Basics of feedback control systems.

MT ARD ST3 Annual Meeting

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Introduction

Goals for todays tutorial

- **Tutorial goal**
  - Finish in 45-60 minutes
  - Introduce system descriptions in Laplacian domain
  - Introduce classical PID feedback control for single input single output systems

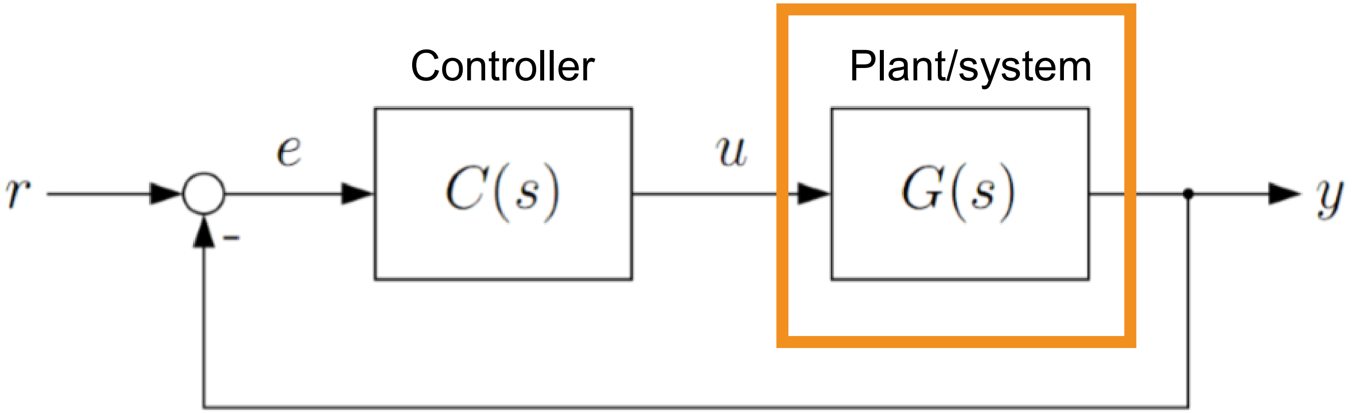
- **What this tutorial will cover**
  - Basic understanding of system dynamics and basic feedback setup

- **What this tutorial will NOT cover**
  - Detailed theory and implementations
  - Multiple input – multiple output systems
  - Non-linear systems
  - …

Outlook

Closed loop system analysis

1. **System description and modelling**
2. Controller design/analysis
3. PID controller
4. Outlook



Systems

- C(s) … feedback controller
- G(s) … plant / system

Basic loop signals

- r … reference or set-point
- y … output signal
- e = r-y … error between reference and output
- u … control signal

In addition and not shown yet

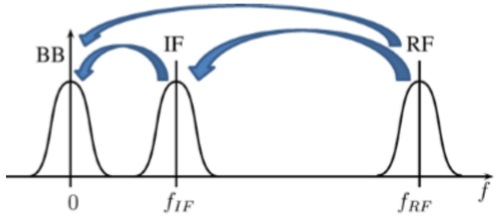
- Disturbance to system/measurement
- Noise to measurement
- …

System Description

System Description

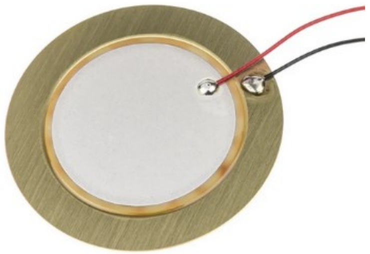
Low-pass with time delay

- Main focus on systems based on low-pass characteristic with time delay
- Why?
  - Good approximation for many systems
  - Valid for most of the RF structures which are down-converted into base-band

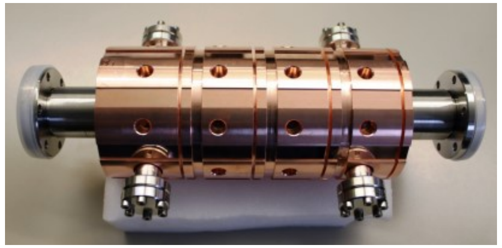


  - What differs for the systems are the bandwidth and delay

- Superconducting TESLA type cavities
  - 10 – 500 Hz half-bandwidth

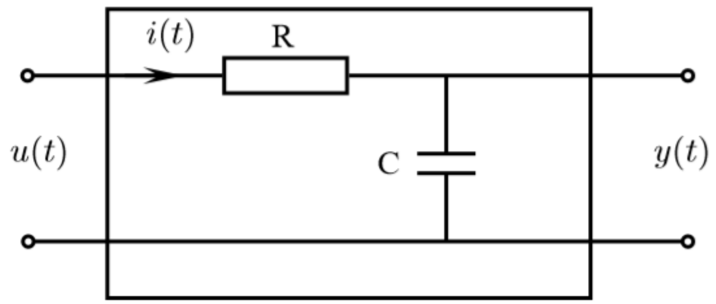


- Piezo regulation (Laser, Cavity, …)
  - ~ 10 kHz half-bandwidth
- Normal-conducting copper cavities
  - Standing/travelling wave
  - 50 kHz – 1 MHz half-bandwidth
  - E.g. RF-gun, BACCA, … ARES TWS1/2 structure
- …



System Description

Example: Low-pass without time-delay



$$Ri(t) + y(t) = u(t) \quad ; \quad C\frac{dy(t)}{dt} = i(t)$$

$$RC\frac{dy(t)}{dt} + y(t) = u(t)$$

Assume $u(t) = U_0$ and split in homogeneous and non homogeneous case.
Approach homogeneous case ($y(t) = e^{\lambda t}$) or solve it directly.

$$RC\lambda e^{\lambda t} + e^{\lambda t} = 0 \quad ; \quad \lambda = -\frac{1}{RC}$$

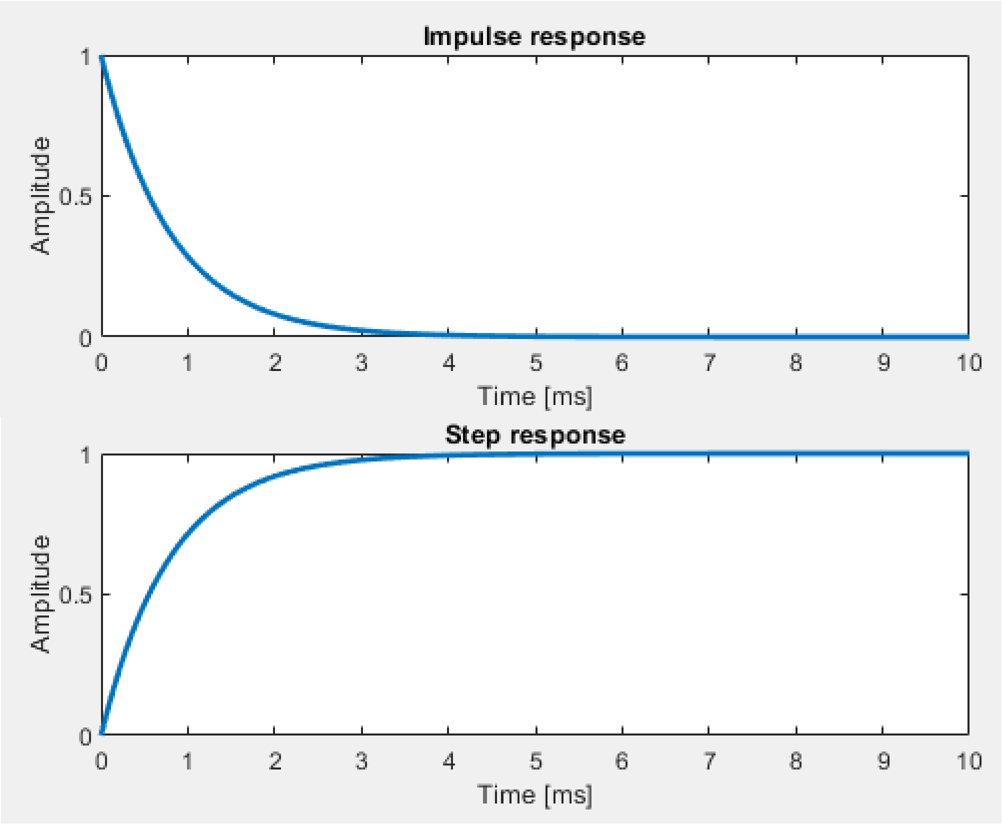
Assume u(t) = 0

$$y(t)_{hom} = Ke^{-\frac{1}{RC}t}$$

Impulse response g(t)

$$y(t) = y(t)_{hom} + y(t)_{nonhom} = U_0(1 - e^{-\frac{1}{RC}t})$$

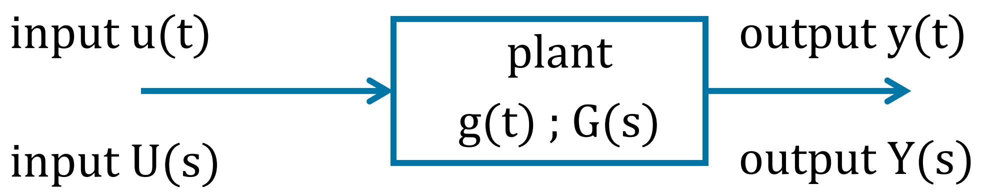
by initial conditions where $u(t) \neq 0$.



The impulse response is needed for system analysis. The output signal y(t) for any input signal u(t) is computed by the convolution of g(t)*u(t).

System Description

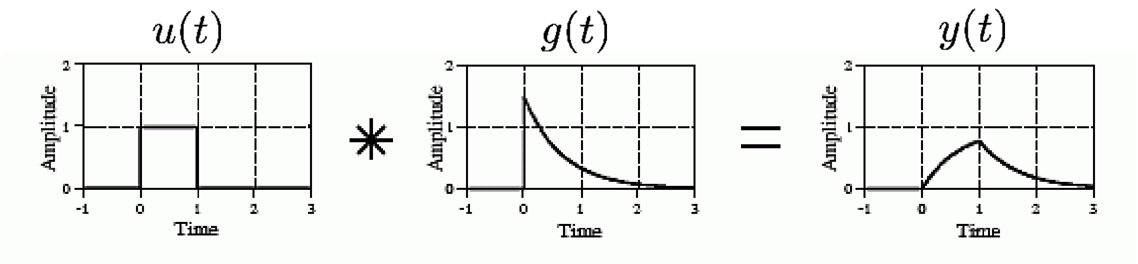
Time and frequency domain



Time domain

- Convolution of impulse response g(t) and input u(t)

$$y(t) = g(t) * u(t)$$



- Makes system analysis very complicated

Frequency domain

- Laplace transformation widely used in system analysis

$$s := \sigma + j\omega$$

- Multiplication of impulse response G(s) and input U(s)

$$Y(s) = G(s) \cdot U(s)$$

- System analysis much easier

System Description

Transformation into Frequency Domain

Fourier transformation

$$F(f) = \int_{t=-\infty}^{\infty} f(t) \cdot e^{-i2\pi f t} dt$$

- Defined for all t

for completeness

Laplace transformation $s := \sigma + j\omega$

- Defined for all t ≥ 0 (causal system)

$$F(s) = \int_{t=0}^{\infty} e^{-st} f(t)\, dt$$

$$f(t) = 0, \forall t < 0$$

Inverse Laplace transformation

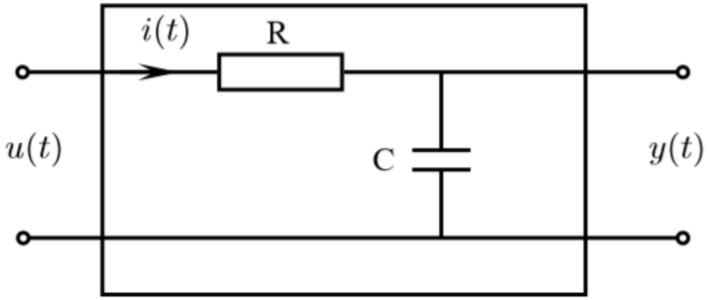
$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{s=\alpha-j\infty}^{\alpha+j\infty} F(s) \cdot e^{st}\, ds$$

System Description

Example: Low-pass filter

Find transformation as table in literature

No.	Time Domain $f(t)$	Frequency Domain $F(s)$
1	Impulse response $\delta(t)$	1
2	Unit step $\sigma(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	t^n	$\frac{n!}{s^{n+1}}$
5	$\frac{df}{dt} = \dot{f}(t)$	$sF(s) - f(0)$ ←
6	$\ddot{f}(t)$	$s^2F(s) - sf(0) - f^{'}(0)$
7	$\int_0^t f(t)$	$\frac{1}{s}F(s)$
8	e^{at}	$\frac{1}{s-a}; s > a$
9	$t^n e^{at}$	$\frac{n!}{(s-a)n+1}; s > a$
10	$\sin at$	$\frac{a}{s^2+a^2}; s > 0$
11	$\cos at$	$\frac{s}{s^2+a^2}; s > 0$
...	...	...



Previous example:

$$RC\frac{dy(t)}{dt} + y(t) = u(t) \circ\!\!-\!\!\bullet RCsY(s) + Y(s) = U(s)$$

$$= (RCs+1)Y(s) = U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{RCs+1}$$

Initial conditions are zero!

System Description

Example: Low-pass filter

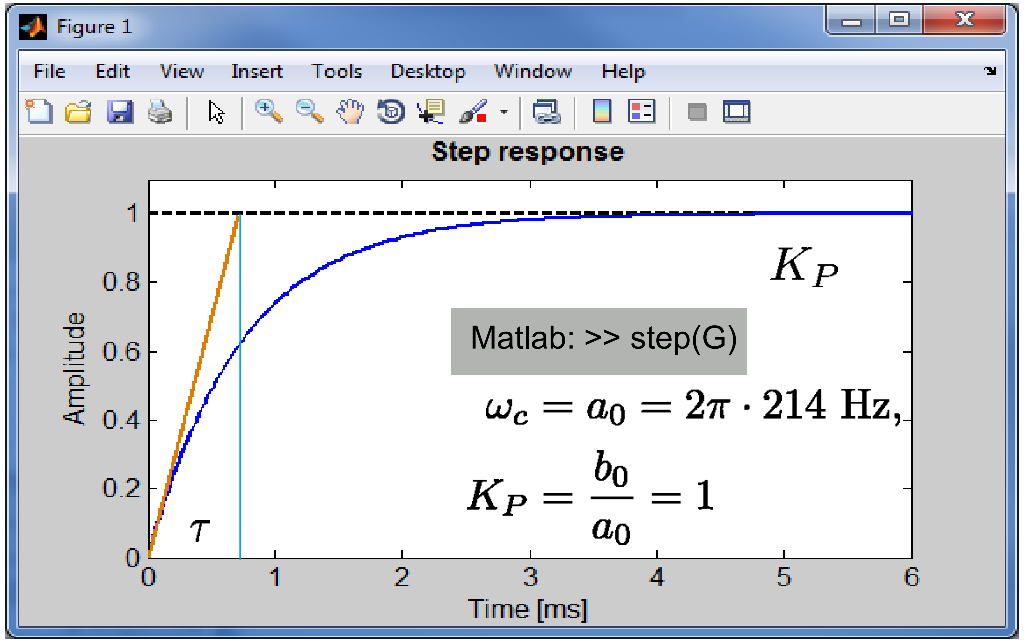
First order system:

$$G(s) = \frac{b_0}{s+a_0}$$

Static gain: $K_P = b_0/a_0$ for s $\to 0$

Time constant: $\tau = 1/a_0$

Step response: $y(t) = K_p(1 - e^{-t/\tau})$

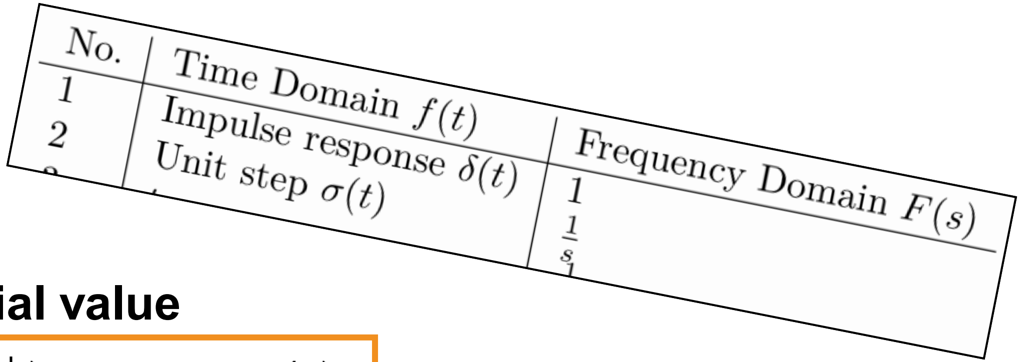


Laplace transformation properties

- **Final value**

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

- impulse response: $\lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{b_0}{s+a_0} \cdot 1 = 0$
- for unit step: $\lim_{s\to 0} sG(s) \cdot \frac{1}{s} = \frac{b_0}{a_0}$



No.	Time Domain $f(t)$	Frequency Domain $F(s)$
1	Impulse response $\delta(t)$	1
2	Unit step $\sigma(t)$	$\frac{1}{s}$

- **Initial value**

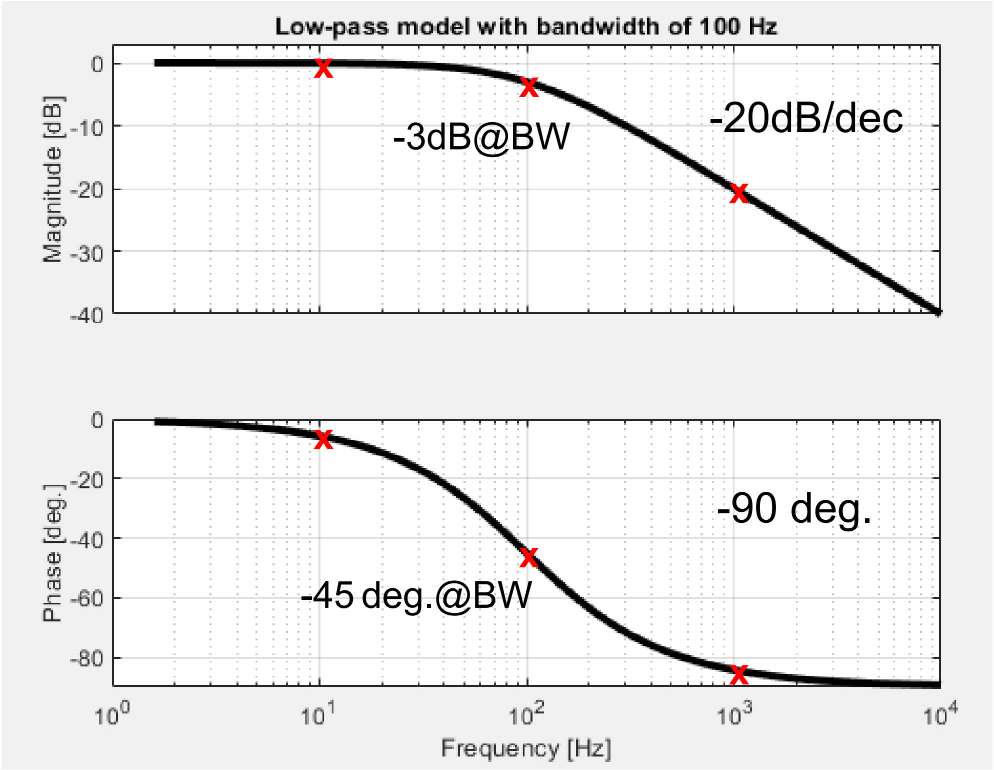
$$x(0^+) = \lim_{s\to\infty} sX(s)$$

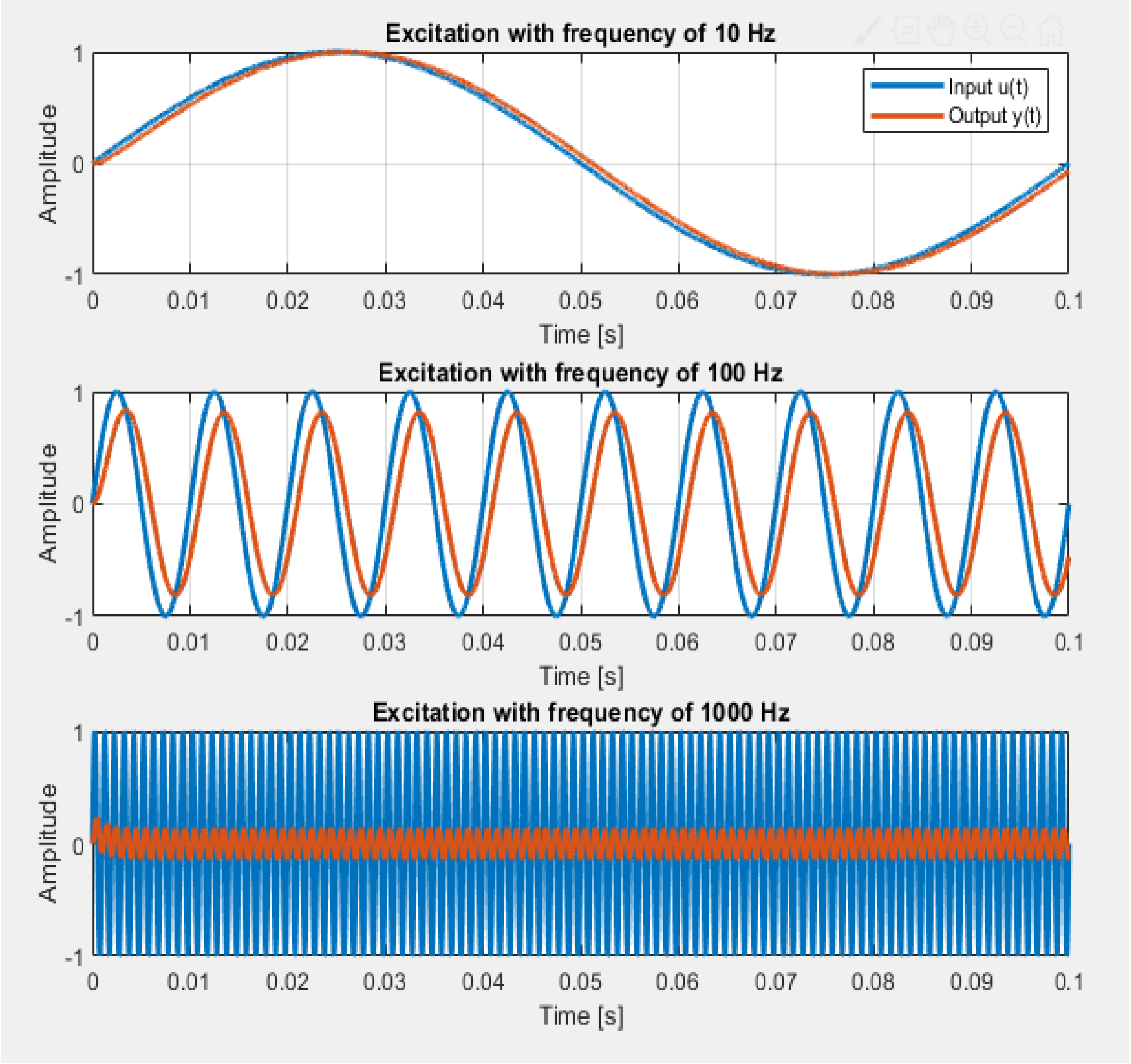
System Description

Low-pass response in time and frequency

- **First order low-pass characteristic (BW 100Hz)**
- Sine as input, corresponding output analyzed
- I/O ratio and phase shift is plotted

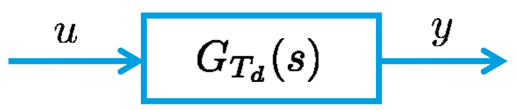
→ **Bode diagram**





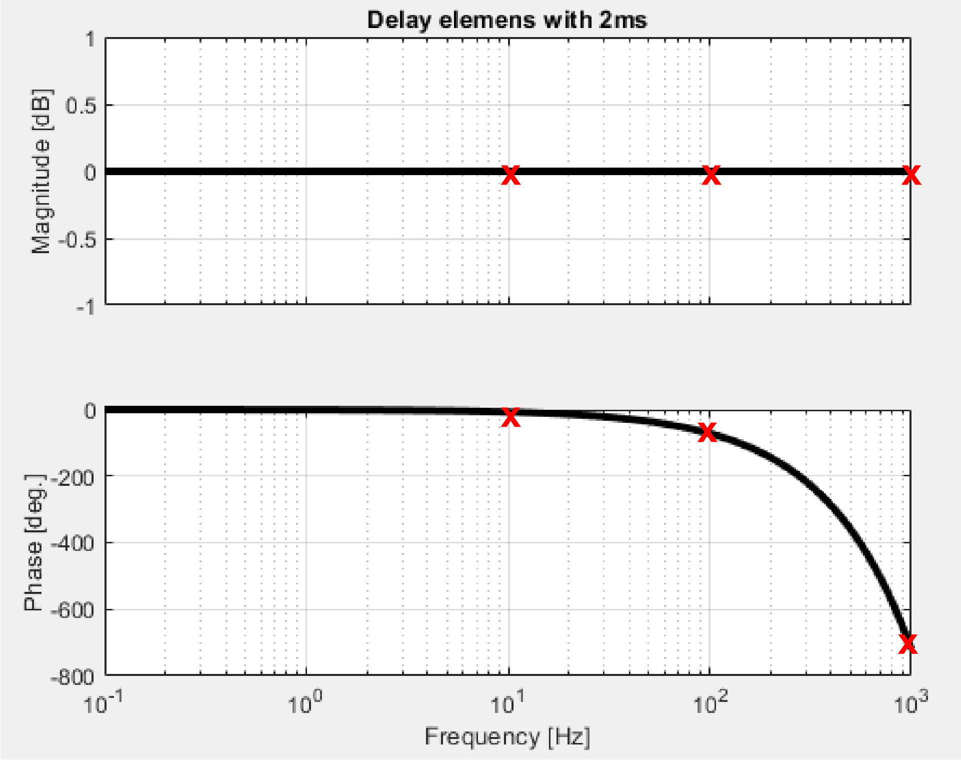
System Description

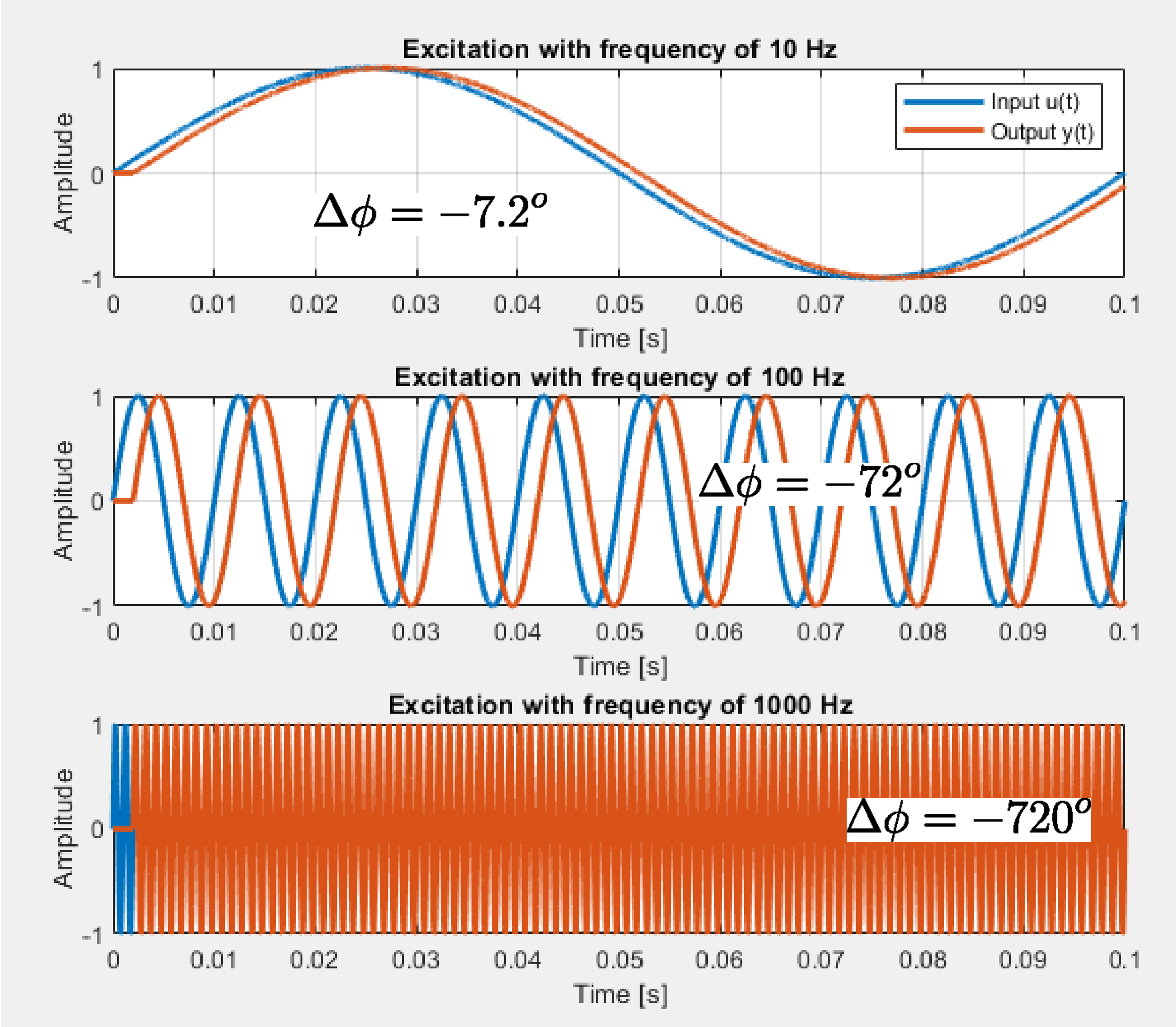
Time delay response in time and frequency



- **Time delay of 2ms**
- Sine as input, corresponding output analyzed
- I/O ratio and phase shift is plotted

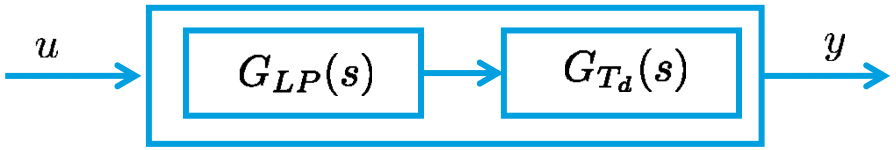
→ **Bode diagram**





System Description

Time delay in time and frequency domain



$$G(s) = G_{T_d}(s) \cdot G_{LP}(s)$$

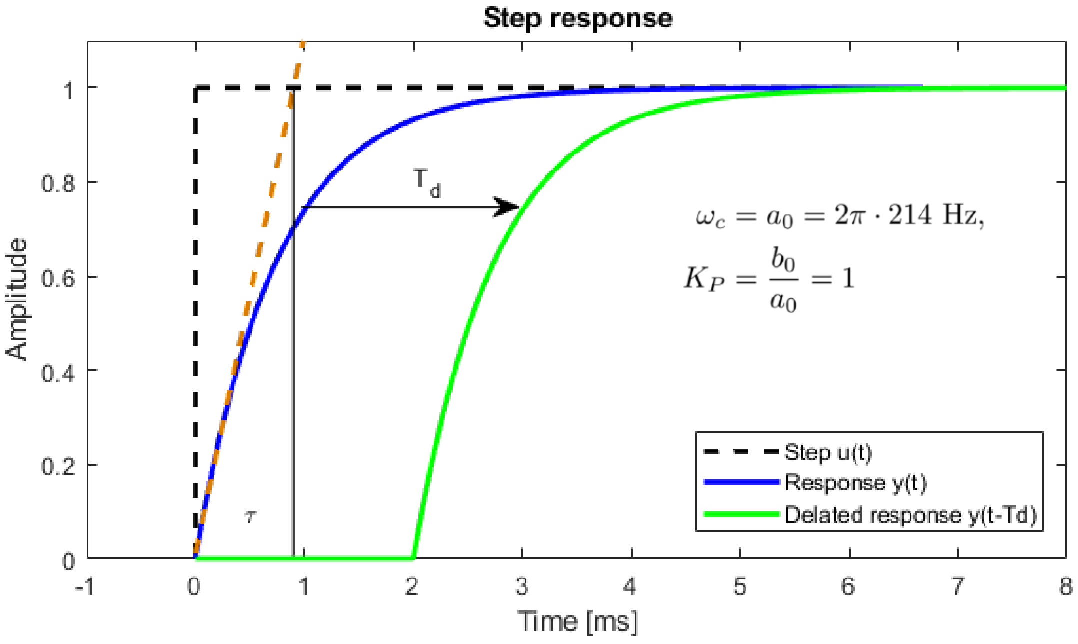
System with time delay T_d:

$$y(t - T_d) \circ\!\!-\!\!\bullet\, G_{T_d}(s) = e^{-T_d \cdot s}; \quad (y = 0;\ \forall t \in [0 \leq t < T_d])$$

(compare to $\vec{v} = |v| e^{i\phi_v}$ and set $s = j\omega$)

$$|e^{-T_d \cdot s}| = 1 \text{ and } \angle(e^{-T_d \cdot s}) = \phi(\omega) = -T_d\omega \qquad [rad]$$

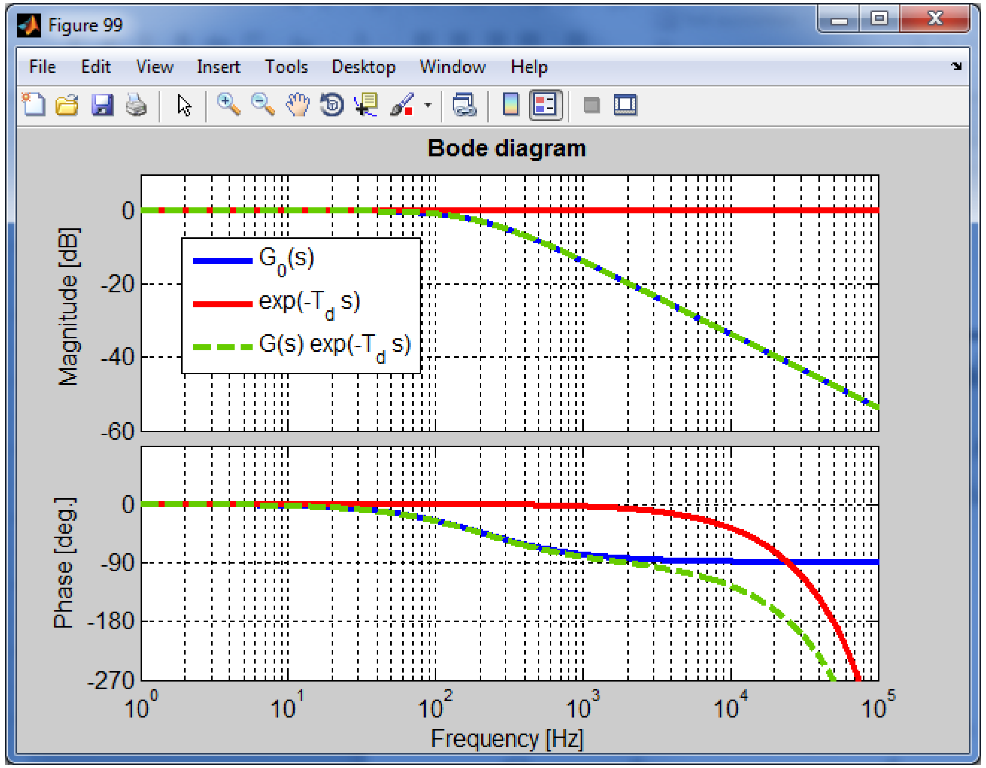
$$= -T_d\omega \frac{180}{pi} \ [deg]$$



System with time delay T_d:

$$G(s) = G_0(s) \cdot e^{-T_d \cdot s}$$

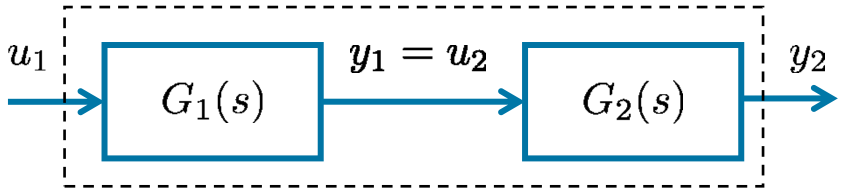
- $G_0(s)$ ... time delay-free system
- Time delay with gain of 1 and phase

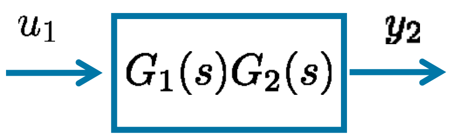


System Description

Serial, parallel and feedback connection of blocks

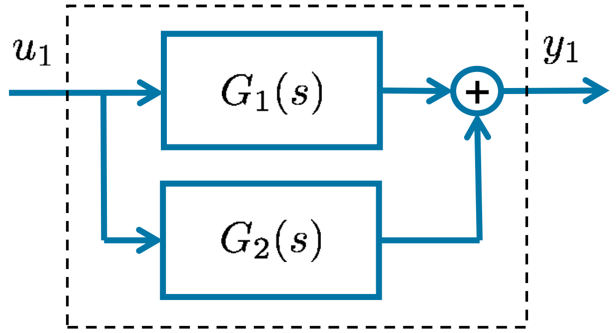
- **Serial connection**

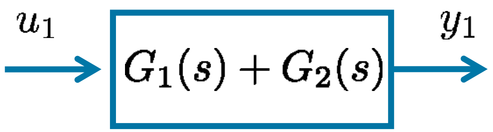




$$Y_2(s) = G_1(s) \cdot G_2(s) \cdot U_1(s)$$

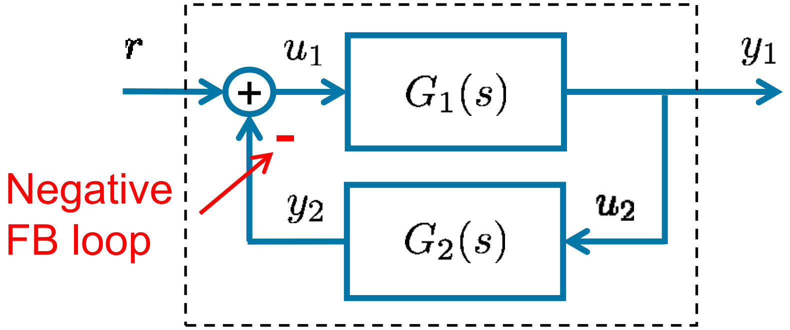
- **Parallel connection**

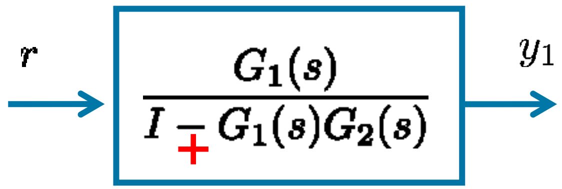




$$Y_1(s) = (G_1(s) + G_2(s)) \cdot U_1(s)$$

- **Feedback**





$$Y_1(s) = \frac{G_1(s)}{I \mp G_1(s)G_2(s)} \cdot R(s)$$

$$G_{cl} = \frac{Y_1(s)}{R(s)} = \frac{\text{forward gain}}{I \mp \text{loop gain}}$$

System Description

Time vs. frequency domain

Time domain

- Differential equation to n first order equations in state space representation

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$$G(s) = C(sI - A)^{-1}B + D$$

Frequency domain

- Transfer function of n-th order

$$G(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_s s + b_0}{s^n + + a_{n-1}s^{n-1} + \ldots + a_s s + a_0}$$

(2) Map the transfer function in frequency domain into special structure for A,B,C,D matrices
→ Keyword from frequency to state space :

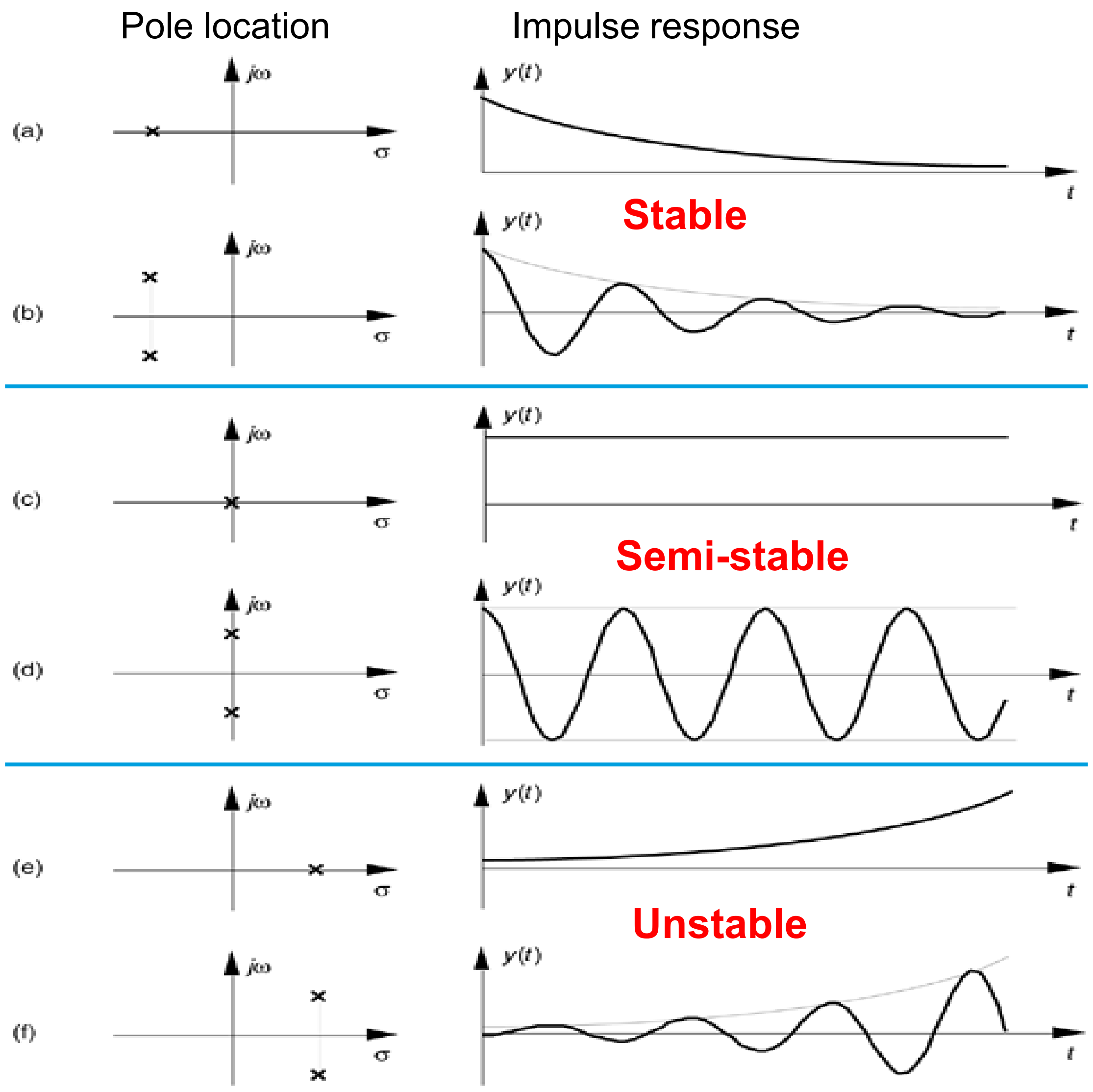
- Controllable canonical form
- Observable canonical form

(1) Map TF into time domain as differential equation

No	Time domain $f(t)$	Frequency domain $F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $\sigma(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	t^n	$\frac{n!}{s^{n+1}}$
5	$\frac{\mathrm{d}f}{\mathrm{d}t} = \dot{f}(t)$	$sF(s) - f(0)$
6	$\ddot{f}(t)$	$s^2F(s) - sf(0) - f'(0)$
7	e^{at}	$\frac{1}{s-a}; s > a$

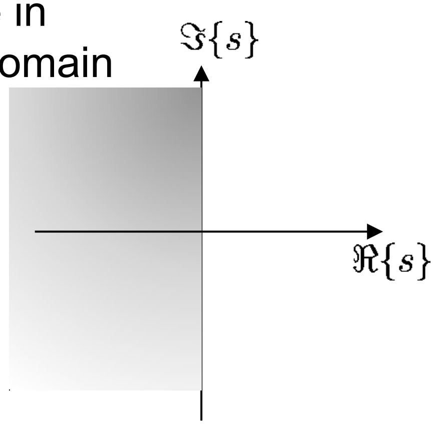
System Description

Stability - Is my system itself stable?

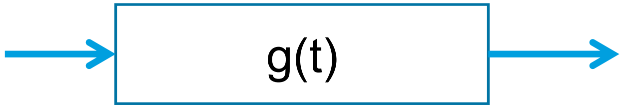


Understanding Digital Signal Processing (2nd Edition)
Mar 25, 2004by Richard G. Lyons

Stable left half plane in cont. time Laplace domain



A system is stable if for a given bounded input signal the output signal is bounded and finite (BIBO stable); if not, the system is called unstable



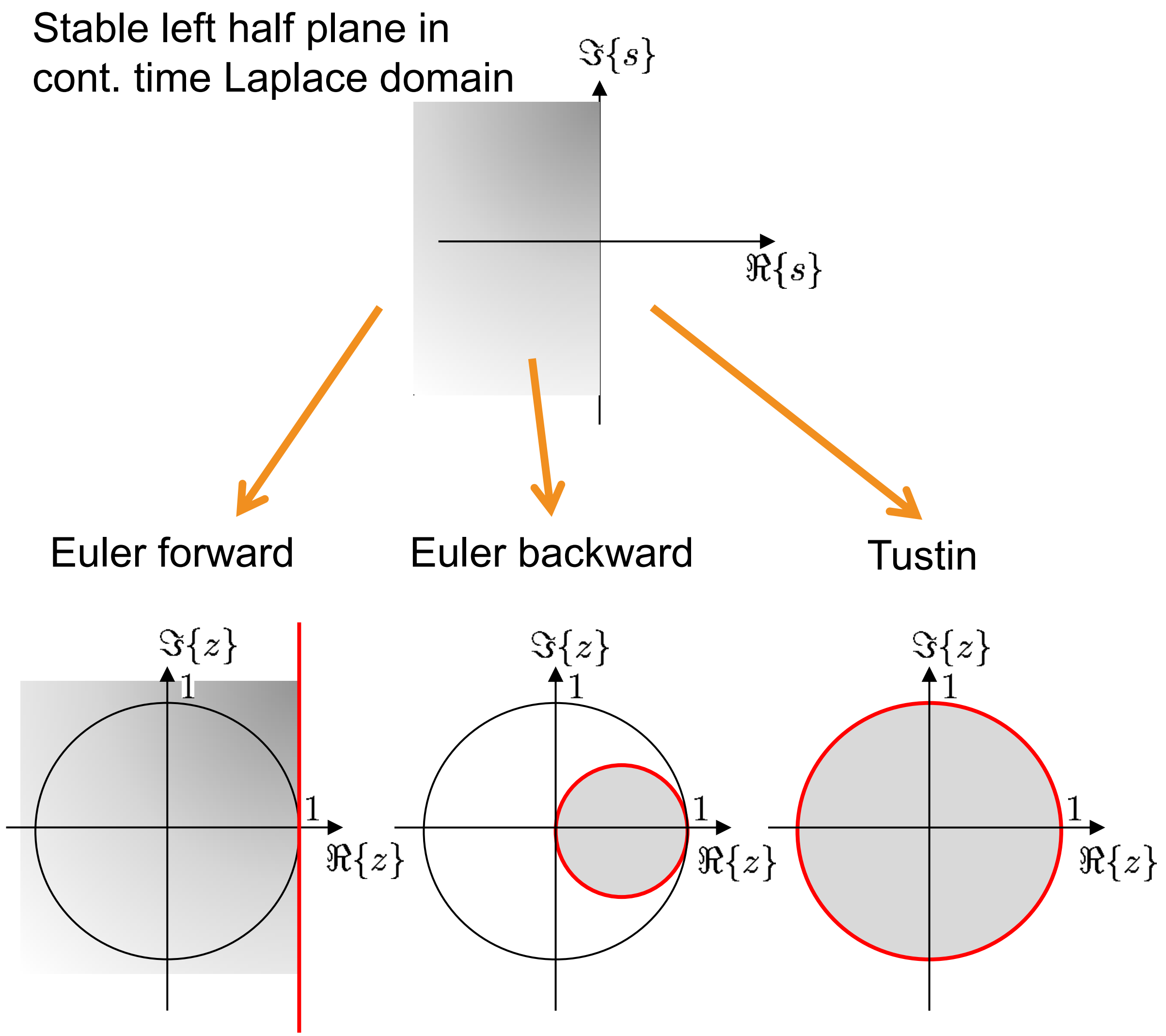
$$\int_{t=-\infty}^{\infty} |g(t)|dt < \infty$$

System Description

Discrete time representation for digital systems

Translate continuous to discrete time model/controller by finding an approximate showing the same characteristics over the frequency range of interest

- **Pole-zero matching (exact approximation)**
  - Map pole/zero by $z_i = e^{s_i T_s}$ with location s_i
  - Find appropriate gain (at critical frequencies (s→0))
  - Only for SISO systems
- **Numerical integration**
  - Euler forward/backward $s \mapsto \frac{z-1}{T_s}; \quad s \mapsto \frac{1-z^{-1}}{T_s}$
  - Tustin (bilinear transformation) $s \mapsto \frac{2}{T_s}\frac{z-1}{z+1}$
- Hold equivalents (discretization in the time domain)
  - ZOH, FOH

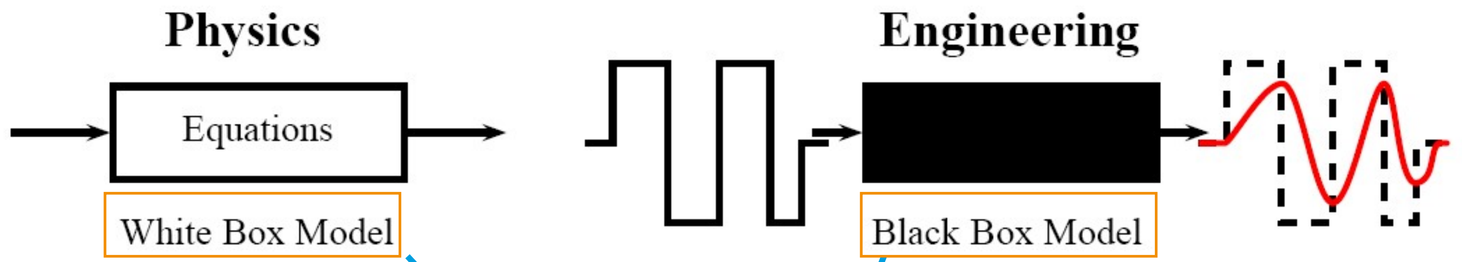


Stability: Poles in RHP → outside unit circle are unstable poles!

System Modelling

System Modelling

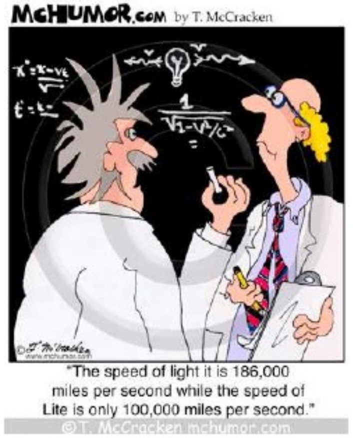
System identification using special input signals



Fully based on physical description e.g. cavity equation

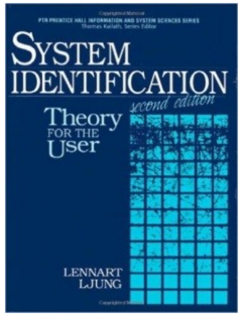
Identification Methods

Usage of defined input signal u(t)
→ observation of output signal y(t)
→ G(s) = Y(s)/U(s)



Grey Box Model

Parameter estimation based on physical equation
→ Find some free parameters



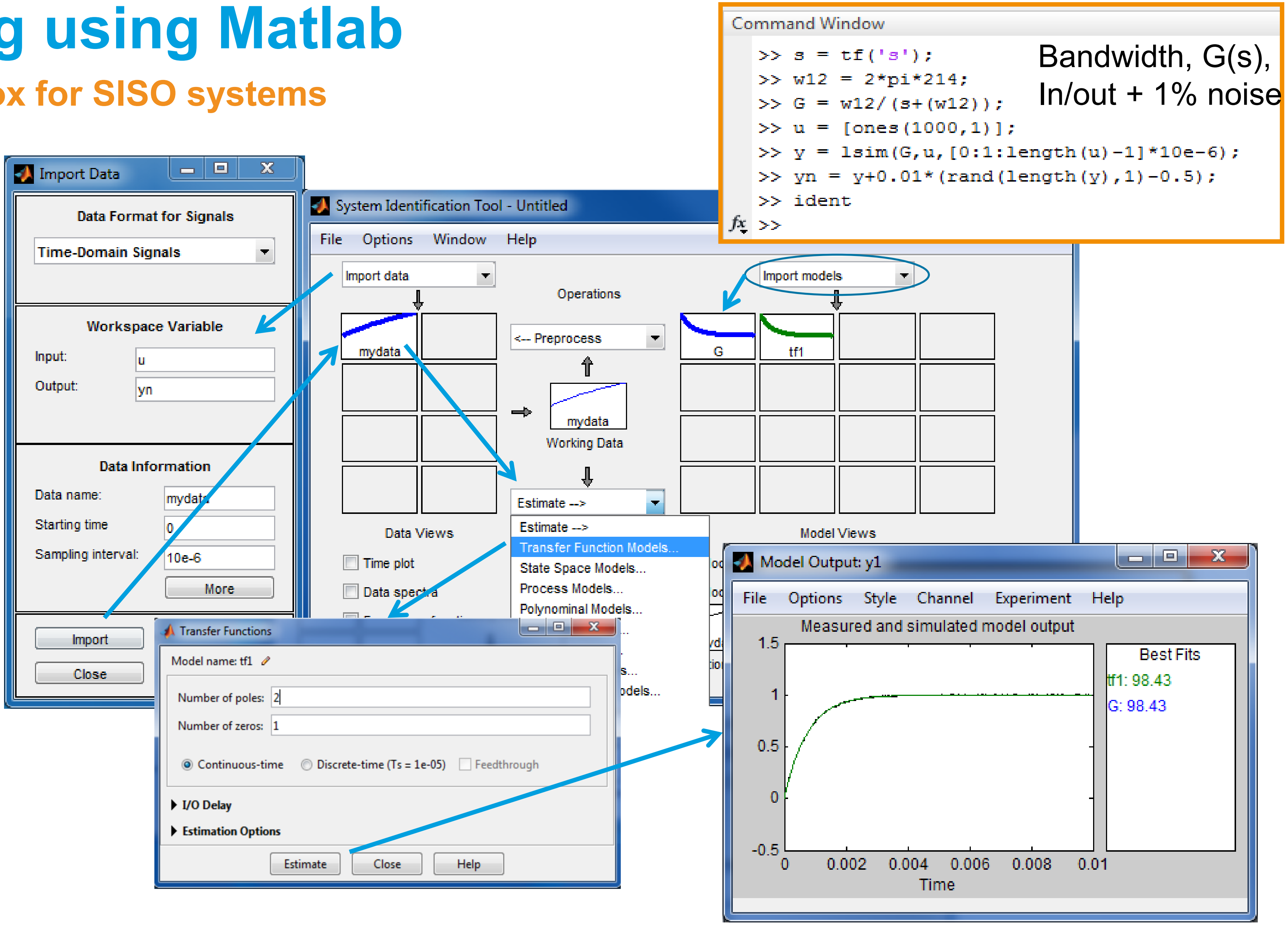
[Ljung.1999]

A system model is a simplified representation or abstraction of the reality. Reality is generally too complex to copy exactly. Much of the complexity is actually irrelevant in problem solving, e.g. controller design.

System Modelling using Matlab

System Identification Toolbox for SISO systems

- System delay estimated and extracted or approximated by poles/zeros (Pade approximation) increasing the model order
- Hint: Try to identify delay free systems by aligning the responses and add the delay later
- Keep system model order low to reduce complexity and to characterize main system characteristics
- Try to exploit physical knowledge about system

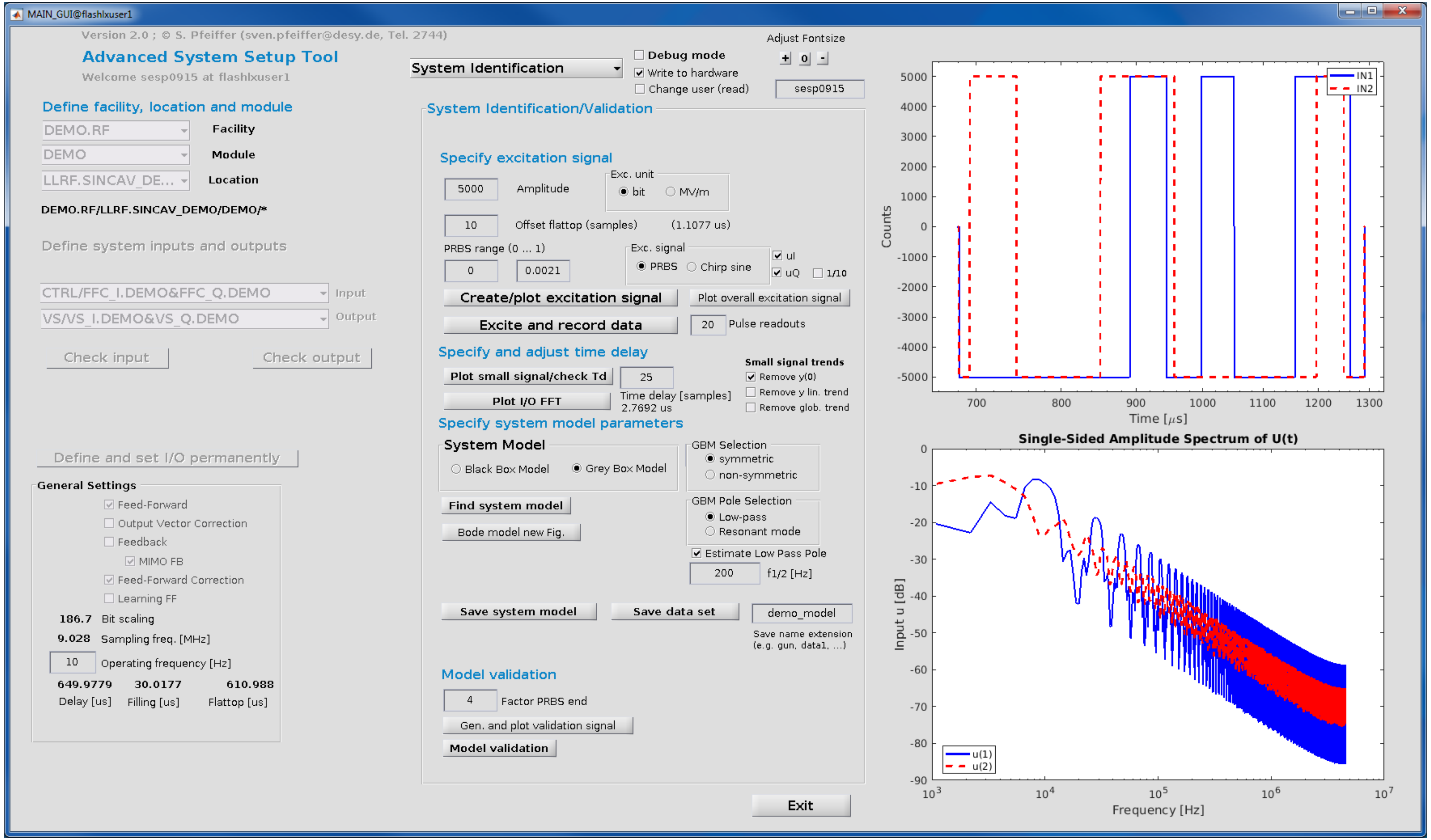


System Modelling using Matlab

Advanced LLRF System Setup Tool

Features:

- Select facility
- Select subsystem
- **System identification**
  - **Delay estimation**
  - **Grey box**
  - **Black box**
- FB Controller design
- Learning feed-forward
- Smith predictor setup



“Advanced LLRF System Setup Tool for RF Field Regulation of SRF Cavities”
S. Pfeiffer, et al., Proceedings of SRF2019, Dresden, Germany

System Modelling for SRF cavity

Remark: Piecewise reconstruction is often NOT the transfer function!

Piecewise reconstruction with G(w) = Y(w)/U(w)

- Chirp sine excitation with range of 500kHz each

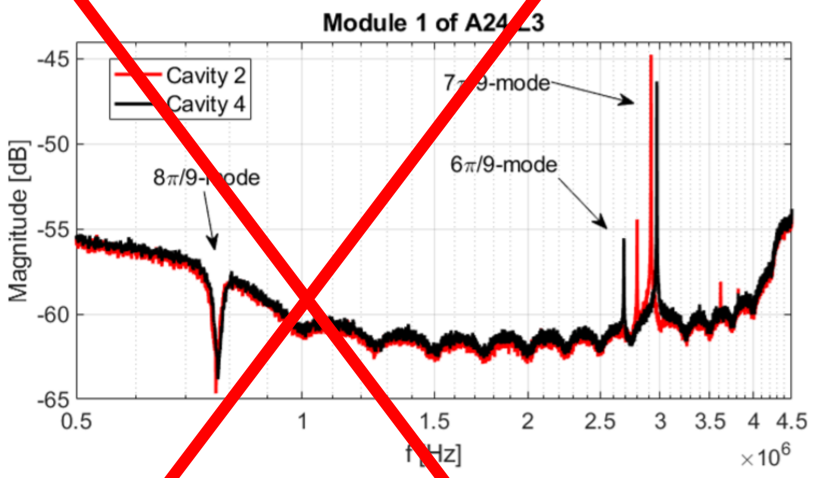


Figure 4: Piecewise reconstruction of transfer function

- This approach works only if the steady state condition is reached!!!
- Applies also for other systems

System identification

- Chirp sine signal in given range around passband mode

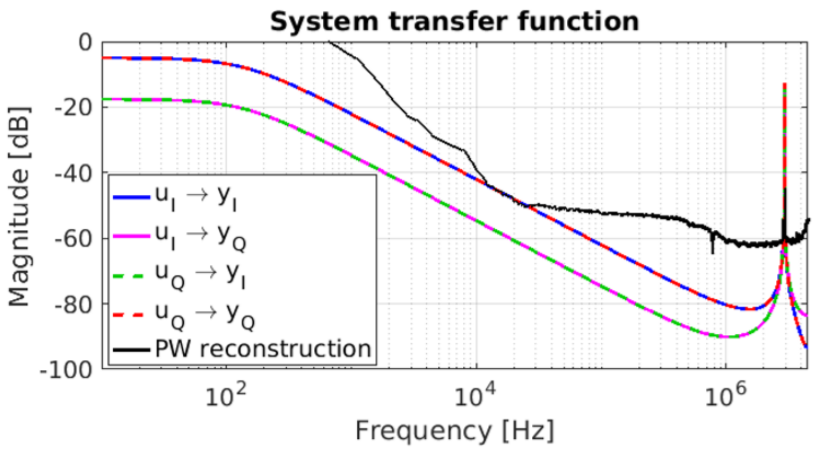


Figure 5: Grey box model identification with low-pass characteristic and $7\pi/9$-mode modelling for C2.M1.A24.L3. The magnitude plot using piecewise (PW) reconstruction have been added for completeness.

See: S.Pfeiffer et. al, *Advanced LLRF System Setup Tool for RF Field Regulation of SRF Cavities*, SRF2019,

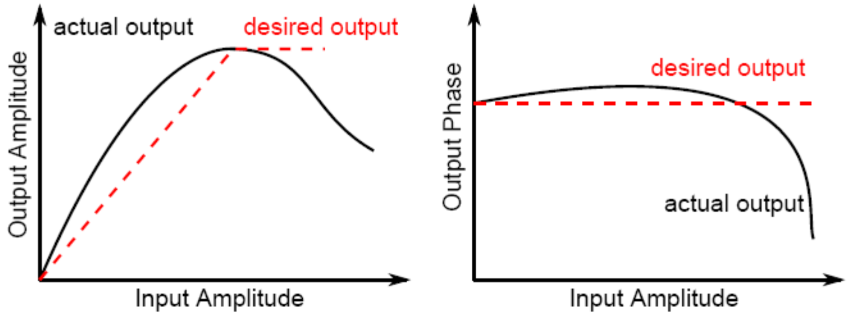
System Modelling

Additional effects

- Non-linear elements
  - E.g. klystron I/O characteristic
    - Quadratic/polynomial approximation
  - Saturation
  - Dead-zone
  - Hysteresis
  - …
- Time varying elements
  - Characteristic changes over time

Example Klystron

- Non-linear behavior in amplitude (e.g. saturation at max. output) and phase
- Linearization of static characteristic curve
- Bandwidth usually tens of MHz (>> cavity BW)



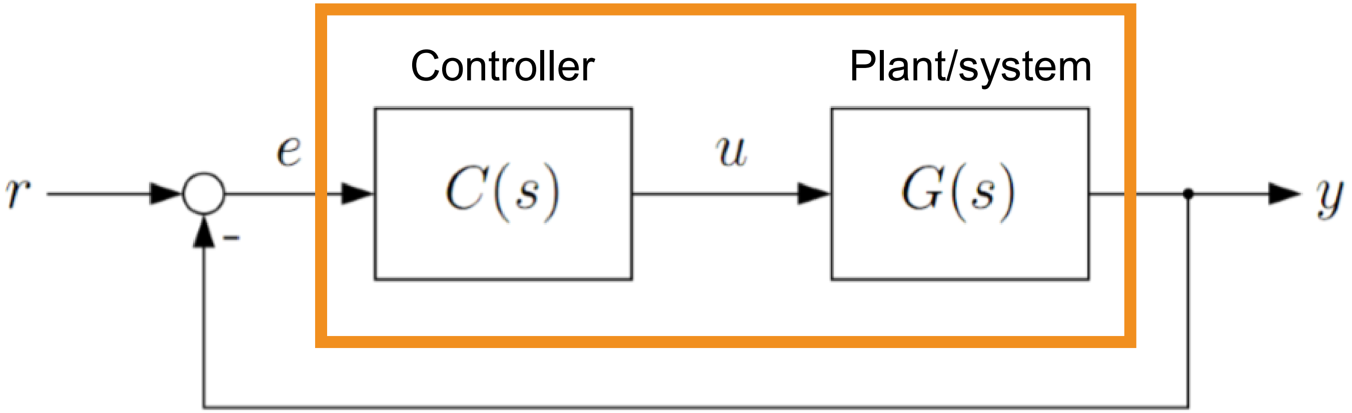
Output amplitude and phase is function of input amplitude

Outlook

Closed loop system analysis

1. System description and modelling
2. **Controller design/analysis**
3. PID controller
4. Outlook

- Ways to control
- Stability
- Control objectives

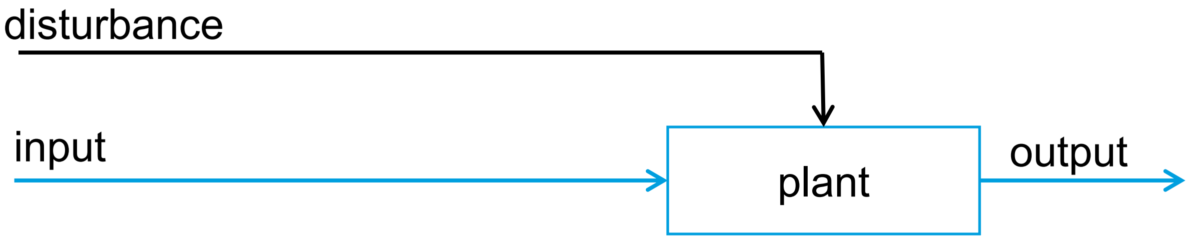


Ways to control

Ways to Control

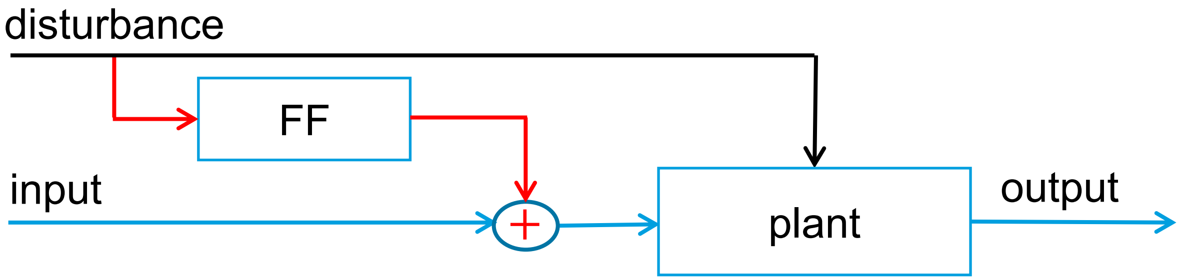
Regulation choices

Open loop (simple)



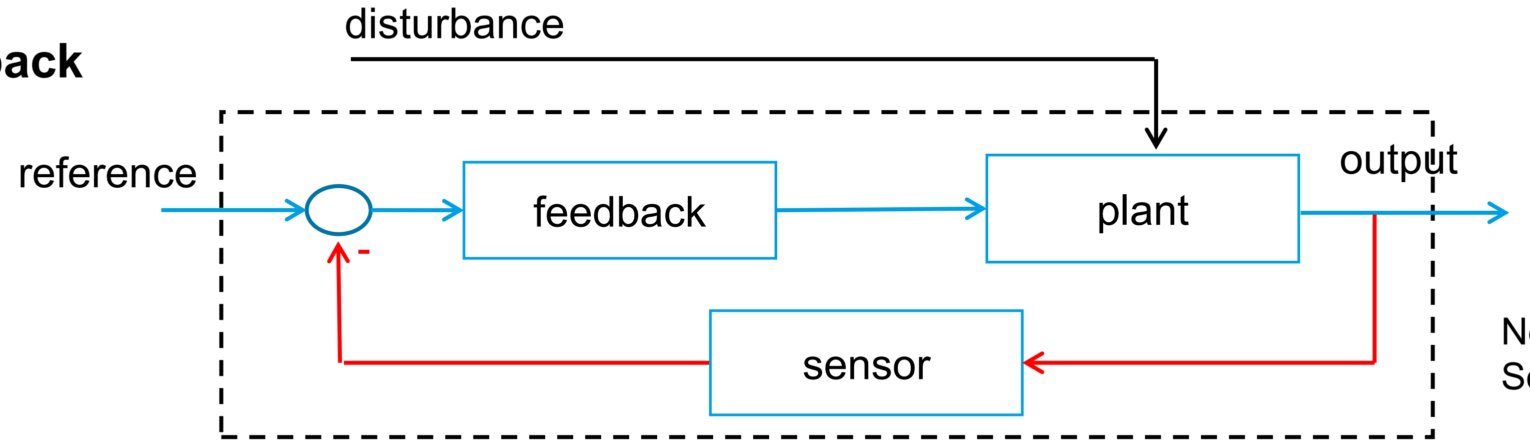
Precise knowledge on I/O behavior;
No action on disturbances

Feed forward



Precise knowledge on I/O behavior;
Act by feedforward e.g. on disturbances
→ *No action on signal to be controlled*

Feedback

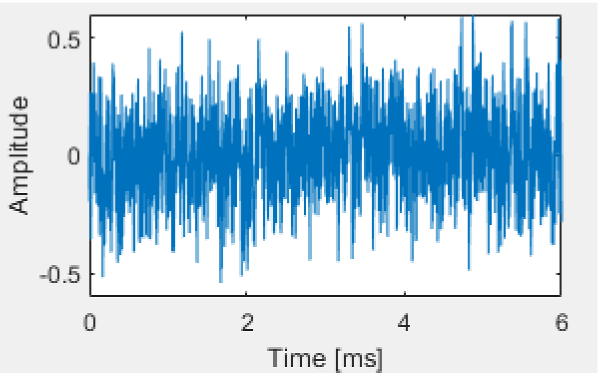


Feedback and regulate the signal to be controlled by acting on the input

New system with new properties !
See: connection of systems

Ways to Control

Regulation choices - Examples

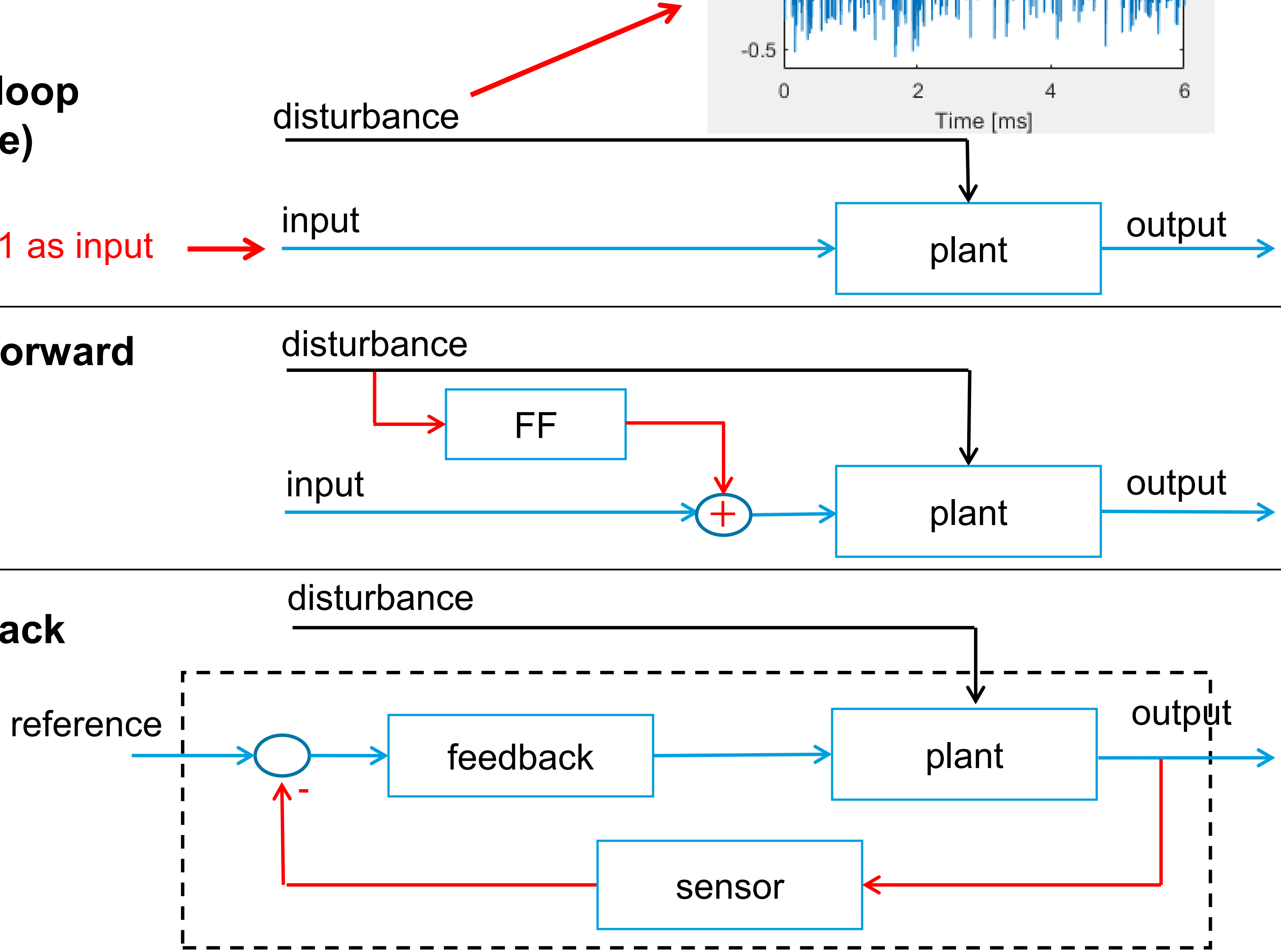


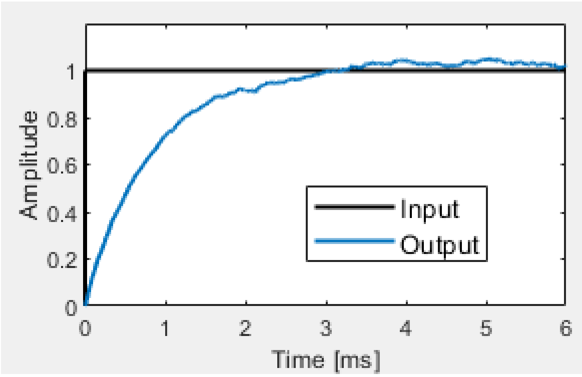
Open loop (simple)

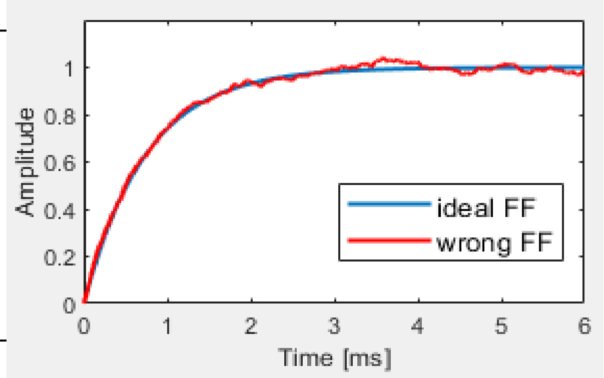
Step to 1 as input

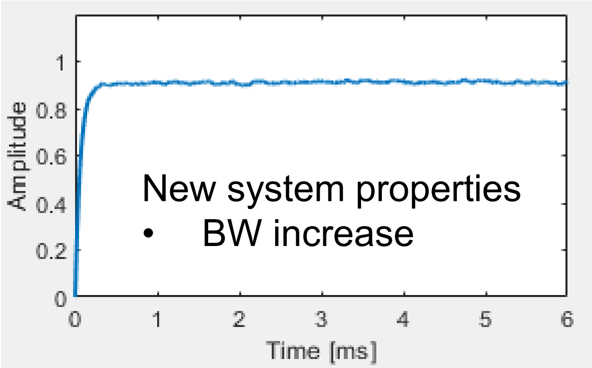
Feed forward

Feedback



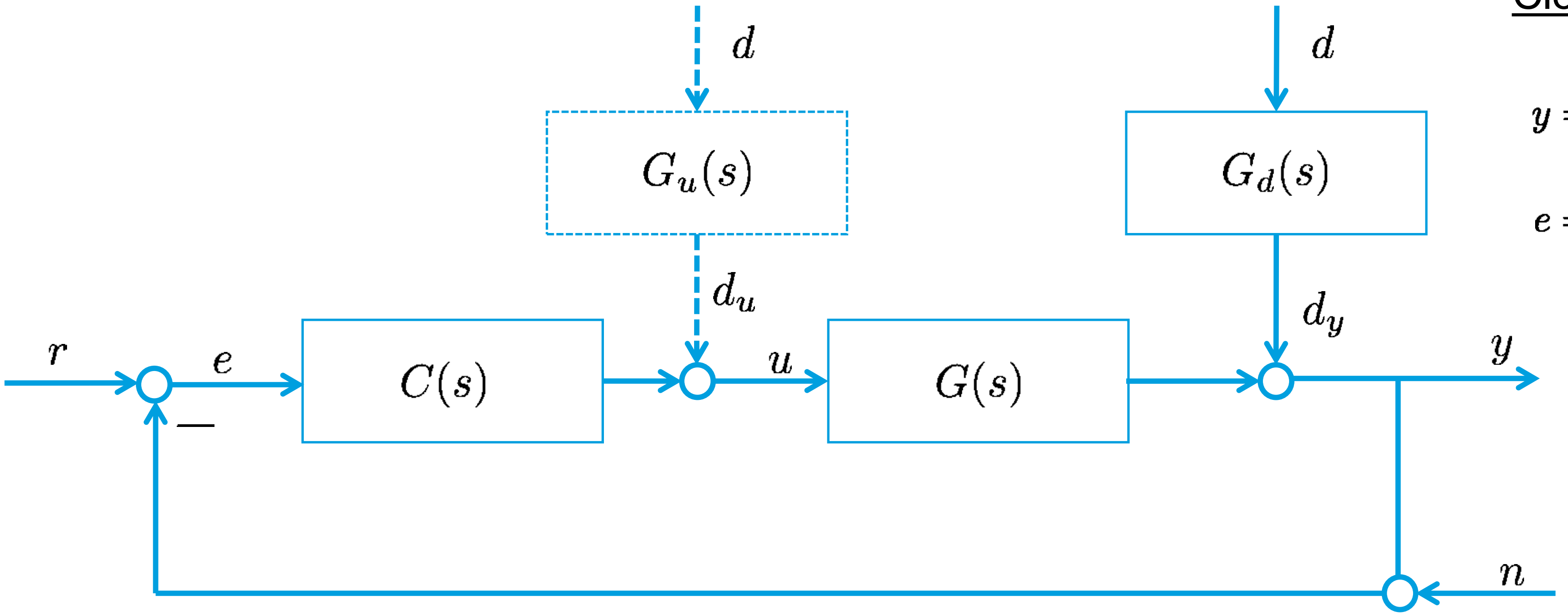






Ways to Control

General Feedback Control Loop



Closed-loop transfer function

$$y = \frac{GC}{1+GC}(r-n) + \frac{1}{1+GC}d_y$$

$$e = \frac{1}{1+GC}(r-n-d_y)$$

Sensitivity function

$$S = \frac{1}{1+GC}$$

Complementary sensitivity function

$$T = \frac{GC}{1+GC}$$

$$y = T(r-n) + Sd_y$$

$$e = S(r-n-d_y)$$

Both depend on the loop transfer function $L = GC$

Coupling $S + T = 1$

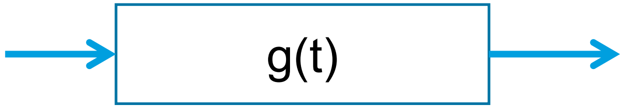
Stability

Stability Criteria's (incomplete!)

Stable if impulse response absolutely integrable and bounded

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_s s + b_0}{s^n + +a_{n-1} s^{n-1} + \ldots + a_s s + a_0}$$

A system is stable if for a given bounded input signal the output signal is bounded and finite (BIBO stable); if not, the system is called unstable



$$\int_{t=-\infty}^{\infty} |g(t)| dt < \infty$$

Stable or unstable linear systems

- Open loop or closed loop
- Unstable open loop: Stabilize closed loop system behavior using feedback controller

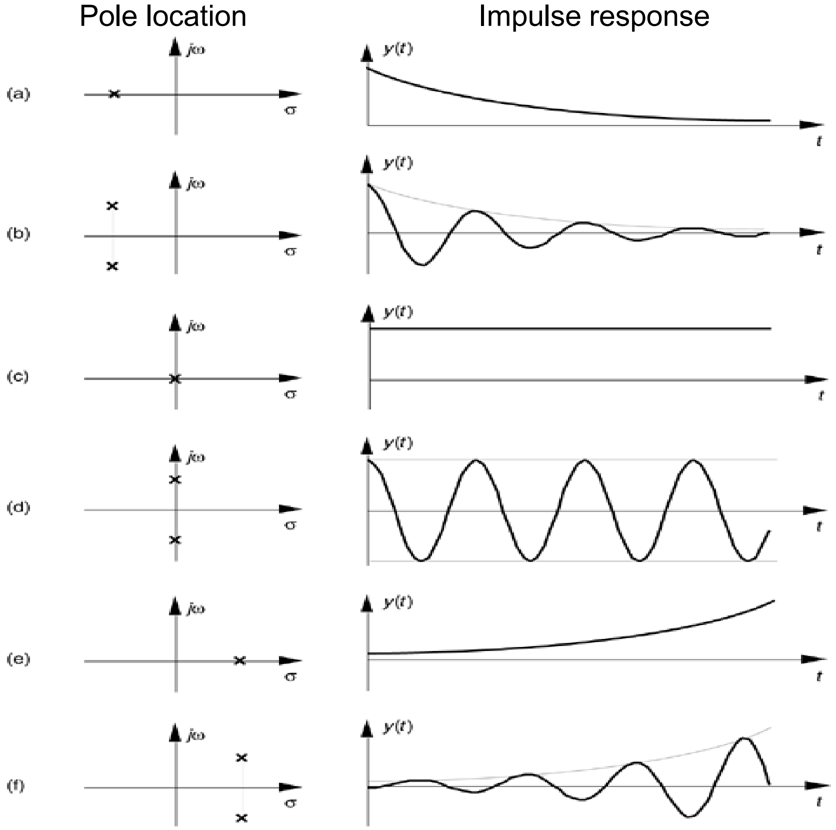
Stability check in s-domain by e.g.:

- **Pole location (all poles in left half plane)**
- Bode diagram
- Nyquist plot
- H-infinity norm for MIMO systems

Non-linear systems → harmonic balance

→ ***Check stability for "Gang of four" (internal stability)***

This system has *n* poles and *m* zeros, and if it is physically realizable we have $n \geq m$.



Understanding Digital Signal Processing (2nd Edition)
Mar 25, 2004by Richard G. Lyons

Stability Criteria's (incomplete!)

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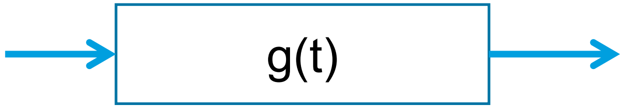


$$\int_{t=-\infty}^{\infty} |g(t)|dt < \infty$$

Stable or unstable linear systems

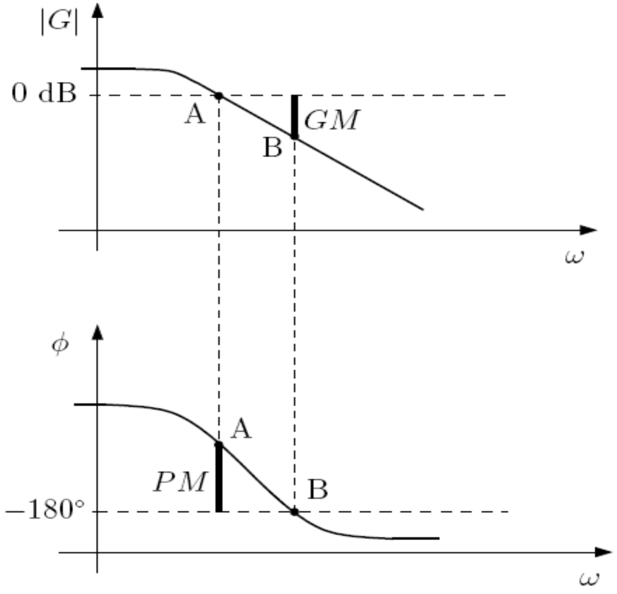
- Open loop or closed loop
- Unstable open loop: Stabilize closed loop system behavior using feedback controller

Stability check in s-domain by e.g.:

- Pole location (all poles in left half plane)
- **Bode diagram** → Open-loop Bode/Nyquist of L=CG to analyze closed-loop stability
- **Nyquist plot**
- H-infinity norm for MIMO systems

Non-linear systems → harmonic balance

→ ***Check stability for "Gang of four" (internal stability)***



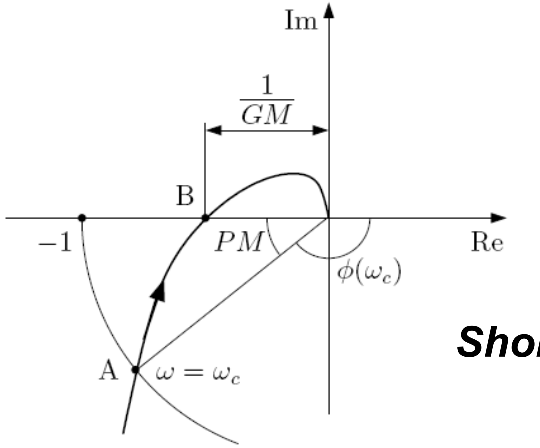
Bode diagram

Gain margin (at -180deg.)

$$K_P < \mathrm{GM}$$

Phase margin (at 0 dB)

$$\Delta\phi < \mathrm{PM}$$



Nyquist plot

Characteristic polynomial: $1 + GC$

$$GC \neq -1$$

Short: Do not encircle -1!

Robustness margins

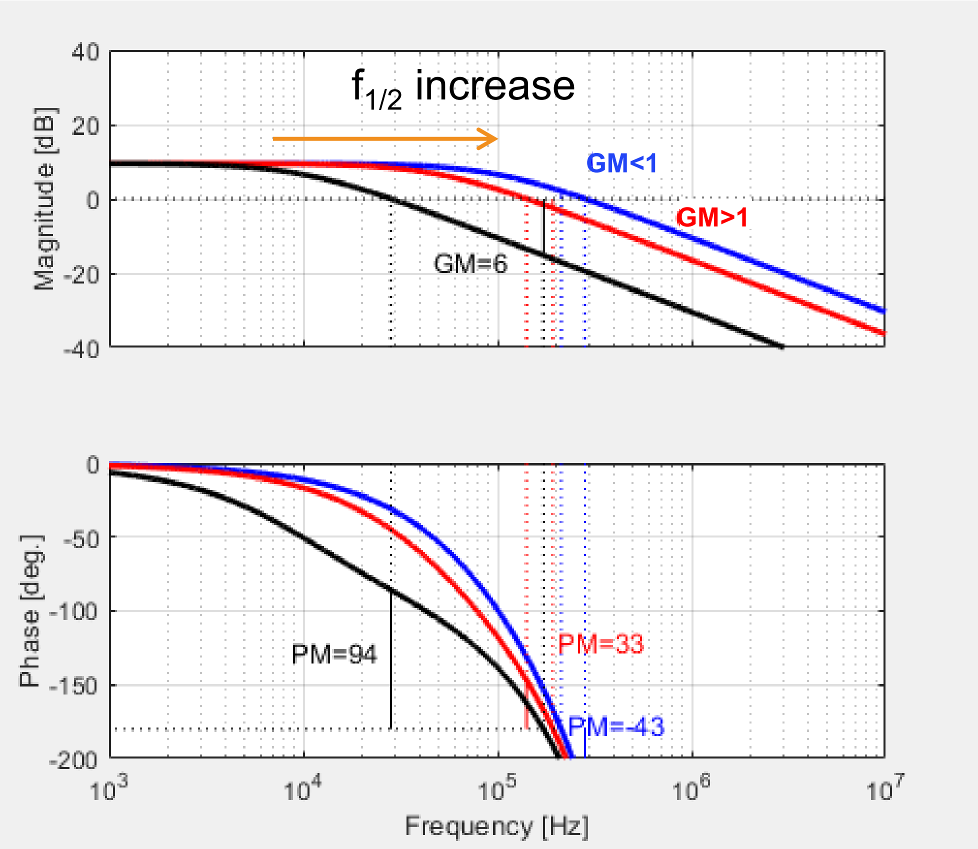
How close is a system to instability

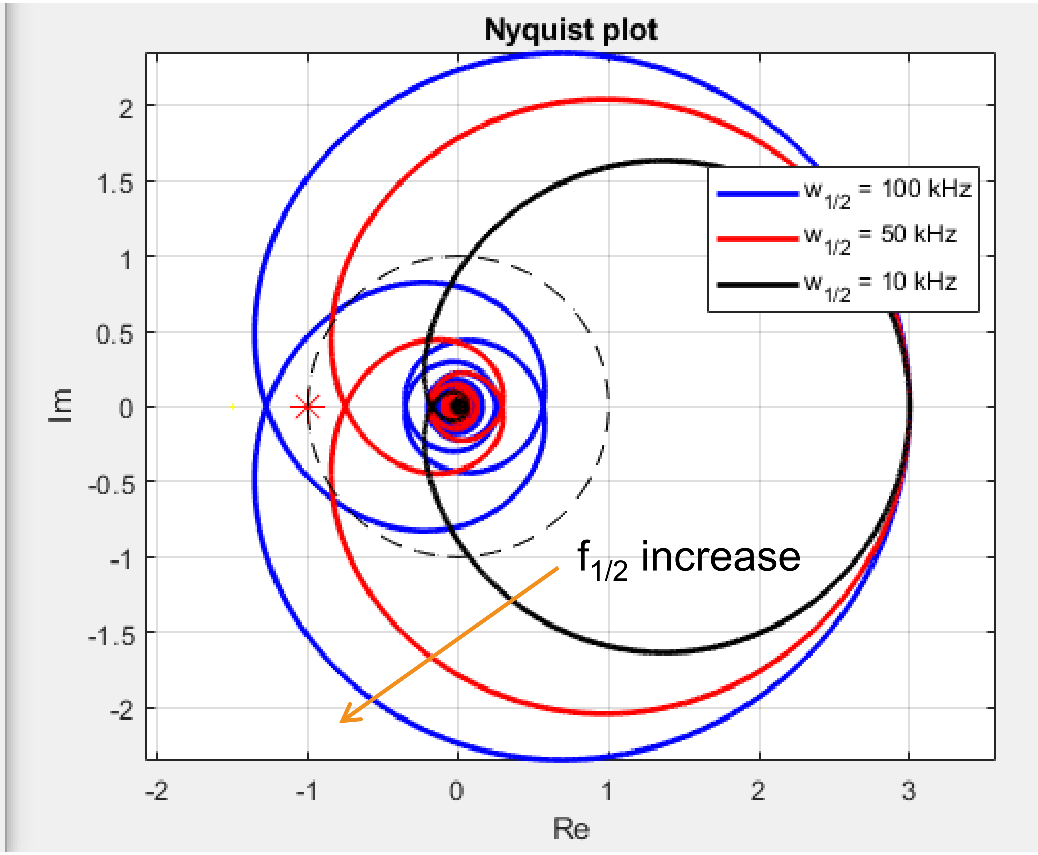
Example RF-gun @ Eu-XFEL and FLASH

Plant BW variation

- $K_P = 3$
- $T_d = 1.5\ \mu s$

GM and PM reduction by band-width increase of plant





Robustness margins

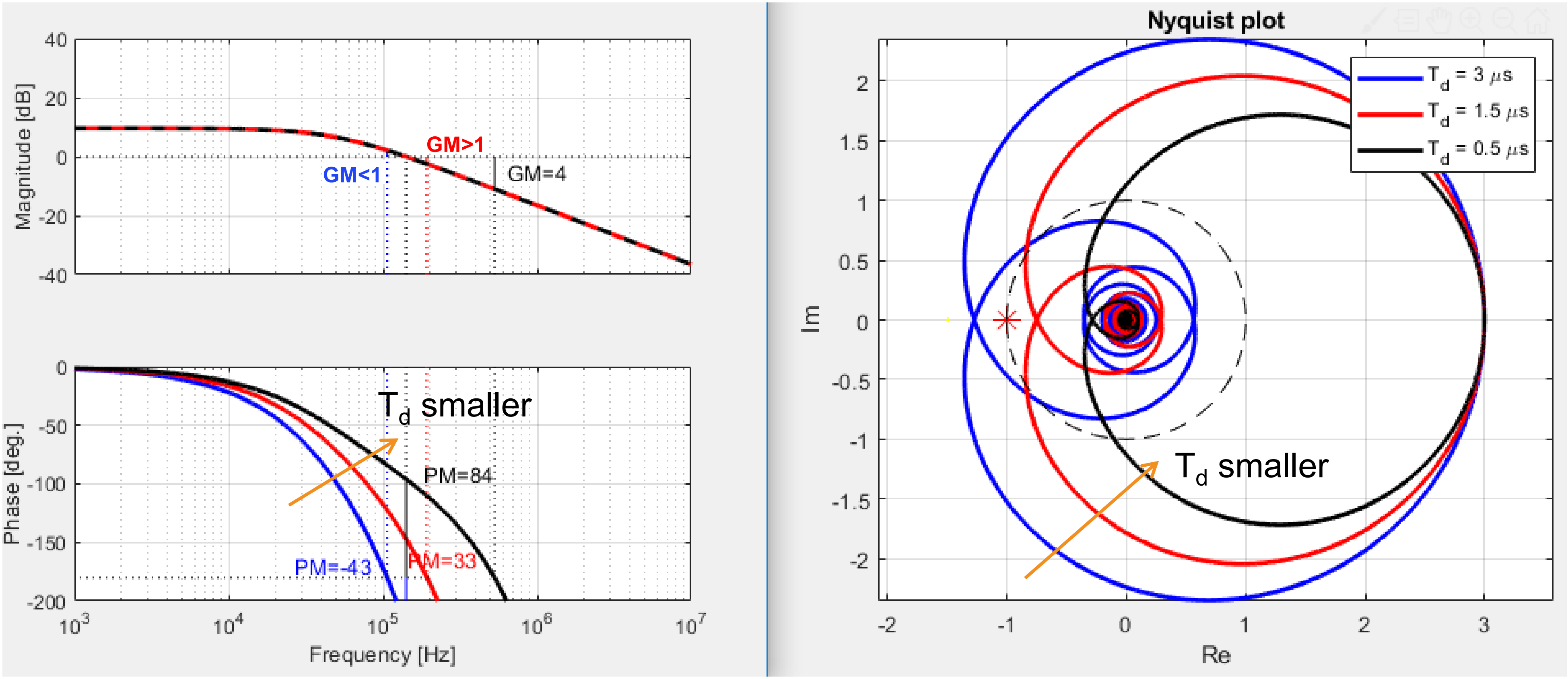
How close is a system to instability

Example RF-gun @ Eu-XFEL and FLASH

Delay variation

- $f_{1/2}$ = 50 kHz
- K_P = 3

GM and PM increase by reduction of the time delay.



Robustness margins

How close is a system to instability

Example RF-gun @ Eu-XFEL and FLASH

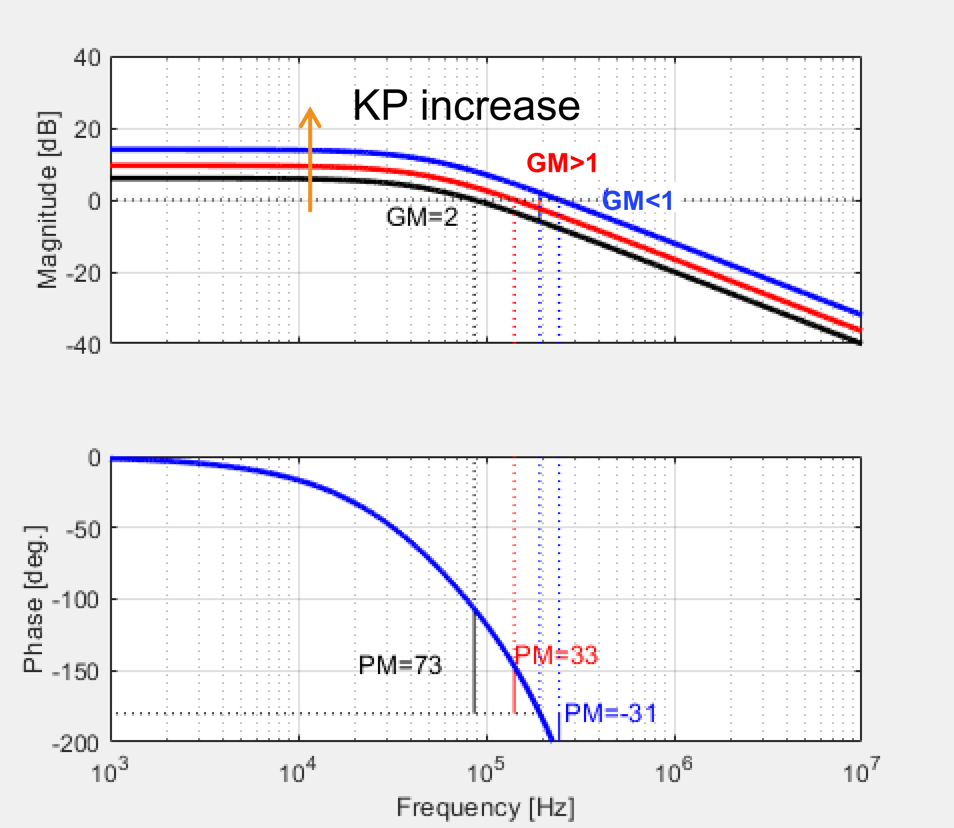
→ **Trade-off: robustness vs. performance**

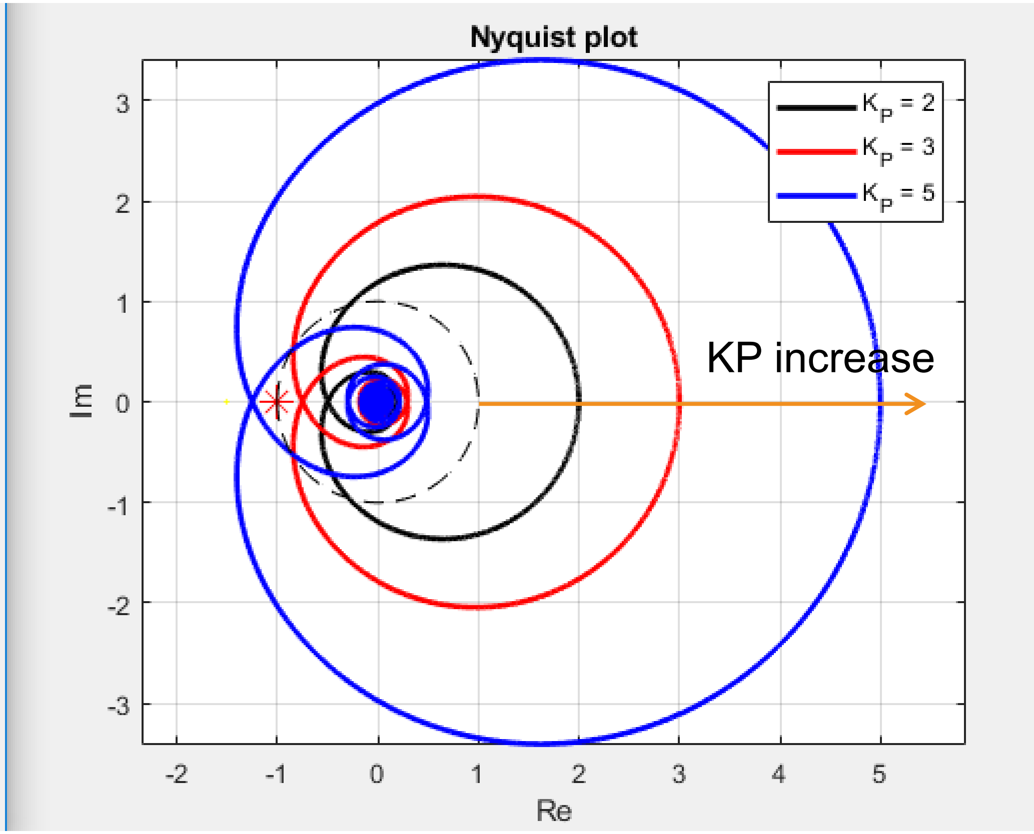
e.g. in case of plant variations, delay changes and FB gain variations

FB gain variation

- $f_{1/2}$ = 50 kHz
- T_d= 0.5 µs

GM and PM reduction by FB gain increase.



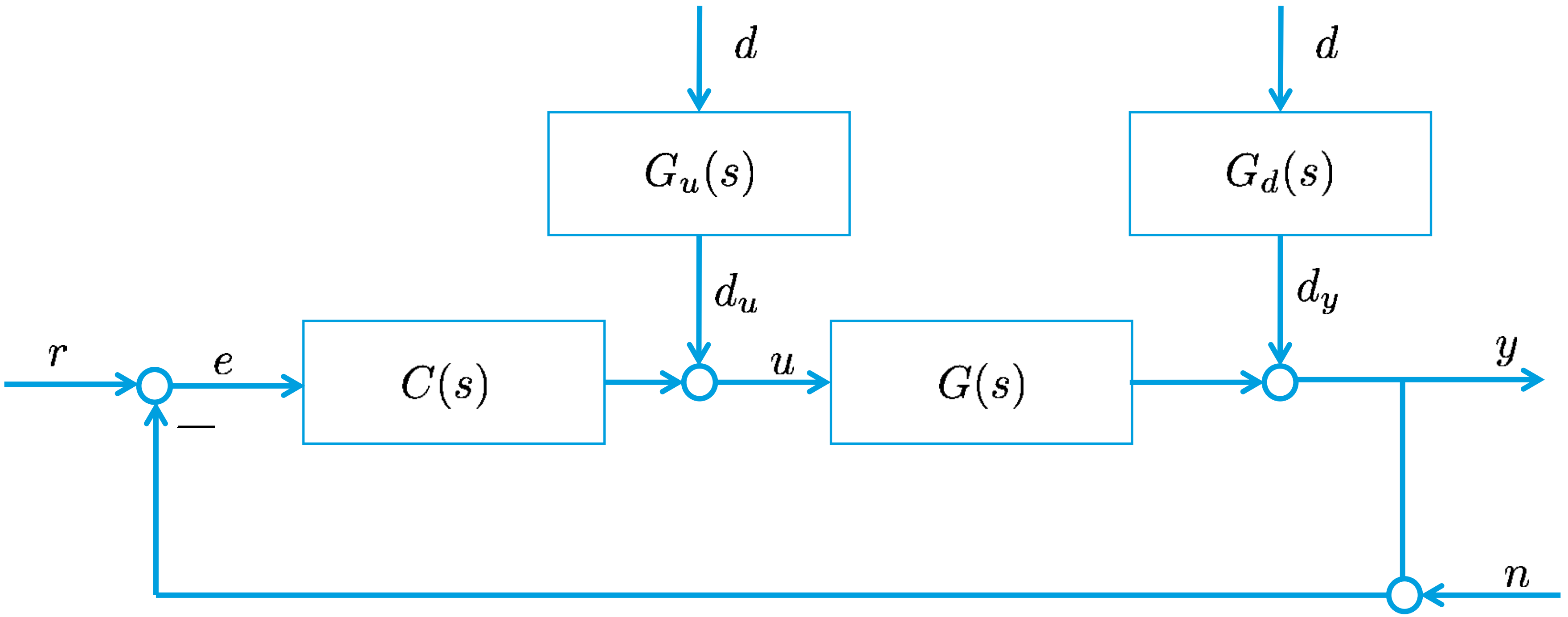


Gang of Four

Internal stability

The closed-loop system is internally stable (if no unstable hidden modes in C and G) if and only if all four transfer functions are stable:

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{1+GC} & -\frac{C}{1+GC} \\ \frac{G}{1+GC} & \frac{1}{1+GC} \end{bmatrix} \begin{bmatrix} d_u \\ d_y \end{bmatrix} = \begin{bmatrix} S & -CS \\ GS & S \end{bmatrix} \begin{bmatrix} d_u \\ d_y \end{bmatrix}$$



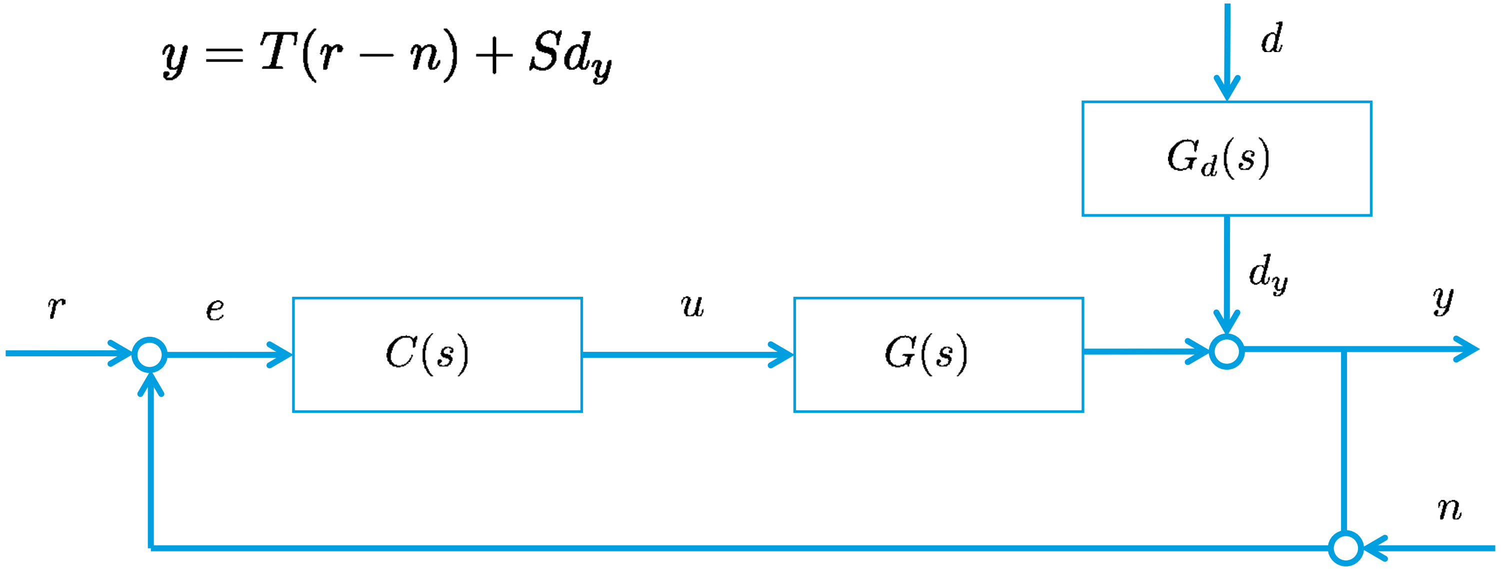
Control objectives

Closed-loop Performance

Performance measures in time-domain

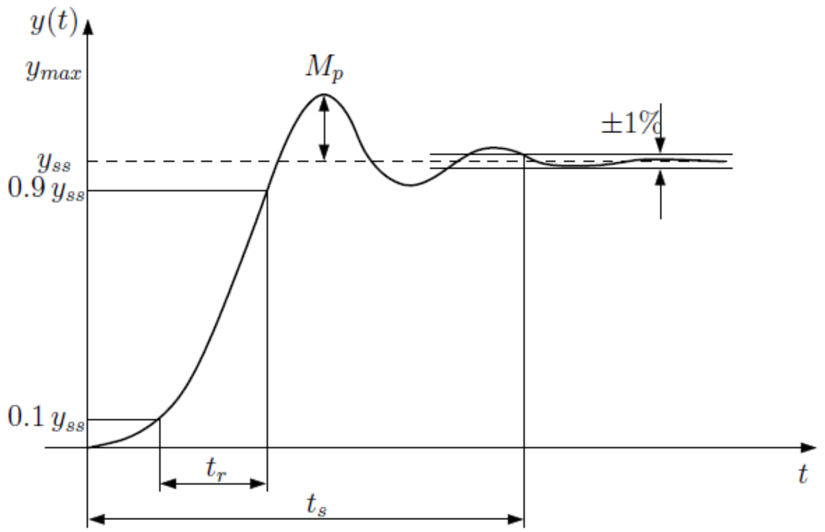
- ***Servo problem*** (reference tracking without disturbance and noise)
  - Manipulate u(t) to keep the output y(t) close to the reference r(t)

$$y = T(r - n) + Sd_y$$



Step response

r(t) = unit step, n(t) = 0, d(t) = 0



M_p overshoot
t_r rise time
t_s settling time

→ For good performance all should be small!

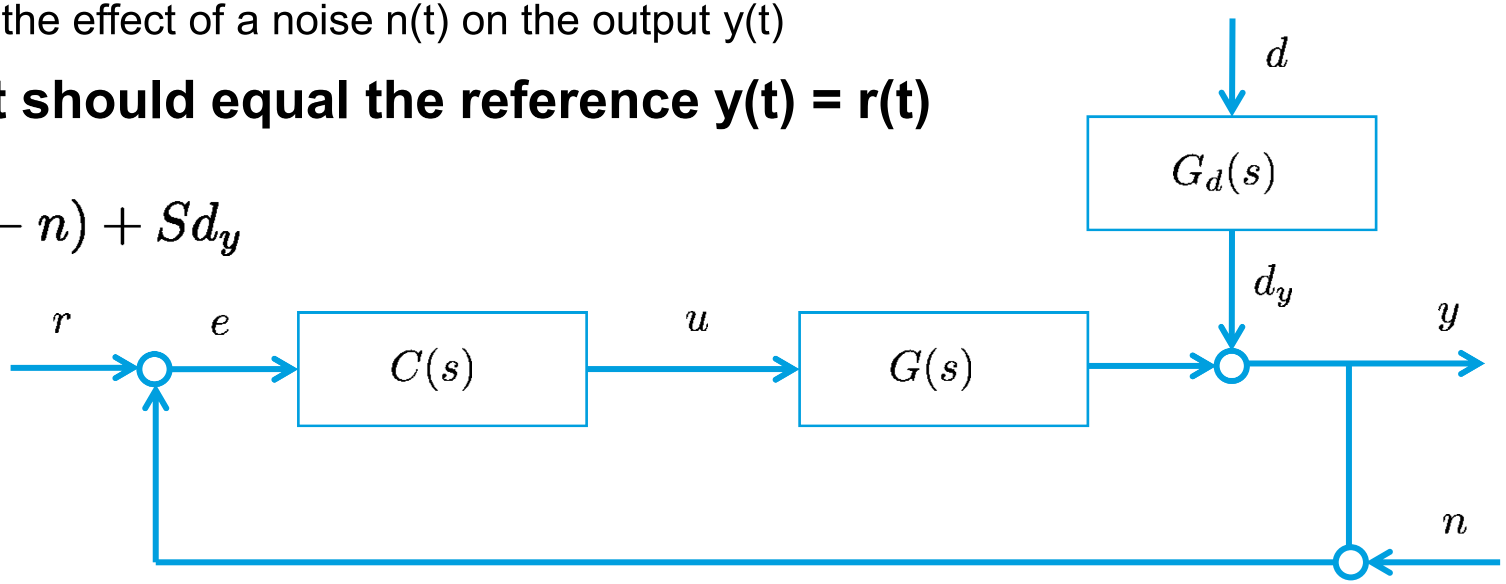
Objective of a Feedback Control Problem

Make the output y(t) behave in a desired way by manipulating the plant input u(t)

- ***Regulator problem*** (output disturbance rejection with constant reference and without noise)
  - Counteract the effect of a disturbance d(t)
- ***Servo problem*** (reference tracking without disturbance and noise)
  - Manipulate u(t) to keep the output y(t) close to the reference r(t)
- ***Noise rejection*** (noise rejection with constant reference and without disturbance)
  - Counteract the effect of a noise n(t) on the output y(t)

Goal: output should equal the reference y(t) = r(t)

$$y = T(r - n) + Sd_y$$



Objective of a Feedback Control Problem

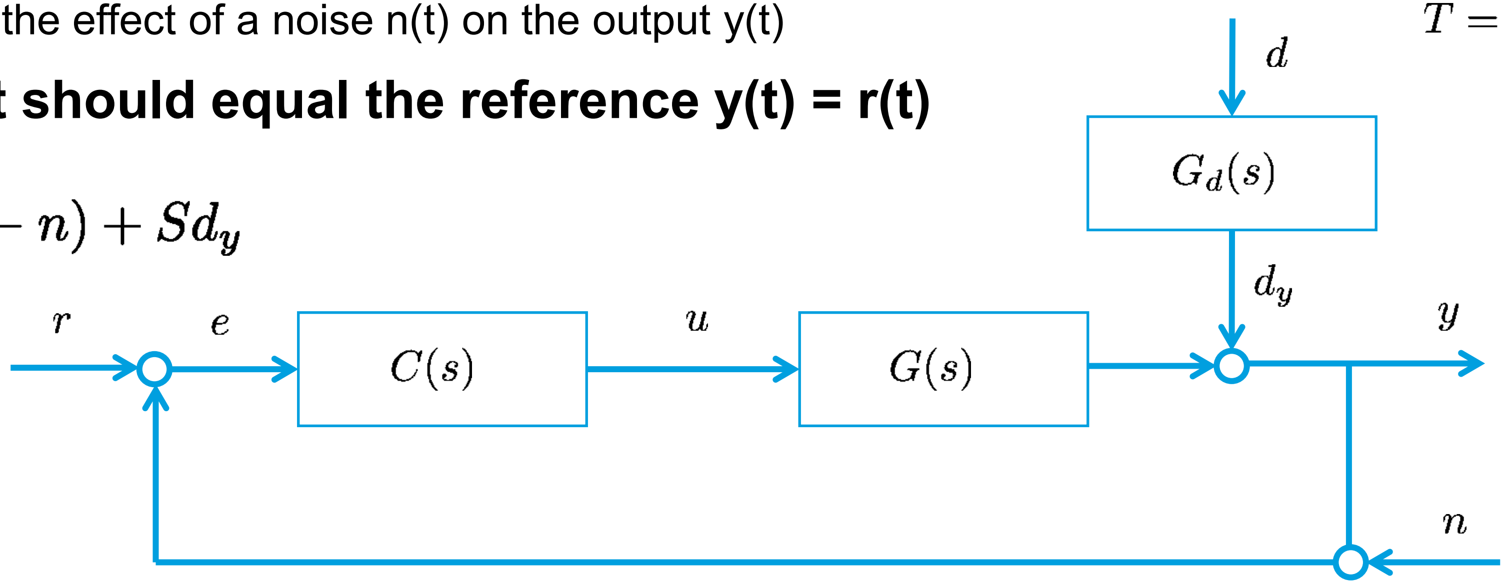
Make the output y(t) behave in a desired way by manipulating the plant input u(t)

- ***Regulator problem*** (output disturbance rejection with constant reference and without noise)
  - Counteract the effect of a disturbance d(t) $S = 0$
- ***Servo problem*** (reference tracking without disturbance and noise)
  - Manipulate u(t) to keep the output y(t) close to the reference r(t) $T = 1$
- ***Noise rejection*** (noise rejection with constant reference and without disturbance)
  - Counteract the effect of a noise n(t) on the output y(t) $T = 0$

Conflicting goals

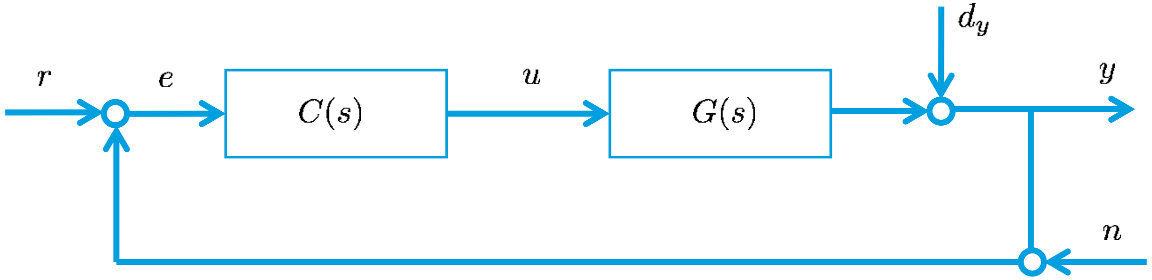
Goal: output should equal the reference y(t) = r(t)

$$y = T(r - n) + Sd_y$$



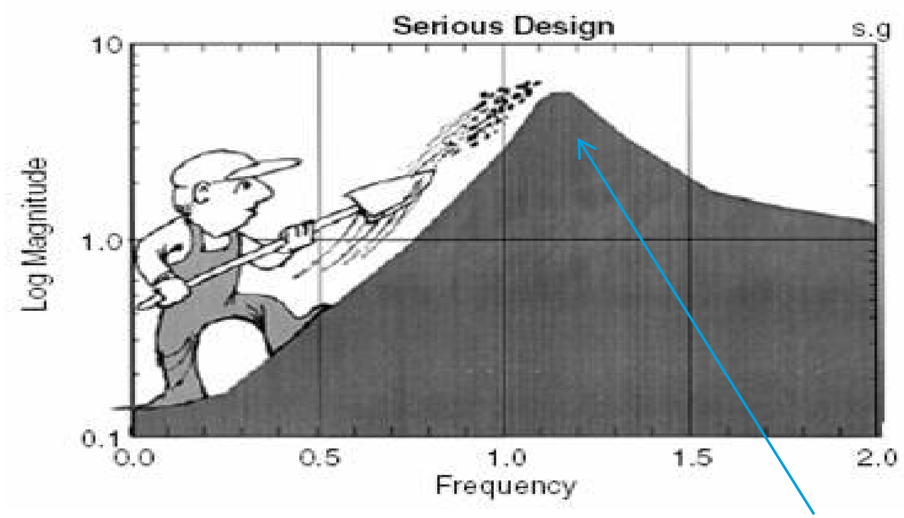
Fundamental Limitations on Sensitivity

Waterbed effect



- Conflicting goals:
  - S = 0 for disturbance rejection
  - T = 1 for reference tracking
  - T = 0 for noise rejection
- Typically:
  - High frequency noise signal n
  - Low frequency disturbance d
  - Low frequency reference signal r
- Choose: $S(0) = \text{small}; \quad S(\infty) = 1$

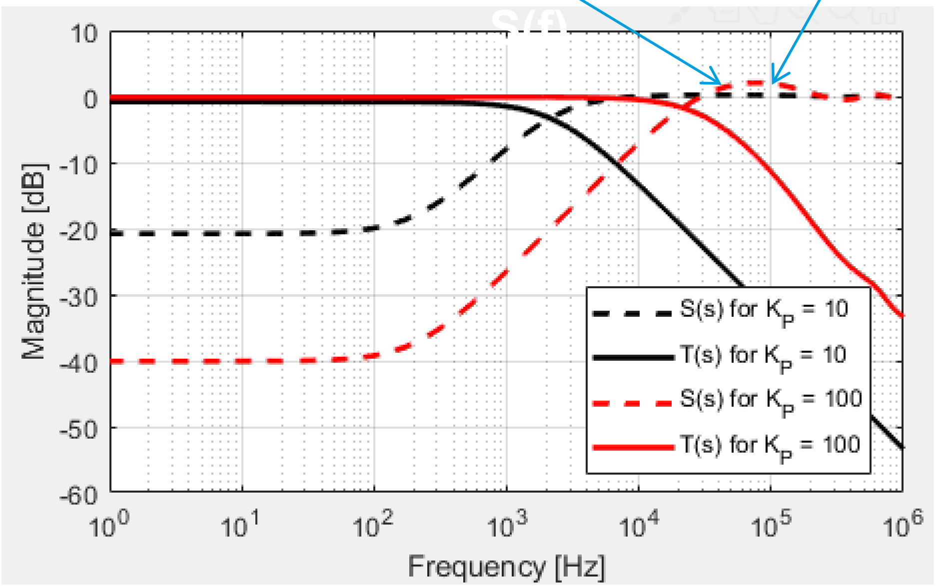
  $T(0) = 1; \quad T(\infty) = \text{small}$



2005 - "Respect the Unstable," IEEE Control Systems Magazine, Vol. 23, No. 4, pp. 12-25, August 2003.

Waterbed effect

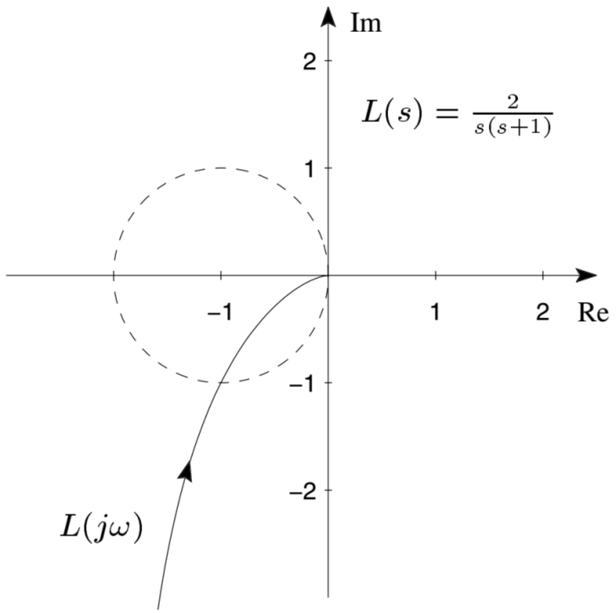
Oscillations at high frequencies



Fundamental Limitations on Sensitivity

Waterbed effect

Multivariable Feedback Design, Analysis and Design (2nd Ed.), 2005 by S. Skogestad and I. Polethwaite



|S|>1 whenever the Nyquist plot of L is inside the circle:

$$|S(j\omega)| > 1$$

$$\Longleftrightarrow$$

$$\frac{1}{|1+G(j\omega)C(j\omega)|} > 1$$

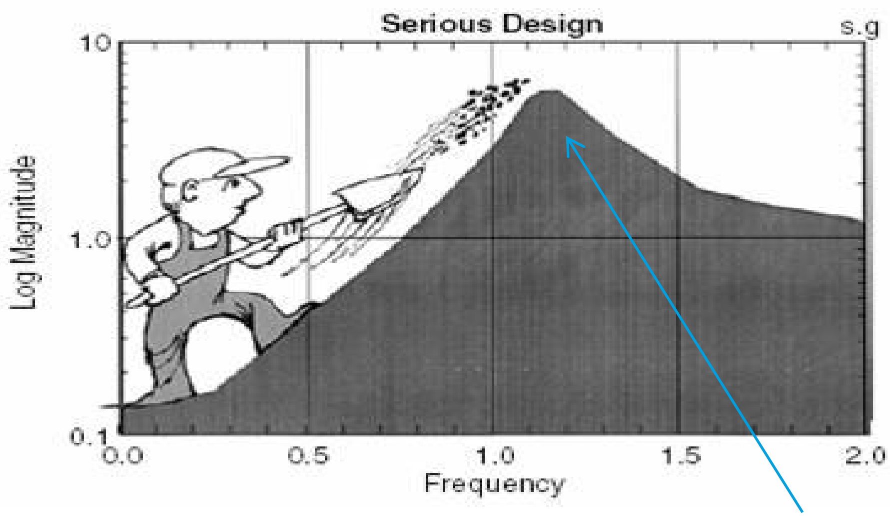
$$\Longleftrightarrow$$

$$1 > |1+G(j\omega)C(j\omega)|$$

If the loop transfer function L=GC is stable, than

$$\int_0^\infty \ln|S(j\omega)|d\omega = 0$$

If we push |S(jω)| down at low frequencies → it must pop up somewhere else (at high frequencies)



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Oscillations at high frequencies

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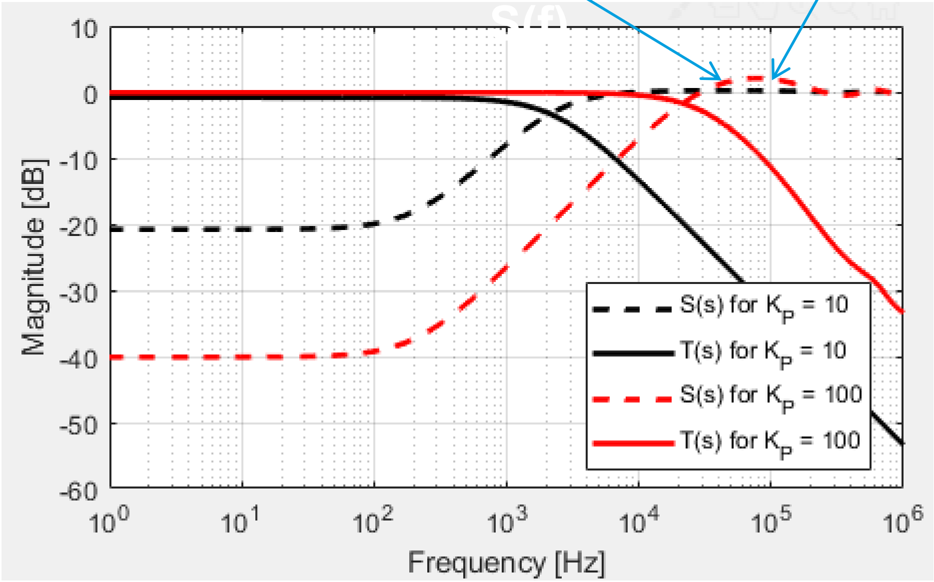
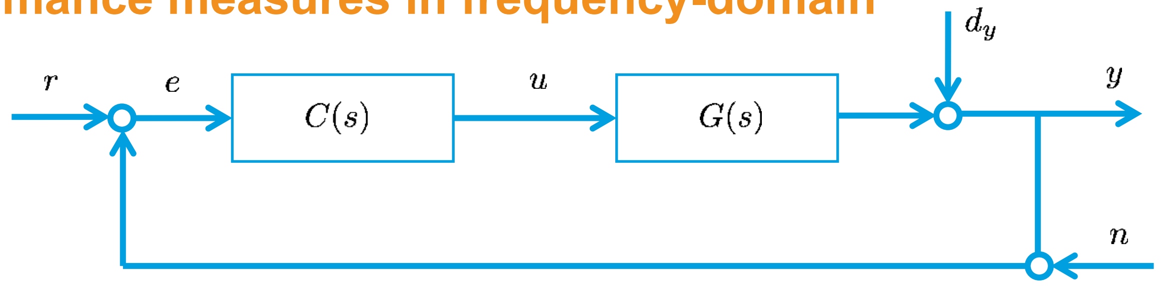
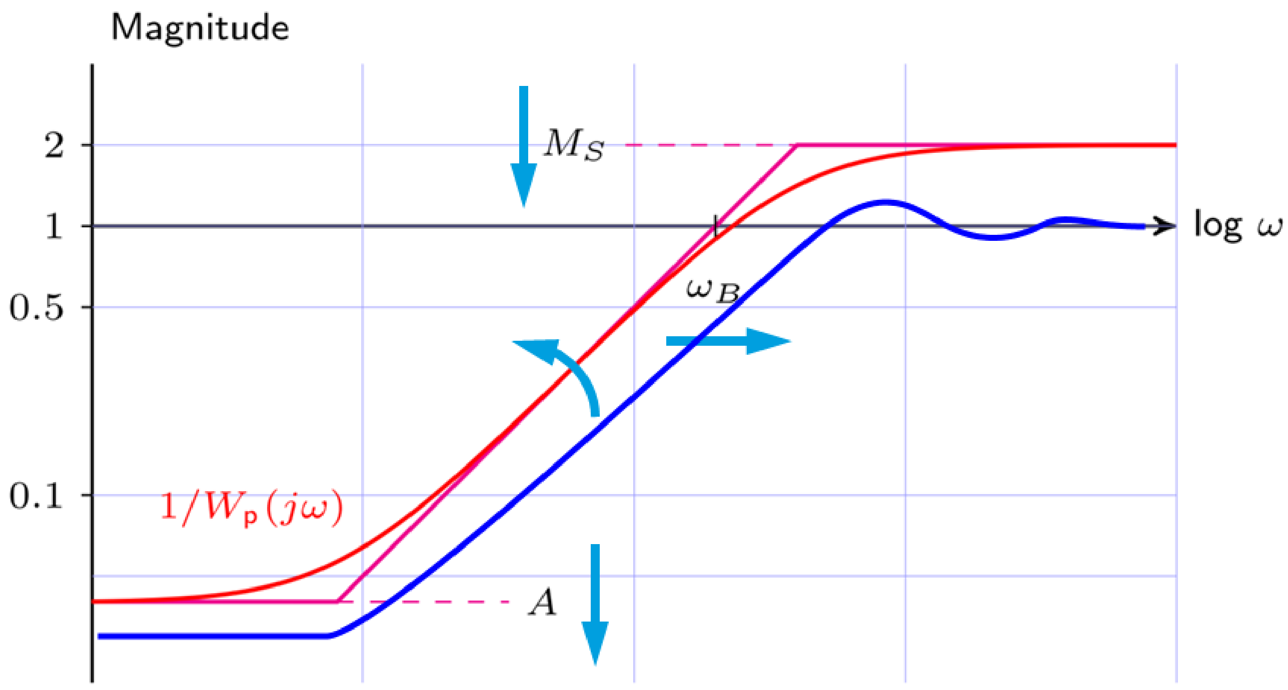


Closed-loop Performance

Performance measures in frequency-domain





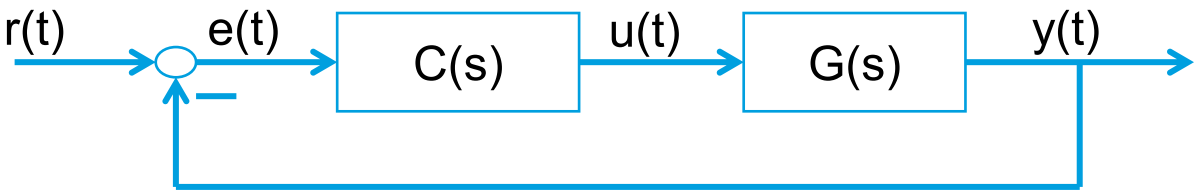
Desired bound sensitivity function

$$S(0) = \text{small}; \qquad S(\infty) = 1$$
$$T(0) = 1; \qquad T(\infty) = \text{small}$$

- M_s : push down for better robustness margins
- ω_b : increase the bandwidth
- A : push down for smaller steady state error
- Increase slope for better transient behavior

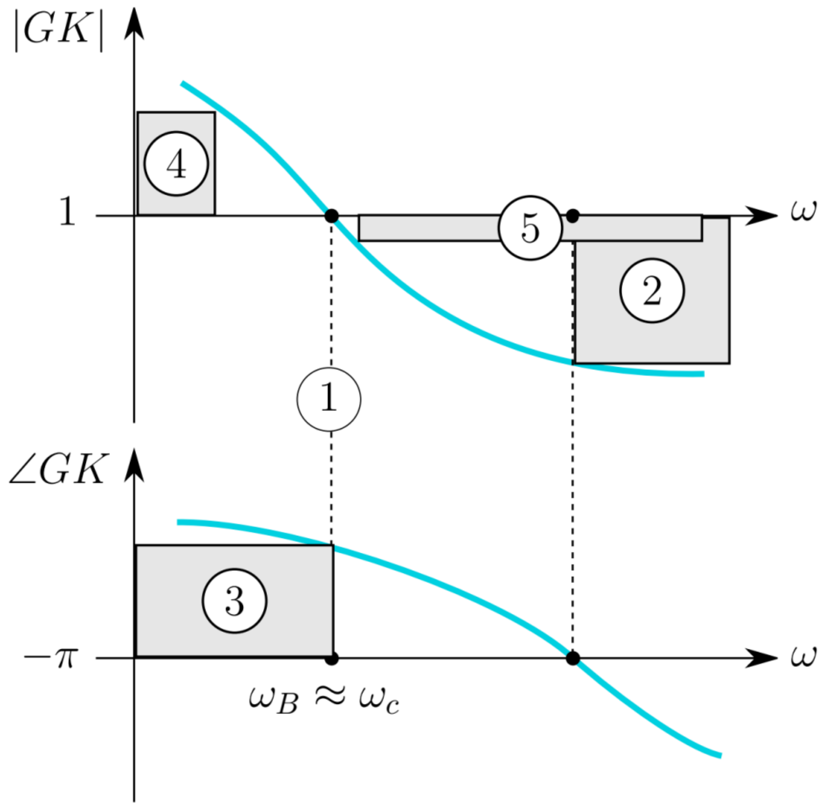
Classical FB Control

Frequency domain analysis



Loop Shaping

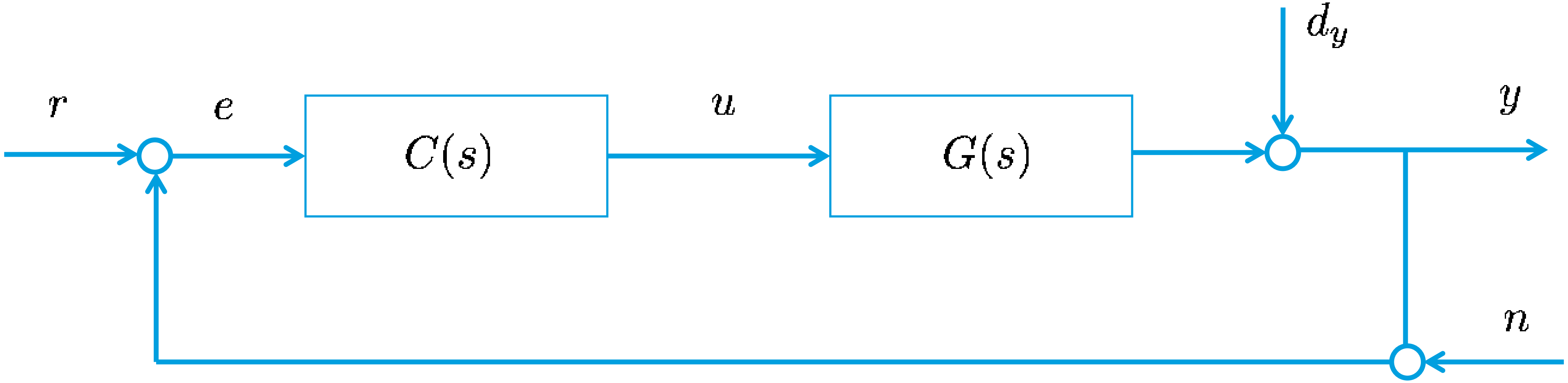
modify the open-loop system $L(s) = C(s)G(s)$ to ensure effective control in closed loop



1. crossover frequency (for large bandwidth)
2. gain margin (GM) (for closed-loop robustness)
3. phase margin (PM) (for closed-loop robustness)
4. $|GK|_{s=0} \gg 1$ (for small stationary control error)
5. nominal stability of closed loop (use Nyquist/Bode/small gain criterion)

Closed-loop Performance

Robustness vs. Performance



- In general: Trade-off between robustness and performance

  → Example with proportional controller K_P

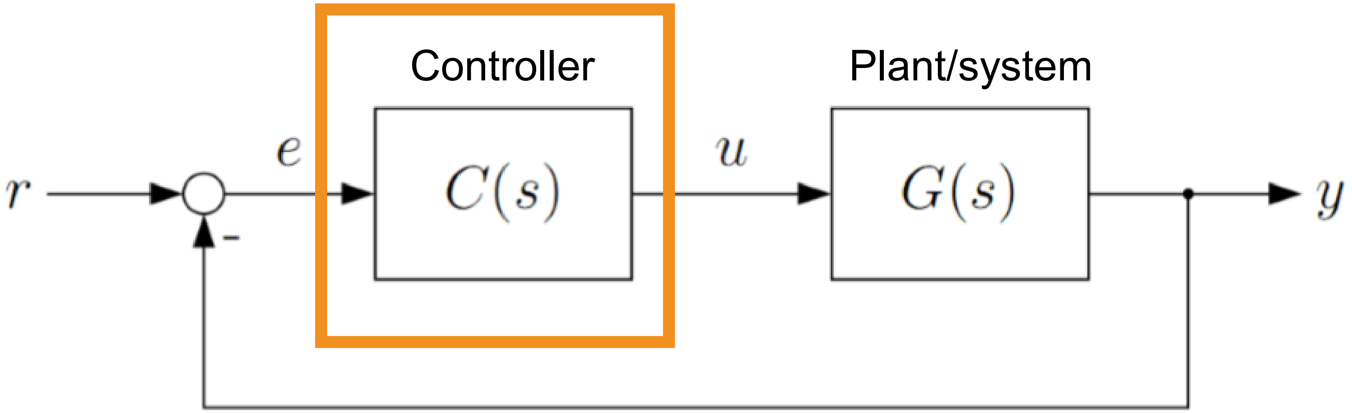
- Way out: more complicated controller structure

  → Offer more degrees of freedom to improve both

Outlook

Closed loop system analysis

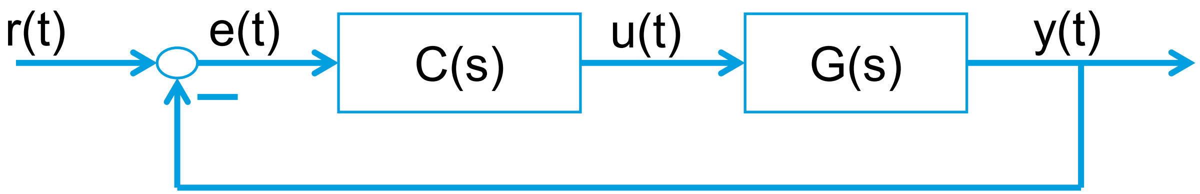
1. System description and modelling
2. Controller design/analysis
3. **PID controller**
4. Outlook



PID Controller

PID Controller

Influence of the PID components

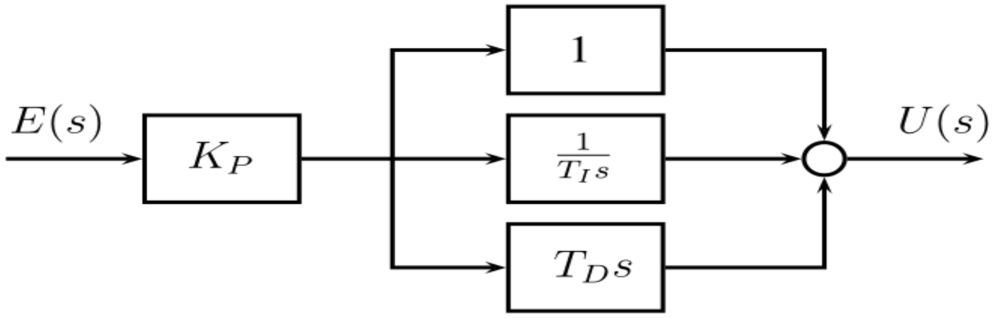


PID-Control

$$\frac{U(s)}{E(s)} = C(s) = K_P$$

$$u(t) = K_P \left(e(t) + \frac{1}{T_I} \int_{t_0}^{t} e(\tau) d\tau + T_D \dot{e}(t) \right)$$

$$U(s) = K_P \left[1 + \frac{1}{T_I s} + T_D s \right] E(s)$$



Proportional P

changes the magnitude and not the phase

- increasing P decrease steady state error
- increasing P decreases robustness margins

Integral I

magnitude → infinity for frequencies → 0

- no steady state error
- in case of saturation → loop is cut open (integrator keeps integrating → bad transients)
  (→ Integrator windup)

Derivative D

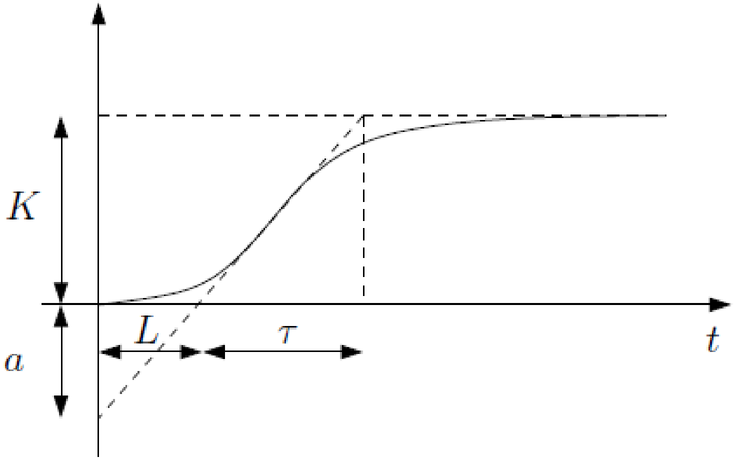
Introduces a 90° phase lead at high frequencies

- can improve phase margin (smaller overshoot)
- sensitive at high frequencies → noise amplification

PID Controller Tuning

Ziegler-Nichols tuning rules

Ziegler Nichols tuning rules 1 (step response)



	K_P	T_I	T_D
P	$1/a$	∞	0
PI	$0.9/a$	$3L$	0
PID	$1.2/a$	$2L$	$l/2$

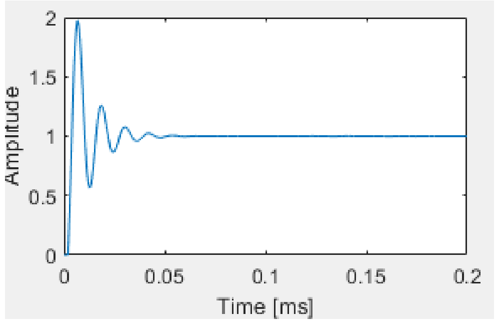
Knowledge of G(s) not required!

Tuning rules for example system

Rule 1

$$K_P = 335$$

$$T_I = 6 \times 10^{-6}$$

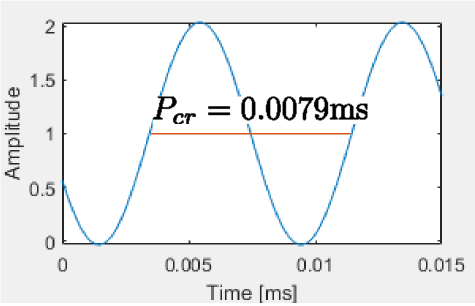


Ziegler Nichols tuning rules 2 (stable oscillation)

- Close the loop with proportional controller
- Increase controller gain to the *critical gain* K_{cr} (steady state oscillation)
- Period of oscillation is *critical period* P_{cr}

	K_P	T_I	T_D
P	$0.5 \cdot K_{cr}$	∞	0
PI	$0.45 \cdot K_{cr}$	$0.83 \cdot P_{cr}$	0
PID	$0.6 \cdot K_{cr}$	$0.5 \cdot P_{cr}$	$0.125 \cdot P_{cr}$

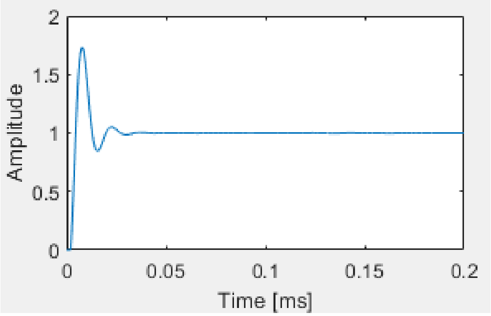
Knowledge of G(s) not required! Time-consuming and dangerous driving close to instability



Rule 2

$$K_P = 263$$

$$T_I = 6.5 \times 10^{-6}$$



PID Controller Tuning

More tuning rules

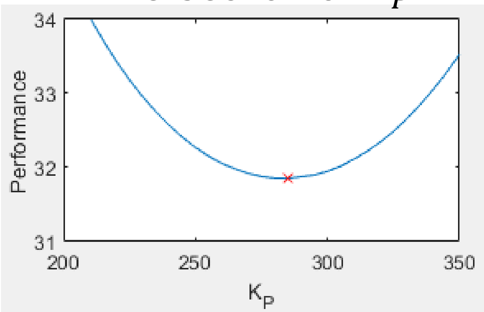
Practical tuning

- Define performance criteria (RMSE, overshoot, rise time…)
- Increase K_P with $1/T_I$=0 until performance gets worse
- Increase $1/T_I$ with the K_P found before until performance gets worse

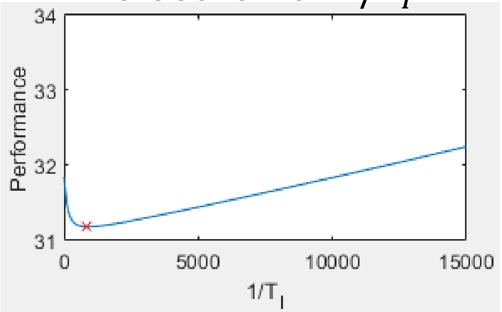
Knowledge of G(s) is not required!
Time-consuming

Tuning rules for example system

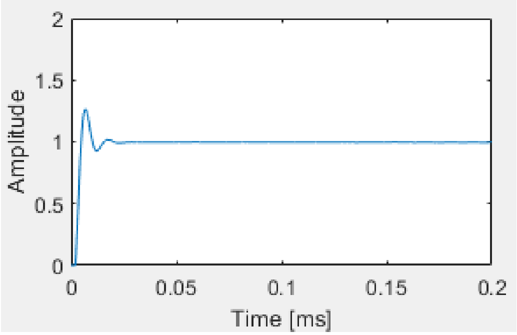
Line search of K_P



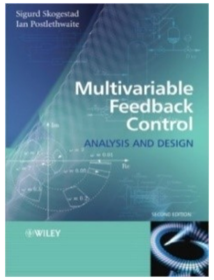
Line search of $1/T_I$



$$K_P = 285$$
$$T_I = 0.012$$



Skogestad for PI controller [Skogestad.2003]



- For first (and second) order systems with delay

$$G(s) = \frac{b_0}{s + a_0} e^{-T_d s}$$

$$K_p = \frac{1}{b_0} \frac{1}{\tau_c + T_d}$$
$$T_I = \min\left(\frac{1}{a_0}, 4(\tau_c + T_d)\right)$$

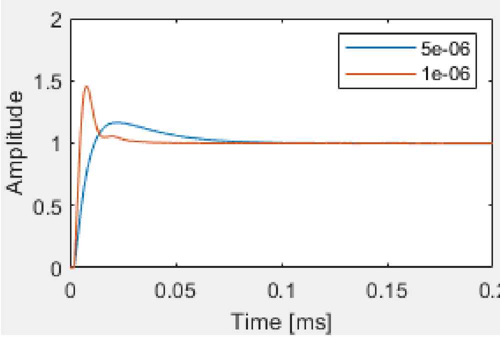
- Desired rise time τ_c

Knowledge of G(s) required!

$$\tau_c = 5 \times 10^{-6}$$
$$K_P = 106$$
$$T_I = 28 \times 10^{-6}$$

$$\tau_c = 10^{-6}$$
$$K_P = 248$$
$$T_I = 12 \times 10^{-6}$$

Different choices of τ_c



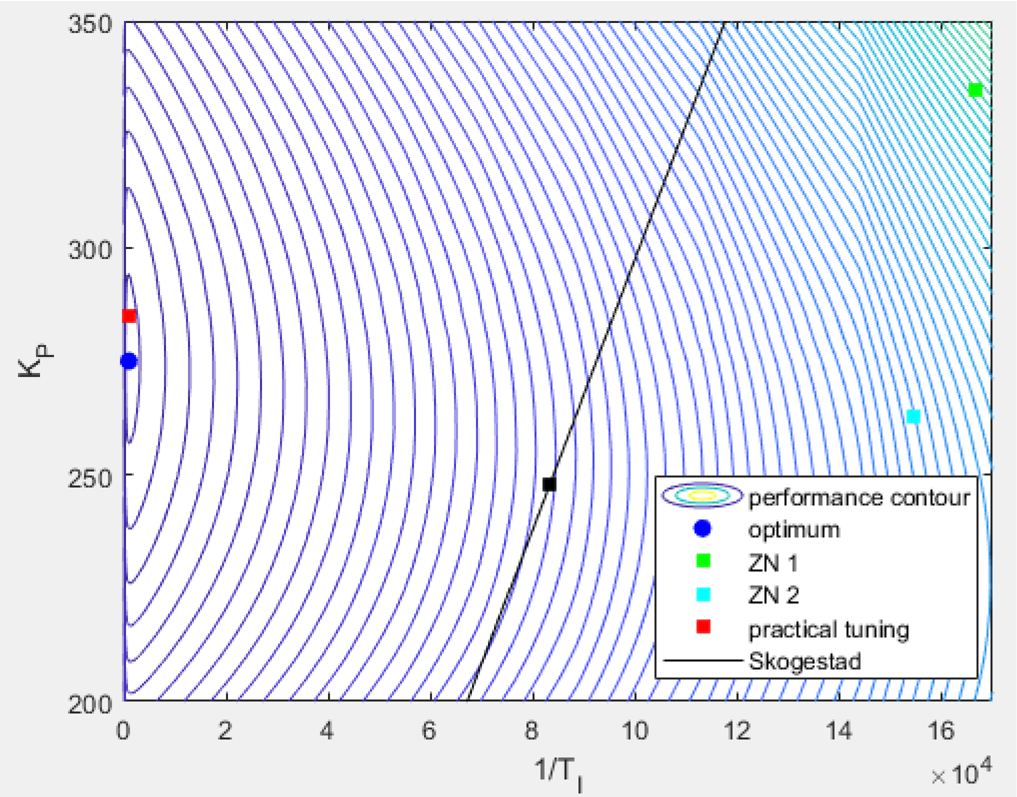
PID Controller Tuning

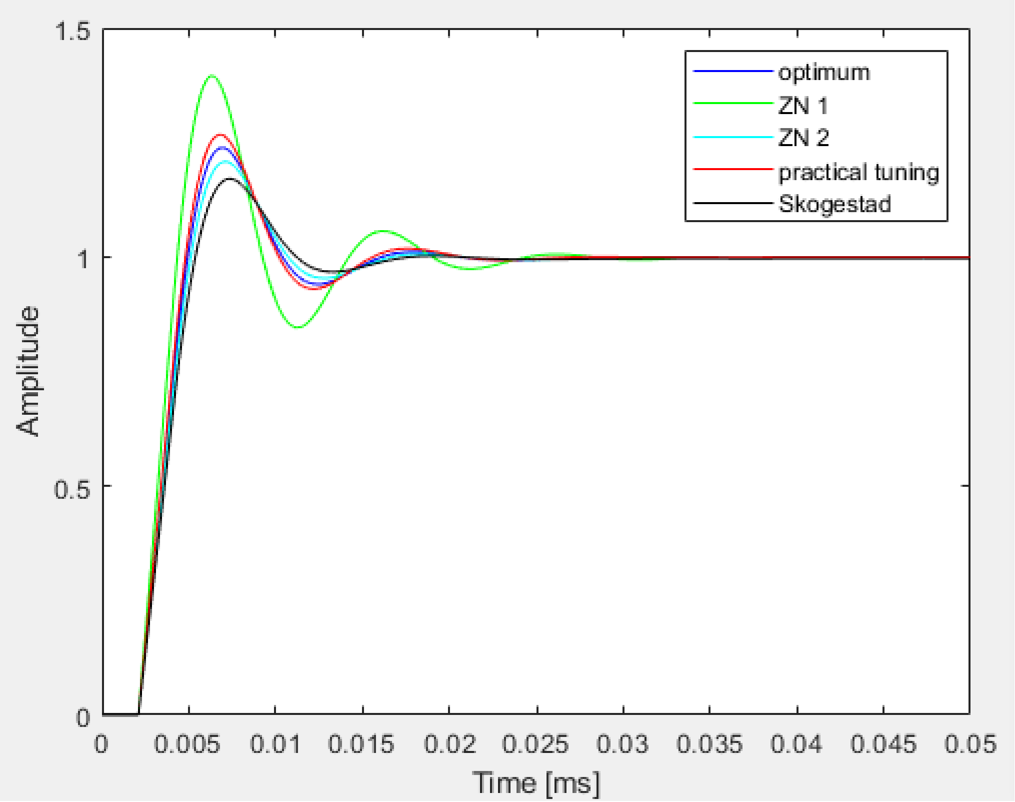
Comparison for the example system

$$\text{Cost function} = \int_{t=0}^{6\text{ms}} e(t)^2$$

Performance contour (optimum)

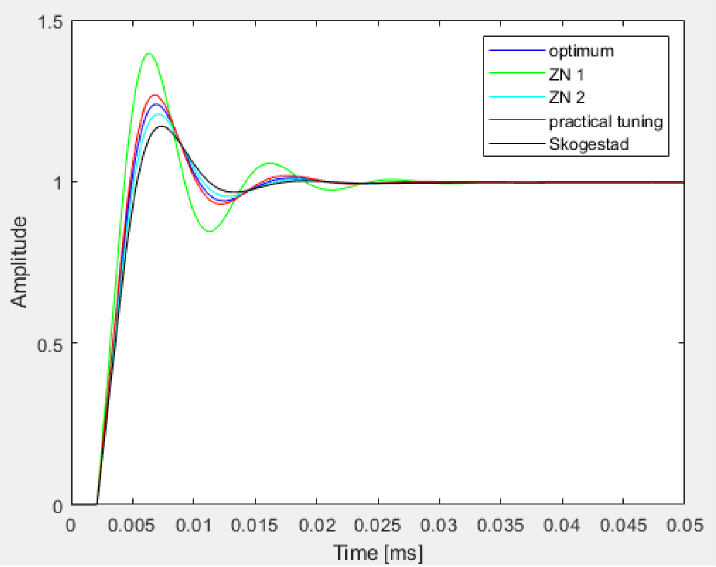
- strongly depends on chosen cost function
- strongly depends on system type





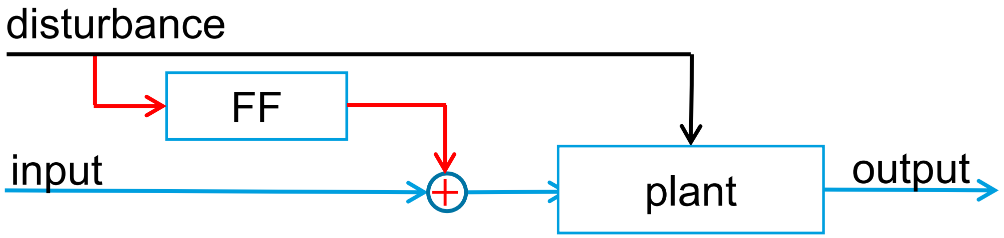
PID Controller Tuning

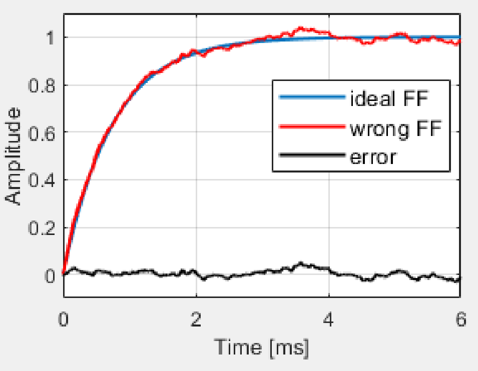
Comparison for the example system

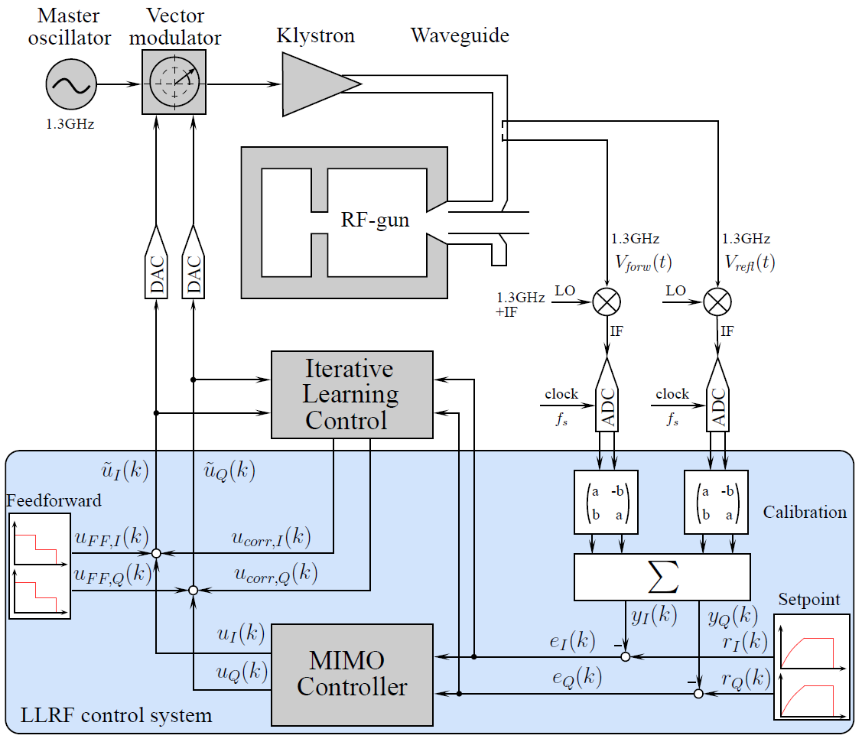


- Optimization using this cost function:
  - Looks fine in terms of reference tracking, but does not penalize the available input signal/power (in FB 150x more drive signal needed compared to FF only)
    - SRF cavity with $w_{1/2}$=214 Hz reaches steady state in 20 µs…
- How to overcome this?
  - Choose proper feedforward signal and adjust the reference correspondingly

Feed forward





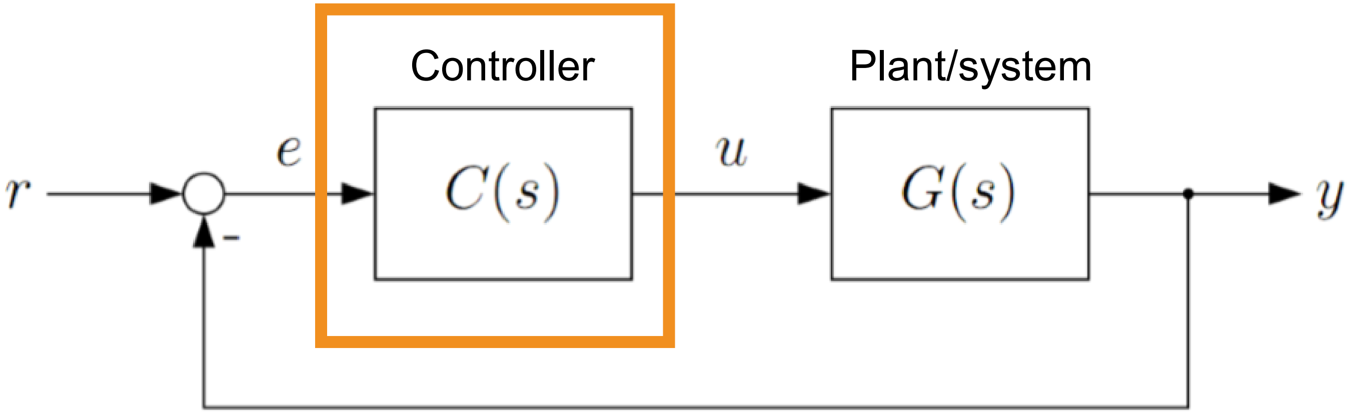


- PID tuning rules does not apply for all practical problems
  - Be careful, start conservative and/or talk to an expert

Outlook

Closed loop system analysis

1. System description and modelling
2. Controller design/analysis
3. PID controller
4. **Outlook**



Outlook

Types of Feedback Control

Classical FB Control

Frequency domain analysis

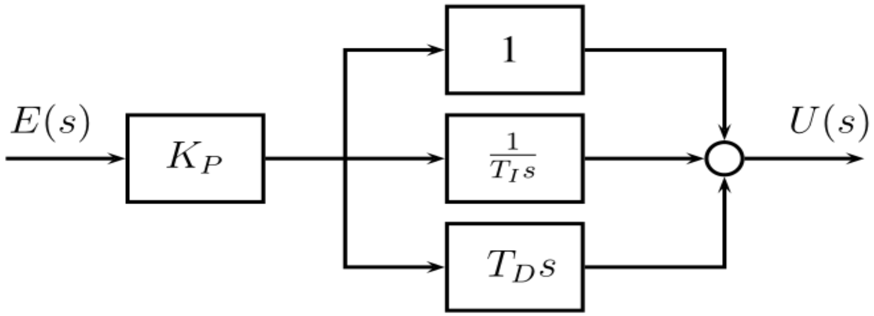
→ Bode Diagram, Nyquist Plot

PID-Control

$$\frac{U(s)}{E(s)} = C(s) = K_P$$

$$u(t) = K_P\left(e(t) + \frac{1}{T_I}\int_{t_0}^{t} e(\tau)d\tau + T_D\dot{e}(t)\right)$$

$$U(s) = K_P\left[1 + \frac{1}{T_I s} + T_D s\right]E(s)$$



Modern FB Control

Time domain analysis

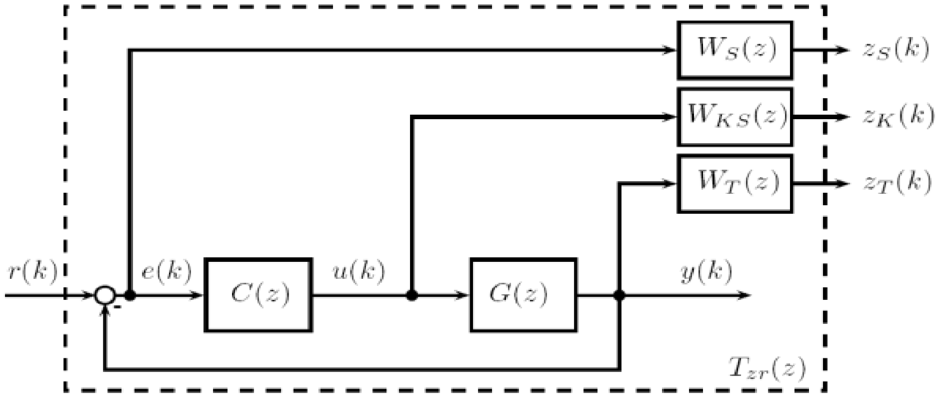
→ State space representation

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

→ Linear-quadratic regulator (LQR) etc.

$$u(t) = -K \cdot x(t)$$

→ H-infinity optimization by shaping the sensitivity and complementary sensitivity function



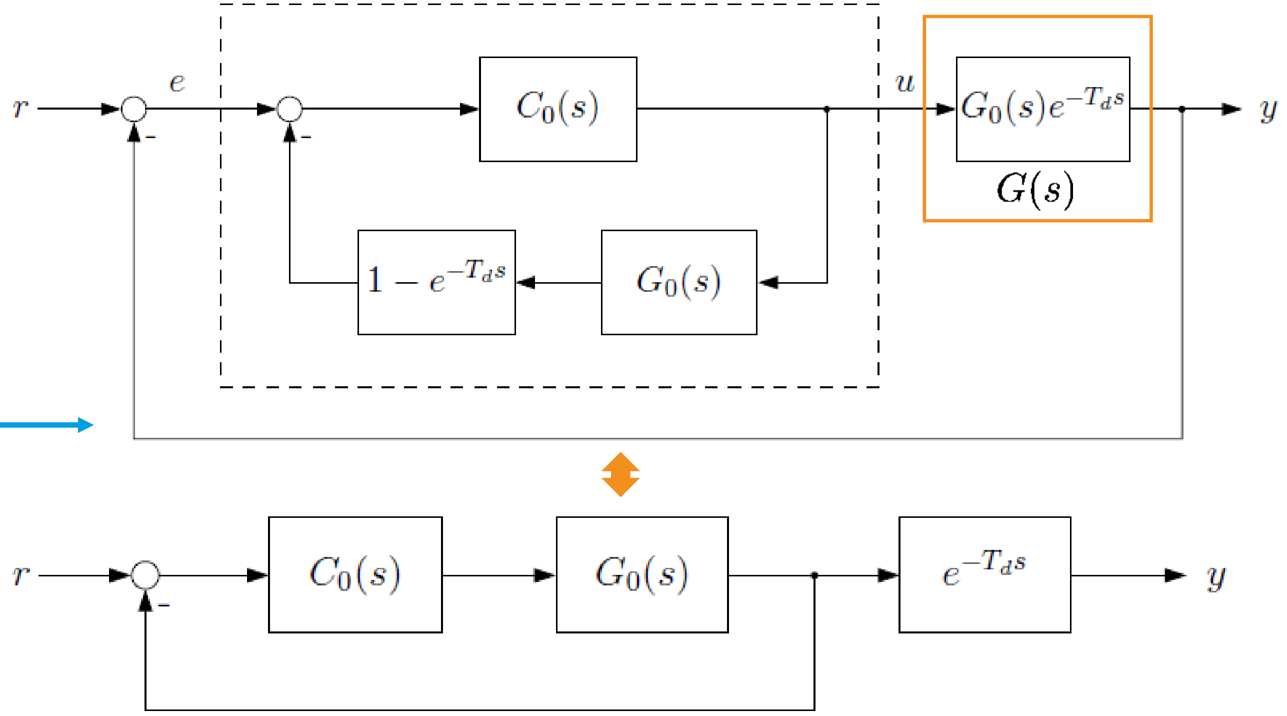
$$\|T_{zr}(z)\|_\infty = \left\|\begin{bmatrix} W_S(z)\cdot S(z) \\ W_{CS}(z)\cdot C(z)S(z) \\ W_T(z)\cdot T(z)\end{bmatrix}\right\|_\infty < 1$$

Outlook

How to deal with delays?

Smith Predictor

- Design $C_0(s)$ for the system $G_0(s)$ without time delay
- Use the controller structure
- Achieve desired closed-loop with delay



MIMO systems (multi-input multi-output systems)

- Plant with m inputs and l outputs → $G(s)$ is a $l \times m$ transfer matrix
- Stability is in terms of eigenvalues of A matrix, generalized Nyquist (Bode does not generalize)
- Change one input affects all output → coupling between inputs and outputs
- Manual controller tuning gets tedious → optimization based controller synthesis (see **modern FB control**)

Thank you for your attention!

Any Questions?

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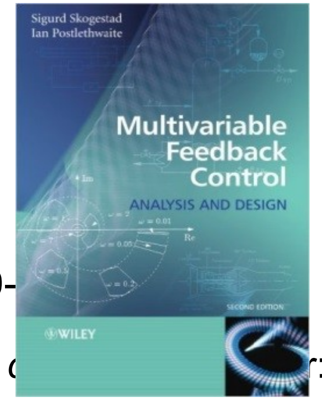

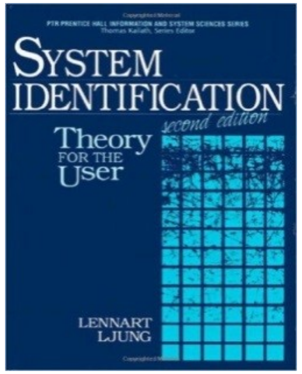

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