powered propeller and a set of sails. A ship with a very eventful history too, whose main legacy might be the submarine telegraph cables she laid. To wrap up, Scott Russell built the cupola for a building called Rotunda in Vienna in 1873 which was the biggest cupola in the world for nearly a century.

B Sidney Coleman: a short biography

I find very hard to close this Workshop Seminar series without writing a few lines about Sidney Coleman, the discoverer of Q-balls and one of the most influential theoretical physicists of the second half of the 20th century.

Sidney Richard Coleman was born in Chicago on 7 March 1937. He grew up in a tough neighbourhood of Chicago and developed an early interest in the construction of the atomic bomb, which gave him his ambition to become a physicist. Without losing time, he had already built a primitive computer in high school[18].

Coleman did his undergrad studies in the Illinois Institute of Technology and then moved to Caltech, where he was taught by Feynman and Gell-Mann. The later was his PhD supervisor. After receiving his PhD in 1962, he moved to Harvard where he stayed the rest of his career. During the following 3 decades, he produced several remarkable works. Among his contributions to Physics, we find the Coleman-Mandula theorem, the Coleman theorem, the CCWZ formalism, the Coleman-Weinberg potential, Q-balls, studies on the cosmological constant and tadpoles and many others.

He received the Dirac Medal, the NAS Award for Scientific Reviewing and the David Heineman Prize. But his reputation among his colleagues was much higher that the amount of awards. He was considered the "physicist's physicist" and "the Oracle", with a huge encyclopedic knowledge and a deep understanding of Physics. Remarkably, he is pretty much unknown out of the field of theoretical particle physics.

Coleman was also a legendary teacher. His lectures at Harvard were praised by his students. And his series of lectures at the Erice (Sicily, Italy) Summer School were put altogether in the remarkable book "Aspects of Symmetry". He had several well-known graduate students, like Erick Weinberg, David Politzer, Jeffrey Mandula and Anthony Zee. And there are lot of stories about him, stressing his special

[18] The information for this appendix was extracted from Wikipedia, the obituary published by Harvard (https://news.harvard.edu/gazette/story/2007/11/sidney-coleman-dies-at-70/) and the Sidneyfest webpage (http://media.physics.harvard.edu/QFT/sidneyfest.htm)

character, like refusing to teach at 9 A.M. saying “I can’t stay up that late”. He was also a huge fan of science fiction (he even founded a publisher), poker and hiking.

Coleman stopped teaching and working in 2003. In 2005, with him present, Harvard’s Physics Department organized a celebration of his life and work, called Sidneyfest. This meeting turned out to be a meeting of all the brilliant theoretical physicists of the last decades. Coleman died 2 years later, on 7 November 2007, at 70.