

Spinning black holes versus quantum higher-spin particles

Justin Vines

Max Planck Institute for Gravitational Physics (AEI) Potsdam

From Classical Gravity to Quantum Amplitudes and Back:
post-Newtonian, post-Minkowskian,
effective one-body, self-force, ...

Kolleg Mathematik Physik Berlin

Overview

- Review of results for classical scattering of spinning black holes

$$p_1, S_1, \ p_2, S_2, \ b \rightarrow p'_1, S'_1, \ p'_2, S'_2, \ b'$$

in the post-Minkowskian approximation to general relativity

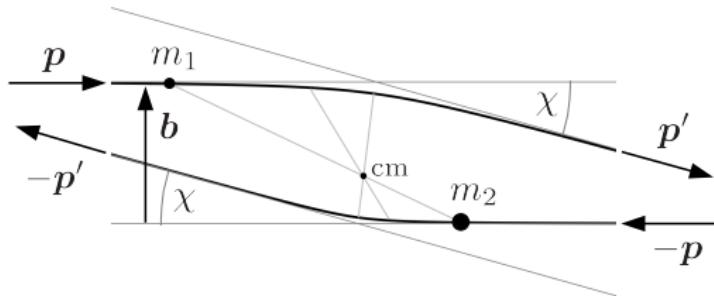
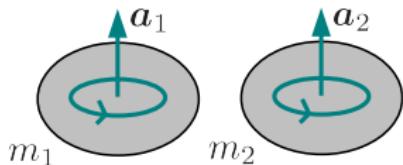
$$\begin{aligned} \frac{\Delta \mathbf{p}}{|\mathbf{p}|} = & \ \textcolor{brown}{c}_{10} \frac{GM}{b} + \left(\frac{GM}{b} \right)^2 \left(\textcolor{brown}{c}_{20} + \textcolor{brown}{c}_{11} \hat{a} \right) \quad [\text{Siemonsen-JV 1909.07361}] \\ & + \left(\frac{GM}{b} \right)^3 \left(\textcolor{violet}{c}_{30} + \textcolor{brown}{c}_{21} \hat{a} + \textcolor{blue}{c}_{12} \hat{a}^2 \right) \\ & + \left(\frac{GM}{b} \right)^4 \left(\textcolor{red}{c}_{40} + \textcolor{red}{c}_{31} \hat{a} + \textcolor{violet}{c}_{22} \hat{a}^2 + \textcolor{blue}{c}_{13} \hat{a}^3 \right) + \dots \end{aligned}$$

$$\mathbf{c}_{k\ell} = \mathbf{c}_{k\ell} \left(\left\{ \frac{p^\mu}{m}, \frac{S^\mu}{S} \right\}, \frac{b^\mu}{b}, \frac{m_2 - m_1}{m_2 + m_1} \right), \quad \left\{ \hat{a} = \frac{S}{Gm^2} \in [0, 1) \right\}$$

confirmed complete and generic; conjectural/partial/specialized; no results

- Amplitudes for massive higher-spin particles exchanging gravitons
- A spinning test black hole (?) moving in an exact Kerr background

Elastic aligned-spin scattering of two black holes



masses m_1, m_2 ($c = 1$)

$$\text{spins } S_1 = m_1 a_1 = Gm_1^2 \hat{a}_1$$

$$S_2 = m_2 a_2 = Gm_2^2 \hat{a}_2$$

“proper” impact parameter $b_{(\text{cov})}$

$$\text{relative velocity } v_{(\infty)} : \frac{1}{\sqrt{1 - v^2}} = \gamma$$

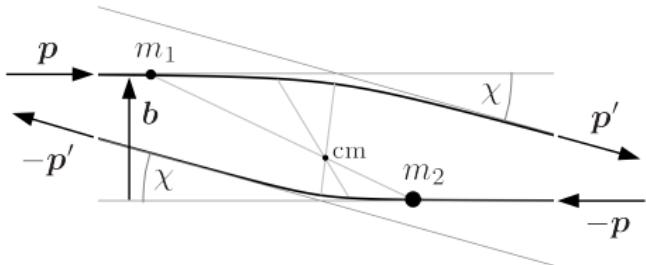
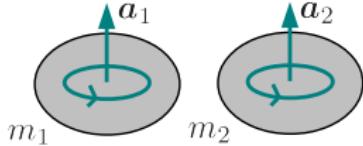
$$s = E^2 = m_1^2 + m_2^2 + 2m_1m_2\gamma$$

scattering angle: [..., Westpfahl '85, ..., Bini-Damour '17-18, ...]

$$\chi = \frac{GE}{b} \left[2 \frac{1 + v^2}{v^2} + 3\pi \frac{4 + v^2}{4v^2} \frac{G(m_1 + m_2)}{b} \right. \\ \left. - \frac{4}{v} \frac{a_1 + a_2}{b} - \pi \frac{2 + 3v^2}{v^3} \left(\frac{G(4m_1 + 3m_2)a_1}{b^2} + 1 \leftrightarrow 2 \right) + \dots \right]$$

{↓ spin-orbit: aligned ↔ generic}

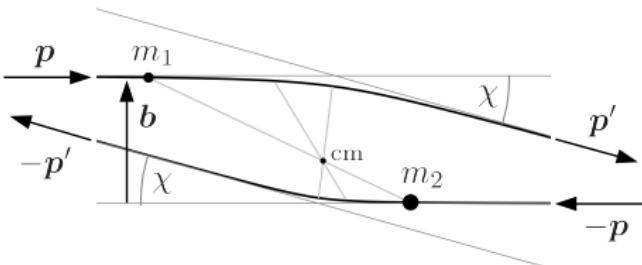
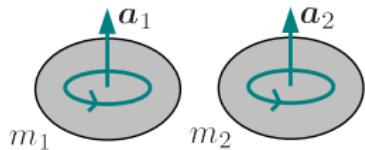
Elastic aligned-spin scattering of two black holes



[..., **PN**, ..., Bini-Damour '17-18, ..., Guevara, JV, Steinhoff, Buonanno, Ochirov, (Chung, Huang, Kim, Lee,) Maybee, O'Connell, '17-19, ...]

$$\begin{aligned} \chi = \frac{GE}{b} & \left[2 \frac{1+v^2}{v^2} + 3\pi \frac{4+v^2}{4v^2} \frac{G(m_1+m_2)}{b} \right. \\ & - \frac{4}{v} \frac{a_1+a_2}{b} - \pi \frac{2+3v^2}{v^3} \left(\frac{G(4m_1+3m_2)a_1}{b^2} + 1 \leftrightarrow 2 \right) \\ & \left. + 2 \frac{1+v^2}{v^2} \frac{(a_1+a_2)^2}{b^2} - \frac{4}{v} \frac{(a_1+a_2)^3}{b^3} + \dots \right] \quad (\text{also generic } \not\leftrightarrow \text{ aligned}) \end{aligned}$$

Elastic aligned-spin scattering of two black holes

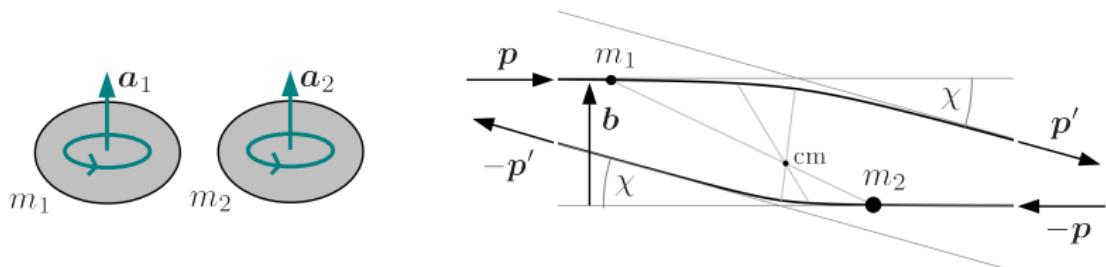


[..., **PN**, ..., Bini-Damour '17-18, ..., Guevara, JV, Steinhoff, Buonanno, Ochirov, (Chung, Huang, Kim, Lee,) Maybee, O'Connell, '17-19, ...]

$$\chi = \frac{GE}{v^2} \left[\frac{(1+v)^2}{b+a_1+a_2} + \frac{(1-v)^2}{b-a_1-a_2} + 3\pi \frac{4+v^2}{4} \frac{G(m_1+m_2)}{b^2} - \pi \frac{2+3v^2}{v} \left(\frac{G(4m_1+3m_2)a_1}{b^3} + 1 \leftrightarrow 2 \right) + \dots \right]$$

$$\begin{aligned} L_{\text{can}} &= \frac{m_1 m_2 \gamma v}{E} b + \frac{E - m_1 - m_2}{2E} \left[E(a_1 + a_2) - (m_1 - m_2)(a_1 - a_2) \right] : \text{SR} \\ &= J - S_1 - S_2 \end{aligned}$$

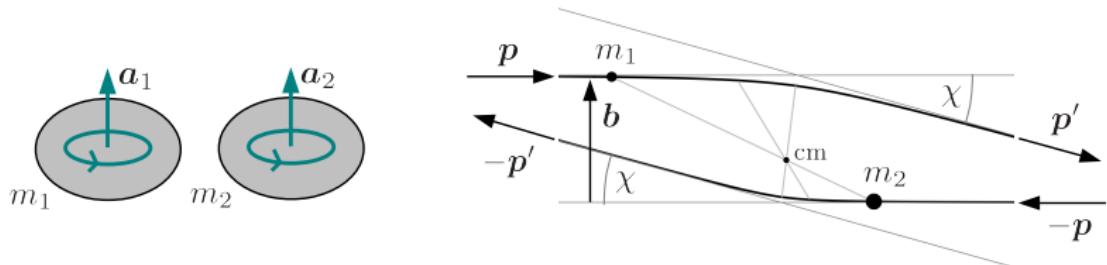
Elastic aligned-spin scattering of two black holes



[..., **PN** (... Levi-Steinhoff NNLO- S^2), ..., **BD**, ..., **Guevara, JV, Steinhoff, Siemonsen, Buonanno, Ochirov, Chung, Huang, Kim, Lee, Maybee, O'Connell**]

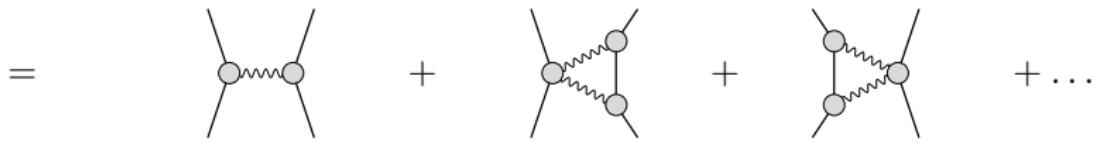
$$\chi = \frac{GE}{v^2} \left[\sum_{\pm} \frac{(1 \pm v)^2}{b \pm a_1 \pm a_2} + 3\pi \frac{4+v^2}{4} \frac{G(m_1 + m_2)}{b^2} + c_{30}^{\text{BCRSSZ}}(v, \nu) \frac{G^2 M^2}{b^3} \right. \\ \left. - \pi \frac{2+3v^2}{v} \left(\frac{Gm_1(4a_1 + 3a_2)}{b^3} + 1 \leftrightarrow 2 \right) \right. \\ \left. - \left(\frac{Gm_1 [c_{22}^0(v) a_1^2 + c_{22}^1(v) a_1 a_2 + c_{22}^2(v) a_2^2]}{b^4} + 1 \leftrightarrow 2 \right) + \dots \right]$$

Two-BH χ through $\mathcal{O}(G^2\sigma^4)$ from a test BH in Kerr?



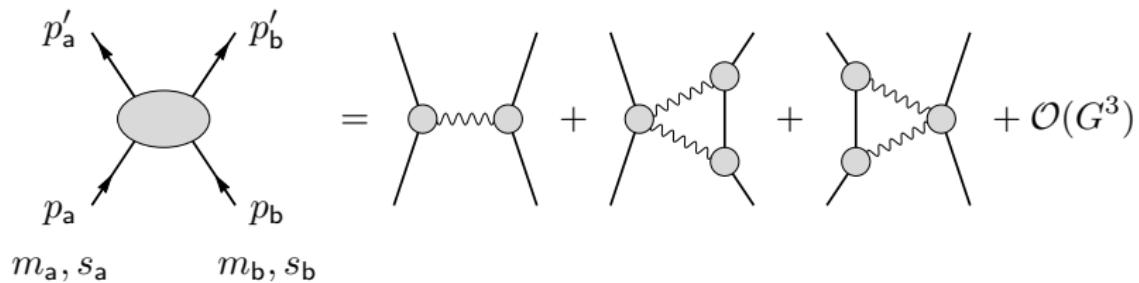
[..., **PN** (...NNLO- S^2), **BD**, ... **A**rkani-Hamed **HH**, **GOV**, **CHKL**, **K**osower **MO**, ++ ...]

$$\chi = \frac{GE}{v^2} \left[\sum_{\pm} \frac{(1 \pm v)^2}{b \pm a_1 \pm a_2} + \left(Gm_2 F(v, b, a_1, a_2) + 1 \leftrightarrow 2 + \mathcal{O}(a_{1,2}^5) \right) + \dots \right]$$



$$F(v, b, \sigma, a) = \frac{\pi}{2v^2} \frac{\partial}{\partial b} \oint \frac{d\zeta}{2\pi i} \frac{(1-v\zeta)^4}{(\zeta^2-1)^{3/2}} \left(b - \zeta a - \frac{\zeta-v}{1-v\zeta} \sigma \right)^{-1} + \mathcal{O}(\sigma^5) ?$$

Amplitudes for massive spin- s particles and gravitons



“Minimal coupling” (high-energy limit) [AHH] [Guevara, GOV, CHKL, ++]

$$\text{1} \quad \begin{array}{c} \diagdown \\ \text{2} \end{array} \quad \text{3}^+ = \frac{1}{m_P} \left(\frac{\langle \zeta | p_1 | 3 \rangle}{\langle \zeta | 3 \rangle} \right)^2 \left(\frac{\langle \mathbf{12} \rangle}{m} \right)^{2s}, \quad \text{for any } s,$$

$$\text{4} \quad \begin{array}{c} \diagup \\ \text{1} \end{array} \quad \text{3}^- \quad = \frac{-\langle 3 | p_1 | 2 \rangle^4}{m_P^2 t(s-m^2)(u-m^2)} \left(\frac{\langle 43 | \mathbf{12} \rangle + \langle 13 | \mathbf{42} \rangle}{\langle 3 | p_1 | 2 \rangle} \right)^{2s}.$$

2⁺ for $s \leq 2$.

Nonspinning black holes as point masses (monopoles)

Effective action W
for spacetime metric g
and masses m_a with worldlines $x = z_a(\tau_a)$:

$$W = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R - \sum_a m_a \int d\tau_a.$$

$$\frac{\delta W}{\delta z_a} = 0 \quad \Rightarrow \quad \text{geodesic eqs. for worldlines } z_a.$$

$$\frac{\delta W}{\delta g} = 0 \quad \Rightarrow \quad \text{Einstein field eq. for metric } g_{\mu\nu}, \text{ with}$$

$$T^{\mu\nu} = \sum_a \int d\tau_a \dot{z}^\mu \dot{z}^\nu \frac{\delta^4(x - z_a)}{\sqrt{-g}} \quad (\text{when } g_{\mu\nu} \dot{z}^\mu \dot{z}^\nu = -1)$$

... good for test bodies in a background.

... for self-gravitating bodies? ...

Nonspinning black holes as point masses (monopoles)

Effective action W
for spacetime metric g
and masses m_a with worldlines $x = z_a(\tau_a)$:

$$W = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R - \sum_a m_a \int d\tau_a.$$

$$\frac{\delta W}{\delta z_a} = 0 \quad \Rightarrow \quad \text{geodesic eqs. for worldlines } z_a.$$

$$\frac{\delta W}{\delta g} = 0 \quad \Rightarrow \quad \text{Einstein field eq. for metric } g_{\mu\nu}, \text{ with}$$

$$T^{\mu\nu} = \sum_a \int d\tau_a \dot{z}^\mu \dot{z}^\nu \delta_4(x, z_a) \quad (\text{when } g_{\mu\nu} \dot{z}^\mu \dot{z}^\nu = -1)$$

... good for test bodies in a background.

... for self-gravitating bodies? ...

Perturbative approximation schemes

... point masses:
$$W = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R - \sum_a m_a \int d\tau_a \quad ...$$

- post-Newtonian: $g_{\mu\nu} dx^\mu dx^\nu = -(c^2 + 2U) dt^2 + \delta_{ij} dx^i dx^j + \mathcal{O}(c^{-2})$.
- post-Minkowskian: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2), \quad h \sim \mathcal{O}(G)$.
- post-test-body:
 - self-force results: motion is geodesic in an effective regularized perturbed metric satisfying the vacuum field eq.
[..., Detweiler, Whiting, Wald, Gralla, Poisson, Pound, Harte, ...]

Spinning bodies and higher multipoles

Minimal degrees of freedom (others “integrated out”): worldline $z(\tau)$
plus local Lorentz frame (“body-fixed” vierbein) $\Lambda_a{}^\mu(\tau)$.

Internal Lorentz symmetry \Rightarrow dependence only on $\Omega^{\mu\nu} := \Lambda^{a\mu} \frac{D\Lambda_a{}^\nu}{d\tau}$,

$$W = \int d\tau \mathcal{L}(z, \dot{z}, \Omega)[g] + \text{constraints.}$$

: equivalently, phase-space action : general covariance

$$W = \int d\tau \left\{ p_\mu \dot{z}^\mu + \frac{1}{2} S_{\mu\nu} \Omega^{\mu\nu} - \beta^\mu S_{\mu\nu} p^\nu - \frac{\alpha}{2} [p^2 + \mathcal{M}^2(p, S, z)] \right\},$$

momentum p_μ and spin $S_{\mu\nu}$ varied along with z , $\Lambda_a{}^\mu$ (and α , β^μ).

Dynamical mass function: $-p^2 = \mathcal{M}^2(p_\mu, S_{\mu\nu}, g_{\mu\nu}(z), \{R_{\mu\nu\kappa\lambda;N}(z)\})$.
 $-g^{\mu\nu} p_\mu p_\nu =$

$N=(\alpha_1 \dots \alpha_n)$

Spinning bodies and higher multipoles

Dynamical mass: $-p^2 = \mathcal{M}^2 \left(\frac{p_\mu}{\sqrt{-p^2}}, S_{\mu\nu}, g_{\mu\nu}(z), \{R_{\mu\nu\kappa\lambda;N}(z)\} \right)$.

Equations of motion: $0 = S_{\mu\nu} p^\nu,$

$$\frac{D}{d\tau} p_\mu + \frac{1}{2} R_{\mu\nu\kappa\lambda} \dot{z}^\nu S^{\kappa\lambda} = \frac{p \cdot \dot{z}}{2} \frac{D}{Dz^\mu} \ln \mathcal{M}^2,$$

$$\frac{D}{d\tau} S^{\mu\nu} - 2p^{[\mu} \dot{z}^{\nu]} = p \cdot \dot{z} \left(p^{[\mu} \frac{\partial}{\partial p_{\nu]}} + 2S^{[\mu}{}_\rho \frac{\partial}{\partial S_{\nu]\rho}} \right) \ln \mathcal{M}^2$$

: Mathisson-Papapetrou-Dixon-Harte $\Leftrightarrow \nabla_\mu T^{\mu\nu} = 0$ with

$$T^{\mu\nu}(x) = \int d\sigma \left\{ p^{(\mu} \dot{z}^{\nu)} \delta_4(x, z) - \nabla_\rho \left[S^{\rho(\mu} \dot{z}^{\nu)} \delta_4(x, z) \right] + \frac{p \cdot \dot{z}}{2} \sum_{n=0}^{\infty} \frac{\partial \ln \mathcal{M}^2}{\partial R_{\alpha\beta\gamma\delta;N}} \left[\mathcal{G}^{(\mu\nu)} R_{\alpha\beta\gamma\delta;N} \delta_4(x, z) + \frac{2}{\sqrt{-g}} \frac{\delta R_{\alpha\beta\gamma\delta;N}(z)}{\delta g_{\mu\nu}(x)} \right] \right\}.$$

Spinning bodies and higher (spin-induced) multipoles

$$-p^2 = \mathcal{M}^2(u^\mu, \sigma^\mu, z)[g] = m^2 + \mathcal{O}(R). \quad u^\mu := \frac{p^\mu}{\sqrt{-p^2}}$$

Bare-rest-mass-rescaled spin (ring-radius) vector:

$$\sigma^\mu = -\frac{1}{2m}\epsilon^{\mu\nu\kappa\lambda}u_\nu S_{\kappa\lambda} \quad \Leftrightarrow \quad S_{\mu\nu} = m\epsilon_{\mu\nu\kappa\lambda}u^\kappa\sigma^\lambda.$$

Minimal MPDH eqs.:

$$\frac{D}{d\tau}p_\mu + \frac{1}{2}R_{\mu\nu\kappa\lambda}\dot{z}^\nu S^{\kappa\lambda} = \frac{p \cdot \dot{z}}{2} \frac{D}{Dz^\mu} \ln \mathcal{M}^2,$$

$$\frac{D}{d\tau}S^{\mu\nu} - 2p^{[\mu}\dot{z}^{\nu]} = p \cdot \dot{z} \left(u^{[\mu} \frac{\partial}{\partial u_{\nu]} + \sigma^{[\mu} \frac{\partial}{\partial \sigma_{\nu]}} \right) \ln \mathcal{M}^2.$$

[..Goldberger..Rothstein..Porto.. ..Marsat.. Levi-Steinhoff ...]: Consider

$$\frac{\mathcal{M}^2}{m^2} = 1 + \mathcal{C}_2 R_{u\sigma u\sigma} + \mathcal{C}_3 R_{u\sigma u\sigma; \sigma}^* + \mathcal{C}_4 R_{u\sigma u\sigma; \sigma\sigma} + \dots + \mathcal{O}(\sim R^2).$$

Relevant couplings for a spinning test black hole

- Suppressing indices, $*$'s, u 's, dimensionless coefficients,

$$\begin{aligned}\frac{\mathcal{M}^2}{m^2} = & 1 + R\sigma^2 + \nabla R\sigma^3 \oplus \nabla^2 R\sigma^4 \oplus \nabla^3 R\sigma^5 \oplus \nabla^4 R\sigma^6 \oplus \dots \\ & \oplus R^2\sigma^4 \oplus \nabla R^2\sigma^5 \oplus \nabla^2 R^2\sigma^6 \oplus \dots \\ & \oplus R^3\sigma^6 \oplus \dots\end{aligned}$$

plus many other $R^{\geq 2}$ terms with nonzero powers of m ,

$$\oplus (m\nabla)^k (\sigma\nabla)^l \left(\frac{\sigma}{m}\right)^n \left(m^4 R^2 \oplus m^6 R^3 \oplus \dots\right),$$

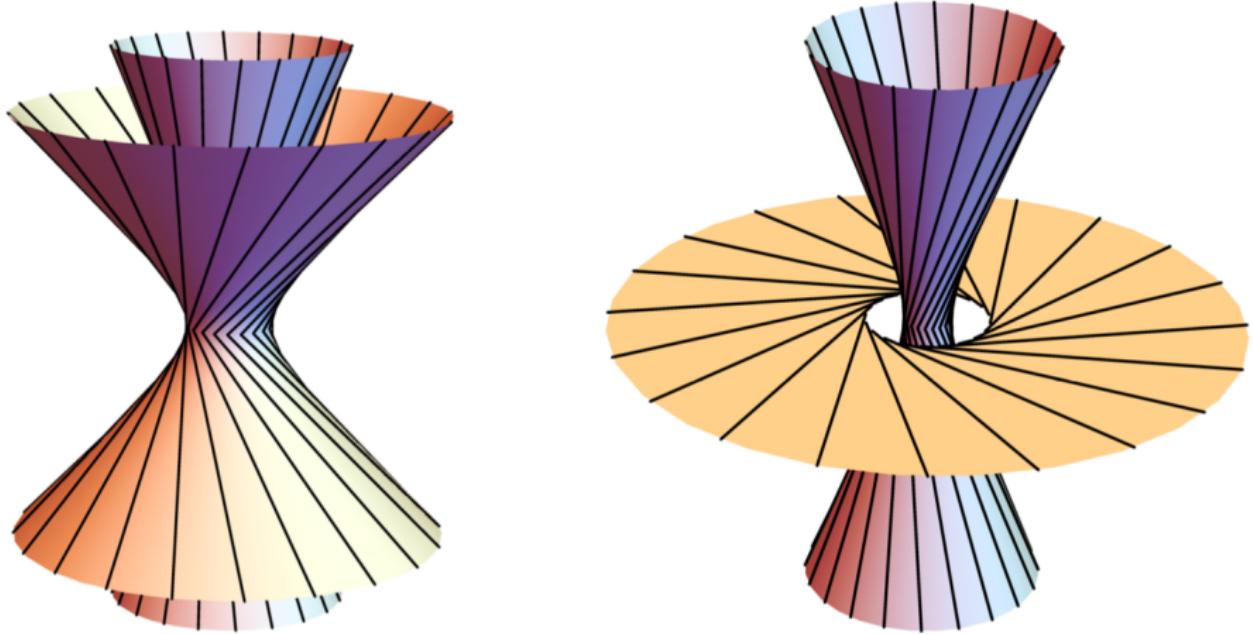
e.g.: $m^4 R^2$ terms, $k, l, n = 0$: leading adiabatic quadrupolar tidal effects

- Reasonable conjecture?:

If a spinning test black hole limit exists, it should have only m^0 terms.

Linearized matching to Kerr

- Exact Kerr in Kerr-Schild form ($l_\mu l^\mu = 0$) : $g_{\mu\nu}^{\text{Kerr}} = \eta_{\mu\nu} + \varphi l_\mu l_\nu$.



Linearized matching to Kerr

- Exact Kerr in Kerr-Schild form ($l_\mu l^\mu = 0$) :

$$g_{\mu\nu}^{\text{Kerr}} = \eta_{\mu\nu} + \varphi l_\mu l_\nu = \eta_{\mu\nu} + h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)},$$

where $h_{\mu\nu}$ is an exact solution (off the disk) of $\square \bar{h}^{\mu\nu} = 0 = \partial_\mu \bar{h}^{\mu\nu}$:

$$\bar{h}^{\mu\nu} = u^\rho u^{(\mu} \exp(\sigma * \partial)^{\nu)}{}_\rho \frac{4Gm}{r}, \quad (\sigma * \partial)^\mu{}_\nu = \epsilon^\mu{}_{\nu\kappa\lambda} \sigma^\kappa \partial^\lambda.$$

- Take $T^{\mu\nu}[z, u, \sigma, \mathcal{M}^2, g]$ with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)$ and $z = \text{geod.}(\eta)$ and solve $\square \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} + \mathcal{O}(h^2)$ for $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\rho{}_\rho$.
- Match, fixing the \sim linear-in-curvature couplings:

$$\frac{\mathcal{M}^2 - m^2}{2m^2} = \left(-\frac{1}{2!} R_{u\sigma u\sigma} + \frac{1}{3!} R_{u\sigma u\sigma; \sigma}^* + \frac{1}{4!} R_{u\sigma u\sigma; \sigma\sigma} + \dots \right) + \mathcal{O}(\sim R^2).$$

Couplings quadratic in curvature and quartic in spin

$$\frac{\mathcal{M}^2}{m^2} = 1 - \frac{2}{2!} R_{u\sigma u\sigma} + \frac{2}{3!} R_{u\sigma u\sigma; \sigma}^* + \frac{2}{4!} R_{u\sigma u\sigma; \sigma\sigma} + \dots$$
$$\oplus \sim R^2 \sigma^4 \quad \oplus \dots$$

In terms of the STF tidal tensors, $\mathcal{E}_{\mu\nu} = R_{\mu\kappa\nu\lambda} u^\kappa u^\lambda$, $\mathcal{B}_{\mu\nu} = R_{\mu\kappa\nu\lambda}^* u^\kappa u^\lambda$,

$$\delta(\frac{\mathcal{M}^2}{m^2})_4 = C_{4A} (\mathcal{E}_{\sigma\sigma})^2 + C_{4D} (\mathcal{B}_{\sigma\sigma})^2$$
$$+ C_{4B} \mathcal{E}_{\sigma\mu} \mathcal{E}_{\sigma}{}^{\mu} \sigma^2 + C_{4E} \mathcal{B}_{\sigma\mu} \mathcal{B}_{\sigma}{}^{\mu} \sigma^2$$
$$+ C_{4C} \mathcal{E}_{\mu\nu} \mathcal{E}^{\mu\nu} \sigma^4 + C_{4F} \mathcal{B}_{\mu\nu} \mathcal{B}^{\mu\nu} \sigma^4 + C_{4G} R_{u\sigma u\sigma; uu} \sigma^2.$$

Aligned-spin scattering in Kerr at $\mathcal{O}(G^2 \sigma^4)$ depends only on

$$C_{4a} = C_{4A} + C_{4B}, \quad C_{4c} = C_{4C}, \quad C_{4e} = C_{4E} + \frac{C_{4F}}{2}.$$

Couplings quadratic in curvature and quartic in spin

Effective test black hole:

$$\begin{aligned}\frac{\mathcal{M}^2}{m^2} = & \ 1 - \frac{2}{2!} R_{u\sigma u\sigma} + \frac{2}{3!} R_{u\sigma u\sigma;\sigma}^* + \frac{2}{4!} R_{u\sigma u\sigma;\sigma\sigma} + \dots \\ & + C_{4..} \cdot R^2 \sigma^4 \oplus \dots\end{aligned}$$

Aligned-spin scattering angle for the test black hole
in a Kerr background,
at the leading order in the ultrarelativistic limit $v \rightarrow 1$:

$$\begin{aligned}\chi = & -\frac{315\pi(Gm)^2(5b-4a)\sigma^4}{256(b+a)^4(b^2-a^2)^{3/2}} \frac{C_{4a} + 2C_{4c} + C_{4e}}{1-v^2} + \mathcal{O}(1-v^2)^0 \\ & + \mathcal{O}(G^2\sigma^5) + \mathcal{O}(G^3).\end{aligned}$$

⇒ we want $C_{4a} + 2C_{4c} + C_{4e} \equiv 0$?

Matching to perturbed Kerr (self-force)

[Siemonsen-JV 1909.07361]

- For an arbitrary-mass two-black-hole system, assume the validity of

$$\chi \doteq \frac{GE}{v^2} \left[\sum_{\pm} \frac{(1 \pm v)^2}{b \pm a_1 \pm a_2} + \left(Gm_2 F(v, b, a_1, a_2) + 1 \leftrightarrow 2 + \mathcal{O}(a_{1,2}^5) \right) + \dots \right]$$
$$\doteq \quad \begin{array}{c} \text{Diagram 1: Two black holes (circles) connected by a wavy line (self-force), with two external legs.} \\ [1ex] + \end{array} \quad \begin{array}{c} \text{Diagram 2: Two black holes (circles) connected by a wavy line, with three external legs (one internal).} \\ [1ex] + \end{array} \quad \begin{array}{c} \text{Diagram 3: Two black holes (circles) connected by a wavy line, with four external legs (two internal).} \\ [1ex] + \dots \end{array}$$

with $F(v, b, \sigma, a)$ determined by a consistent test-black-hole limit

Matching to perturbed Kerr (self-force)

[Siemonsen-JV 1909.07361]

- For an arbitrary-mass two-black-hole system, assume the validity of

$$\chi \doteq \frac{GE}{v^2} \left[\sum_{\pm} \frac{(1 \pm v)^2}{b \pm a_1 \pm a_2} + \left(Gm_2 F(v, b, a_1, a_2) + 1 \leftrightarrow 2 + \mathcal{O}(a_{1,2}^5) \right) + \dots \right]$$

with $F(v, b, \sigma, a)$ determined by a consistent test-black-hole limit

- Assume the existence of a local-in-time canonical Hamiltonian H for the two-body conservative dynamics—determined modulo gauge by χ —, and employ the associated first law of spinning binary mechanics [Le Tiec+] (circular, aligned-spin),

$$dE = \Omega dL + \sum_a \left(z_a dm_a + \Omega_a dS_a \right),$$

to compute the (Detweiler) redshift invariants $z_a = \frac{\partial H}{\partial m_a}$

and the spin precession frequencies $\Omega_a = \frac{\partial H}{\partial S_a}$

Matching to perturbed Kerr (self-force)

[Siemonsen-JV 1909.07361]

- For an arbitrary-mass two-black-hole system, assume the validity of

$$\chi \doteq \frac{GE}{v^2} \left[\sum_{\pm} \frac{(1 \pm v)^2}{b \pm a_1 \pm a_2} + \left(Gm_2 F(v, b, a_1, a_2) + 1 \leftrightarrow 2 + \mathcal{O}(a_{1,2}^5) \right) + \dots \right]$$

with $F(v, b, \sigma, a)$ determined by a consistent test-black-hole limit.

- ... local-in-time canonical Hamiltonian ... first law (aligned, circular) ...

$$dE = \Omega dL + \sum_a \left(z_a dm_a + \Omega_a dS_a \right),$$

- Take the $m_2 \gg m_1$ limit and compare to self-force results
for z_1 [Kavanagh, Ottewill, Wardell]
and Ω_1 [Bini, Damour, Geralico, Kavanagh, van de Meent]

$$\Rightarrow C_{4a} + 6C_{4c} \doteq 0, \quad C_{4a} + 3C_{4e} \doteq 0.$$

Matching to perturbed Kerr (self-force)

[Siemonsen-JV 1909.07361]

$$\chi \doteq \frac{GE}{v^2} \left[\sum_{\pm} \frac{(1 \pm v)^2}{b \pm a_1 \pm a_2} + \left(Gm_2 F(v, b, a_1, a_2) + 1 \leftrightarrow 2 + \mathcal{O}(a_{1,2}^5) \right) + \dots \right]$$

The AHH-Guevara-GOV result,

$$F(v, b, \sigma, a) = \frac{\pi}{2v^2} \frac{\partial}{\partial b} \oint \frac{d\zeta}{2\pi i} \frac{(1-v\zeta)^4}{(\zeta^2 - 1)^{3/2}} \left(b - \zeta a - \frac{\zeta - v}{1 - v\zeta} \sigma \right)^{-1} + \mathcal{O}(\sigma^5)?$$

corresponds precisely to $C_{4a} \doteq C_{4a} \doteq C_{4e} \doteq 0$, which is the unique solution of the two equations from circular self-force matching and the one from a finite ultrarelativistic limit ($v \rightarrow 1$).

Further comments:

- eccentric self-force results
- applicability of the first law
- higher orders in spin
- $\sqrt{\text{Kerr}}$
- G^3