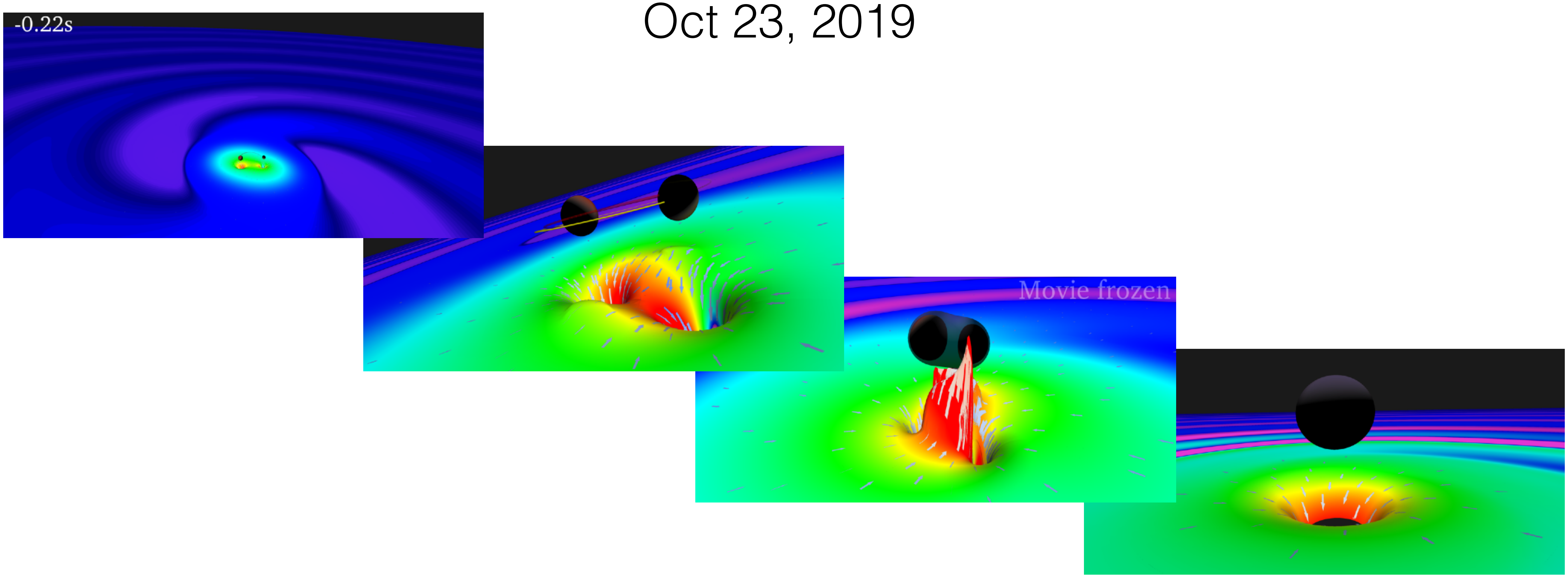
Numerical-relativity simulations of binary black holes in the 3G era

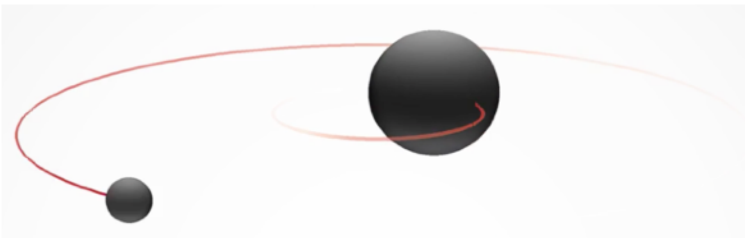
Harald Pfeiffer
AEI Potsdam

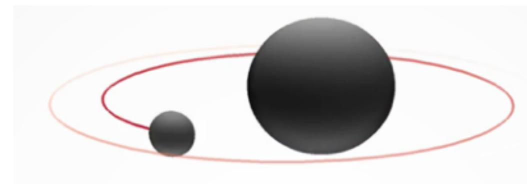
The Science of 3rd Generation GW Detectors
Berlin-Brandenburg Academy of Sciences & Humanities
Oct 23, 2019



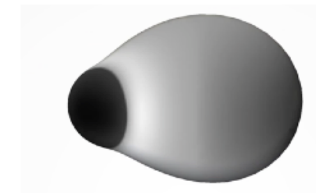
Methods for modeling BBH

Inspiral

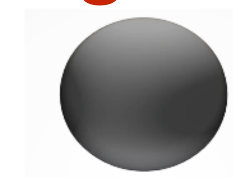




Merger



Ringdown



frequency

Methods for modeling BBH

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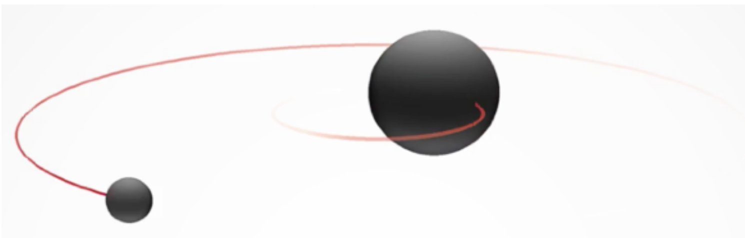
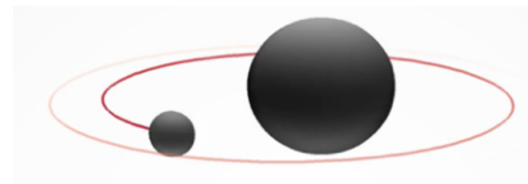
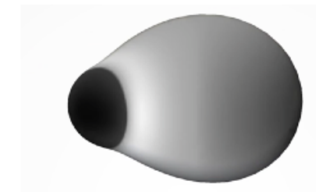
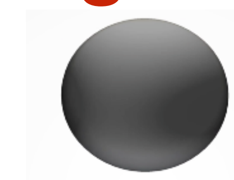
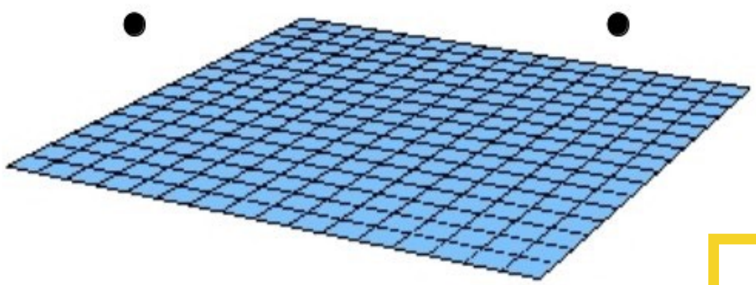










post-Newtonian theory

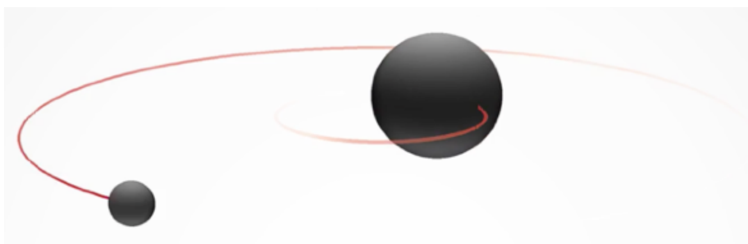
Perturbative expansion in v/c

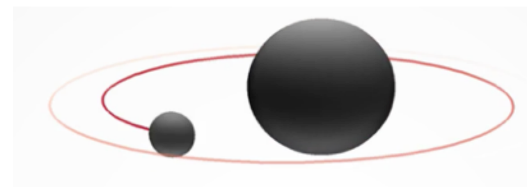
$$\begin{aligned}\phi = -\frac{x^{-5/2}}{32\nu}\Bigg\{ & 1 + \left(\frac{3715}{1008} + \frac{55}{12}\nu\right) x - 10\pi x^{3/2} \\ & + \left(\frac{15293365}{1016064} + \frac{27145}{1008}\nu + \frac{3085}{144}\nu^2\right) x^2 + \left(\frac{38645}{1344} - \frac{65}{16}\nu\right) \pi x^{5/2} \ln\left(\frac{x}{x_0}\right) \\ & + \left[\frac{12348611926451}{18776862720} - \frac{160}{3}\pi^2 - \frac{1712}{21}\gamma_\mathrm{E} - \frac{856}{21}\ln(16\,x) \right. \\ & \qquad \left. + \left(-\frac{15737765635}{12192768} + \frac{2255}{48}\pi^2\right)\nu + \frac{76055}{6912}\nu^2 - \frac{127825}{5184}\nu^3\right] x^3 \\ & + \left(\frac{77096675}{2032128} + \frac{378515}{12096}\nu - \frac{74045}{6048}\nu^2\right) \pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\},\end{aligned}$$

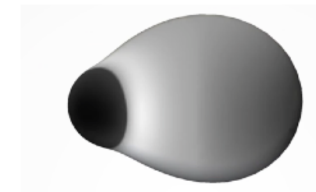
Blanchet, Living Reviews

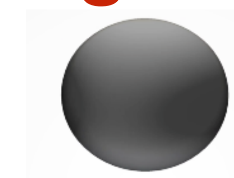
Methods for modeling BBH

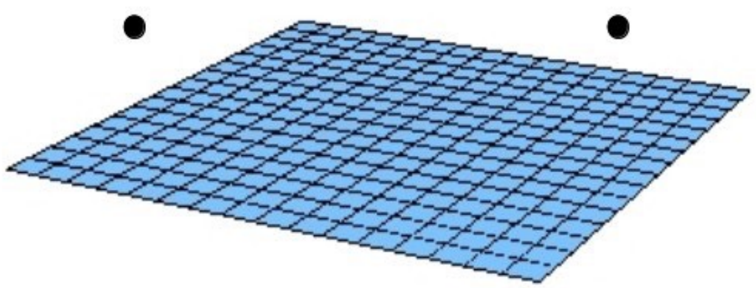
Inspiral **Merger** **Ringdown**



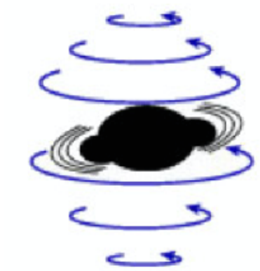




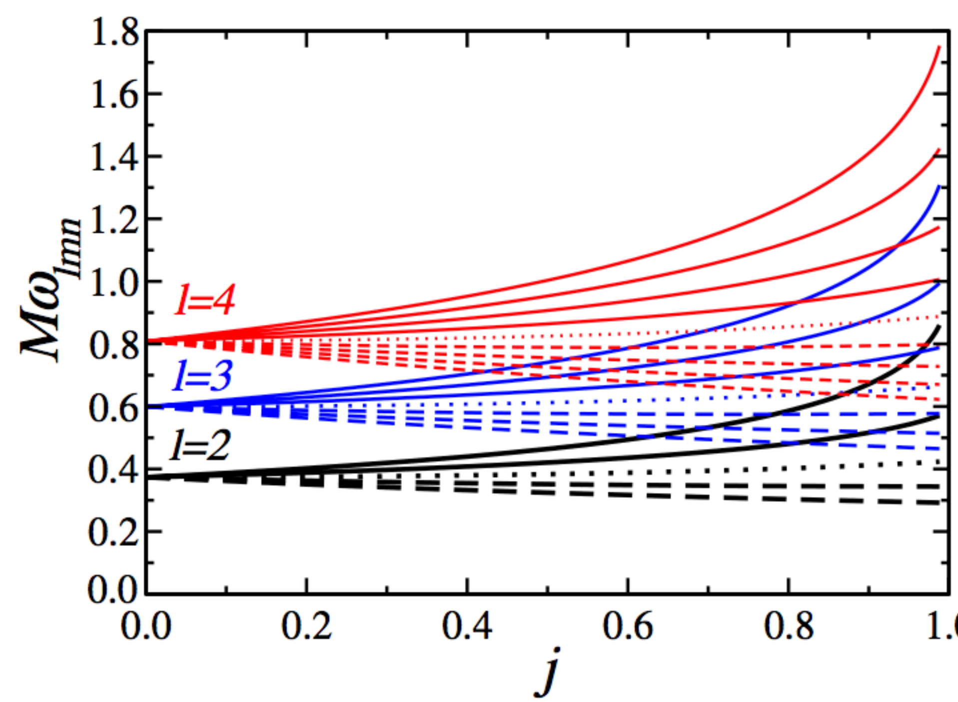




post-Newtonian theory



BH perturbation theory



Berti+ 06

Methods for modeling BBH

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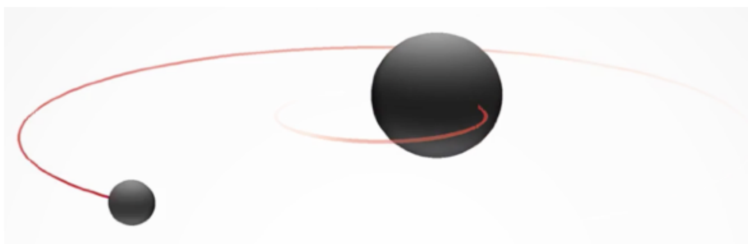
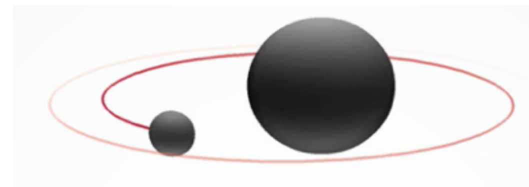
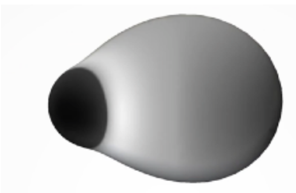
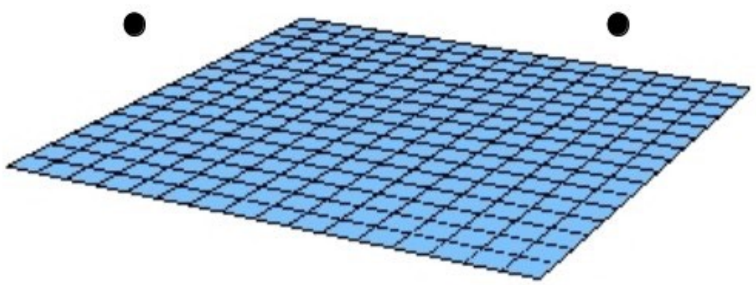


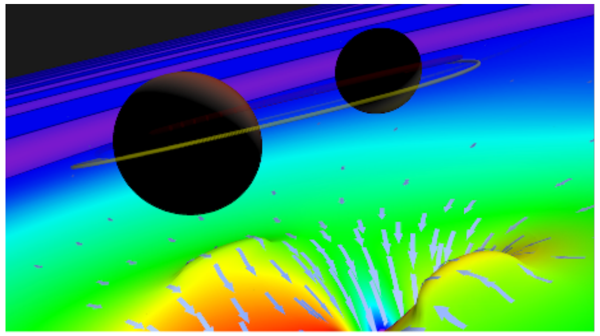


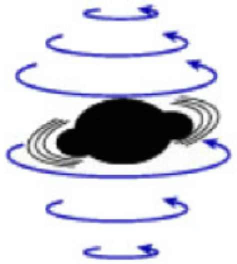




post-Newtonian theory

Numerical Relativity

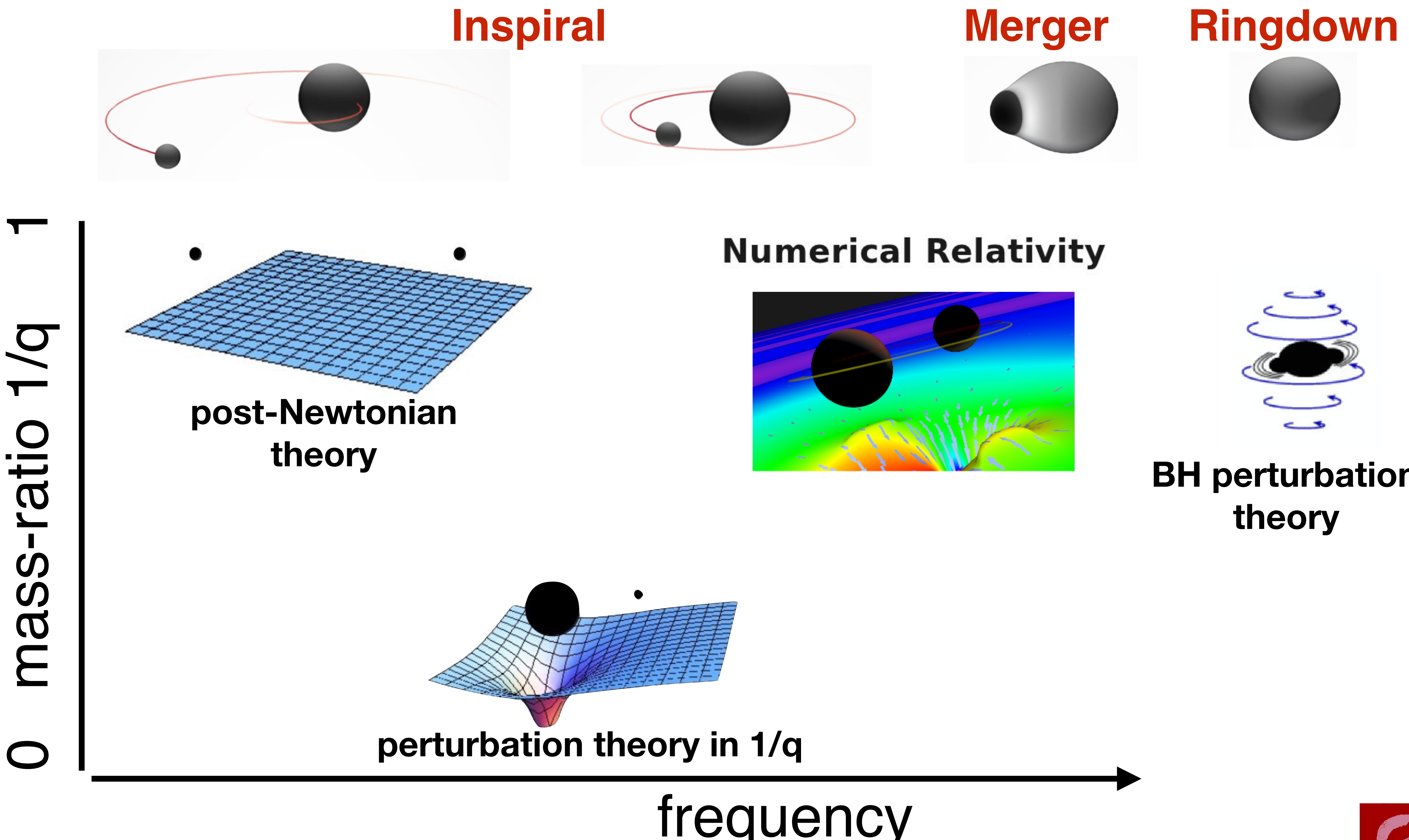




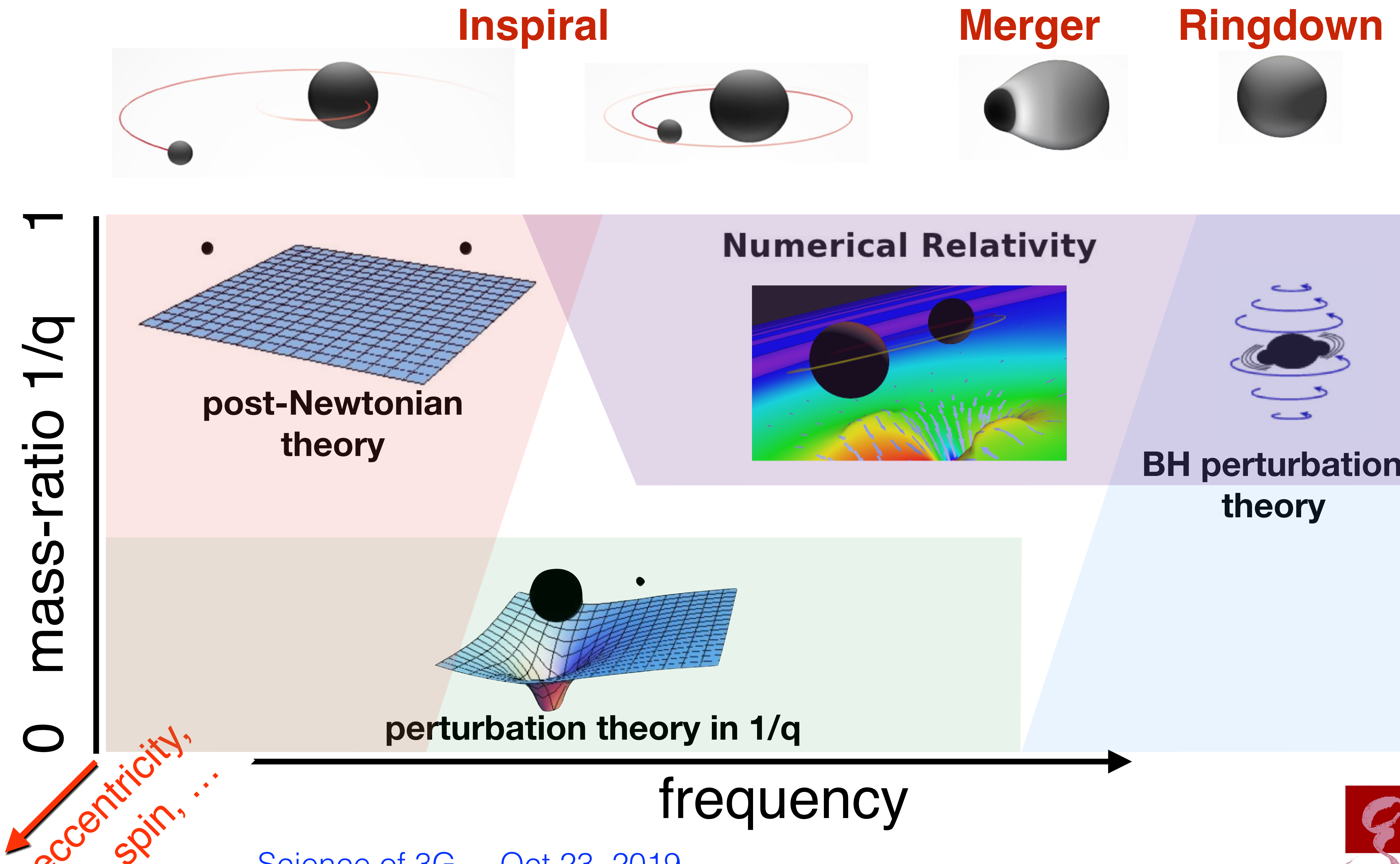
BH perturbation theory

frequency

Methods for modeling BBH



Methods for modeling BBH



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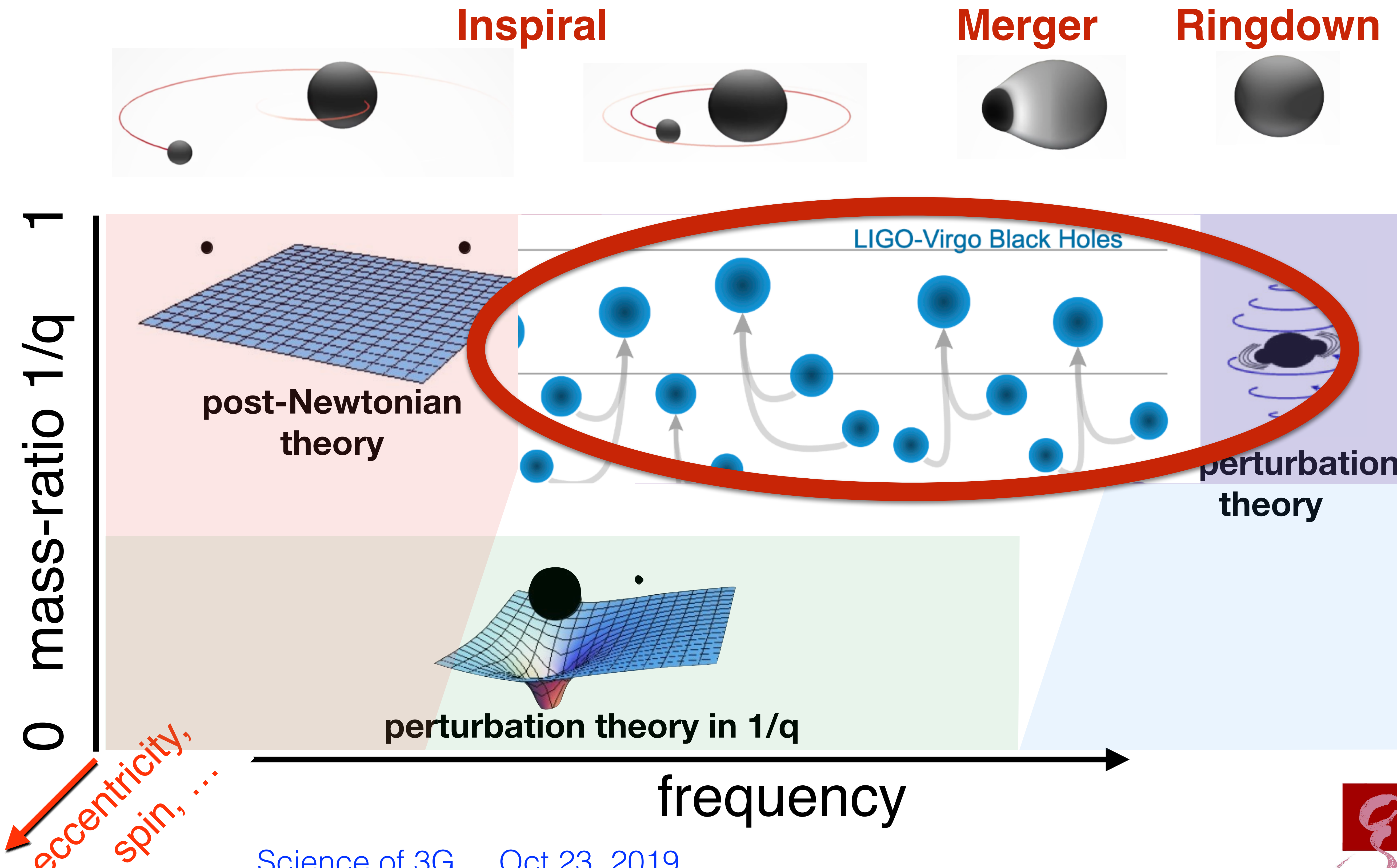
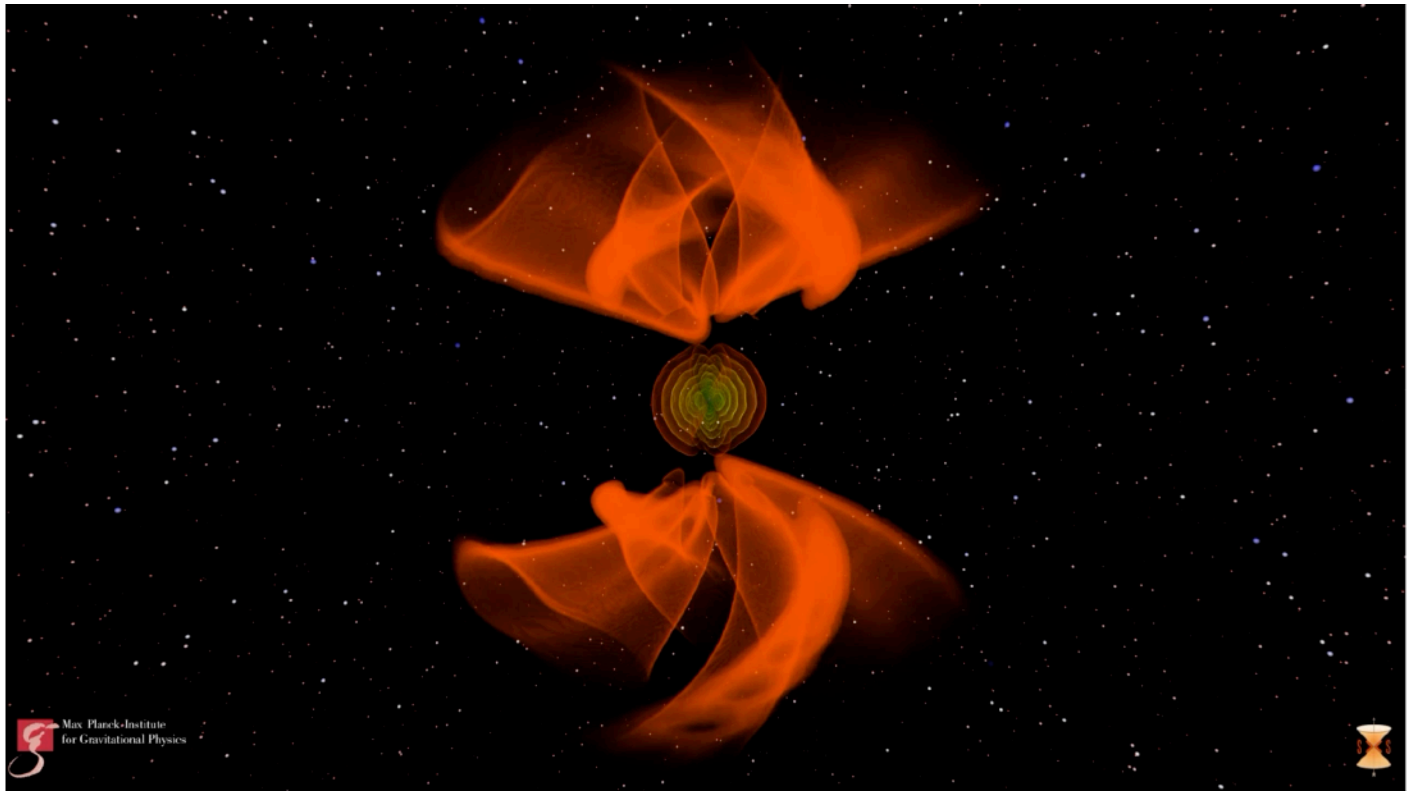


Role of NR



- **Solution of GR**
  for late inspiral + merger
- Provide **error estimates**
- Determine **regions of validity** of perturbative methods
  - all available perturbation orders needed **for science** (3.5PN, 2SMR). No extra order for error estimate

⇒ complete waveform models for GW data-analysis

➔ Alessandra's talk

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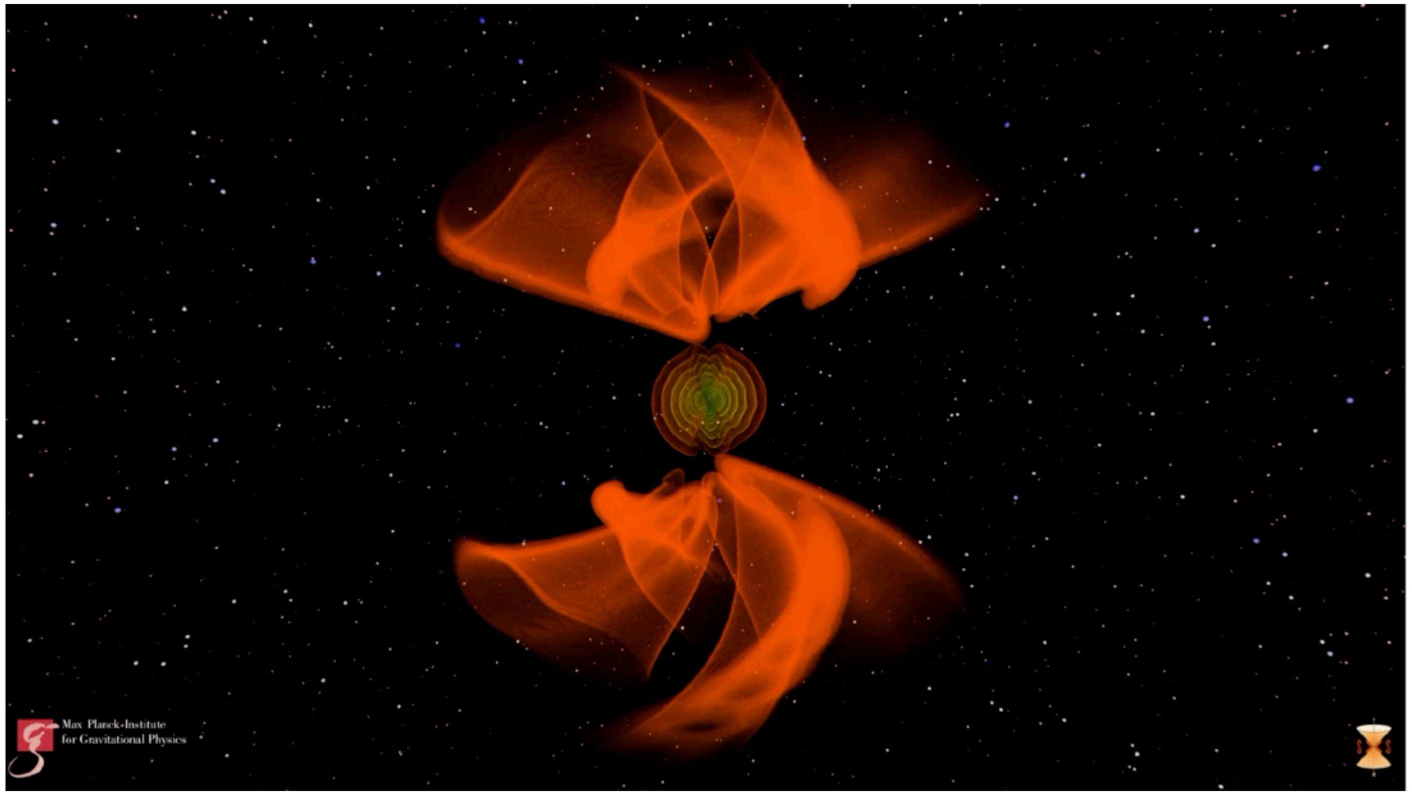


⇒ complete waveform models for GW data-analysis

➔ Alessandra's talk

The very beginning

ANNALS OF PHYSICS: **29**, 304–331 (1964)

The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

International Business Machines Corporation, New York, New York

AND

RICHARD W. LINDQUIST

The numerical calculations were carried out on an IBM 7090 electronic computer. The parameters a and μ_0 were both set equal to unity; the mesh lengths were assigned the values $h_1 = 0.02$, $h_2 = \pi/150 \approx 0.021$, yielding a 51 × 151 mesh. The calculations of all unknown functions, including a great number of input-output operations and some built-in checking procedures, took approximately four minutes per time step. Different check routines indicated that results close to the point $\mu = 0$, $\eta = 0$ lost accuracy fairly quickly. Since these would, in the long run, influence meshpoints further away, the computations were stopped after the 50th time step, when the total time elapsed was approximately 1.8. Some of the results are shown in Table I.

The first 50 Years of numerical relativity for BBH

1962 *ADM*
3+1 formulation

1964
Hahn-Lindquist
2 wormholes

1984
Unruh
excision

1992,3
Choptuik;
Abrahams+Evans
critical phenomena

1997
Brandt-
Brügmann
puncture data

1994-98
BBH Grand Challenge

1999-00
AEI/PSU
grazing collisions

2000-04
AEI/UTB-NASA
revive crashing
codes (Lazarus)

~2000 *Choptuik;*
Schnetter;Brügmann
mesh refinement

2005
Gundlach ea
constraint damping

2005 *Pretorius*
inspiral-merger-
ringdown (IMR)
w/ harmonic

2005-06
Campanelli+; Baker+
IMR w/ BSSN &
moving punctures

2006-08
Scheel..HP+ *SXS*
IMR w/ spectral

2007-
Ajith, AEI, Jena
phenom GW models

2009-
UMD, SXS
EOB GW models

2011
Schmidt ea;
Boyle ea
Radiation aligned
frame

2011
Lousto ea
q=100

2014-
precessing
GW models

2015
Szilagyi ea
175 orbits

1964 ~1999 ~2005 2015

1975-77
Smarr-Eppley
head-on
collision

1979 *York*
kinematics and
dynamics of GR

1989-95
Bona-Masso
modified ADM,
(hyperbolicity)

1994 *Cook*
Bowen-York
initial data

1994-95
NCSA-WashU
improved
head-on collision

1999
BSSN
evolution
system

1999 *York*
conformal thin
sandwich ID

1999-2005 *York,*
Cornell, Caltech, LSU
hyperbolic formulations

2000-02
Alcubierre
gauge conditions

2004
Brügmann ea
one orbit

2003-08
Cook, Pfeiffer ea
improved ID

2000 *Ashtekar*
isolated horizons

2006,07
Baker ea;
Gonzalez ea
non-spinning BBH
kicks

2007 *SXS*
PN-NR
comparison

2007-11
RIT; Jena; AEI;…
BBH superkicks

2008
all of NR
NINJA

2009-11
Bishop, …
Cauchy
characteristic
extraction

2010
Bernuzzi ea
C4z

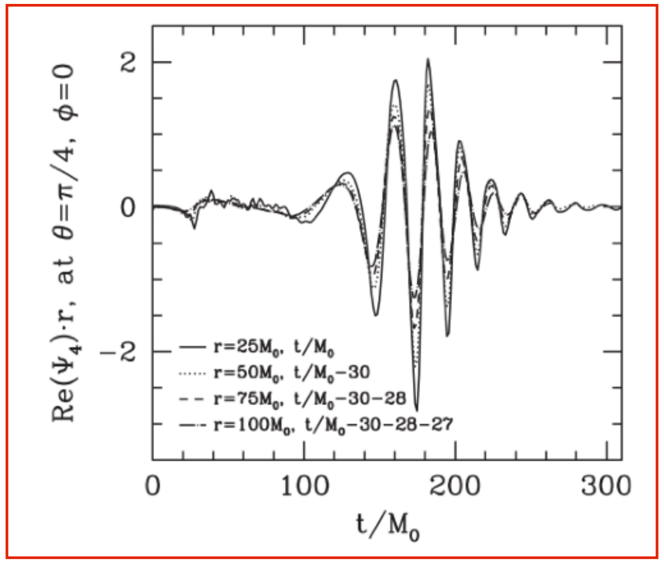
2011
Lovelace ea
S/M^2=0.97

2011-
Le Tiec ea
self-force studies

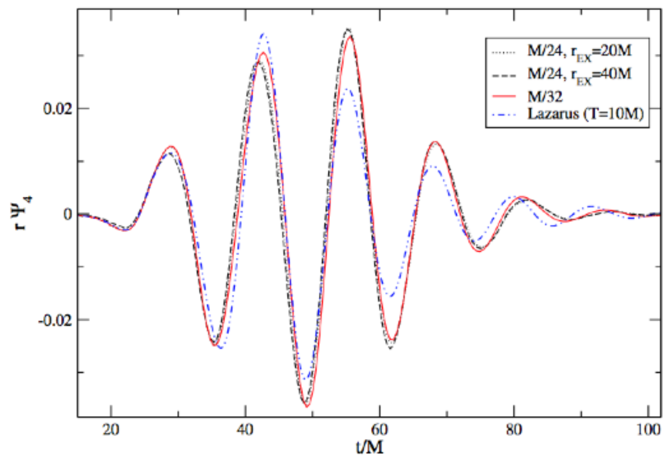
2013
GaTech; SXS
Precessing
parameter
studies

Courtesy Carlos Lousto,
updated by HP

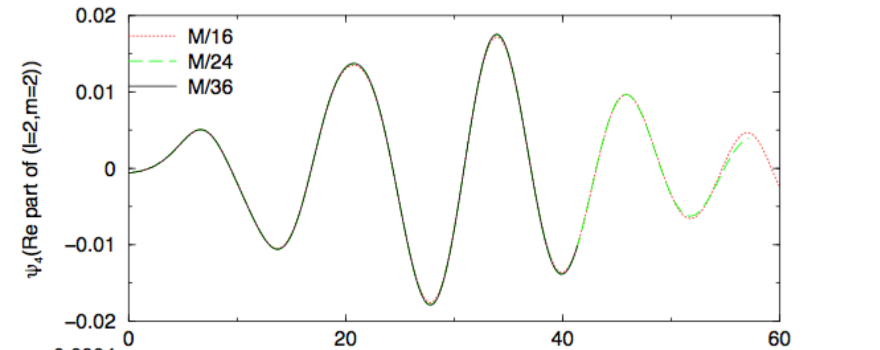
2005: First working BBH inspirals



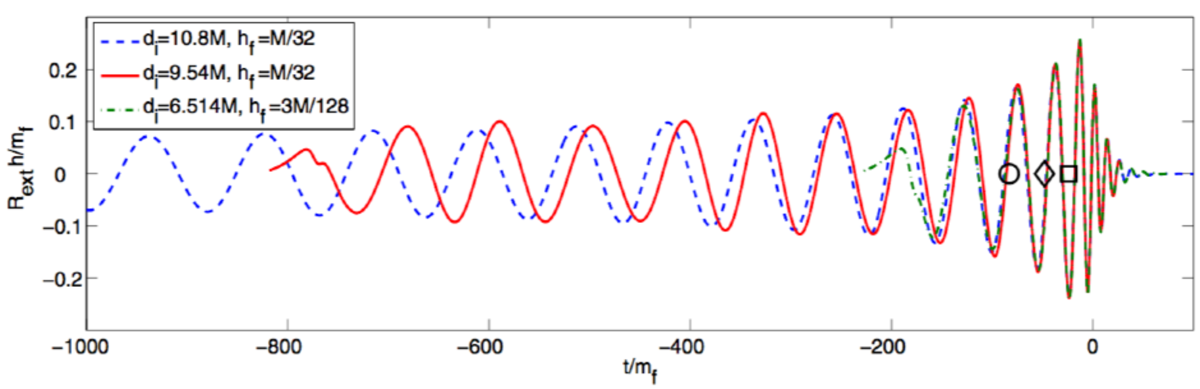
Pretorius 05



Baker+06



Campanelli+06



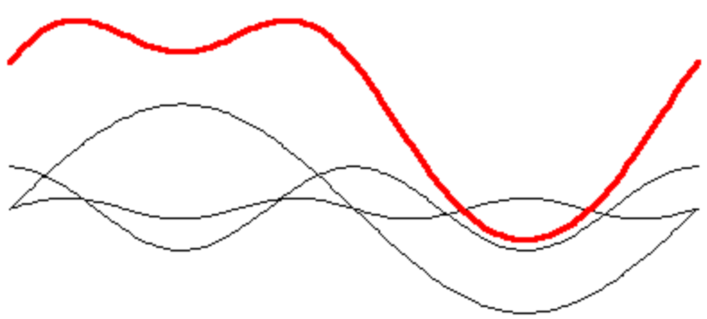
Baker+07

Important early result:
Simplicity of merger
Continuous transition
inspiral → ringdown

Spectral Einstein Code (SpEC)

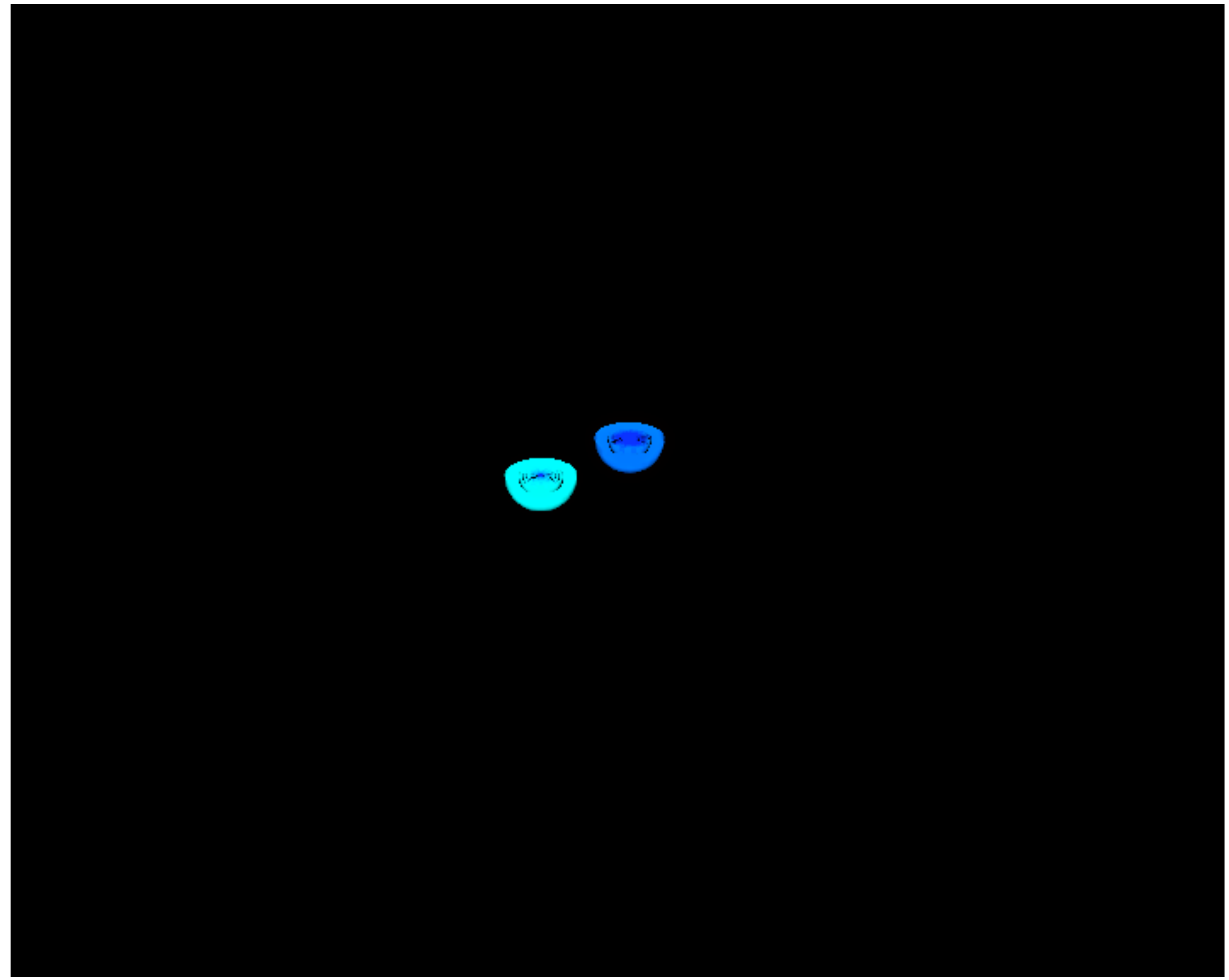
- Expand in **basis-functions**

$$u(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x)$$



- Compute derivatives **analytically**

$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x)$$



Simulations of Extreme Spacetimes (SXS) collaboration

http://www.black-holes.org/SpEC.html

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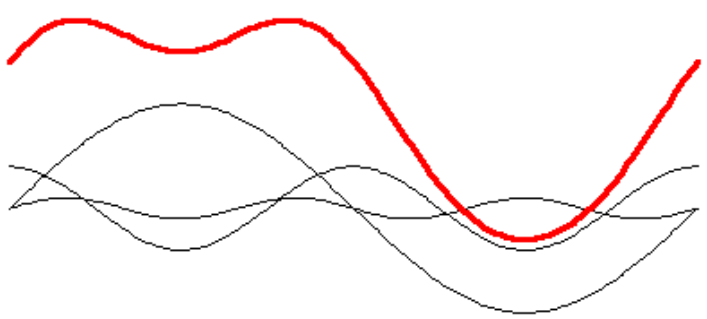

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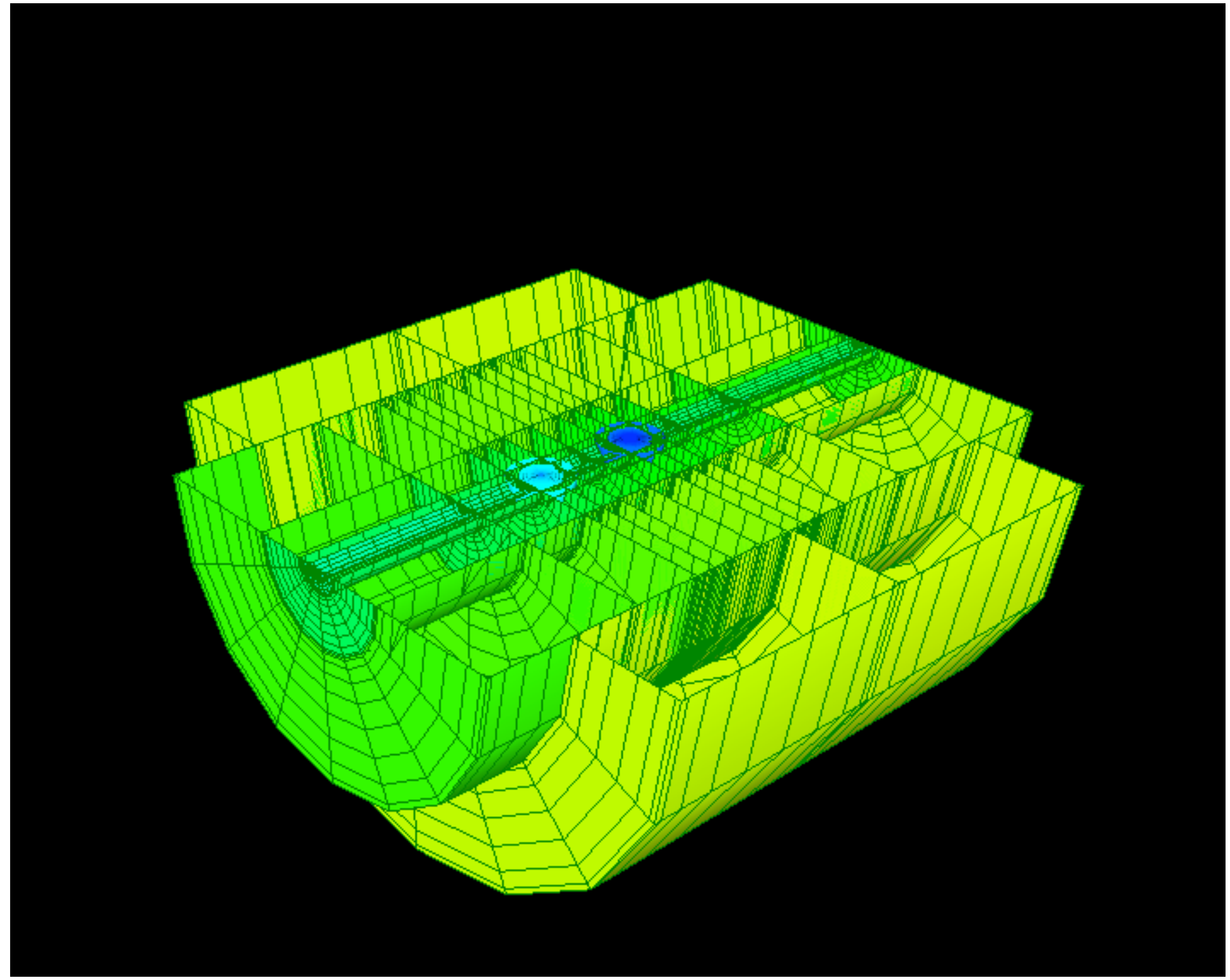

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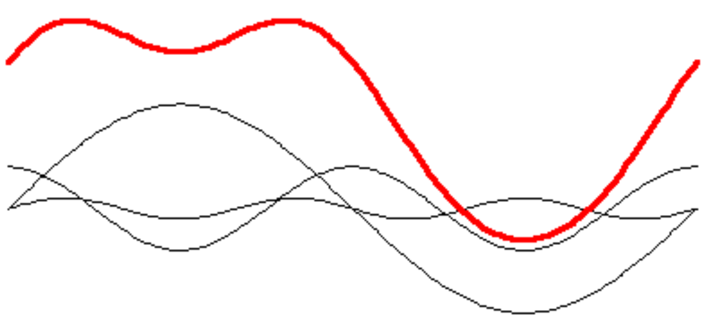

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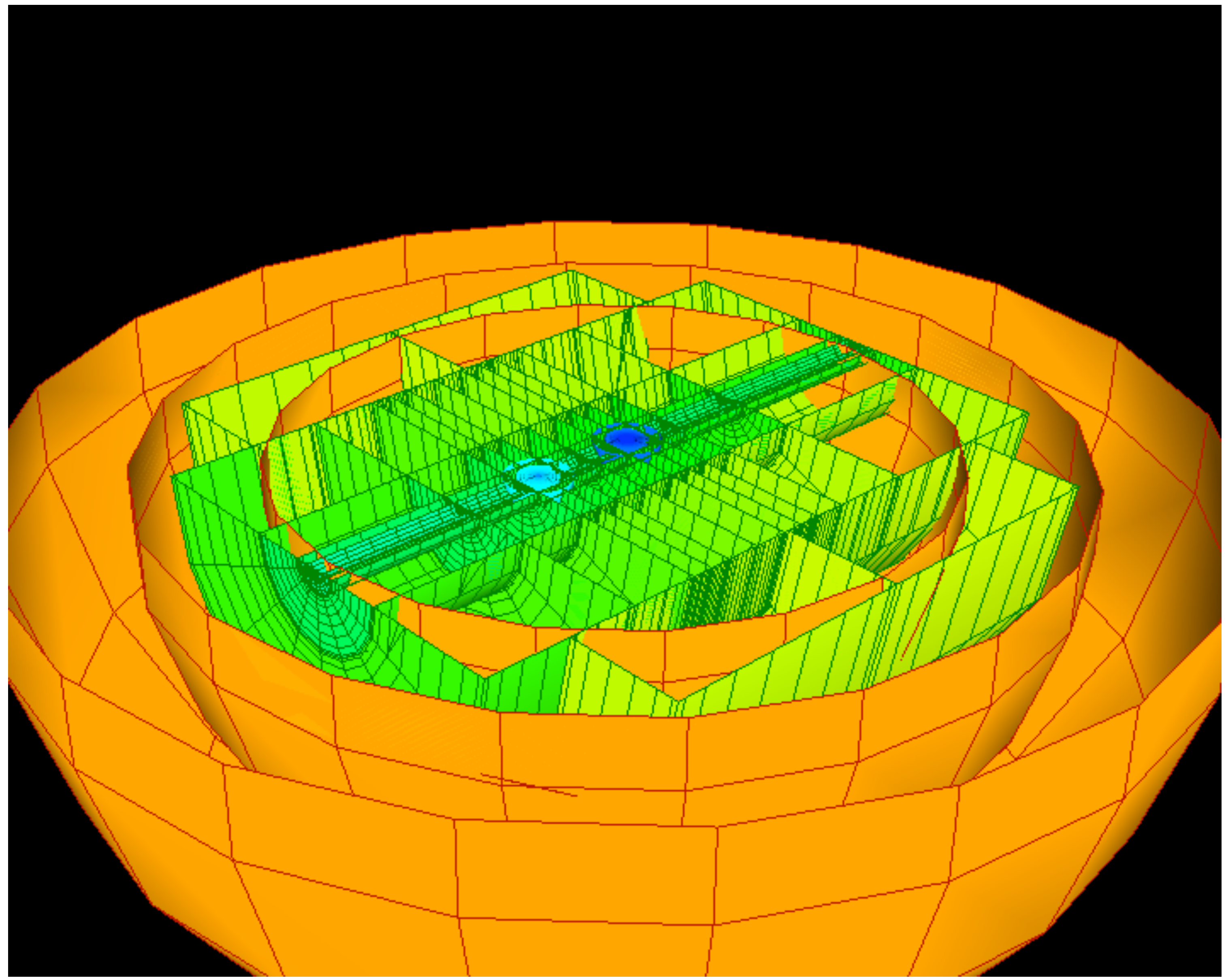

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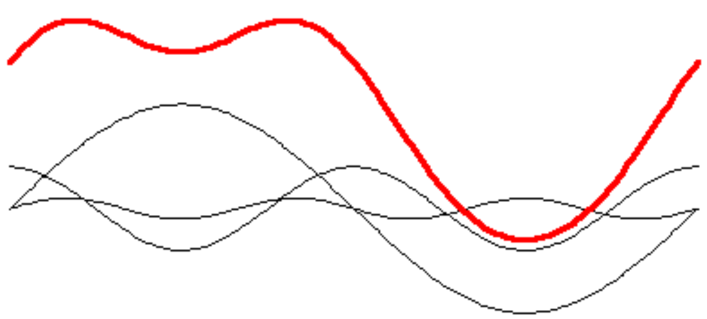

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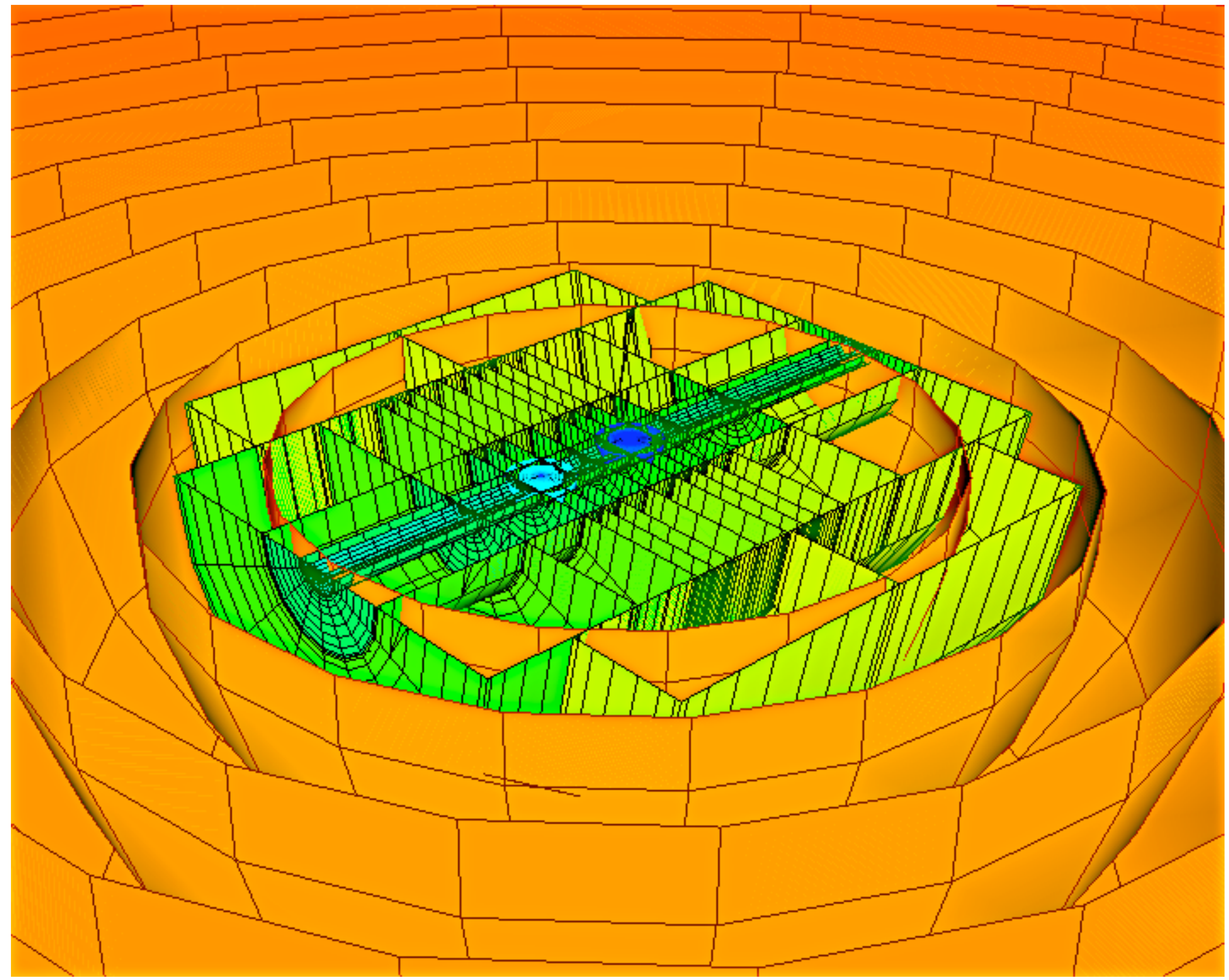

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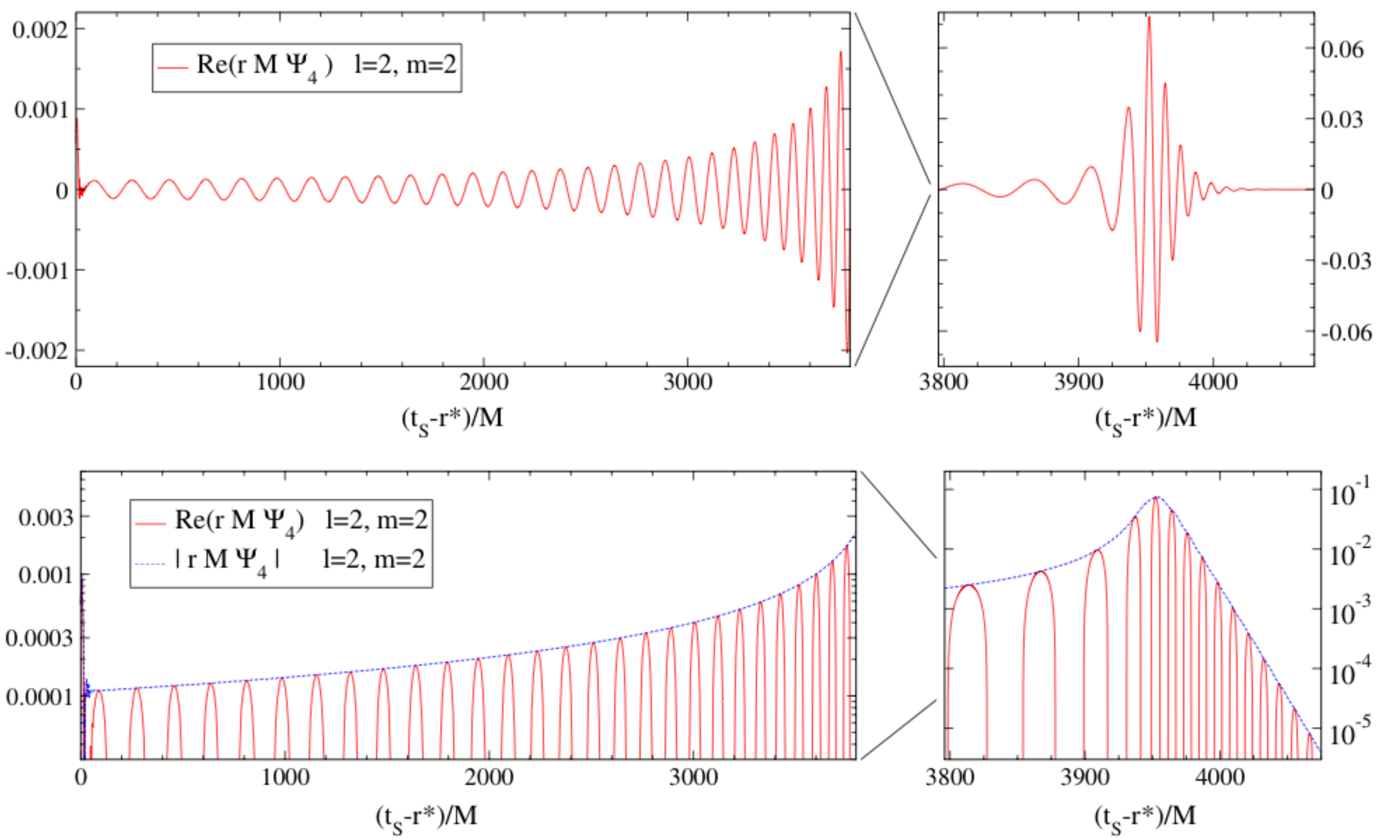
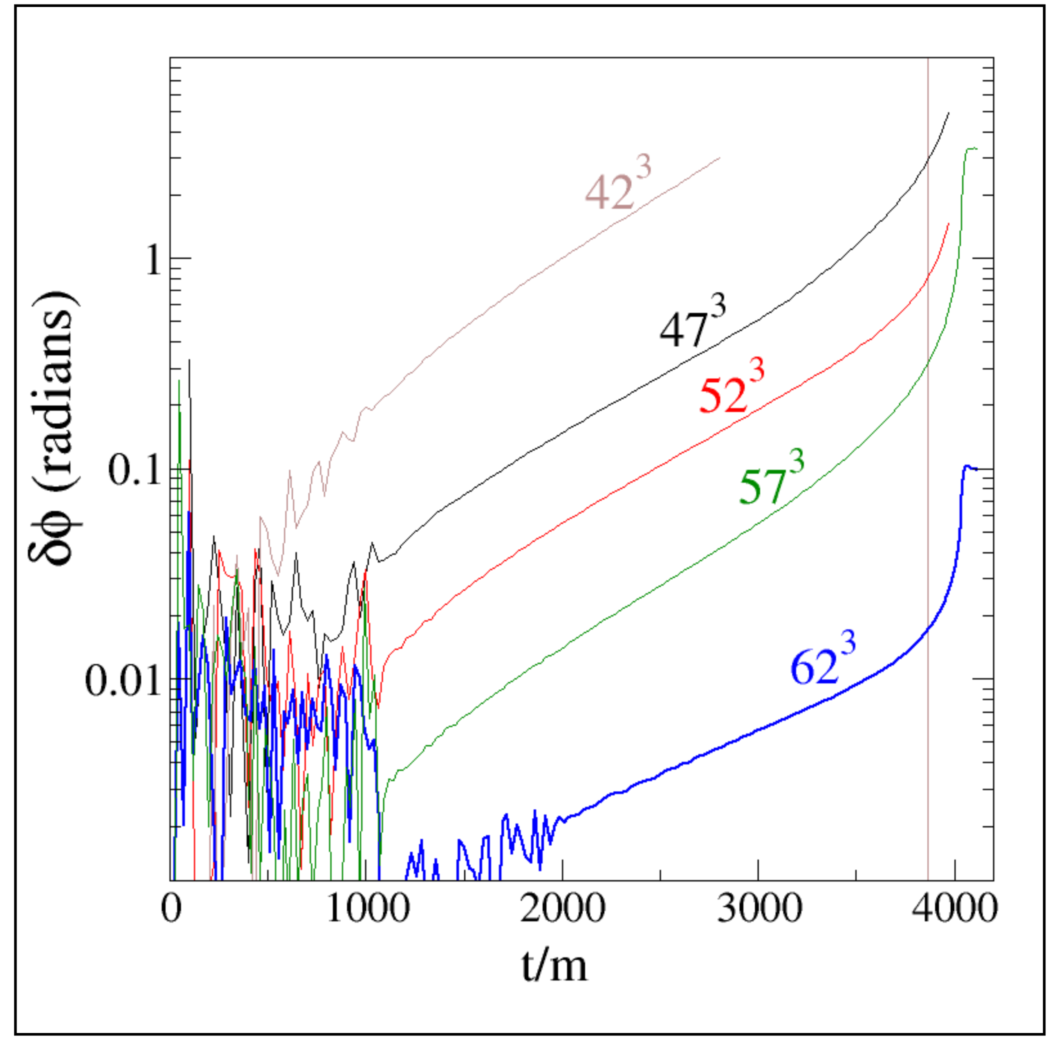
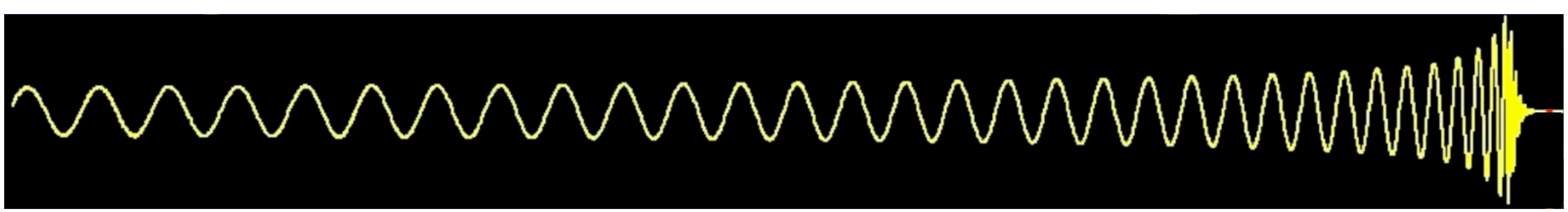
Accuracy of SpEC

GW precision data







Scheel,HP+ 09

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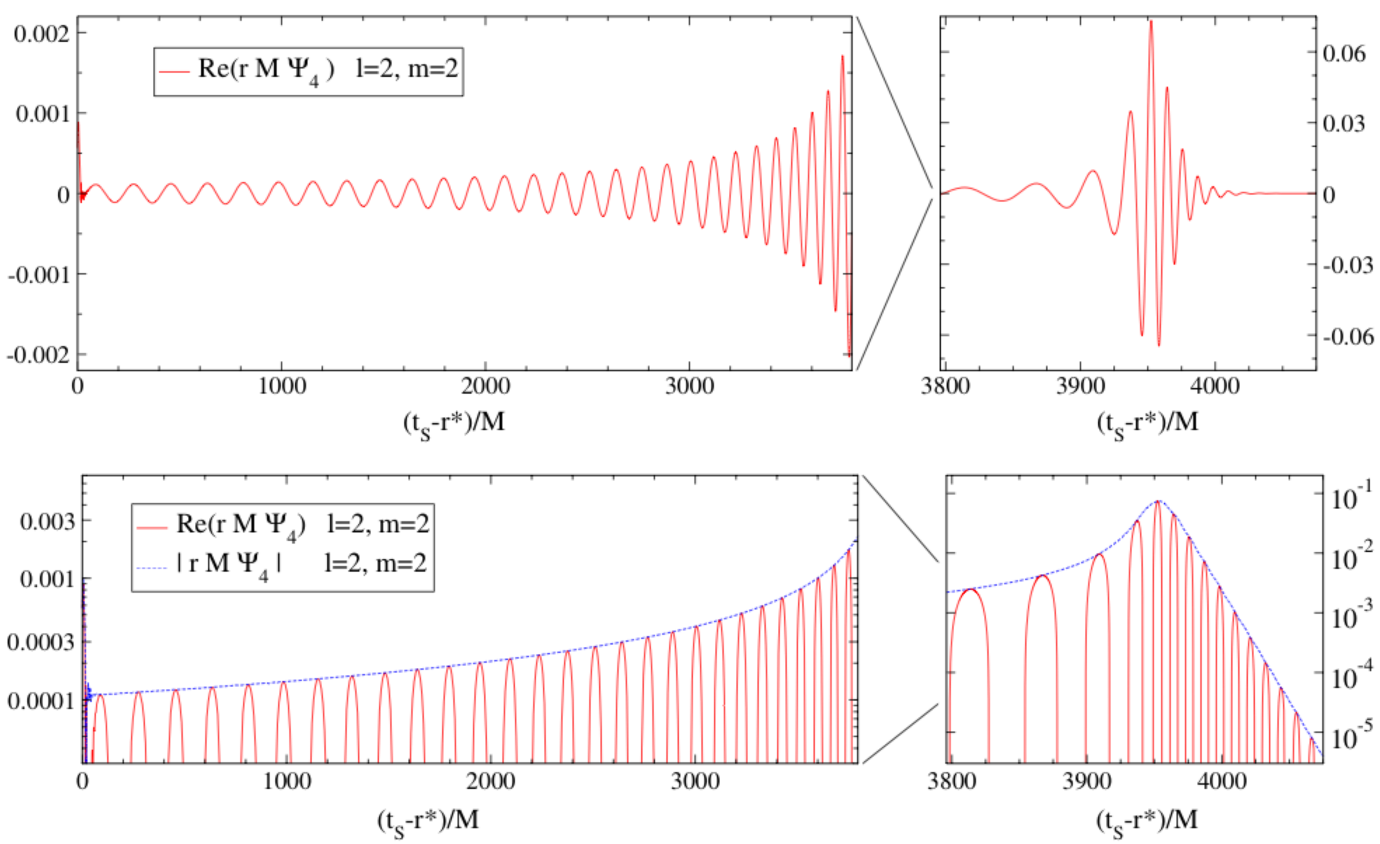
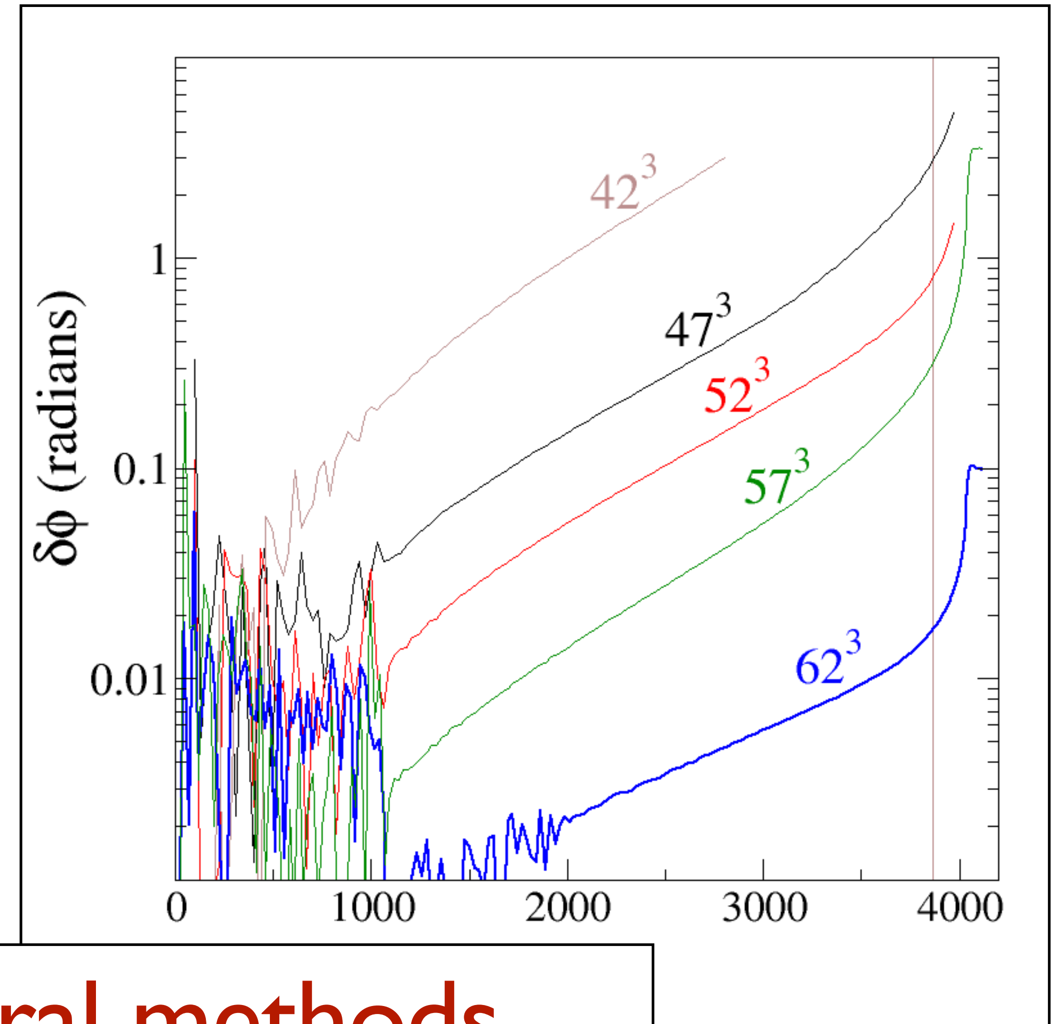
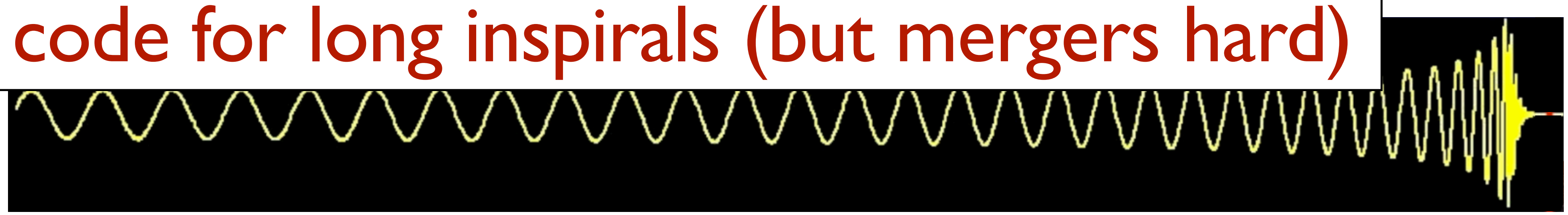


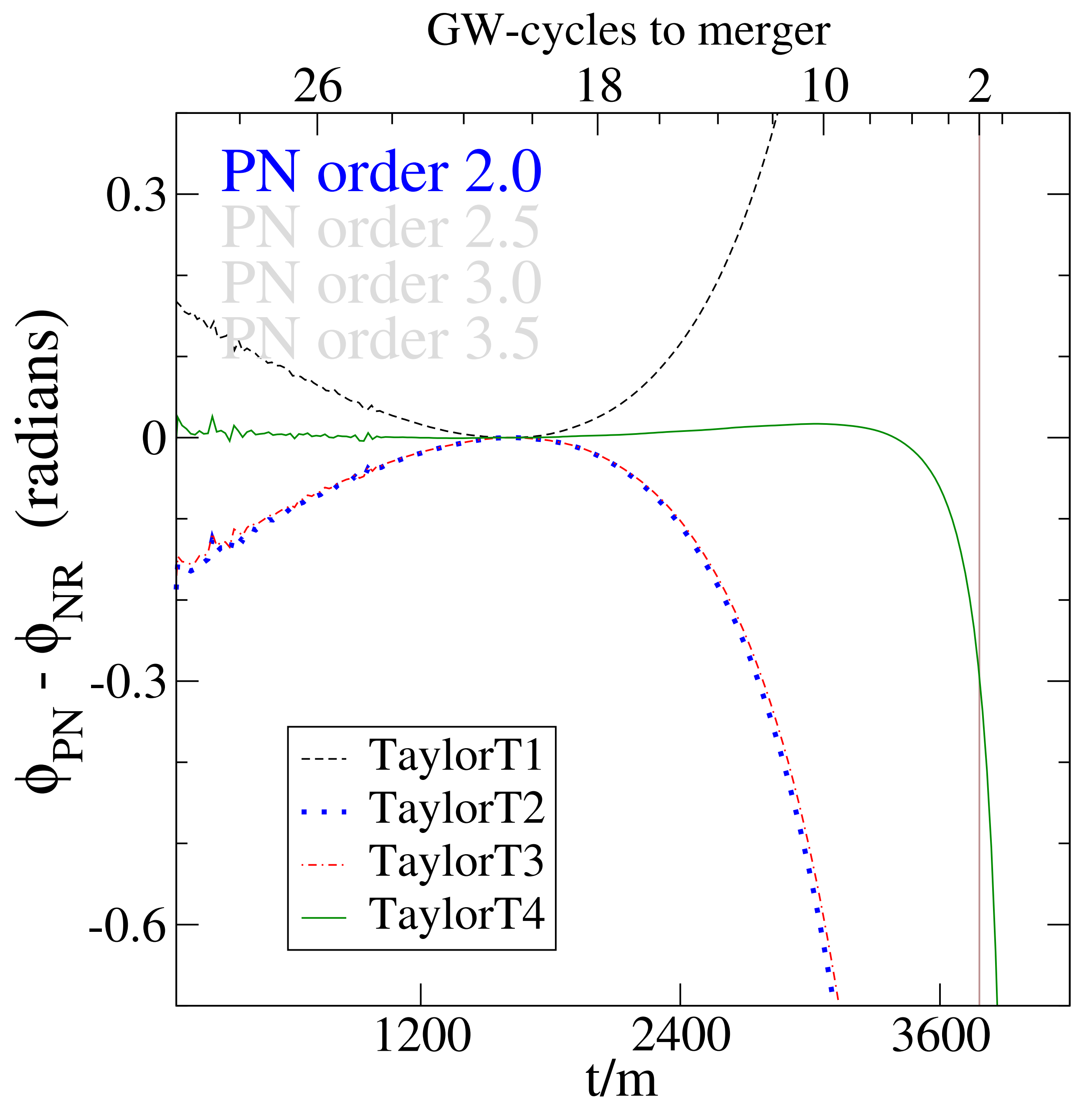


- Rapid convergence due to spectral methods
- Small errors due to moving grid
- Best code for long inspirals (but mergers hard)



Scheel,HP+ 09

post-Newtonian vs. NR

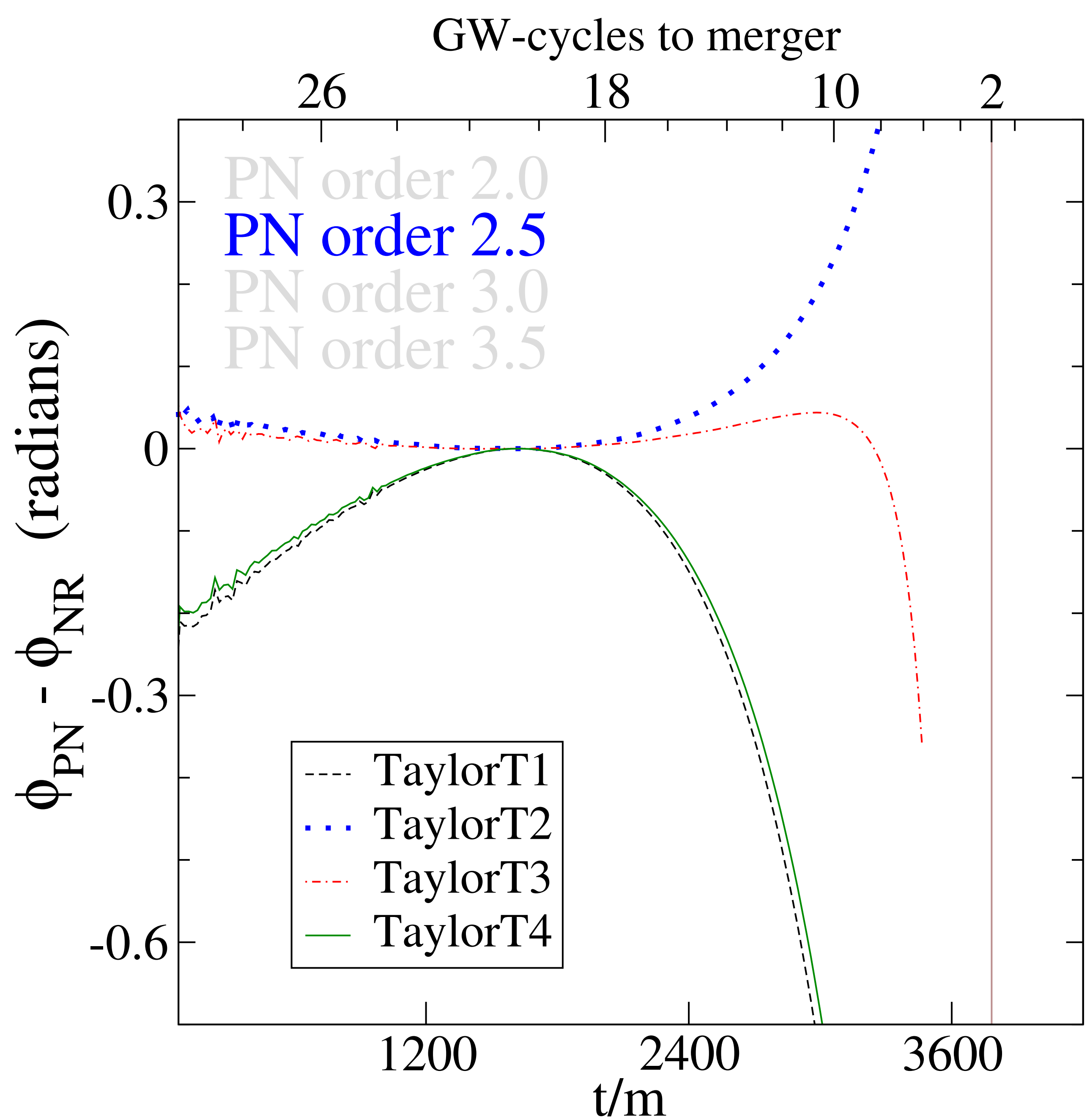


Boyle..HP+ 07

PN approximants
Equally justified approaches to derive inspiral rate from energy balance

$$\frac{dE}{dt} = -F_{\mathrm{GW}}$$

post-Newtonian vs. NR



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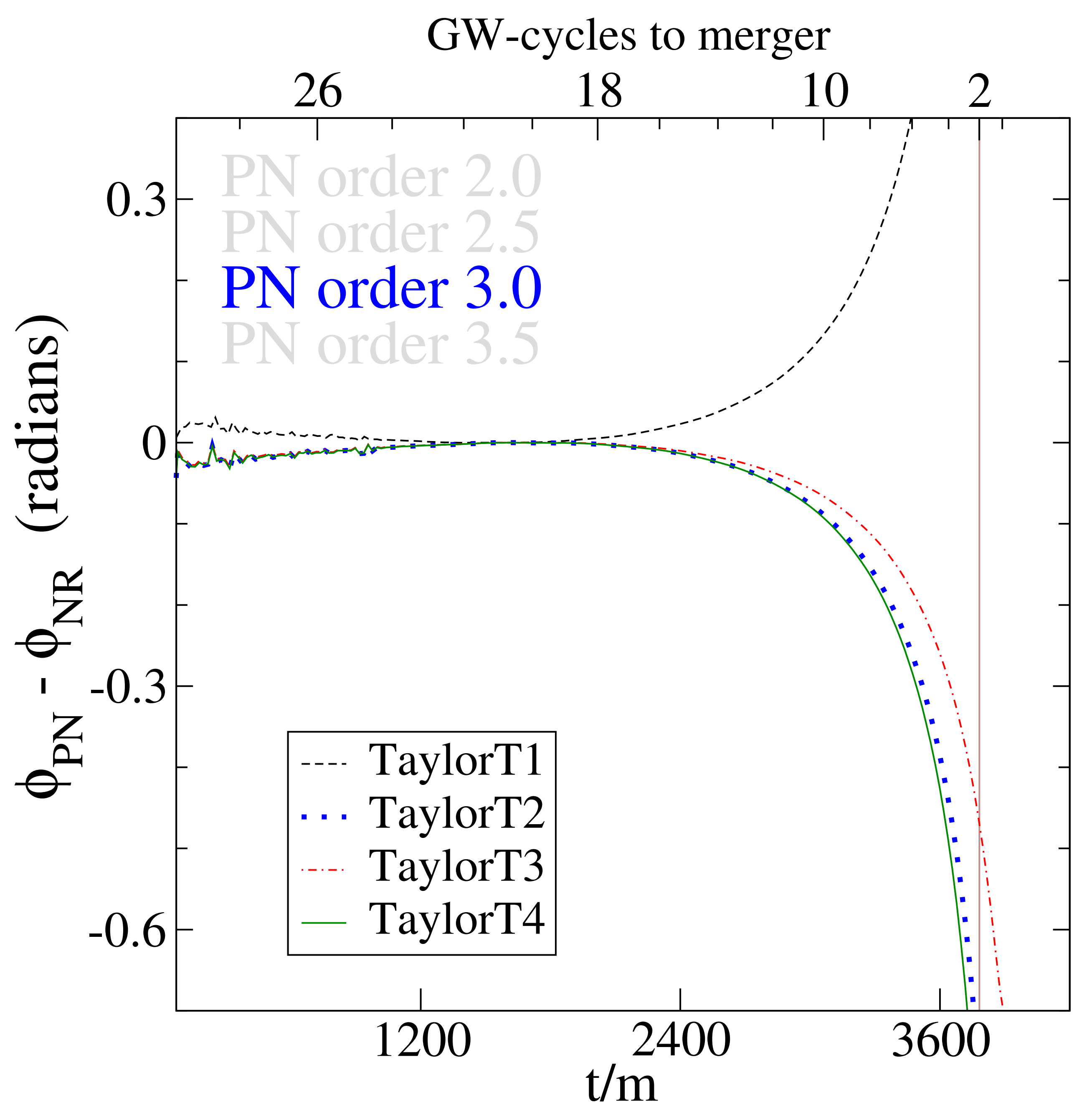

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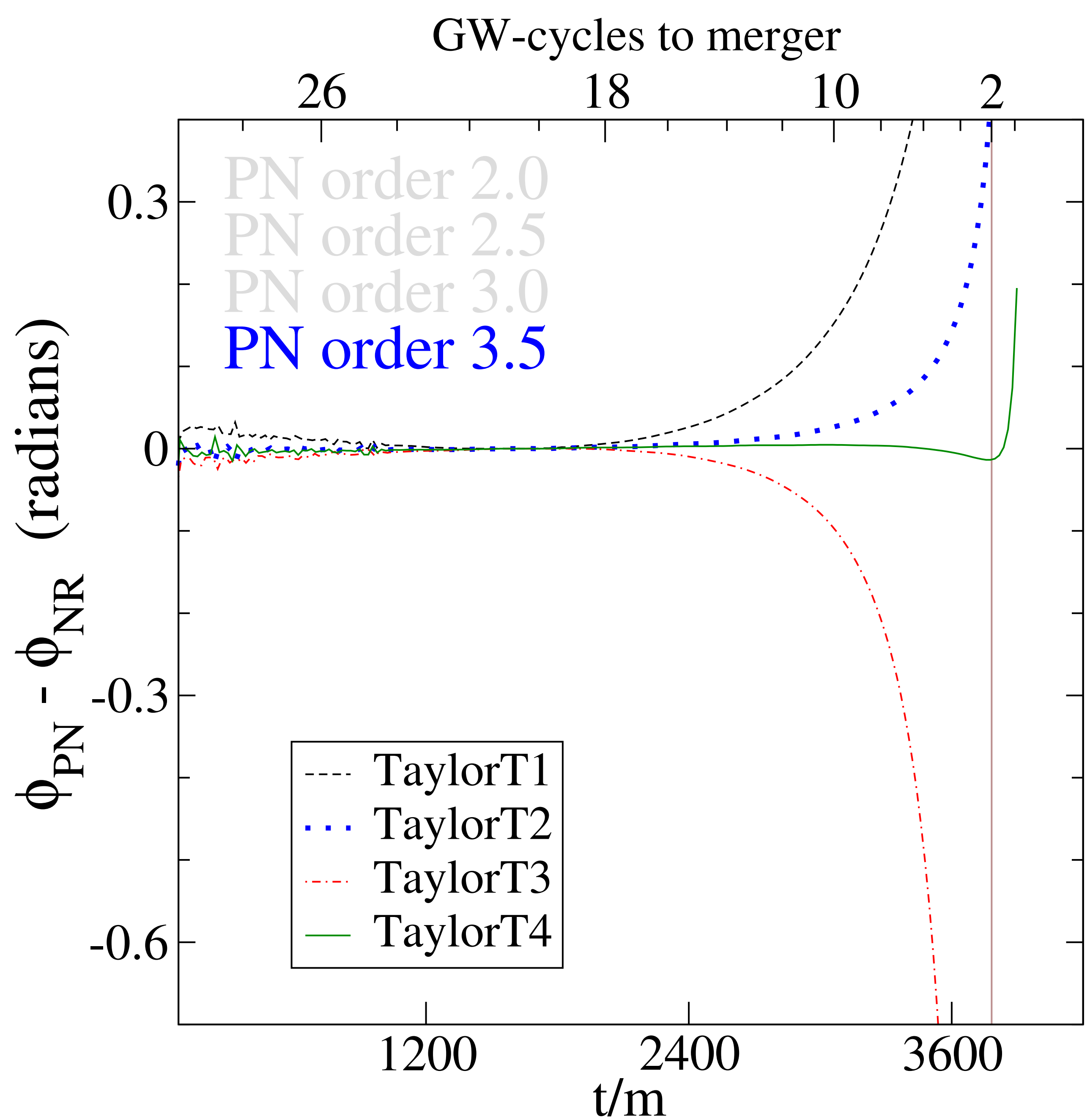

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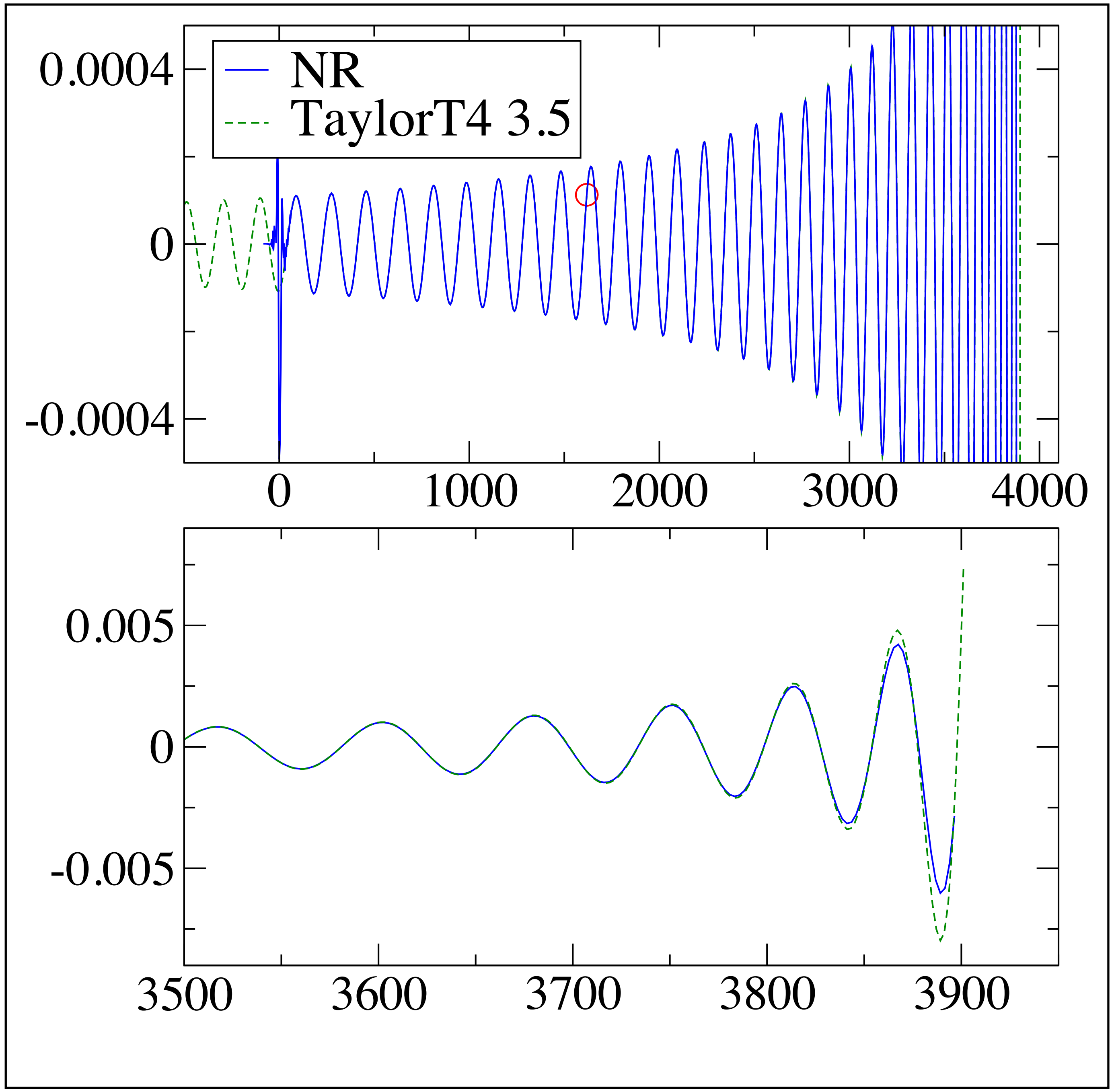
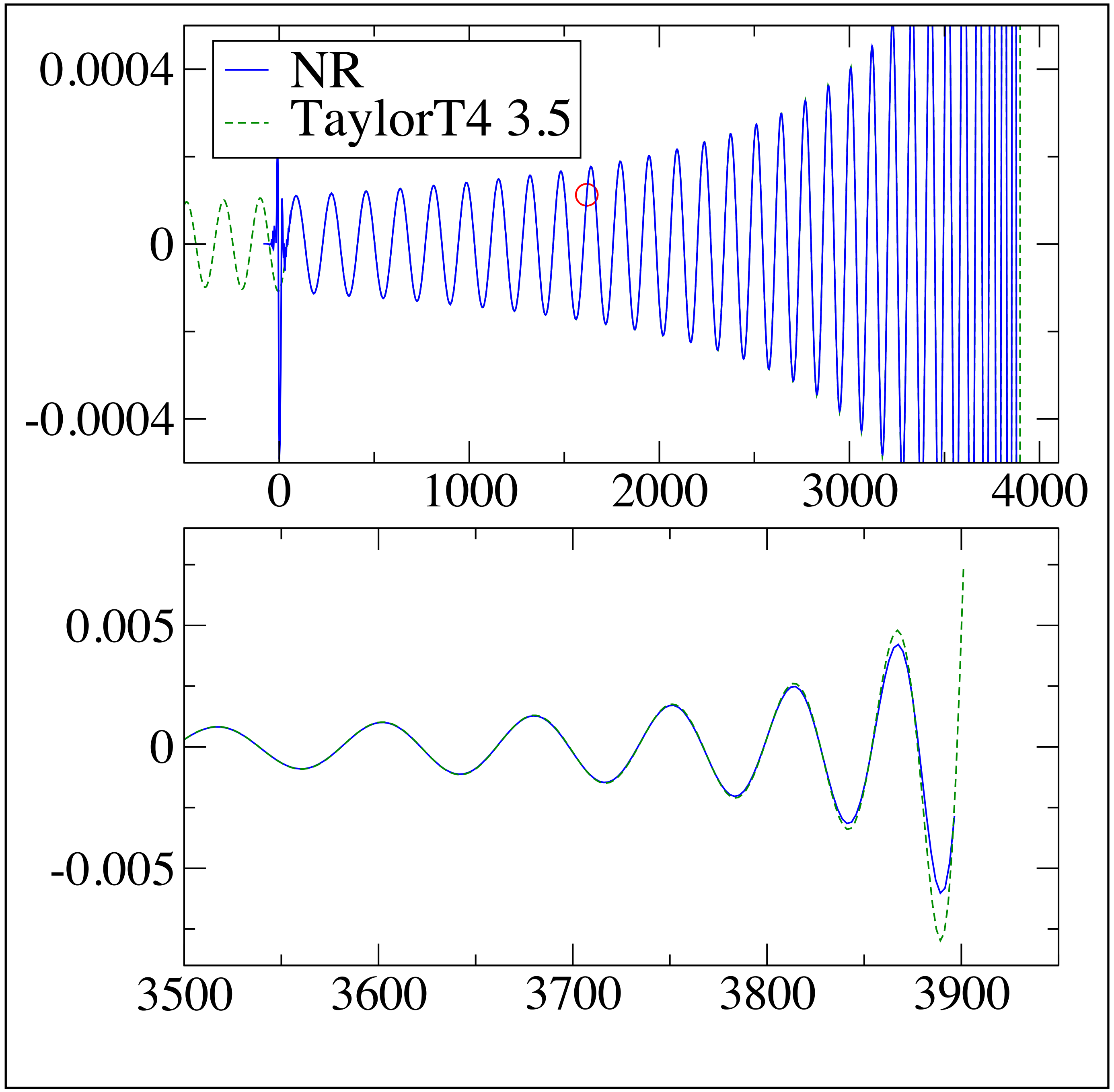


Boyle..HP+ 07

post-Newtonian vs. NR

- NR & PN agree!



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post-Newtonian vs. NR

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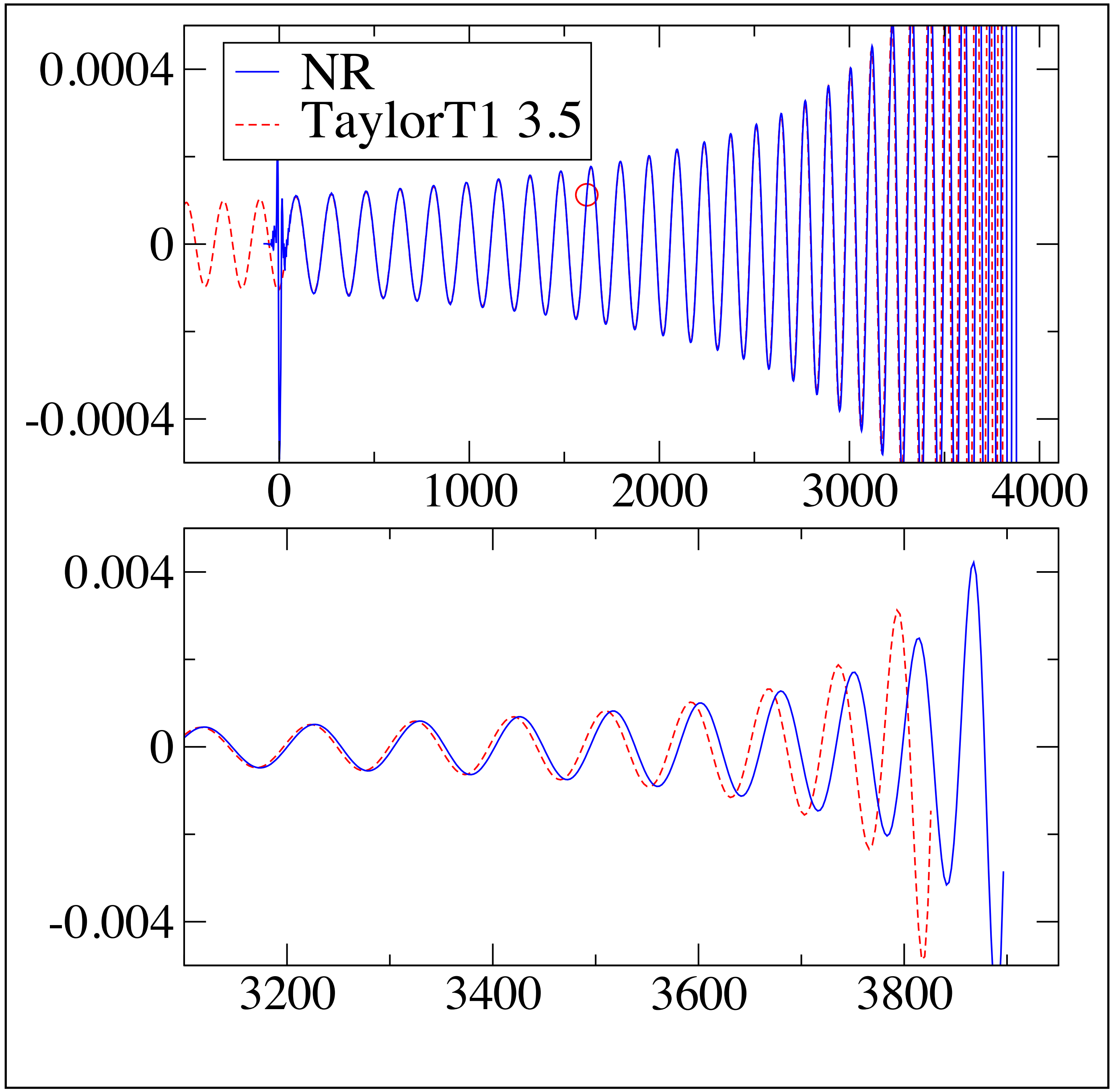
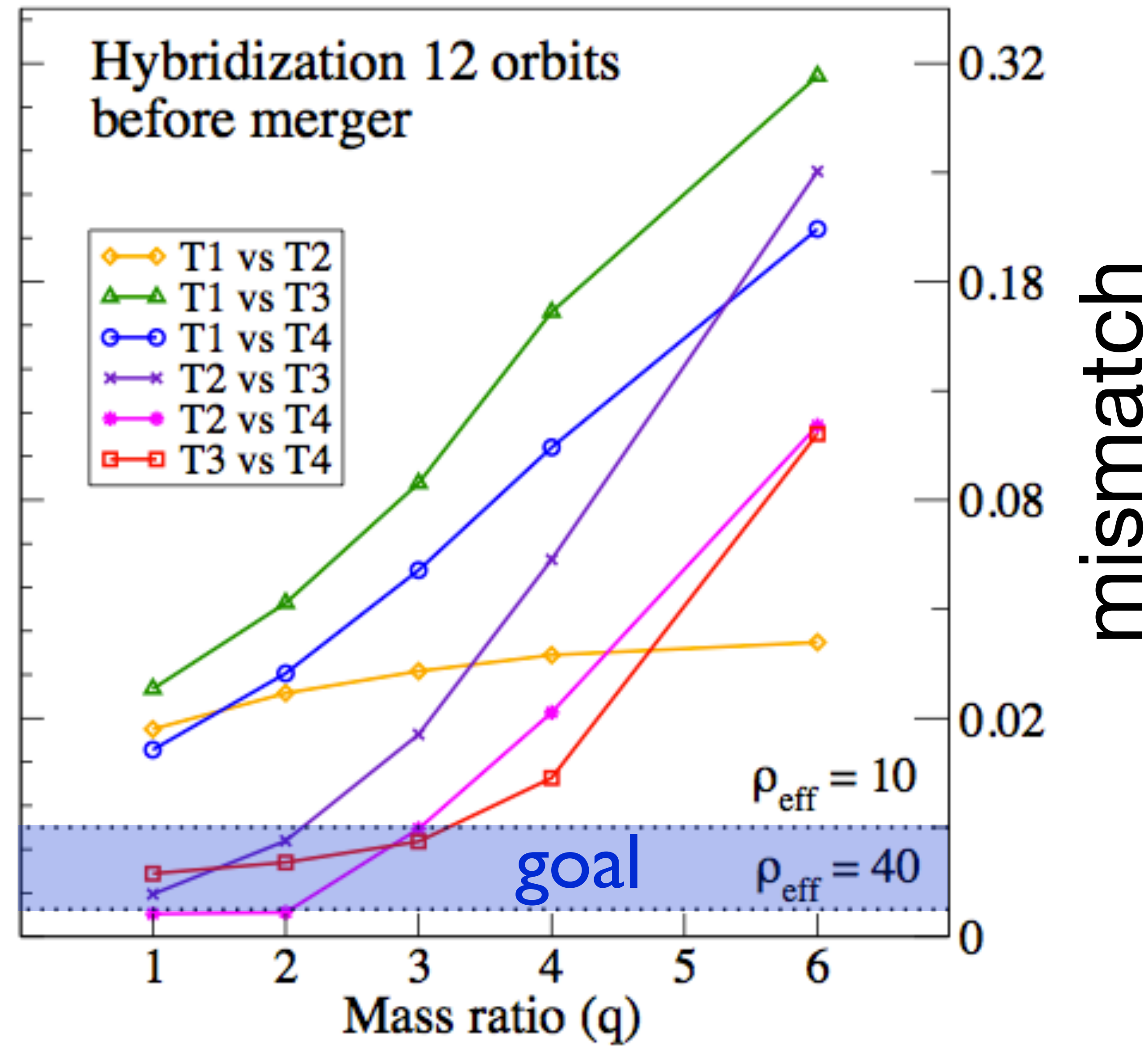
- Or do they?
  - Some versions of PN match well
  - No **a priori** knowledge which ones work (if any)



Boyle..HP+ 07

post-Newtonian vs. NR

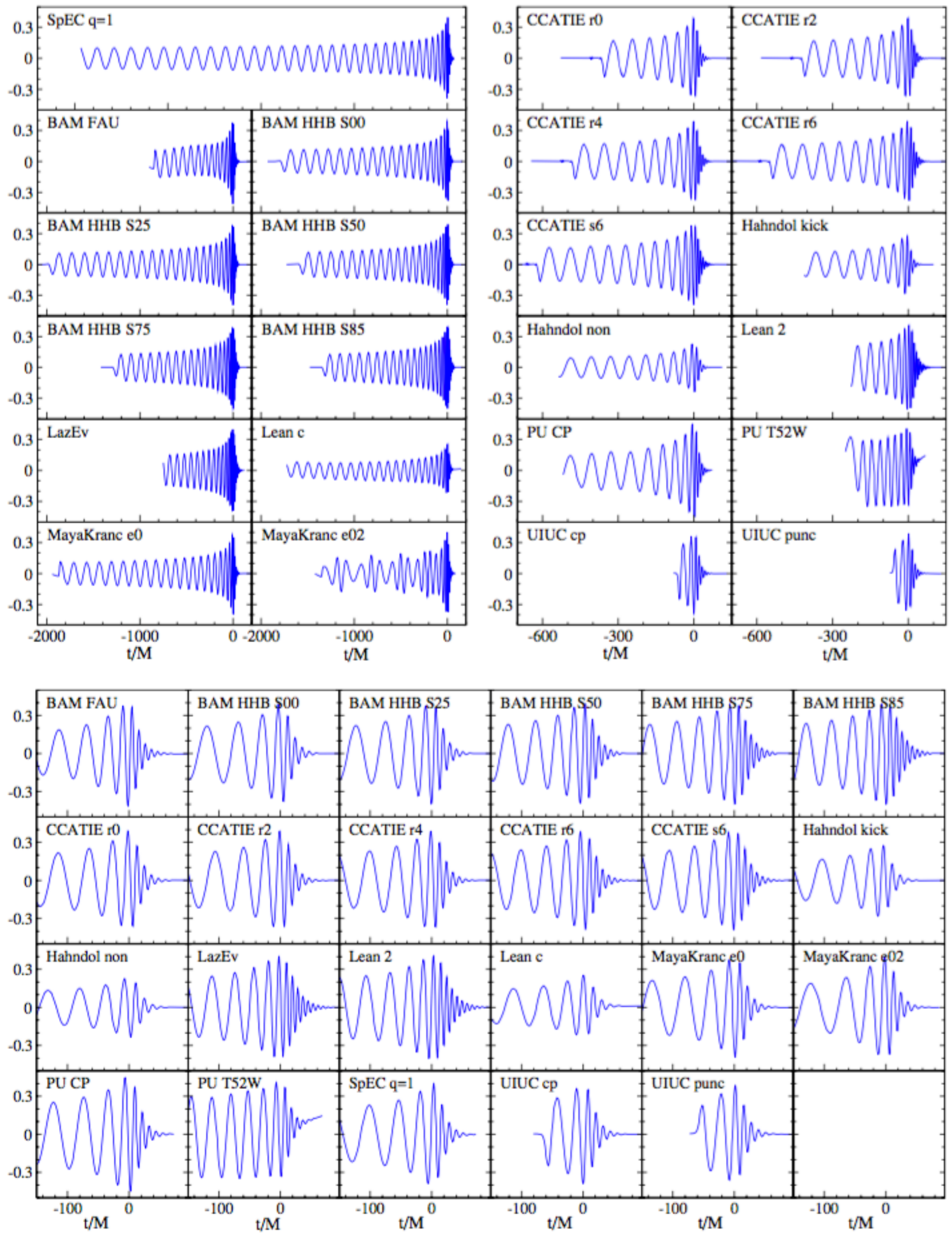
- NR & PN agree!
- Or do they?
  - Some versions of PN match well
  - No **a priori** knowledge which ones work (if any)
- **q=1, S=0 best case**
  - unequal masses and/or spinning BH give larger deviations



MacDonald..HP+ 12
see also Ohme+ 11

Parameter space exploration

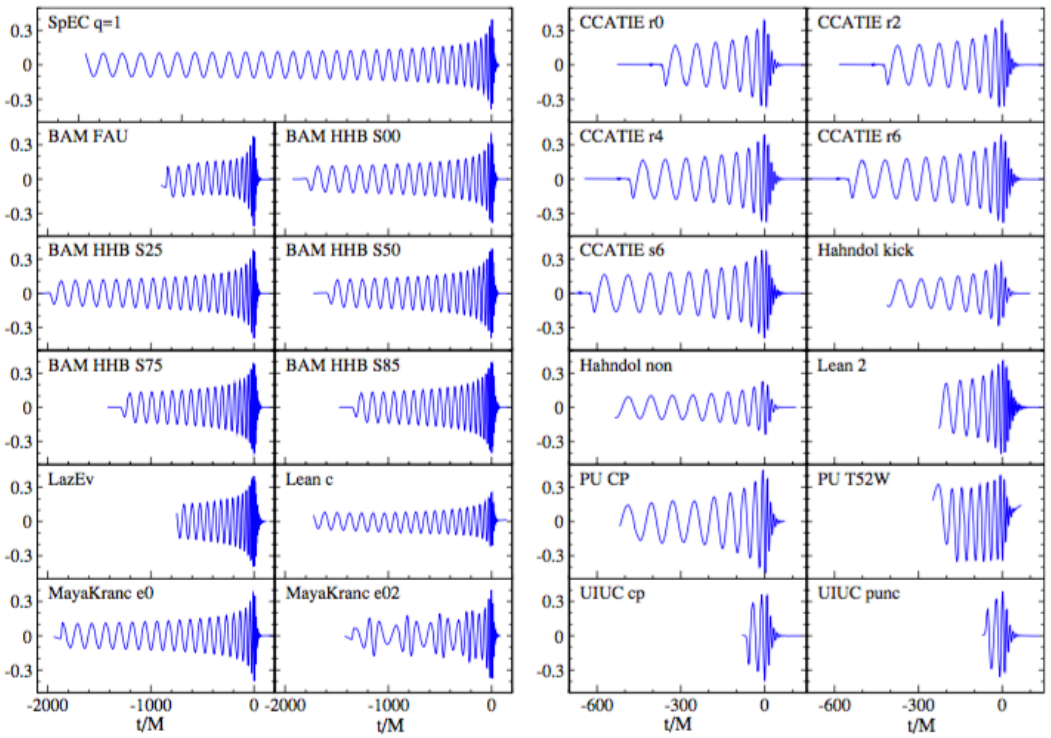
NINJA *Aylott .. HP+ 09*

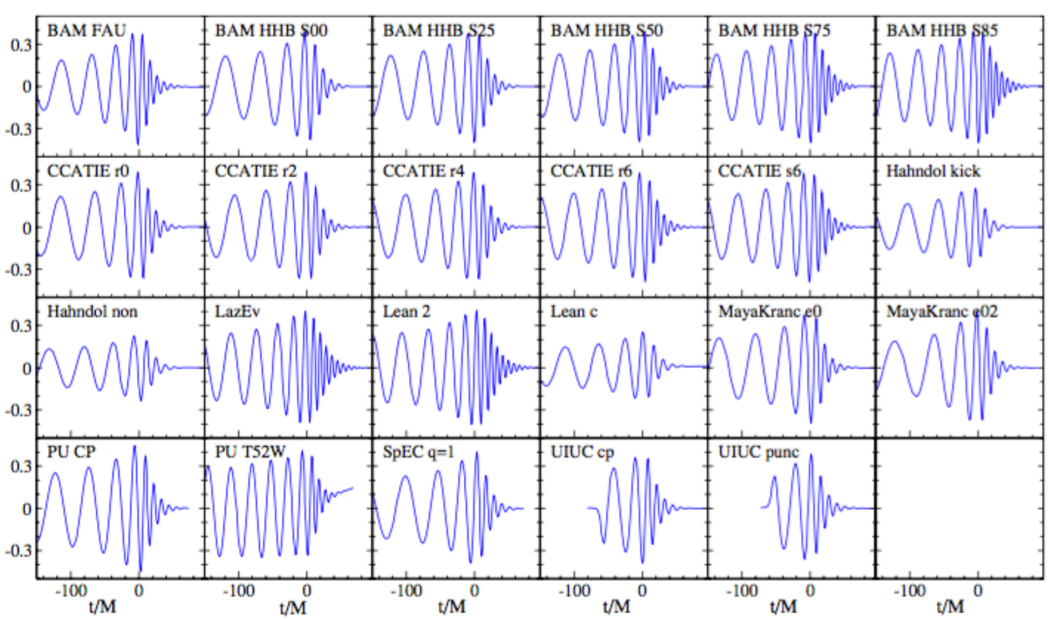


Parameter space exploration

NINJA

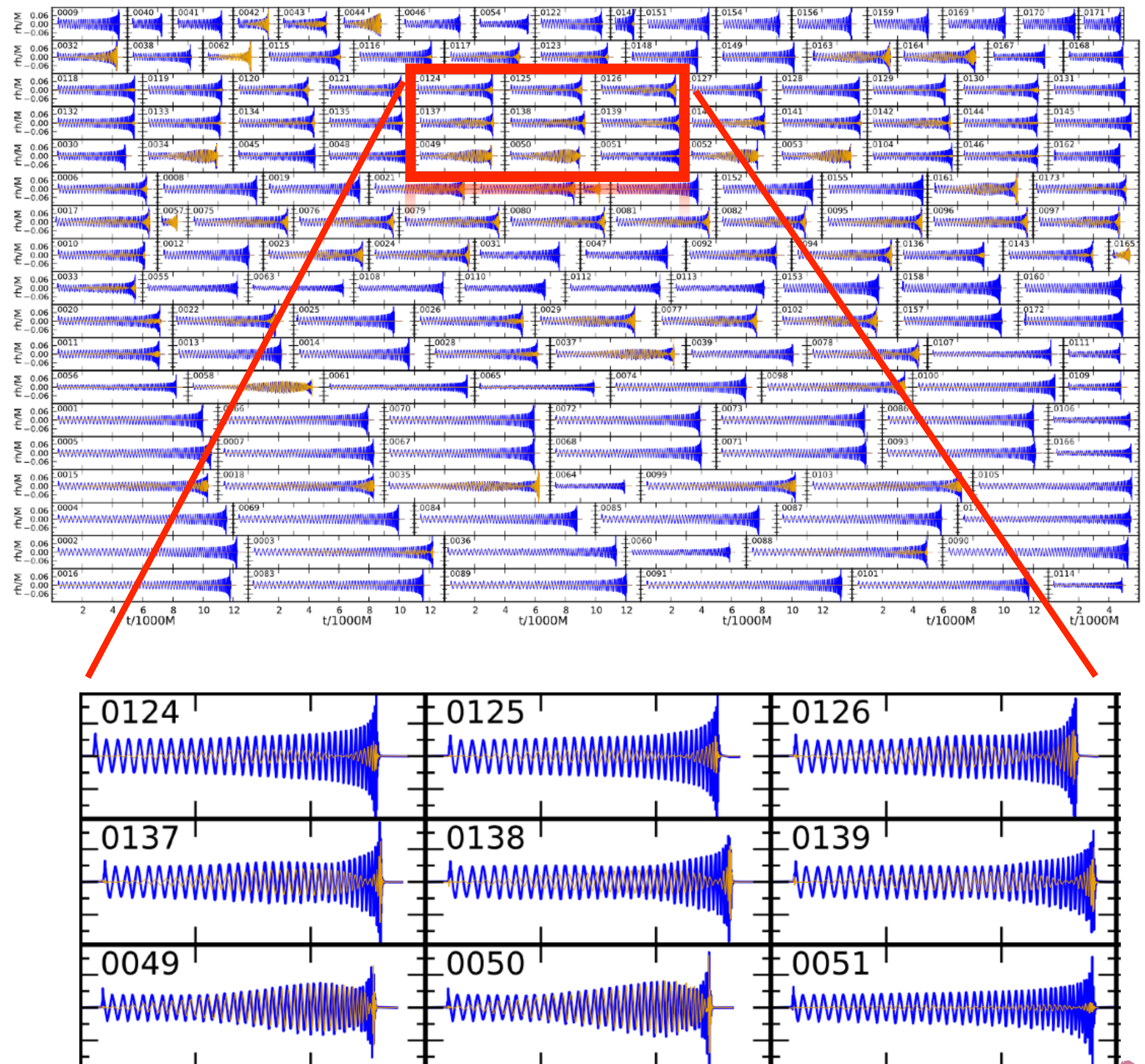
Aylott .. HP+ 09





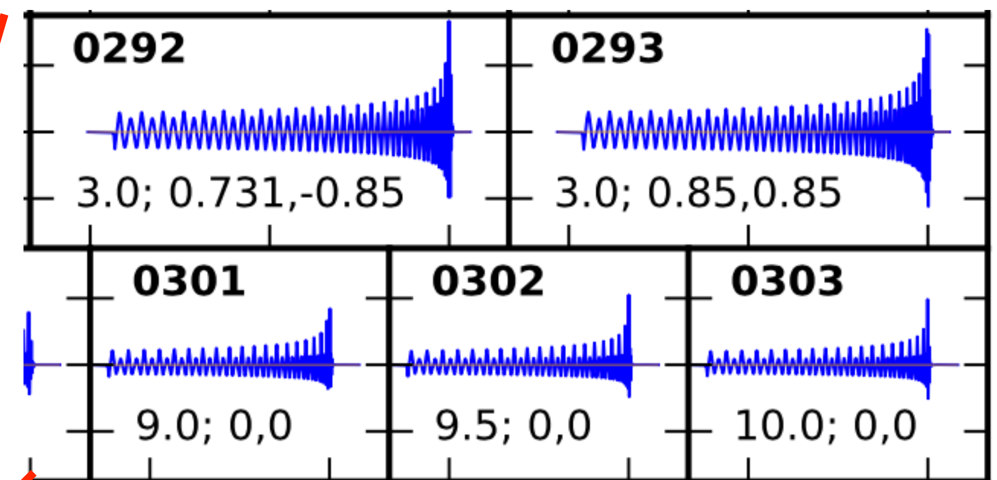
1st SXS Catalog

Mroue .. HP+ 13

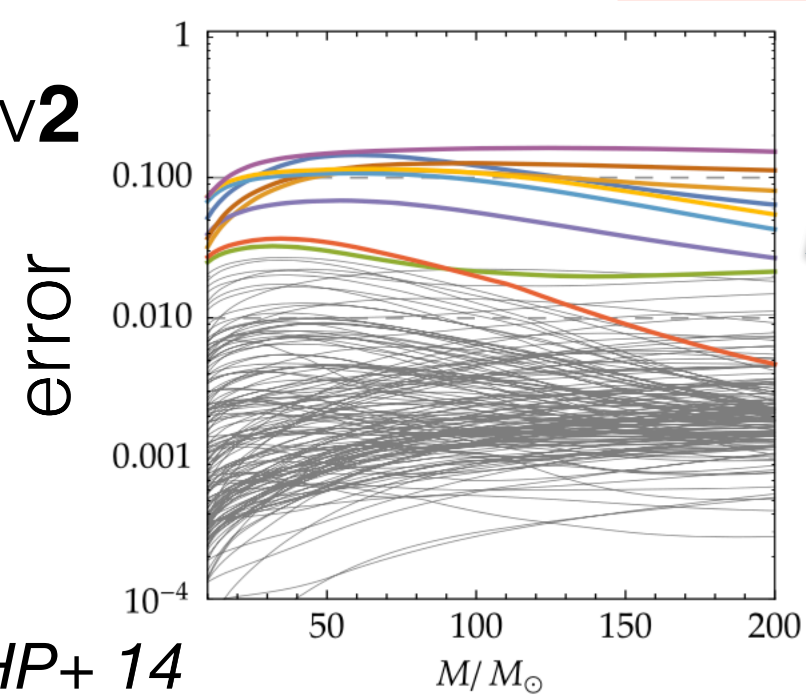


Improve analytical waveform models

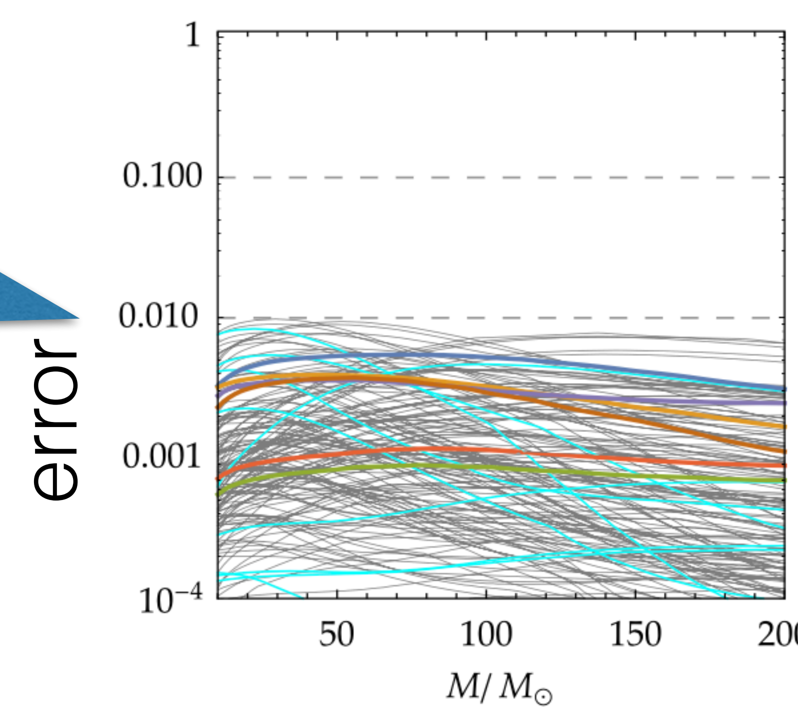
2nd SXS Catalog *Chu .. HP+ 15*



SEOBNRv**2**



Taracchini..HP+ 14

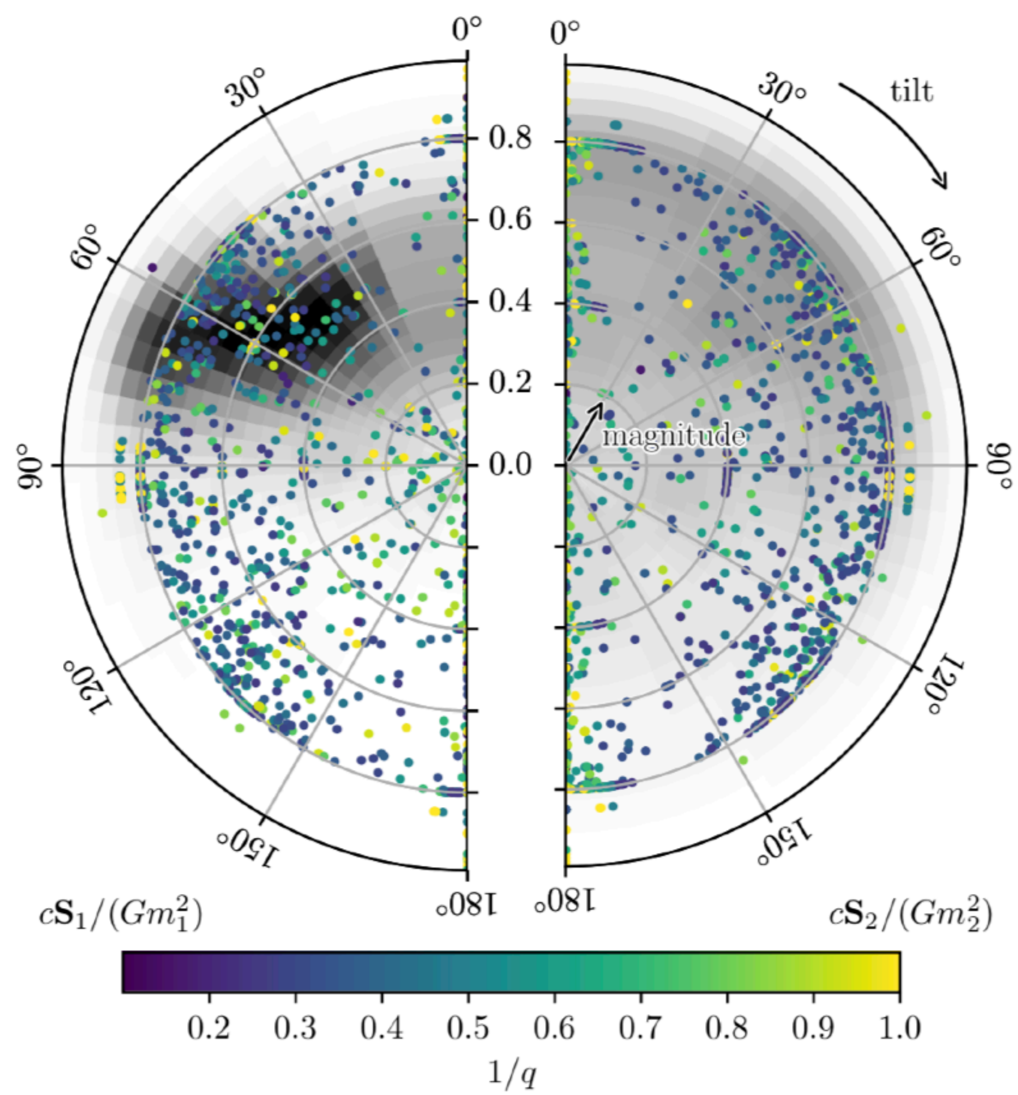


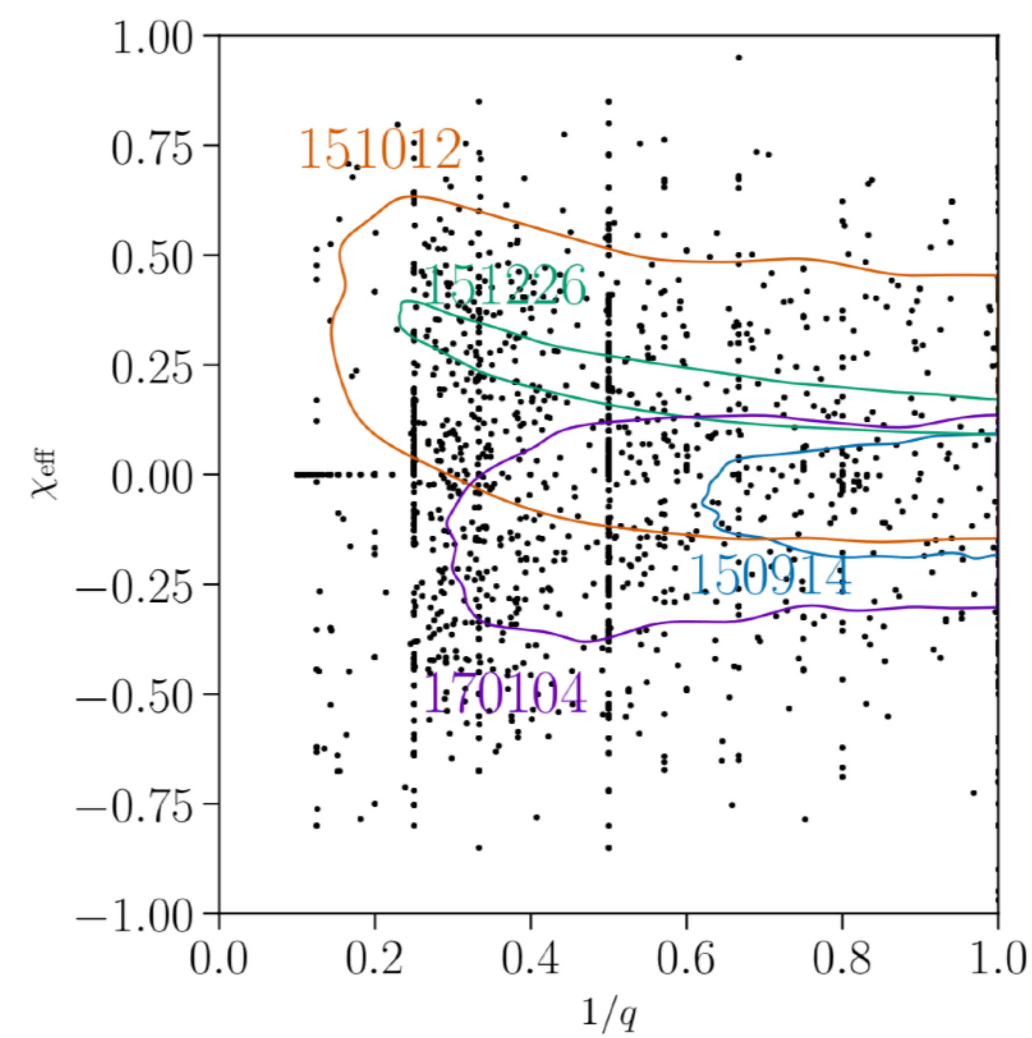
SEOBNRv**4**

Bohe..HP+ 17

SXS waveform catalog 2019 edition

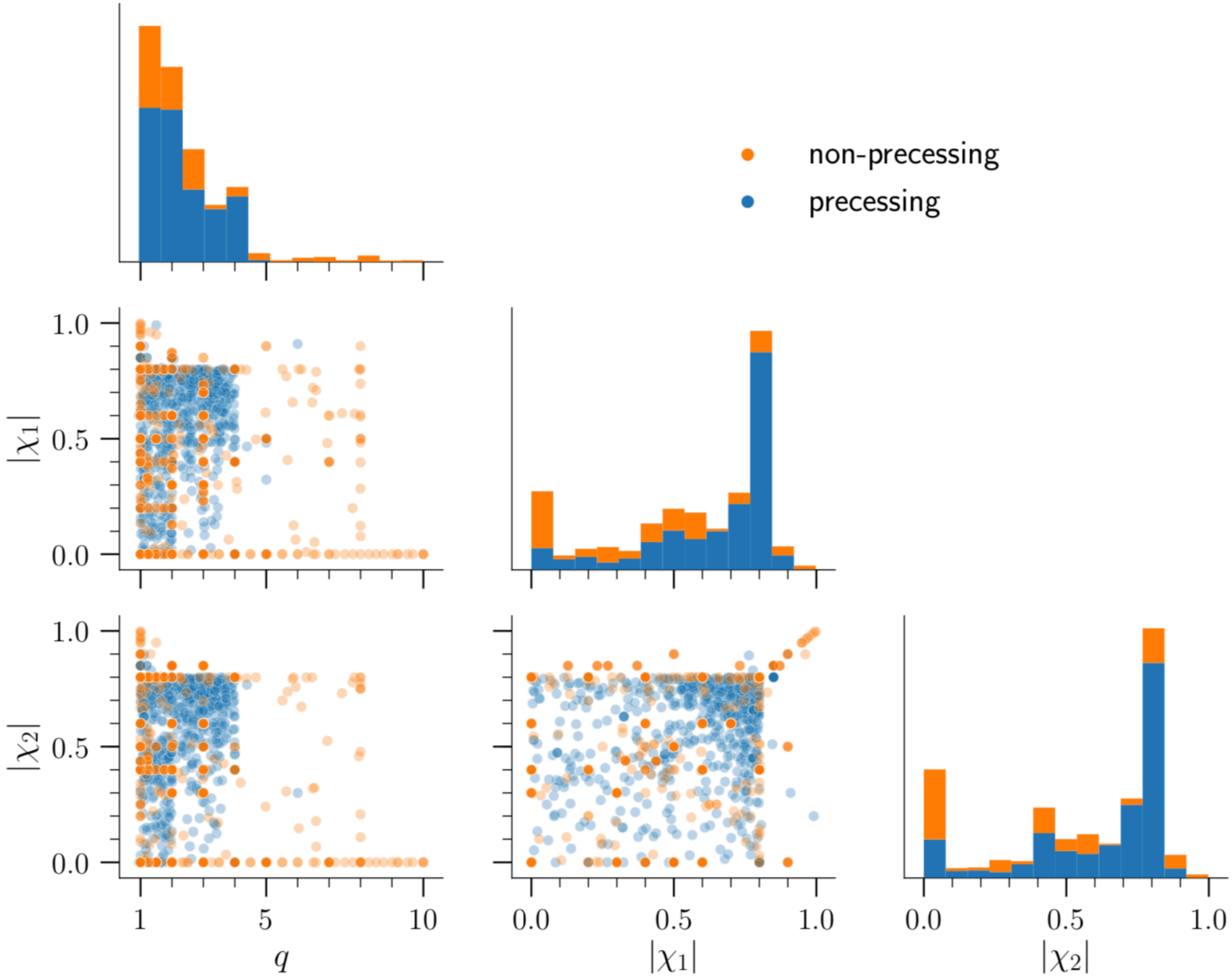
- 2018 simulations; params over-plotted on GW events



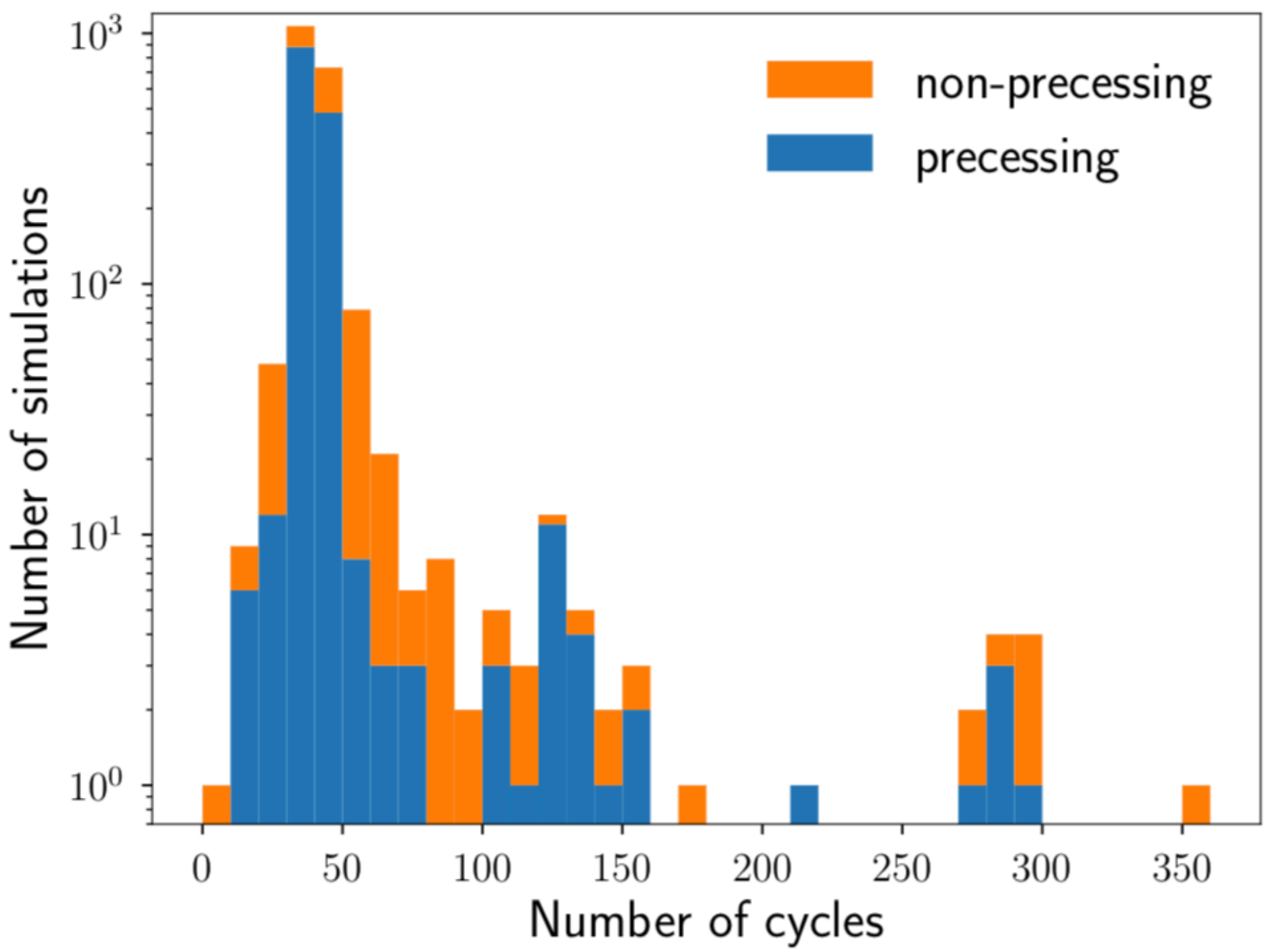


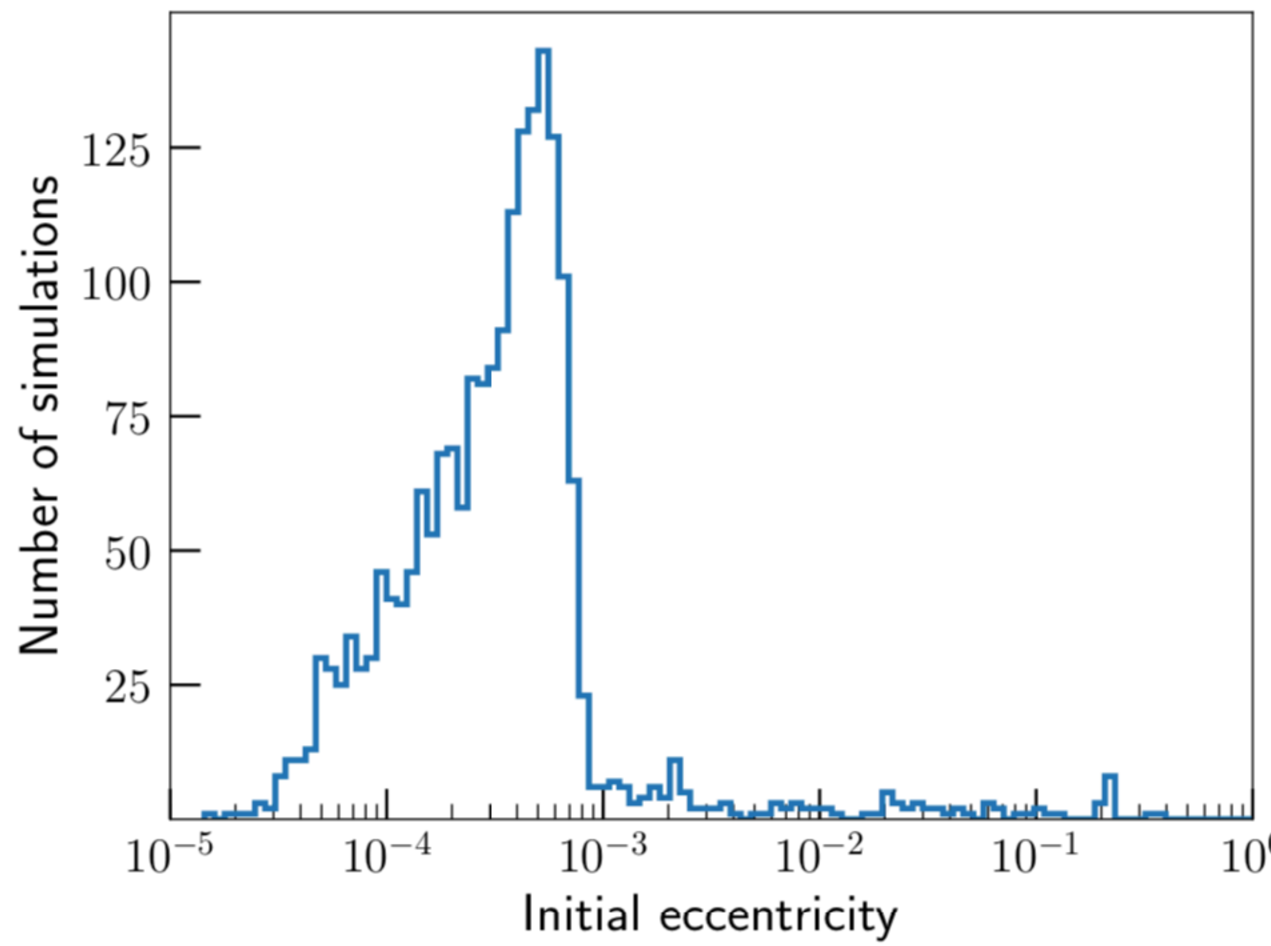
SXS Collaboration (Boyle, ..HP+), CQG 2019 (1904.04831)

SXS waveform catalog 2019 edition



SXS waveform catalog 2019 edition





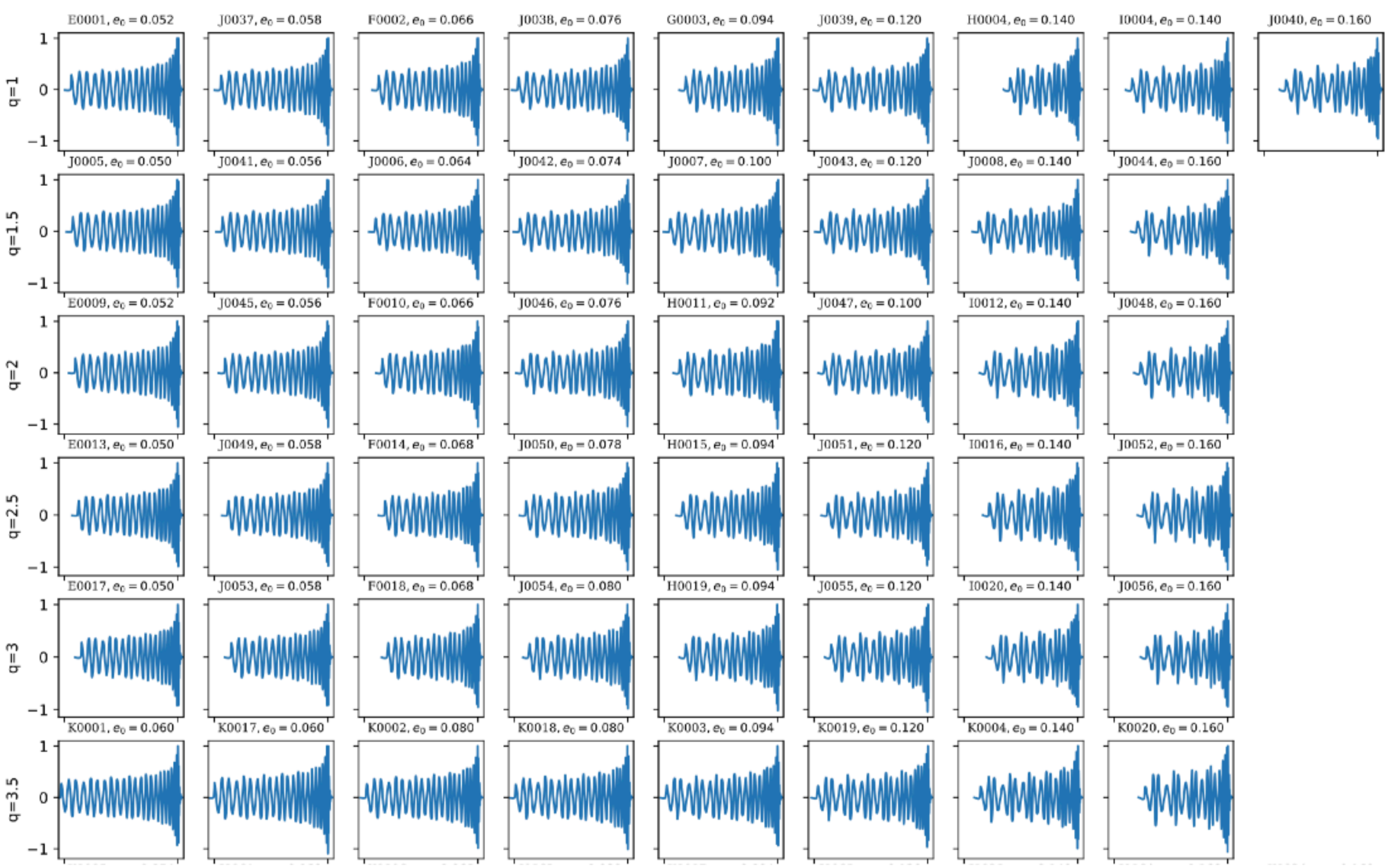
More exploration efforts

Catalog	Started	Updating?	Simulations	m_1/m_2 range	$\lvert\chi_1\rvert$ range	$\lvert\chi_2\rvert$ range	Precessing?	Median $N_{\rm cyc}$	Public?
NINJA [98, 115]	2008	✗	63	1–10	0–0.95	0–0.95	✗	15	✗
NRAR [120]	2013	✗	25	1–10	0–0.8	0–0.6	✓	24	✗
Georgia Tech [122]	2016	✓	452	1–15	0–0.8	0–0.8	✓	4	✓
RIT (2017) [123]	2017	✓	126	1–6	0–0.85	0–0.85	✓	16	✓
RIT (2019) [124]	2017	✓	320	1–6	0–0.95	0–0.95	✓	19	✓
NCSA (2019) [125]	2019	✗	89	1–10	0	0	✗	20	✗
SXS (2018)	2013	✓	337	1–10	0–0.995	0–0.995	✓	23	✓
SXS (2019)	2013	✓	2018	1–10	0–0.998	0–0.998	✓	39	✓

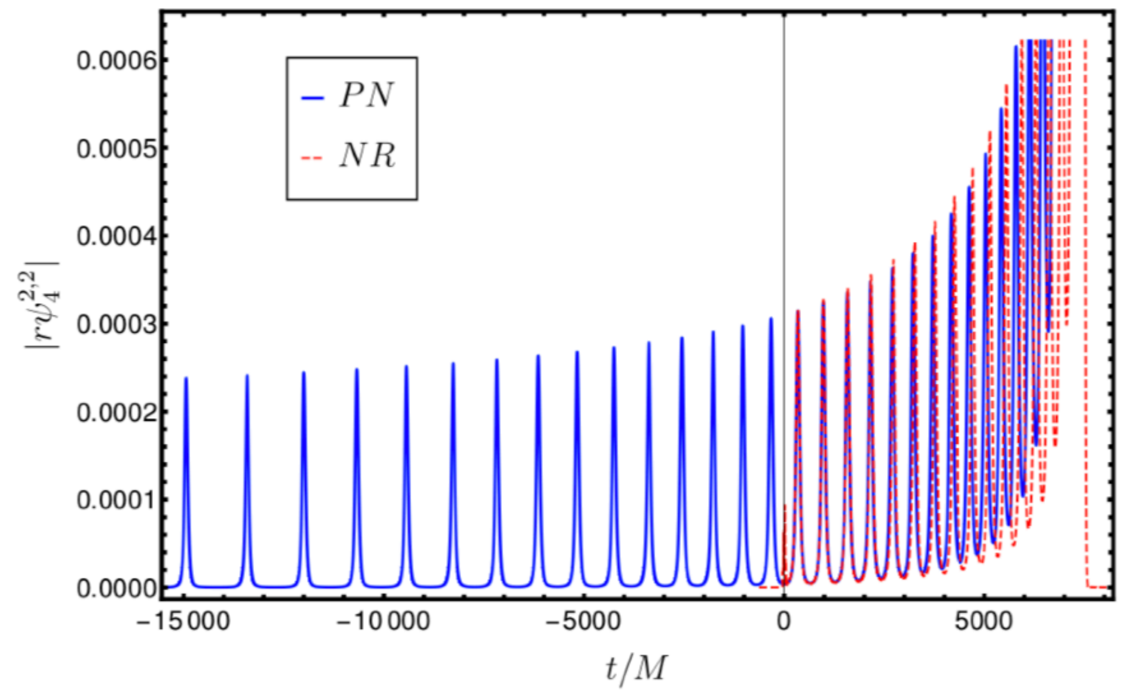
SXS Collaboration (Boyle, ..HP+), CQG 2019 (1904.04831)

And Palma group (Husa+), which hasn't published a catalog

Beginning to explore eccentricity (q~1)



Huerta+ 1901.07038



Ramos-Buades+ 1909.11011

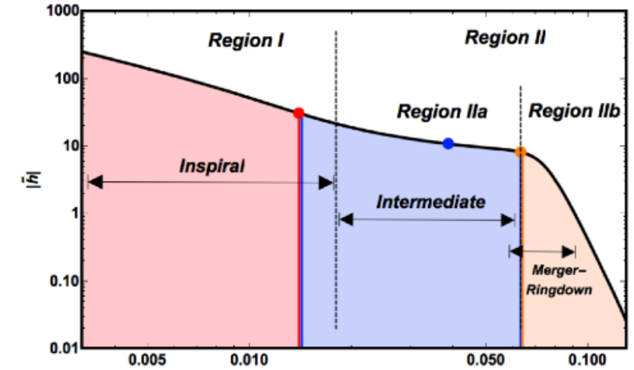
Two families of BH-BH waveform models

Effective one body (EOB)

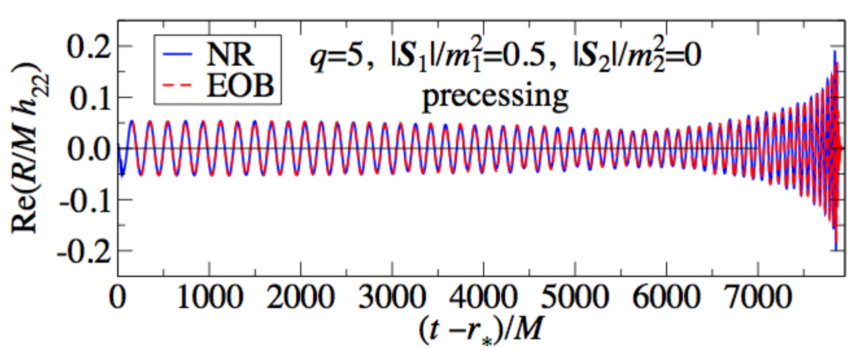
$$H = \mu \sqrt{p_r^2 + A(r)\left[1 + \frac{p_r^2}{r^2} + 2(4-3\nu)\nu\frac{p_r^4}{r^2}\right]}$$

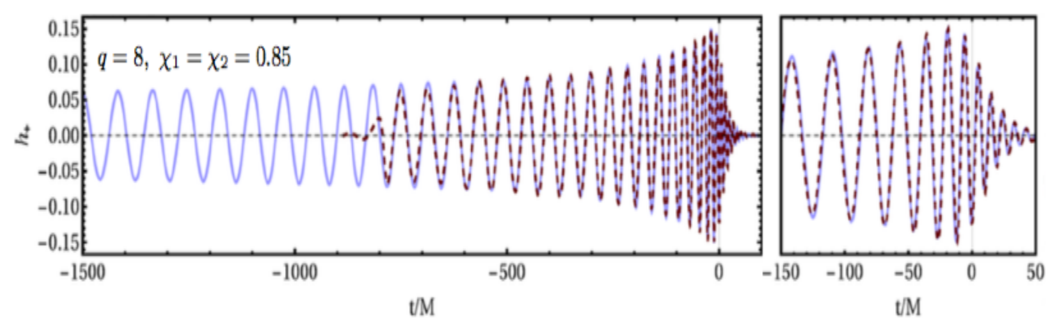
Buonanno, Damour 99

Phenomenological (Phenom)



Calibrated against **numerical simulations**
circular orbits only





- Most accurate inspiral
- Full precession $\vec{S}_1, \vec{S}_2$

E.g. Pan..HP+ 14, Taracchini..HP+ 14, Bohe..HP+, 17

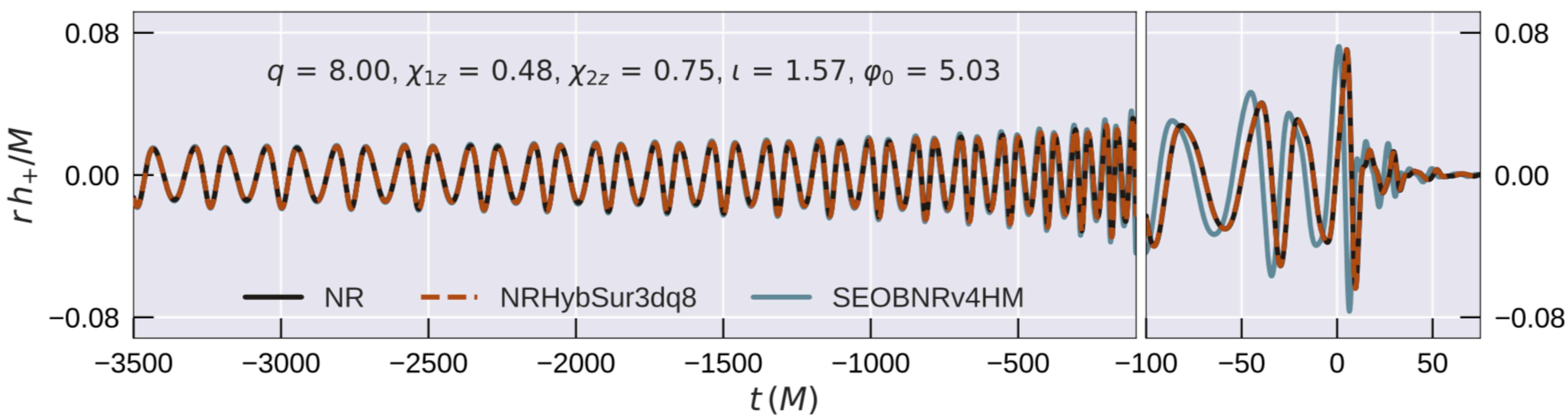
- Fast evaluation
- Approximate precession

e.g. Hannam+13, Khan+15

BH-BH waveform modeling continues

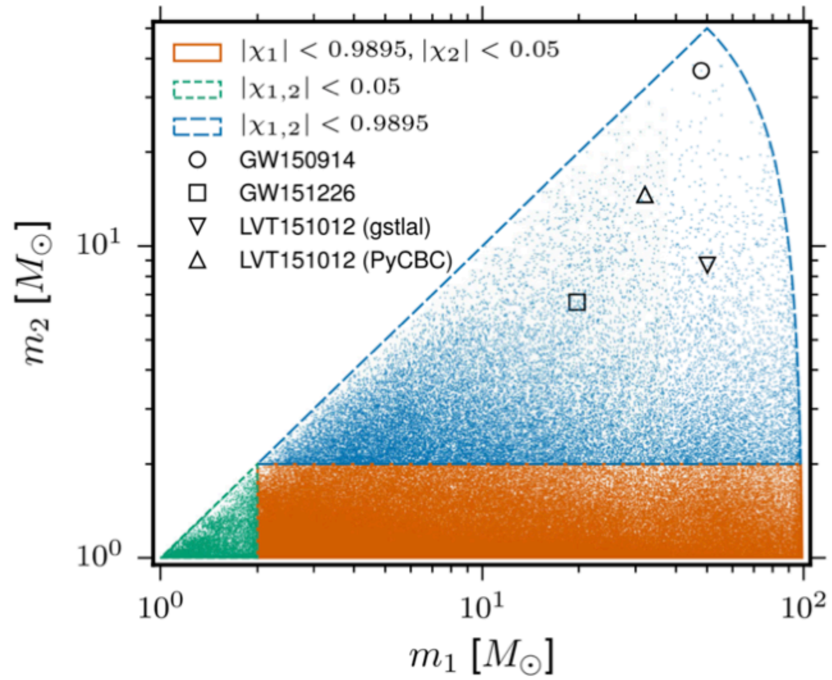
- state of the art: **Precession** and **higher modes**
- EOB models, Phenom models
- new kid on the block: **NR surrogate models**
  - need O(1000) NR sims
  - nearly "automatic" model construction
  - model-accuracy ~ NR-accuracy

Varma..HP 1812.07865
and refs therein



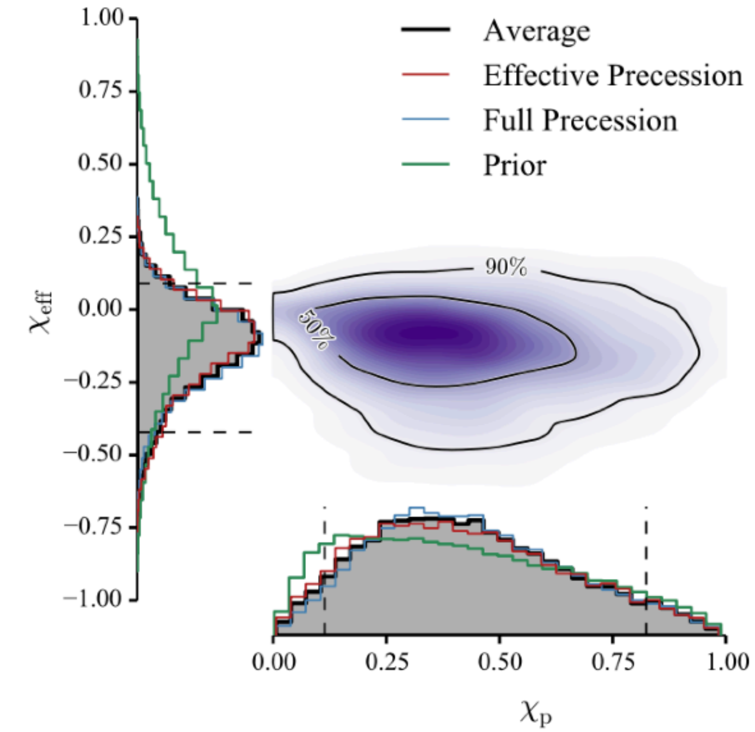
Waveform knowledge ***underpins GW astronomy***

Detection by matched filtering



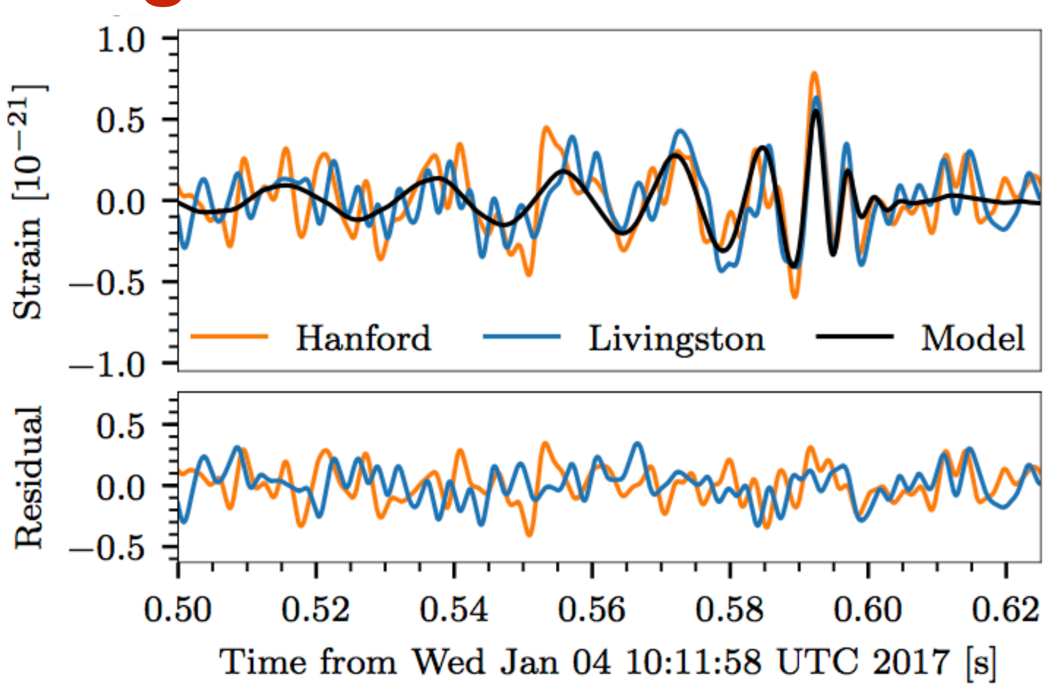
LIGO+Virgo, PRX 2016 (1606.04856)

Parameter estimation



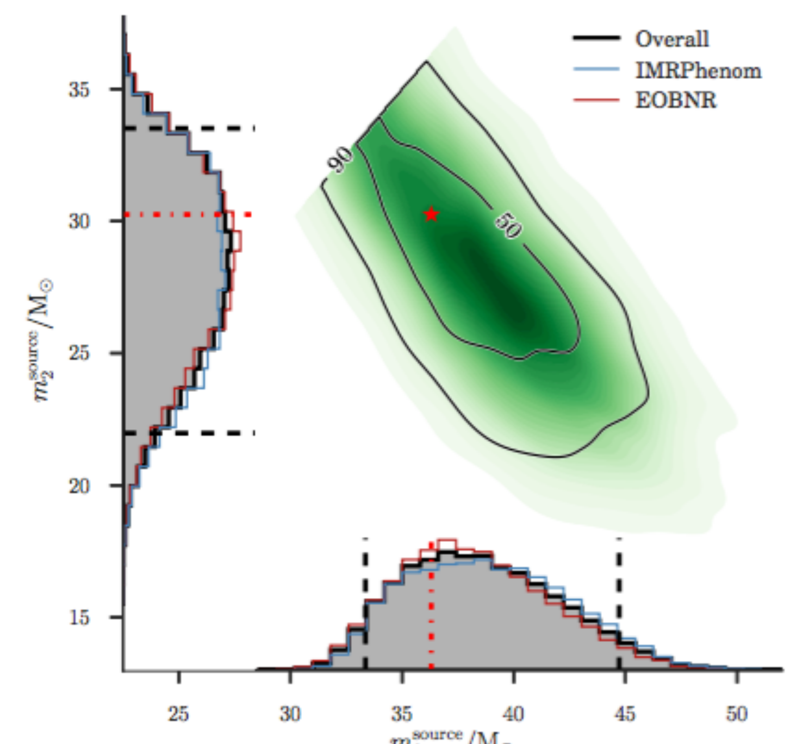
"GW170104" Abbott+ PRL 118, 221101 (2017)

Testing GR



LIGO+Virgo, PRL 2017 (1706.01812)

Validation



LIGO & Virgo: CQG 2017 (1611.07531)

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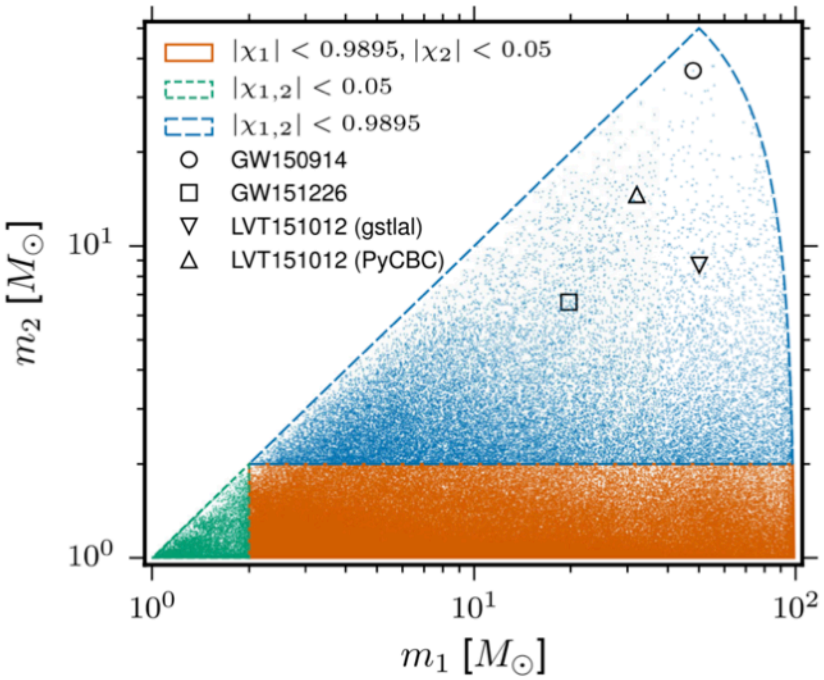
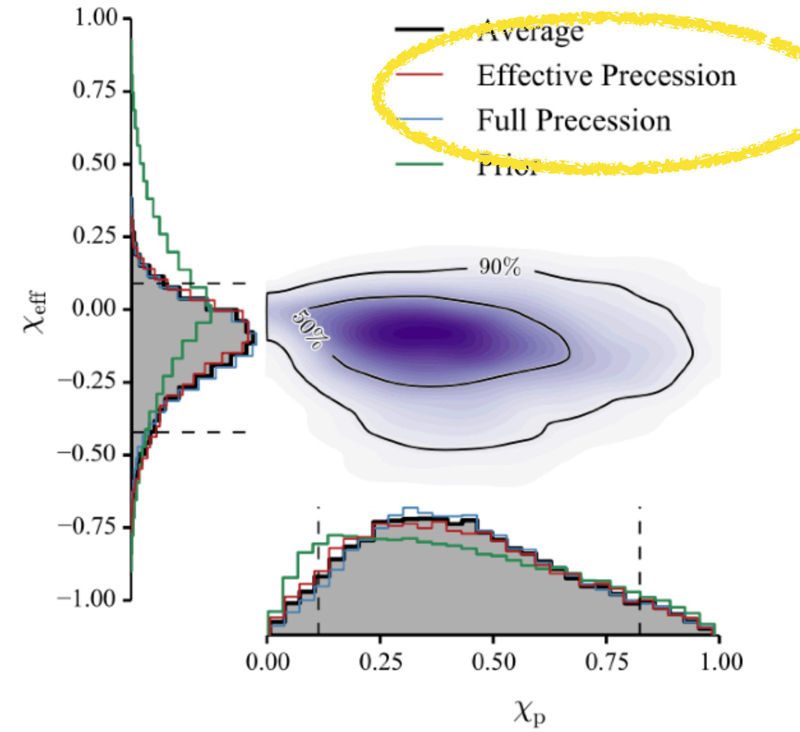
Detection by matched filtering



LIGO+Virgo, PRX 2016 (1606.04856)

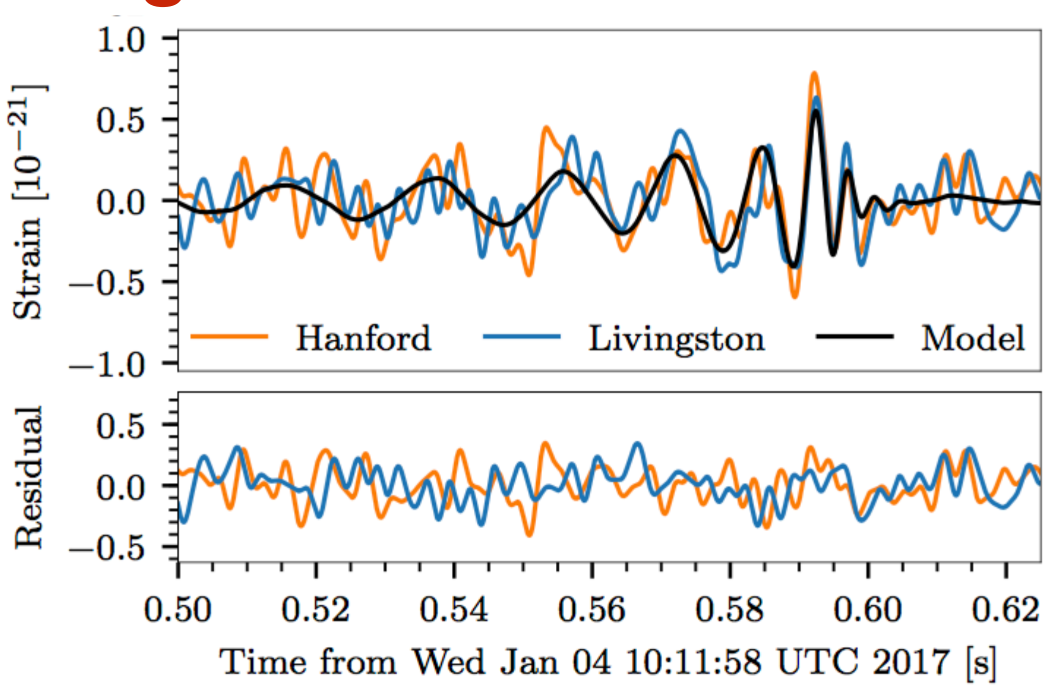
Parameter estimation



Two models Agreement

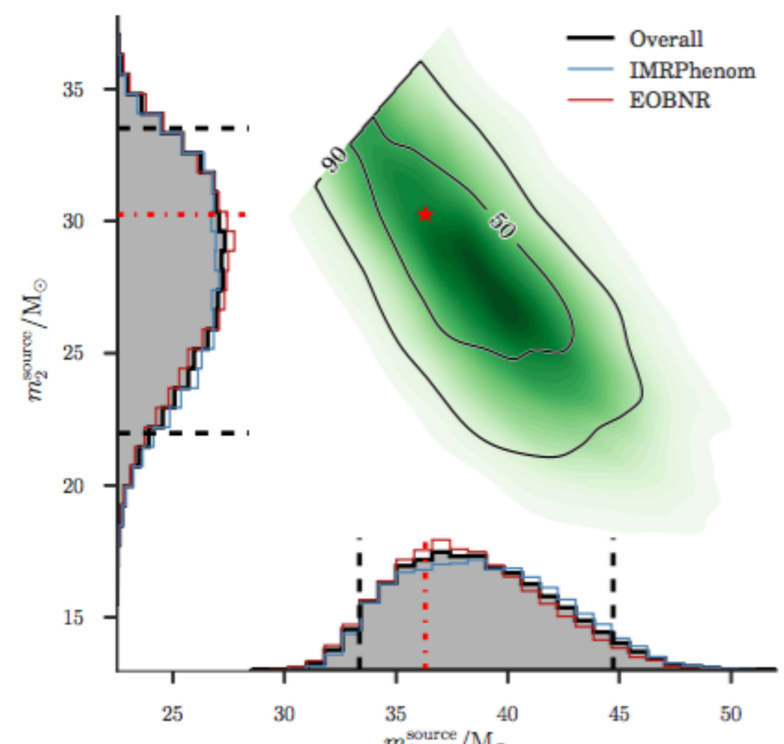
"GW170104" Abbott+ PRL 118, 221101 (2017)

Testing GR



LIGO+Virgo, PRL 2017 (1706.01812)

Validation

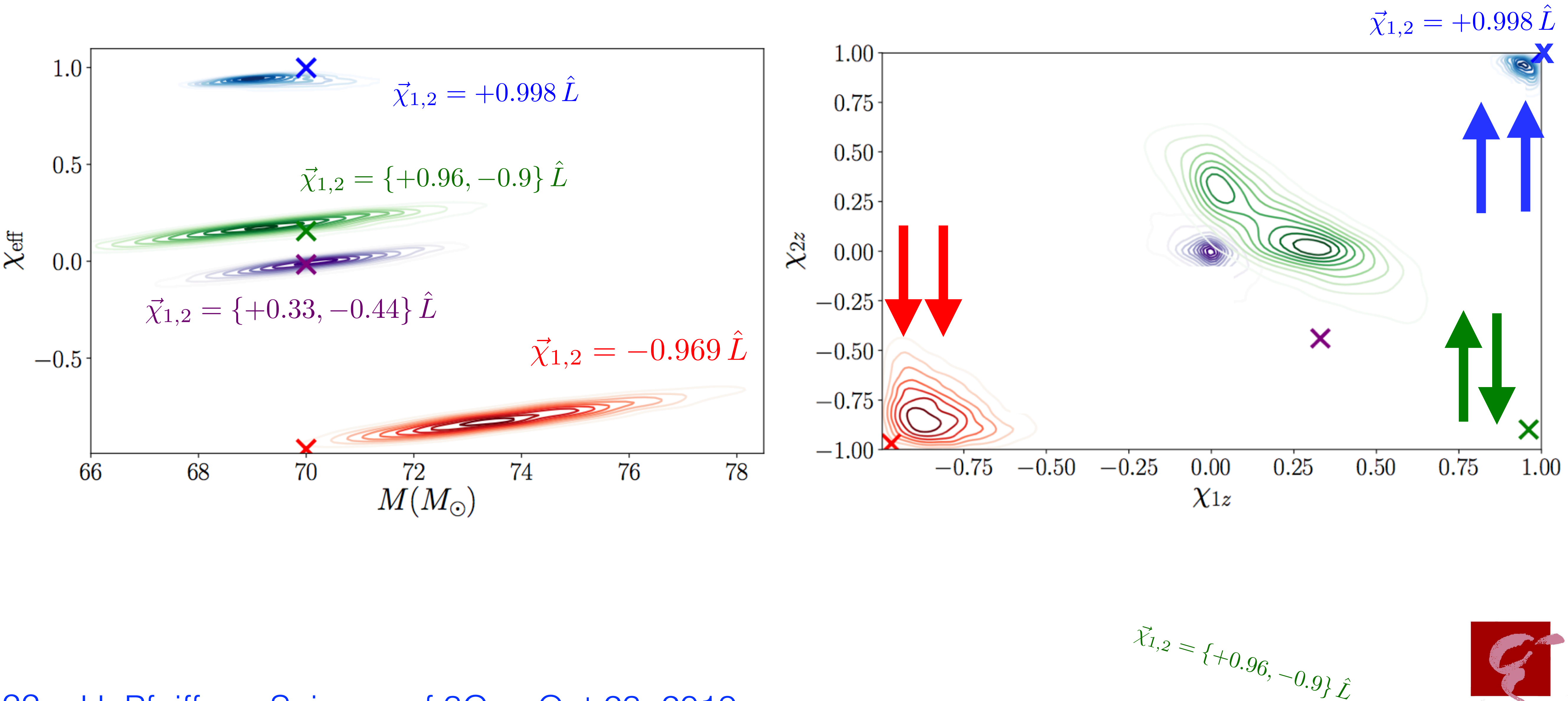


LIGO & Virgo: CQG 2017 (1611.07531)

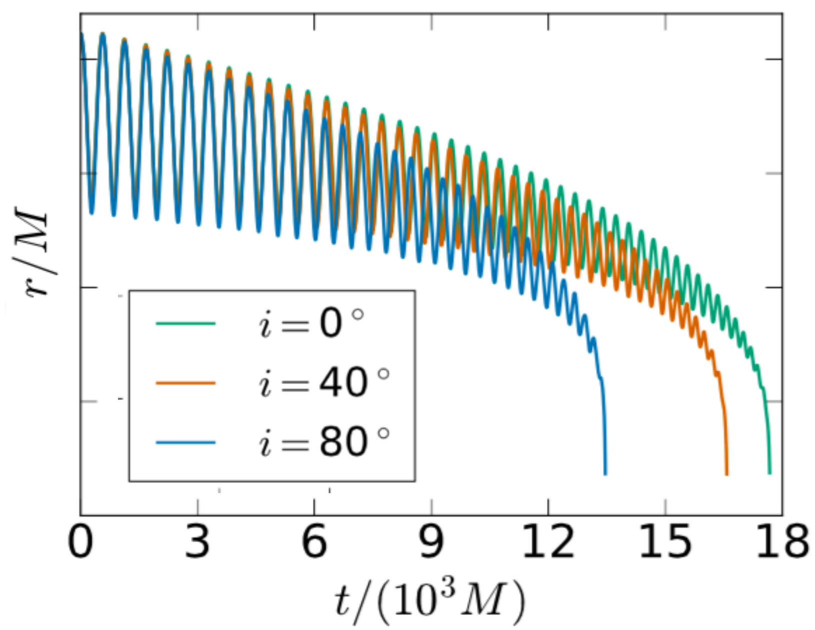
Nearly extremal BBH

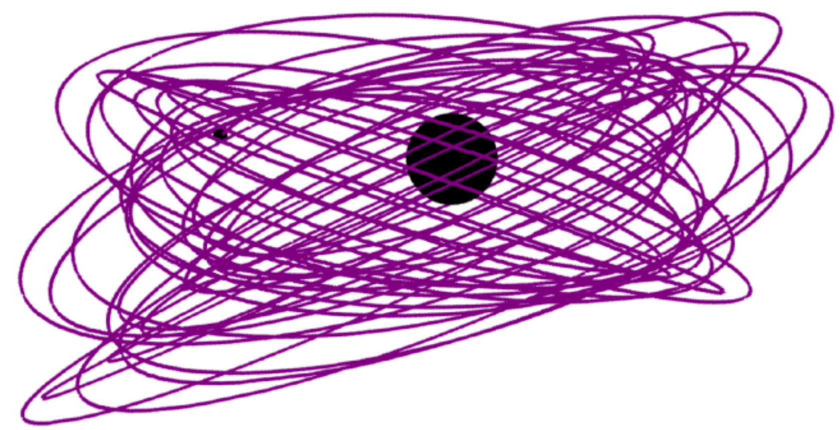
- NR sims with aligned & nearly extremal spins
- Study GW parameter recovery

Chatziiouannou,Lovelace..HP+
PRD 2018 (0804.03704)



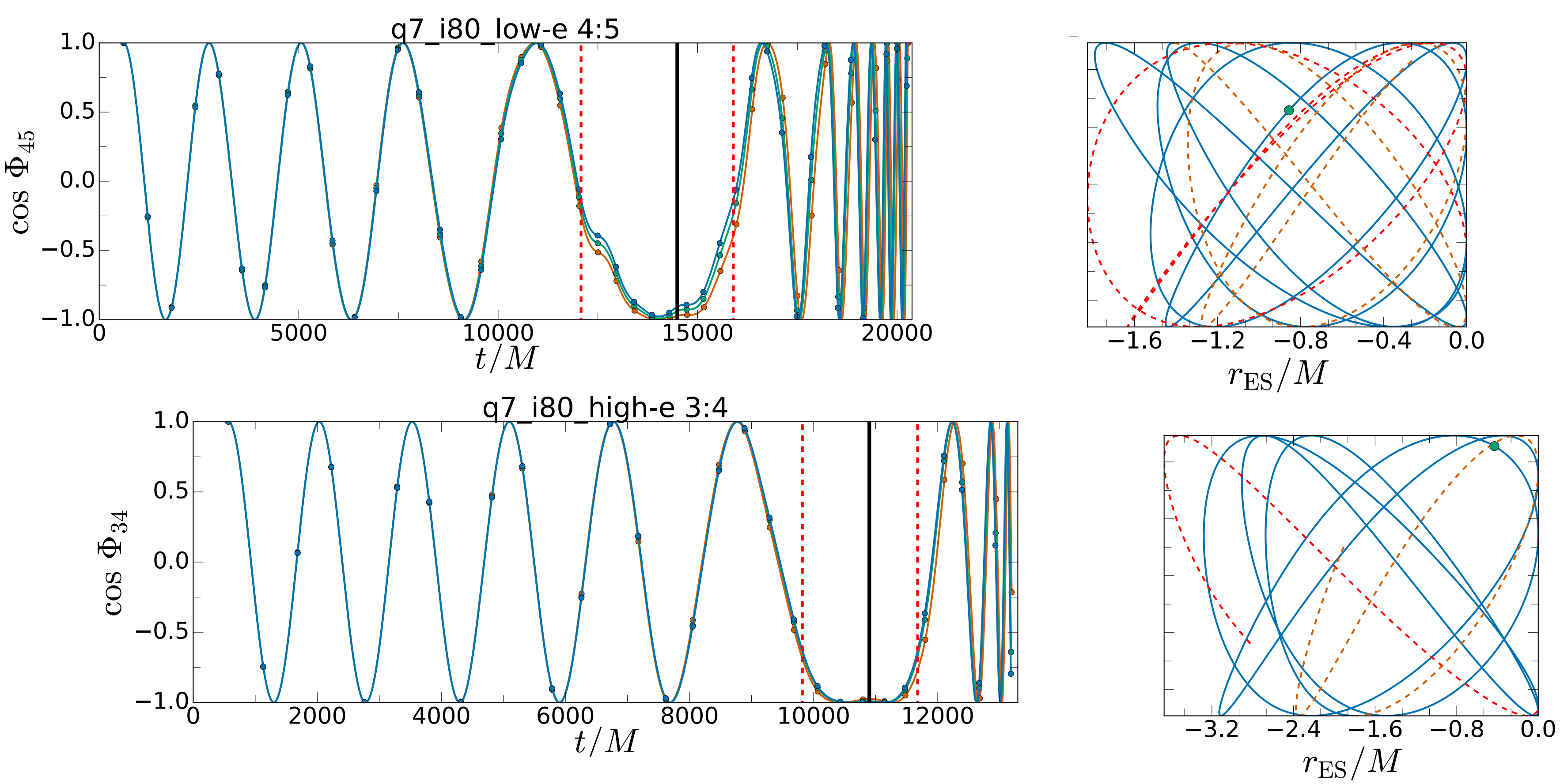
Fully generic (eccentric & precessing) BBH





- **Ω^θ- Ω^r resonances**
  - At extreme mass-ratios: strong impact on GW-phase

Flanagan,Hinderer 12
Flanagan+ 14



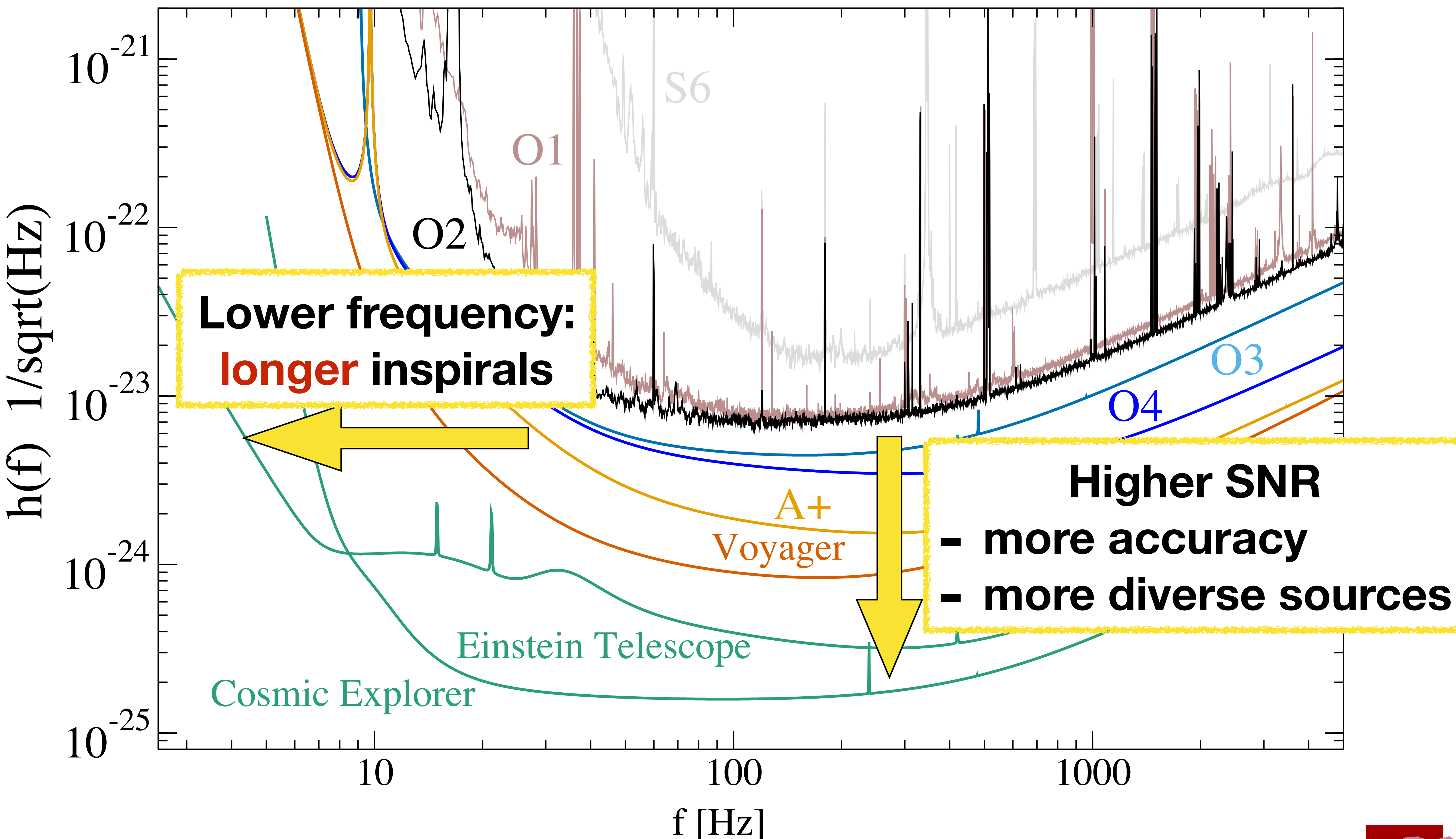
  - At our q=1/7: no discernible impact

Lewis, Zimmerman, HP 17

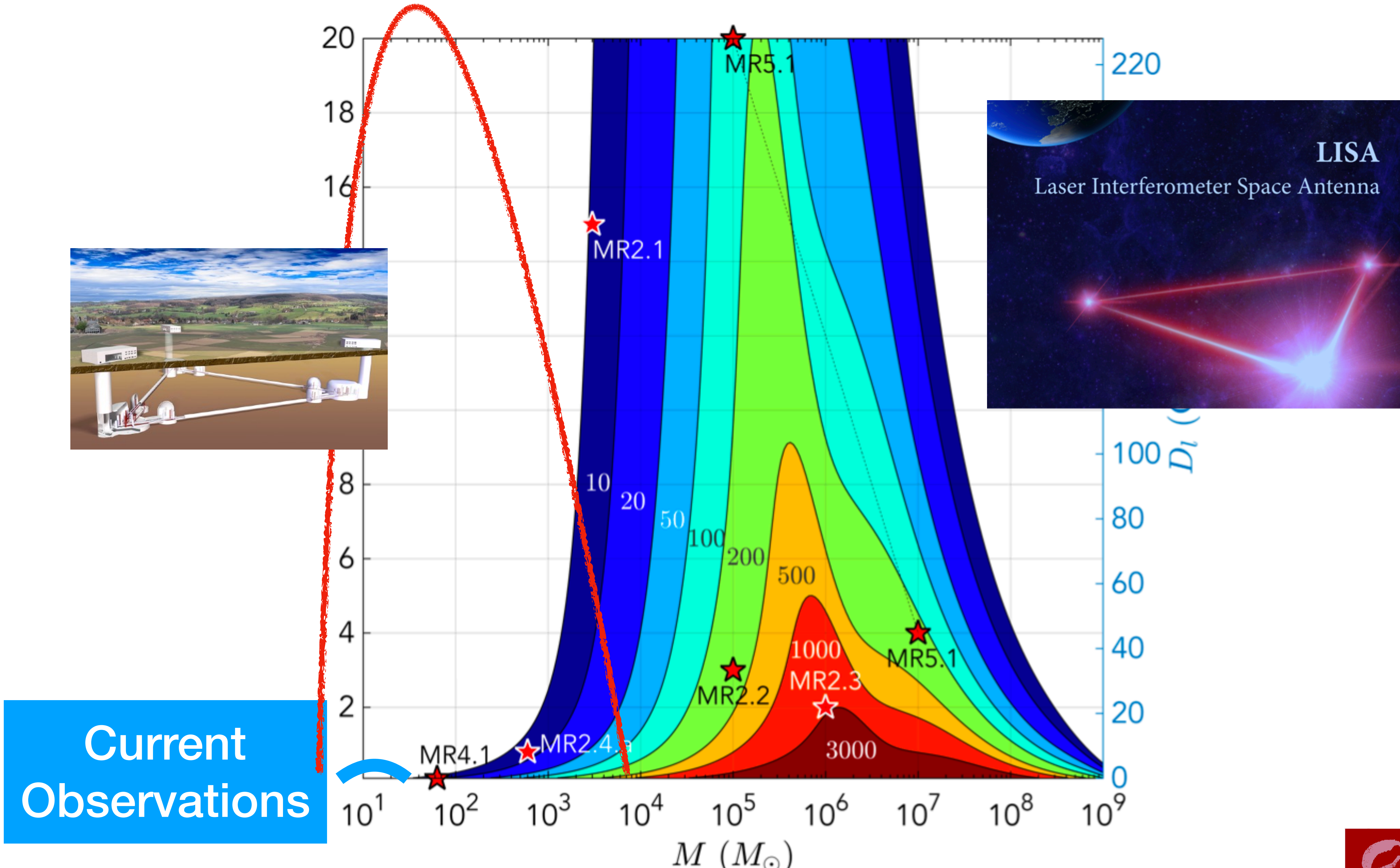
NR in the 3G and LISA era

vacuum GR is mass-invariant

Ground-based Interferometers

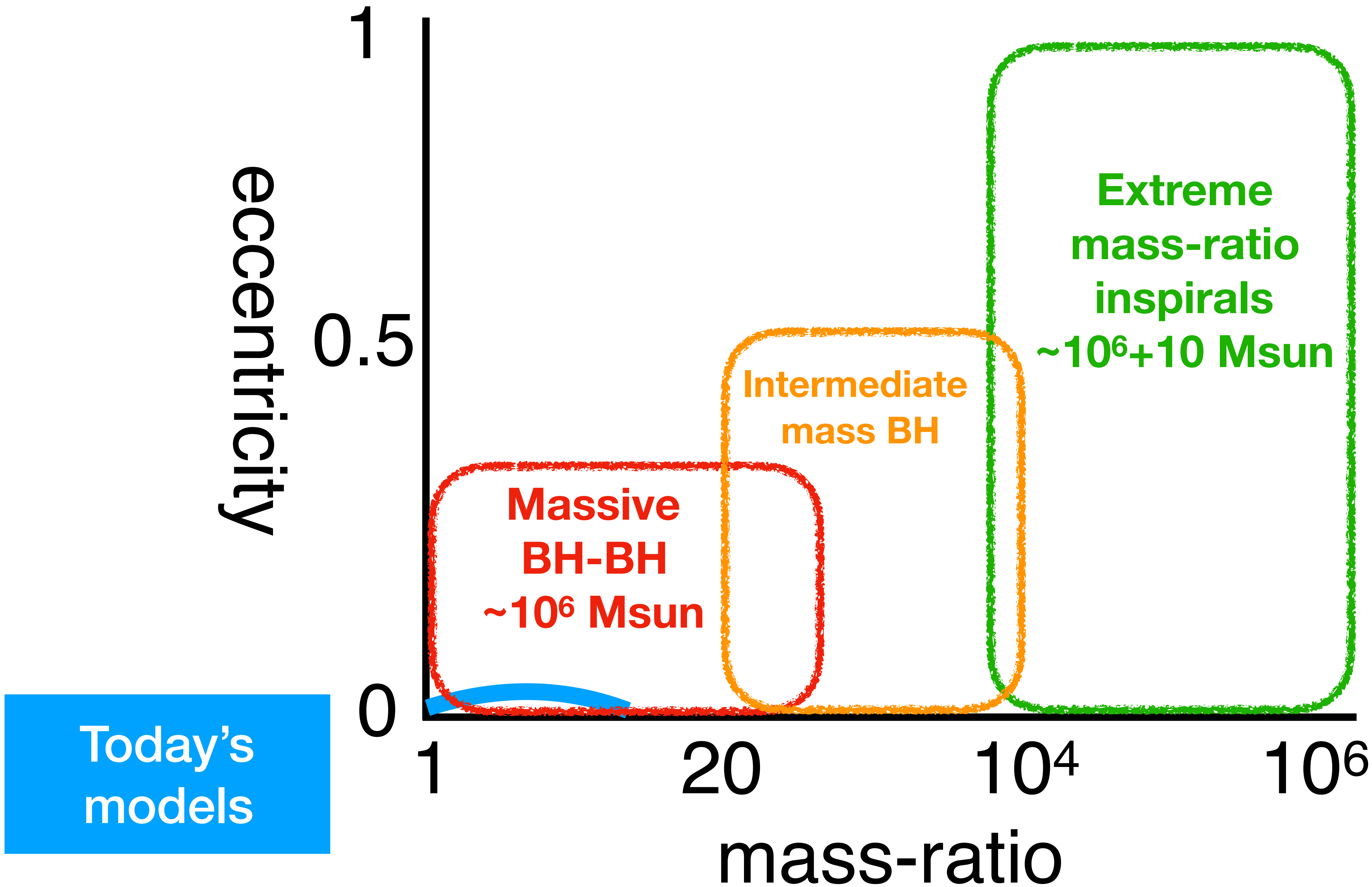


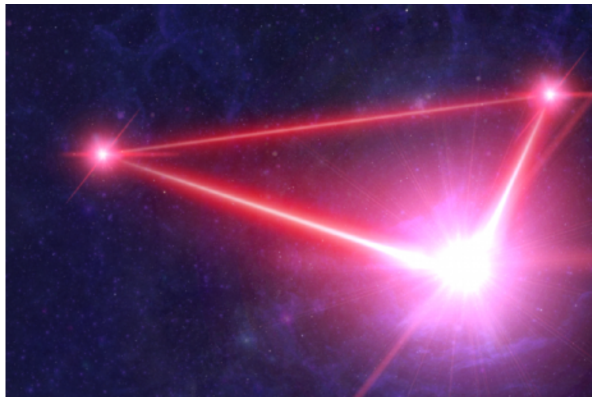
Exploring *all* BH through *all* of universe



LISA (in particular): ***all mass-ratios***, eccentricity

- **more diverse & louder events**
  ⇒ ***<u>much</u>* broader param' coverage & higher accuracy**



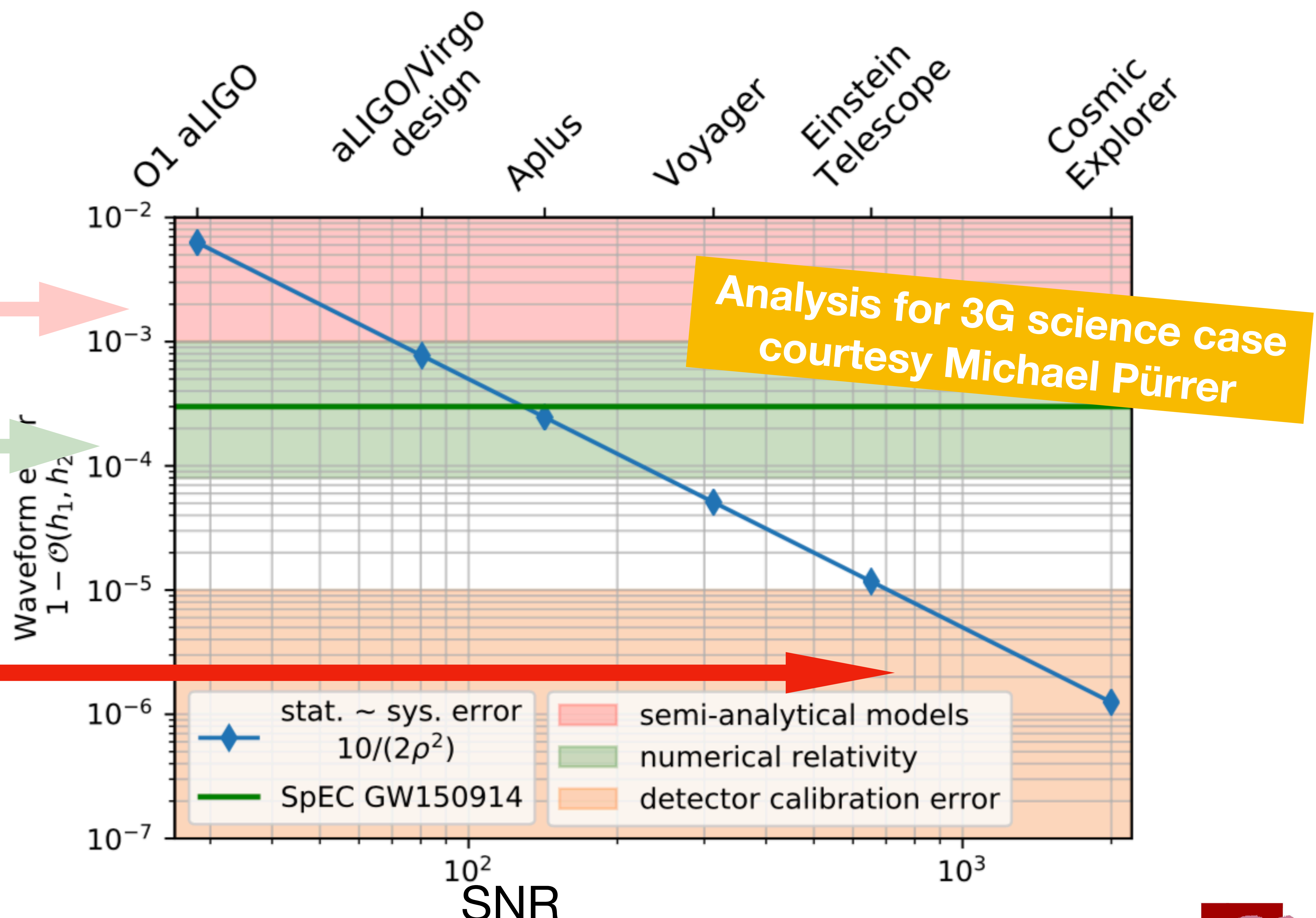


Required accuracy s.t. **systematic errors ≲ statistical errors**

Current models
EOB, Phenom
—circular only—

Current NR
q~1

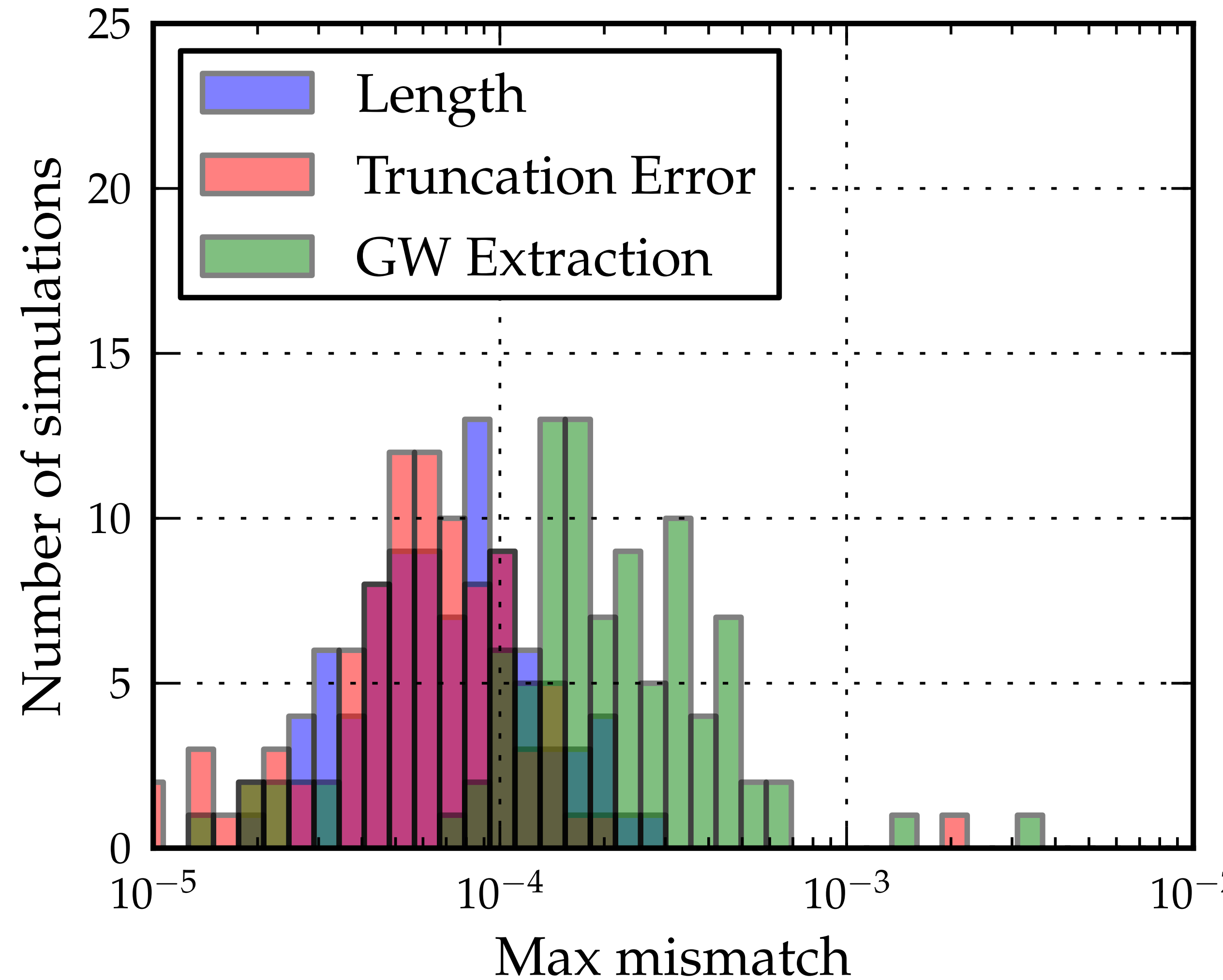
Needed for
SNR=1000



Accuracy

- good for detection (3G+LISA)
- good for *today's* LIGO/Virgo events

- Accuracy improvement **tedious**
  - must carefully control *several* error sources



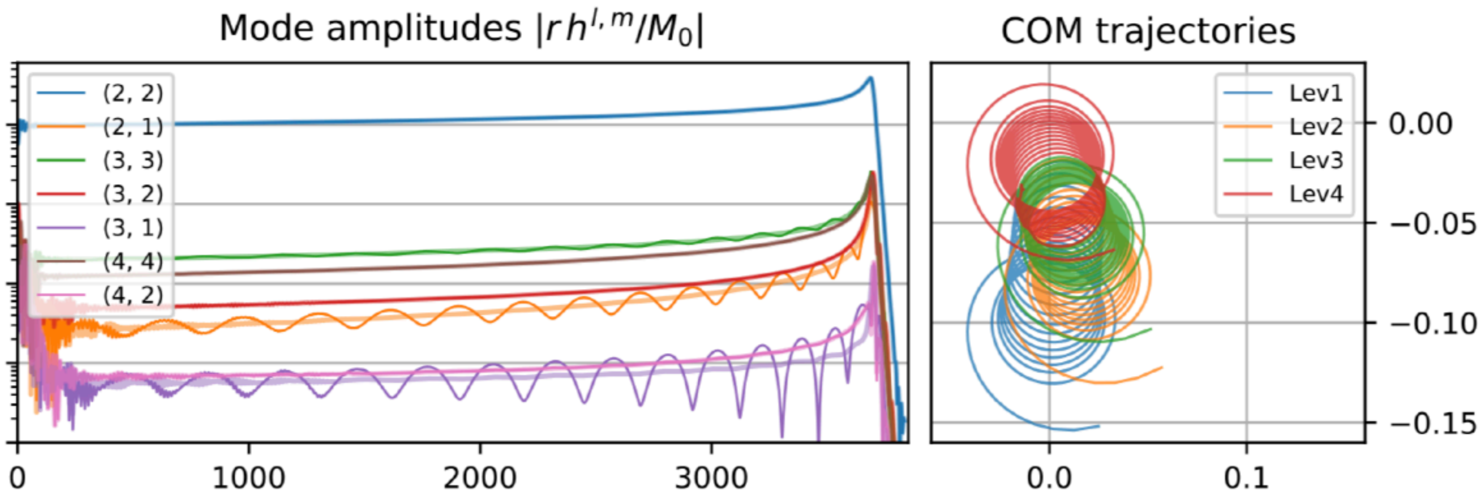
Chu, Fong, Kumar, HP+SXS 2016
— (2,2) mode —

- No apparent difficulty with precession, eccentricity, higher modes, ring-down

Improving NR accuracy (some efforts)

Centre of mass correction

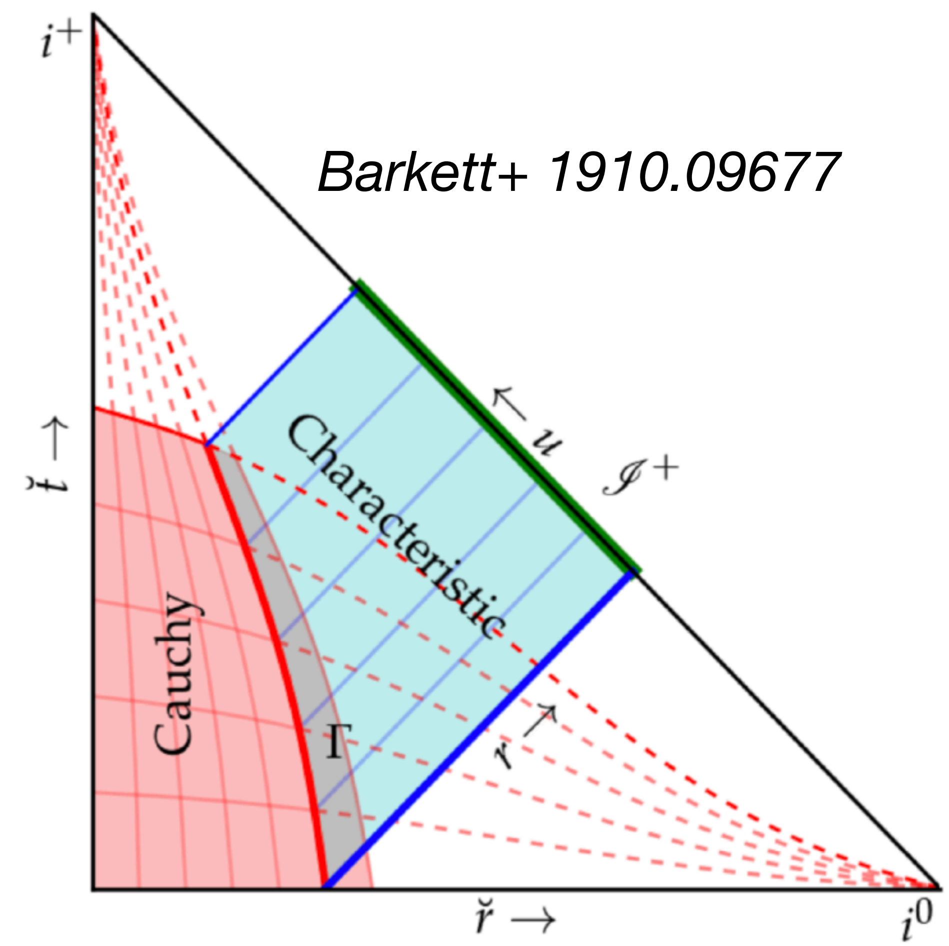
cleaner higher modes



Woodford, Boyle, HP 1904.04842

Spectral Cauchy Characteristic Extraction

remove gauge-effects

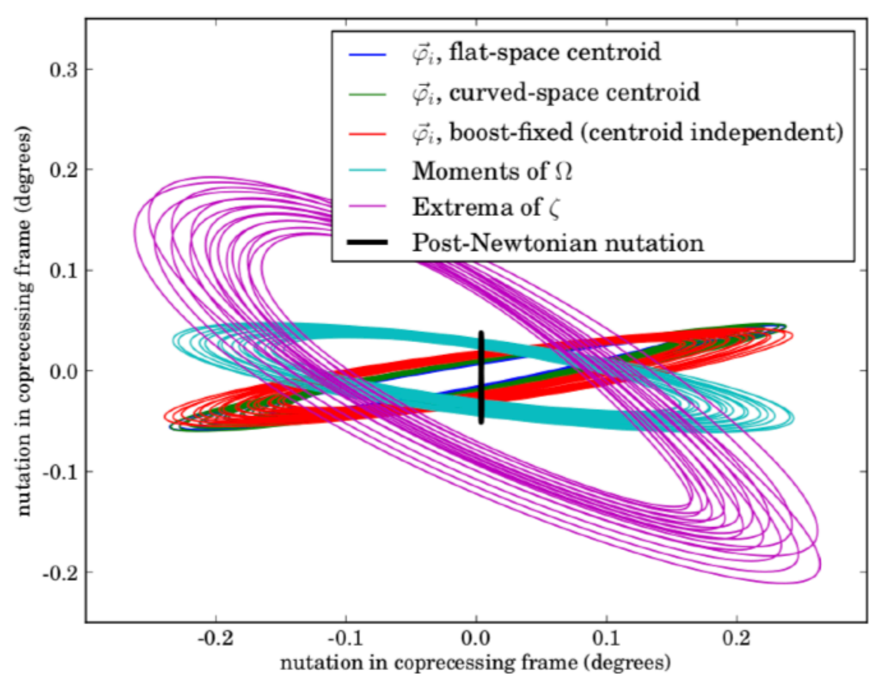


Barkett+ 1910.09677

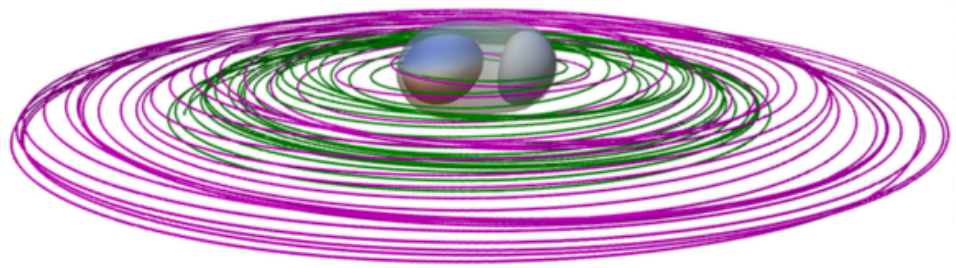
Definition of spin in NR

reproduce PN nutations

Owen+ PRD 2019 (1708.07325)



NR records

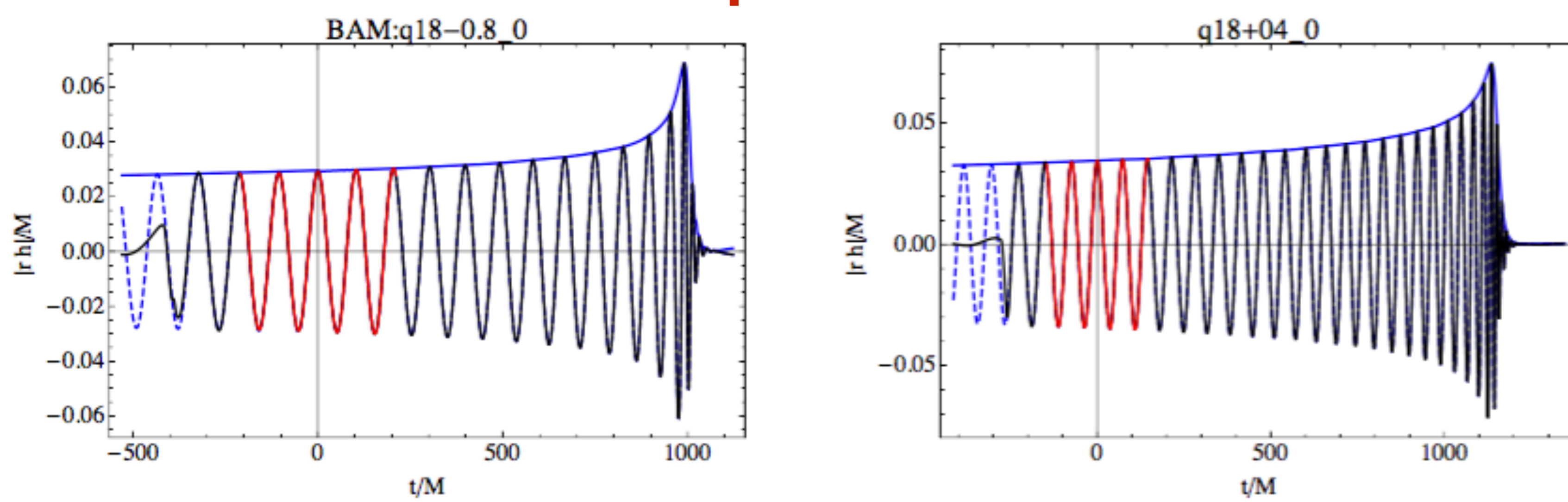


q=1: **S/M²=0.994**
q<1: S/M²=0.95

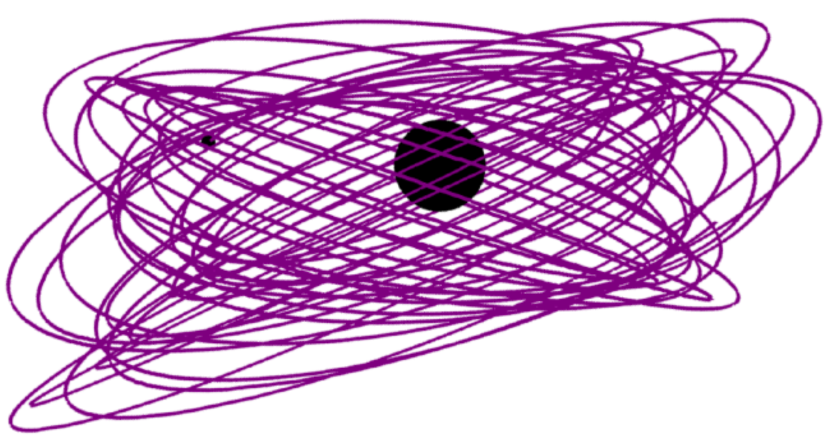
Scheel+ 14
Lovelace..HP+15

Husa+ 15

q=1/18

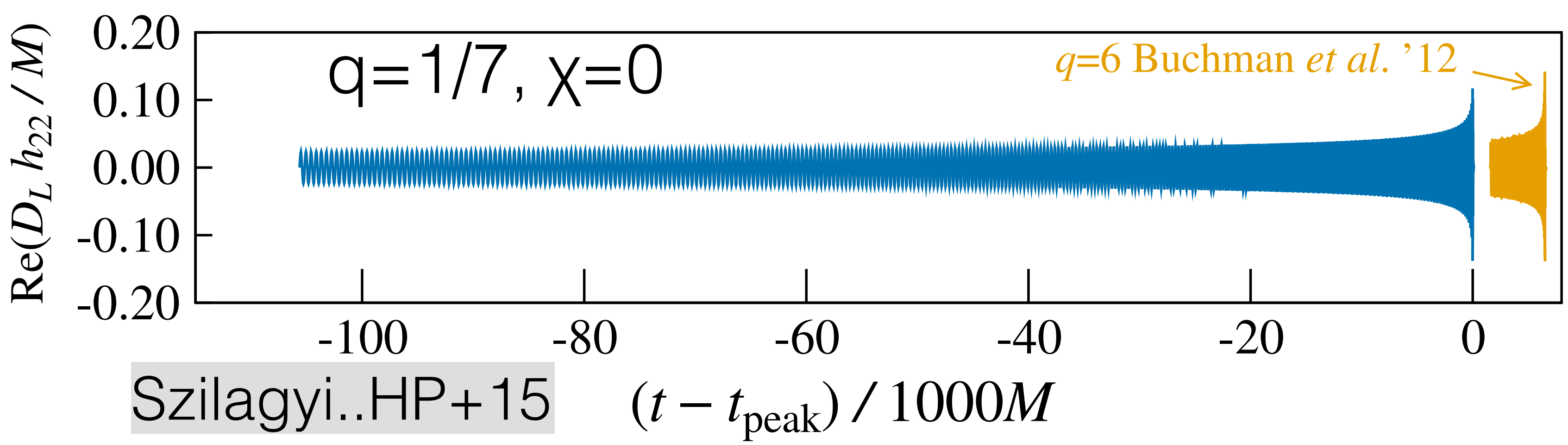


eccentric, precessing q=7



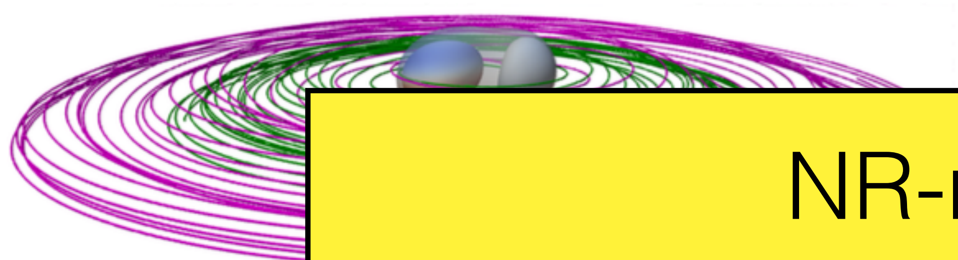
Lewis, Zimmerman, HP 17

350GW cycles

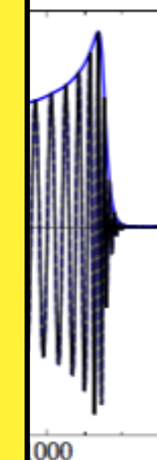


Szilagyi..HP+15

NR records



Husa+ 15



NR-records several years old.
Since then:
denser & improved sampling of “easy” regions
little progress with “hard” regions

hard: two of $\{q \gtrsim 5\}$, $\{\chi_1 \gtrsim 0.9\}$, $\{\chi_2 \gtrsim 0.5\}$, $\{N_{cycle} \gtrsim 50\}$

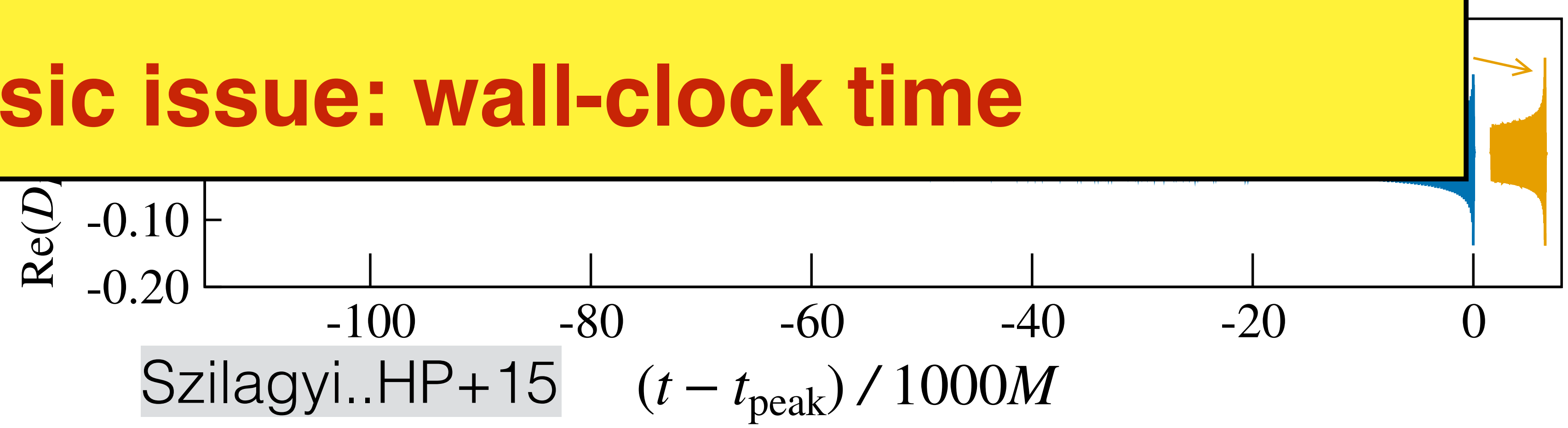
Basic issue: wall-clock time

q=1:
q<1:

Scheel+ 14
Lovelace..H

eccentric

Lewis, Zimmerman, HP 17



Szilagyi..HP+15

Hardest limitation: ***wall-time***

q — more steps per orbit
(Courant limit — *numerics*)
q — more orbits per inspiral
(physics)
$(M\Omega)^{8/3}$ — start frequency

$\chi \gtrsim 0.6$: extra factor ~$1/(1-\chi_1)(1-\chi_2)$
χ_2 larger impact than χ_1

- Scaling of number of time-steps

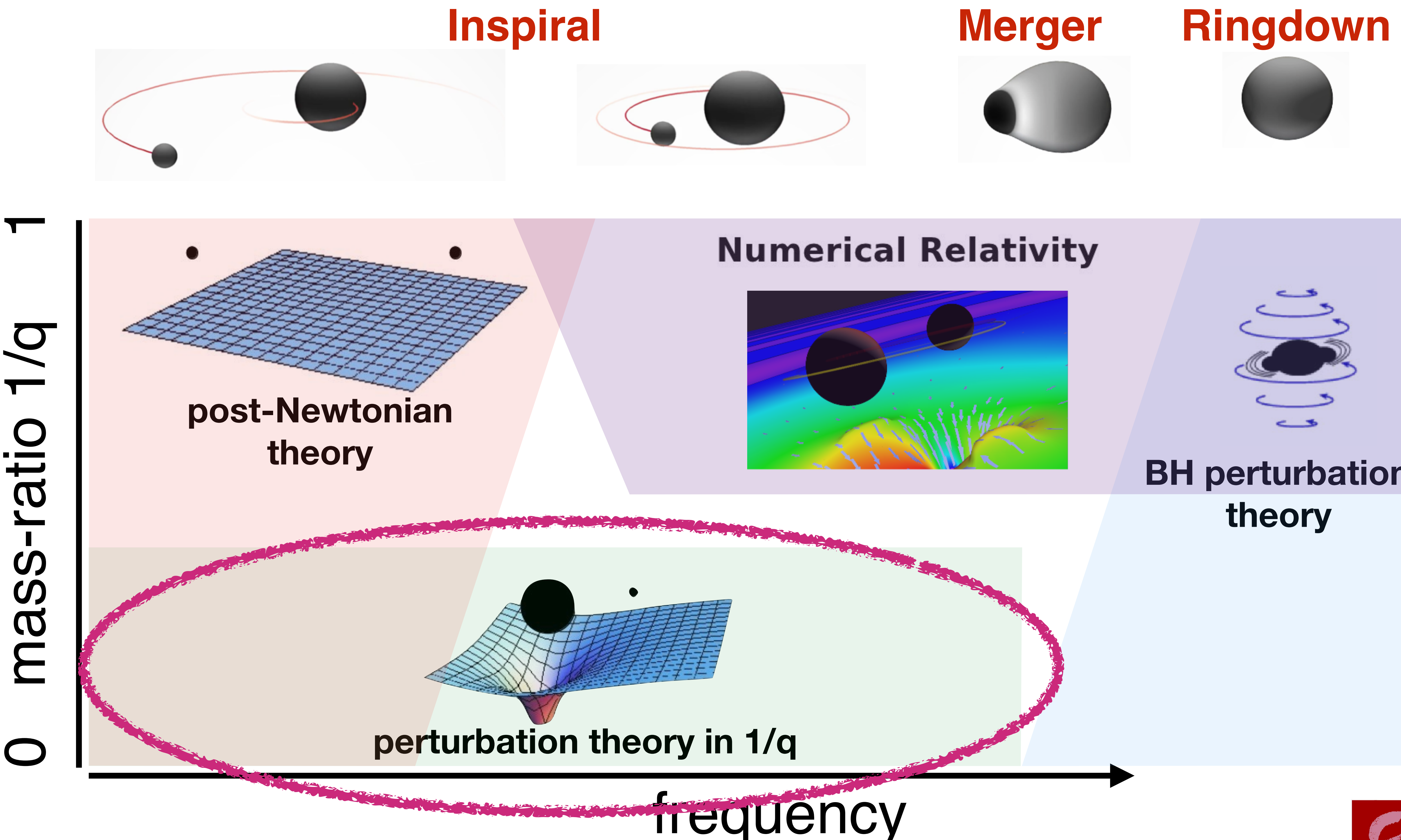
$$N_{\text{steps}} \propto \frac{q^2}{(M\Omega_i)^{8/3}}$$

- Factor 2 in mass-ratio, factor 2 in low-frequency, higher accuracy
  ➔ **O(100) increase in wall-time (with current codes)**

- Need:
  - Better parallel scaling ➔ reduce constant of proportionality
  - Circumvent small BH courant limit ➔ mitigate q-scaling
  - ***Either requires nearly complete re-development of NR code***

- ***... and help from perturbative methods***
  - high order PN and PM; resummation through EOB ➔ larger Ω_i
  - 2nd order small-mass-ratio perturbation theory ➔ less extreme q

Methods for modeling BBH



Small mass-ratio limit (SMR a.k.a. “grav. self-force”)

$$\Phi(M\omega) = \frac{1}{\nu}\Phi_0(M\omega) + \Phi_1(M\omega) + \nu\,\Phi_2(M\omega) + \ldots$$

First order:
generic orbits known
van de Meent 2017

Third order:
no efforts to calculate

Second order:
circular around Schwarzschild
Pound+ 1908.07419

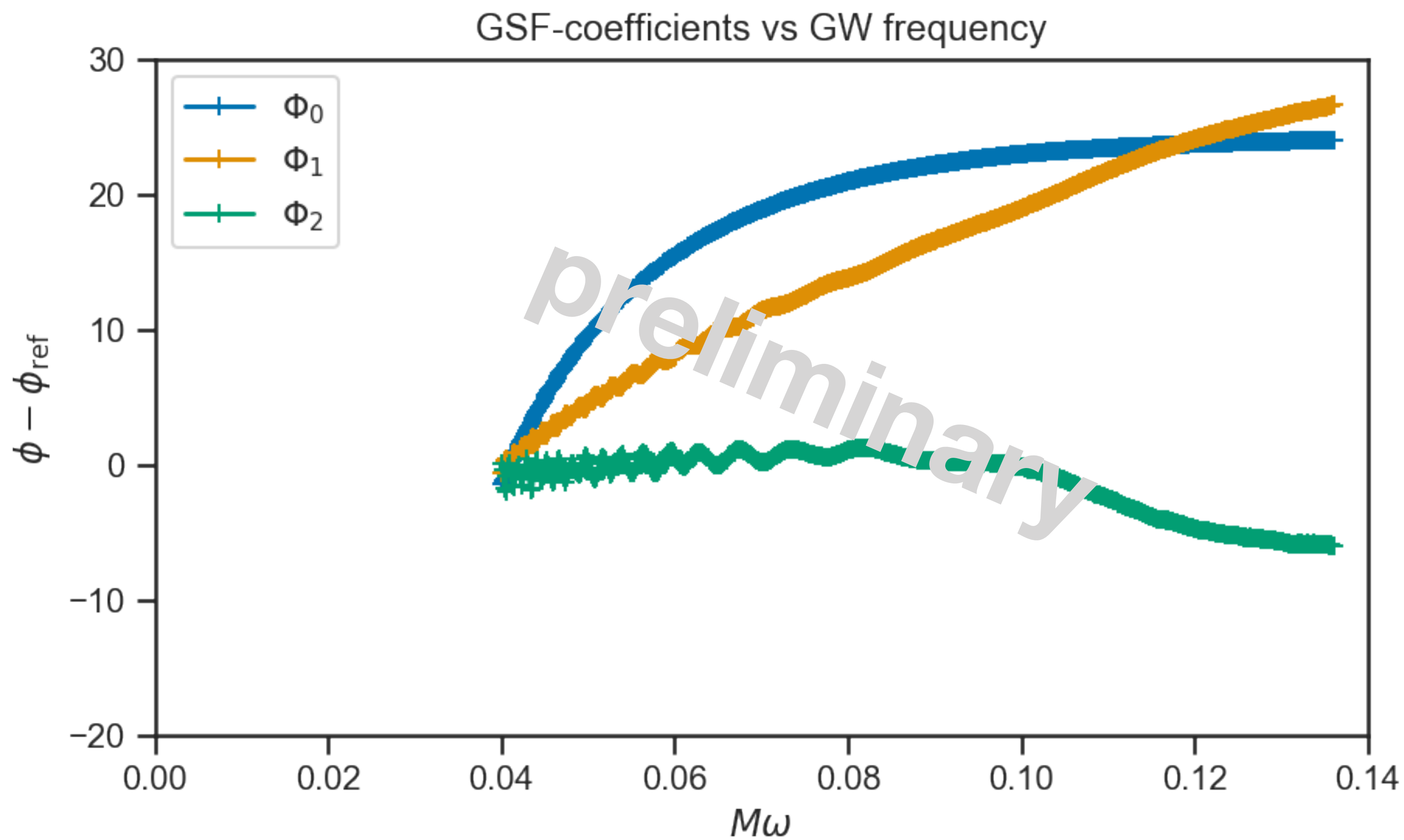
Small mass-ratio (SMR) from NR

$$\Phi(M\omega) = \frac{1}{\nu}\Phi_0(M\omega) + \Phi_1(M\omega) + \nu\,\Phi_2(M\omega) + \dots$$

- Fit GW-phase Φ to NR simulations up to q=10
- Φ_2 is *remarkably small*

- ***Regions of validity of NR and 2nd order SMR remarkably close***

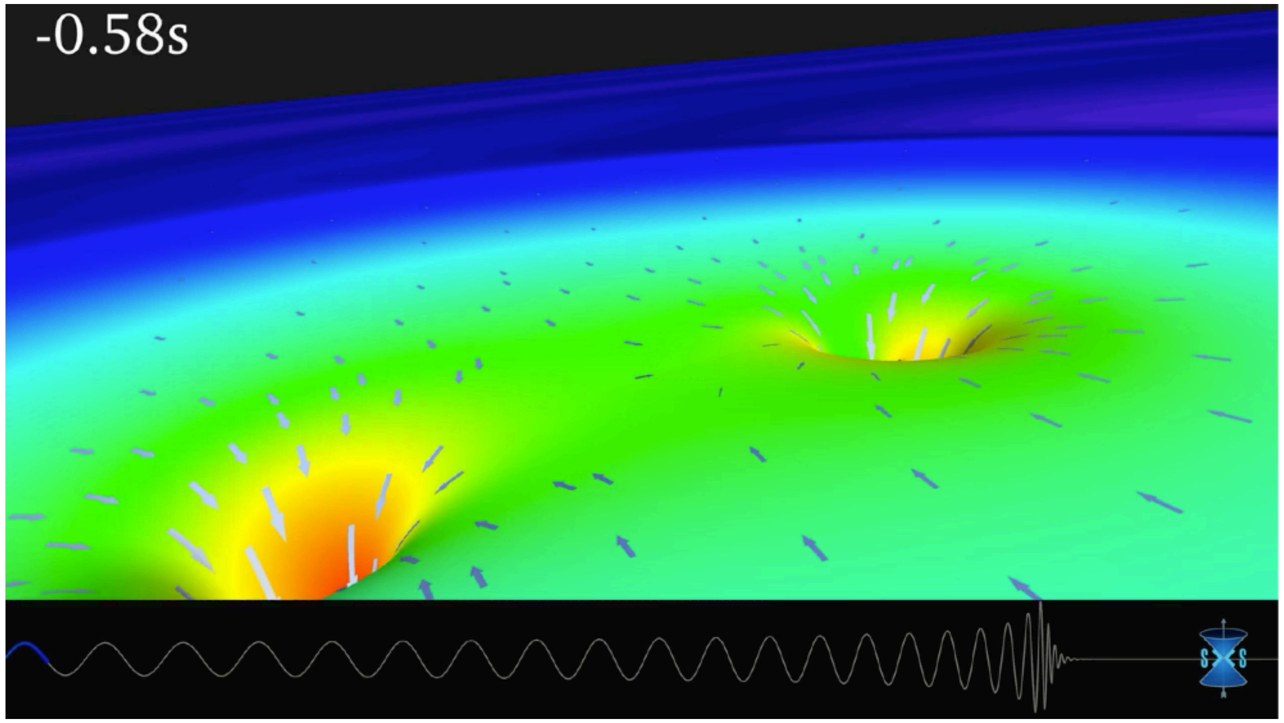
- Caveat: Analysis only for non-spinning and circular binaries.



van de Meent & HP, in prep

Summary

- 3G detectors & LISA require **significant but likely possible improvements** over state-of-the-art:
  - accuracy
  - length
  - parameter space
  - high spins
- **Biggest challenge**
  - **high mass-ratio**



- The **mass-ratio gap** between NR and 2nd order SMR may be **small or even absent**

Summary

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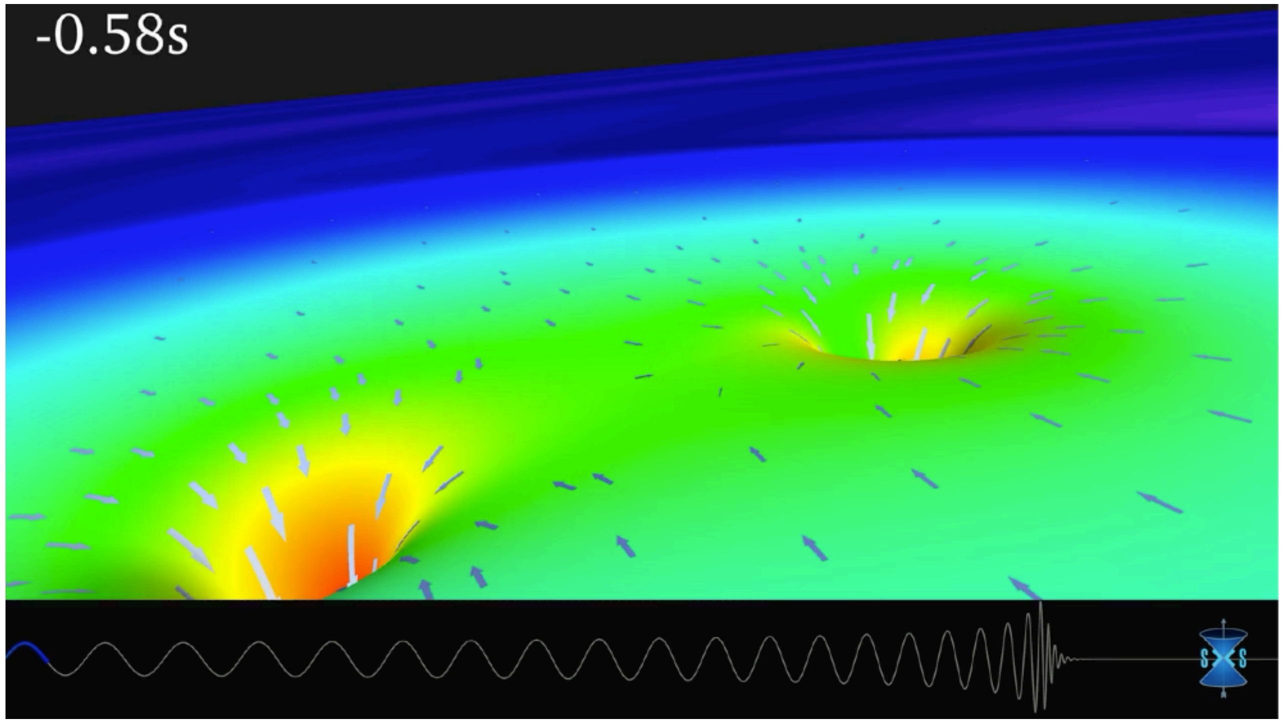

- The **mass-ratio gap** between NR and 2nd order SMR may be **small or even absent**