Evaluating a Fit Method for an Online Orbit-Response-Matrix Model at DELTA

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Abstract

At DELTA, a 1.5-GeV electron storage ring operated by the TU Dortmund University, an online orbit-response-matrix model based on the bilinear-exponential with dispersion (BE+d model) is under development. The online fit will be fed with data generated by the slow orbit feedback. The BE+d model depends on the beta functions, the betatron phases and the tunes in both planes as well as a scaled dispersion. After a new fitting recipe had been introduced to obtain estimates of the aforementioned quantities, this work focuses on investigating the measurement-over-measurement error of the fitted beta functions. The scaled dispersion output is also evaluated. The presented research is based on measurement results.

Plan for an Online Orbit-Response-Matrix Model [1]

The corrected orbit displacement $\Delta\vec{\kappa}_n$ and the applied steering angles $\Delta\vec{\theta}_n$ for each orbit correction n are to be stored in a ring buffer

$$\left[\begin{pmatrix}\Delta\vec{\kappa}_0\\ \Delta\vec{\theta}_0\end{pmatrix}, \begin{pmatrix}\Delta\vec{\kappa}_1\\ \Delta\vec{\theta}_1\end{pmatrix} \cdots \begin{pmatrix}\Delta\vec{\kappa}_N\\ \Delta\vec{\theta}_N\end{pmatrix}\right]$$

of length N. An online fit

$$\chi^2 = \sum_n^N \left|\Delta\vec{\kappa}_n - \mathbf{R}_{\mathrm{BE+d}}\Delta\vec{\theta}_n\right|^2 \to \min$$

then continuously updates an instance of the bilinear-exponential model with dispersion $\mathbf{R}_{\mathrm{BE+d}}$ [2] on the ring buffer. The model instance contains

1. twiss-parameter information and
2. can be used for orbit correction.

Bilinear-Exponential Model with Dispersion (BE+d Model) [2]

According to the BE+d model, a single orbit-response matrix element, which is orbit displacement $\Delta\kappa_{wj}$ at BPM j in plane w divided by steering angle $\Delta\theta_k$ applied at steering magnet k, is given by

$$\frac{\Delta\kappa_{wj}}{\Delta\theta_k} = \sum_m^{M=2} \Re\left\{\sqrt{I_m\beta_{mwj}}e^{i\Phi_{mwj}}e^{-i\pi q_m \operatorname{sign}(s_j-s_k)}A^*_{mk}\right\} + d_{wj}b_k.$$

β_{mwj}	Coupled beta function	$m \in M$	Modes
Φ_{mwj}	Coupled betatron phase	$w \in W$	Planes
d_{wj}	Scaled dispersion	$j \in J$	BPMs
q_m	Tune	$k \in K$	Steering Magnets
I_m	Courant-Snyder invariant		
b_k	Energy diff./steering ang.		
A_{mk}	Steerer parameter		

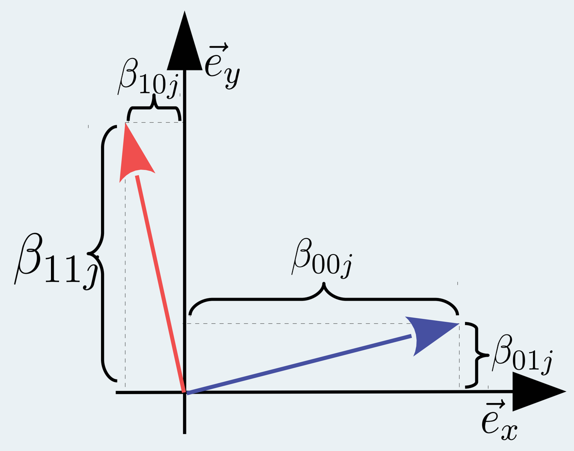

Figure 1: Beta function values β_{mwj} of coupled betatron oscillations at BPM j. The first mode ($m = 0$) is mostly horizontal. The second mode ($m = 1$) is mostly vertical.

The BE+d-model representation of an orbit-response matrix has

$$D = 2M(WJ + K) + K + WJ - M - 1$$

degrees of freedom.

> Only ordering of BPMs and steering magnets along the beam path is required as lattice information.

Fitting Procedure [1]

1. Fit decoupled BE+d model with a single mode ($M = 1$) and without dispersion in both planes separately (166 degrees of freedom in the horizontal plane and 158 degrees of freedom in the vertical plane).
2. Fit complete BE+d model but vertical constant tunes (815 degrees of freedom).
3. Fit complete BE+d model including tunes (817 degrees of freedom).

The idea for this approach is based on inter-plane coupling and dispersion having only small effects on the overall calculation outcome. It requires only measured tunes as additional input. The remaining quantities are initialized random.

> The closed-orbit bilinear-exponential analysis (COBEA) algorithm is available to fit the BE+d model onto a measured orbit-response matrix [3]. It can be indirectly applied to the ring buffer with the detour of a linear fit ($W \cdot J \cdot K = 6048$ degrees of freedom) [1]. COBEA was validated with LOCO results at MLS and BESSY II.

Maschine and Measurement Statistics

machine		
num. of BPMs	J	54
num. of h. steerers	K_h	30
num. of v. steerers	K_v	26

set of measurements	ONE	TWO
SAW status	OFF	ON
num. of measurements	188	250
avg. orbit displacement $\bar{\kappa}$	566 μm	536 μm
avg. steering angle per orbit displacement $\bar{\theta}$	0.024 mrad	0.022 mrad
avg. num. of steerers per orbit displacement	31	33

References

[1] S. Kötter and T. Weis, "Towards an adaptive orbit-response-matrix model for twiss-parameter diagnostics and orbit correction at DELTA", in Proc. IBIC'19, paper WECO02, Malmö, Sweden, Sep. 2019.

[2] B. Riemann, "The bilinear-exponential closed-orbit model and its application to storage ring beam diagnostics", dissertation, TU Dortmund University, Apr. 2016.

[3] B. Riemann, S. Kötter, S. Khan, and T. Weis, "COBEA - optical parameters from response matrices without knowledge of magnet strengths", in Proc. IPAC'17, paper MOPIK066, Copenhagen, Denmark, May 2017.

Beta-Function Comparison for Fits over 60 Random Orbit Displacements

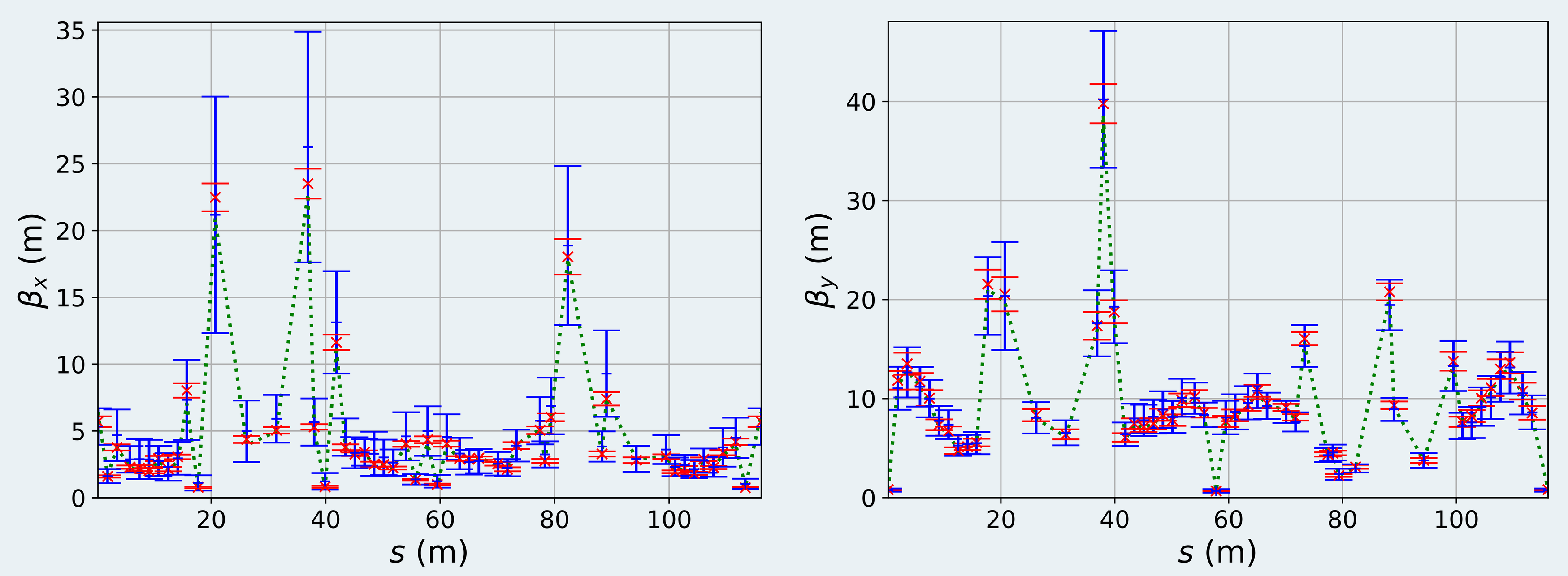

Figure 2: Horizontal beta function (left) and vertical beta function (right) along the beam path of the storage ring for about 100 direct fits (blue) about 100 indirect fits with COBEA (red) of randomly selected sets of 60 orbit displacements from set **ONE**. Errorbars mark standard deviations across all fits. The direct fit over the complete set (green) is given as reference. The results for set **TWO** are very similar and are therefore not presented.

Beta-Function Std. Dev. for Fits over Varying Numbers of Orbit Displacements

- Direct fits perform significantly better for smaller set sizes but produce a larger number of misfits for larger set sizes.
  → Misfits should decrease for a smaller gradient break-off condition!
- Indirect fits produce similar results as direct fits for large set sizes.
- The size of the dispersion term does not have a big impact on standard deviations.

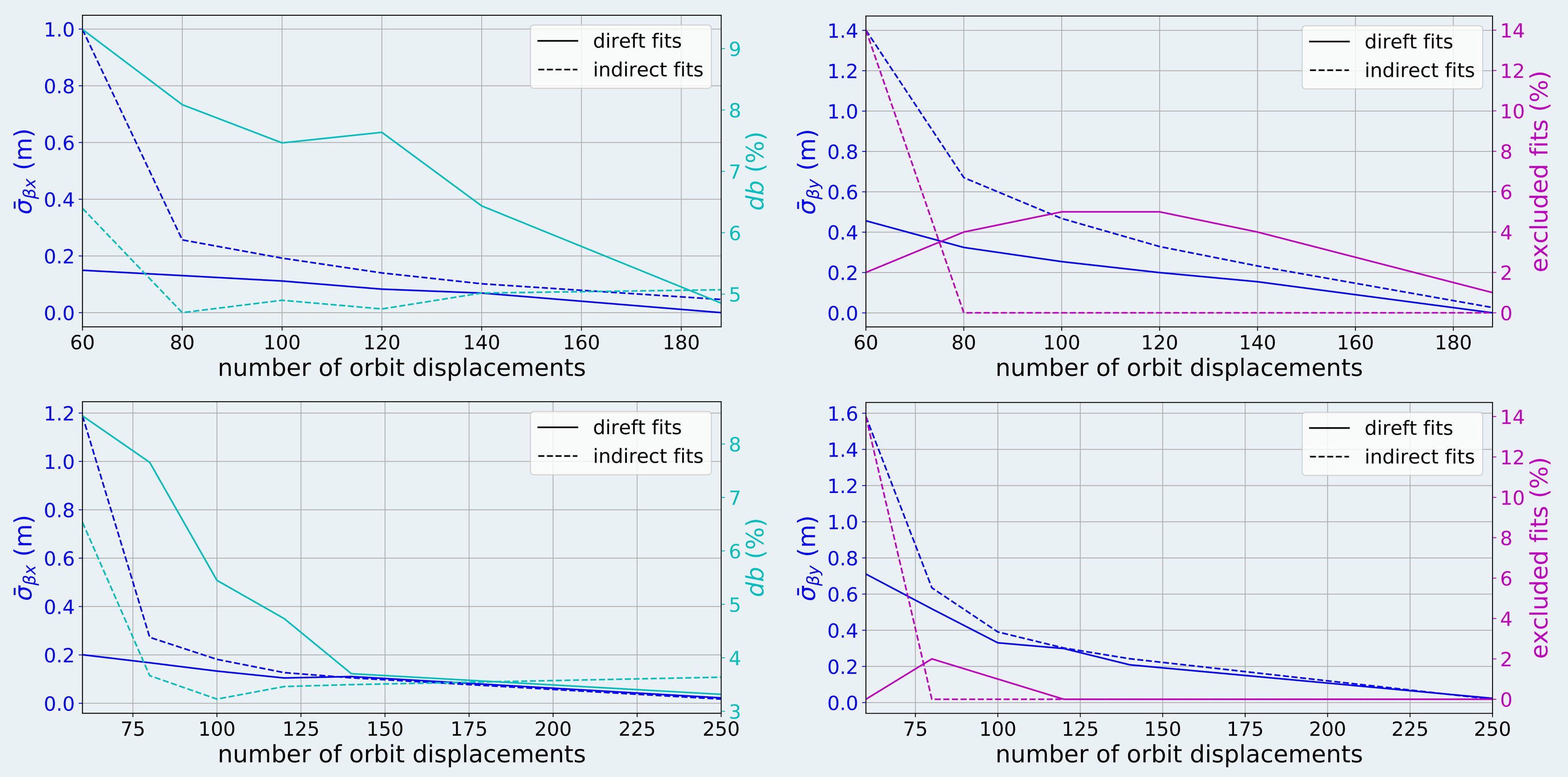

Figure 3: Average standard deviations of the horizontal beta-function fits $\sigma_{\beta x}$ (left) and the vertical beta-function fits $\sigma_{\beta y}$ (right) along the beam path of the storage ring for about 100 direct fits and about 100 indirect fits with COBEA for varying numbers of randomly selected orbit displacements from set **ONE** (left) and set **TWO** (right). Plots include the size of the dispersion term db and the number of fits which were excluded from analysis because the average coupled beta function in the non-native plane of either mode was greater than 5 m.

Evaluation of Scaled Dispersion Output

- Indirect fits do not match the recorded scaled dispersion at all.
- Direct fits produce better scaled dispersion output but require many orbit displacements as input.

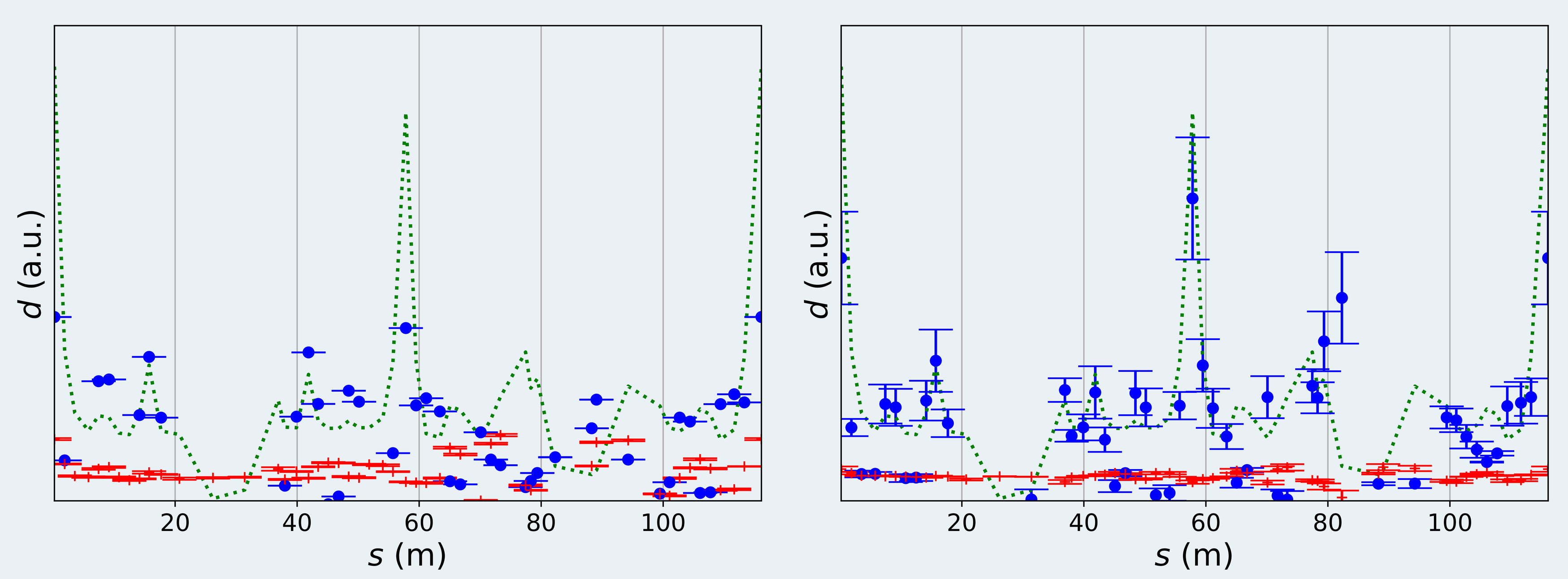

Figure 4: Scaled dispersion along the beam path of the storage ring for 100 direct fits (blue) and 100 indirect fits with COBEA (red) for the complete set of orbit displacements from set **ONE** (left) and set **TWO** (right). Errorbars mark standard deviations across all fits. The reference (green) is a recorded orbit displacement due to rf-frequency modulation.

Summary and Outlook

The average standard diviations of the beta-function output from the direct fits are significatly smaller than the standard deviations of the indirect fits for small sets of orbit displacements and are similar but still smaller for large sets of orbit displacements. The dispersion output from the indirect fits is also better. Therefore, the direct-fit method poses a major advantage over the indirect method.

In the future, a measurement with more orbit displacements should be recorded and analyzed in the same manner to better assess the achievable minimum measurement-over-measurement error. Fits over fewer orbit displacements (< 60) should also be investigated. Finally, the beta-function results for fits of the decoupled BE model should be evaluated to further decreases the required number of orbit displacements to achieve good beta-function estimates.

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