Virtual Pepper-Pot Technique for 4D Phase Space Studies.

G. Z. Georgiev, M. Krasilnikov, DESY, Zeuthen

georgi.georgiev@desy.de

Introduction

There are ongoing beam coupling studies at the Photo Injector Test facility at DESY in Zeuthen (PITZ). Electron beam asymetries have been observed and gun quadrupoles are installed to correct them. A 4D phase space characterization is required to understand the asymmetries in detail.

The Virtual-Pepper Pot (VPP) technique is an analysis technique for measurement of 4D transverse phase space and 4D projected emittance of space-charge dominated electron beams. A step called beamlet crossing is crucial in the analysis. Images of beamlets from single slit scan in both horizontal and vertical direction are combined. Each generated image by beamlet crossing corresponds and resembles an image of a single aperture.

Theory

4D transverse beam matrix

A main theoretical tool to describe the 4D beam dynamics is the 4D transverse beam matrix

$$\sigma^{4D} = \begin{pmatrix} \langle xx\rangle & \langle x'x\rangle & \langle yx\rangle & \langle y'x\rangle \\ \langle xx'\rangle & \langle x'x'\rangle & \langle yx'\rangle & \langle y'x'\rangle \\ \langle xy\rangle & \langle x'y\rangle & \langle yy\rangle & \langle y'y\rangle \\ \langle xy'\rangle & \langle x'y'\rangle & \langle yy'\rangle & \langle y'y'\rangle \end{pmatrix}$$

with elements <*uu*> and <*uv*> representing a variance of *u* and a covariance between *u* and *v* respectively.

Emittance and emittance invariant

The projected horizontal emittance (and analogycally for vertical emittance) is defined as

$$\epsilon_{x,\text{scaled,normalized}} = f_{\text{scaling}}\beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

with relativistic factors product βy and scaling factor defined as the ratio of the full beam size to the reconstructed beam size

$$f_{x,\text{scaling}} = \frac{\sigma_{x,\text{fullbeam}}}{\sigma_{x,\text{slitscan}}}$$

The 4D emittance is defined as

$$\epsilon_{4\text{D,scaled,normalized}} = f_{x,\text{scaling}} f_{y,\text{scaling}} (\beta\gamma)^2 \sqrt{\det(\sigma^{4\text{D}})}$$

A transverse emittance invariant is defined as

$$I_2^{(2)} = \epsilon_x^2 + \epsilon_y^2 + 2\begin{vmatrix} \langle xy\rangle & \langle x'y\rangle \\ \langle xy'\rangle & \langle x'y'\rangle \end{vmatrix} \quad [2]$$

X-Y correlation and coupling factors

A correlation value between horizontal phase space and vertical phase space is introduced as

$$\rho_{4\text{D}} = \sqrt{1 - \left(\frac{\epsilon_{4\text{D}}}{\epsilon_x\epsilon_y}\right)^2}$$

Its value ranges between 0 (no correlation) and 1 (fully correlated) in analogy to Pearson's coefficient.

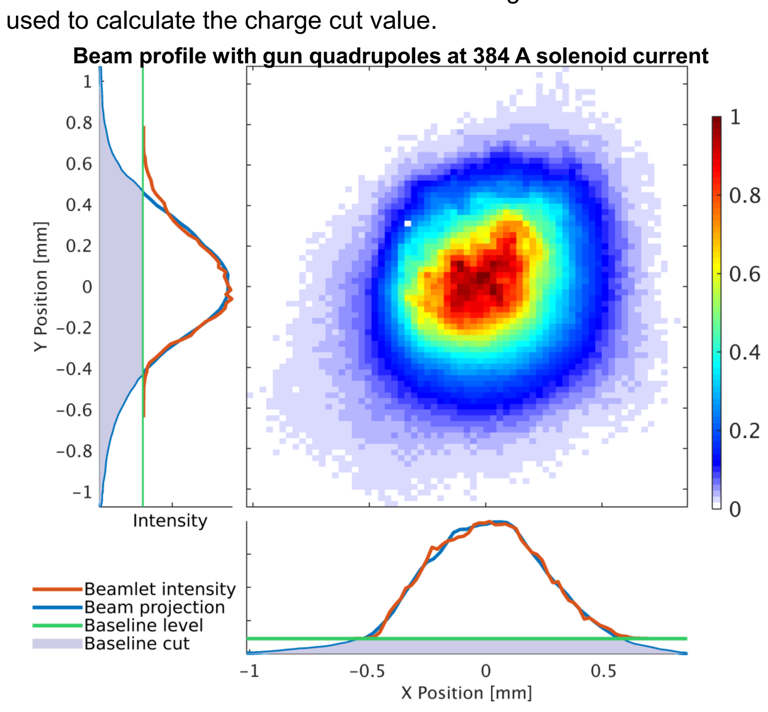
Two coupling factors are shown for comparison.

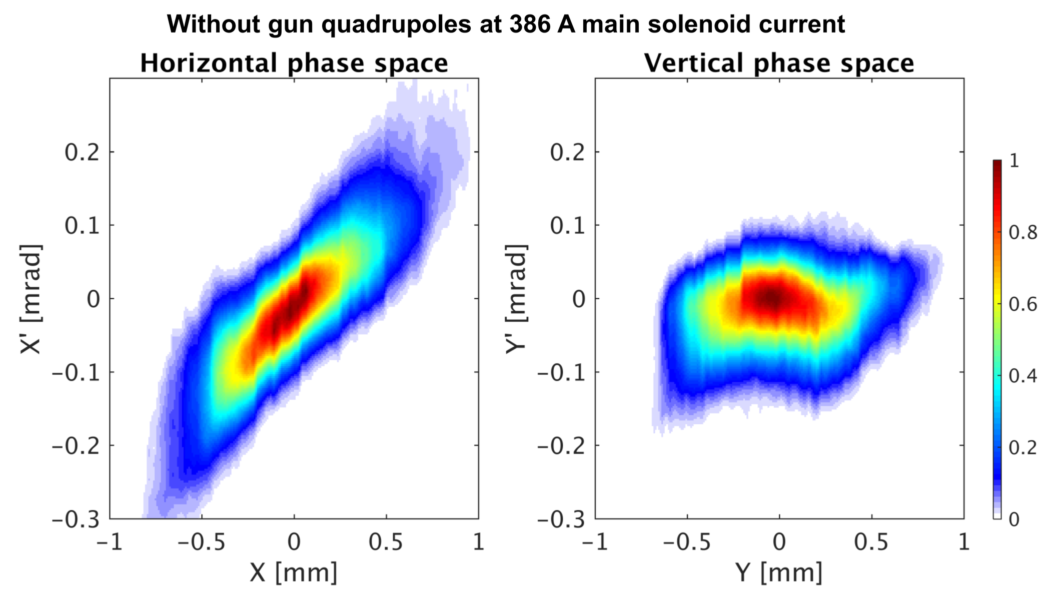
Coupling 1 $$C = \sqrt{\frac{\epsilon_x\epsilon_y}{\epsilon_{4\text{D}}}} - 1 \quad [3]$$

Coupling 2 $$t = \frac{\epsilon_x\epsilon_y}{\epsilon_{4\text{D}}} - 1 \quad [4]$$

Charge cut

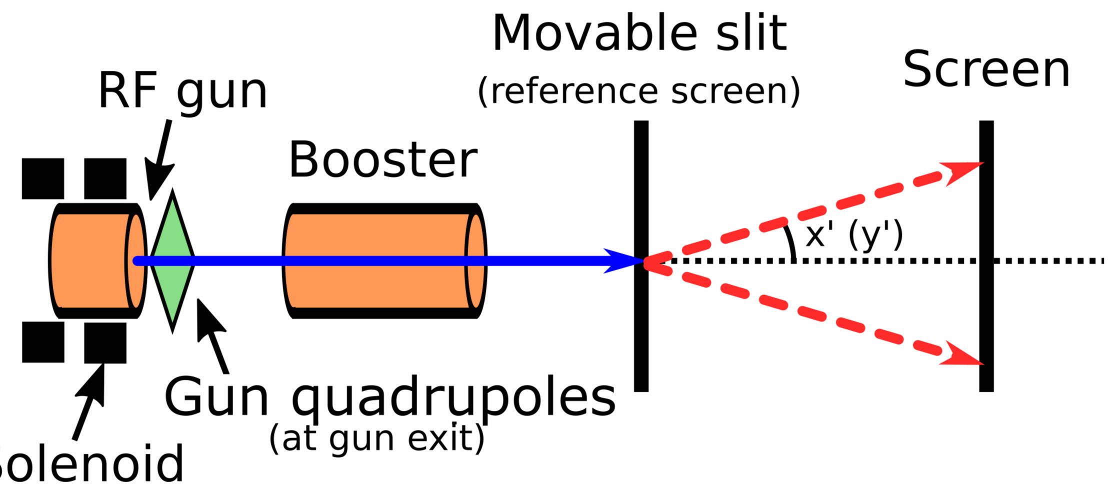
At PITZ, the charge cut value estimates what fraction of the beam missing in analisys because of low signal to noise ratio on the second screen. A fit to a reference beam image on the first screen is used to calculate the charge cut value.





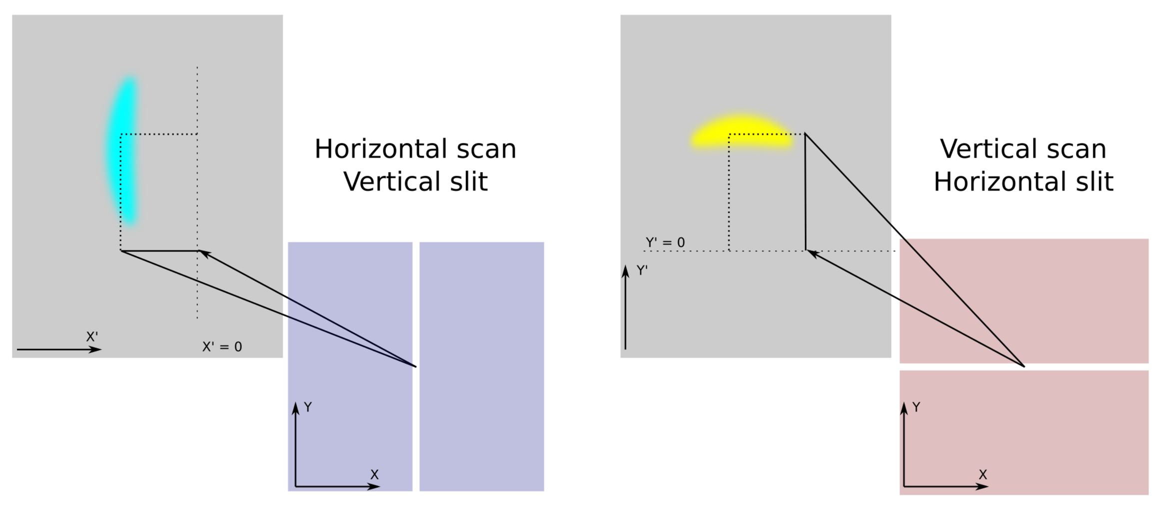
Experimental setup

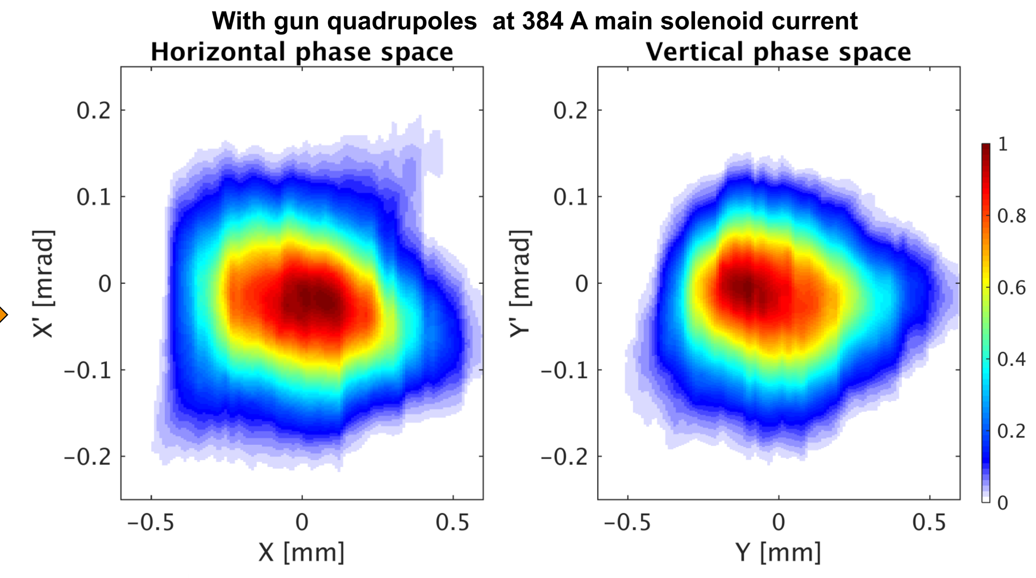
- Sigle slit scan – standard method at PITZ
- Laser pulse: 1.2 mm diameter, FWHM ~11 ps
- 1,6 cell electron gun → 6,5 MeV/c
- Gun solenoid
- Gun quadupoles: 1 normal + 1 skew
- Booster → 22,3 MeV/c at exit
- First station:
  - Slit: 1.0 mm thick, 10 µm opening
  - Screen → reference image
- Vacuum drift 3.13 m
- Second station:
  - Beamlet collector screen
- Beamlet
  - Low charge
  - Emittance dominated
  - Low signal to noise ratio
- Each beamlet → slice of 2D phase space
- 2D phase space → reconstructed from all beamlets



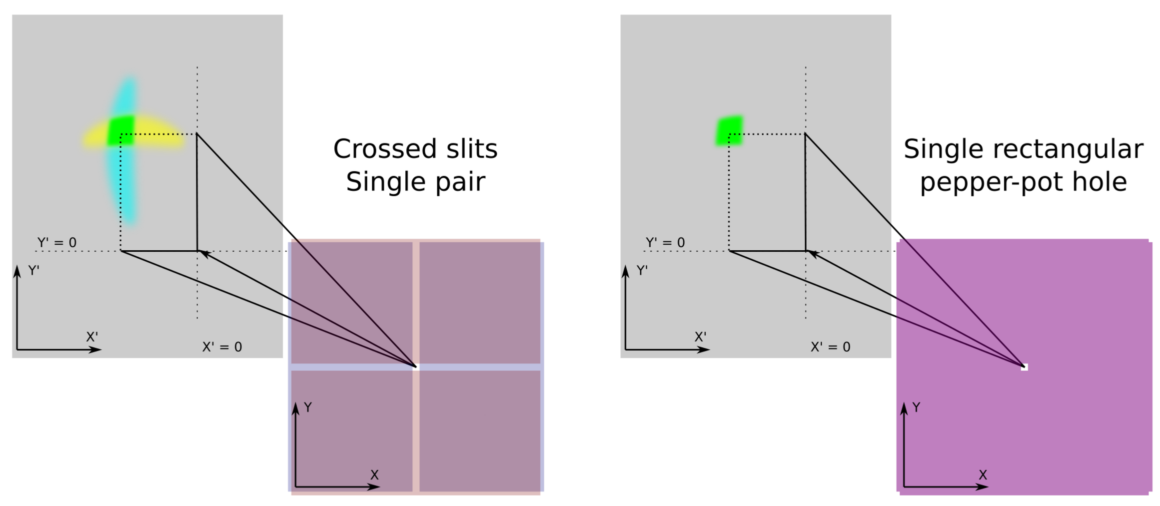
Virtual-Pepper Pot

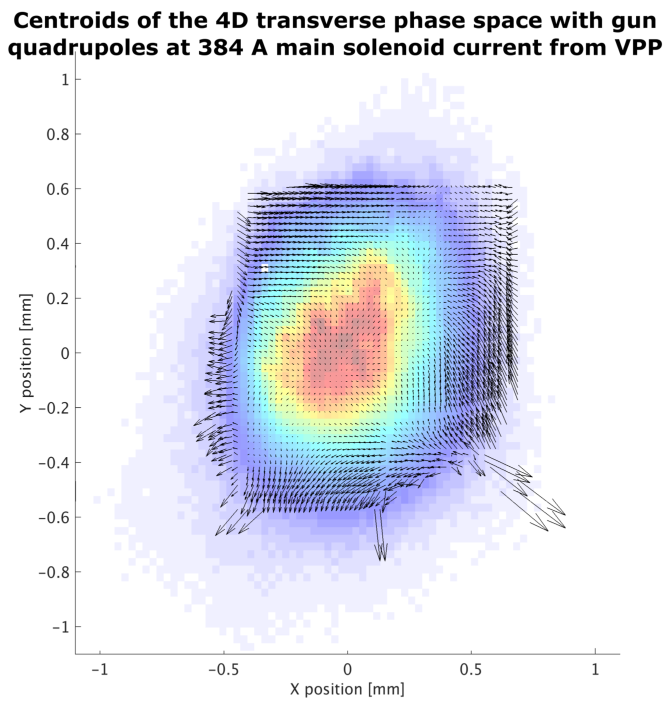
Experiment – horizontal and then vertical single slit scan



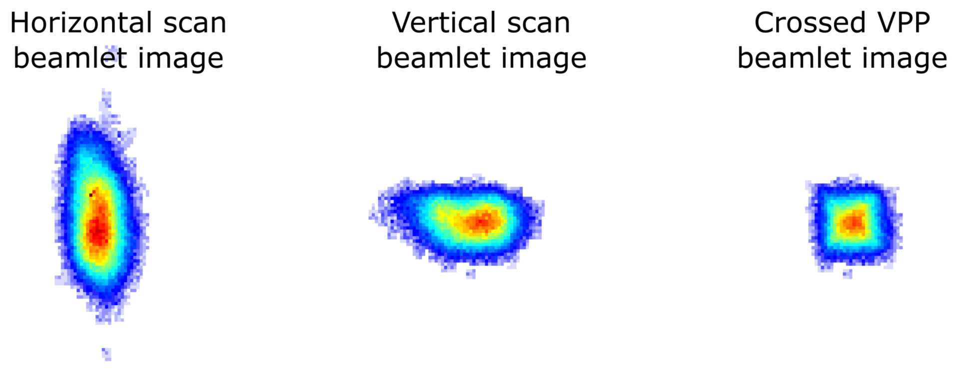


Postprocessing – beamlet crossing for virtual openings





Beamlet crossing

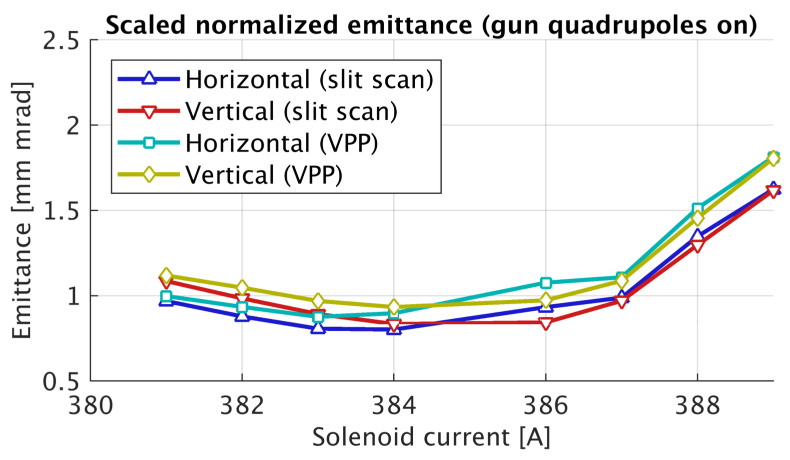


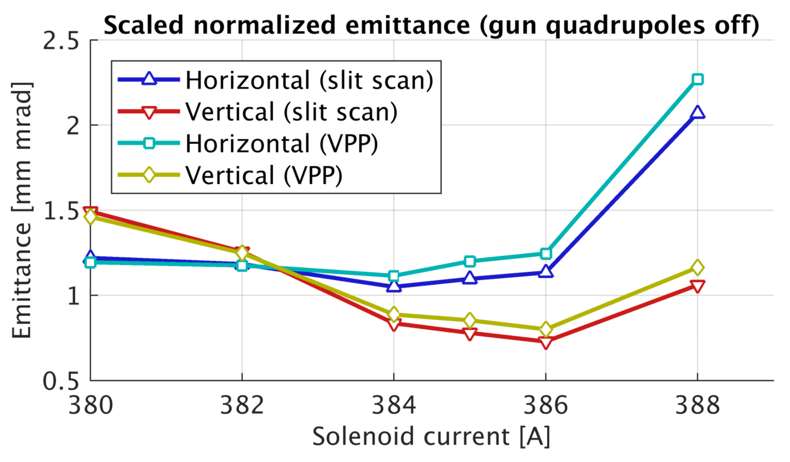
For any virtually crossed slit pair only a beam charge Q_0 passes through to the second screen. During the actual slit scan the Q_0 charge is mixed with the rest of the passing charge. The beamlet crossing step has to separate Q_0 of a pair of slits as much as possible. The pixel-wise minimum is the operation chosen for beamlet crossing.

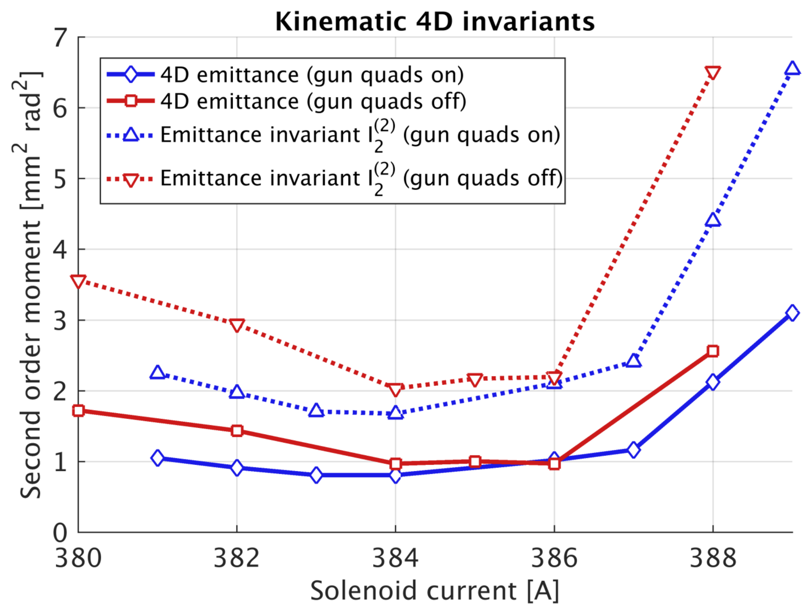
$$Q_{\text{cross}} = \min(Q_x, Q_y) = Q_0 + min(Q_{fx}, Q_{fy})$$

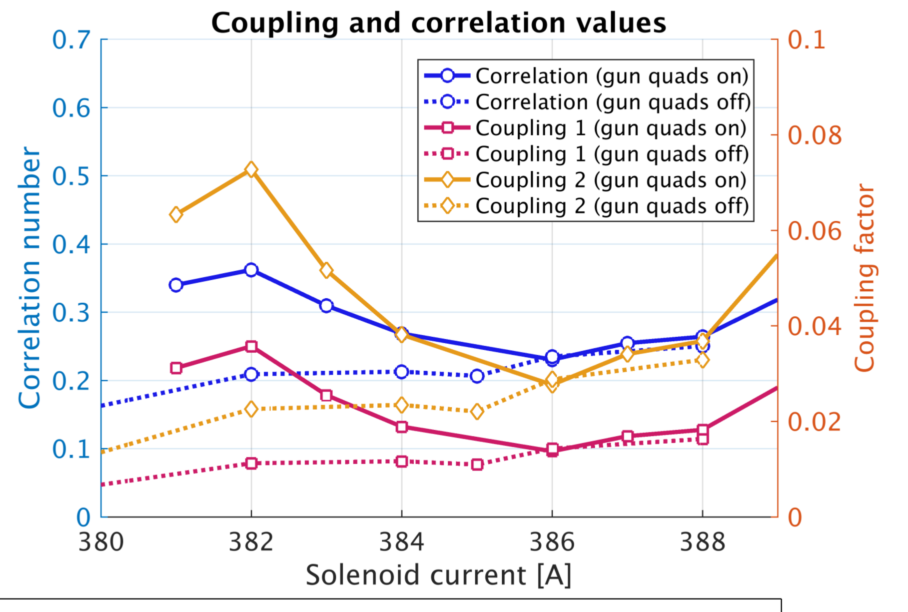
For any pair of beamlet images there are distinct Q_{fx} and Q_{fy}, referred to as foreign charges, of the corresponding vertical and horizontal slit image. The pixel-wise minimum operation minimizes the foreign charge for the crossing. Equivalently normalized beamlet images are required for this operation. The ratio of the 2D horizontal emittance to 2D vertical emittance from the slit scans is used as a reference point for the renormalization.

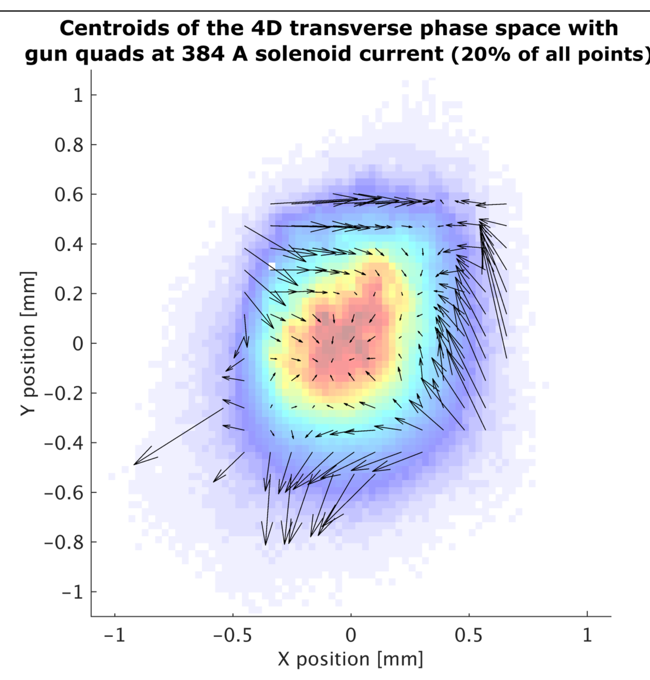
Results [1]

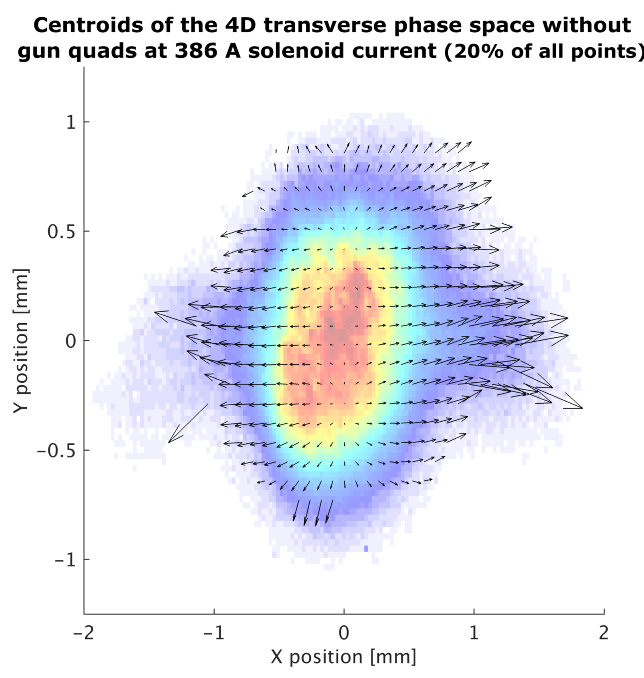












Conclusion

- 4D phase space dynamics observed
- Systematic error is present in projected emittance from VPP to single slit scan results
- Analysis of gun quadrupoles in this setup did not reveal coupling compensation, but 4D emittance reduction is obtained
- Further studies on charge cut effects on 4D phase space, 4D emittance, coupling terms are planned

References

[1] M. Krasilnikov *et al.*, "Electron beam asymmetry compensation with gun quadrupoles at PITZ".
[2] G. Rangarajan, F. Neri and A. Dragt, "Generalized emittance invariants".
[3] E. Prat and M. Aiba, "Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique".
[4] Q. Zhao *et al.*, "Beam Transverse Coupling and 4D Emittance Measurement Simulation Studies for PITZ".