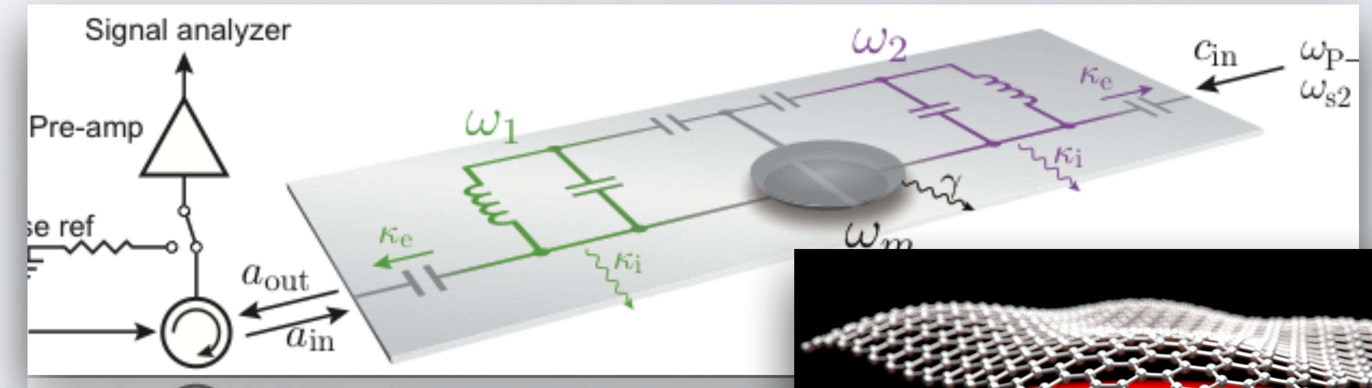
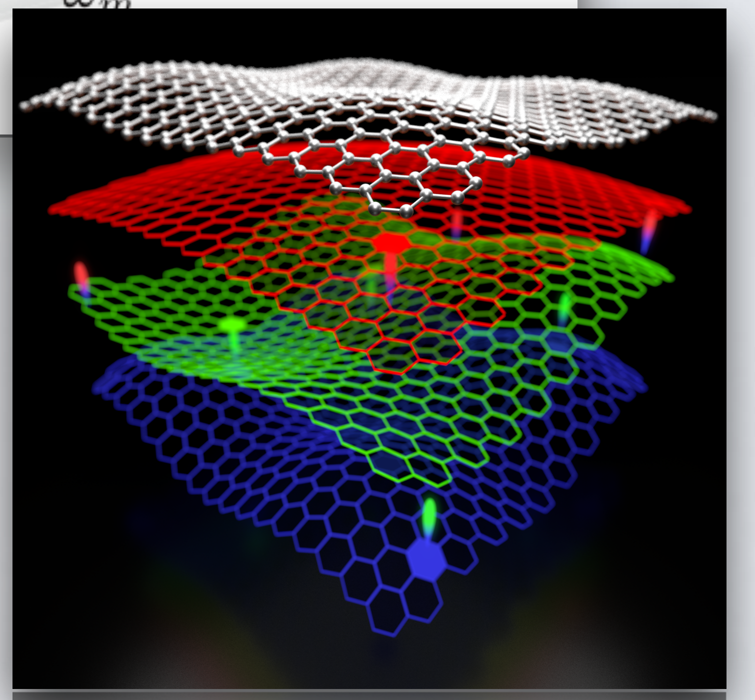
University of
South-Eastern Norway

NANOSCIENCE: QUANTUM PHYSICS GOES MACROSCOPIC

F. Massel

Dept. of Science and Industry systems
University of South-Eastern Norway



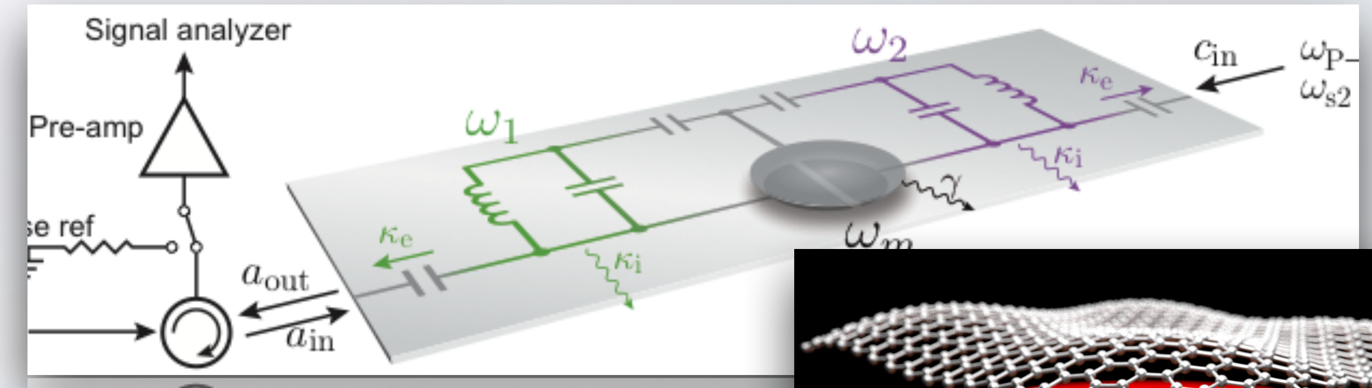


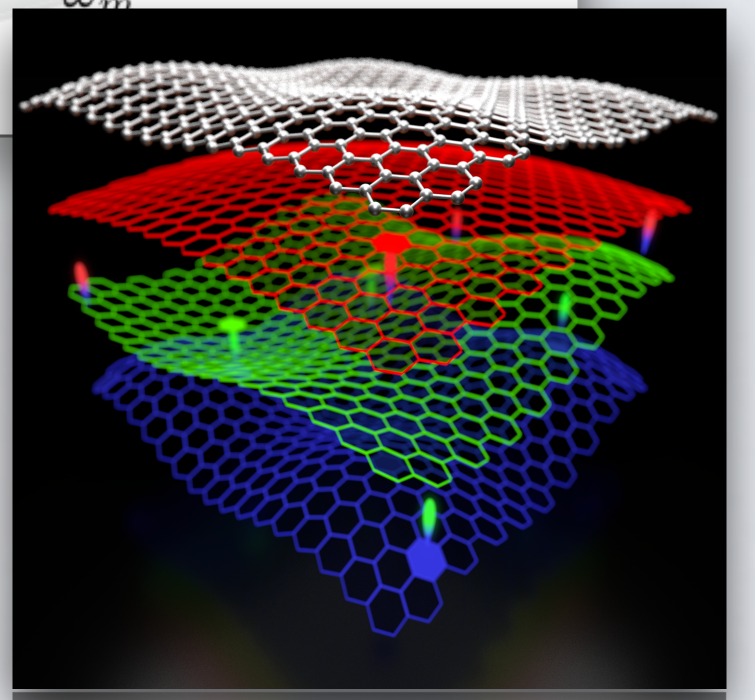
University of
South-Eastern Norway

MACROSCOPIC QUANTUM STATES IN OPTOMECHANICS

F. Massel

Dept. of Science and Industry systems
University of South-Eastern Norway





A WORLD VIEW

How do we perceive the world around us?

A WORLD VIEW

How do we perceive the world around us?

Rather simple answer: through our senses, but combined with our "view" of the world.

A WORLD VIEW

How do we perceive the world around us?

- Rather simple answer: through our senses, but combined with our "view" of the world.
- With modern science, measuring apparatuses appear to enhance our senses and change our world view.

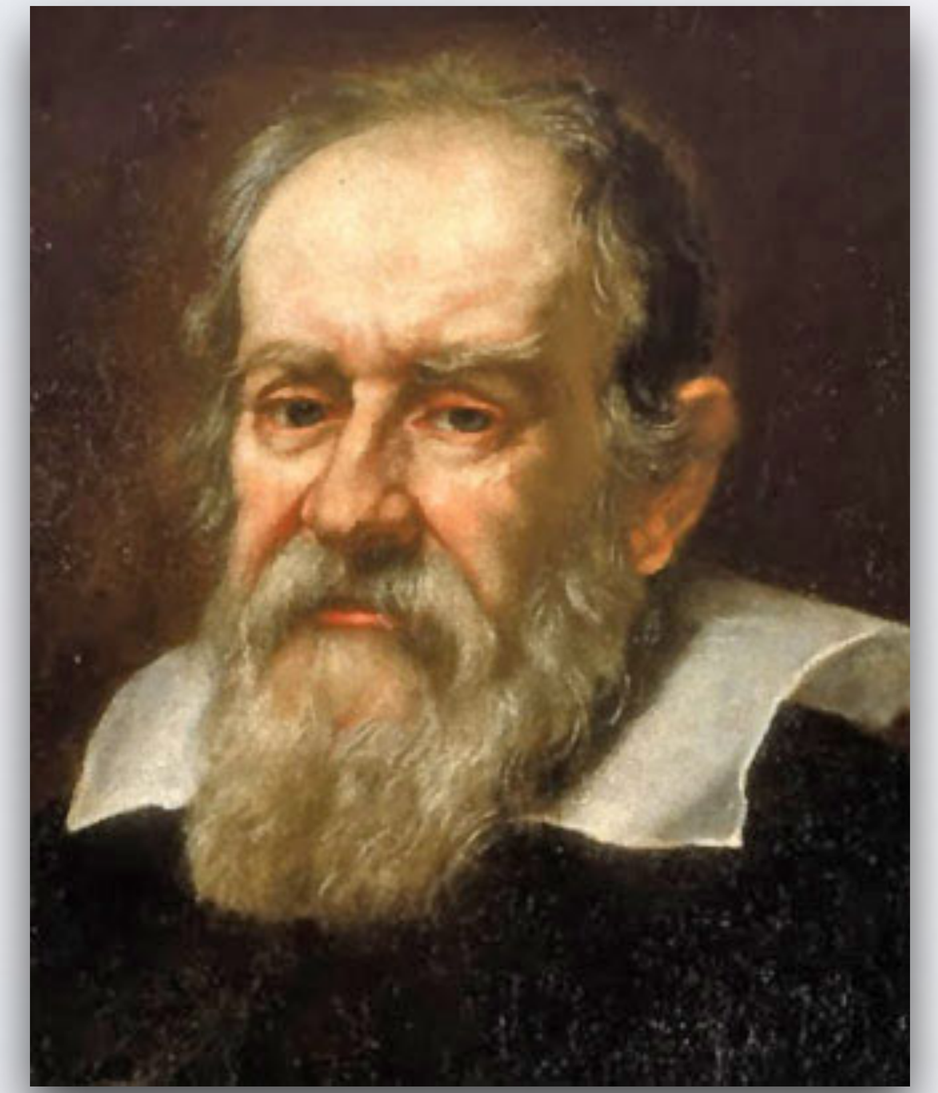
A WORLD VIEW

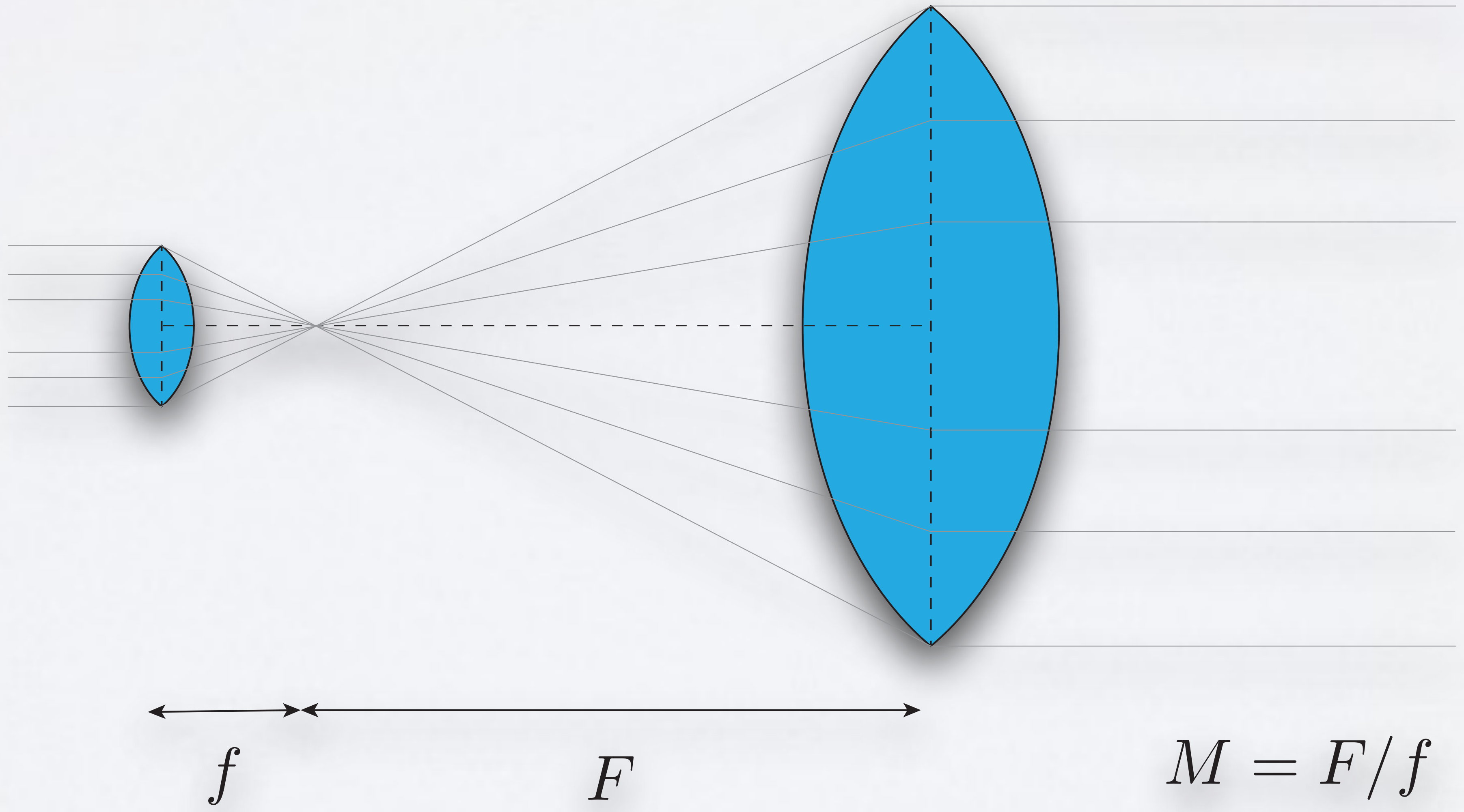
How do we perceive the world around us?

Rather simple answer: through our senses, but combined with our "view" of the world.

With modern science, measuring apparatuses appear to enhance our senses and change our world view.

Galileo with the telescope: observation of the moons of Jupiter: revolution in the picture of the cosmos by Aristoteles





NANO (QUANTUM) PHYSICS PERSPECTIVE

Quantum physics (along with the theory of relativity) is the perfect example of how reality inner mechanisms escape our senses.

NANO (QUANTUM) PHYSICS PERSPECTIVE

Quantum physics (along with the theory of relativity) is the perfect example of how reality inner mechanisms escape our senses.

In the quantum world, things have properties that are qualitatively different from what we are used to:

NANO (QUANTUM) PHYSICS PERSPECTIVE

Quantum physics (along with the theory of relativity) is the perfect example of how reality inner mechanisms escape our senses.

In the quantum world, things have properties that are qualitatively different from what we are used to:

-they can simultaneously behave as particles and waves;

NANO (QUANTUM) PHYSICS PERSPECTIVE

Quantum physics (along with the theory of relativity) is the perfect example of how reality inner mechanisms escape our senses.

In the quantum world, things have properties that are qualitatively different from what we are used to:

-they can simultaneously behave as particles and waves;

-they can travel (unscathed) through walls;

NANO (QUANTUM) PHYSICS PERSPECTIVE

Quantum physics (along with the theory of relativity) is the perfect example of how reality inner mechanisms escape our senses.

In the quantum world, things have properties that are qualitatively different from what we are used to:

-they can simultaneously behave as particles and waves;

-they can travel (unscathed) through walls;

-they can be in two (or more) states —like position and velocity—at the same time.

NANO (QUANTUM) PHYSICS PERSPECTIVE

Quantum physics (along with the theory of relativity) is the perfect example of how reality inner mechanisms escape our senses.

In the quantum world, things have properties that are qualitatively different from what we are used to:

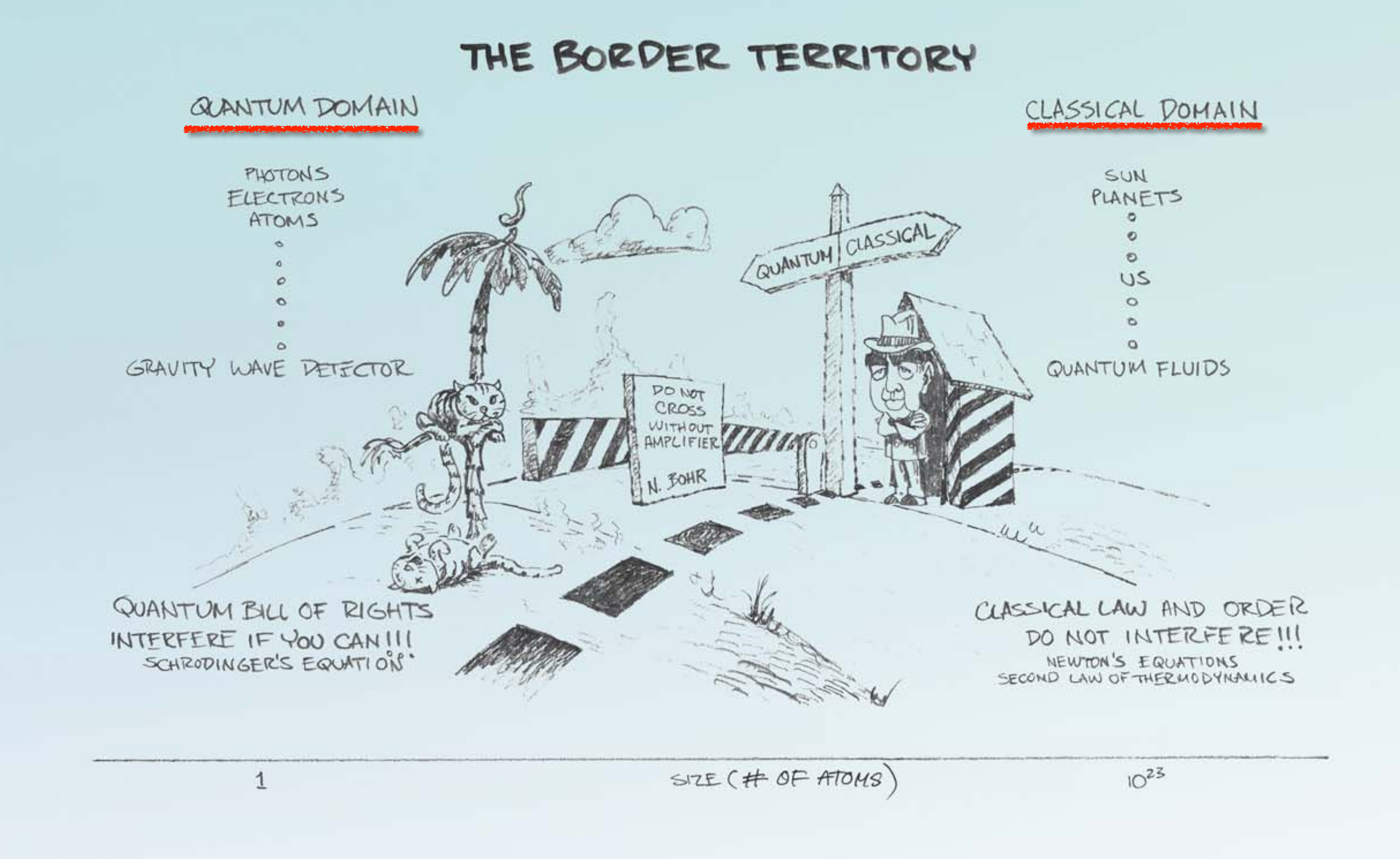
-they can simultaneously behave as particles and waves;

-they can travel (unscathed) through walls;

-they can be in two (or more) states —like position and velocity—at the same time.

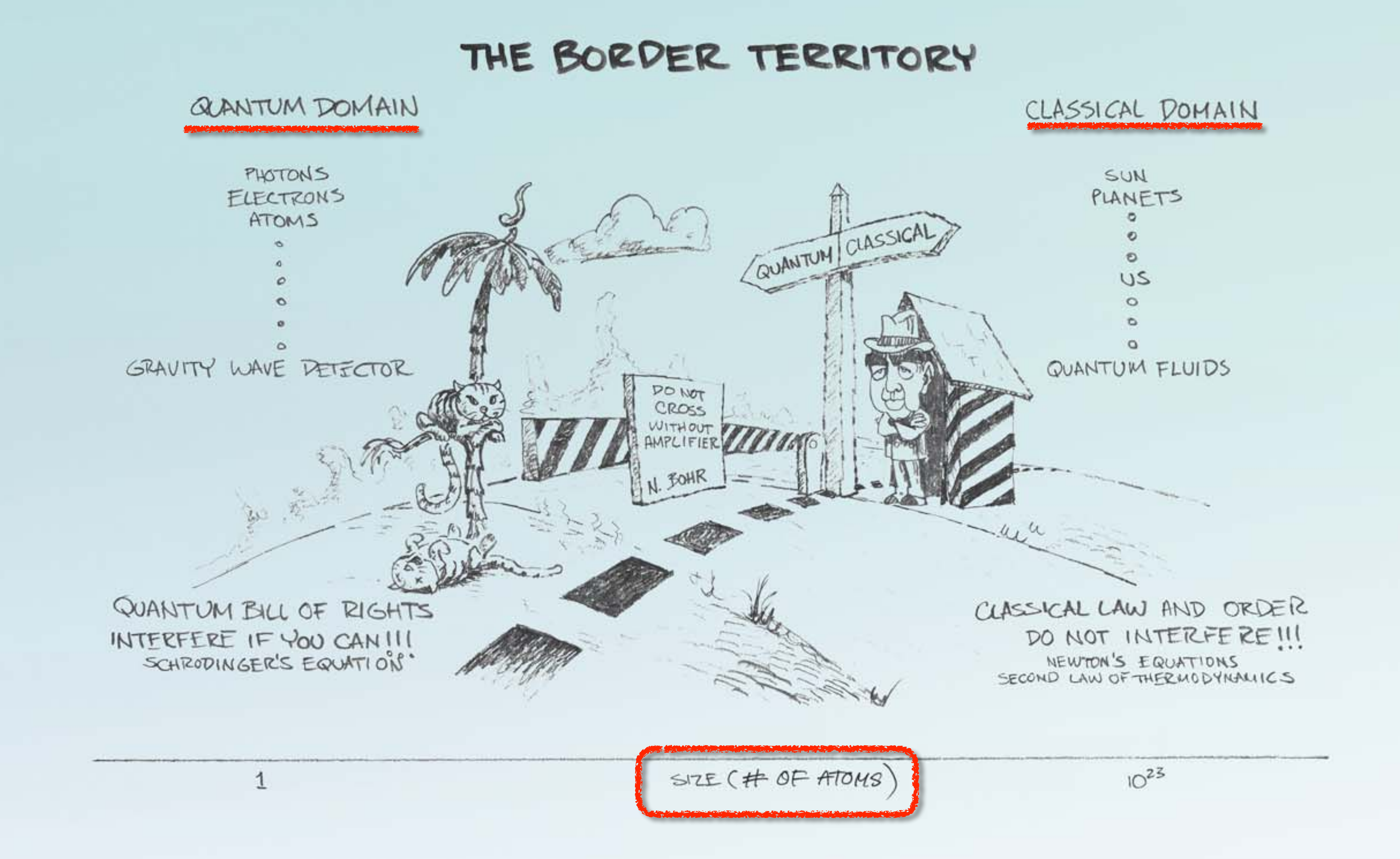
These properties are "weird" to us, because we are not used to them in our experience of the world.

NANO (QUANTUM) PHYSICS PERSPECTIVE



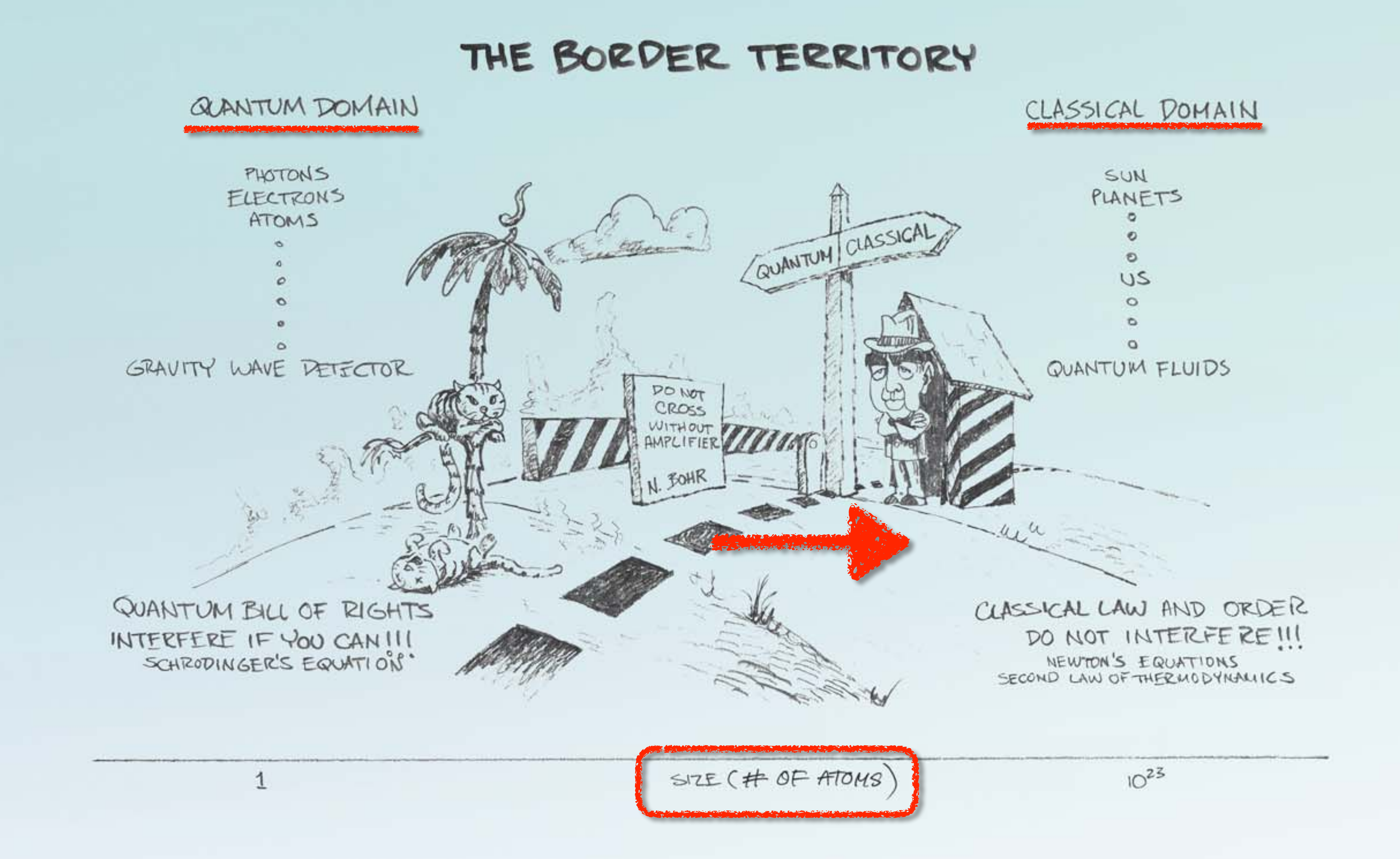
W. Zurek, Phys. Today. 44 10, (1991)

NANO (QUANTUM) PHYSICS PERSPECTIVE



W. Zurek, Phys. Today. 44 10, (1991)

NANO (QUANTUM) PHYSICS PERSPECTIVE



W. Zurek, Phys. Today. 44 10, (1991)

NANO (QUANTUM) PHYSICS PERSPECTIVE

NANO (QUANTUM) PHYSICS PERSPECTIVE

-Large scale, compared to atomic, subatomic or Planck scale

NANO (QUANTUM) PHYSICS PERSPECTIVE

-Large scale, compared to atomic, subatomic or Planck scale

-Border between quantum and classical:

NANO (QUANTUM) PHYSICS PERSPECTIVE

-Large scale, compared to atomic, subatomic or Planck scale

-Border between quantum and classical:

-theoretical interest: how does classical physics emerge from the quantum world;

NANO (QUANTUM) PHYSICS PERSPECTIVE

-Large scale, compared to atomic, subatomic or Planck scale

-Border between quantum and classical:

-theoretical interest: how does classical physics emerge from the quantum world;

-applied interest: q. enhanced measurements:

NANO (QUANTUM) PHYSICS PERSPECTIVE

-Large scale, compared to atomic, subatomic or Planck scale

-Border between quantum and classical:

-theoretical interest: how does classical physics emerge from the quantum world;

-applied interest: q. enhanced measurements:

-Shot noise & SQL vs Heisenberg limit

NANO (QUANTUM) PHYSICS PERSPECTIVE

-Large scale, compared to atomic, subatomic or Planck scale

-Border between quantum and classical:

-**theoretical interest**: how does classical physics emerge from the quantum world;

-**applied interest**: q. enhanced measurements:

-Shot noise & SQL vs Heisenberg limit

-Ligo injection of squeezed light

NANO (QUANTUM) PHYSICS PERSPECTIVE

Two examples:

NANO (QUANTUM) PHYSICS PERSPECTIVE

Two examples:

-superconducting circuits

NANO (QUANTUM) PHYSICS PERSPECTIVE

Two examples:

- superconducting circuits
  - LC circuit quantization
  - charge qubit (Cooper-pair box)

NANO (QUANTUM) PHYSICS PERSPECTIVE

Two examples:

- superconducting circuits
    - LC circuit quantization
    - charge qubit (Cooper-pair box)
- optomechanical systems

NANO (QUANTUM) PHYSICS PERSPECTIVE

Two examples:

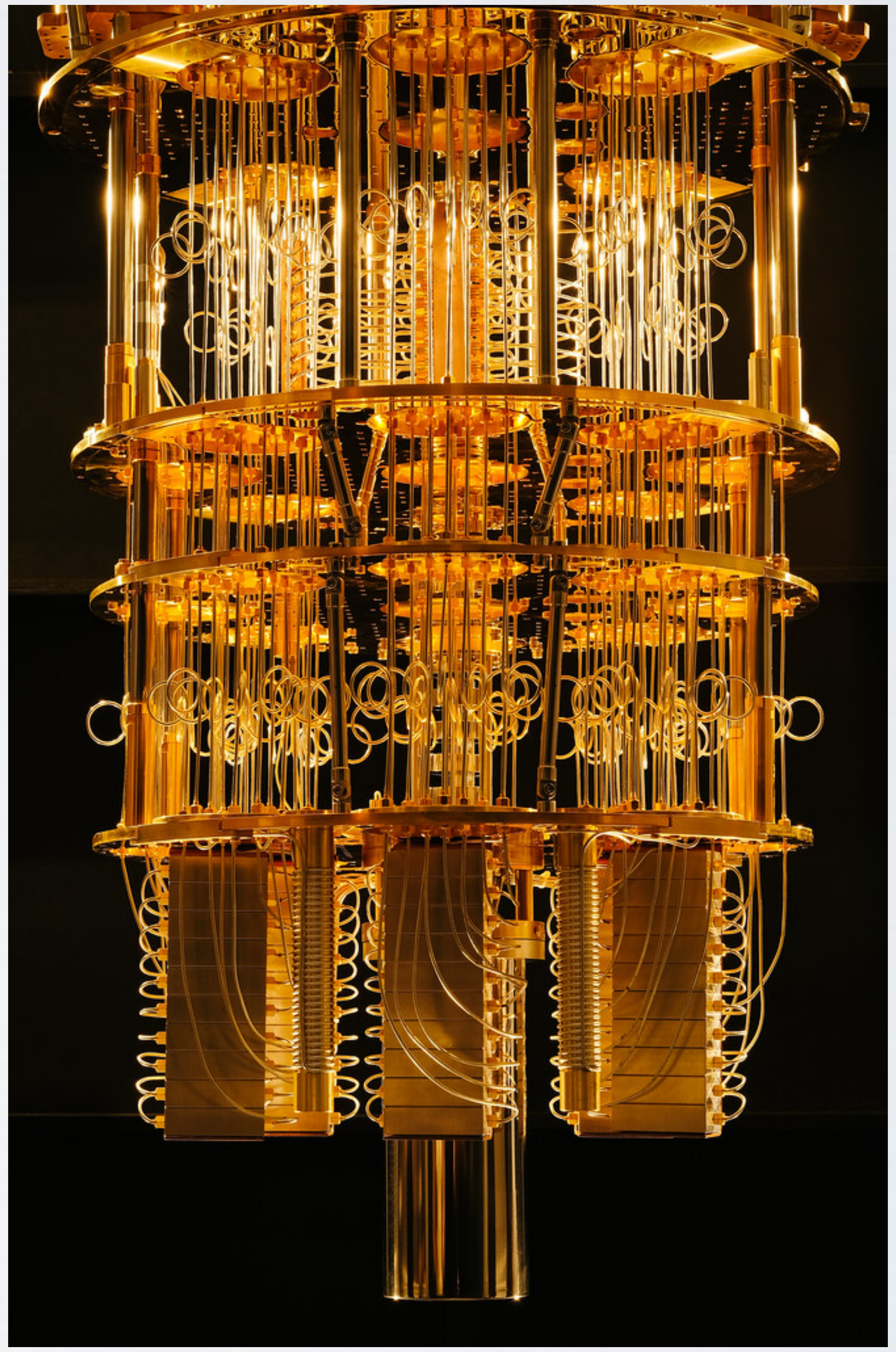
- -superconducting circuits
  - -LC circuit quantization
  - -charge qubit (Cooper-pair box)
- -optomechanical systems
  - -general idea (cooling, amplification, *ad libitum*)

EXAMPLE 1: SUPERCONDUCTING QUBITS

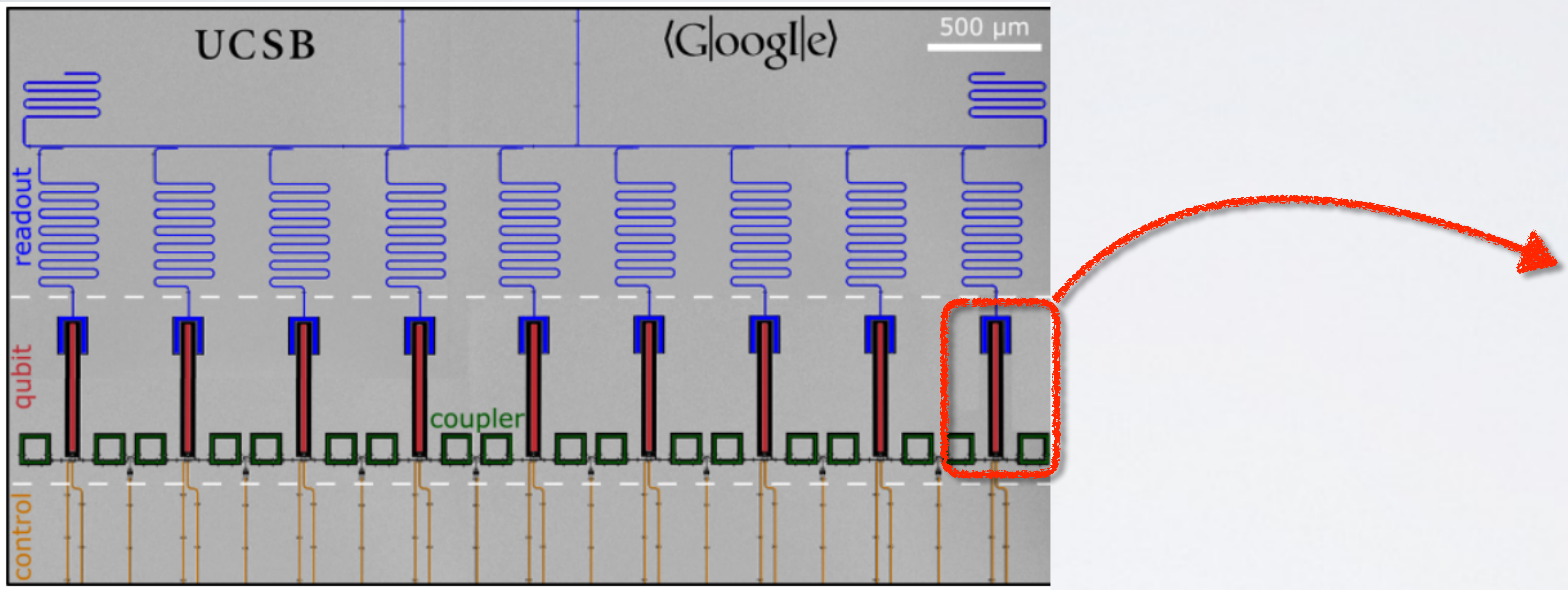
Superconducting circuits ,most promising candidates for quantum computation architectures

Google

IBM

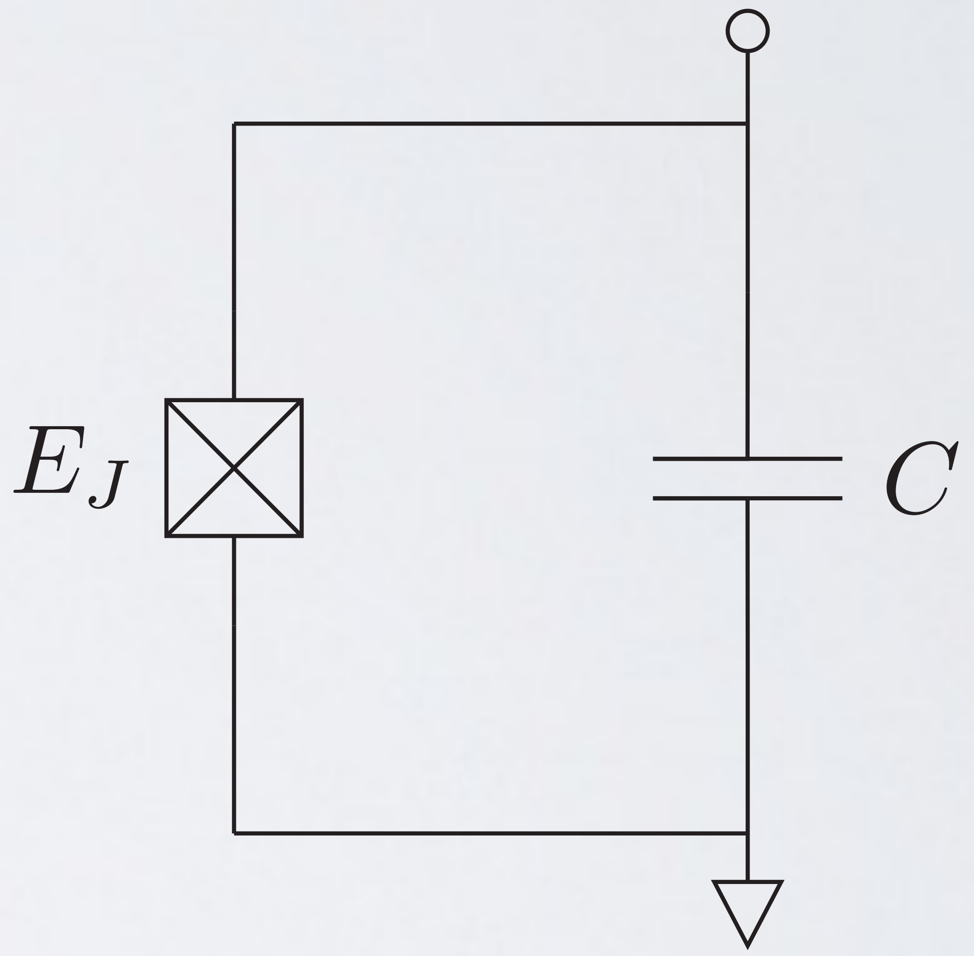


EXAMPLE 1: SUPERCONDUCTING CIRCUITS

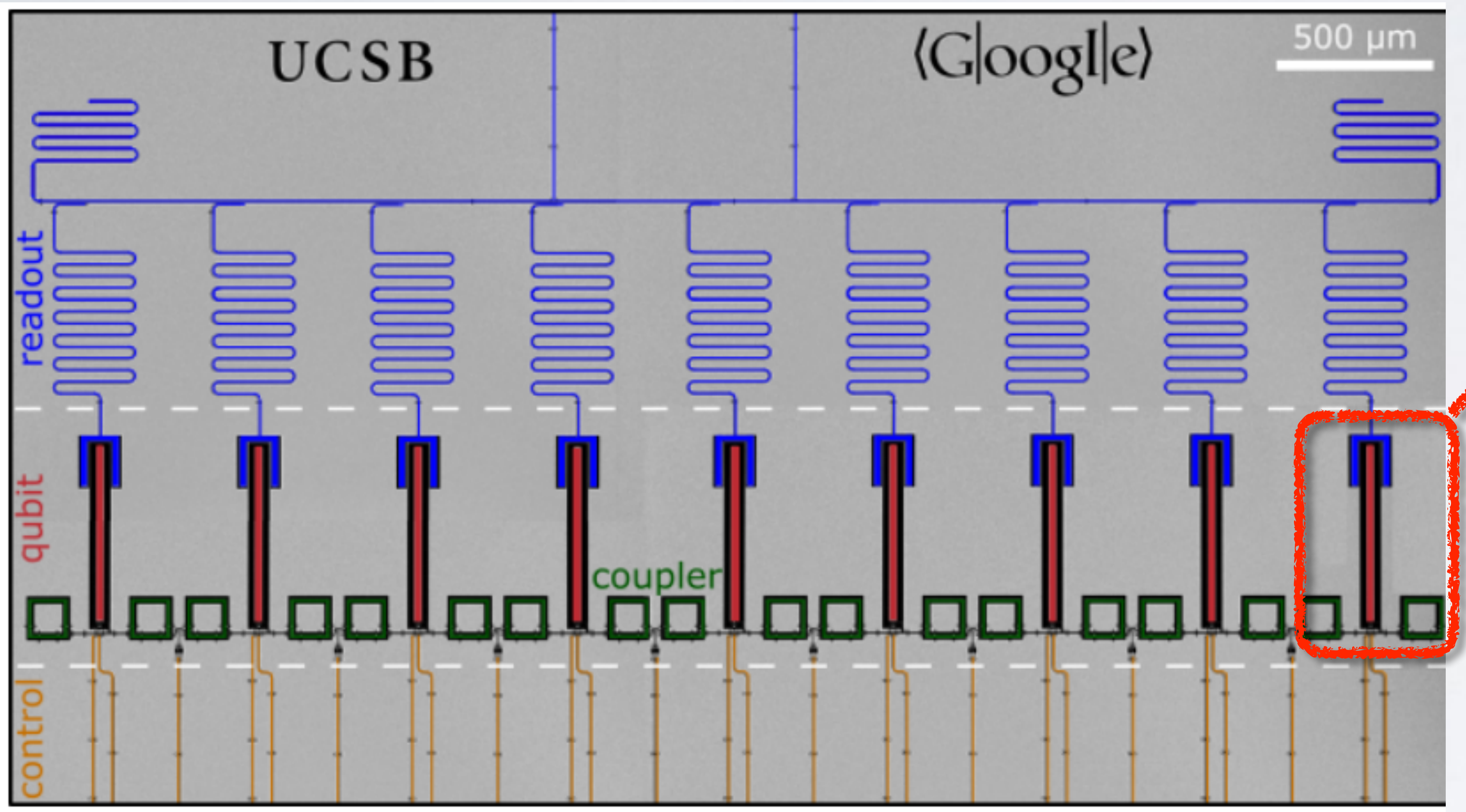


*J. Martinis @ Google & UCSB

$\simeq$ charge qubit
(Cooper-pair box)

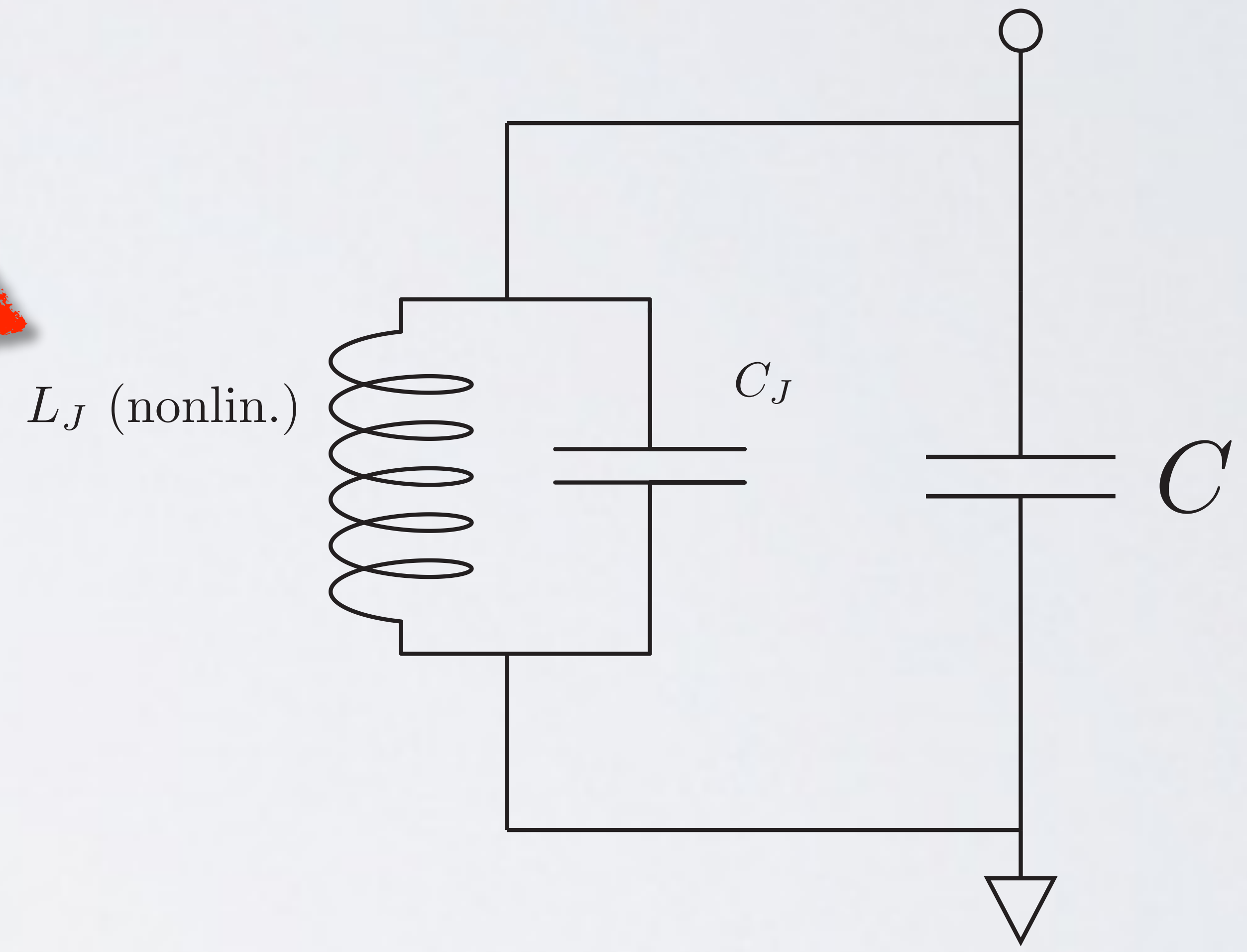


EXAMPLE 1: SUPERCONDUCTING CIRCUITS

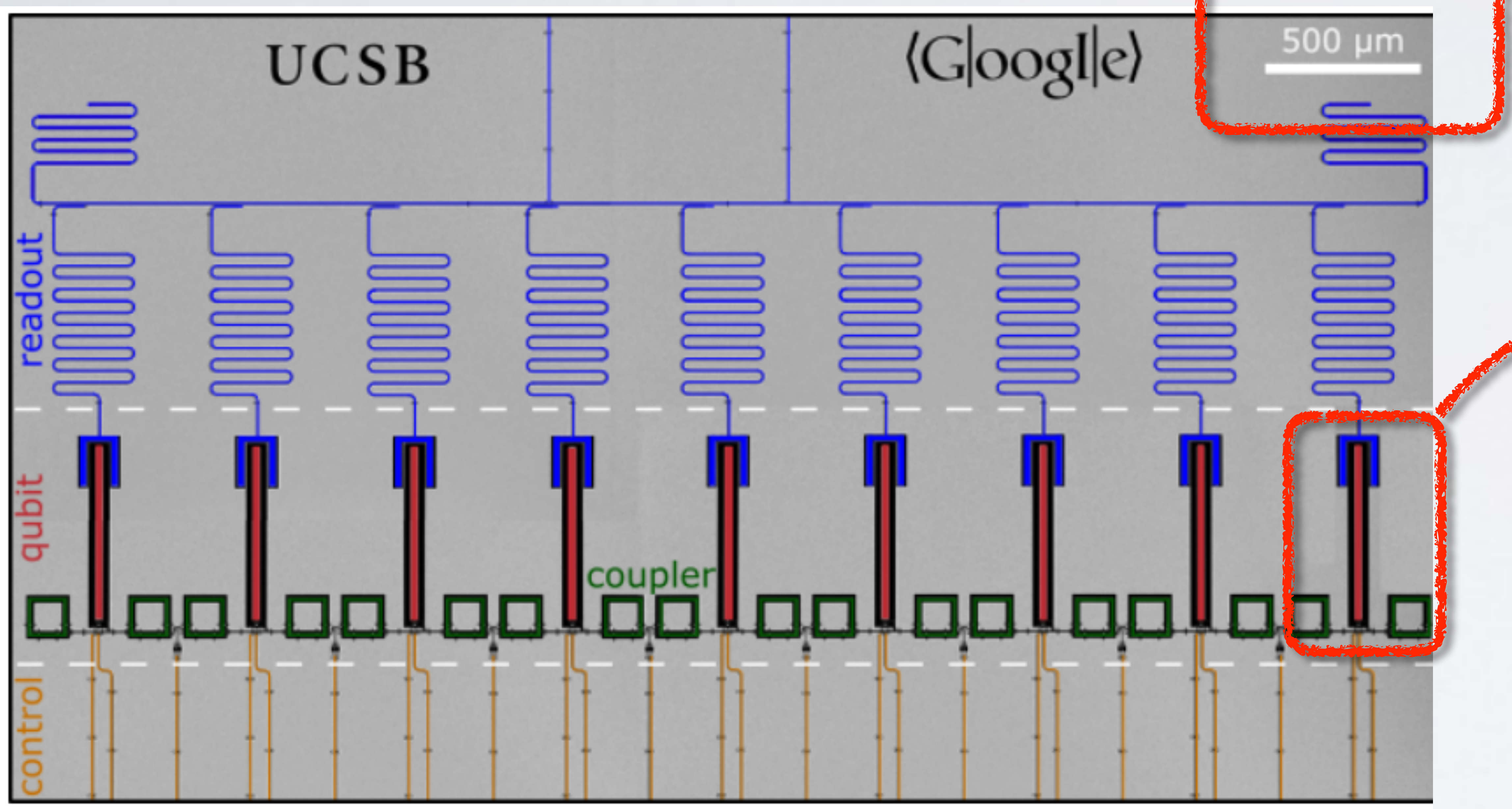


*J. Martinis @ Google & UCSB

≃ charge qubit
(Cooper-pair box)

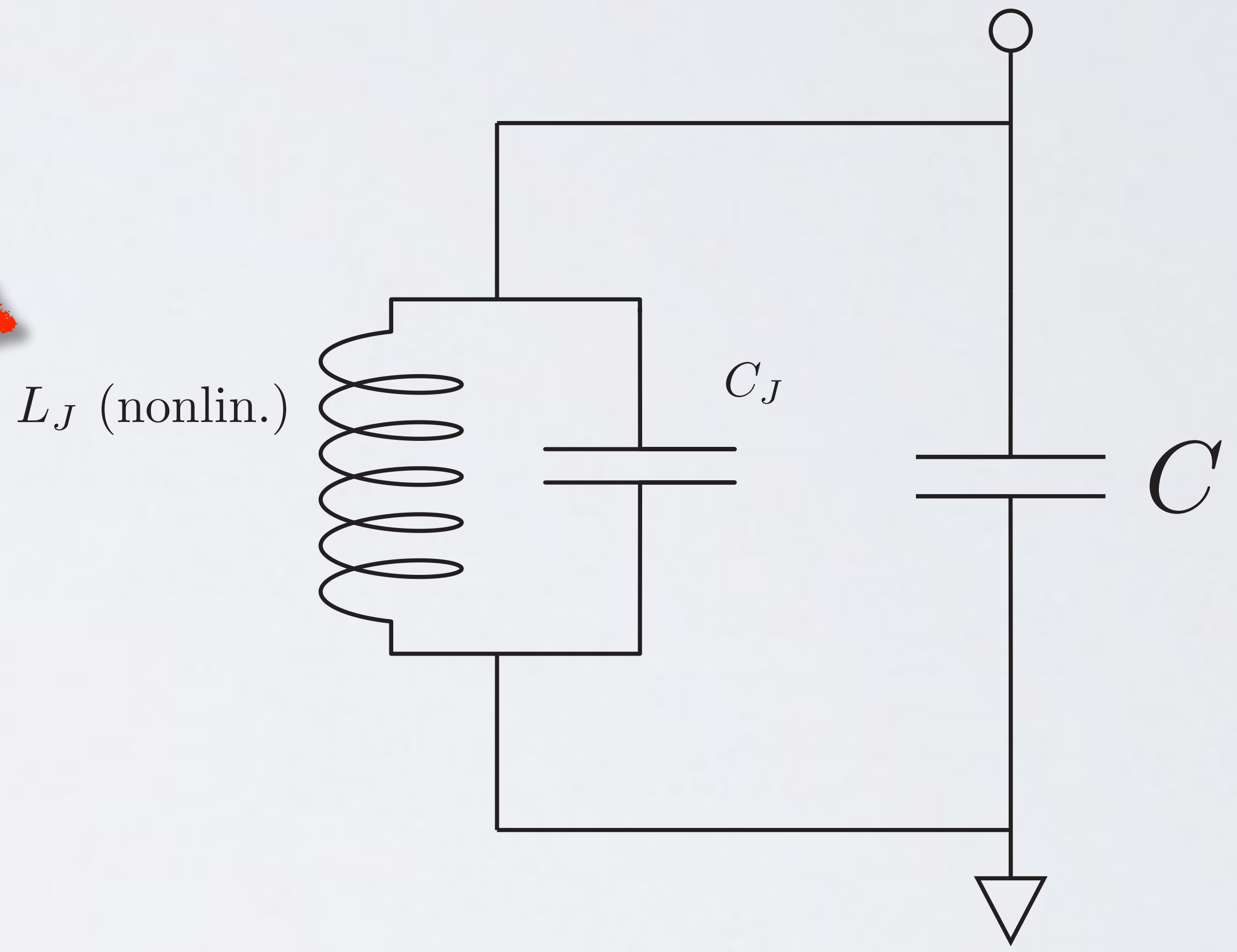


EXAMPLE 1: SUPERCONDUCTING CIRCUITS



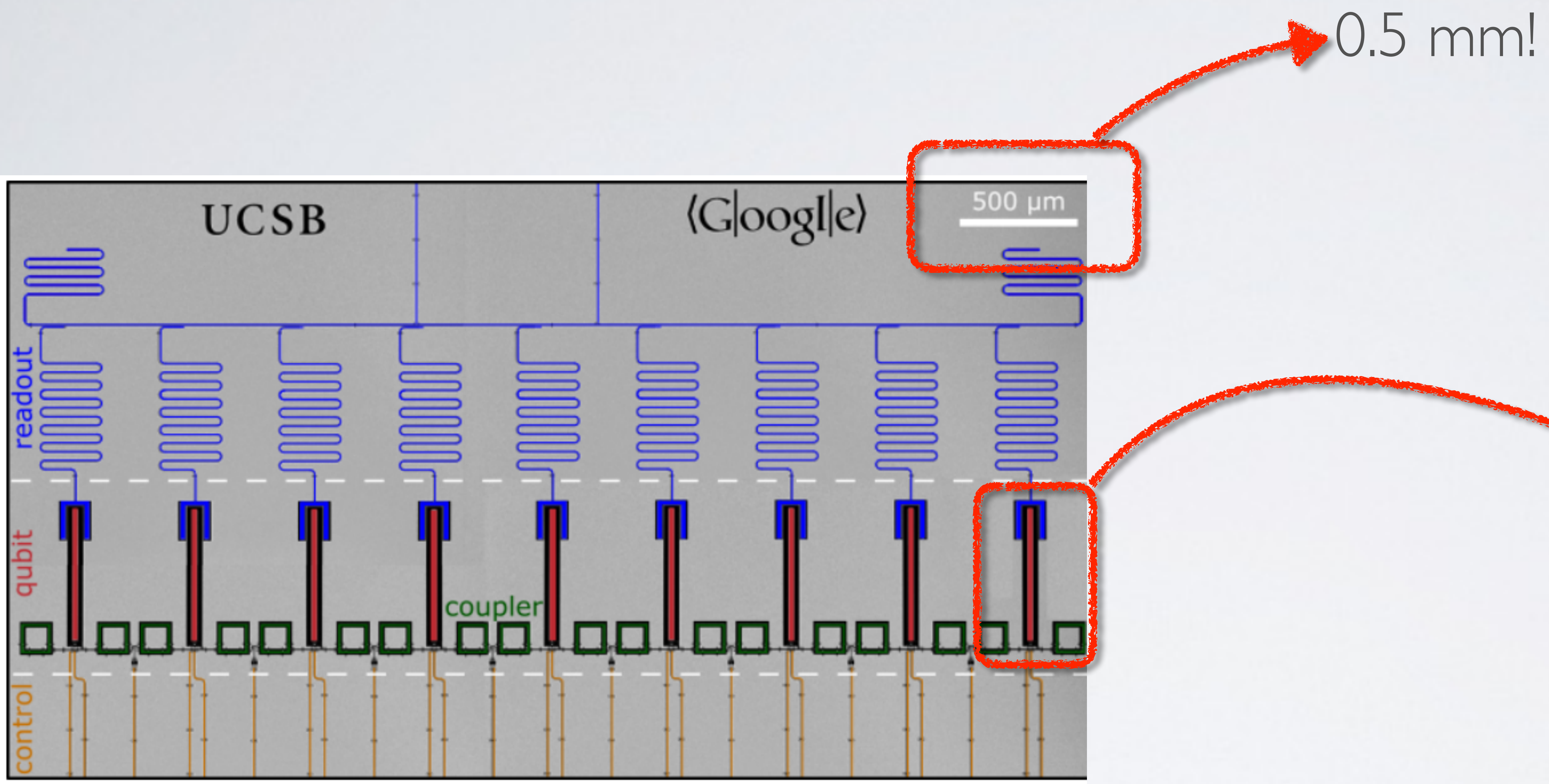
0.5 mm!

≃ charge qubit
(Cooper-pair box)



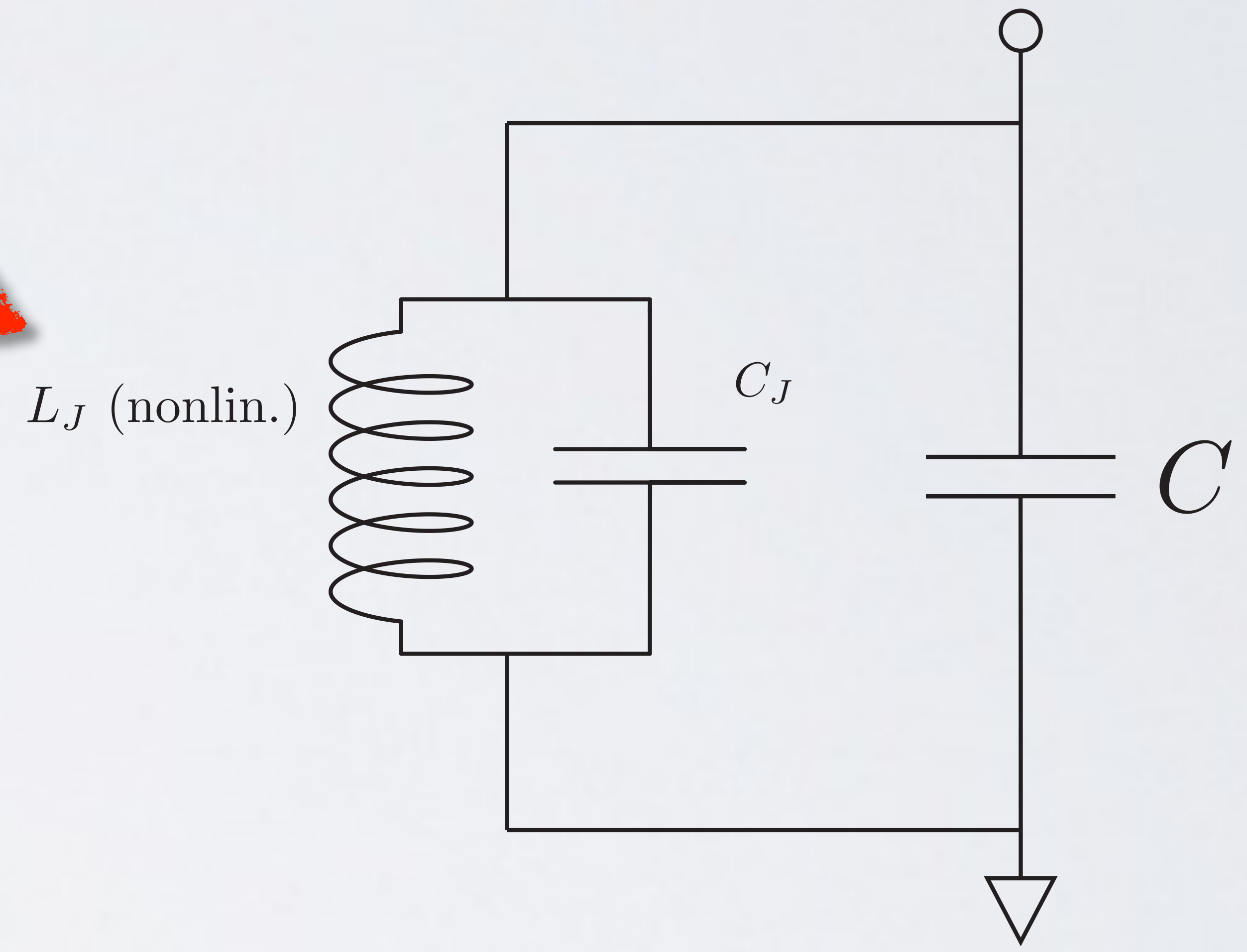
*J. Martinis @ Google & UCSB

EXAMPLE 1: SUPERCONDUCTING CIRCUITS



*J. Martinis @ Google & UCSB

≃ charge qubit
(Cooper-pair box)



Quantum effects on a "macroscopic" scale!

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Quantum (collective) effects on a "macroscopic" scale!

How is this possible?

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Quantum (collective) effects on a “macroscopic” scale!

How is this possible?

1. superconductivity gaps the single particle excitations

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Quantum (collective) effects on a "macroscopic" scale!

How is this possible?

1. superconductivity gaps the single particle excitations
2. (bulk) plasma mode oscillations are (at microwave frequencies) frozen in the ground state

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Quantum (collective) effects on a "macroscopic" scale!

How is this possible?

1. superconductivity gaps the single particle excitations
2. (bulk) plasma mode oscillations are (at microwave frequencies) frozen in the ground state
3. "boundary" conditions of the circuit we are considering impose a low-energy cutoff dictated by the size of the system components

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Quantum (collective) effects on a "macroscopic" scale!

How is this possible?

1. superconductivity gaps the single particle excitations
2. (bulk) plasma mode oscillations are (at microwave frequencies) frozen in the ground state
3. "boundary" conditions of the circuit we are considering impose a low-energy cutoff dictated by the size of the system components

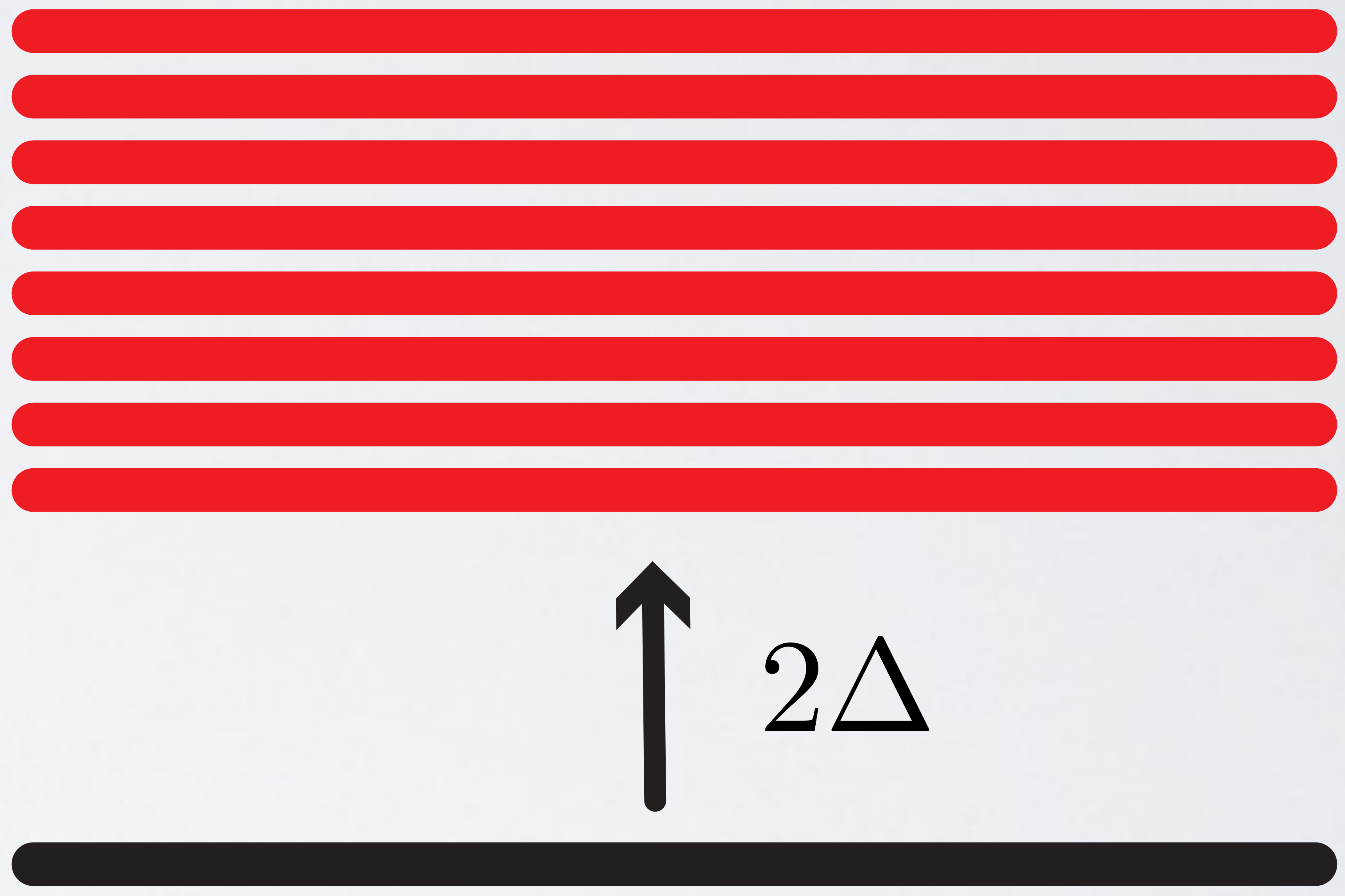
lumped-elements $(\mathbf{L}, \mathbf{C})$ description

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Superconductivity gaps the single-particle excitations

$$2\Delta \sim 1\,\mathrm{K}$$ for Al

Superconductor (resistance $\mathrm{R} = 0$)



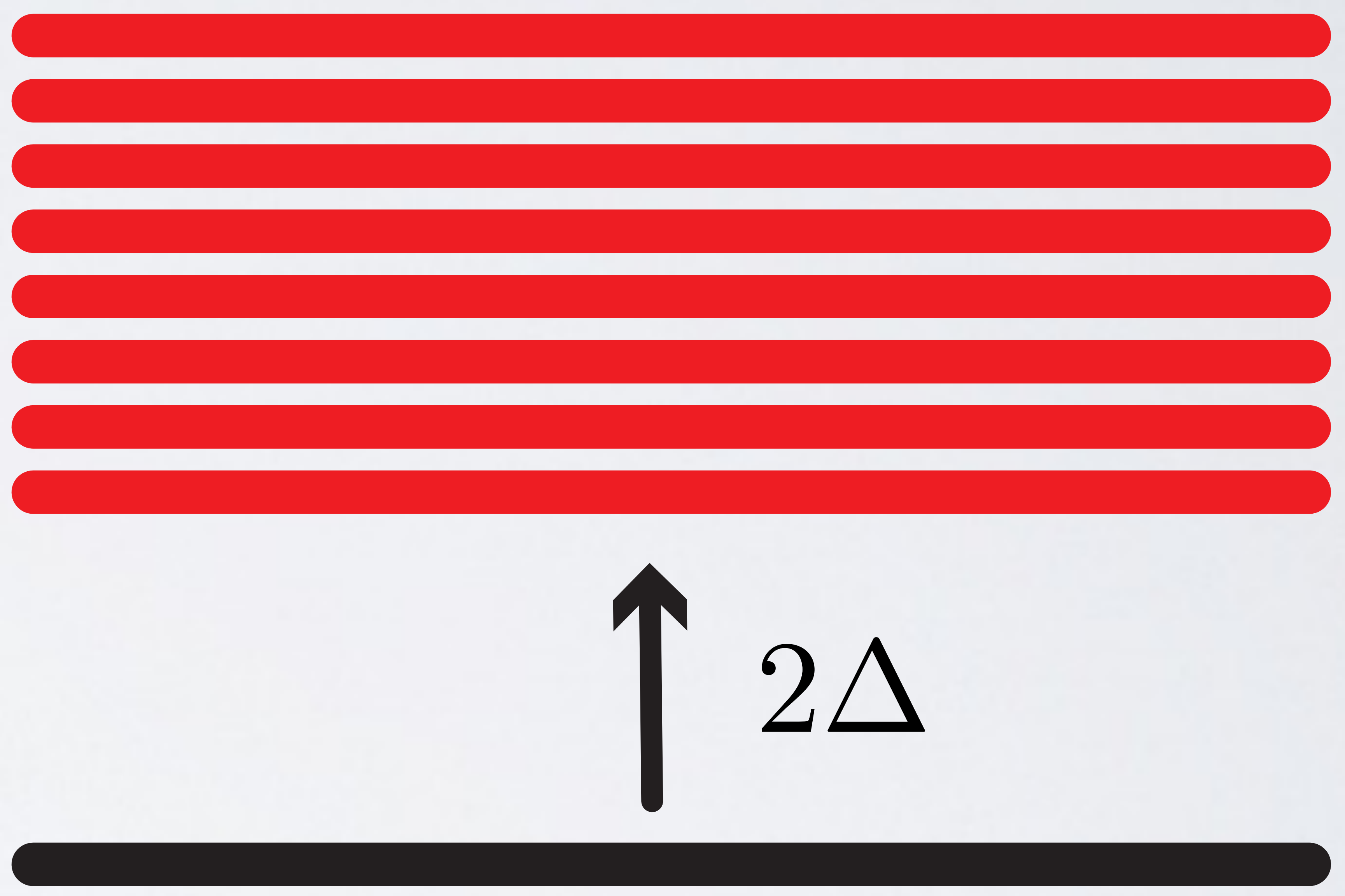
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Superconductivity gaps the single-particle excitations

$$2\Delta \sim 1\,\mathrm{K}$$ for Al

Superconductor (resistance $\mathrm{R} = 0$)

At cryogenic temperatures, the single-particle states occupation can be neglected



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

(Bulk) plasma mode oscillations are (at microwave frequencies) frozen in the ground state

$$\rho(\mathbf{r}) = -e\delta n$$

$$\mathbf{J}(\mathbf{r}) = -en\mathbf{v}(\mathbf{r}, t)$$

$$\partial_t \mathbf{v} = \frac{-e}{m}\mathbf{E}$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

(Bulk) plasma mode oscillations are (at microwave frequencies) frozen in the ground state

$$\rho(\mathbf{r}) = -e\delta n$$

$$\mathbf{J}(\mathbf{r}) = -en\mathbf{v}(\mathbf{r}, t)$$

$$\partial_t \mathbf{v} = \frac{-e}{m}\mathbf{E} \longrightarrow \partial_t \mathbf{J} = \frac{e^2 n}{m}\mathbf{E}$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

(Bulk) plasma mode oscillations are (at microwave frequencies) frozen in the ground state

$$\rho(\mathbf{r}) = -e\delta n$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\mathbf{J}(\mathbf{r}) = -en\mathbf{v}(\mathbf{r}, t)$$

$$\nabla \cdot \mathbf{J} + \partial_t \rho = 0$$

$$\partial_t \mathbf{v} = \frac{-e}{m}\mathbf{E} \longrightarrow \partial_t \mathbf{J} = \frac{e^2 n}{m}\mathbf{E} \longrightarrow \boxed{\frac{\partial^2}{\partial t^2}\rho = -\omega_p^2 \rho}$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

(Bulk) plasma mode oscillations are (at microwave frequencies) frozen in the ground state

$$\rho(\mathbf{r}) = -e\delta n$$

$$\mathbf{J}(\mathbf{r}) = -en\mathbf{v}(\mathbf{r}, t)$$

$$\partial_t \mathbf{v} = \frac{-e}{m}\mathbf{E} \quad \longrightarrow \quad \partial_t \mathbf{J} = \frac{e^2 n}{m}\mathbf{E}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{J} + \partial_t \rho = 0$$

$$\longrightarrow \quad \boxed{\frac{\partial^2}{\partial t^2}\rho = -\omega_p^2 \rho}$$

Plasma frequency: $\omega_p^2 = \frac{ne^2}{m\epsilon_0}$ $(10^{15}\,\mathrm{Hz})$

electromagnetic field is screened

London penetration depth: $\lambda_L = \frac{c}{\omega_p}$ $(14\,\mathrm{nm})$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

What is the message here?

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

What is the message here?

Collective (quantum) degree of freedom for a macroscopic ($n \sim 10^{23}$) number of electrons

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

What is the message here?

Collective (quantum) degree of freedom for a macroscopic ($n \sim 10^{23}$) number of electrons

We can treat these excitations in terms of the “usual” circuit theory

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

What is the message here?

Collective (quantum) degree of freedom for a macroscopic ($n \sim 10^{23}$) number of electrons

We can treat these excitations in terms of the "usual" circuit theory

quantum mechanics for circuits

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

What is the message here?

Collective (quantum) degree of freedom for a macroscopic ($\mathbf{n} \sim 10^{23}$) number of electrons

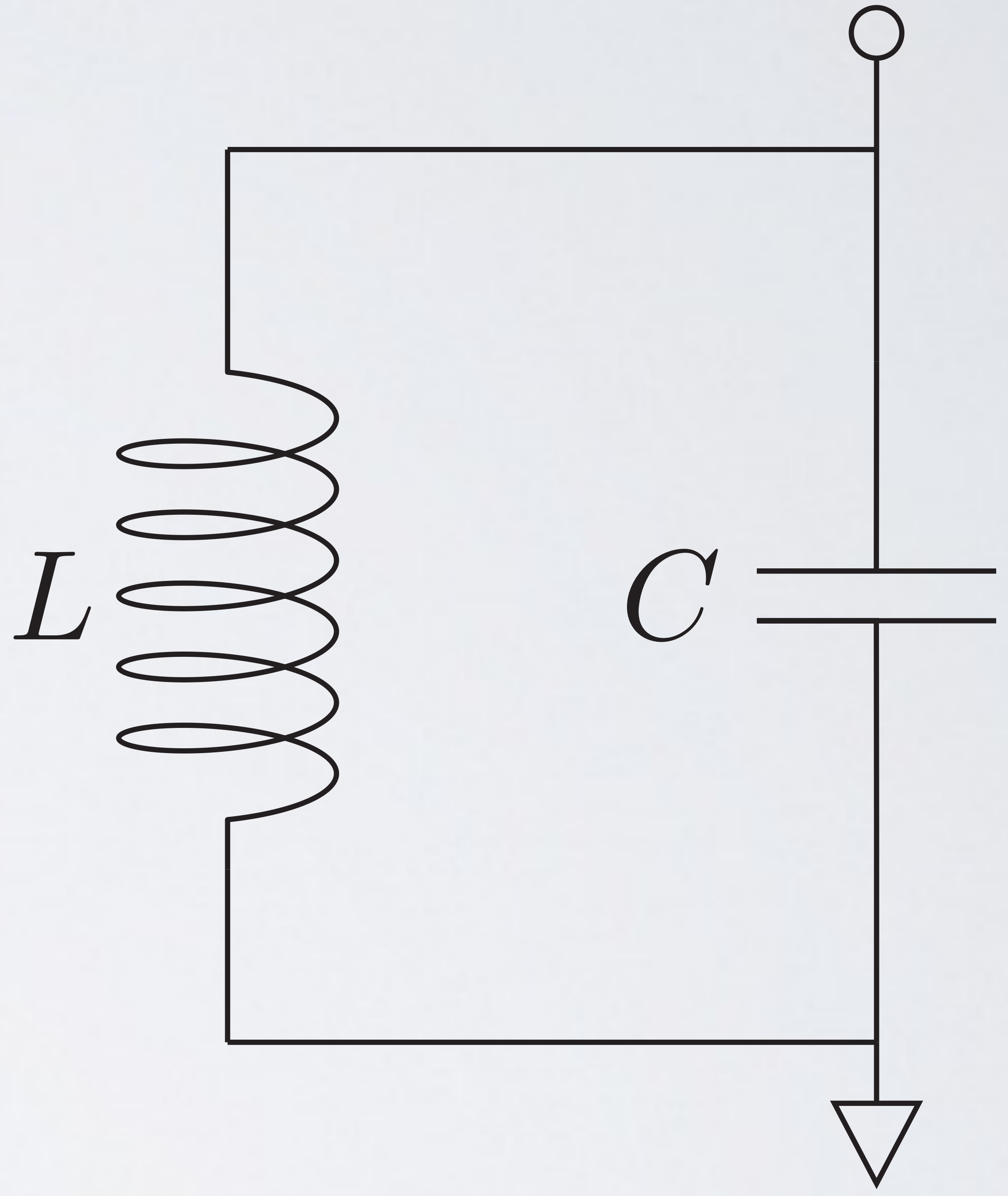
We can treat these excitations in terms of the "usual" circuit theory

quantum mechanics for circuits → circuit quantum electrodynamics

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Let's start from an **LC** circuit:

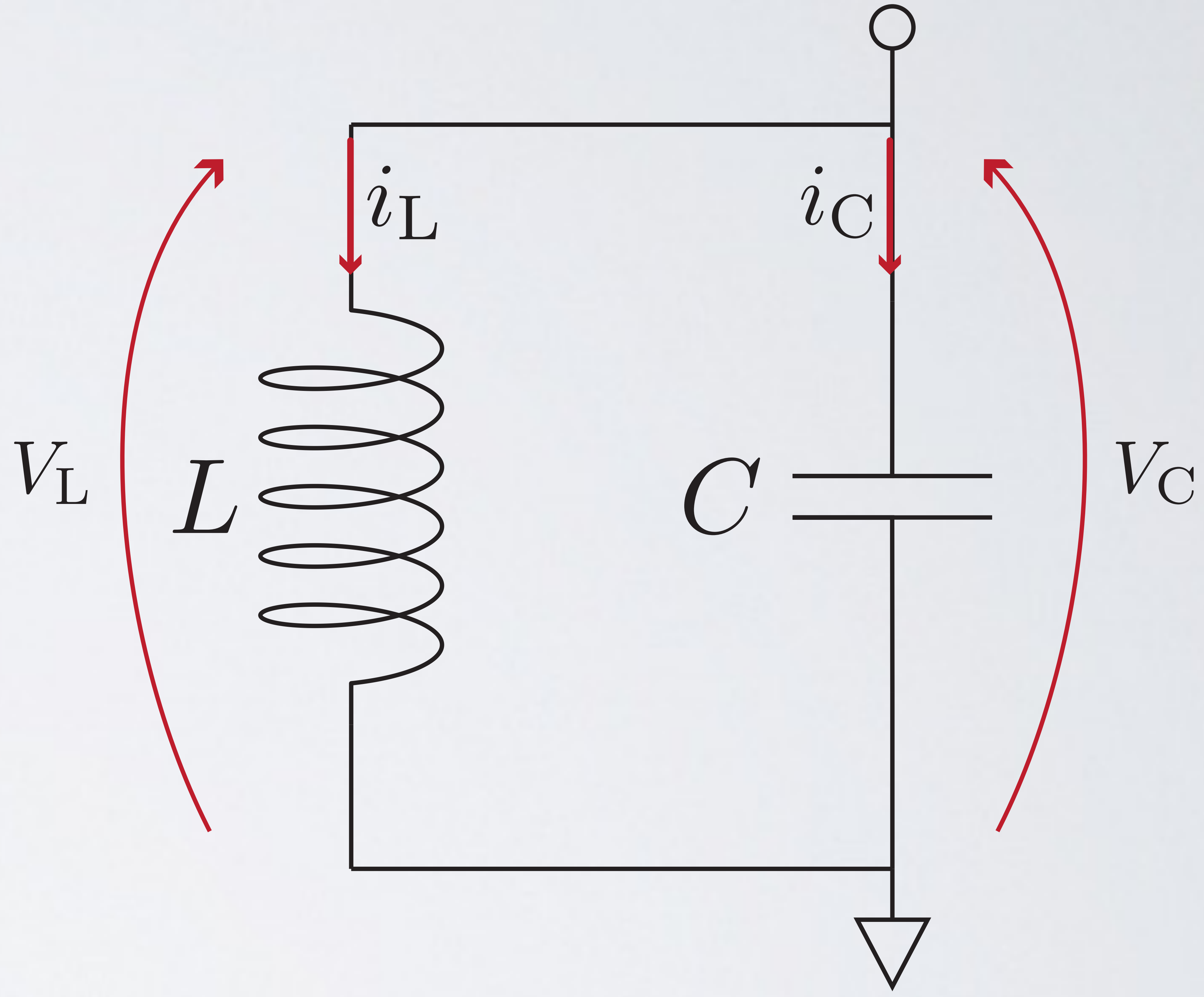
$$V_c = \frac{\int dt\, i_C}{C}, \quad V_L = L\, \partial_t i_L$$



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Let's start from an **LC** circuit:

$$V_c = \frac{\int dt\, i_C}{C}, \quad V_L = L\, \partial_t i_L$$



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

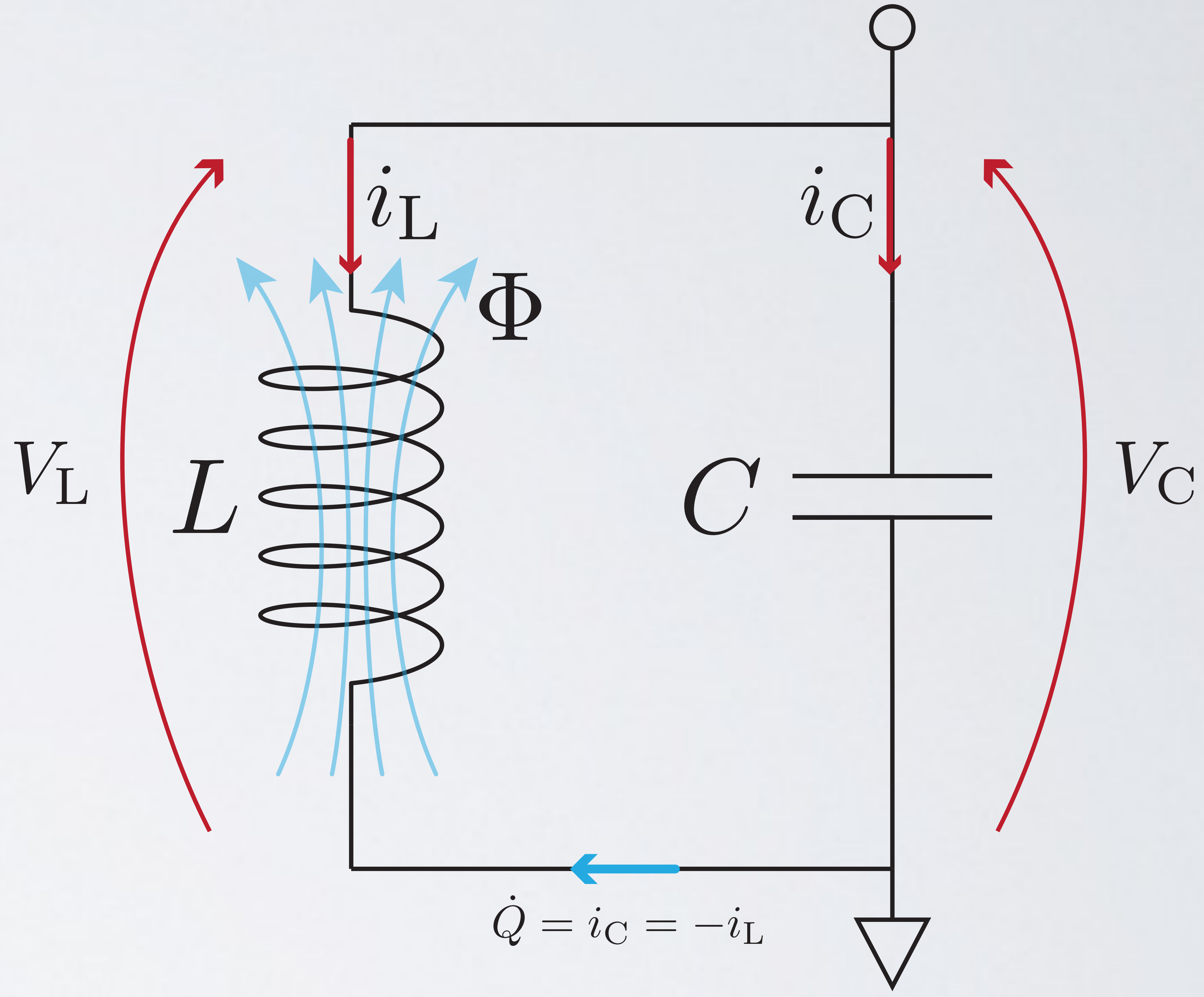
Let's start from an **LC** circuit:

$$V_c = \frac{\int dt\, i_C}{C}, \qquad V_L = L\,\partial_t i_L$$

$$\dot{\Phi} = \frac{Q}{C}, \qquad \Phi = -L\,\dot{Q}$$

$$\Phi = \int_{-\infty}^{t} dt\, V$$

$$Q = \int_{-\infty}^{t} dt\, \dot{Q}$$



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Let's start from an **LC** circuit:

$$V_c = \frac{\int dt\, i_C}{C}, \quad V_L = L\,\partial_t i_L$$

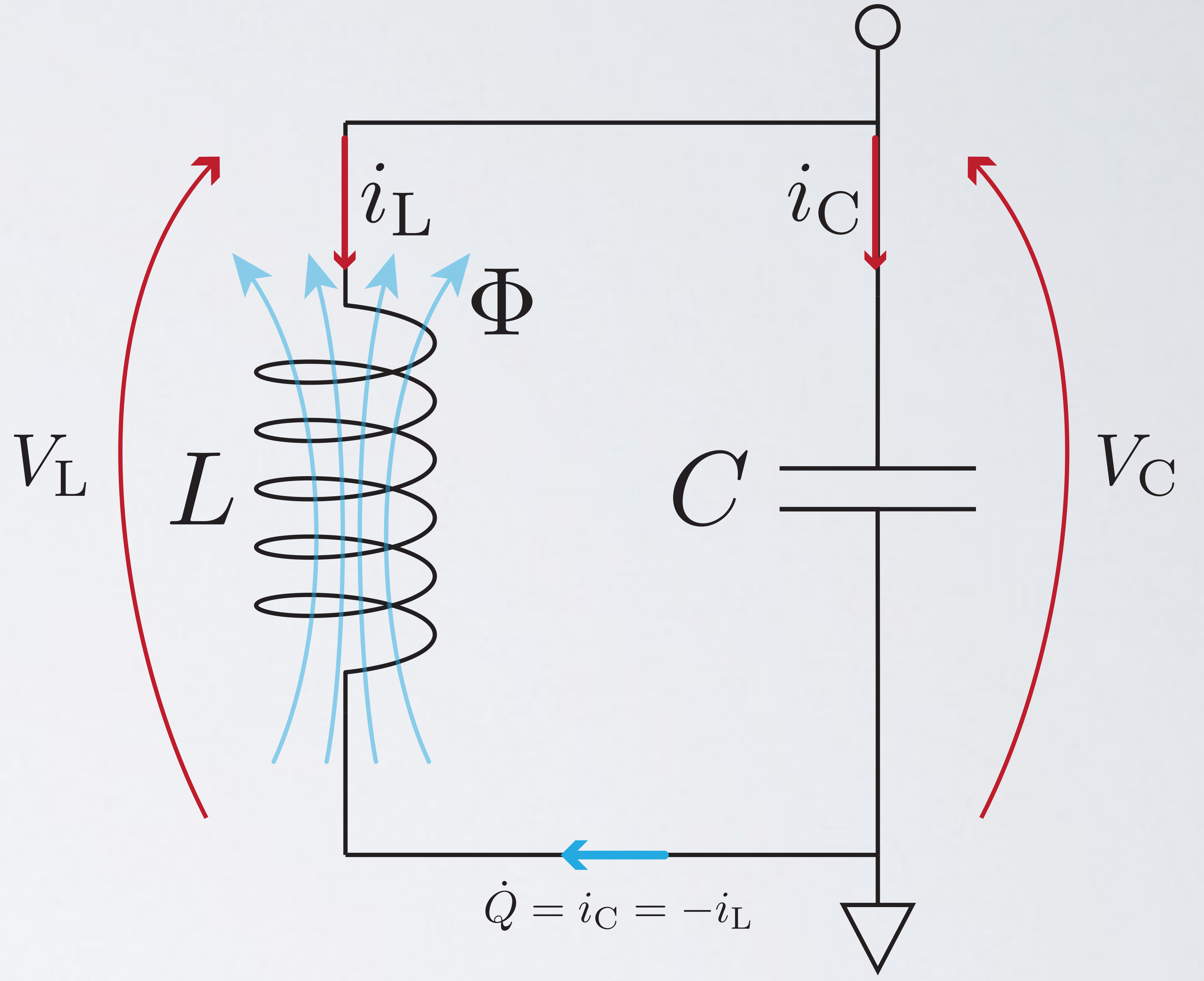
$$\Phi = \int_{-\infty}^{t} dt\, V$$
$$Q = \int_{-\infty}^{t} dt\, \dot{Q}$$

$$\dot{\Phi} = \frac{Q}{C}, \quad \Phi = -L\,\dot{Q}$$

$$\dot{\Phi} = \frac{\partial \mathcal{H}_{el}}{\partial Q}, \quad \dot{Q} = -\frac{\partial \mathcal{H}_{el}}{\partial \Phi} \quad \longleftrightarrow \quad \dot{q} = \frac{\partial \mathcal{H}}{\partial p}, \quad \dot{p} = -\frac{\partial \mathcal{H}}{\partial q}$$

$$\mathcal{H}_{el} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Hamiltonian for the **LC** circuit



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Canonical quantization

Harmonic oscillator

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Canonical quantization

Harmonic oscillator

$$\mathcal{H}_{mech} = \frac{p^2}{2m} + \frac{1}{2}kq^2$$

$$\{q, p\} = 1$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Canonical quantization

Harmonic oscillator

$$\mathcal{H}_{mech} = \frac{p^2}{2m} + \frac{1}{2}kq^2$$

$$\{q, p\} = 1$$

Poisson brackets

$$\{f, g\} \doteq \left(\frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial g}{\partial q}\frac{\partial f}{\partial p}\right)$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Canonical quantization

Harmonic oscillator

$$\mathcal{H}_{mech} = \frac{p^2}{2m} + \frac{1}{2}kq^2 \quad \longrightarrow \quad \hat{H}_{mech} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{q}^2$$

$$\{q,p\} = 1 \qquad\qquad [\hat{q},\hat{p}] = i\hbar$$

Poisson brackets

$$\{f,g\} \doteq \left(\frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial g}{\partial q}\frac{\partial f}{\partial p}\right)$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Canonical quantization

Harmonic oscillator

$$\mathcal{H}_{mech} = \frac{p^2}{2m} + \frac{1}{2}kq^2 \quad \longrightarrow \quad \hat{H}_{mech} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{q}^2$$

$$\{q, p\} = 1 \qquad [\hat{q}, \hat{p}] = i\hbar$$

Poisson brackets

$$\{f, g\} \doteq \left(\frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial g}{\partial q}\frac{\partial f}{\partial p}\right)$$

LC circuit

$$\mathcal{H}_{el} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\{\Phi, Q\} = 1$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Canonical quantization

Harmonic oscillator

$$\mathcal{H}_{mech} = \frac{p^2}{2m} + \frac{1}{2}kq^2 \quad \longrightarrow \quad \hat{H}_{mech} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{q}^2$$

$$\{q, p\} = 1 \qquad\qquad [\hat{q}, \hat{p}] = i\hbar$$

Poisson brackets

$$\{f, g\} \doteq \left(\frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial g}{\partial q}\frac{\partial f}{\partial p}\right)$$

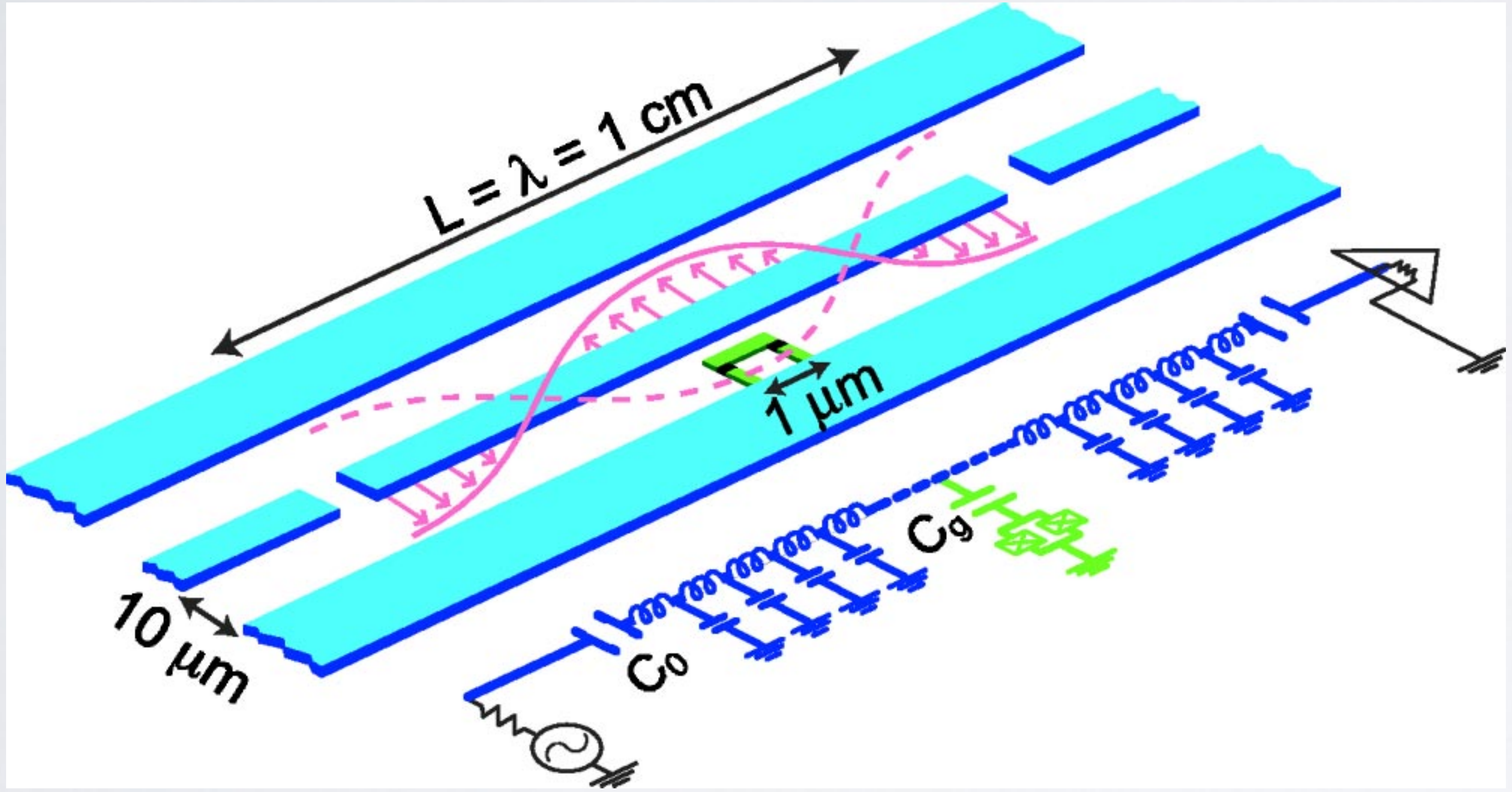
LC circuit

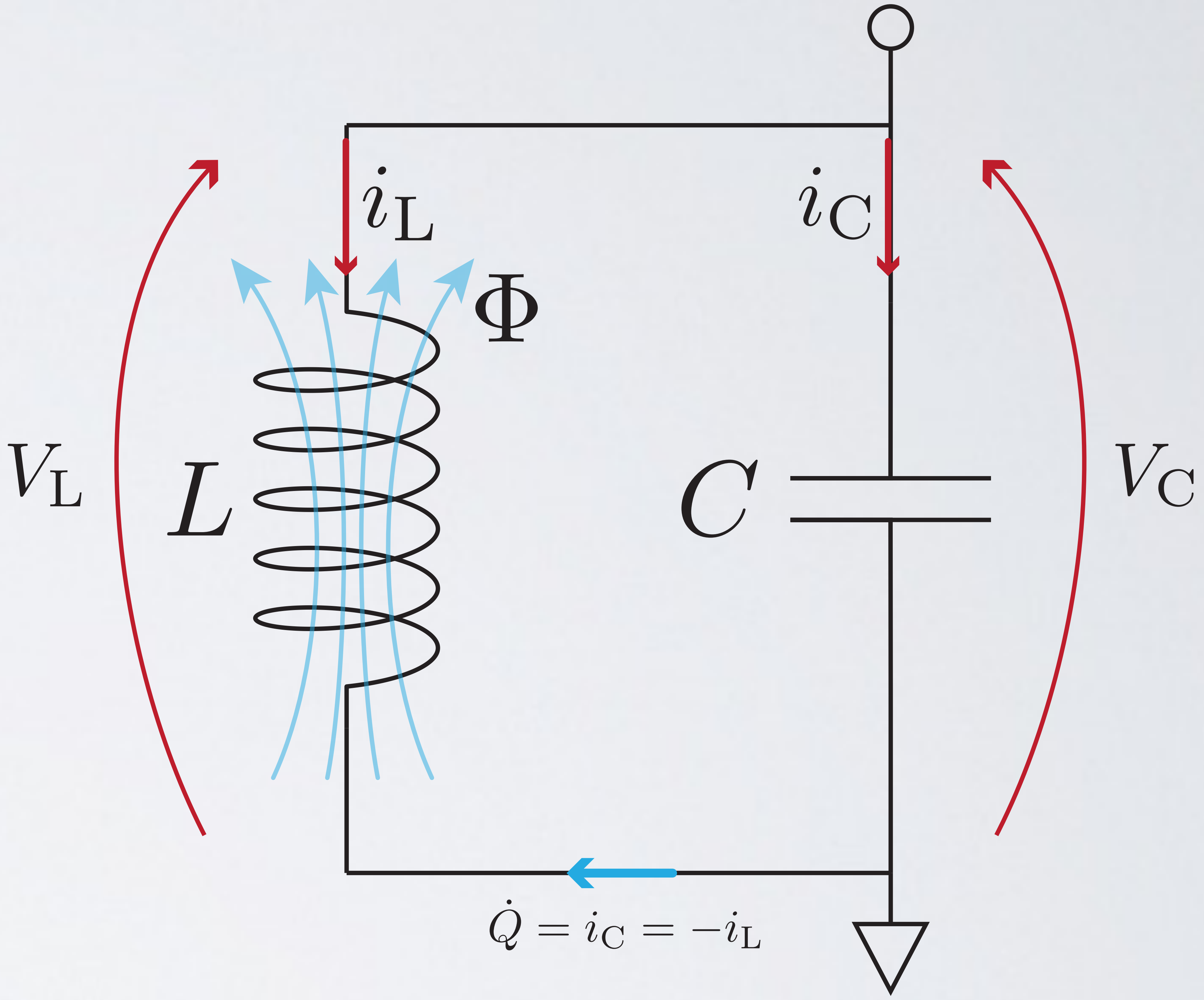
$$\mathcal{H}_{el} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \quad \longrightarrow \quad \hat{H}_{el} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

$$\{\Phi, Q\} = 1 \qquad\qquad \left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

How does the look like "experimentally"





*from Blais *et al.* PRA **69**, 062320 (2004)

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

As for the h.o. Hamiltonian, we can write the $\mathbf{LC}$ circuit Hamiltonian

$$\hat{H}_{el} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \quad \longrightarrow \quad \hat{H}_{el} = \hbar\omega_{el}\left(a^\dagger a + \frac{1}{2}\right)$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

As for the h.o. Hamiltonian, we can write the **LC** circuit Hamiltonian

$$\hat{H}_{el} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \longrightarrow \hat{H}_{el} = \hbar\omega_{el}\left(a^\dagger a + \frac{1}{2}\right)$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

$$\hat{\Phi} = \Phi_0(a^\dagger + a)$$

$$\hat{Q} = iQ_0(a^\dagger - a)$$

$$Q_0 = \sqrt{\frac{\hbar}{2Z}} \qquad \omega_{el} = (LC)^{-1/2}$$

$$\Phi_0 = \sqrt{\frac{\hbar Z}{2}} \qquad Z = \frac{L}{C}$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

As for the h.o. Hamiltonian, we can write the **LC** circuit Hamiltonian

$$\hat{H}_{el} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \longrightarrow \hat{H}_{el} = \hbar\omega_{el}\left(a^\dagger a + \frac{1}{2}\right)$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

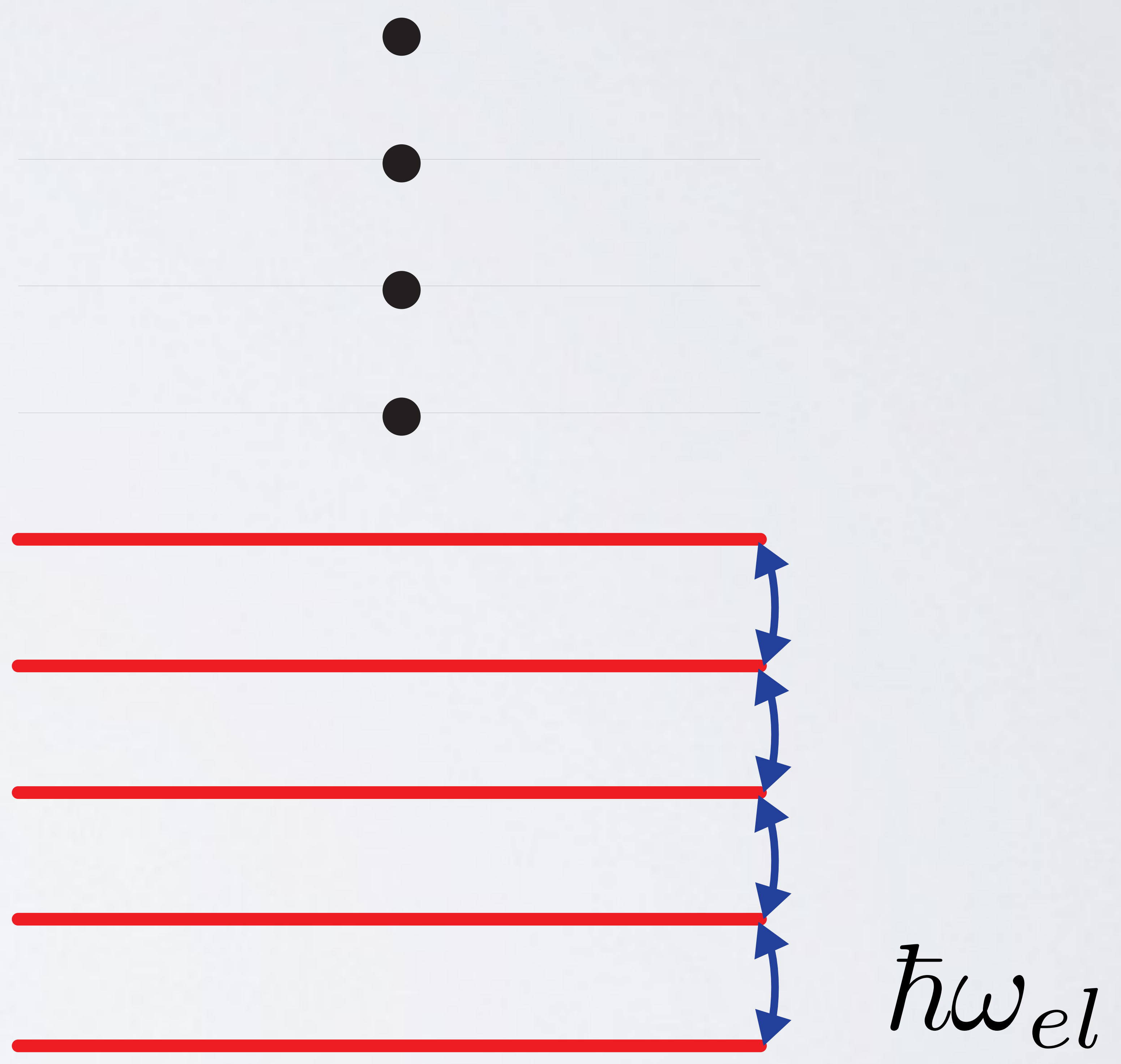
$$\hat{\Phi} = \Phi_0(a^\dagger + a)$$

$$\hat{Q} = iQ_0(a^\dagger - a)$$

$$Q_0 = \sqrt{\frac{\hbar}{2Z}} \qquad \omega_{el} = (LC)^{-1/2}$$

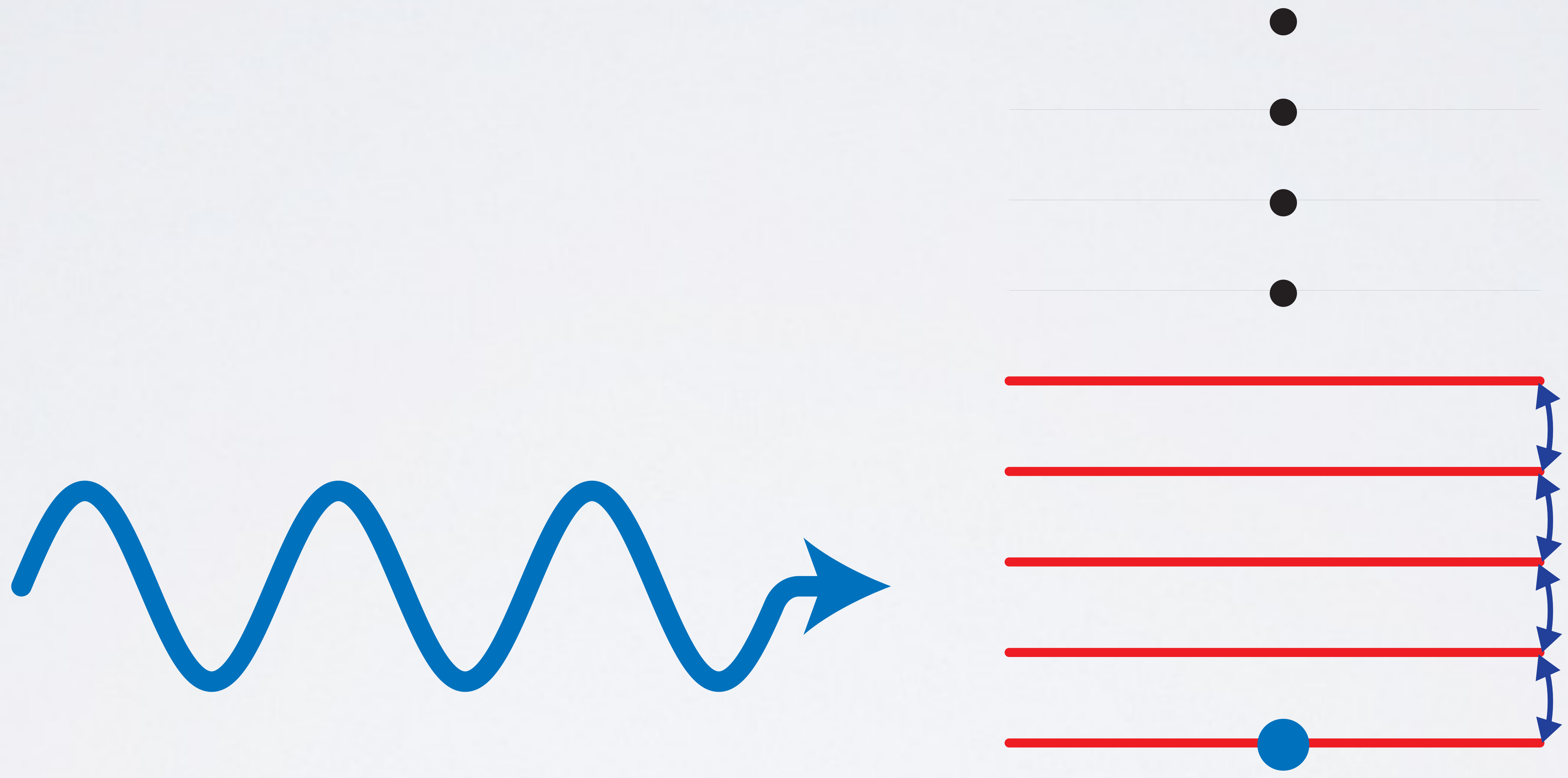
$$\Phi_0 = \sqrt{\frac{\hbar Z}{2}} \qquad Z = \frac{L}{C}$$

Spectrum: →



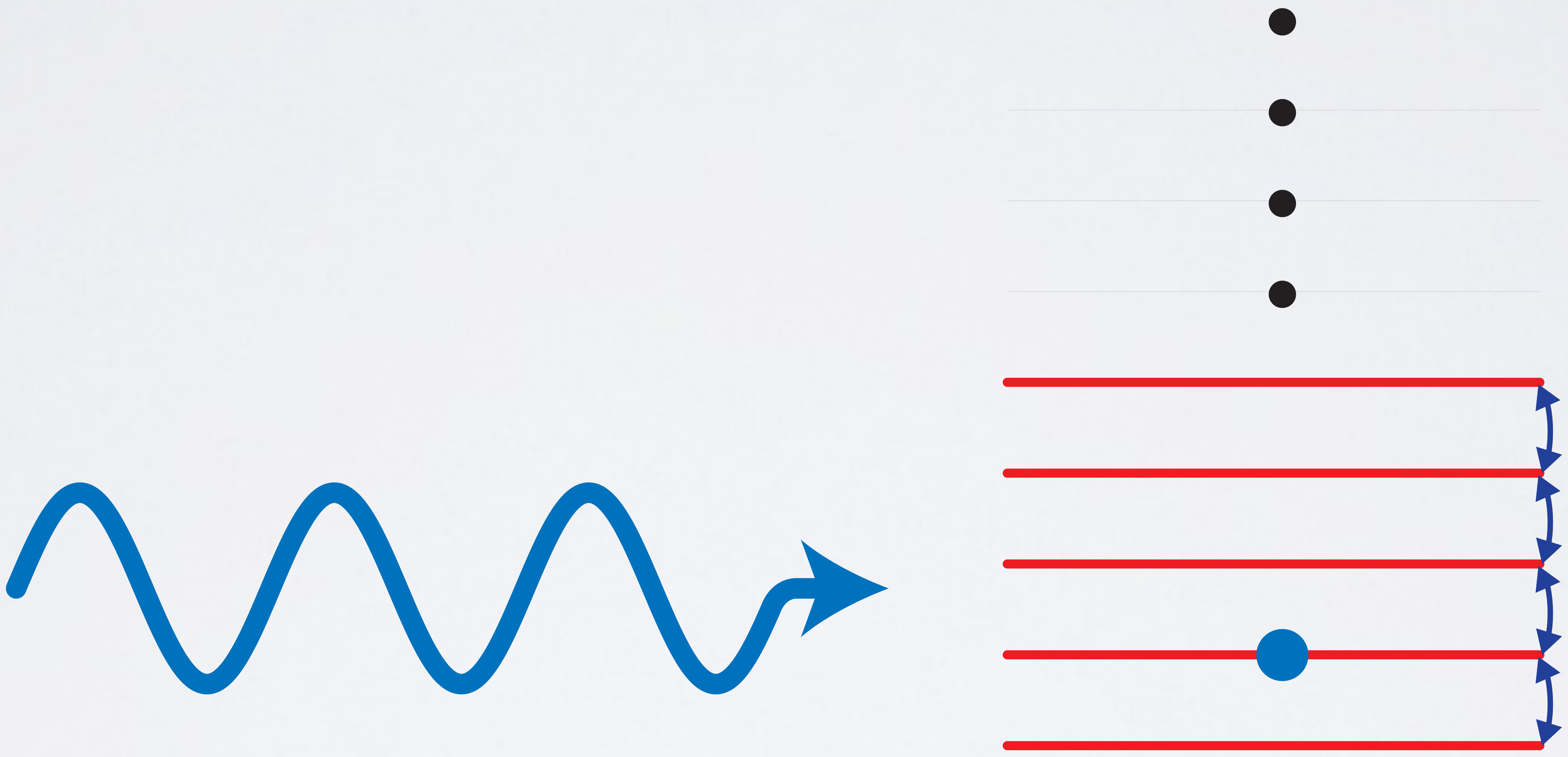
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Spacing between energy levels is constant. Not suitable for a qubit: we want to be able to address selectively the transition between 2 levels.



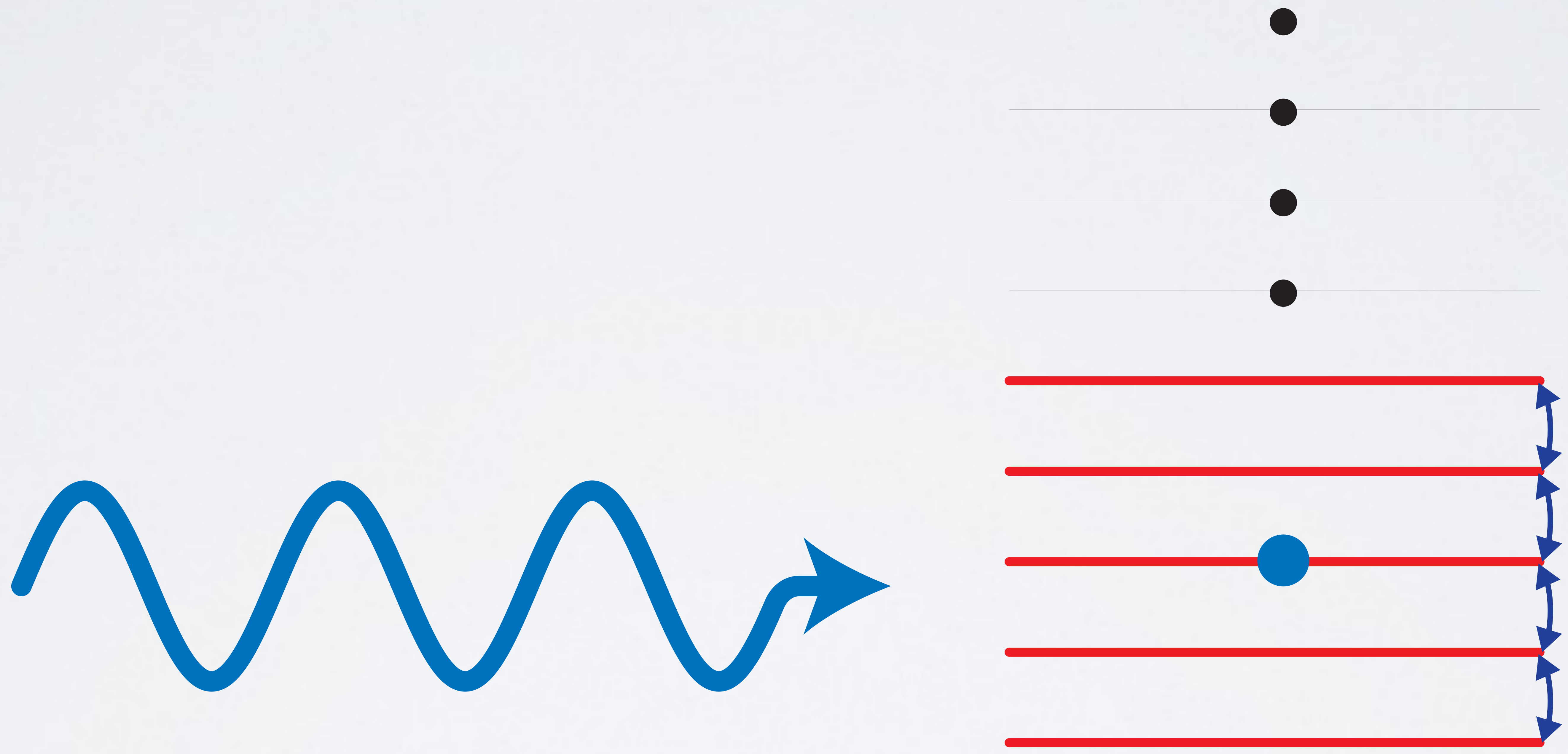
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Spacing between energy levels is constant. Not suitable for a qubit: we want to be able to address selectively the transition between 2 levels.



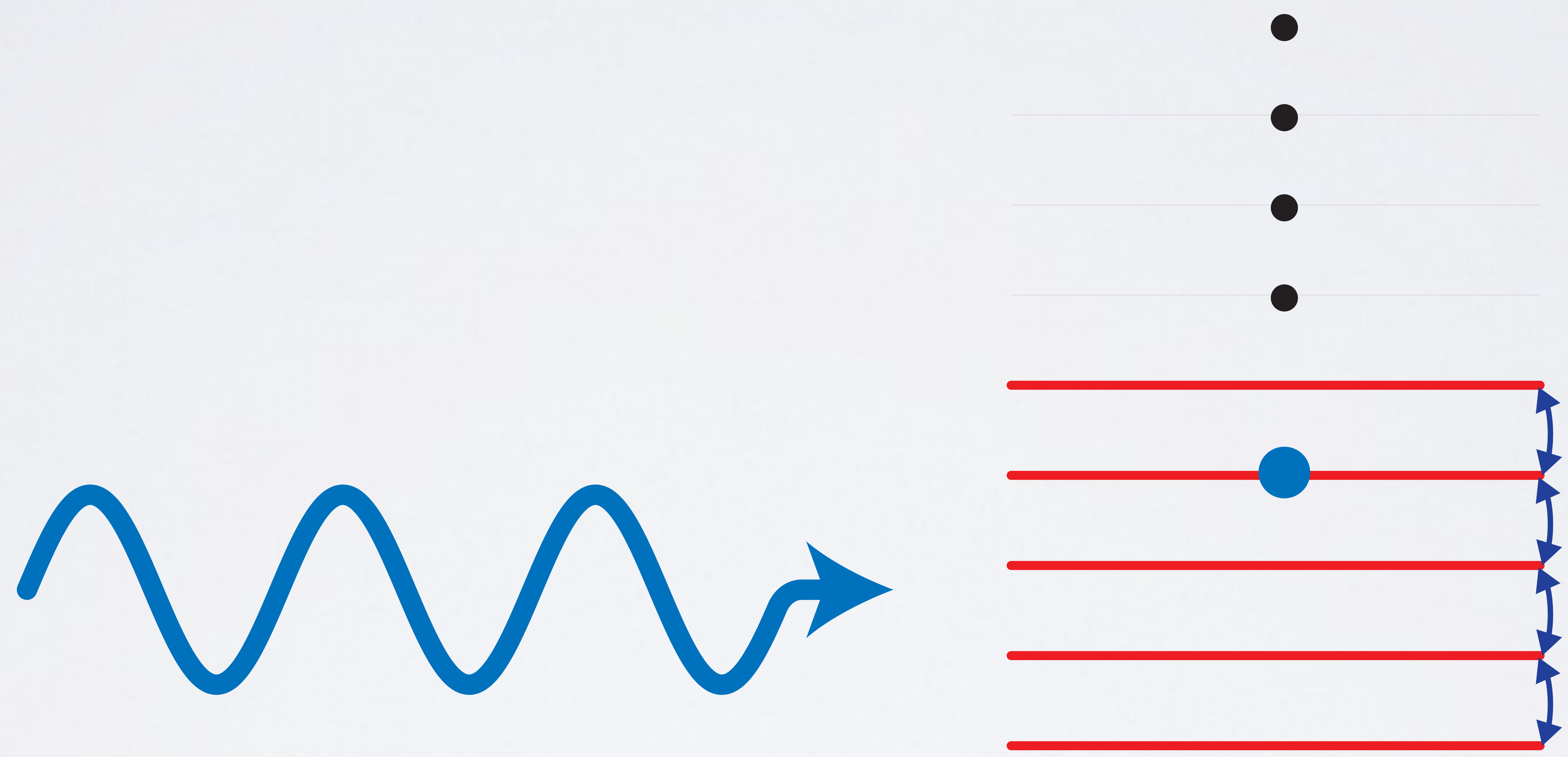
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Spacing between energy levels is constant. Not suitable for a qubit: we want to be able to address selectively the transition between 2 levels.



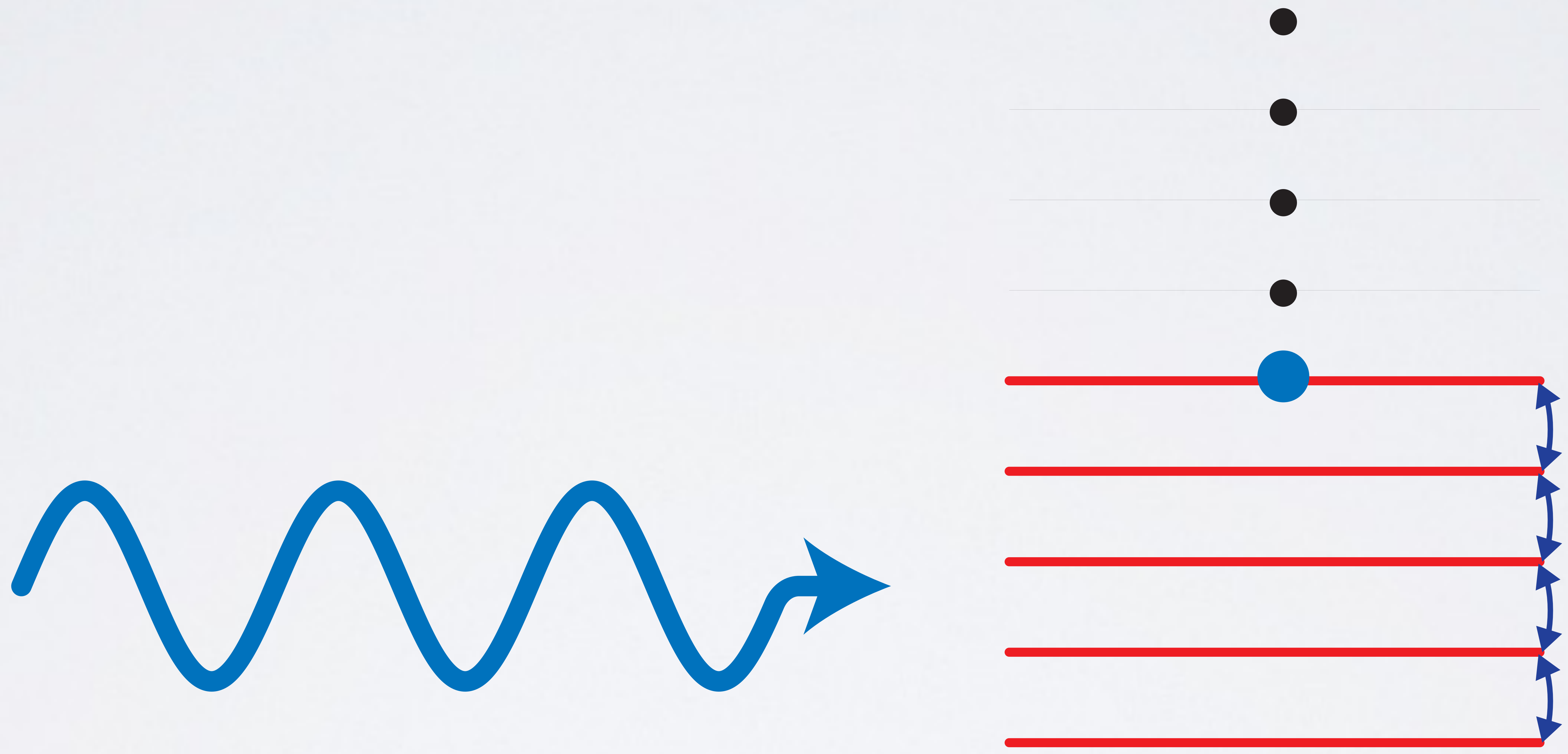
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Spacing between energy levels is constant. Not suitable for a qubit: we want to be able to address selectively the transition between 2 levels.



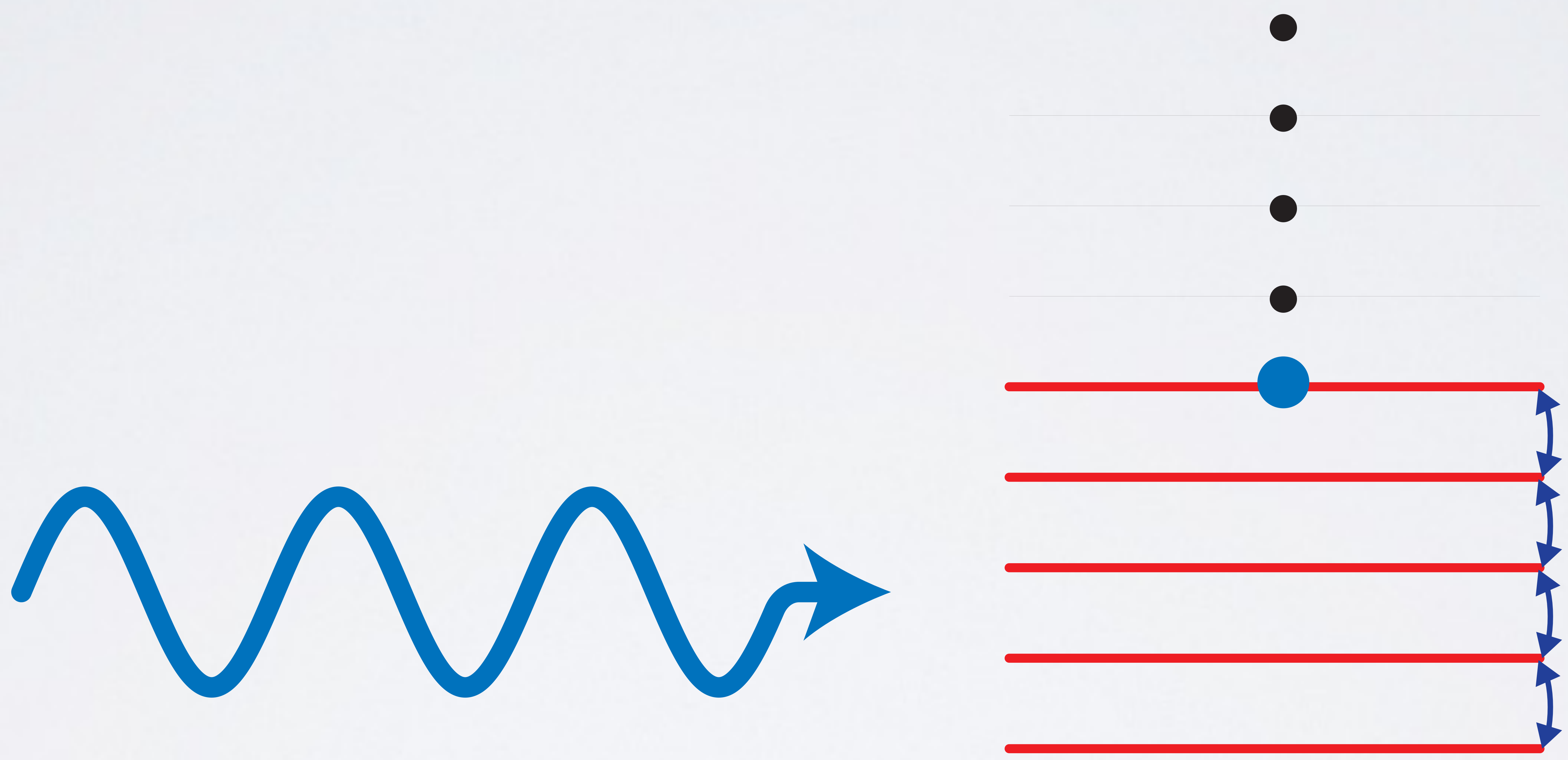
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Spacing between energy levels is constant. Not suitable for a qubit: we want to be able to address selectively the transition between 2 levels.



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

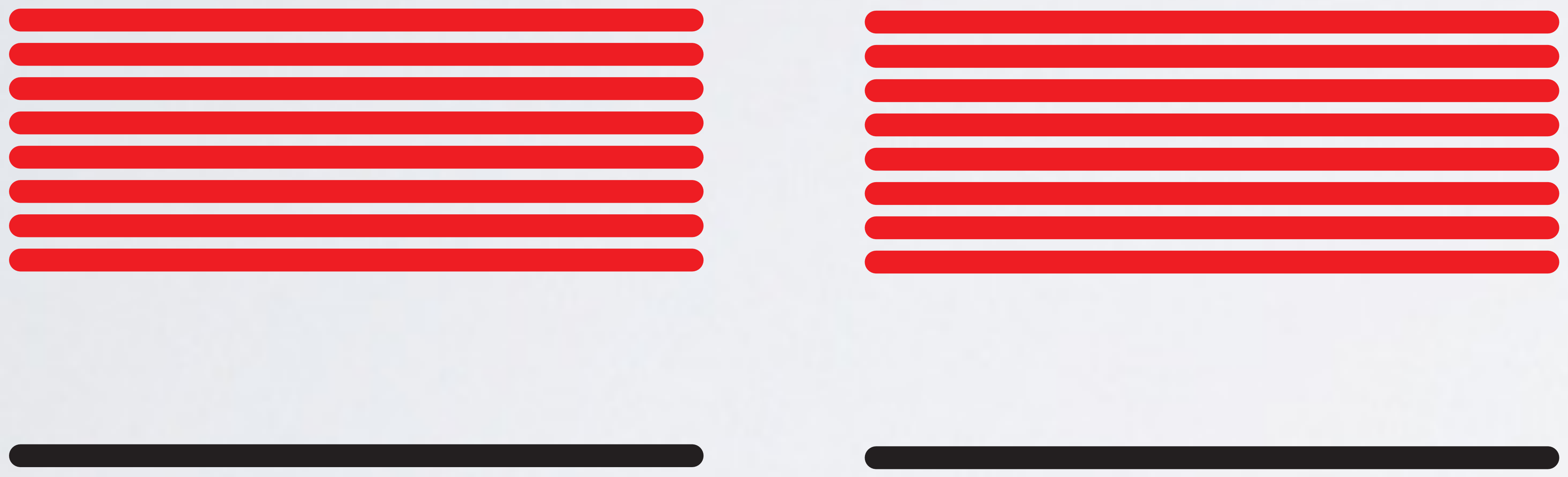
Spacing between energy levels is constant. Not suitable for a qubit: we want to be able to address selectively the transition between 2 levels.



We need something different (for a qubit)

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Two superconductors



$$|N\rangle = |N_l, N_r\rangle$$

$$|N\rangle = |N_l + 1, N_r - 1\rangle$$

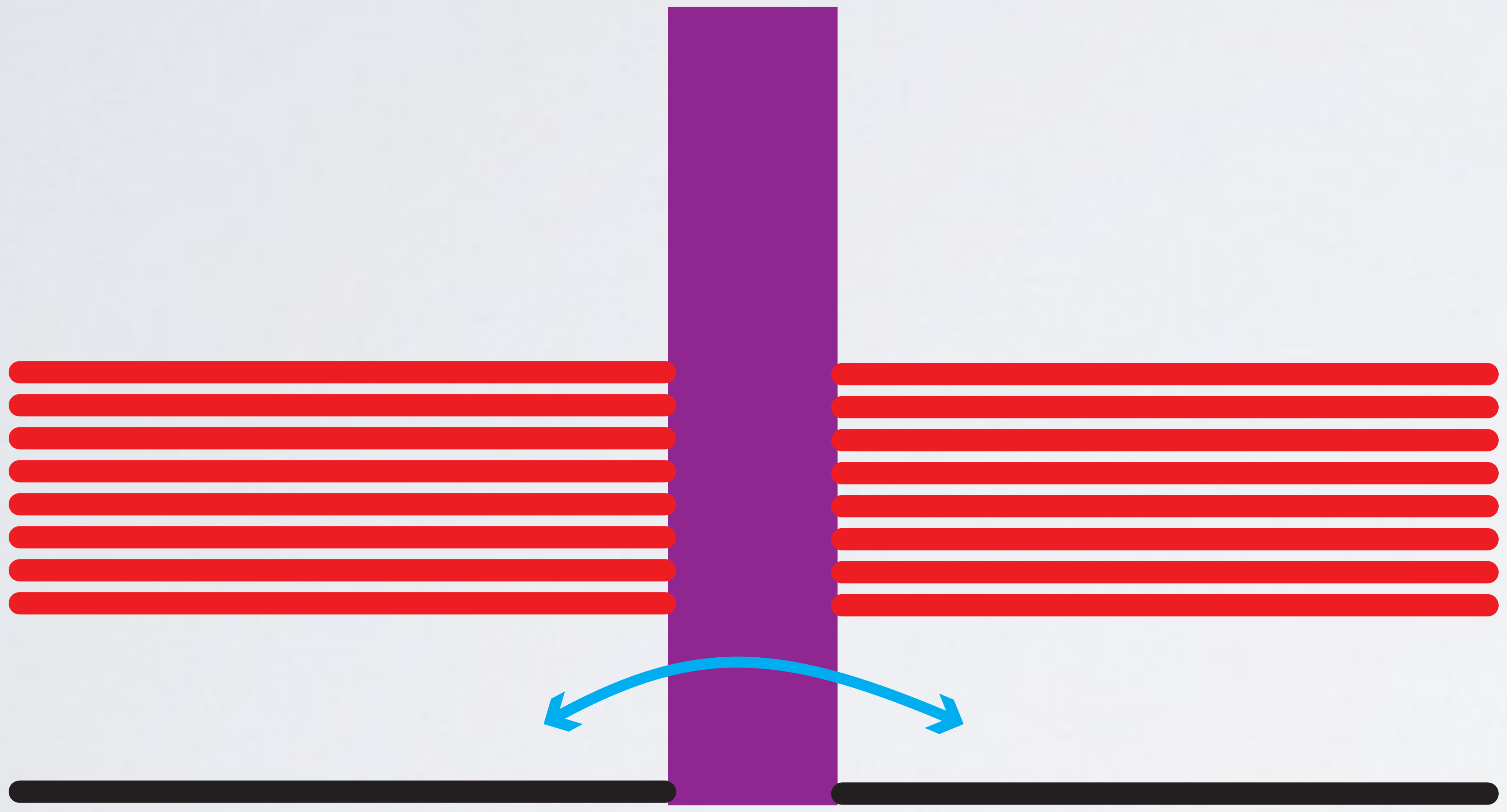
$$\vdots$$

$$|N\rangle = |N_l - 1, N_r + 1\rangle$$

$$|N\rangle = |N_l - n, N_r + n\rangle$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Two superconductors + tunneling junction



$$|N\rangle = |N_l, N_r\rangle$$

$$|N\rangle = |N_l + 1, N_r - 1\rangle$$

•

•

•

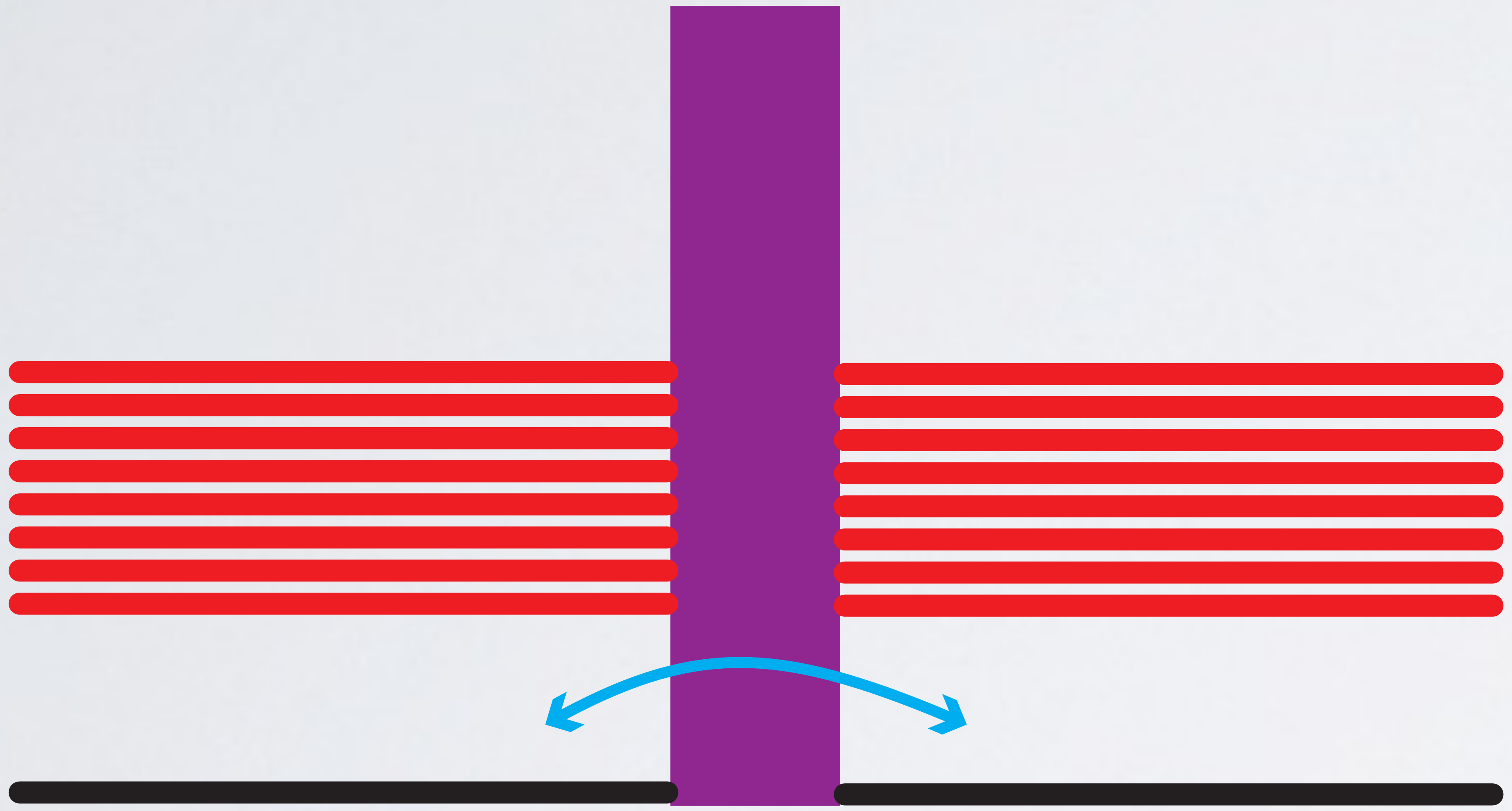
$$|N\rangle = |N_l - 1, N_r + 1\rangle$$

$$|N\rangle = |N_l - n, N_r + n\rangle$$

Tunneling of Cooper pairs

$$\hat{H}_T = -\frac{1}{2} E_J \sum_m |m\rangle \langle m+1| + |m+1\rangle \langle m|$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

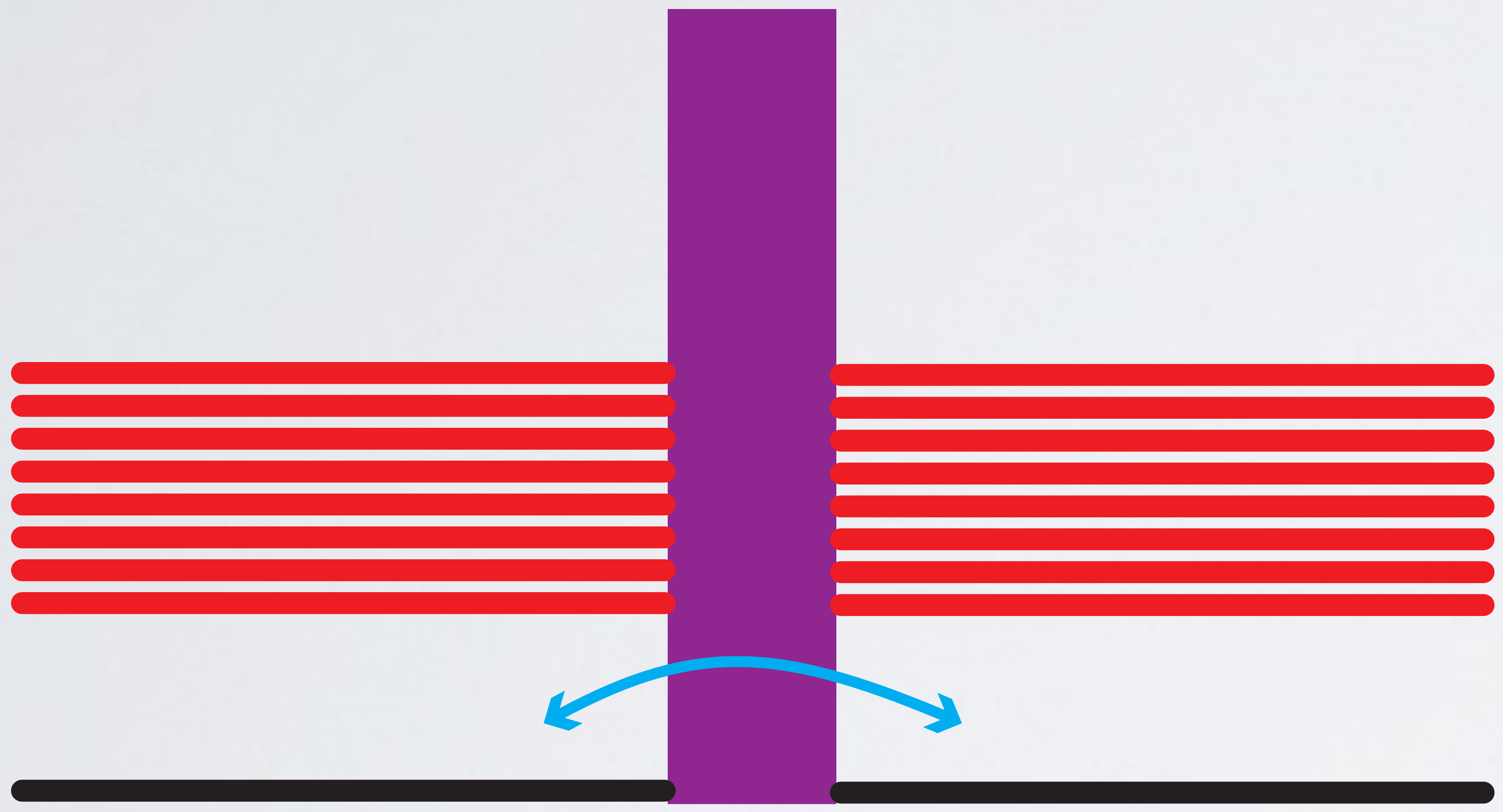


From the expression of $\hat{H}_T$ we can calculate the current operator

$$\hat{I} = 2e\frac{d\hat{n}}{dt} = 2e\frac{i}{\hbar}\left[H_T, \hat{n}\right] =$$

$$= \frac{-ieE_J}{\hbar}\sum |m\rangle\langle m+1| - |m+1\rangle\langle m|$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS



From the expression of $\hat{H}_T$ we can calculate the current operator

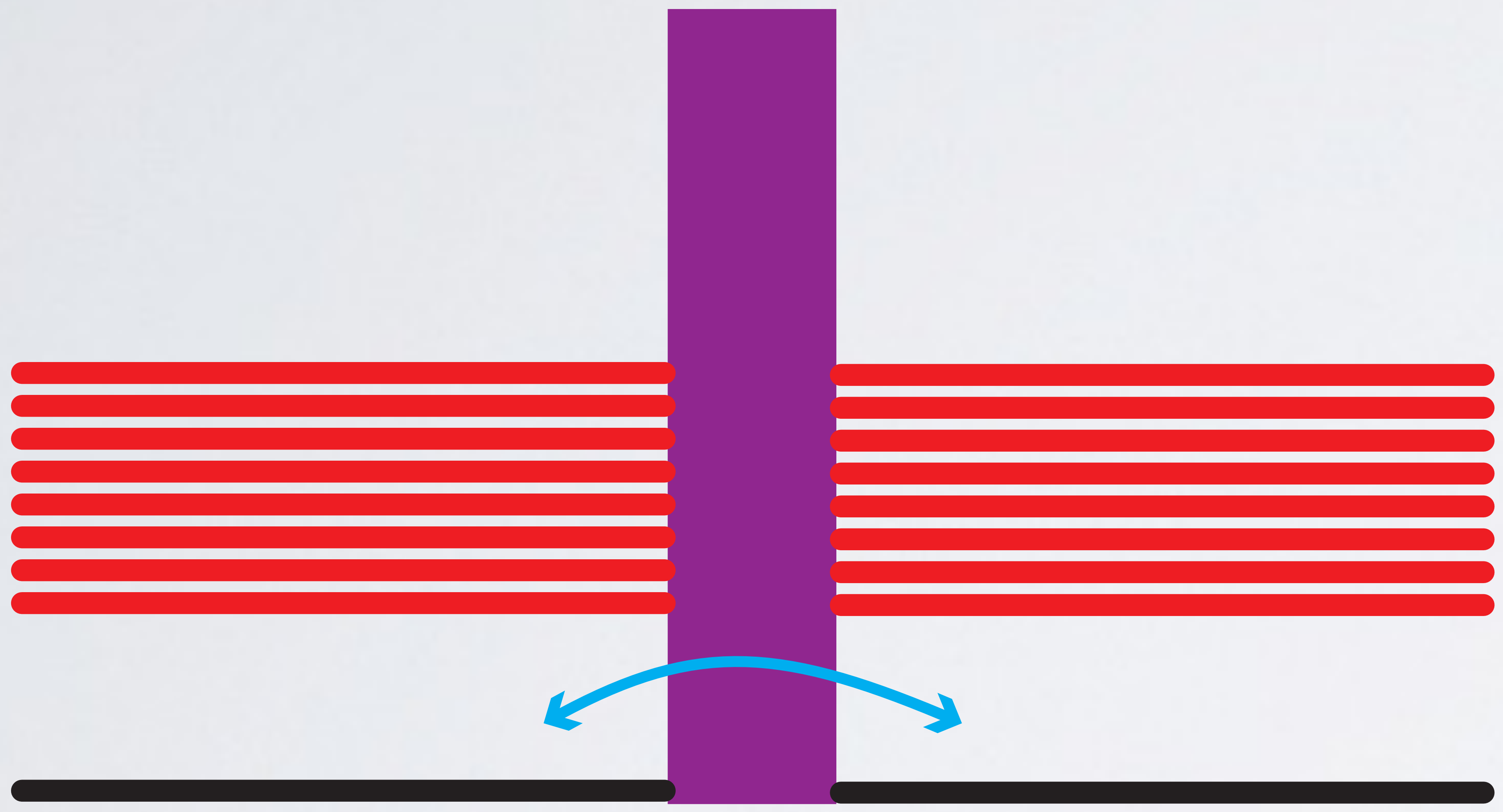
$$\hat{I} = 2e\frac{d\hat{n}}{dt} = 2e\frac{i}{\hbar}\left[H_T, \hat{n}\right] =$$

$$= \frac{-ieE_J}{\hbar}\sum |m\rangle\langle m+1| - |m+1\rangle\langle m|$$

$$\hat{I}\,|\phi\rangle = I_c \sin\phi\,|\phi\rangle$$

$$I_c = \frac{2eE_J}{\hbar}$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS



From the expression of $\hat{H}_T$ we can calculate the current operator

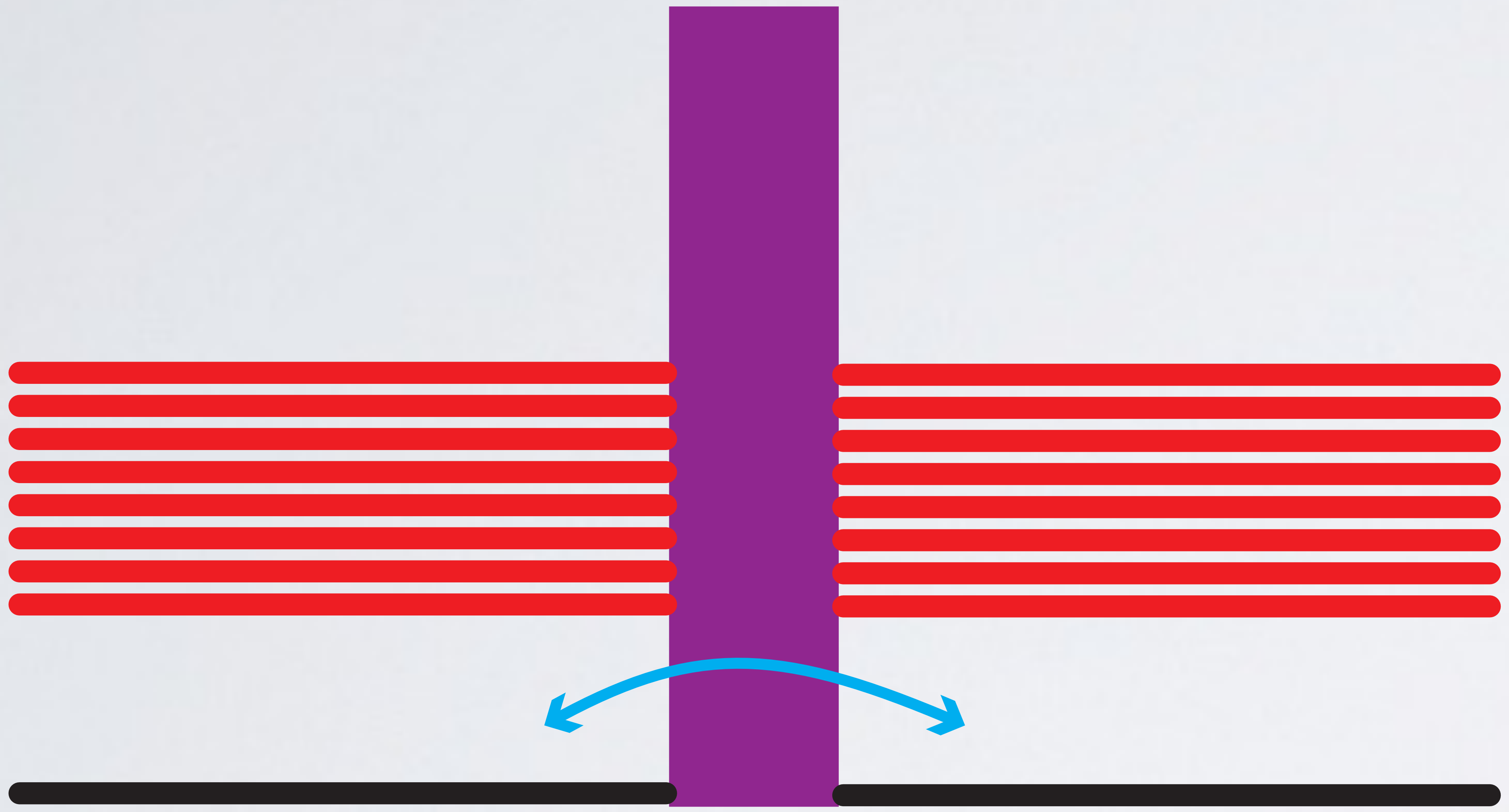
$$\hat{I} = 2e\frac{d\hat{n}}{dt} = 2e\frac{i}{\hbar}\left[H_T, \hat{n}\right] =$$

$$= \frac{-ieE_J}{\hbar}\sum |m\rangle\langle m+1| - |m+1\rangle\langle m|$$

$$\hat{I}\,|\phi\rangle = I_c \sin\phi\,|\phi\rangle$$ First Josephson relation

$$I_c = \frac{2eE_J}{\hbar}$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS



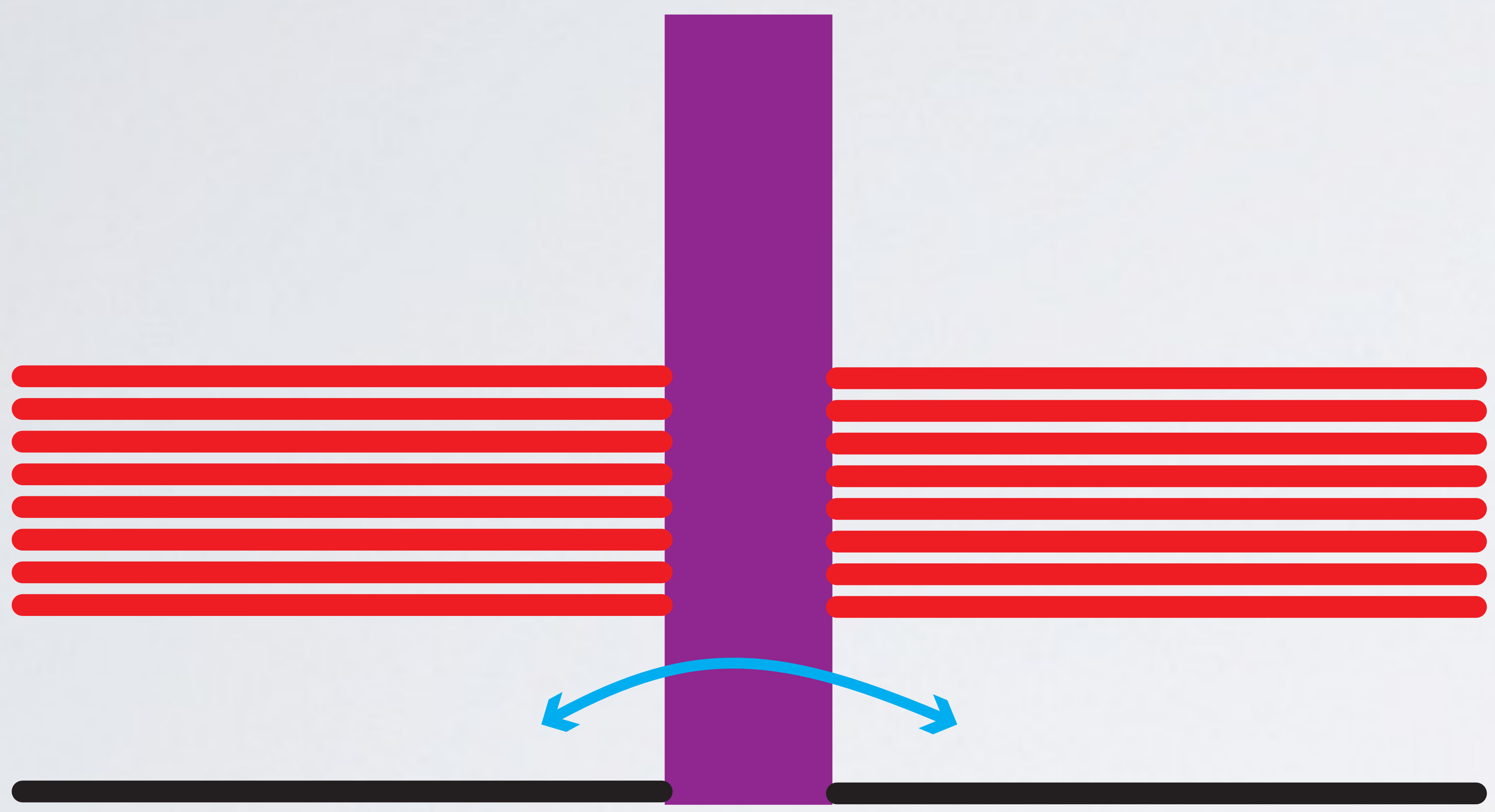
The phase ϕ plays the role of a magnetic field flux $\Phi = \frac{\hbar}{2e}\phi$

$$I = \langle\phi|\,\hat{I}\,|\phi\rangle = I_c \sin\frac{2e\Phi}{\hbar} \simeq \left(\frac{2e}{\hbar}\right)^2 E_J\Phi + O(\Phi^2)$$

…and the Josephson junction con be considered as a nonlinear inductance

(linear term has inductance $\left(\frac{\hbar}{2e}\right)\frac{1}{E_J}$)

EXAMPLE 1: SUPERCONDUCTING CIRCUITS



The phase ϕ plays the role of a magnetic field flux $\Phi = \frac{\hbar}{2e}\phi$

$$I = \langle\phi|\,\hat{I}\,|\phi\rangle = I_c \sin\frac{2e\Phi}{\hbar} \simeq \left(\frac{2e}{\hbar}\right)^2 E_J\Phi + O(\Phi^2)$$

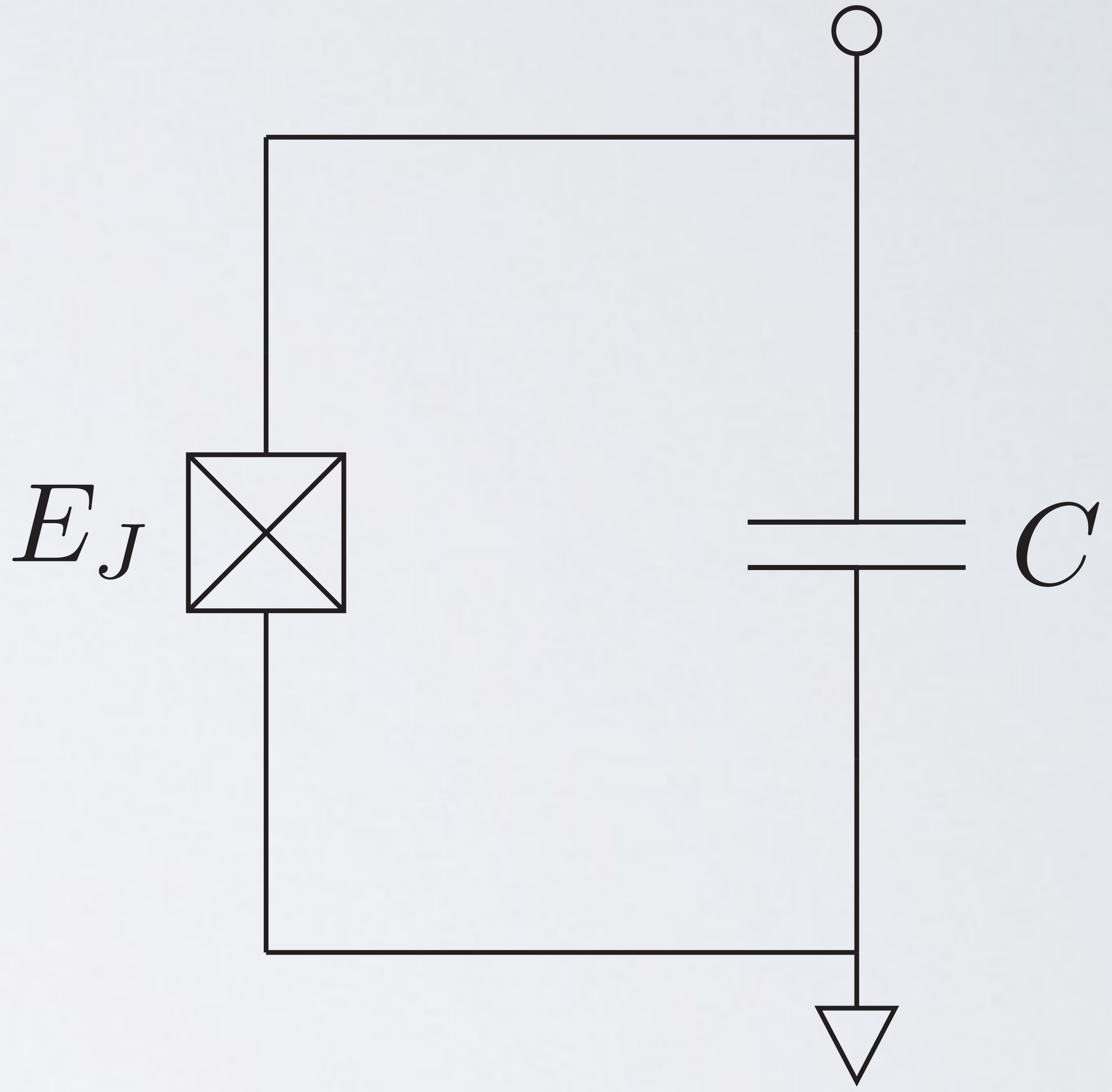
$$\dot{Q} = \Phi/L$$

…and the Josephson junction con be considered as a nonlinear inductance

(linear term has inductance $\left(\frac{\hbar}{2e}\right)\frac{1}{E_J}$)

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Let's add another ingredient…

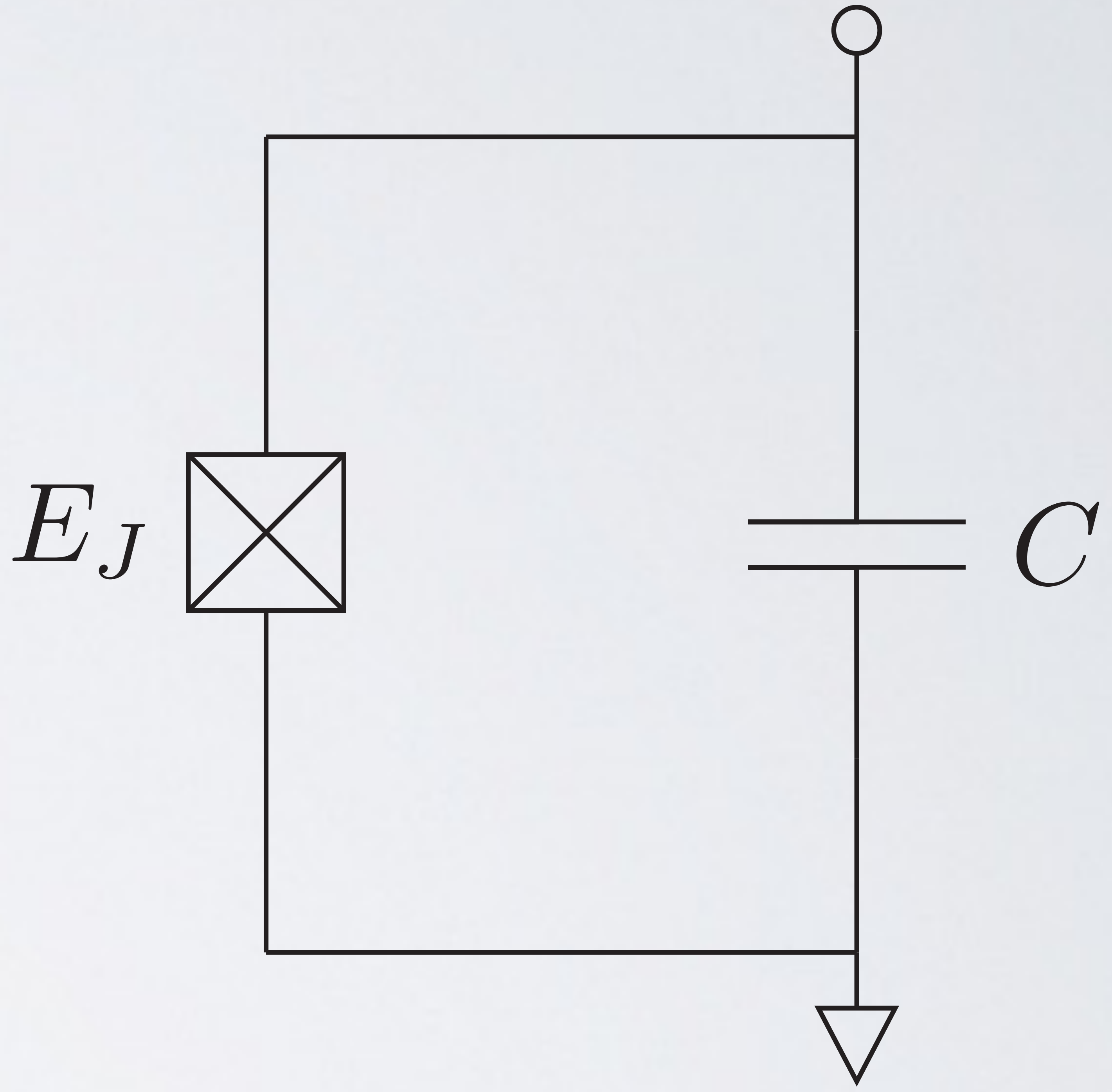


EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Let's add another ingredient…

Josephson junction

$$\hat{H}_T = -\frac{1}{2}E_J \sum_m |m\rangle \langle m+1| + |m+1\rangle \langle m|$$

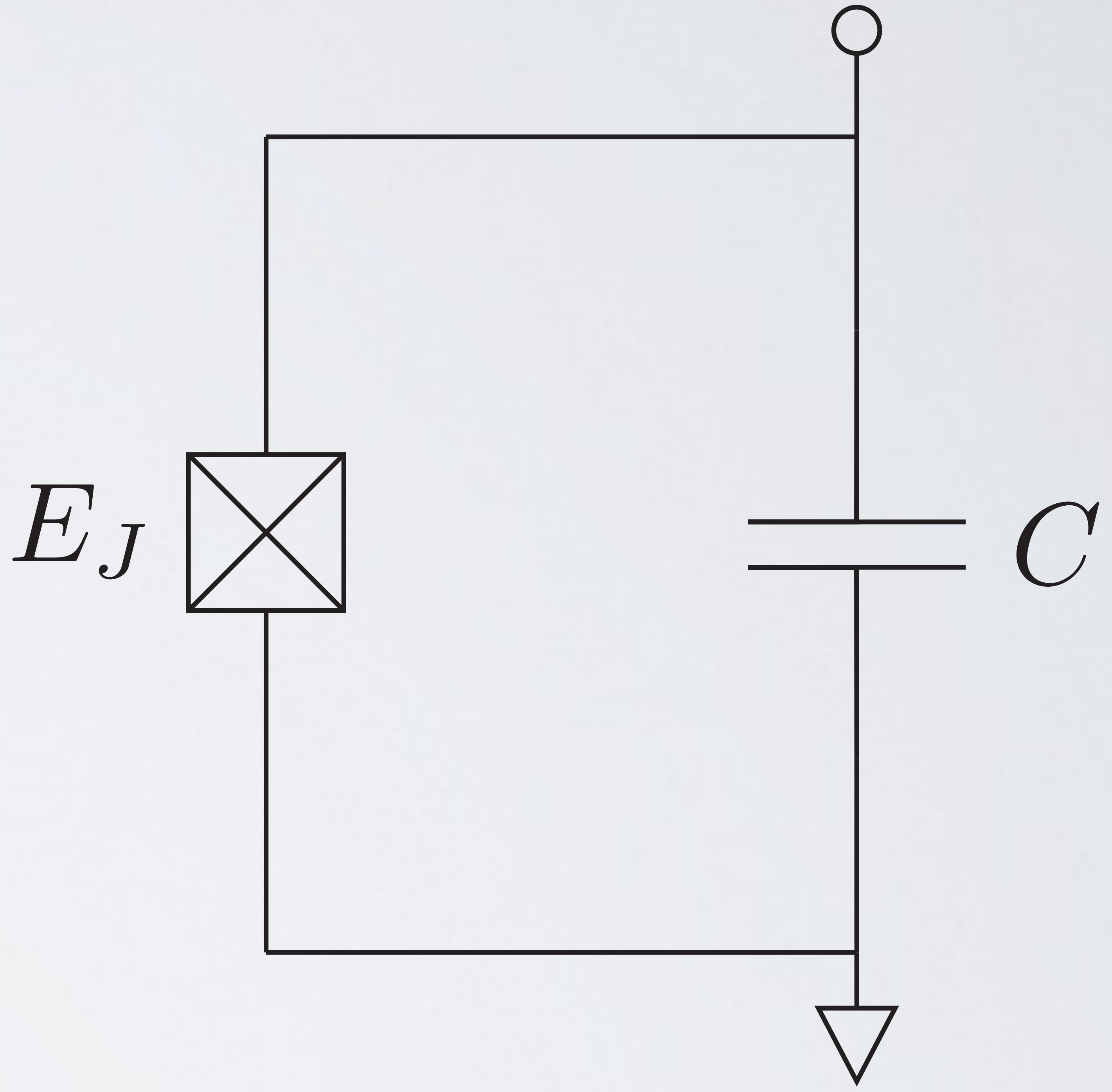


EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Let's add another ingredient…

Josephson junction

$$\hat{H}_T = -\frac{1}{2}E_J \sum_m |m\rangle\langle m+1| + |m+1\rangle\langle m|$$



Capacitance

$$\hat{H}_C = \frac{e^2\hat{n}^2}{2C} \qquad \hat{n} = \sum_m |m\rangle\langle m|$$

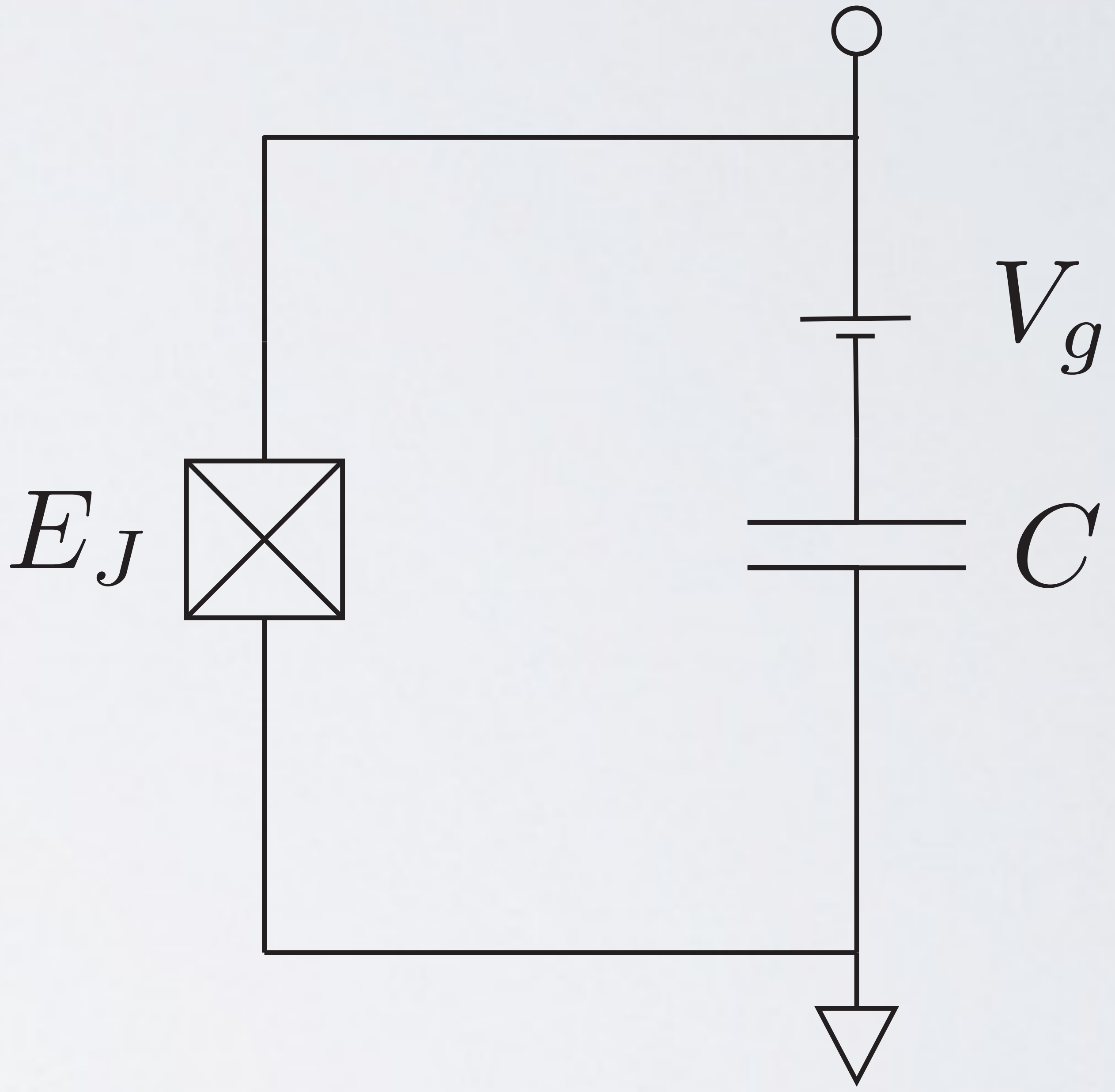
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Josephson junction

$$\hat{H}_T = -\frac{1}{2}E_J \sum_m |m\rangle \langle m+1| + |m+1\rangle \langle m|$$

(Biased) capacitance

$$\hat{H}_C = E_C \left(\hat{n} - n_g\right)^2$$



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

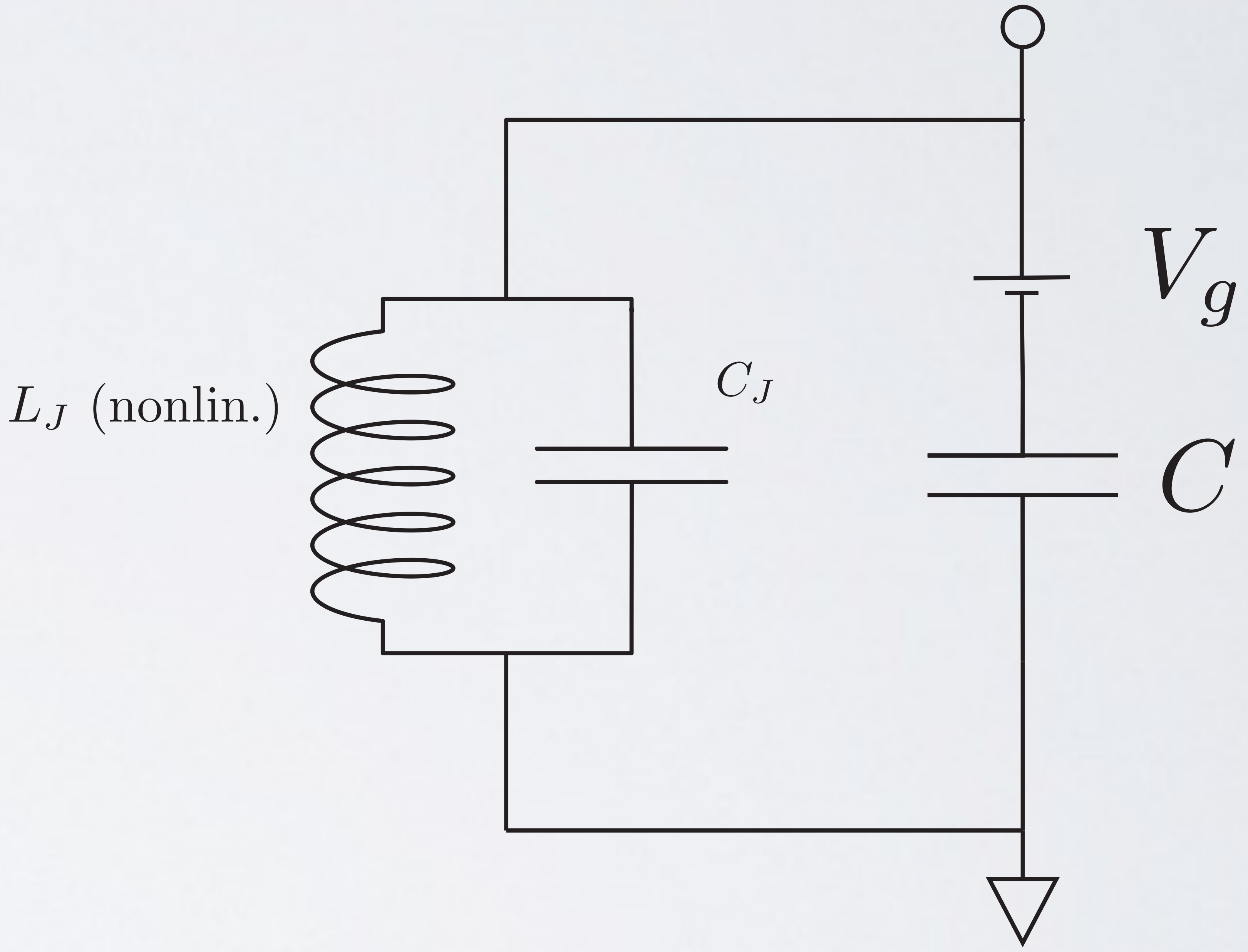
Josephson junction

$$\hat{H}_T = -\frac{1}{2}E_J \sum_m |m\rangle \langle m+1| + |m+1\rangle \langle m|$$

(Biased) capacitance

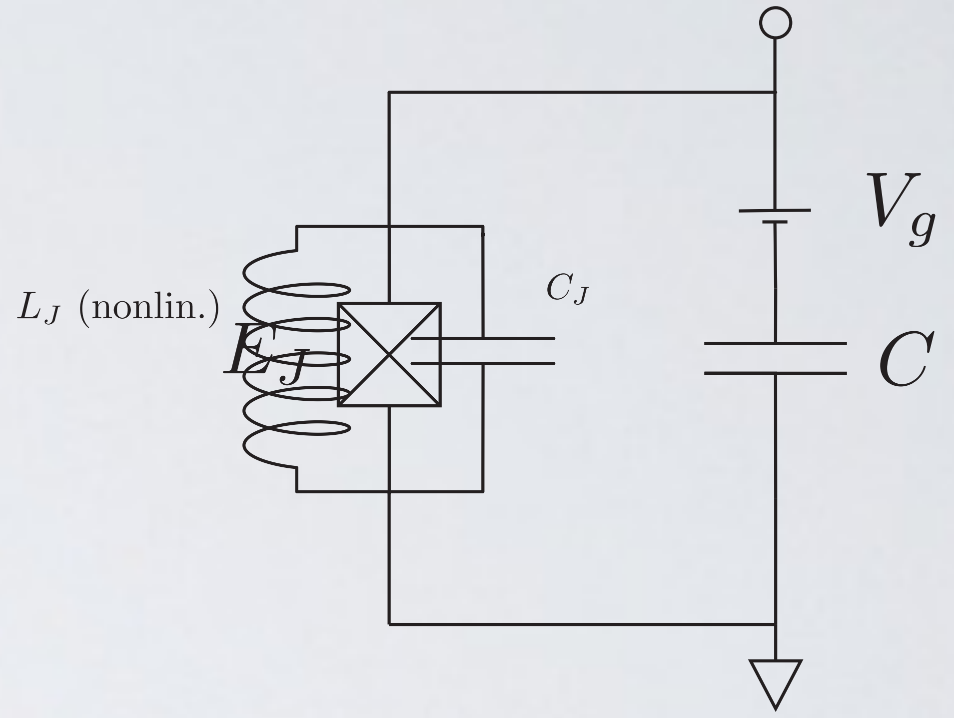
$$\hat{H}_C = E_C \left(\hat{n} - n_g\right)^2$$

$$H = E_c(n - n_g)^2 + E_J \cos\phi$$
$$\simeq E_c(n - ng)^2 + \frac{1}{2L_J}\Phi^2 + A_{nl}\Phi^4 + \dots$$



EXAMPLE 1: SUPERCONDUCTING CIRCUITS

$$\hat{H}_T = -\frac{1}{2} E_J \sum_m |m\rangle \langle m+1| + |m+1\rangle \langle m|$$



$$\hat{H}_C = \frac{e^2}{2C} \left(\hat{n} - n_g\right)^2 \qquad \hat{n} = \sum_m |m\rangle \langle m|$$

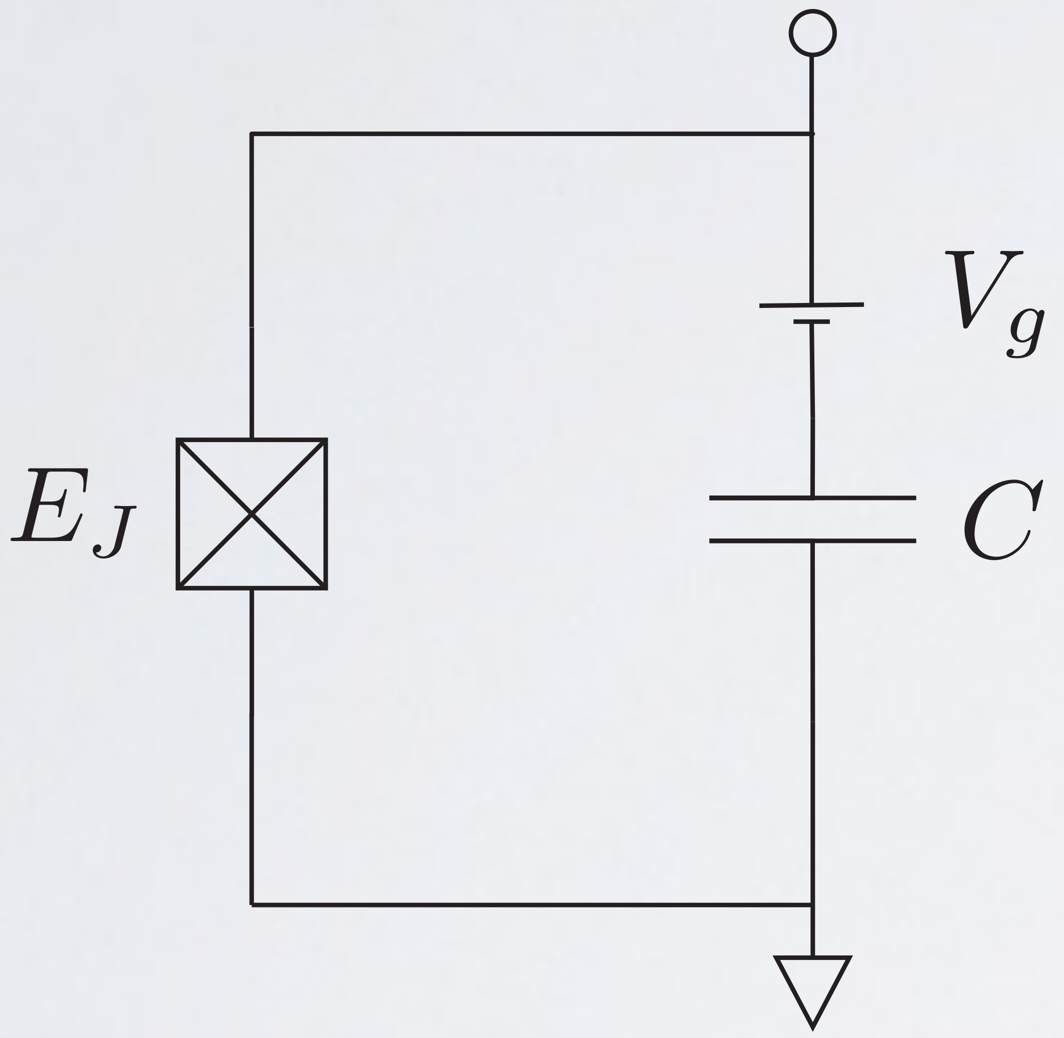
$H = \hat{H}_T + \hat{H}_C$ can be diagonalized (in the "phase representation")

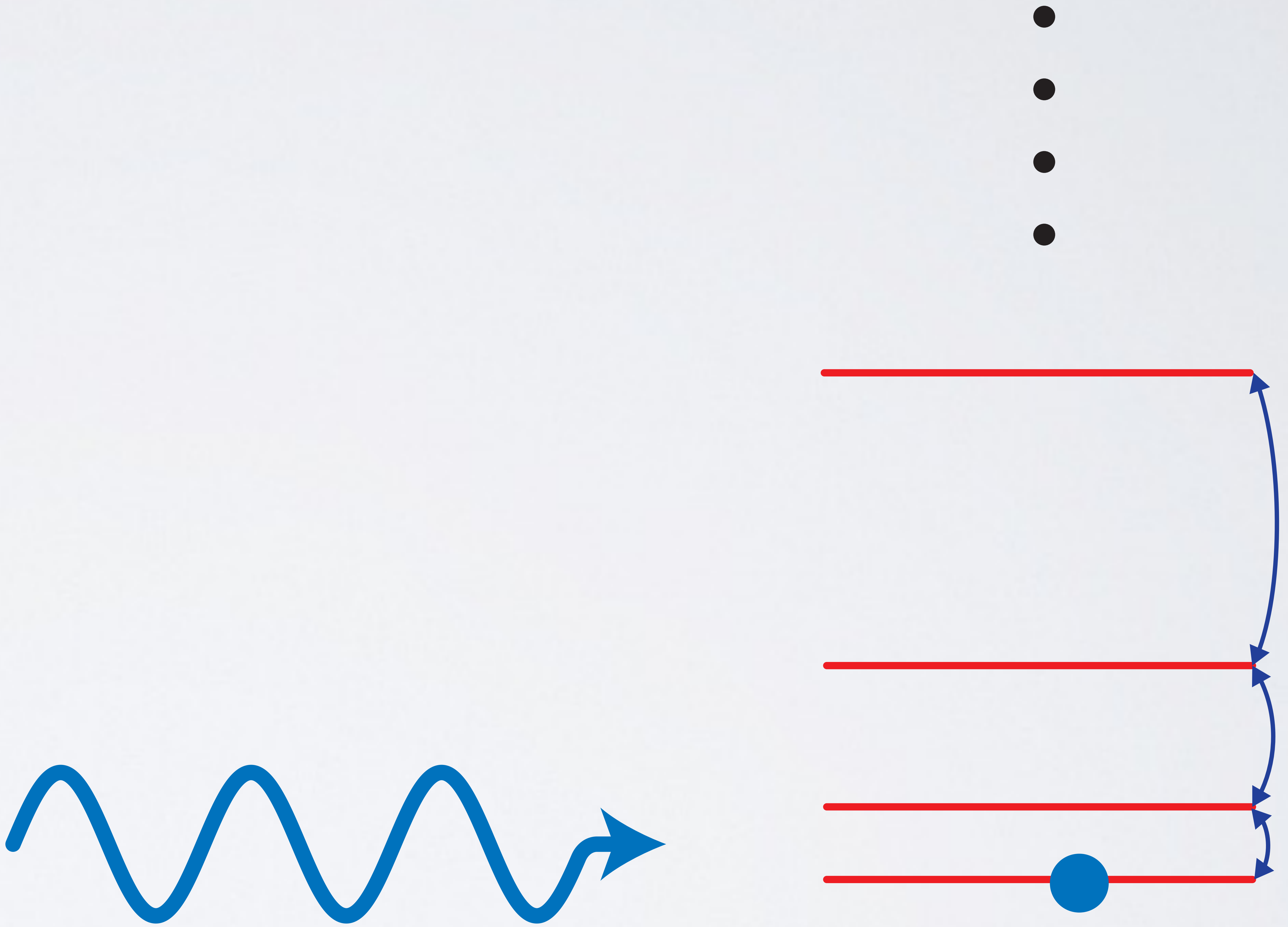
$$\left[\hat{\phi}, \hat{n}\right] = i$$

Nonlinear spectrum

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

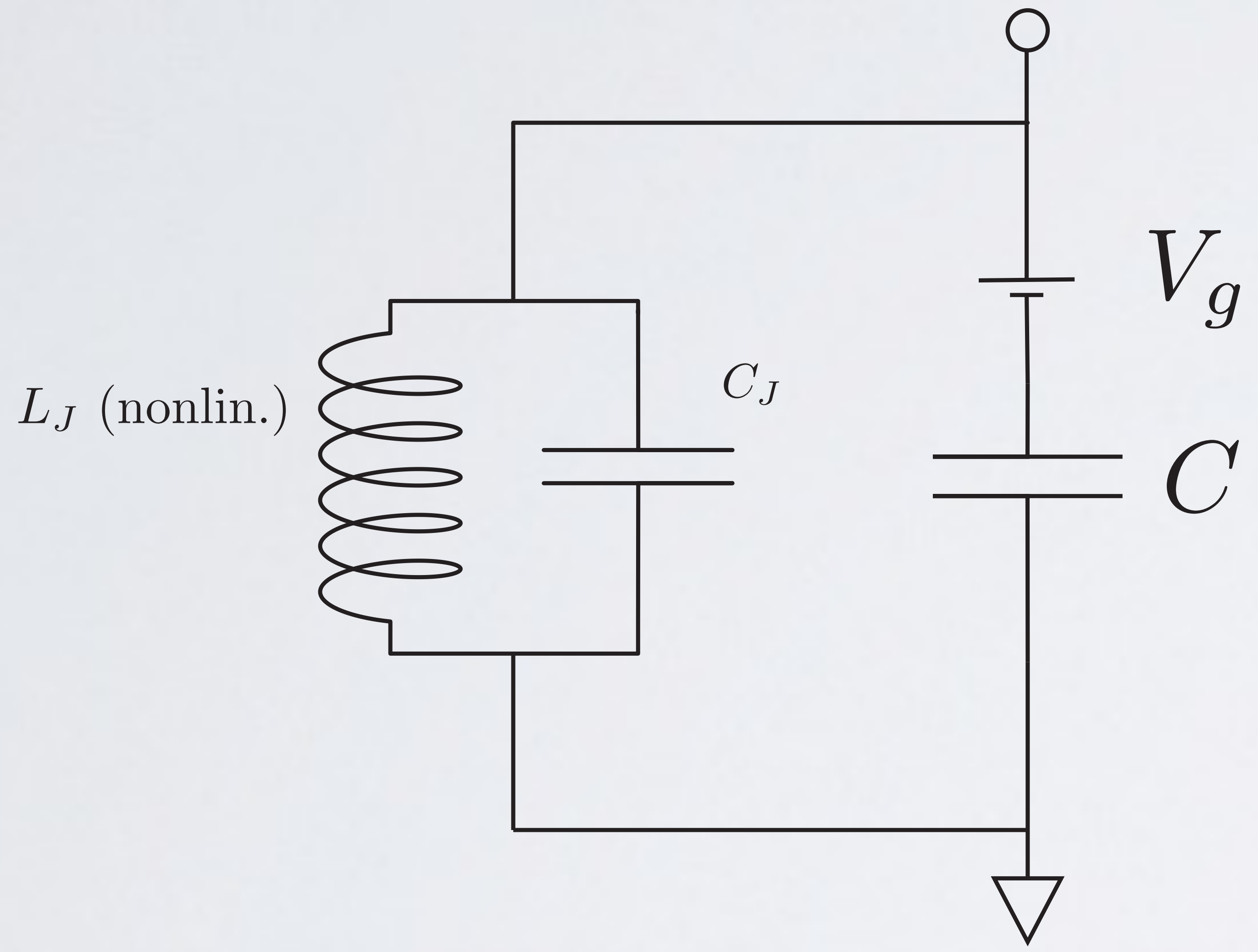
Now we've got what we need

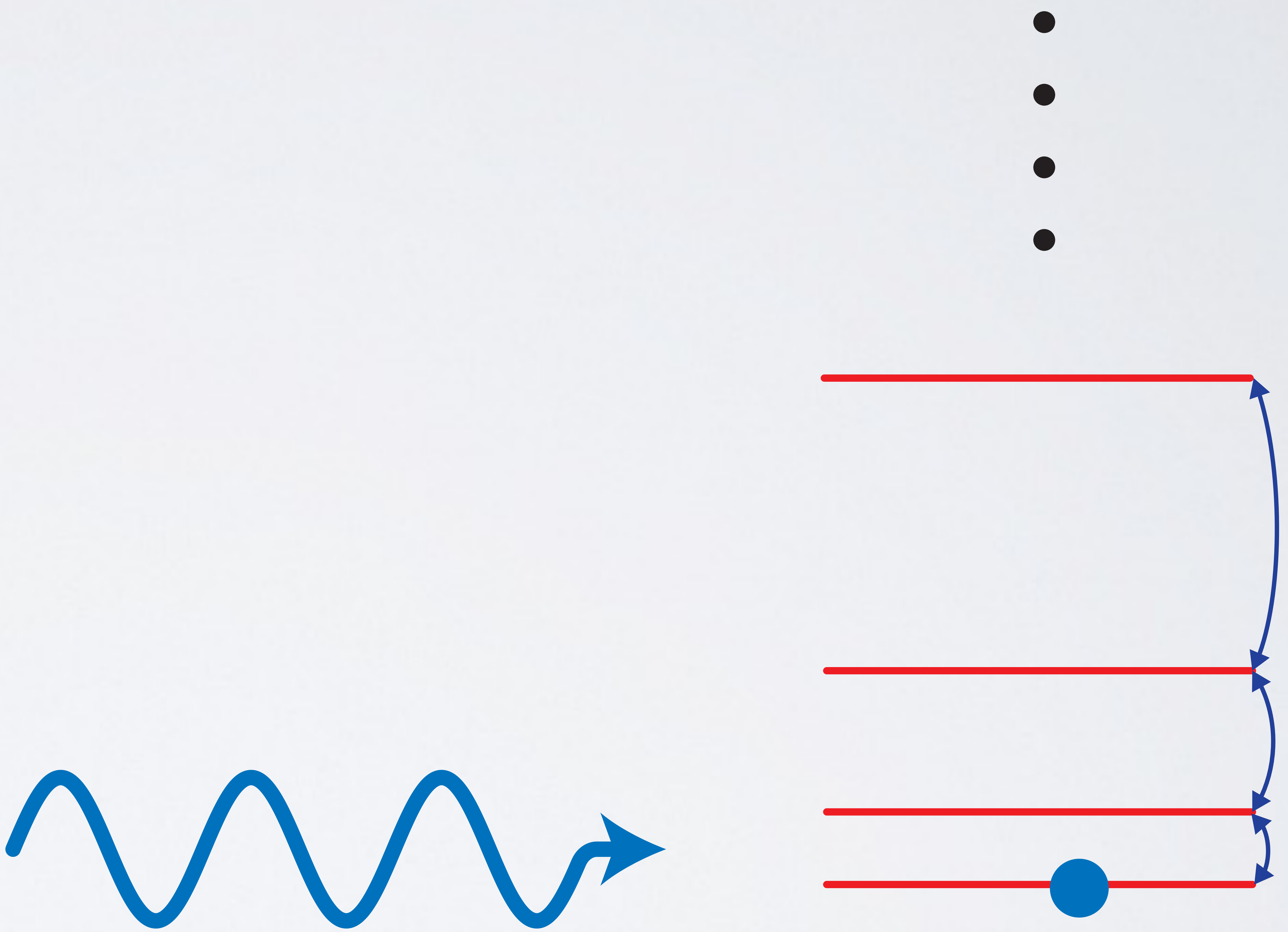




EXAMPLE 1: SUPERCONDUCTING CIRCUITS

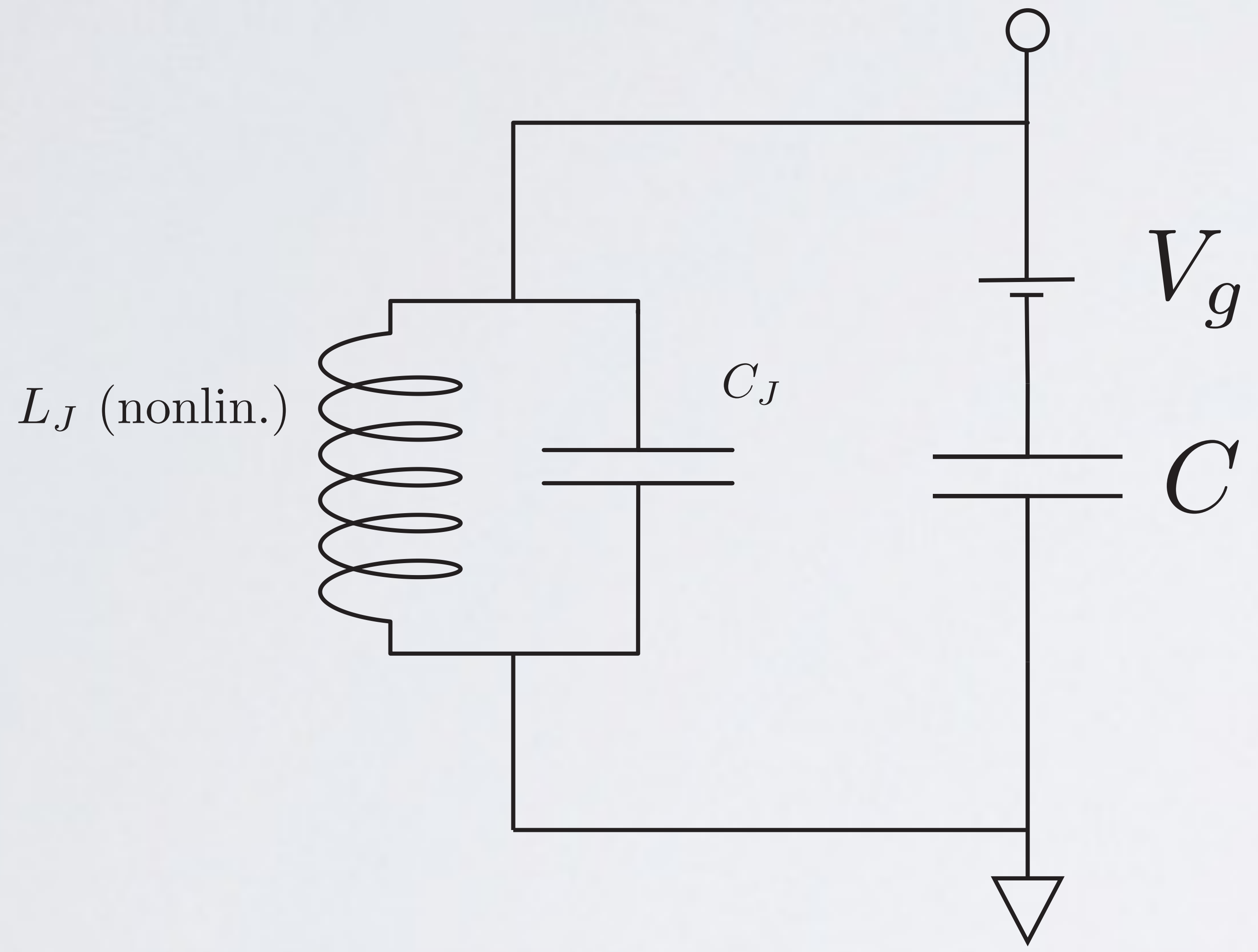
Now we've got what we need

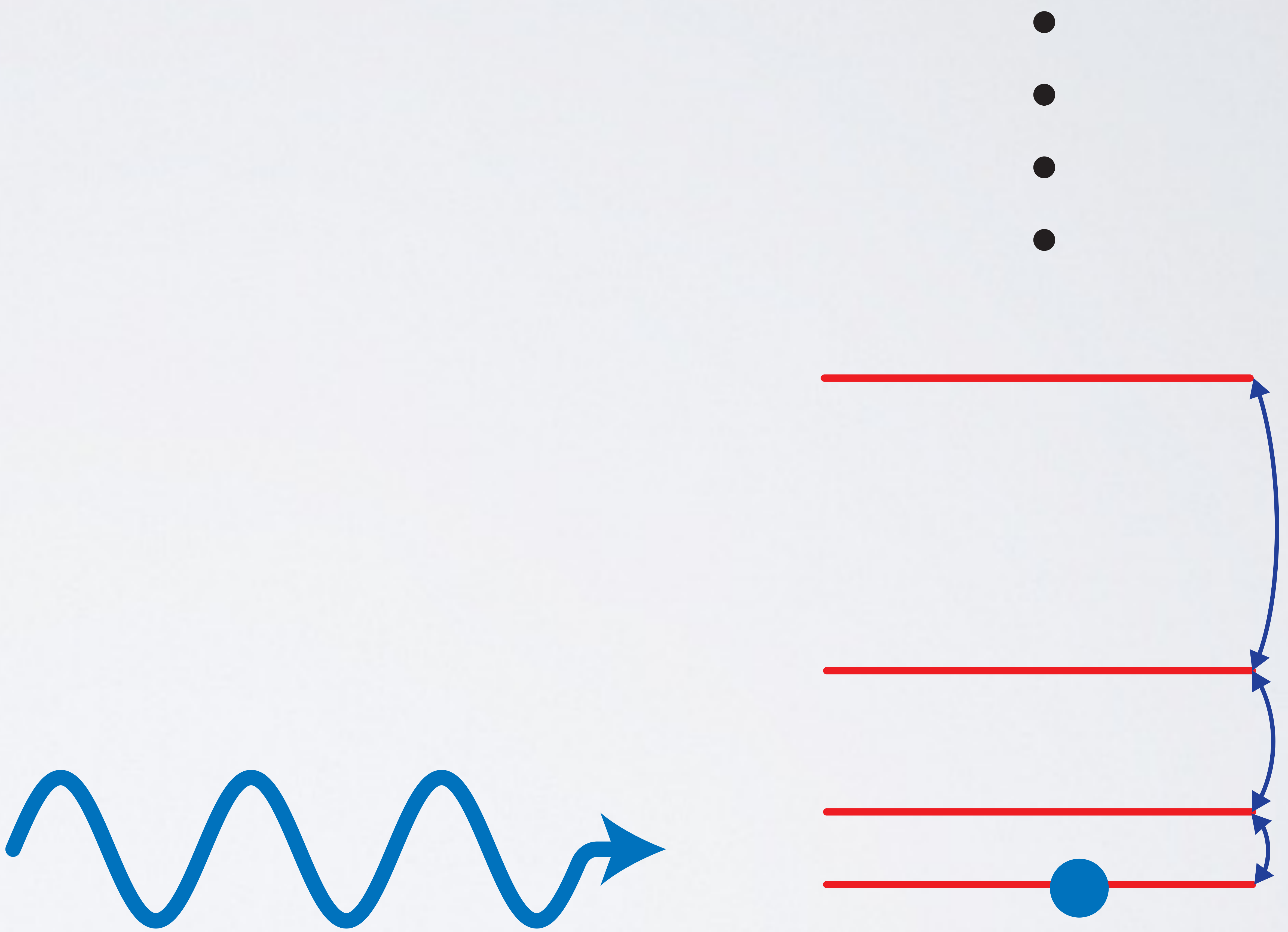




EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Now we've got what we need

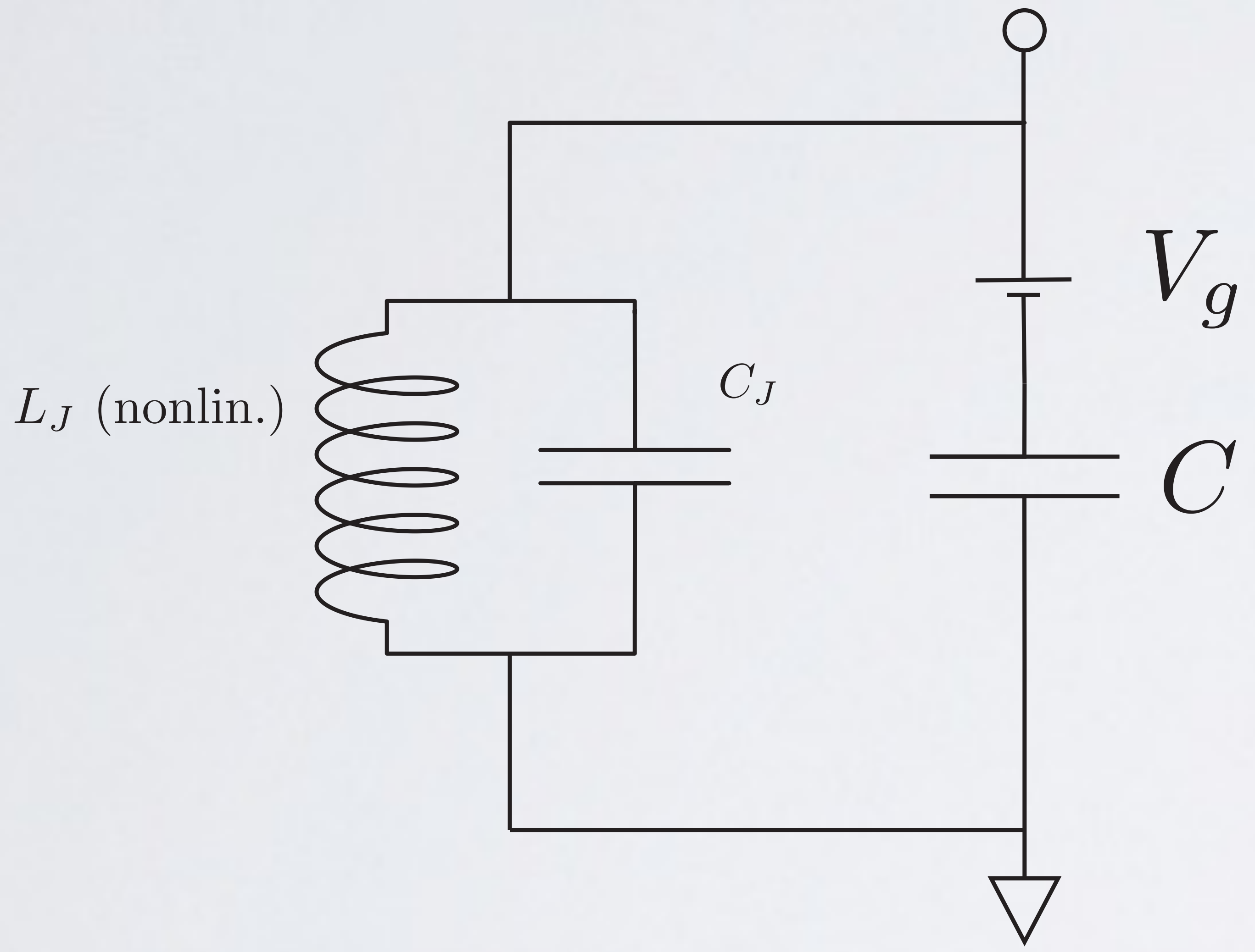


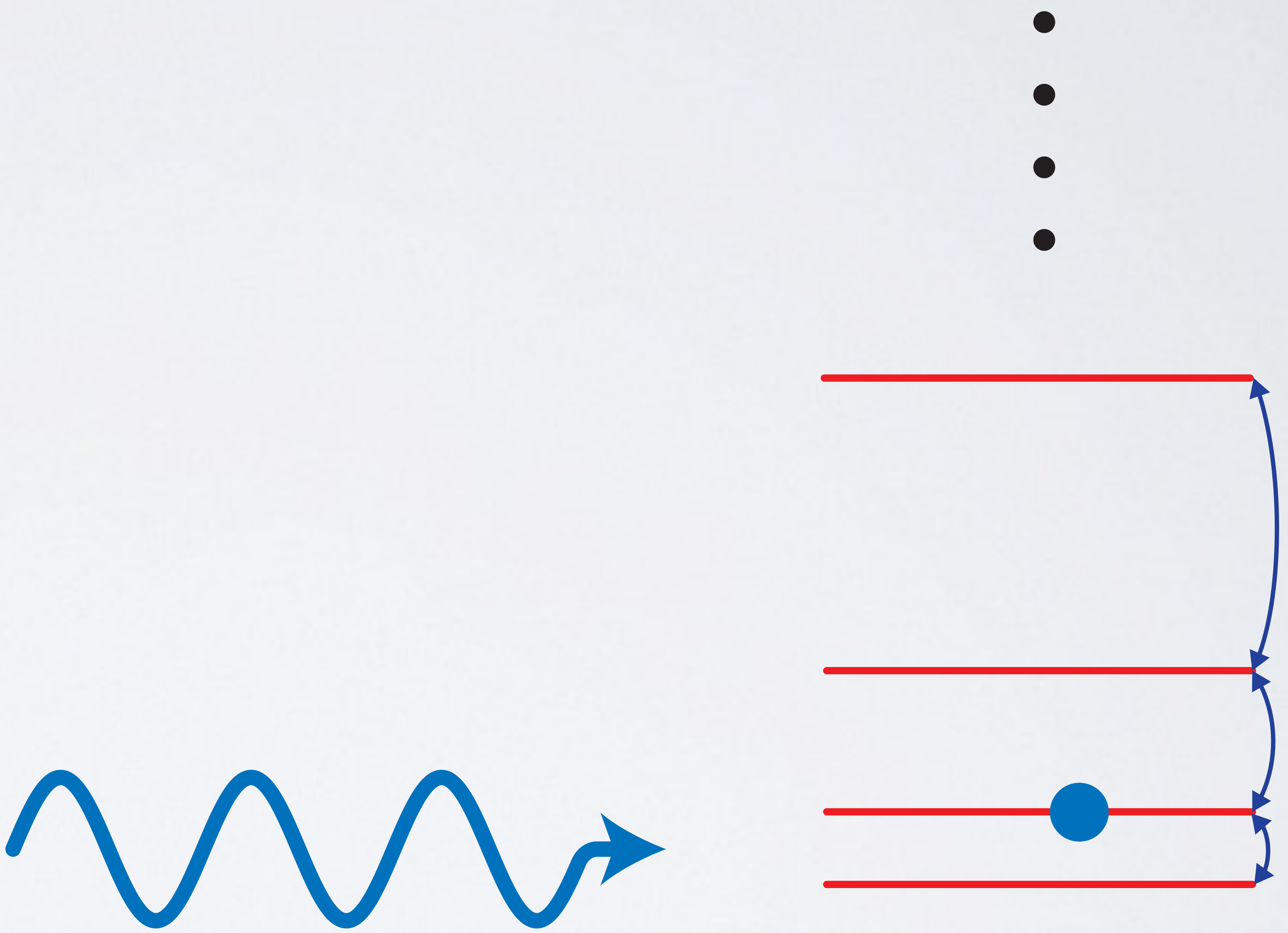


Nonlinear spectrum

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Now we've got what we need

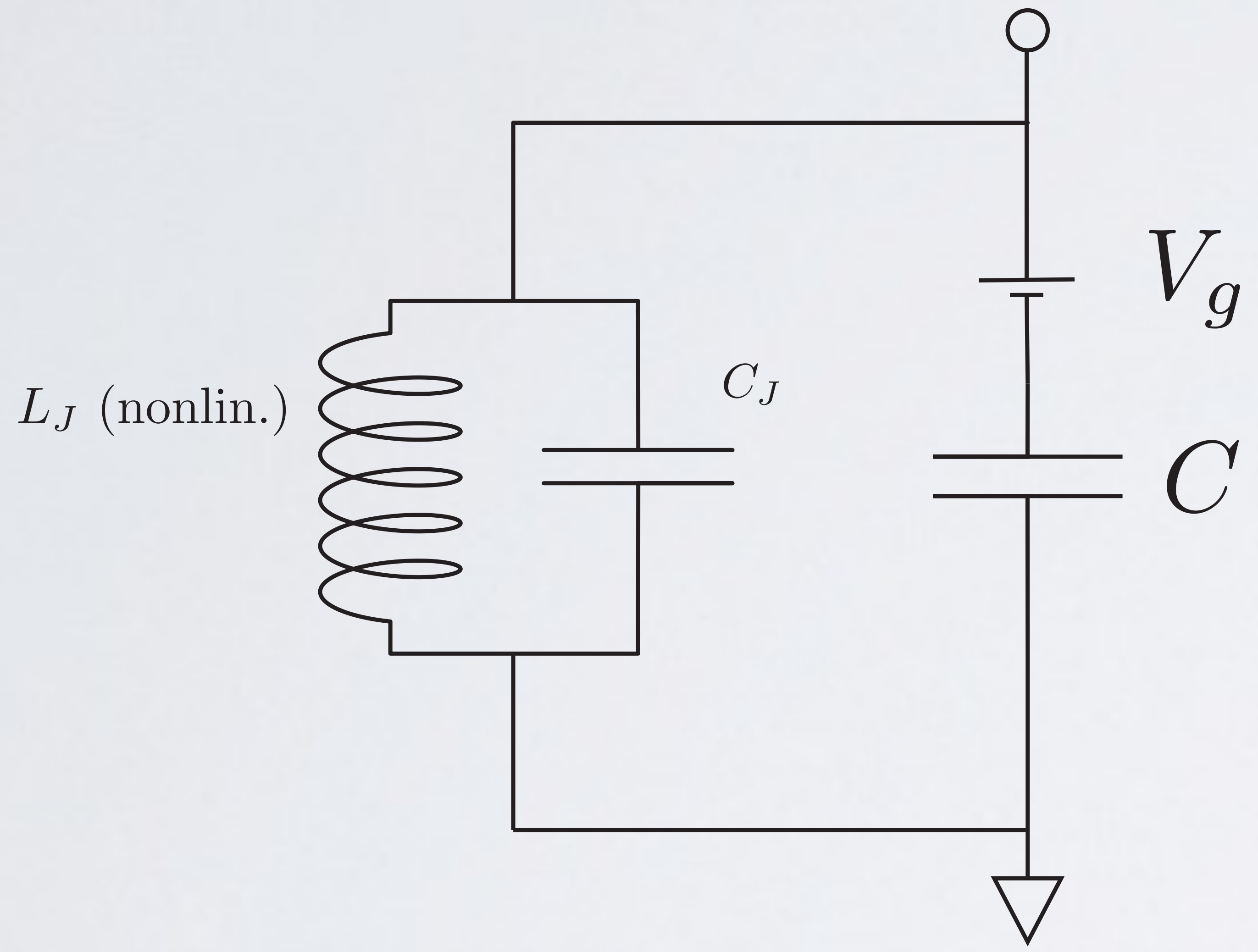


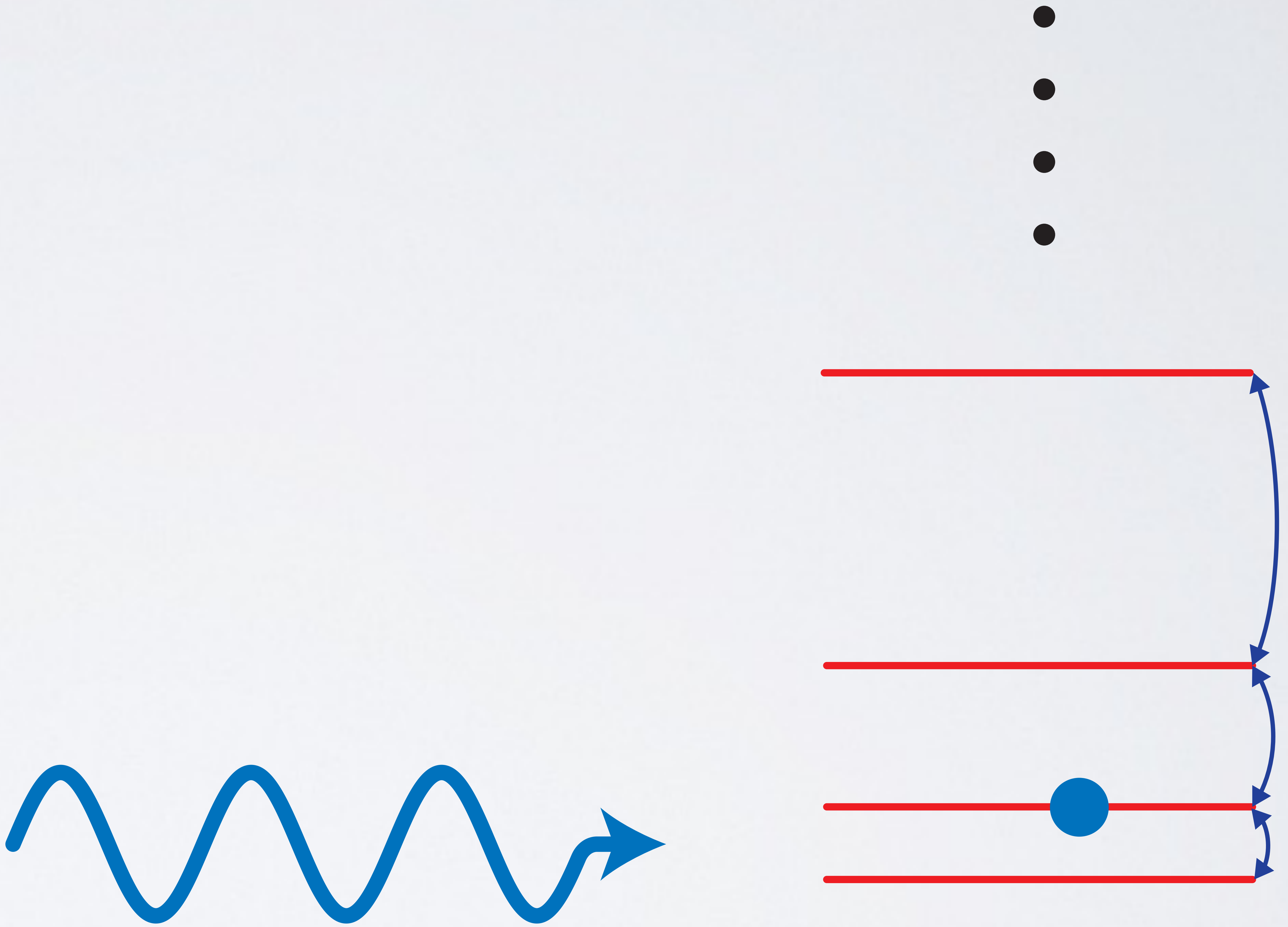


Nonlinear spectrum

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Now we've got what we need

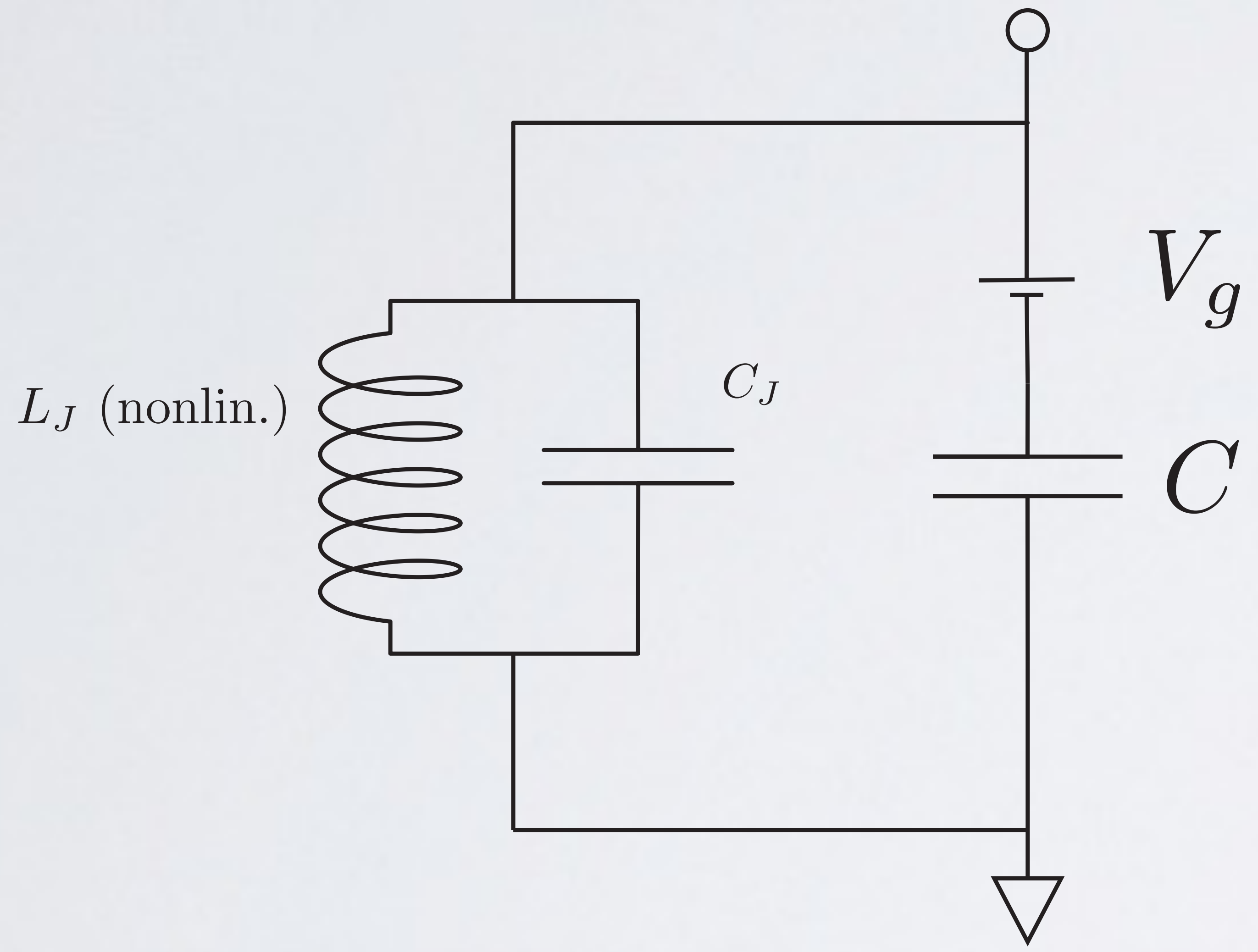




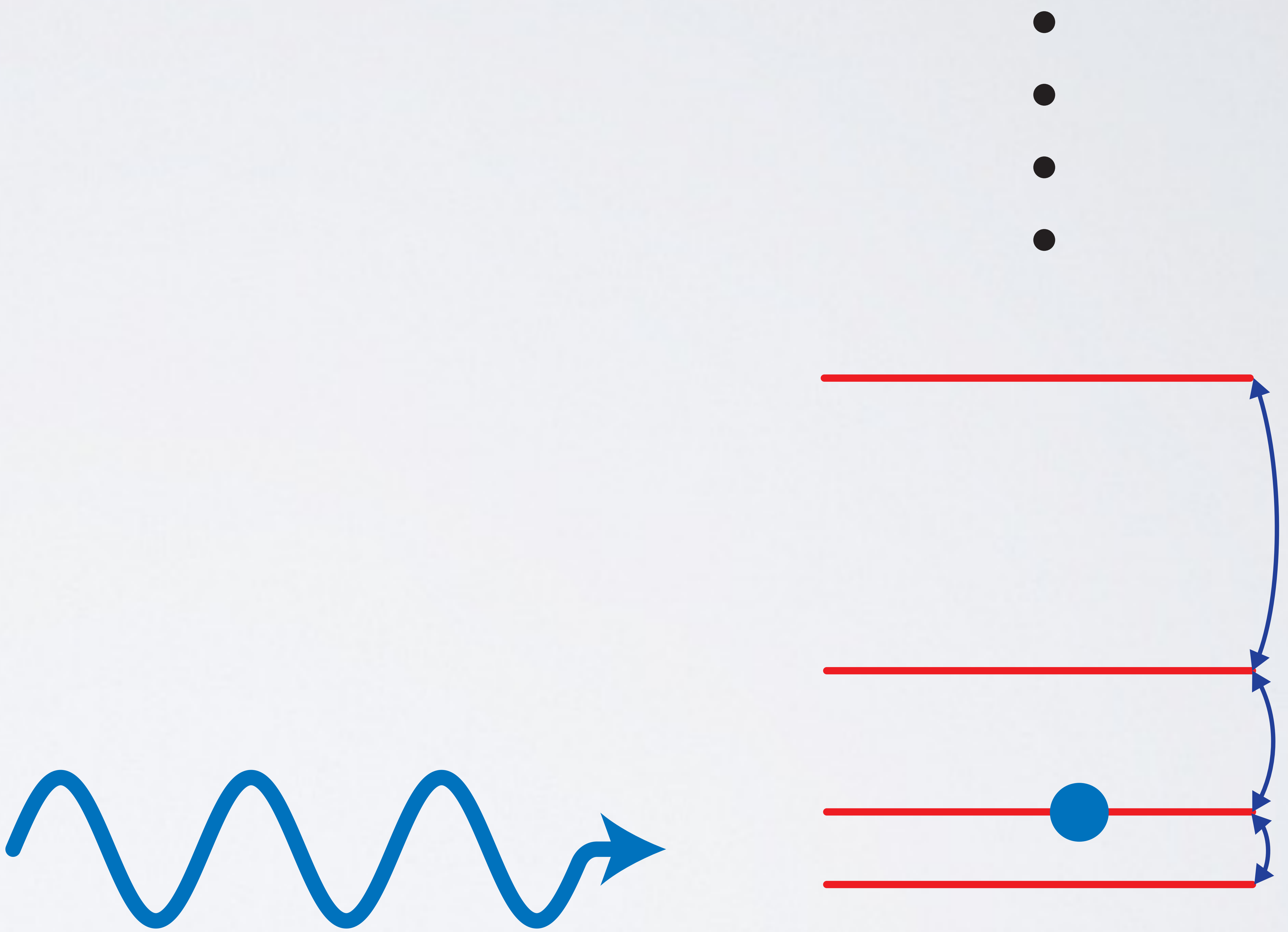
Nonlinear spectrum

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

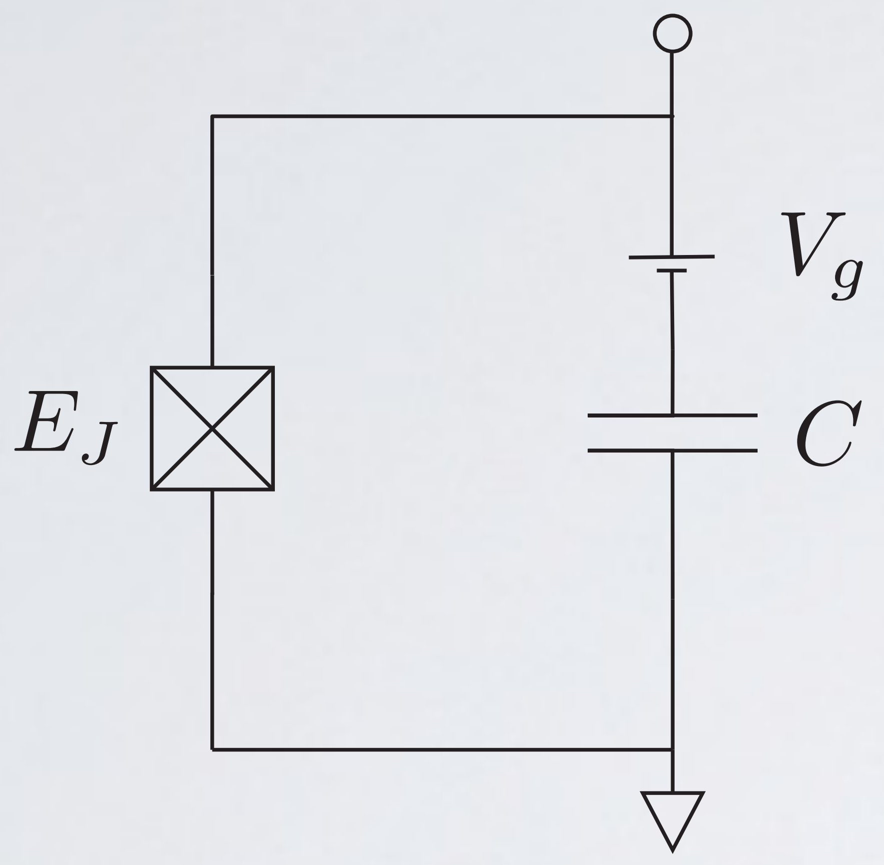
Now we've got what we need

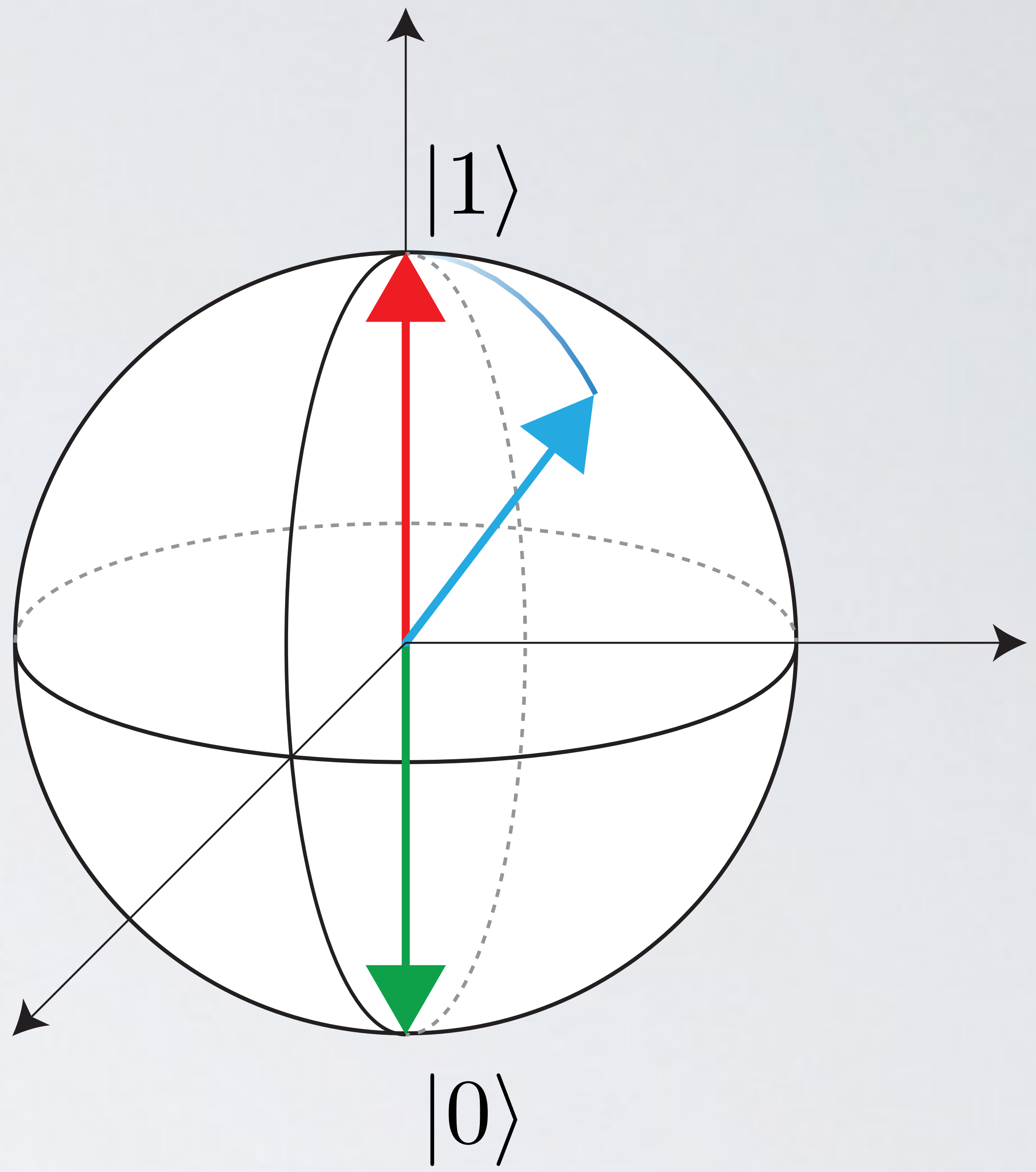


Nonlinear spectrum



EXAMPLE 1: SUPERCONDUCTING CIRCUITS





$$\alpha\,|0\rangle + \beta\,|1\rangle$$

Cooper-pair box

Charge qubit

Transmon qubit ($E_J \gg E_c$)

Xmon qubit

e.g. Google Bristlecore architecture

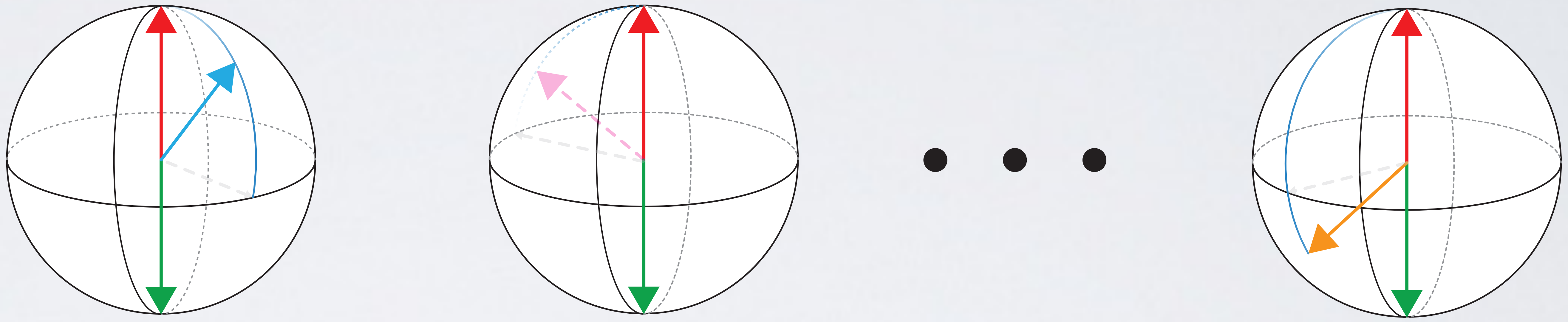
EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Why is the idea so powerful?

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Why is the idea so powerful?

Let's put many of them together

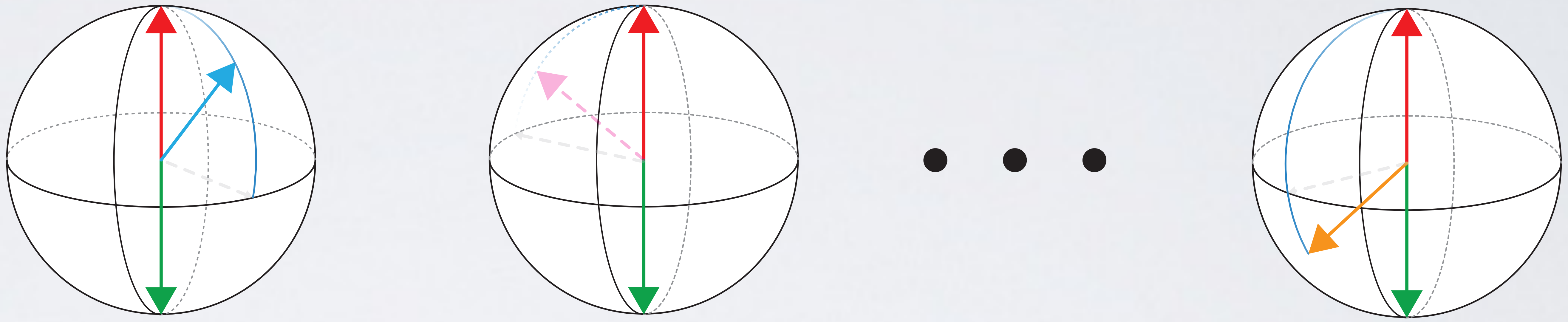


$\alpha_0 |0\rangle + \beta_0 |1\rangle$ $\alpha_1 |0\rangle + \beta_1 |1\rangle$ $\alpha_n |0\rangle + \beta_n |1\rangle$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Why is the idea so powerful?

Let's put many of them together



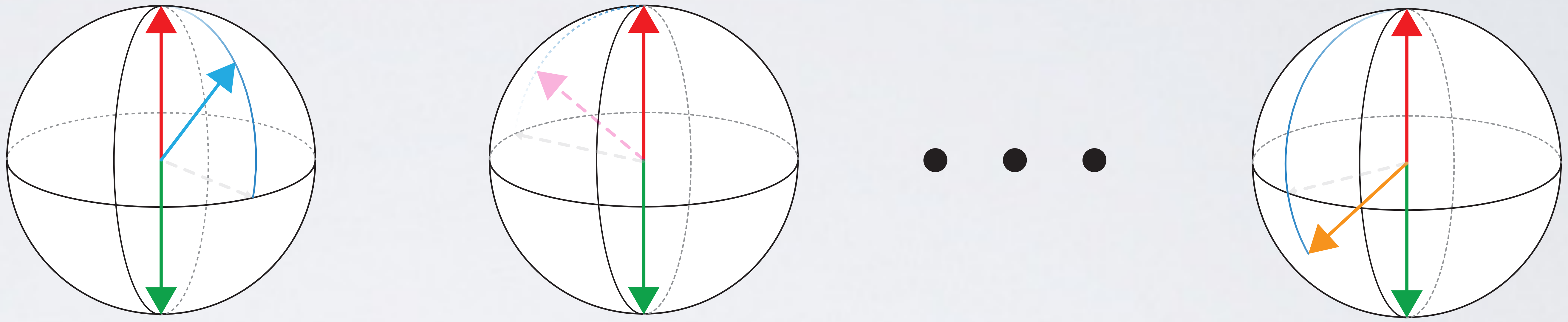
$\alpha_0 |0\rangle + \beta_0 |1\rangle$ $\quad$ $\alpha_1 |0\rangle + \beta_1 |1\rangle$ $\quad$ $\alpha_n |0\rangle + \beta_n |1\rangle$

$$
\begin{aligned}
|\text{qubit}\rangle &= (\alpha_0 |0\rangle + \beta_0 |1\rangle)(\alpha_1 |0\rangle + \beta_1 |1\rangle)\dots(\alpha_n |0\rangle + \beta_n |1\rangle) \\
&= \sum_{i=1}^{2^n} c_i |\text{"bit"}\rangle_i
\end{aligned}
$$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Why is the idea so powerful?

Let's put many of them together



$\alpha_0 |0\rangle + \beta_0 |1\rangle$ $\alpha_1 |0\rangle + \beta_1 |1\rangle$ $\alpha_n |0\rangle + \beta_n |1\rangle$

$$|\text{qubit}\rangle = (\alpha_0 |0\rangle + \beta_0 |1\rangle)(\alpha_1 |0\rangle + \beta_1 |1\rangle)\ldots(\alpha_n |0\rangle + \beta_n |1\rangle)$$
$$= \sum_{i=1}^{2^n} c_i |\text{"bit"}\rangle_i$$

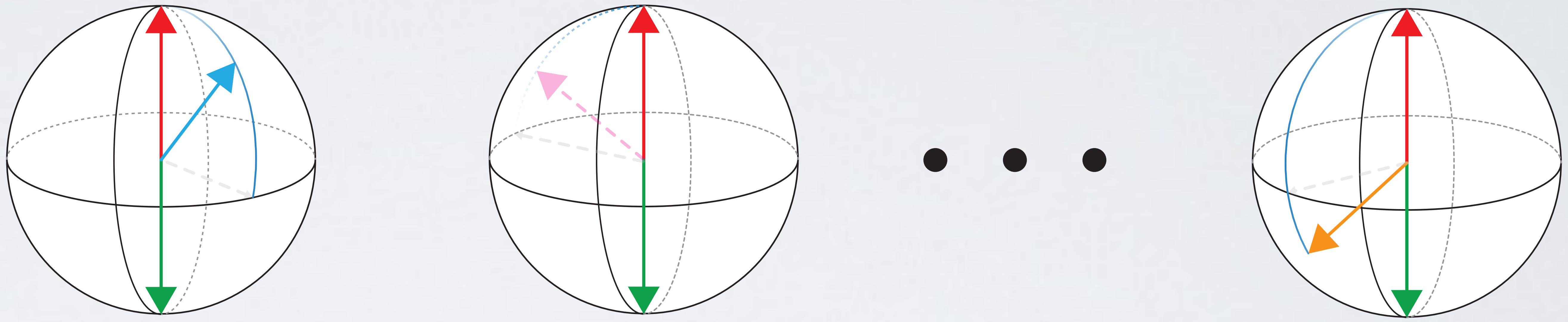
$c_1 = \alpha_0\alpha_1\alpha_2\ldots\alpha_n \quad |\text{"bit"}\rangle_1 = |00\ldots0\rangle$

$c_2 = \beta_0\alpha_1\alpha_2\ldots\alpha_n \quad |\text{"bit"}\rangle_2 = |10\ldots0\rangle$

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Why is the idea so powerful?

Let's put many of them together



$\alpha_0 |0\rangle + \beta_0 |1\rangle$ $\alpha_1 |0\rangle + \beta_1 |1\rangle$ $\alpha_n |0\rangle + \beta_n |1\rangle$

$$|\text{qubit}\rangle = (\alpha_0 |0\rangle + \beta_0 |1\rangle)(\alpha_1 |0\rangle + \beta_1 |1\rangle)\dots(\alpha_n |0\rangle + \beta_n |1\rangle)$$

$$= \sum_{i=1}^{2^n} c_i |\text{"bit"}\rangle_i$$

$c_1 = \alpha_0\alpha_1\alpha_2\dots\alpha_n \quad |\text{"bit"}\rangle_1 = |00\dots0\rangle$

$c_2 = \beta_0\alpha_1\alpha_2\dots\alpha_n \quad |\text{"bit"}\rangle_2 = |10\dots0\rangle$

Quantum algorithm operates on $|\text{qubit}\rangle$ → in parallel on 2^n classical bits

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Wrapping up:

-Metallic properties (screening plasma oscillations)

-Superconductivity (gapping single-particle excitations)

-Artificial atom with tunable properties

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Wrapping up:

-we can engineer the properties of a nearly-macroscopic metal lump to behave quantum mechanically

-Metallic properties (screening plasma oscillations)

-Superconductivity (gapping single-particle excitations)

-Artificial atom with tunable properties

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Wrapping up:

-we can engineer the properties of a nearly-macroscopic metal lump to behave quantum mechanically

-Exploiting

-Metallic properties (screening plasma oscillations)

-Superconductivity (gapping single-particle excitations)

-Artificial atom with tunable properties

EXAMPLE 1: SUPERCONDUCTING CIRCUITS

Wrapping up:

-we can engineer the properties of a nearly-macroscopic metal lump to behave quantum mechanically

-Exploiting

-Metallic properties (screening plasma oscillations)

-Superconductivity (gapping single-particle excitations)

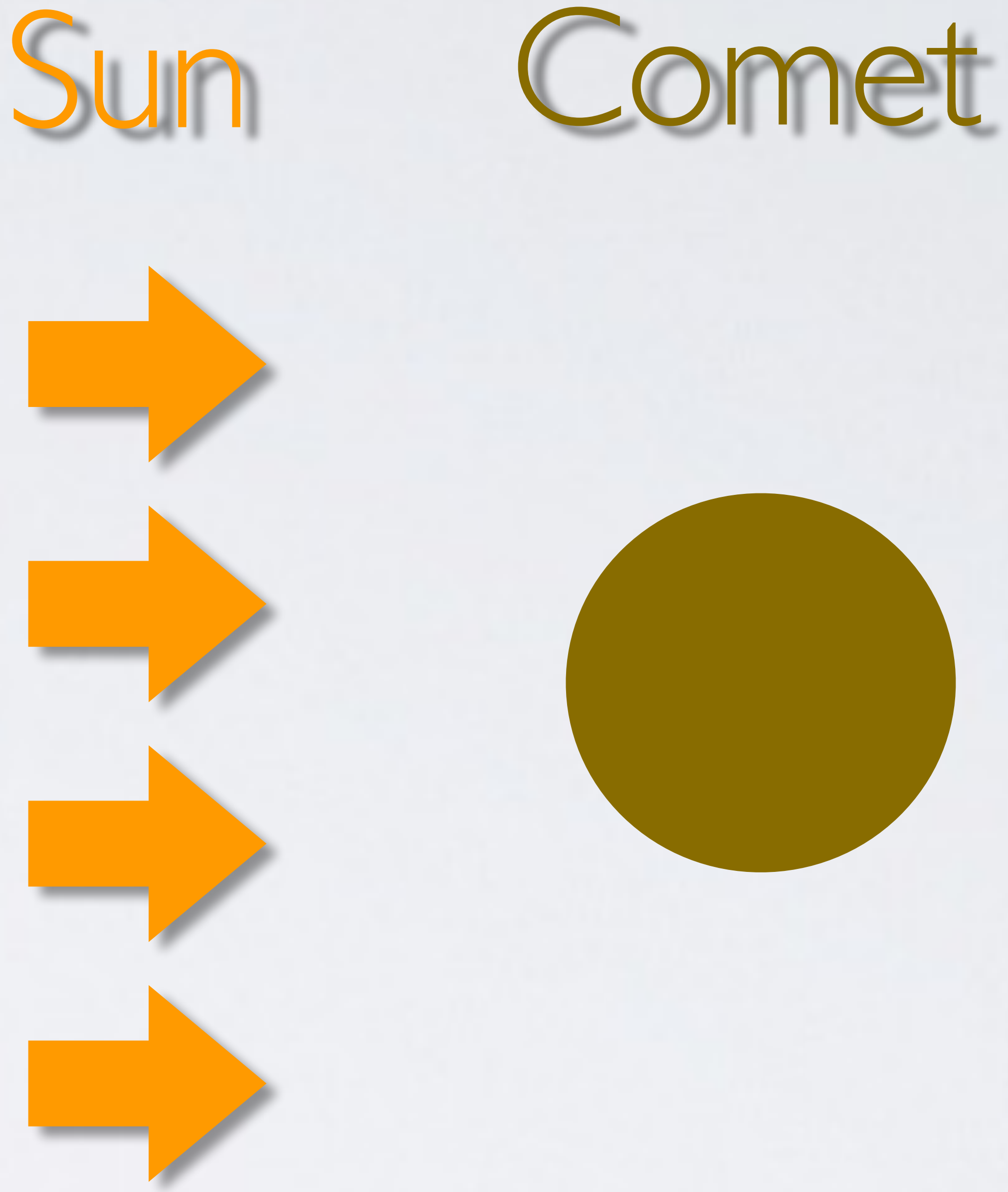
-Low temperatures: needed for sc (but not only…)

-Artificial atom with tunable properties

RADIATION PRESSURE I

First suggestion

J. Kepler
De cometis (1619)



Halley comet (1986)

© Digital Vision Ltd.

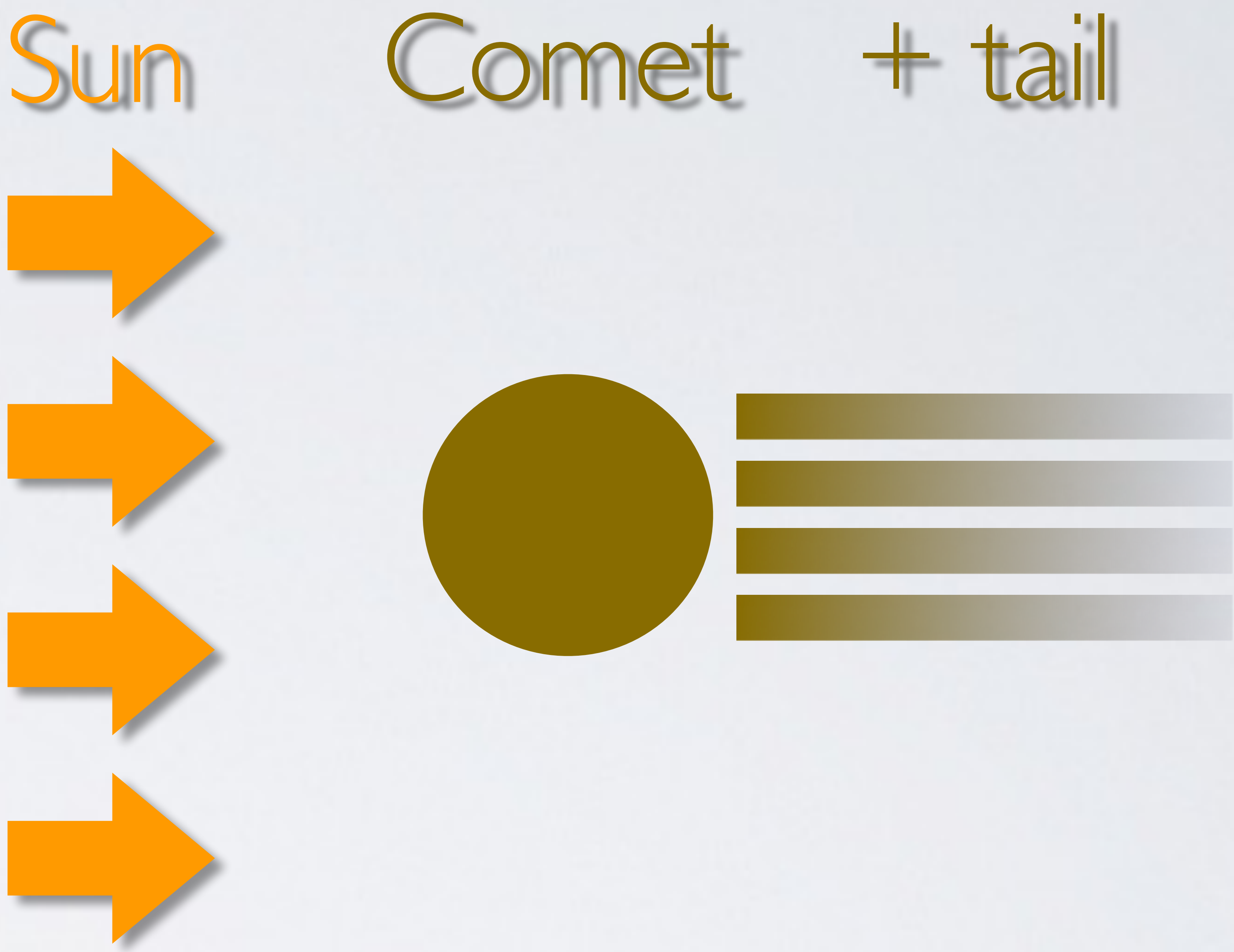
RADIATION PRESSURE I

First suggestion

J. Kepler
De cometis (1619)

Halley comet (1986)

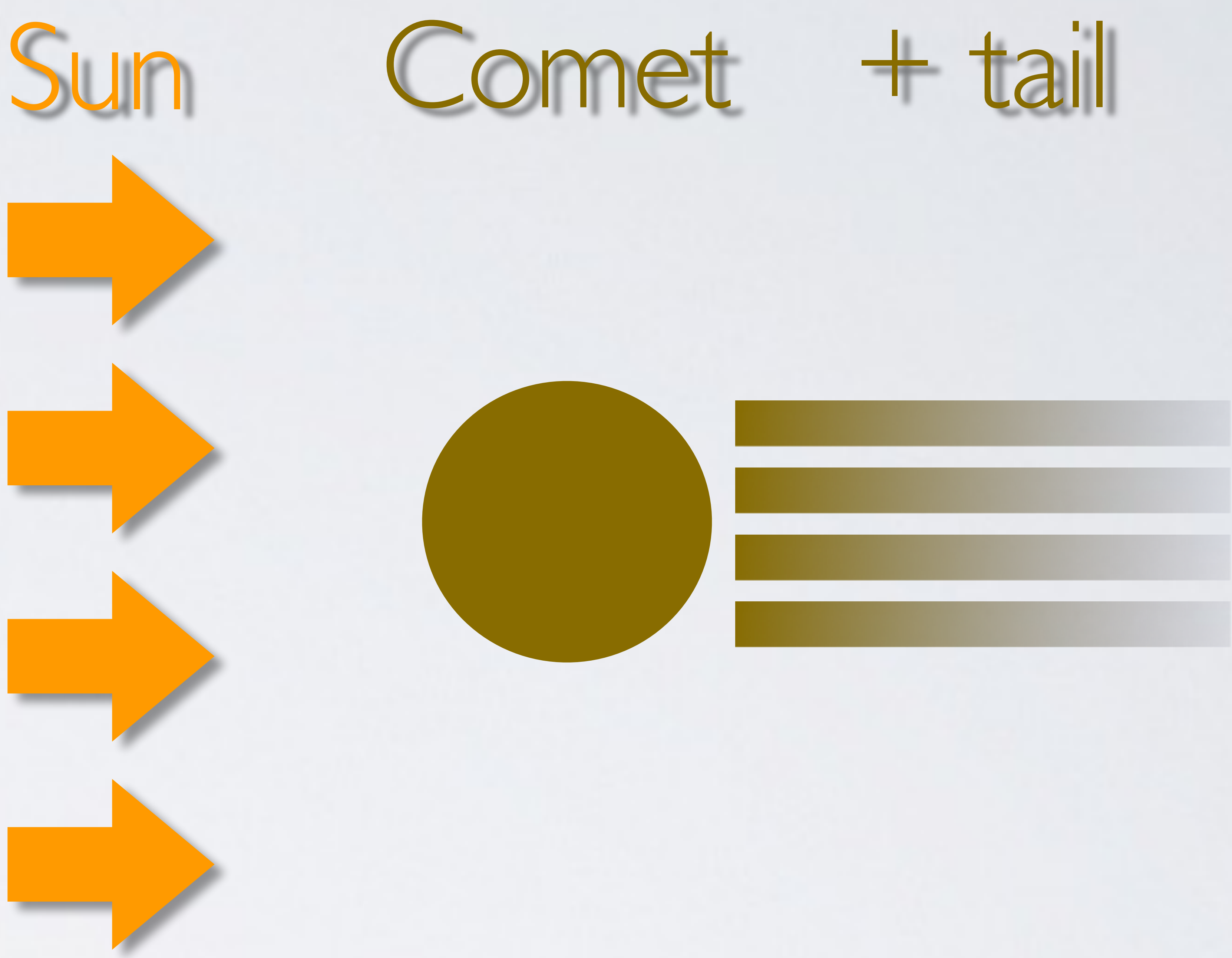
© Digital Vision Ltd.



RADIATION PRESSURE I

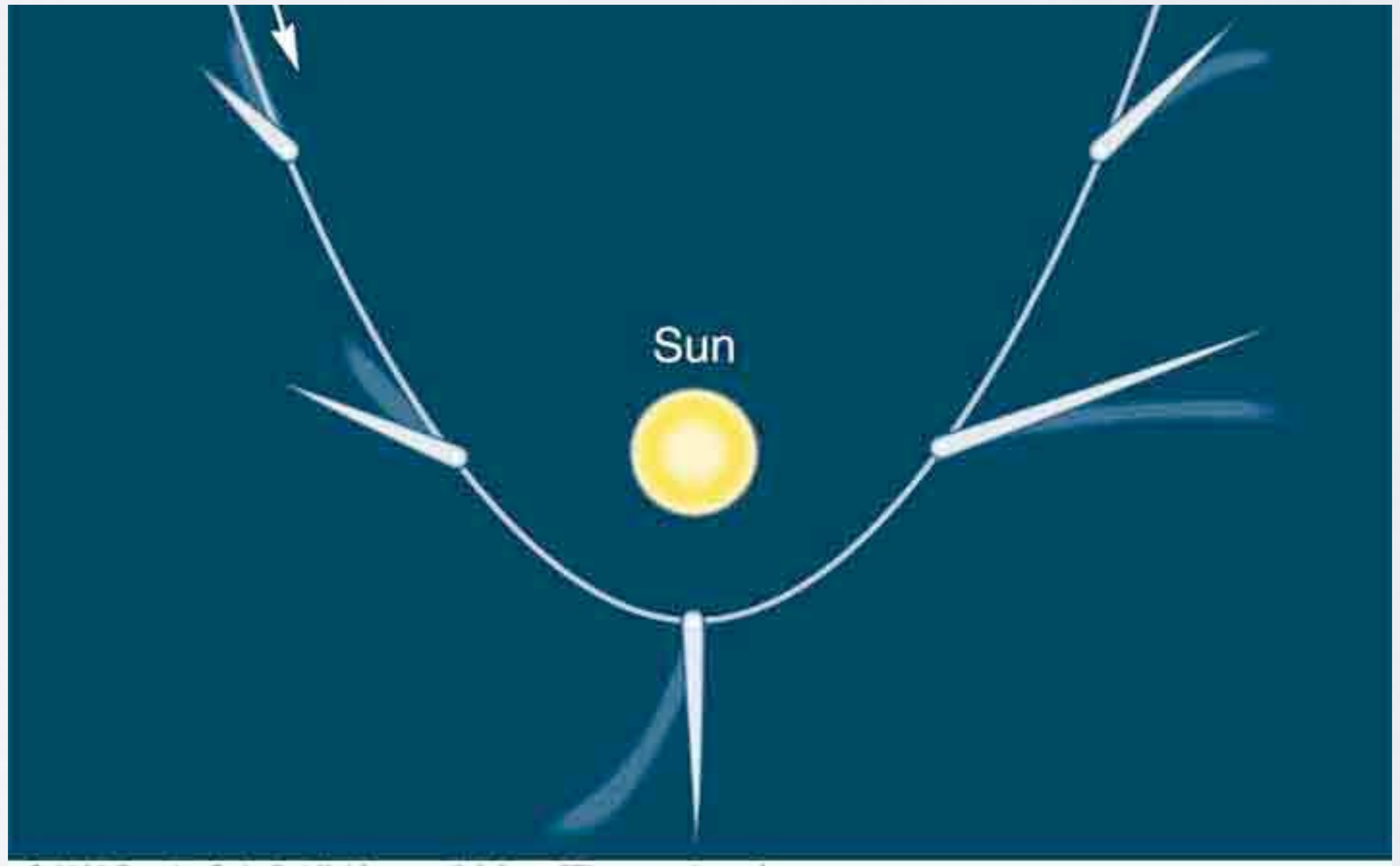
First suggestion

J. Kepler
De cometis (1619)



Halley comet (1986)

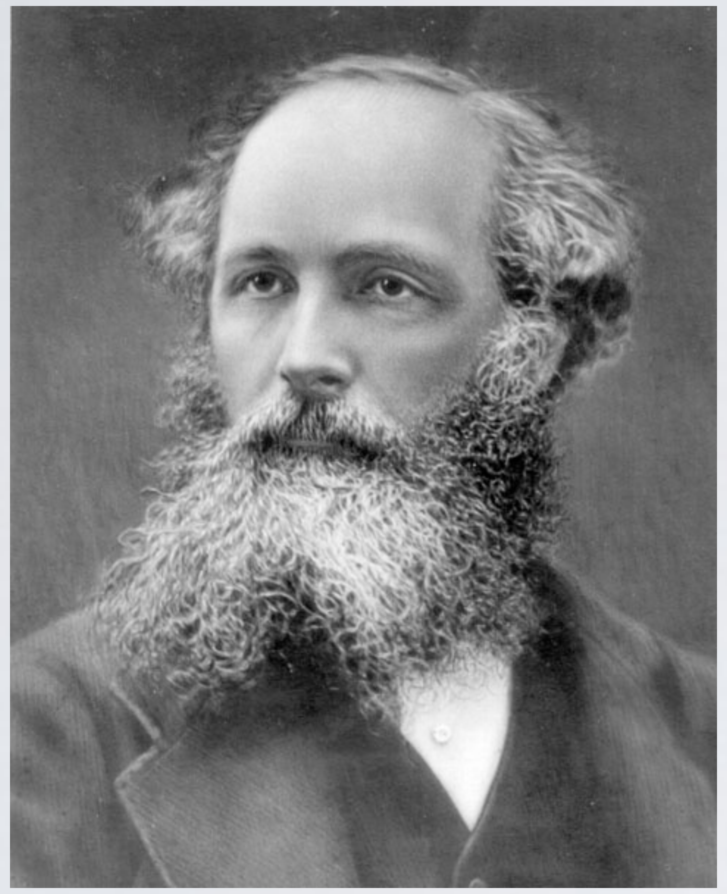
© Digital Vision Ltd.



© 2002 Brooks Cole Publishing - a division of Thomson Learning

RADIATION PRESSURE II

Theoretical description



J. C. Maxwell

Maxwell equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

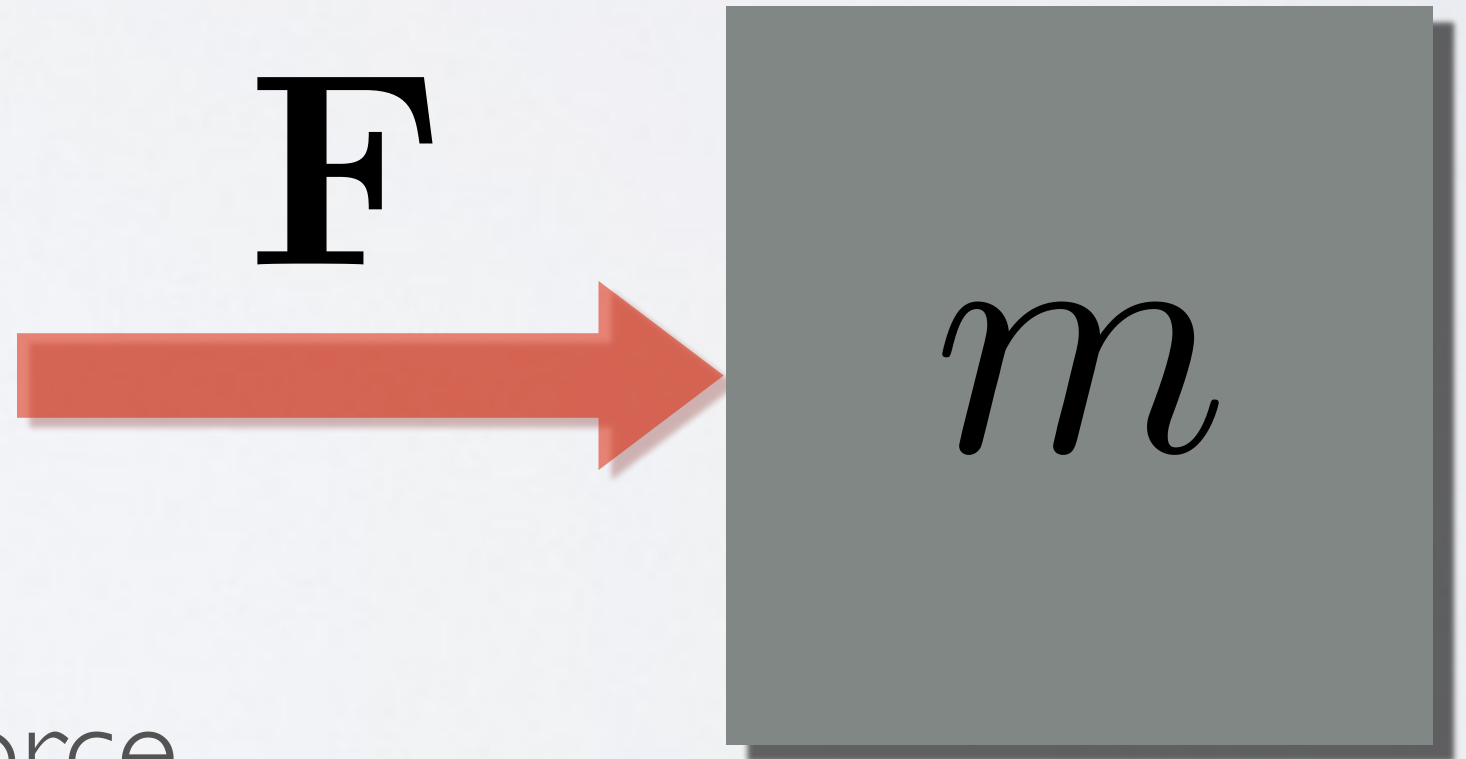
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\left(\frac{\mathbf{J}}{\epsilon_0} + \frac{\partial \mathbf{E}}{\partial t}\right)$$

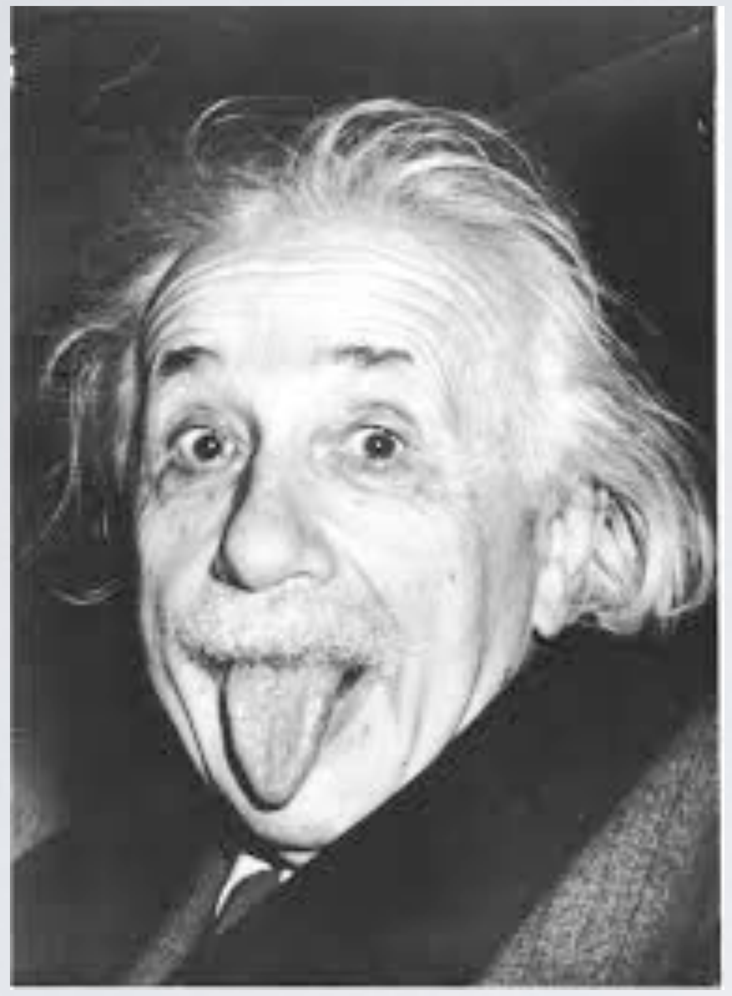
Radiation exerts a force on a material object

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{|\mathbf{E}|^2}{c^2} \propto I$$

radiation pressure force



RADIATION PRESSURE III



A. Einstein

Particle nature of light: *photon*

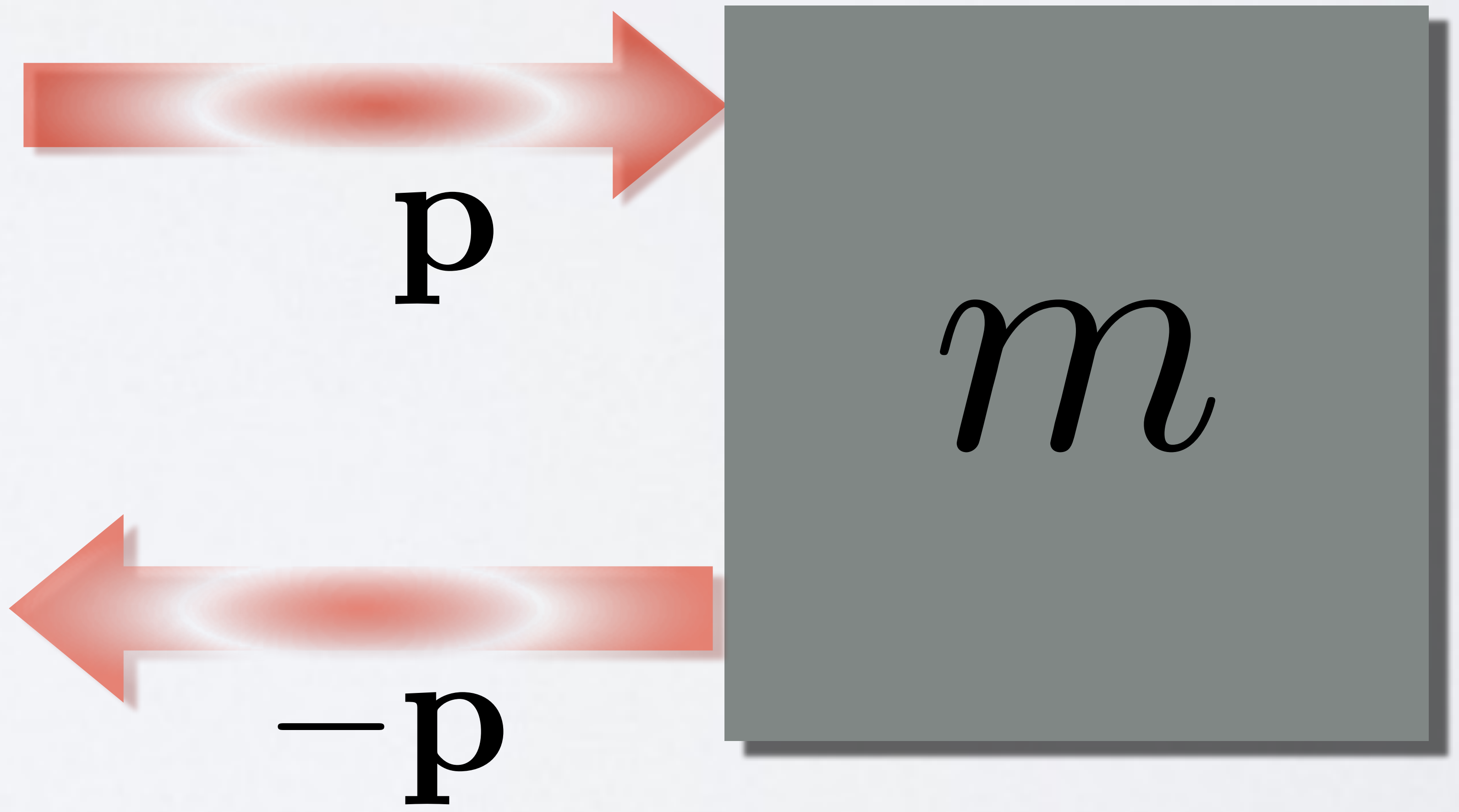
$$\mathbf{p} = \hbar\mathbf{k}$$

Transfer of momentum from the photon to the material object

$$\Delta\mathbf{p} = \mathbf{p} - (-\mathbf{p}) = 2\mathbf{p}$$

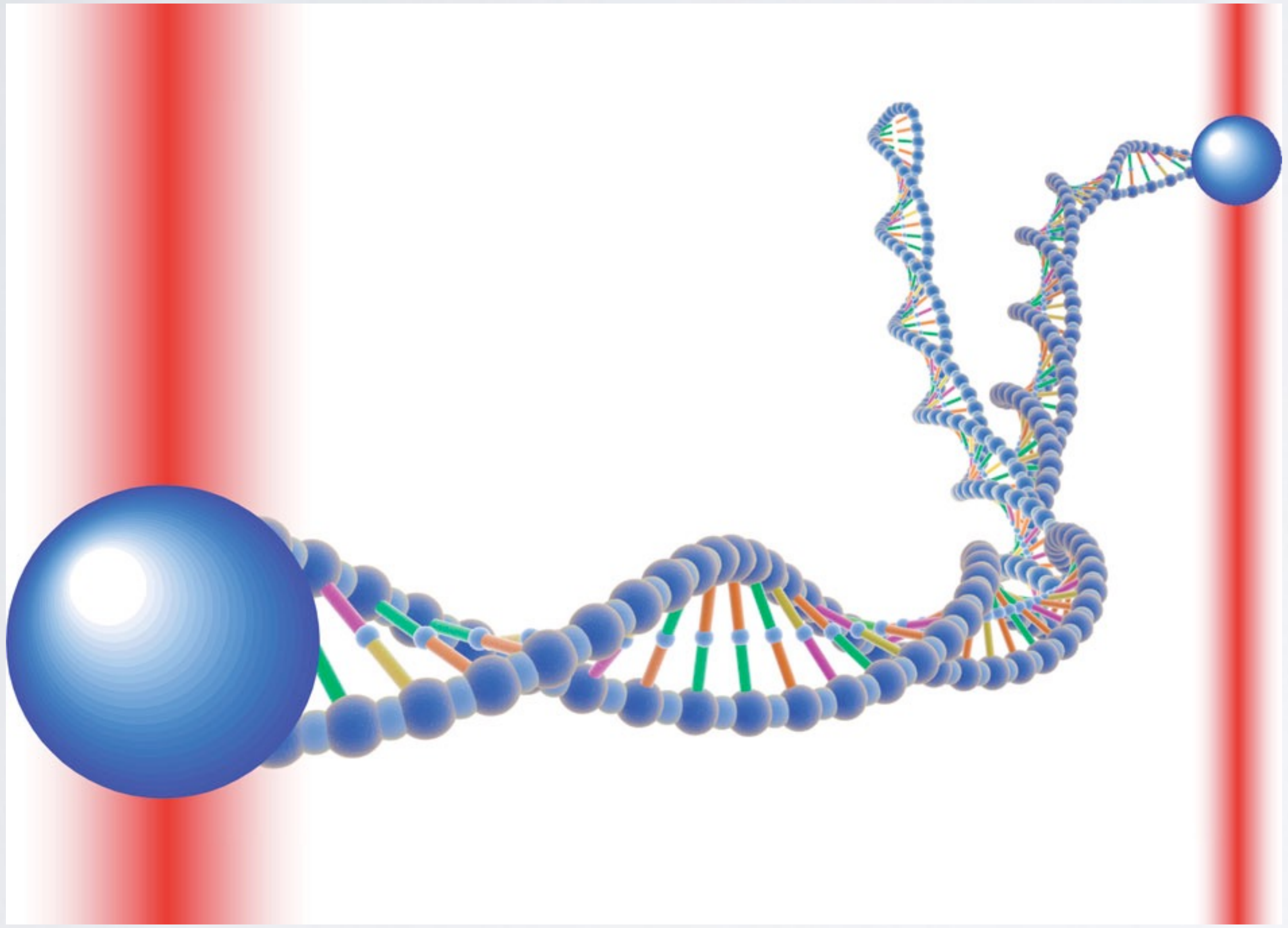
Radiation pressure force

$$\mathbf{F} = \Delta\mathbf{p} \cdot \frac{\#photons}{sec}$$



APPLICATIONS

Optical tweezers:

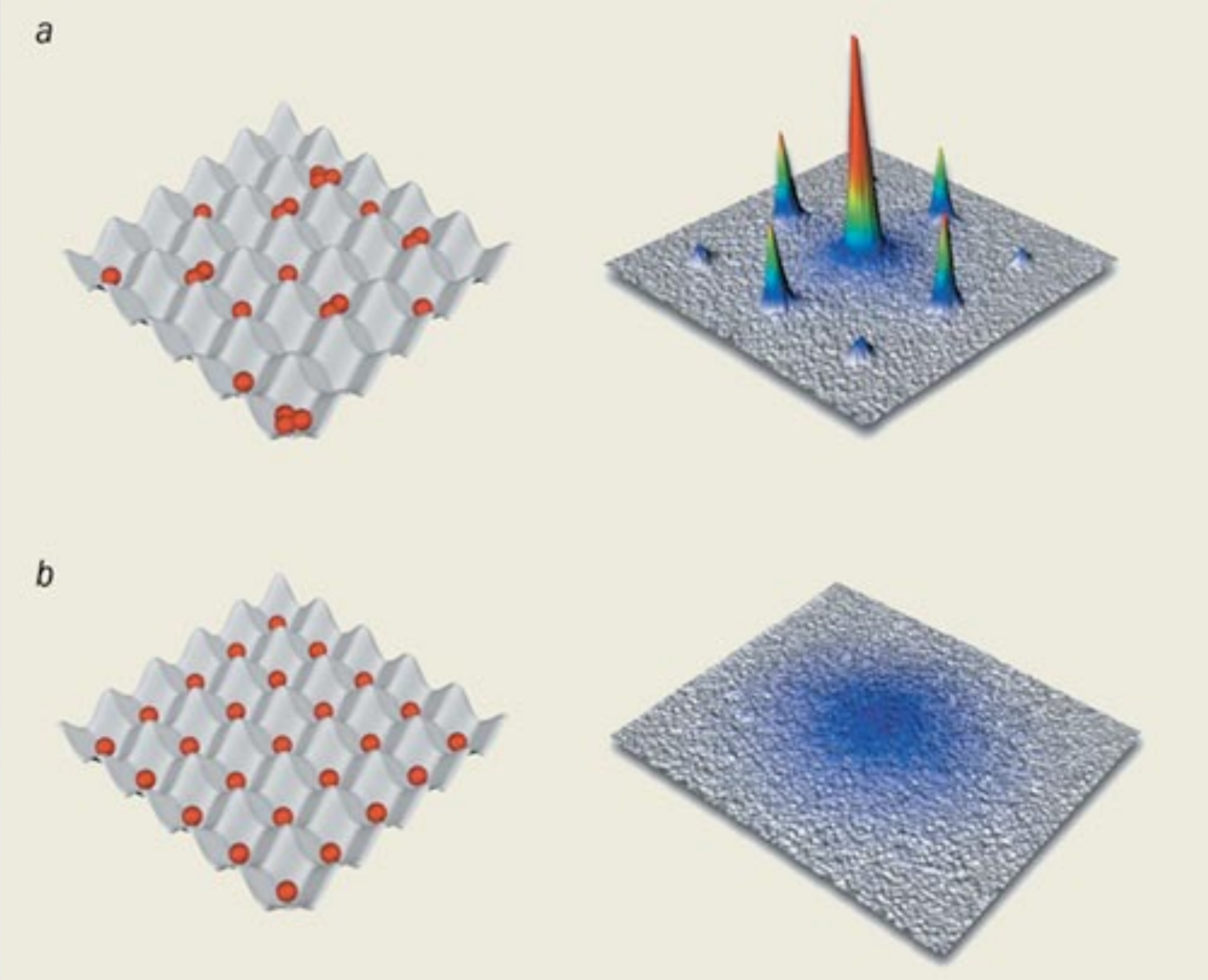


Jarzynski Nat Phys. **7**, 591 (2011)

Manipulation of a DNA string by moving two PS nanobeads with two optical tweezers.

APPLICATIONS

Atom trapping & cooling:



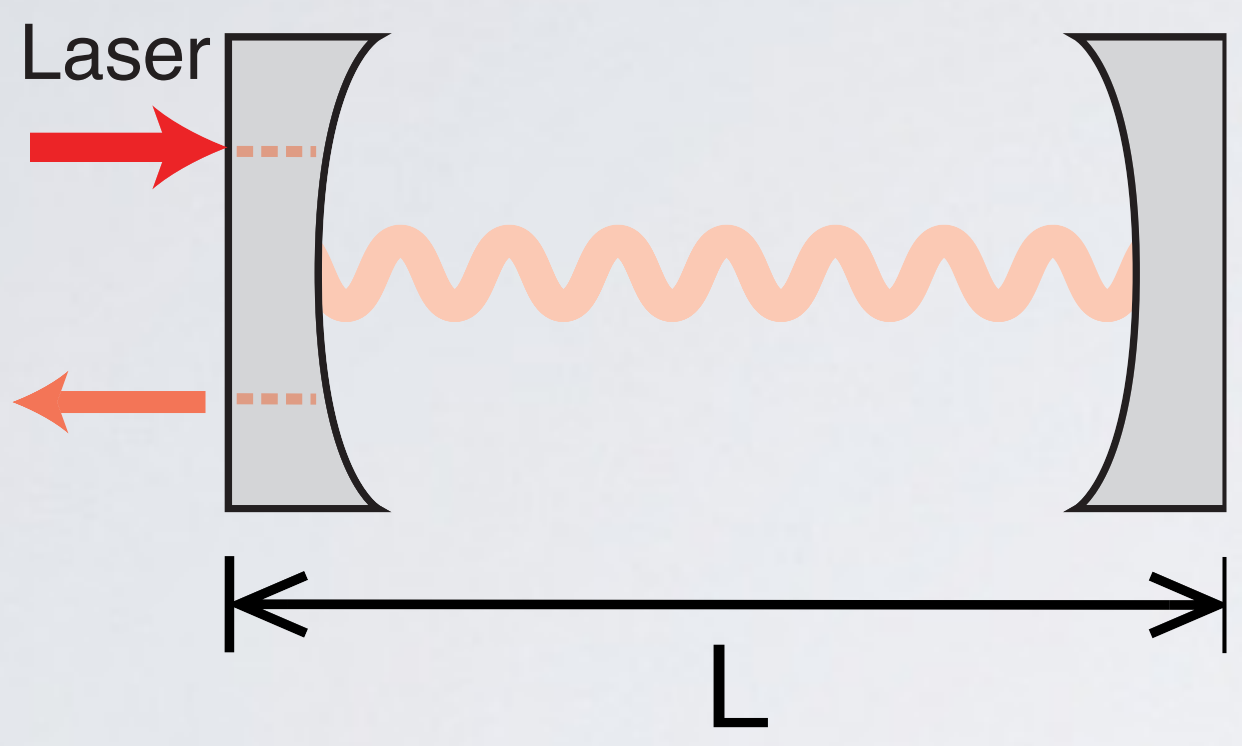
Cooling and trapping neutral atoms in an optical lattice.
Observation of a QPT between a superfluid (a) and a Mott insulator (b).

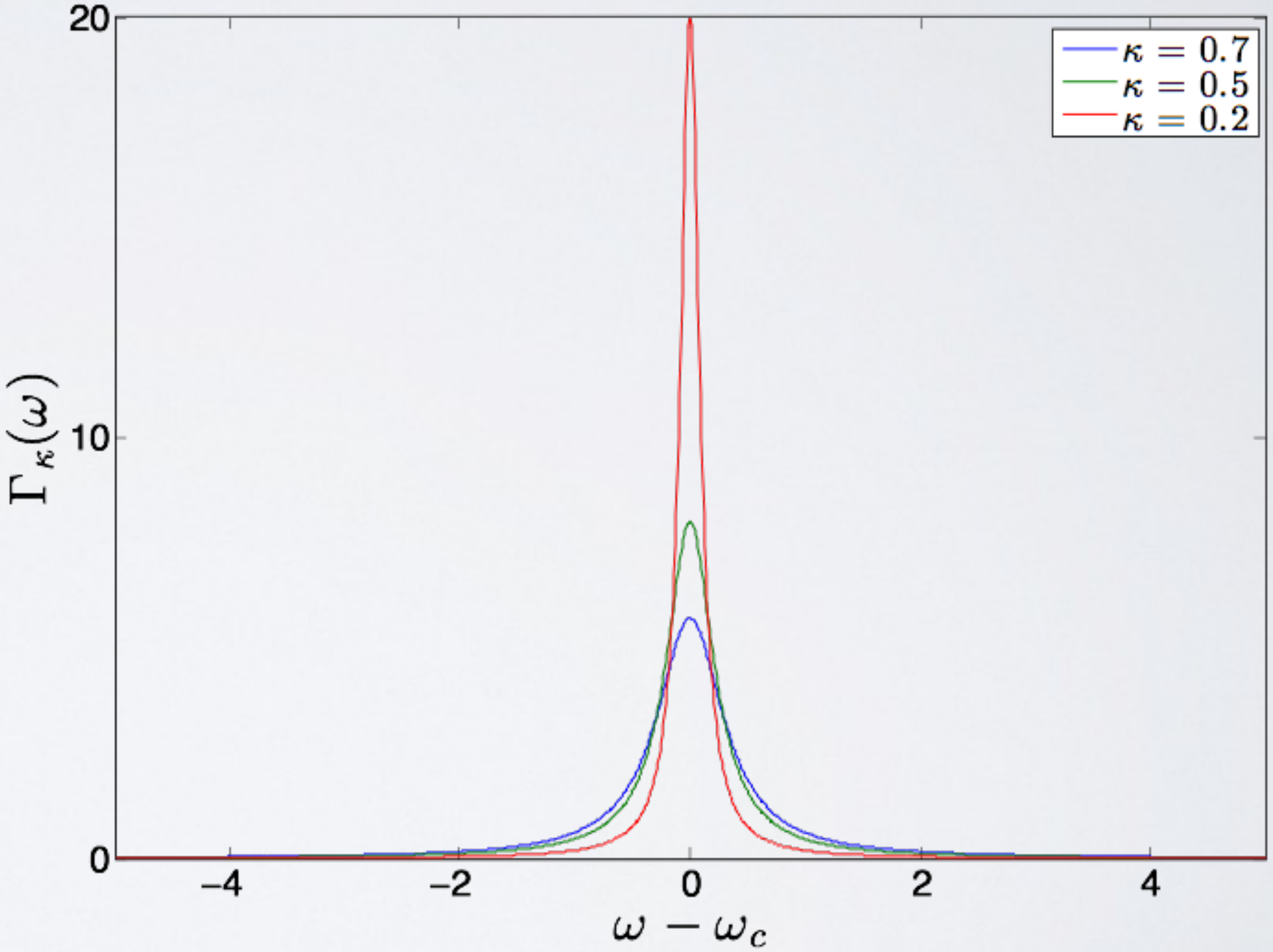
Greiner *et al.* Nature **415**, 39 (2002)

OPTICAL CAVITY

$\Gamma_\kappa(\omega)$: input/output formalism

$$I_c(\omega) = \frac{1}{2\pi}\frac{\kappa}{(\omega-\omega_c)^2+\kappa^2/4}I_i(\omega)$$





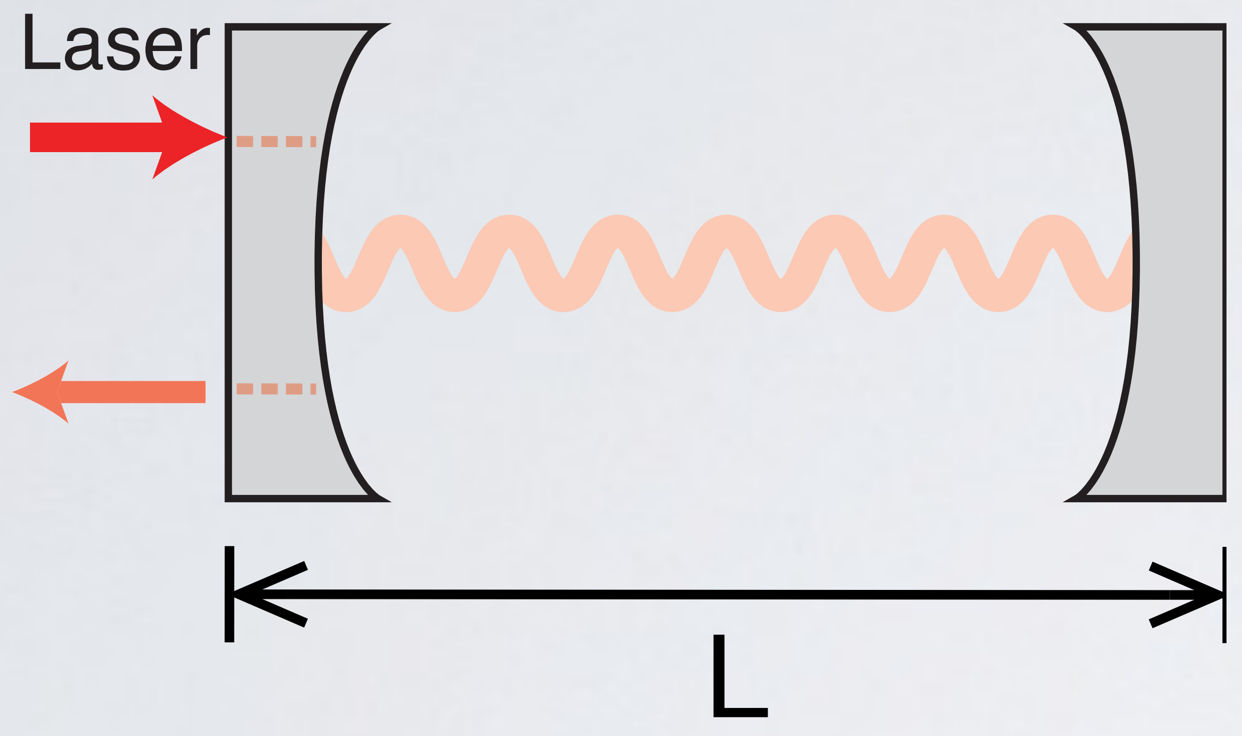
Condition for resonance

$$2L = n\lambda$$

in terms of the frequency $\omega_c = 2\pi/\lambda$

$$\omega_c = \frac{\pi c}{L}$$

OPTICAL CAVITY



Condition for resonance

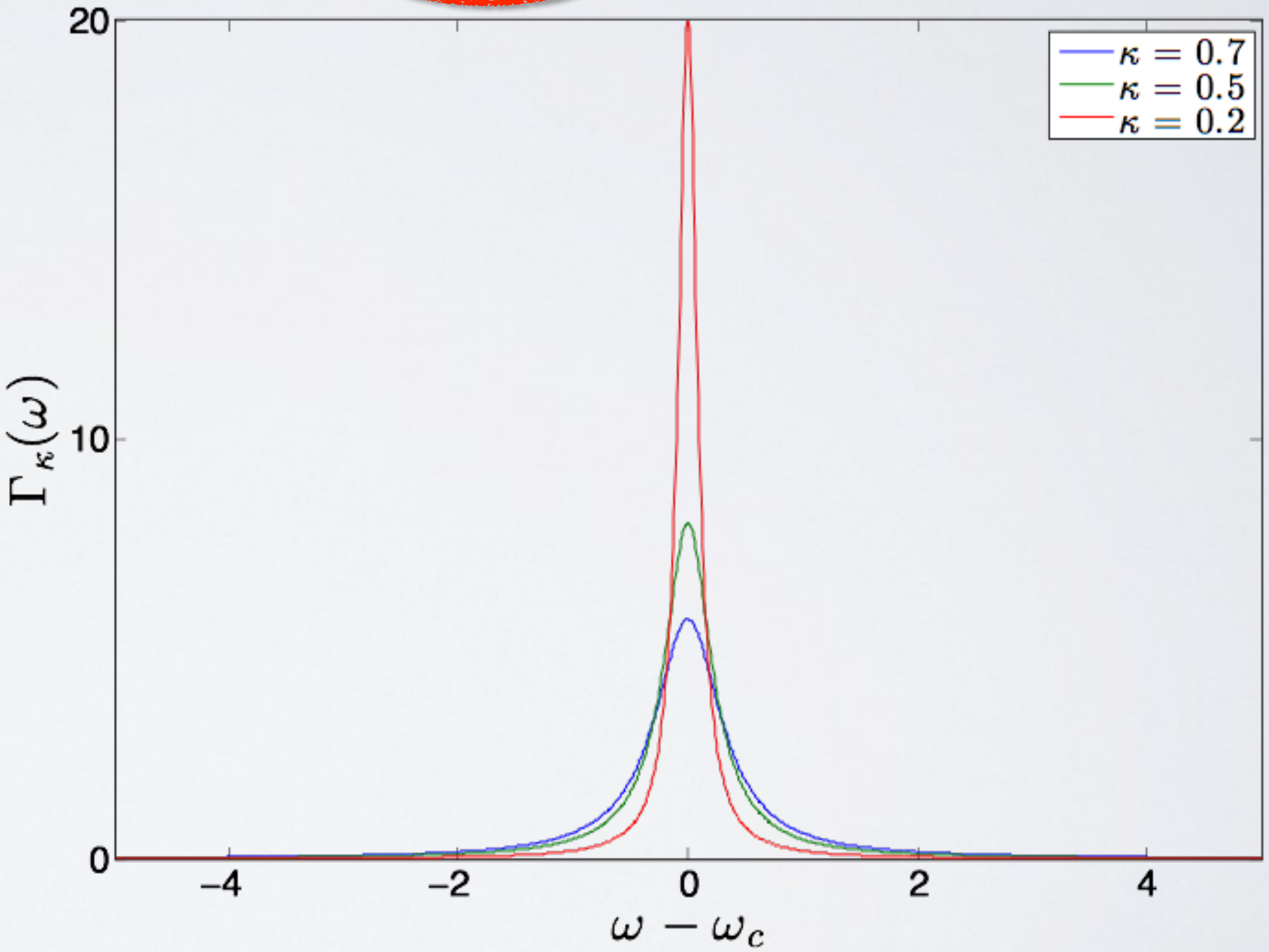
$$2L = n\lambda$$

in terms of the frequency $\omega_c = 2\pi/\lambda$

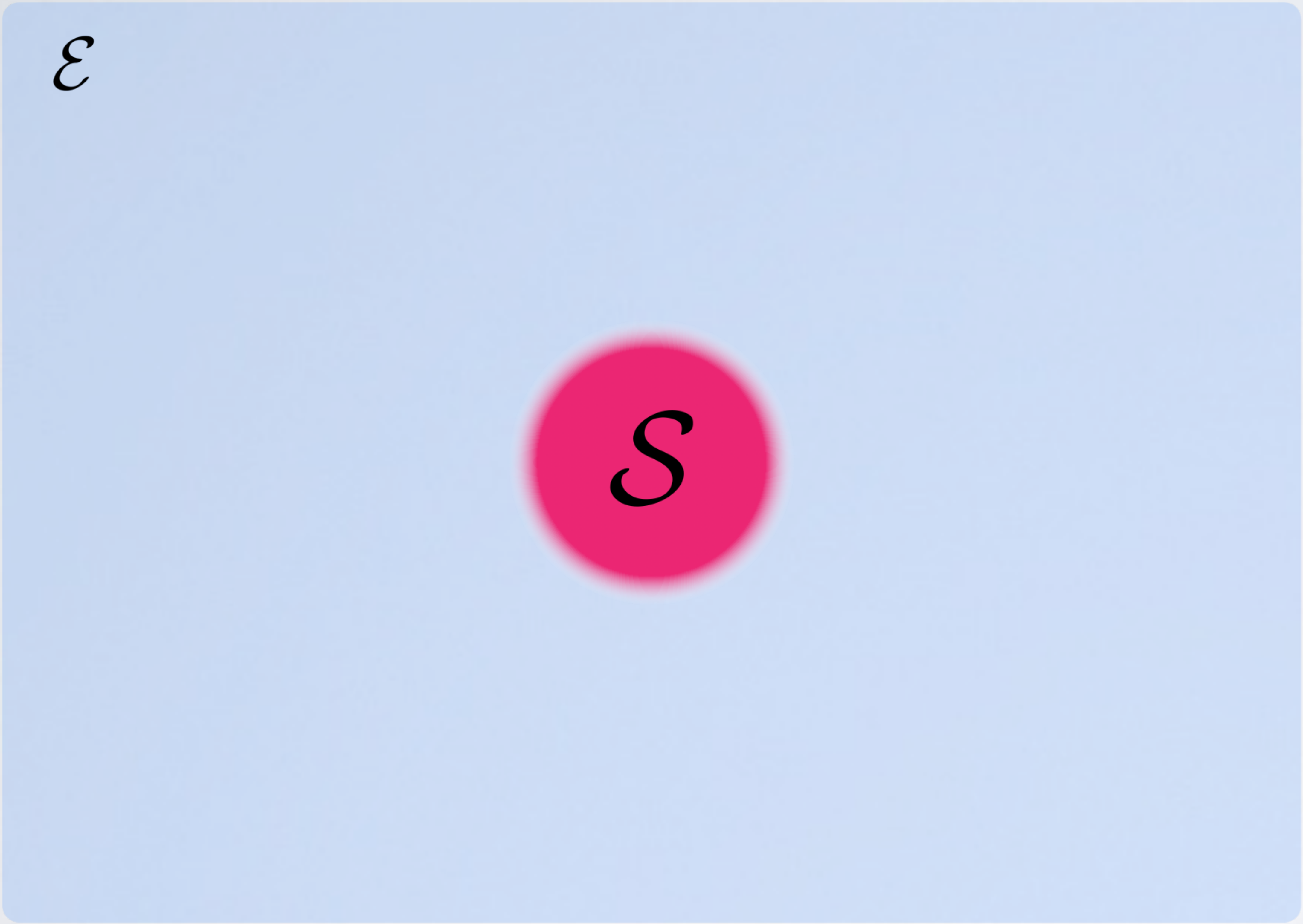
$$\omega_c = \frac{\pi c}{L}$$

$$I_c(\omega) = \frac{1}{2\pi}\frac{\kappa}{(\omega-\omega_c)^2+\kappa^2/4}I_i(\omega)$$

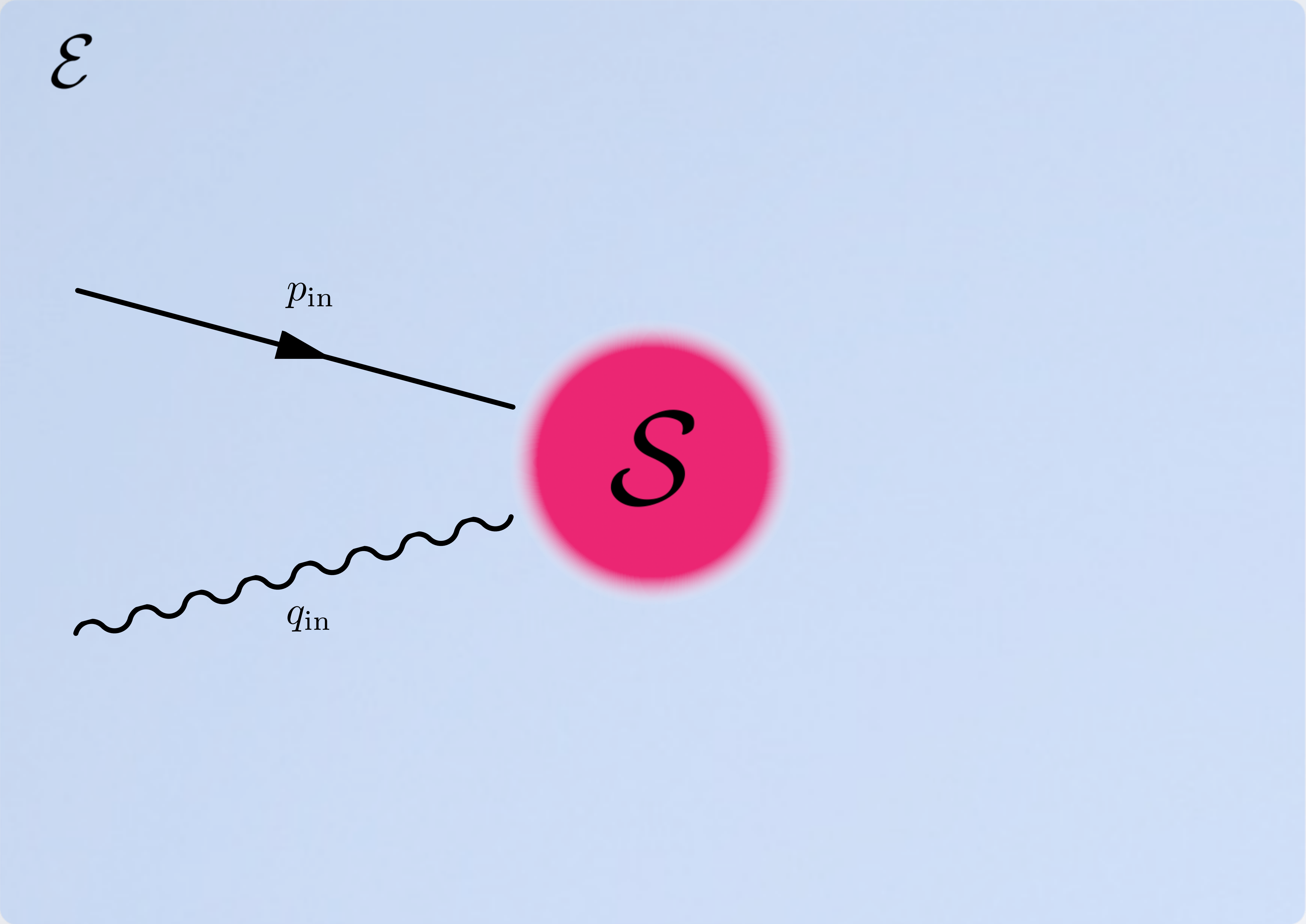
$\Gamma_\kappa(\omega)$: input/output formalism



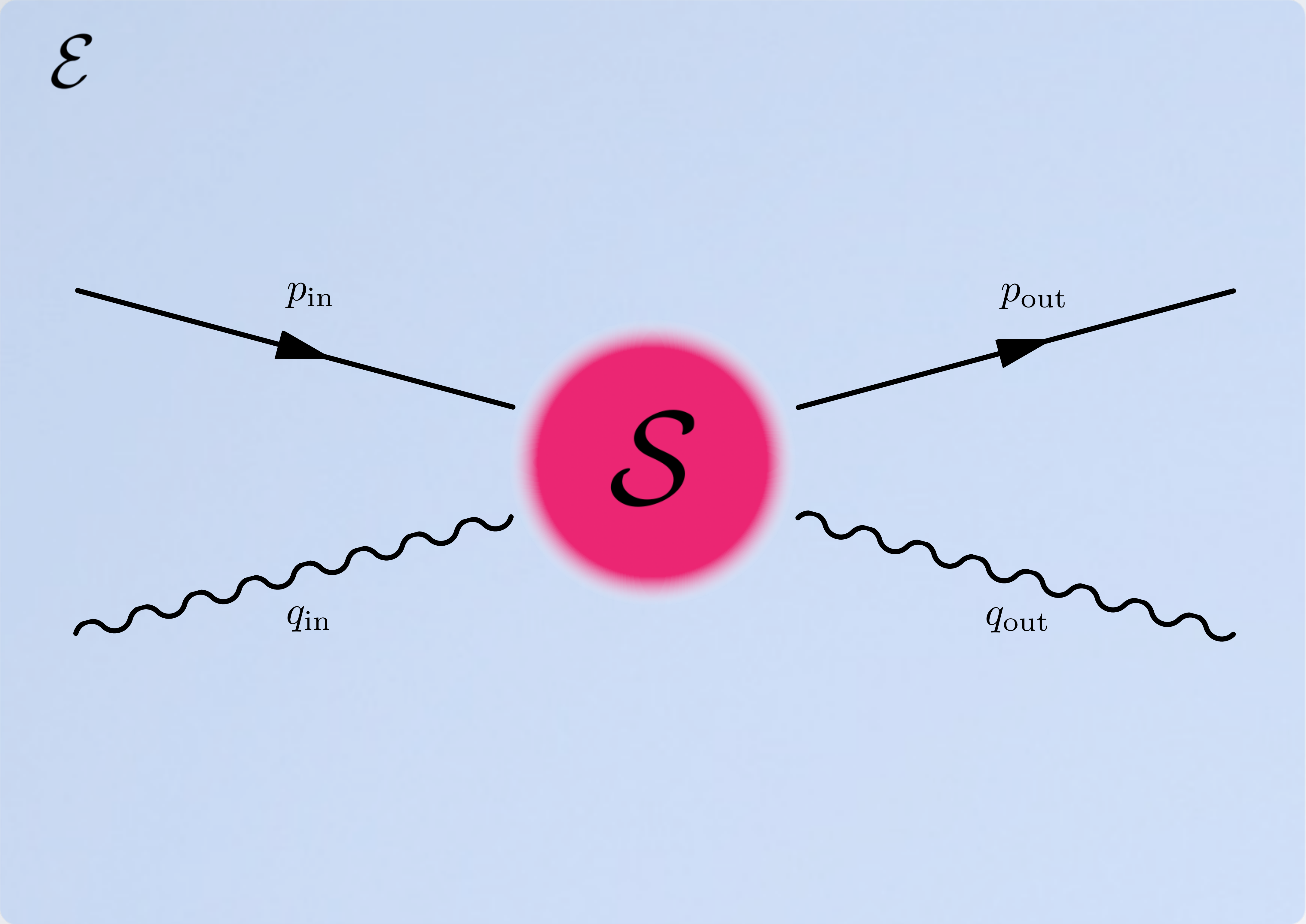
I/O FORMALISM - THE PROBLEM



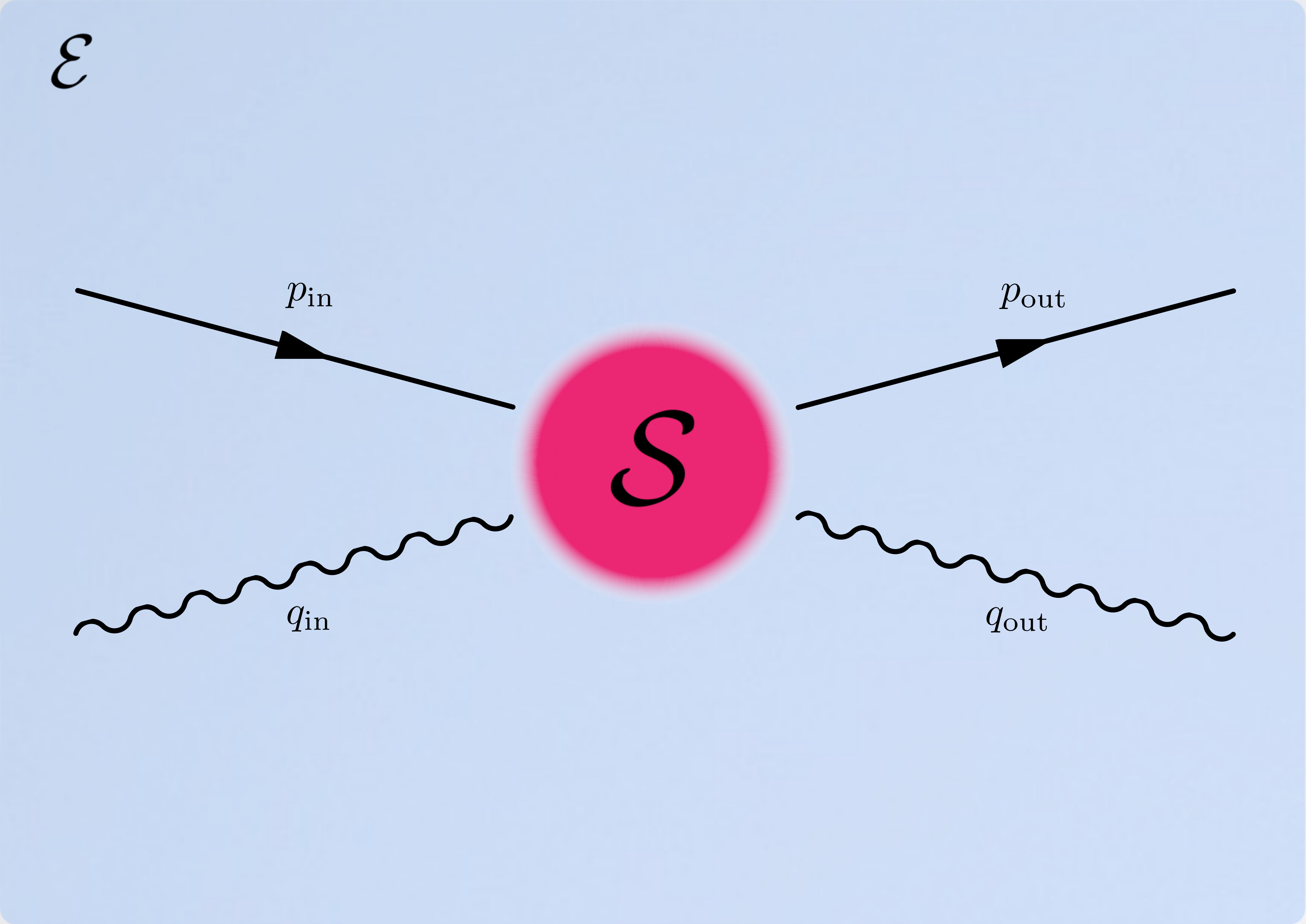
I/O FORMALISM - THE PROBLEM



I/O FORMALISM - THE PROBLEM



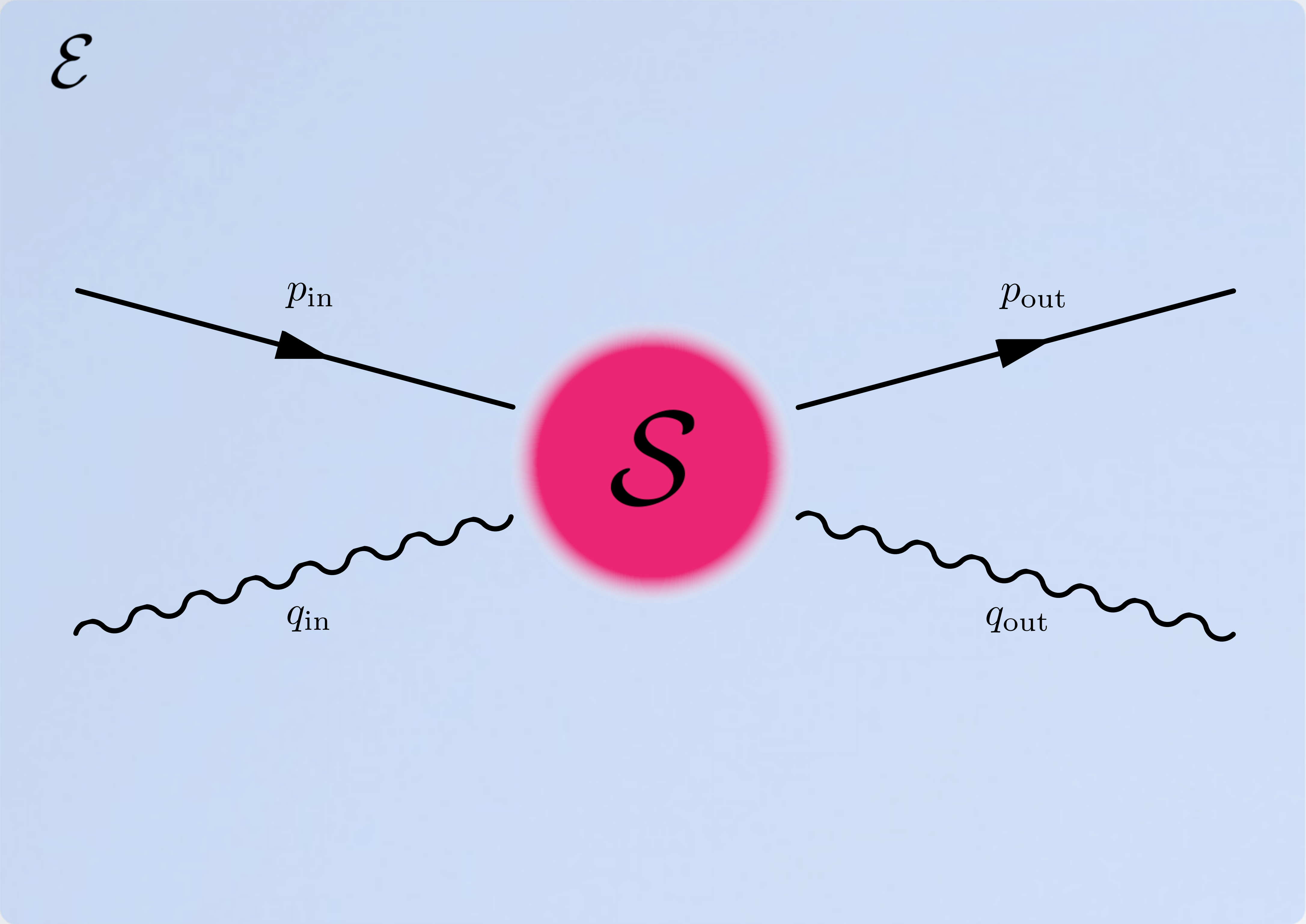
I/O FORMALISM - THE PROBLEM



Environment: non-interacting modes

$\mathcal{E}$ $$H_0 = \sum_{\mathrm{k}} \omega_{\mathrm{k}} b_{\mathrm{k}}^\dagger b_{\mathrm{k}}$$

I/O FORMALISM - THE PROBLEM



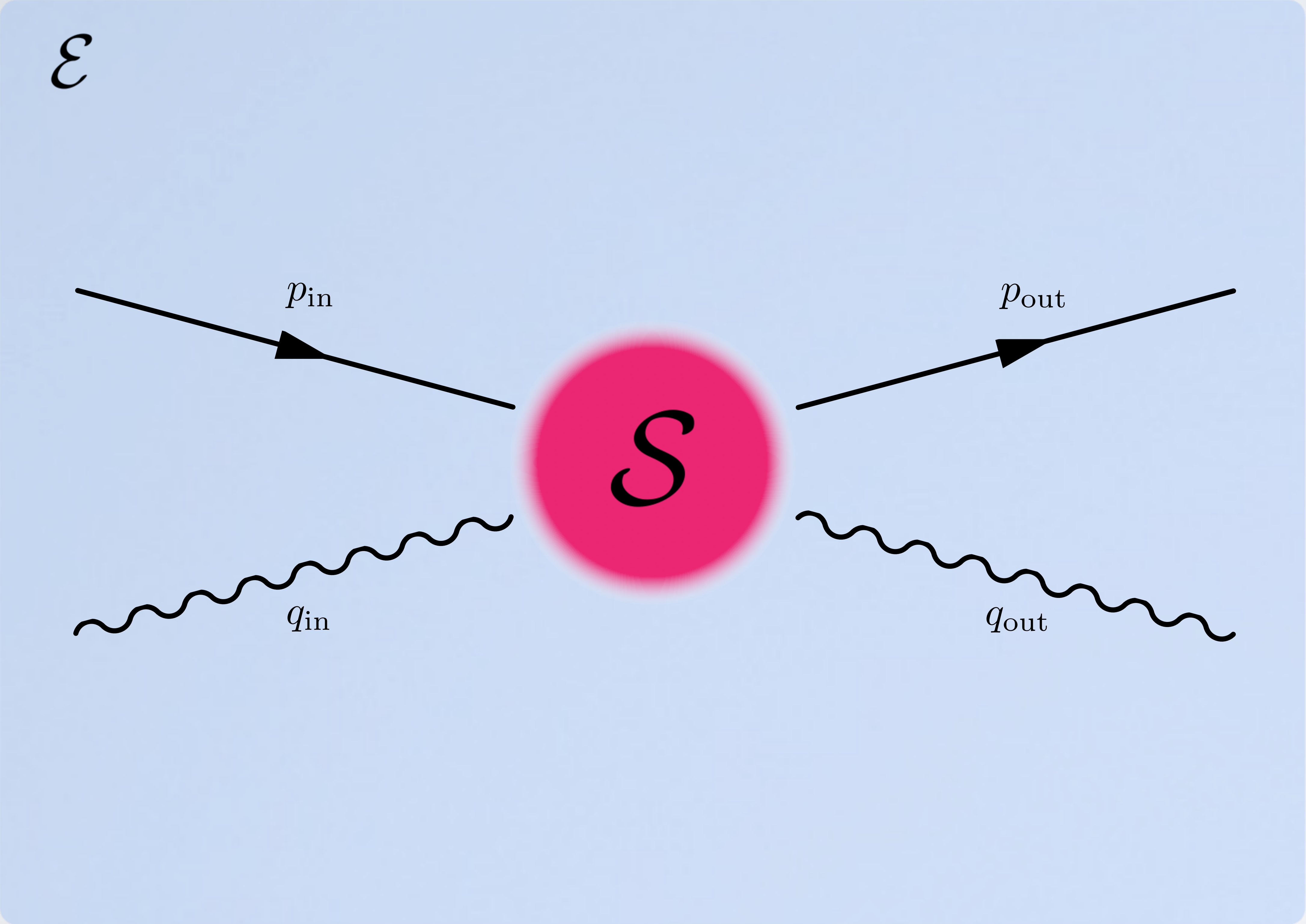
Environment: non-interacting modes

$\mathcal{E} \qquad H_0 = \sum_{\rm k} \omega_{\rm k} b_{\rm k}^\dagger b_{\rm k}$

System:

$\mathcal{S} \qquad H_{\mathcal{S}}$

I/O FORMALISM - THE PROBLEM



Environment: non-interacting modes

$\mathcal{E} \qquad H_0 = \sum_{\rm k} \omega_{\rm k} b_{\rm k}^\dagger b_{\rm k}$

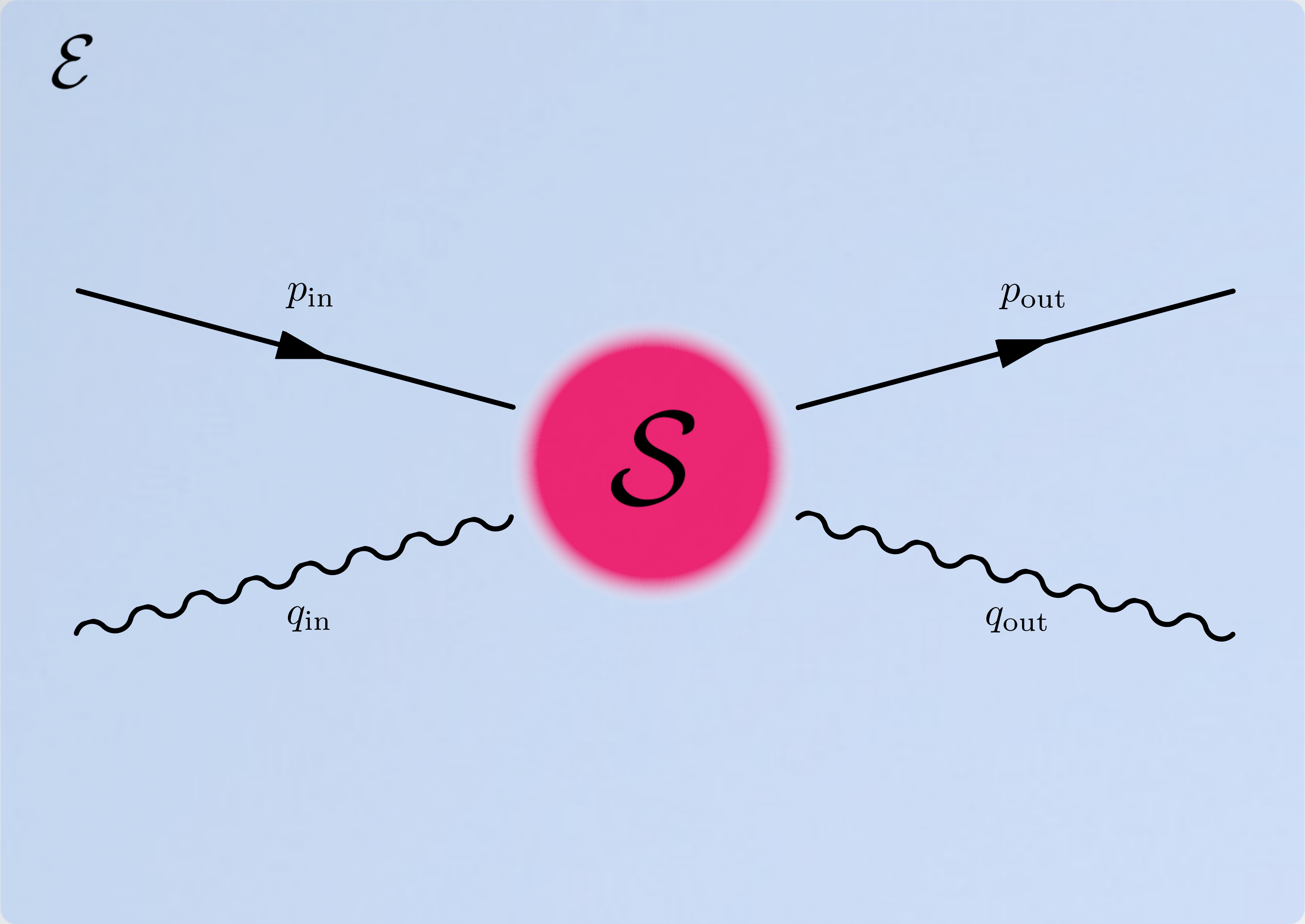
System:

$\mathcal{S} \qquad H_{\mathcal{S}}$

System-environment coupling (e.g.):

$\mathcal{E} - \mathcal{S} \qquad V = \sum_{\rm k} i g_{\rm k} \left(a^\dagger b_{\rm k} - b_{\rm k}^\dagger a \right)$

I/O FORMALISM - SOLUTION



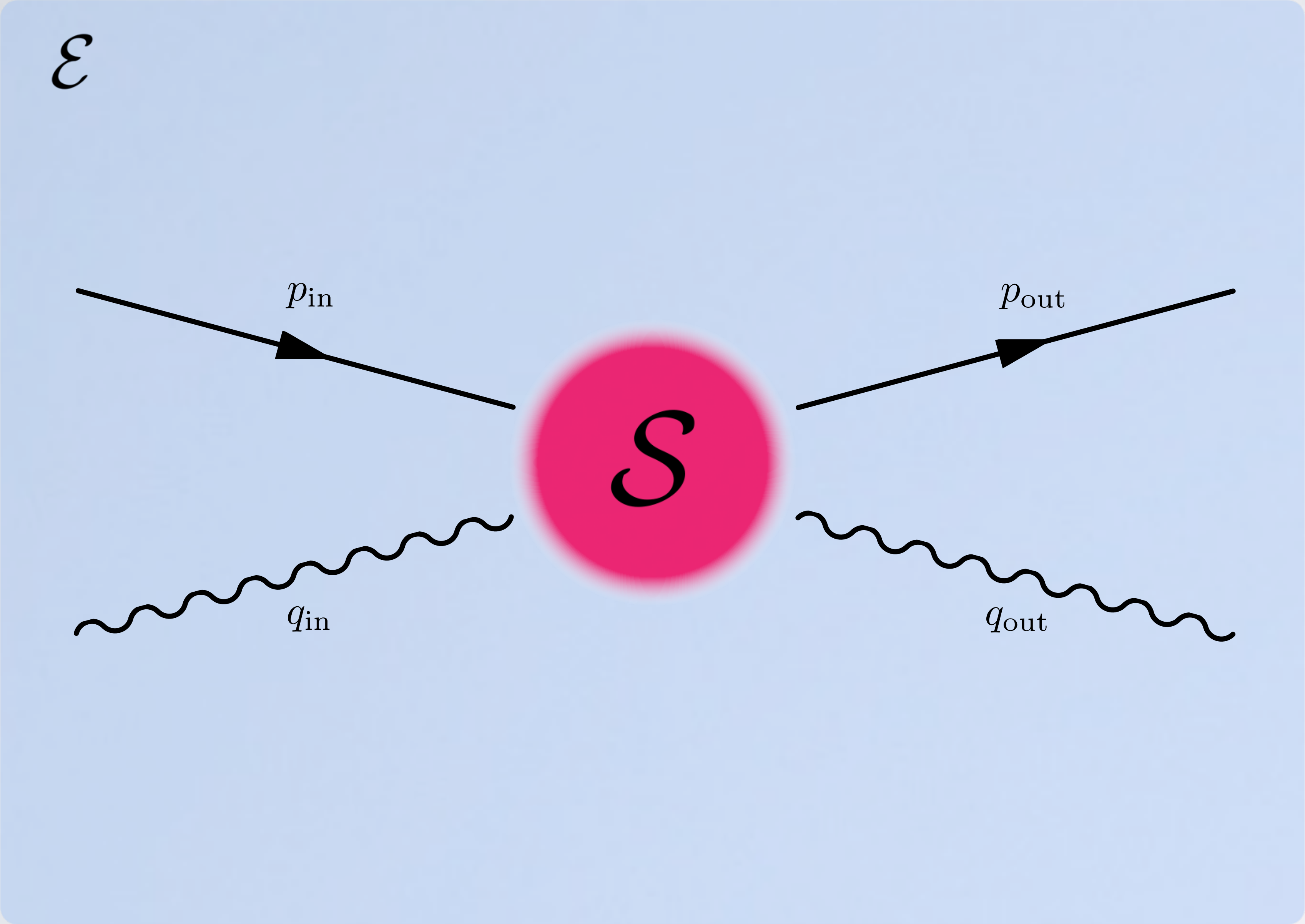
$\mathcal{E}$ $$H_0 = \sum_{\rm k} \omega_{\rm k} b_{\rm k}^\dagger b_{\rm k}$$

$\mathcal{E} - \mathcal{S}$ $$V = \sum_{\rm k} i g_{\rm k} \left(a^\dagger b_{\rm k} - b_{\rm k}^\dagger a \right)$$

$\mathcal{S}$ $$H$$

D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 2007).

I/O FORMALISM - SOLUTION



Solve the EOM for $b_{\rm k}$

$$b_{\rm k}(t) = e^{-i\omega_{\rm k}(t-t_0)}b_{\rm k}(t_0) - g_{\rm k}\int_{t_0}^{t} dt' e^{-i\omega_{\rm k}(t-t')}a(t')dt'$$

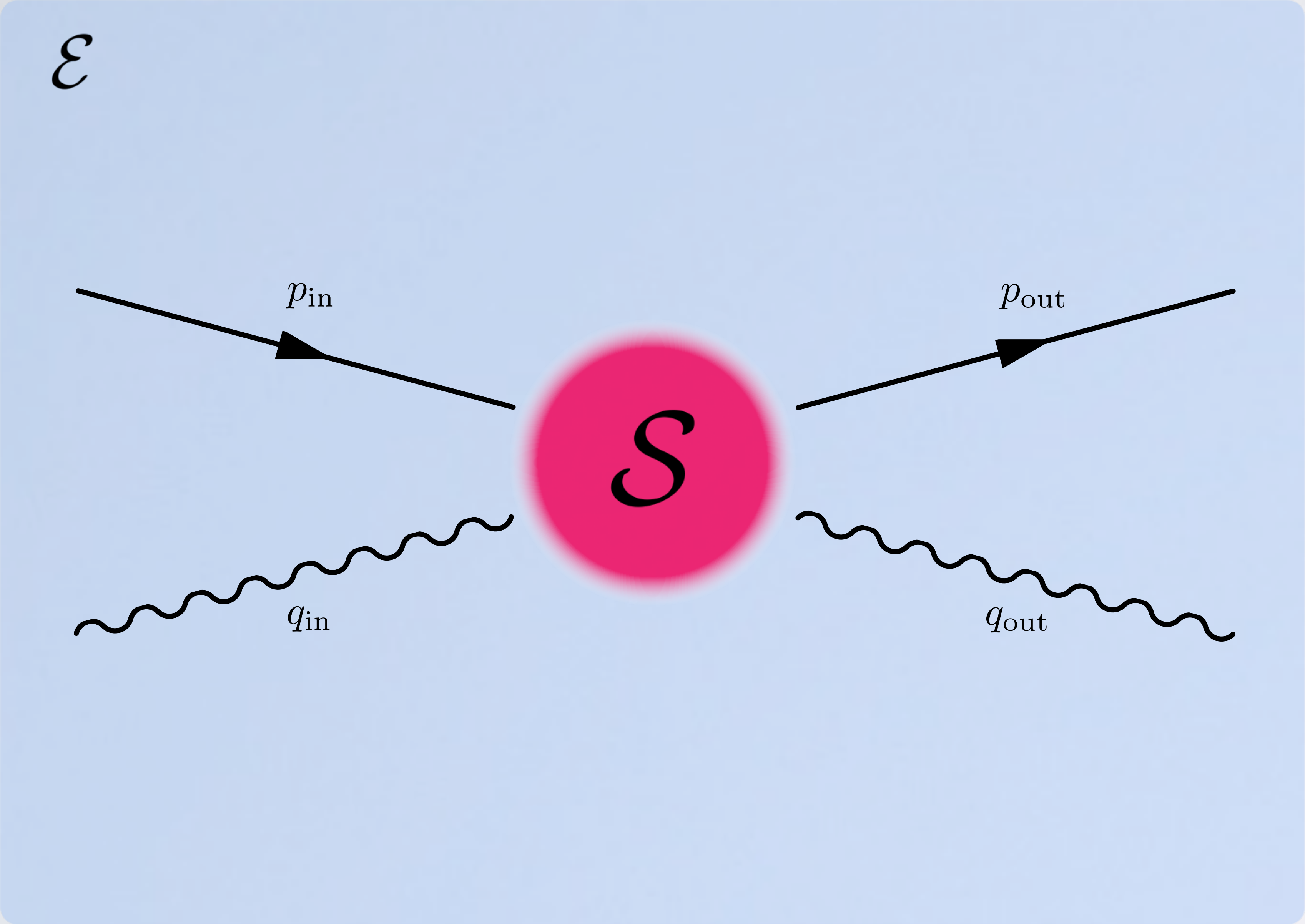
$\mathcal{E}$ $\quad H_0 = \sum_{\rm k} \omega_{\rm k} b_{\rm k}^\dagger b_{\rm k}$

$\mathcal{E} - \mathcal{S}$ $\quad V = \sum_{\rm k} i g_{\rm k}\left(a^\dagger b_{\rm k} - b_{\rm k}^\dagger a\right)$

$\mathcal{S}$ $\quad H$

D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 2007).

I/O FORMALISM - SOLUTION



$\mathcal{E}$ $\quad H_0 = \sum_{\rm k} \omega_{\rm k} b_{\rm k}^\dagger b_{\rm k}$

$\mathcal{E} - \mathcal{S}$ $\quad V = \sum_{\rm k} i g_{\rm k} \left(a^\dagger b_{\rm k} - b_{\rm k}^\dagger a \right)$

$\mathcal{S}$ $\quad H$

Solve the EOM for $b_{\rm k}$

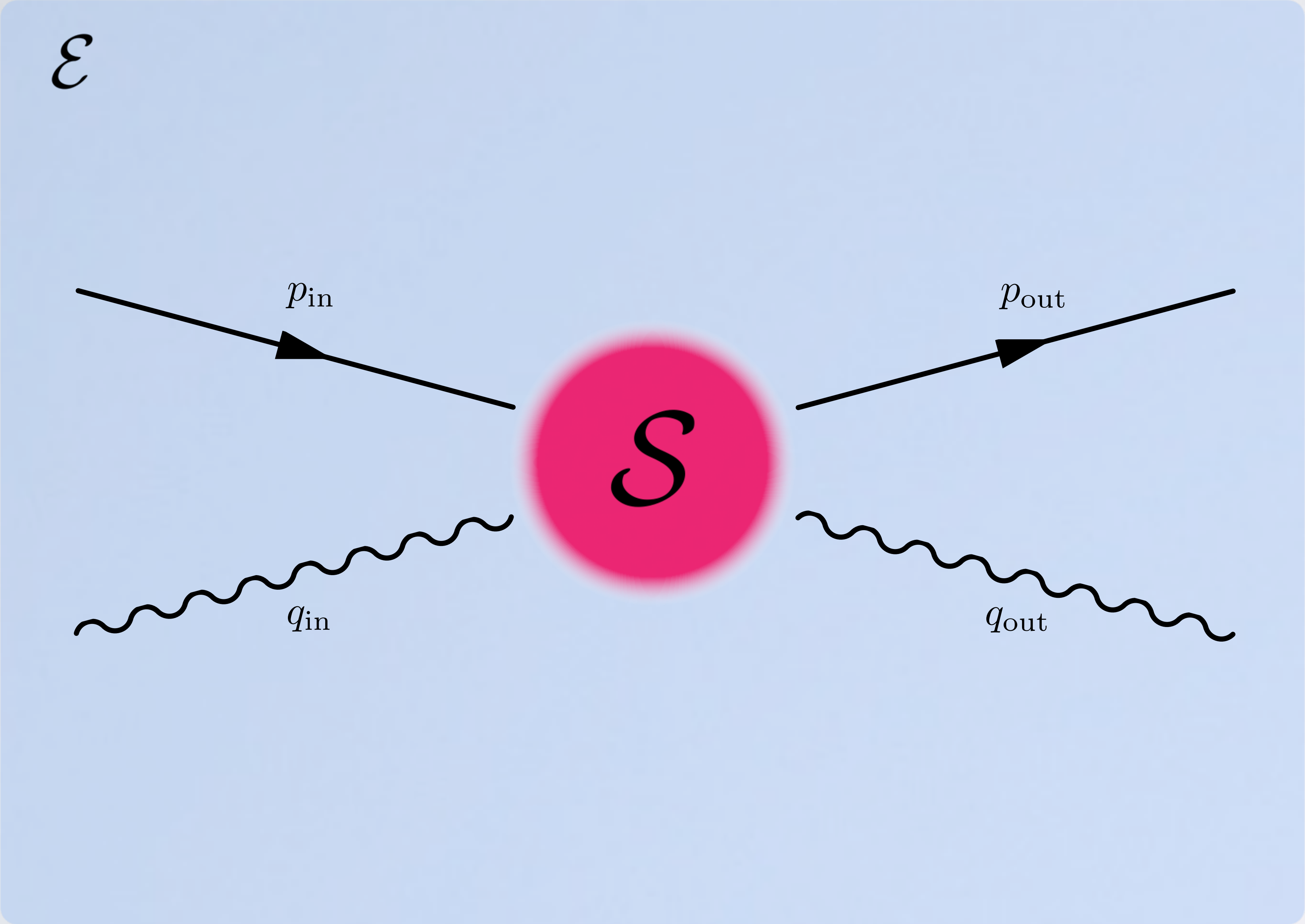
$$b_{\rm k}(t) = e^{-i\omega_{\rm k}(t-t_0)} b_{\rm k}(t_0) - g_{\rm k} \int_{t_0}^{t} dt' e^{-i\omega_{\rm k}(t-t')} a(t') dt'$$

and plug in the EOM for a

$$\dot{a} = i\,[H, a] + \sum_{\rm k} g_{\rm k} e^{-i\omega_{\rm k}(t-t_0)} b_{\rm k}(t_0) - \sum_{\rm k} g_{\rm k}^2 \int_{t_0}^{t} e^{-i\omega_{\rm k}(t-t')} a(t') dt'$$

D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 2007).

I/O FORMALISM - SOLUTION



$\mathcal{E}$ $\quad H_0 = \sum_{\rm k} \omega_{\rm k} b_{\rm k}^\dagger b_{\rm k}$

$\mathcal{E}-\mathcal{S}$ $\quad V = \sum_{\rm k} i g_{\rm k} \left(a^\dagger b_{\rm k} - b_{\rm k}^\dagger a \right)$

$\mathcal{S}$ $\quad H$

Solve the EOM for $b_{\rm k}$

$$b_{\rm k}(t) = e^{-i\omega_{\rm k}(t-t_0)} b_{\rm k}(t_0) - g_{\rm k} \int_{t_0}^{t} dt' e^{-i\omega_{\rm k}(t-t')} a(t') dt'$$

and plug in the EOM for a

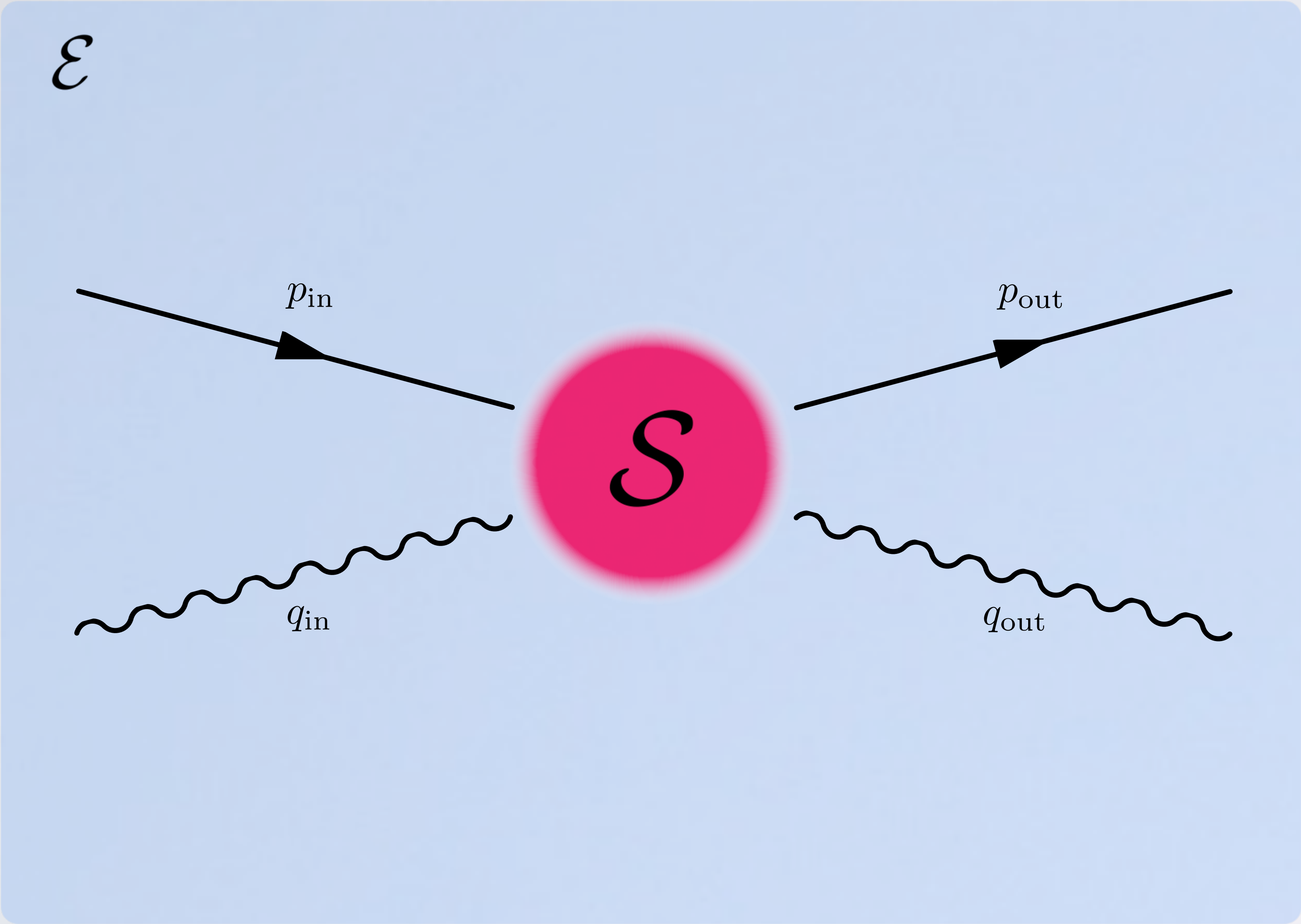
$$\dot{a} = i\left[H, a\right] + \sum_{\rm k} g_{\rm k} e^{-i\omega_{\rm k}(t-t_0)} b_{\rm k}(t_0) - \sum_{\rm k} g_{\rm k}^2 \int_{t_0}^{t} e^{-i\omega_{\rm k}(t-t')} a(t') dt'$$

defining $a_{\rm in} \doteq \int d\omega e^{-i\omega(t-t_0)} b_\omega(t_0)$ and assuming $D g_{\rm k}^2 \simeq \gamma$

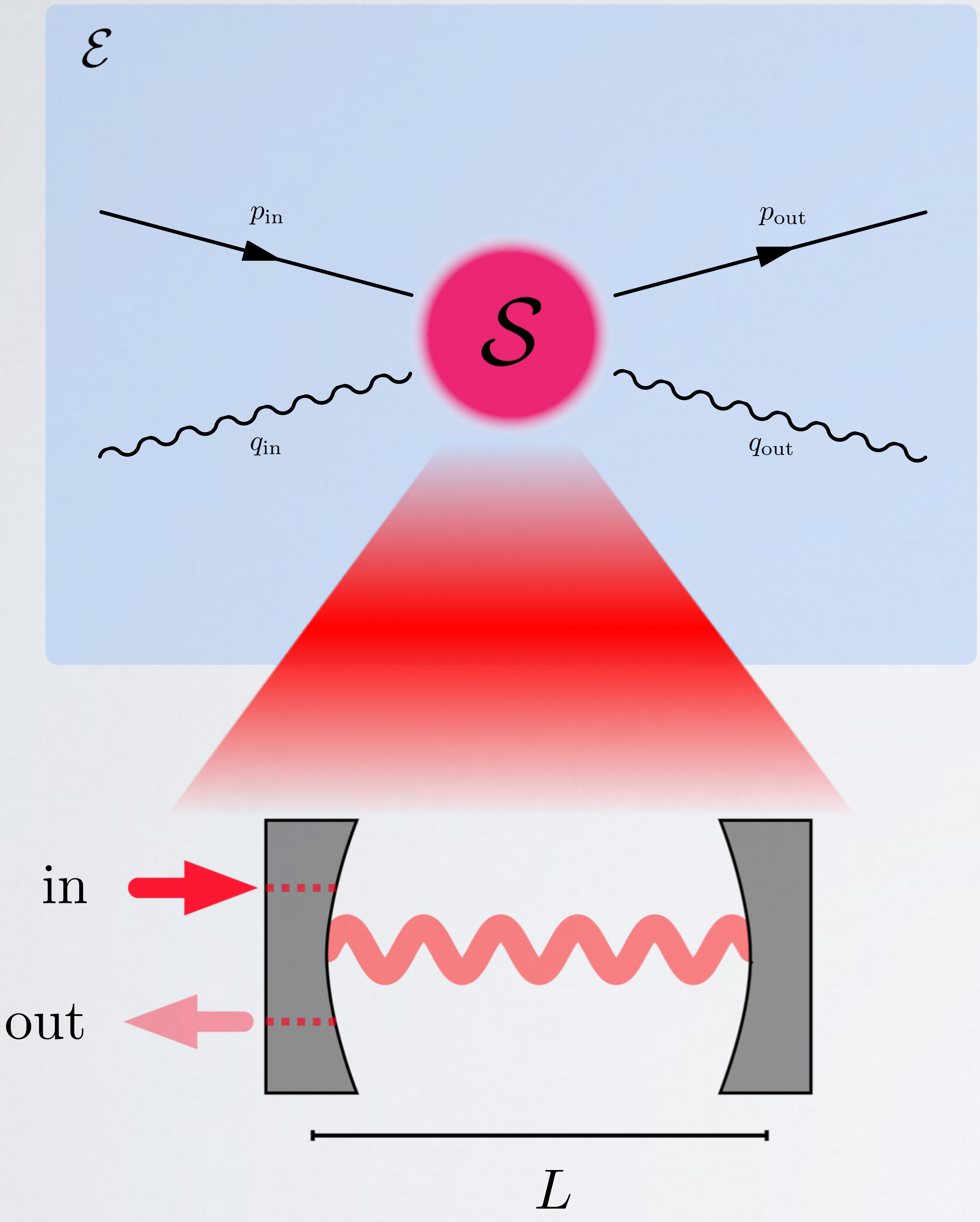
$$\dot{a} = i\left[H, a\right] - \frac{\gamma}{2} a + \sqrt{\gamma} a_{\rm in}$$

D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 2007).

OPTICAL CAVITY



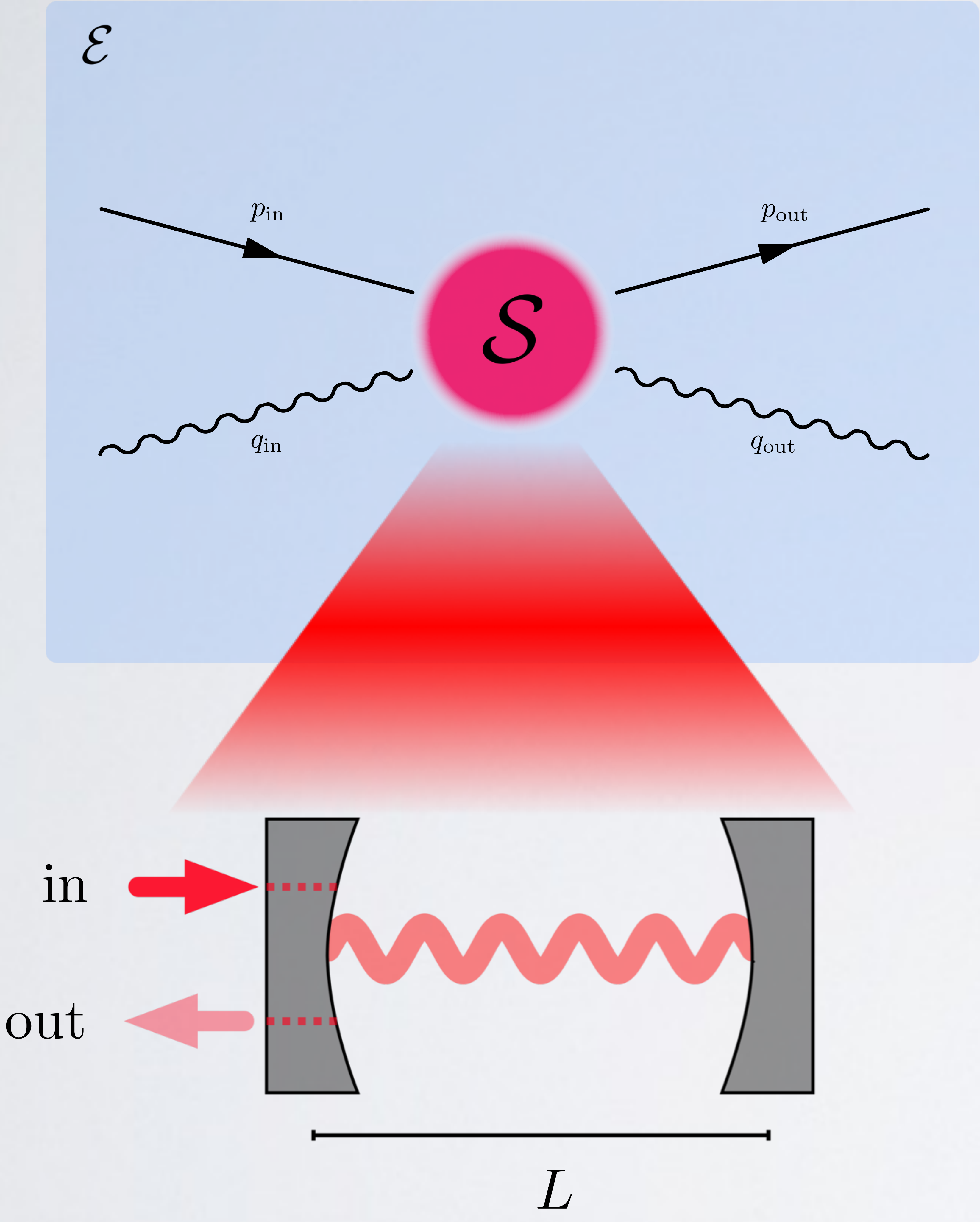
OPTICAL CAVITY



Hamiltonian

$$H = \hbar\omega_c a^\dagger a$$

OPTICAL CAVITY



Hamiltonian

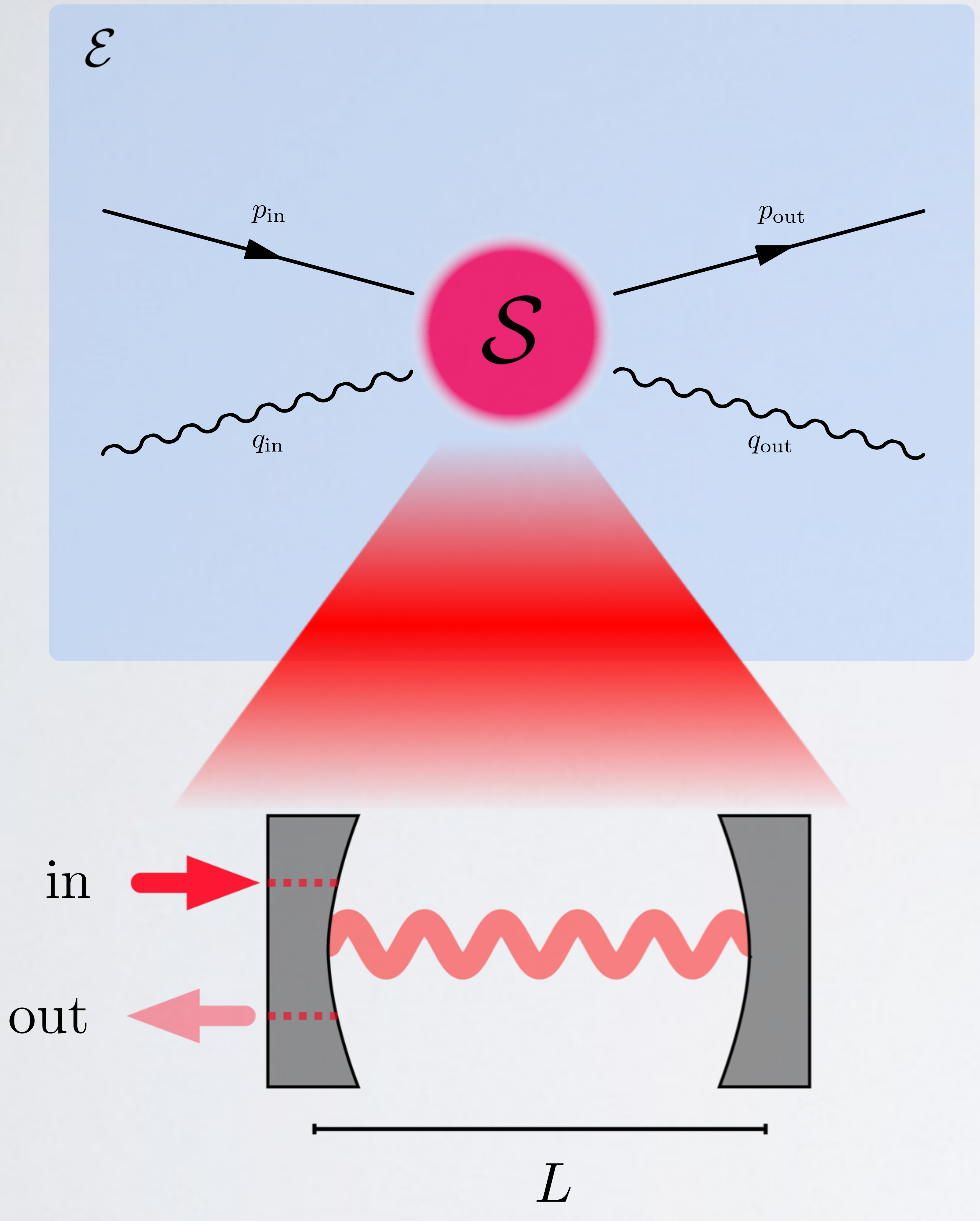
$$H = \hbar\omega_c a^\dagger a$$

$$\dot{a} = -i\omega_c a - \frac{\kappa}{2}a + \sqrt{\kappa}a_{\text{in}}$$

solved by FT

$$a_\omega = \frac{\sqrt{\kappa}}{\frac{\kappa}{2} - i(\omega - \omega_c)}a_{\text{in}}$$

OPTICAL CAVITY



Hamiltonian

$$H = \hbar\omega_c a^\dagger a$$

$$\dot{a} = -i\omega_c a - \frac{\kappa}{2}a + \sqrt{\kappa}a_{\rm in}$$

solved by FT

$$a_\omega = \frac{\sqrt{\kappa}}{\frac{\kappa}{2} - i(\omega - \omega_c)}a_{\rm in}$$

Condition for resonance $2L = n\lambda$

in terms of the frequency $\omega_c = 2\pi/\lambda$

$$\omega_c = \frac{\pi c}{L}$$

OPTICAL CAVITY + MECHANICAL RESONATOR

What if one of the mirrors is allowed to move, e.g. as if connected to a spring?

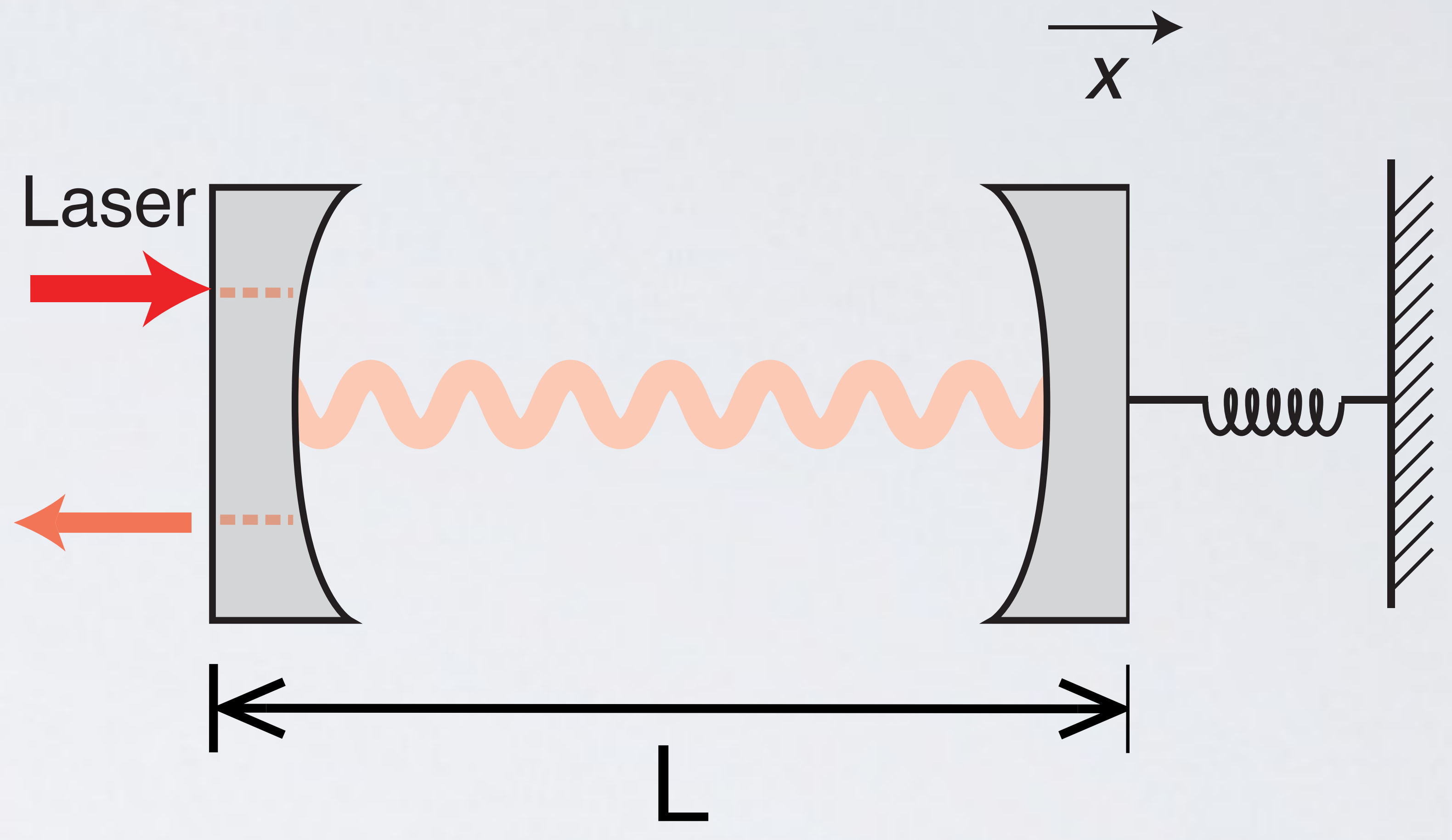
$$\mathbf{F} = -kx$$

a. If $\mathbf{F}$ is the radiation-pressure force, then

$$\mathbf{F} \propto I_c(\omega)$$

b. The cavity deformation leads to a shift in the resonant frequency

$$\omega_c' = \frac{\pi c}{L + x} \simeq \omega_c + \delta\omega(x)$$



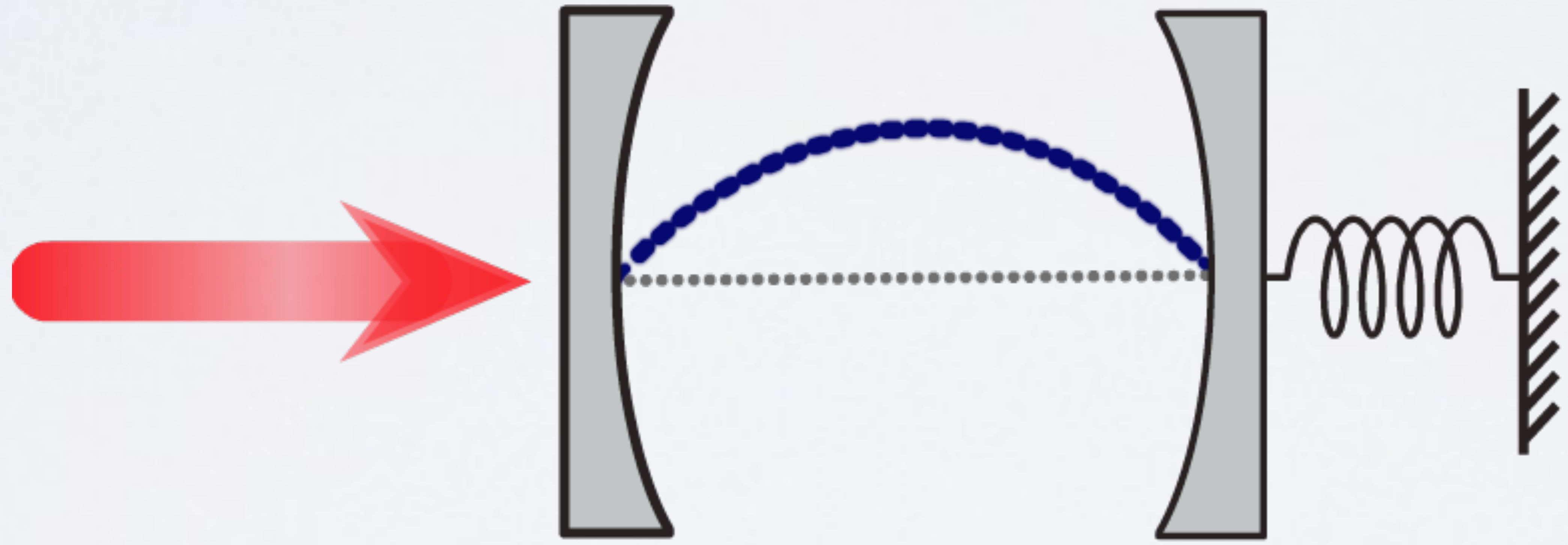
c. Leading to a change in the intensity of the cavity field

$$I_c(\omega) = \frac{1}{2\pi} \frac{\kappa}{(\omega - \omega_c)^2 + \kappa^2/4} I_i(\omega)$$

OPTOMECHANICS

How do we realise this? Small detour in the field of optomechanics…

Cavity optomechanics: electromagnetic field in a resonant cavity coupled to a mechanical degree of freedom through a radiation-pressure term.

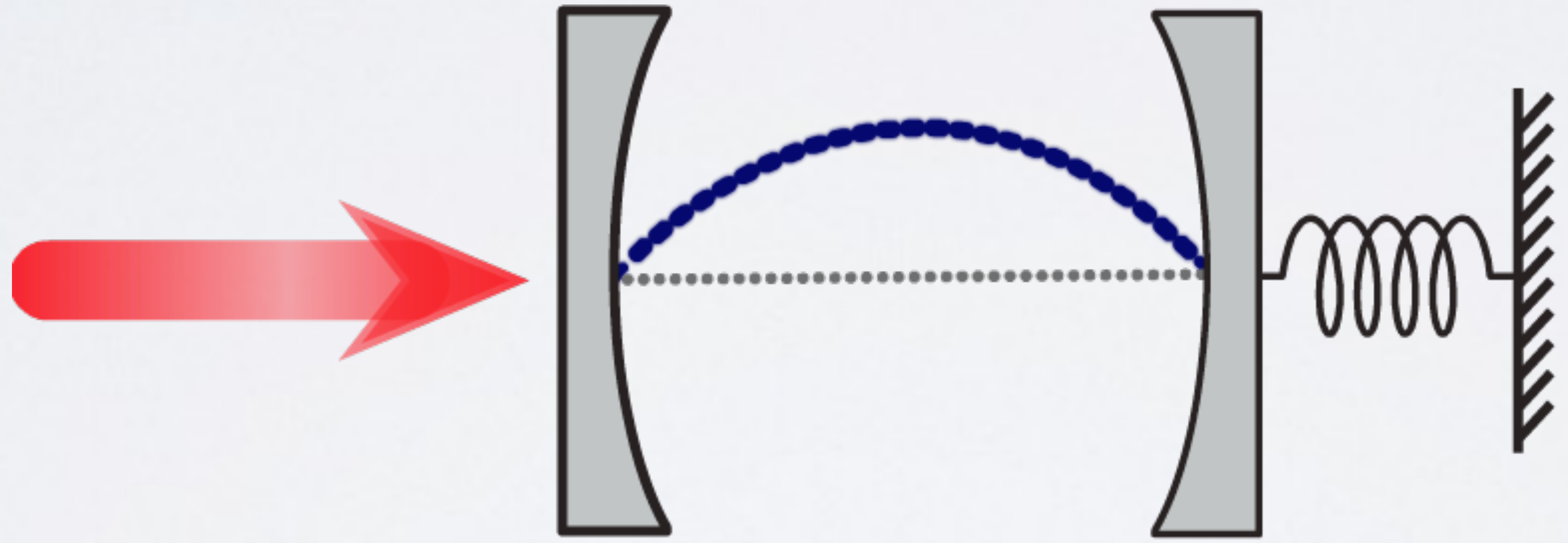


M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014).

OPTOMECHANICS

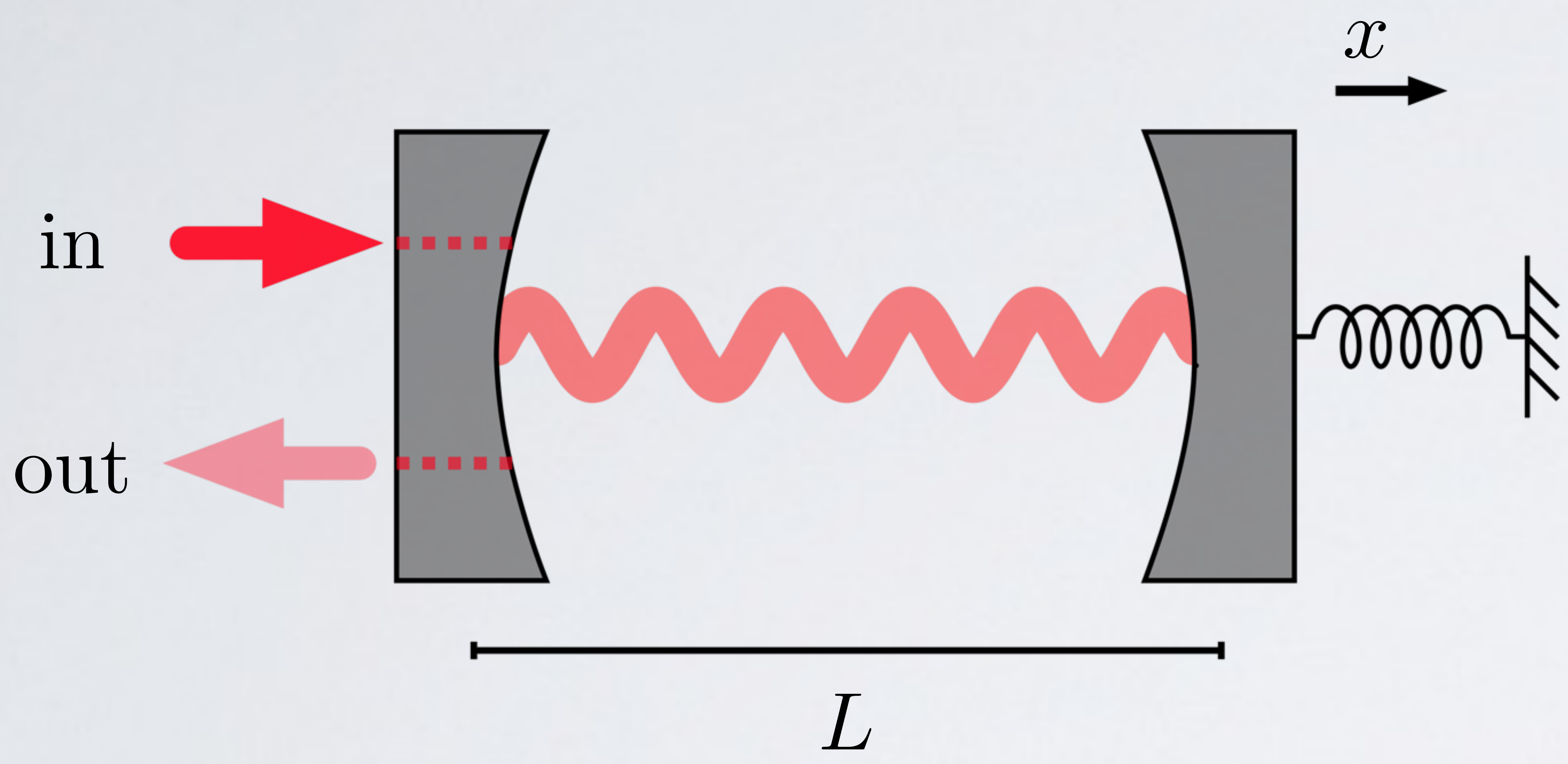
How do we realise this? Small detour in the field of optomechanics…

Cavity optomechanics: electromagnetic field in a resonant cavity coupled to a mechanical degree of freedom through a radiation-pressure term.

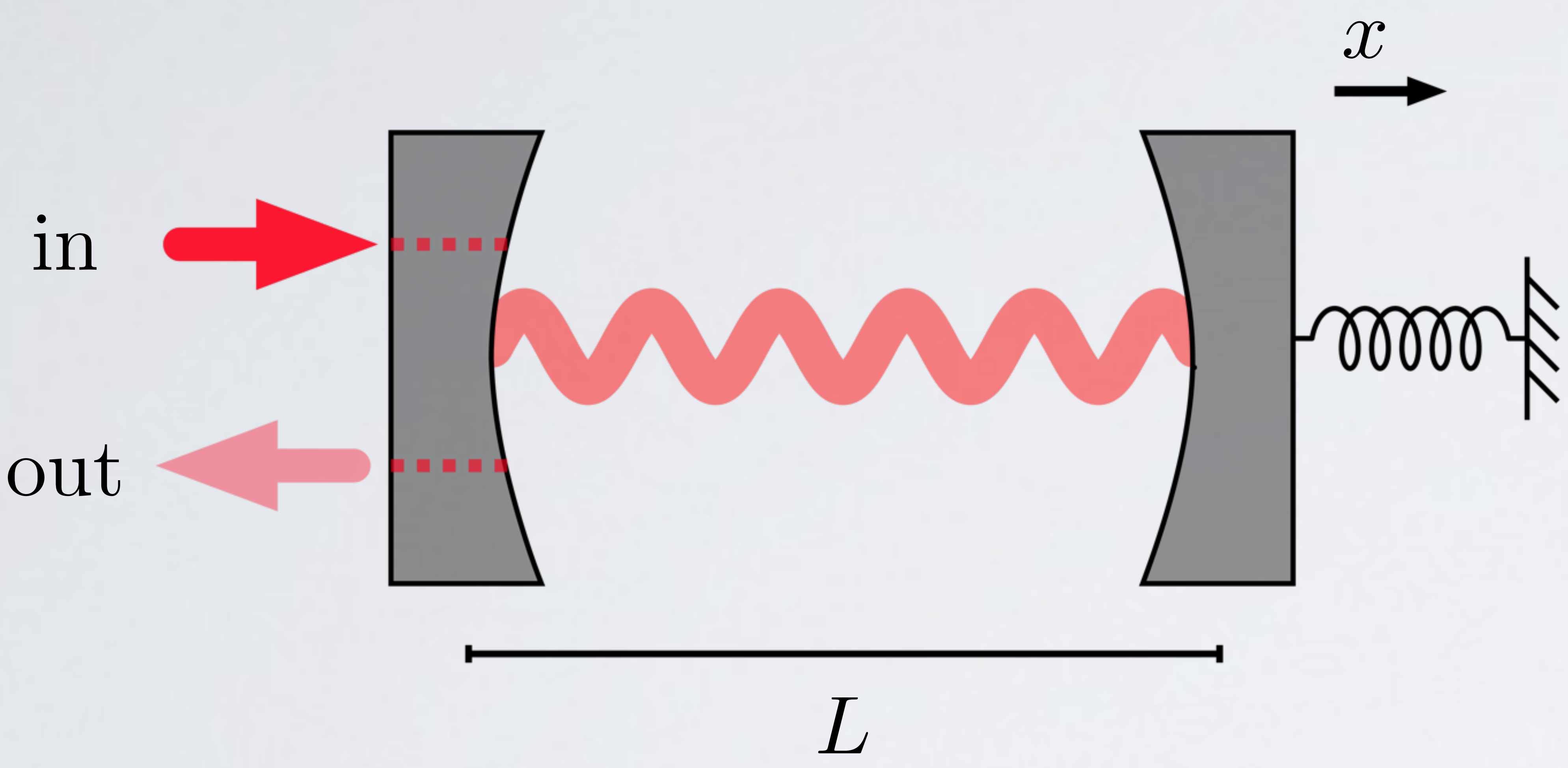


M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014).

OPTOMECHANICAL SYSTEM



OPTOMECHANICAL SYSTEM

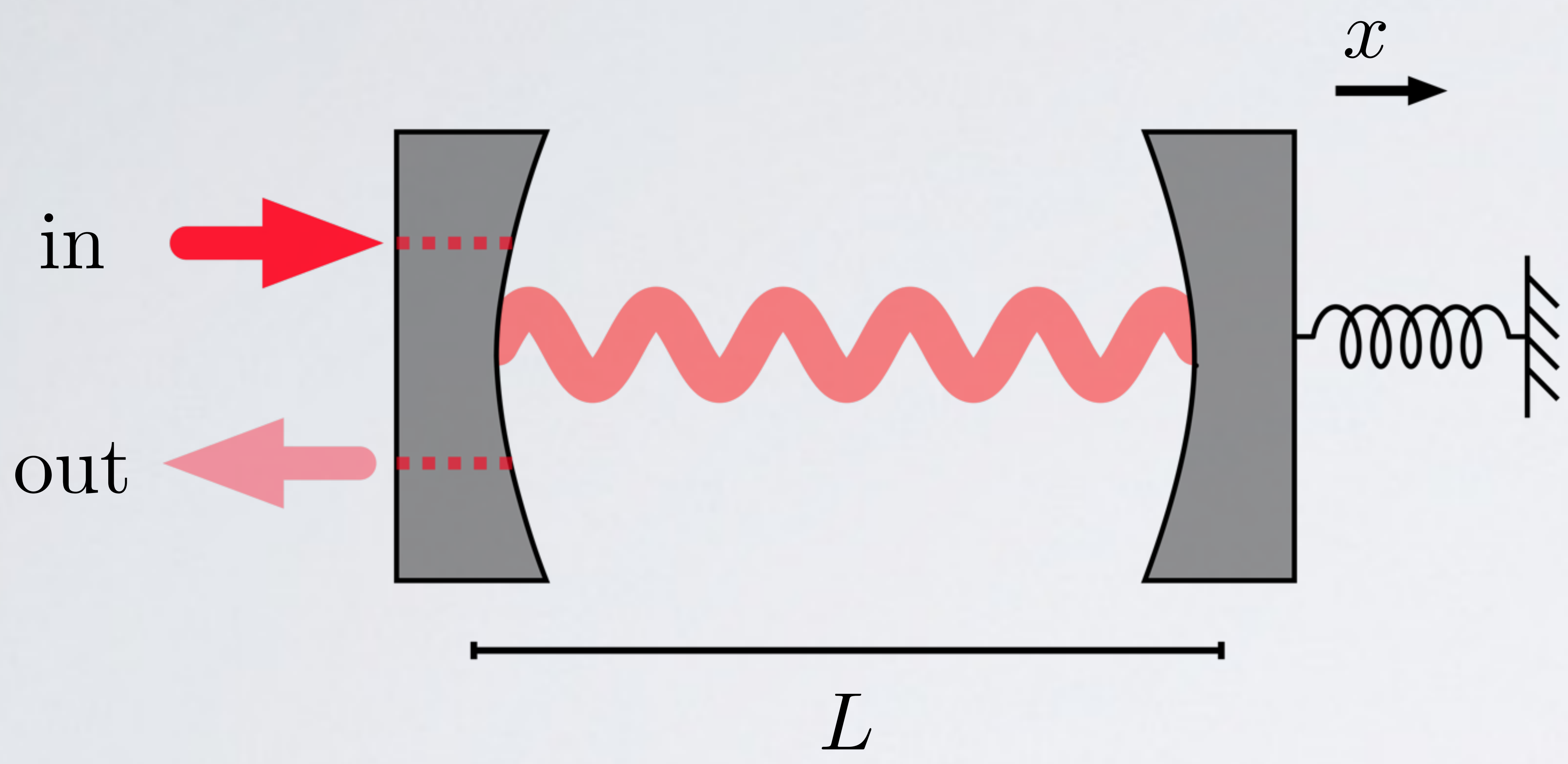


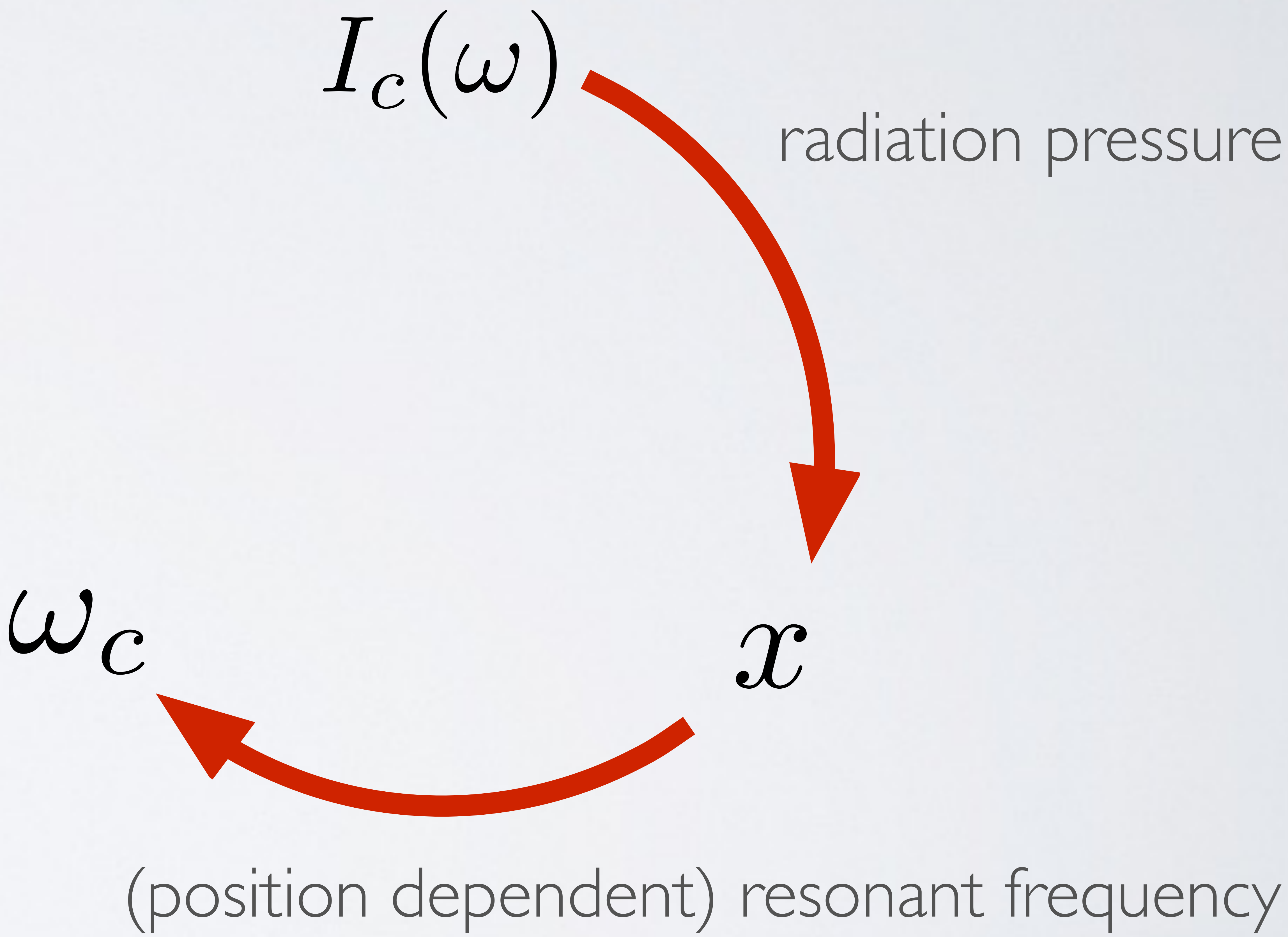
$$I_c(\omega)$$

radiation pressure

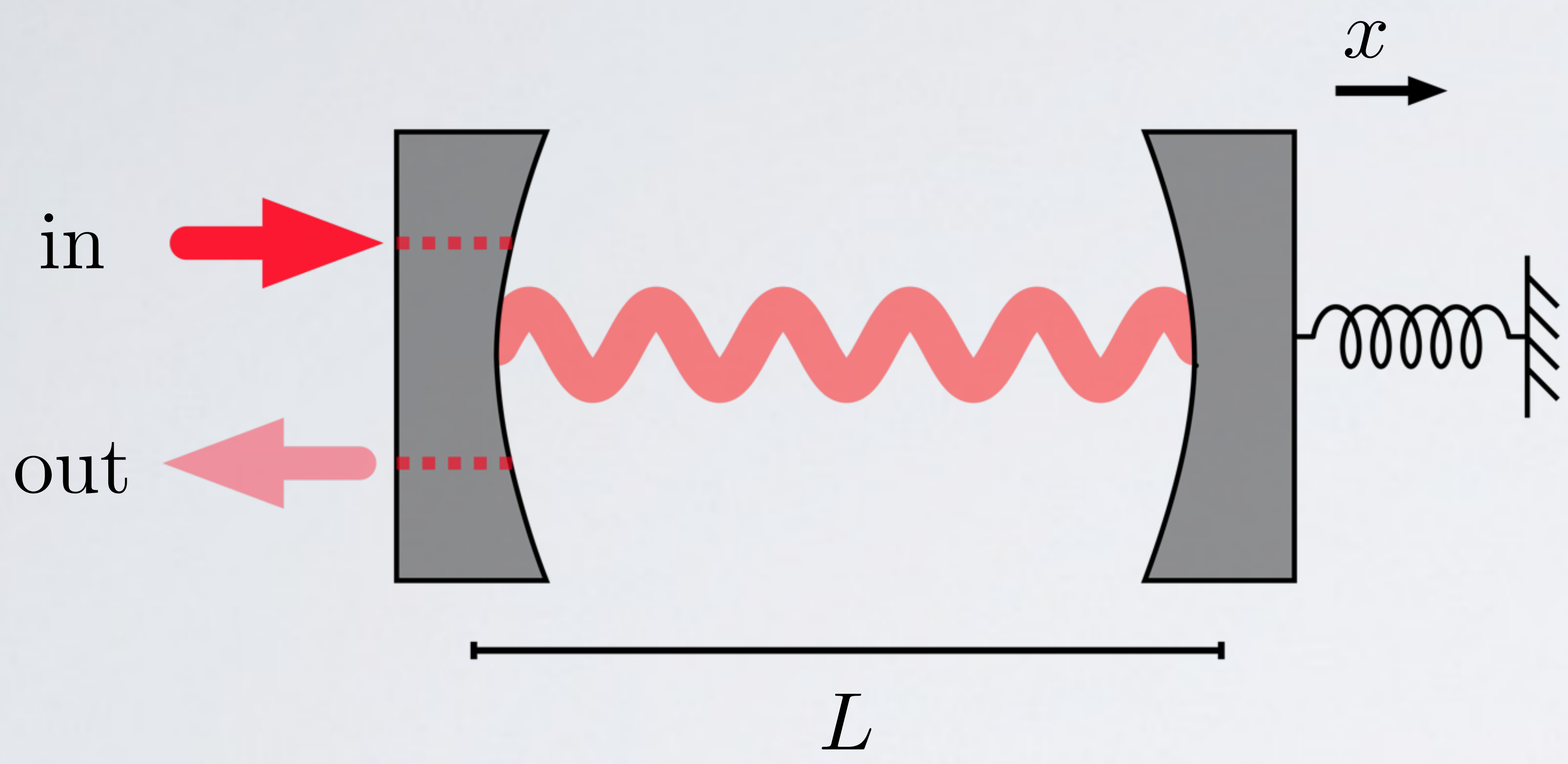
$$x$$

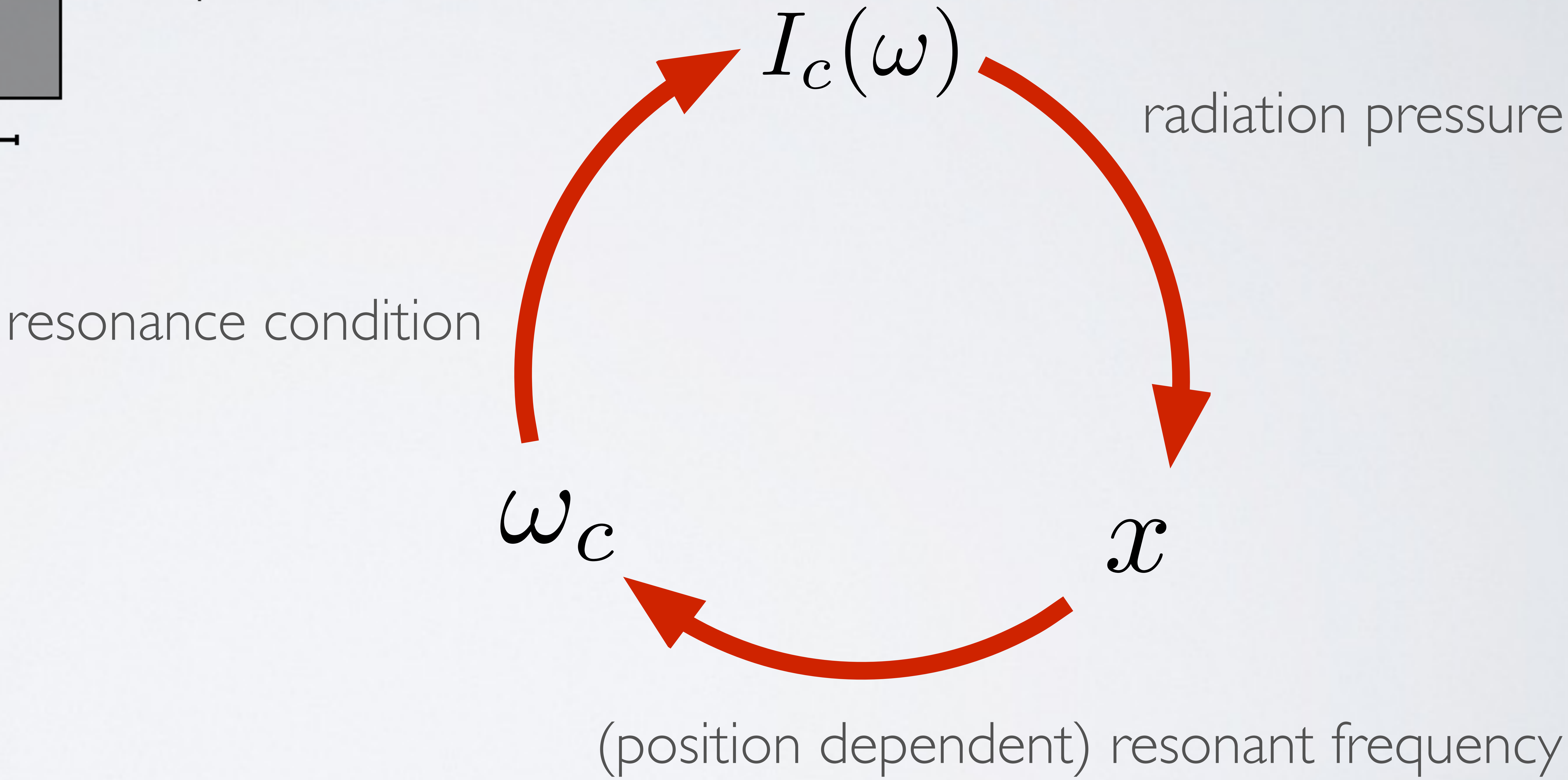
OPTOMECHANICAL SYSTEM



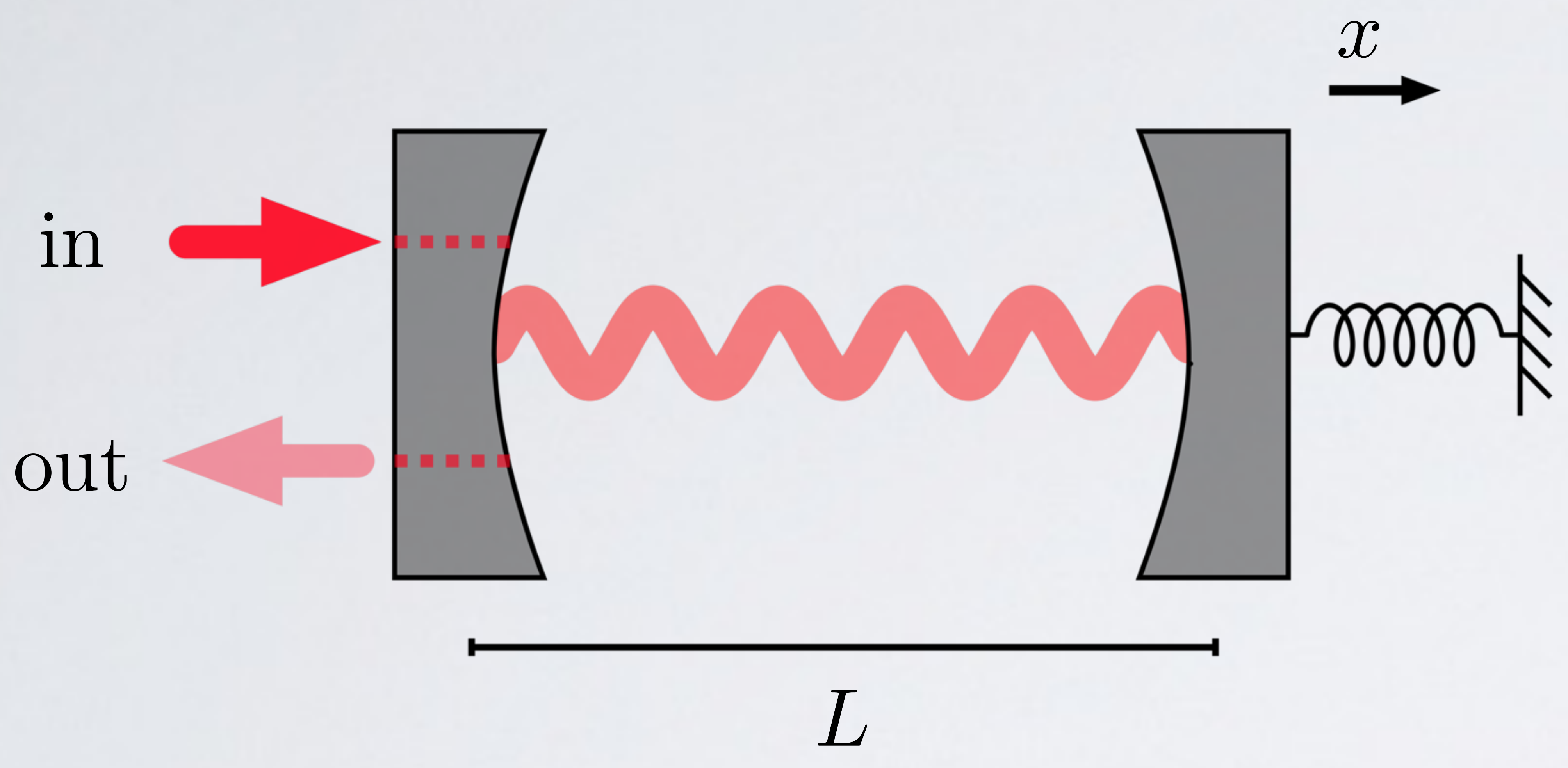


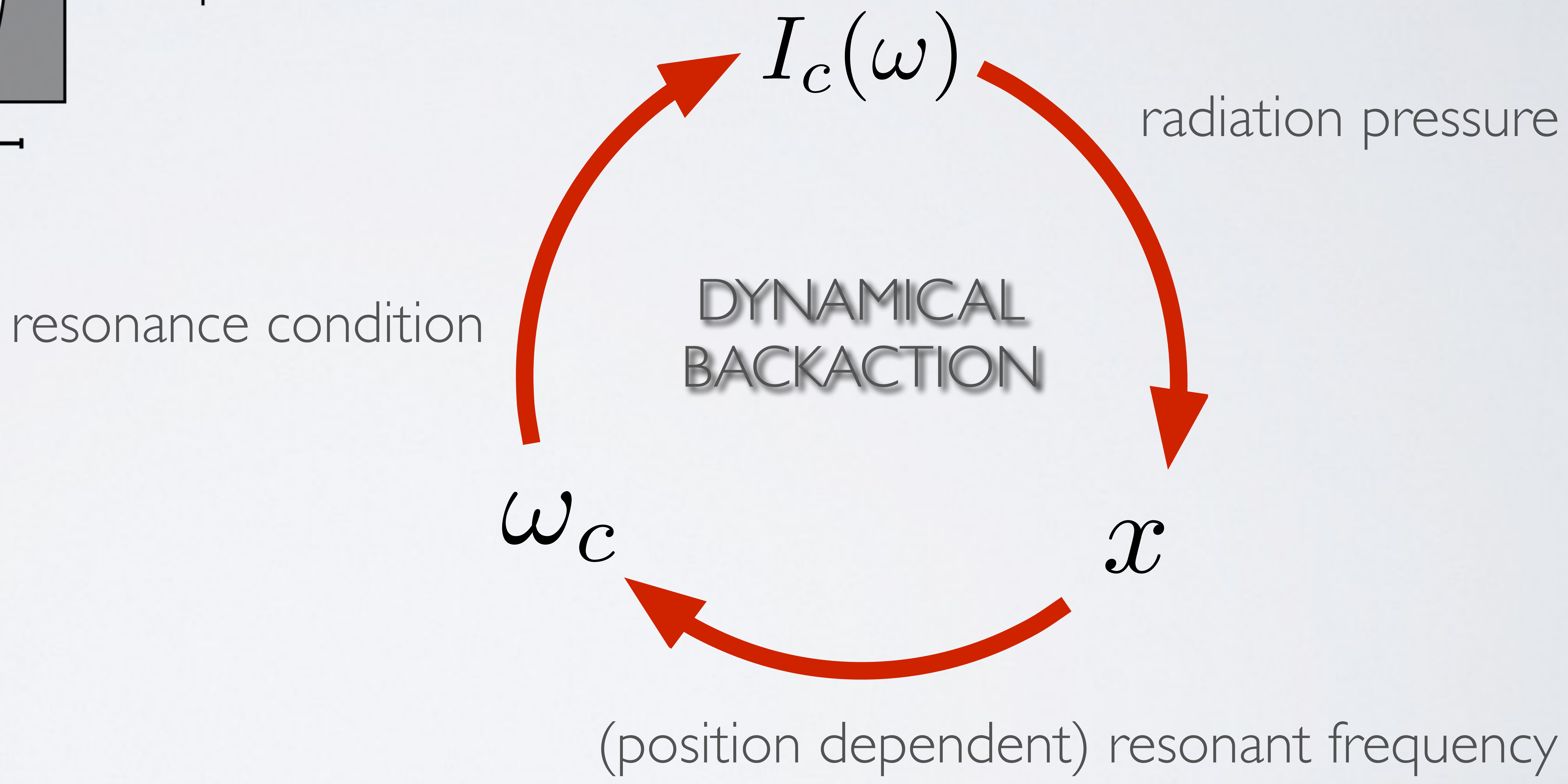
OPTOMECHANICAL SYSTEM





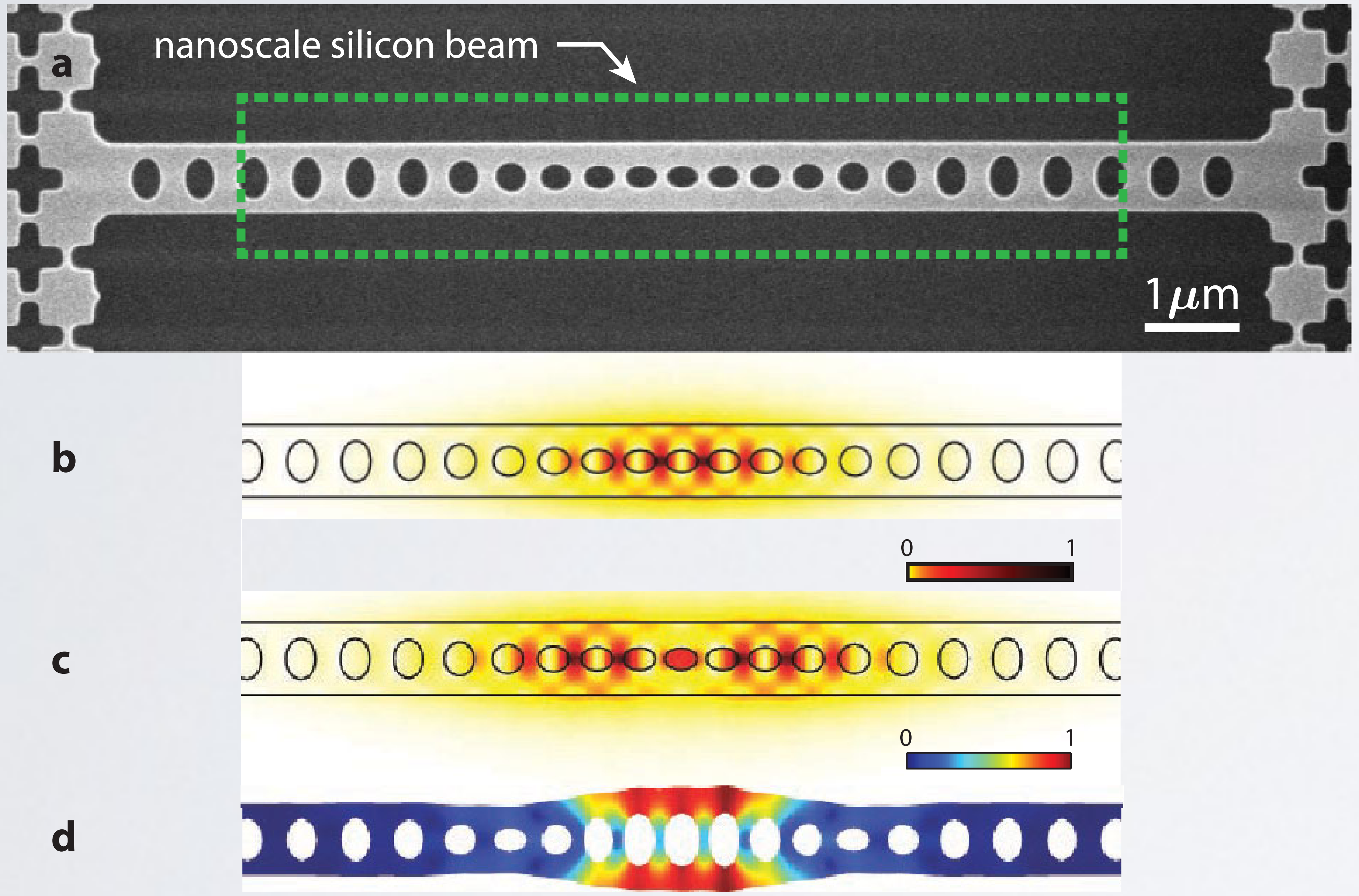
OPTOMECHANICAL SYSTEM



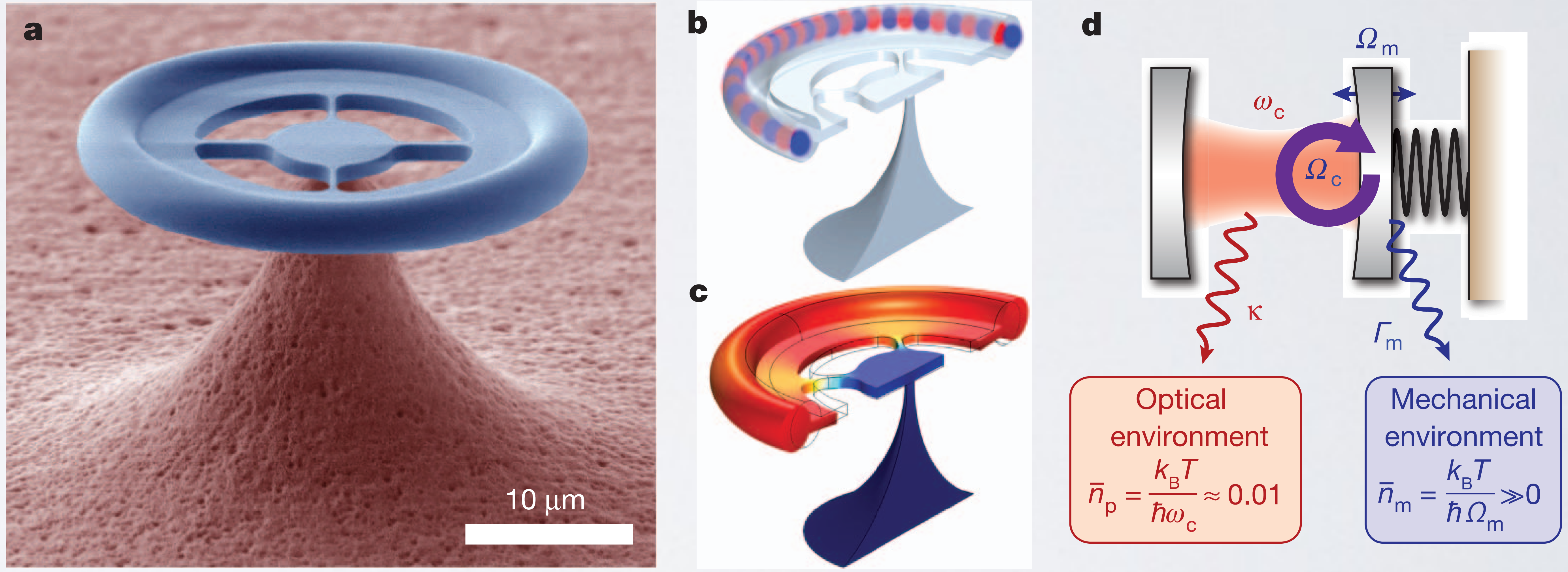


OPTOMECHANICAL SYSTEMS

Optical domain



Safavi-Naeini *et al.* PRL **108**, 033602 (2012)

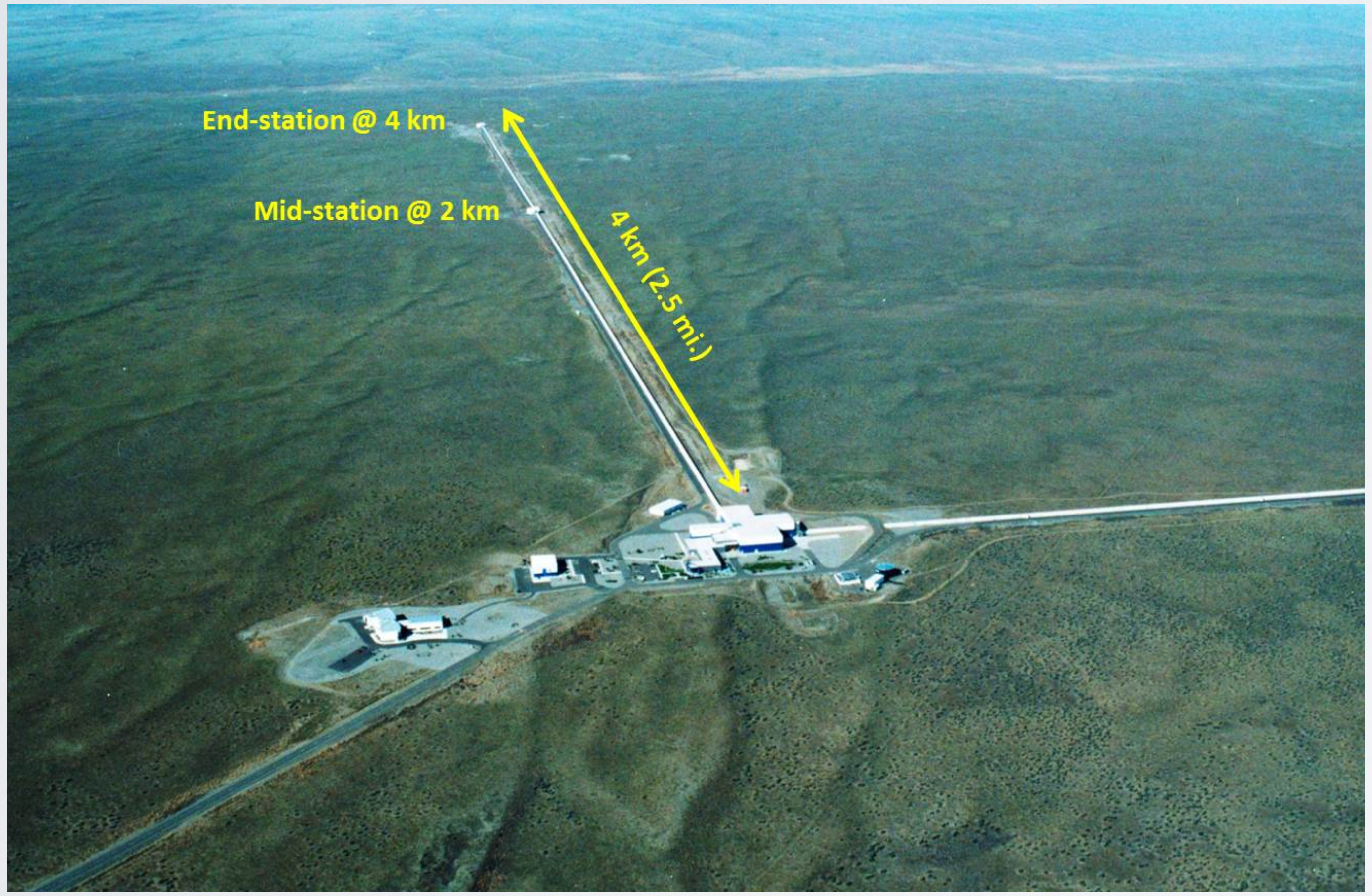


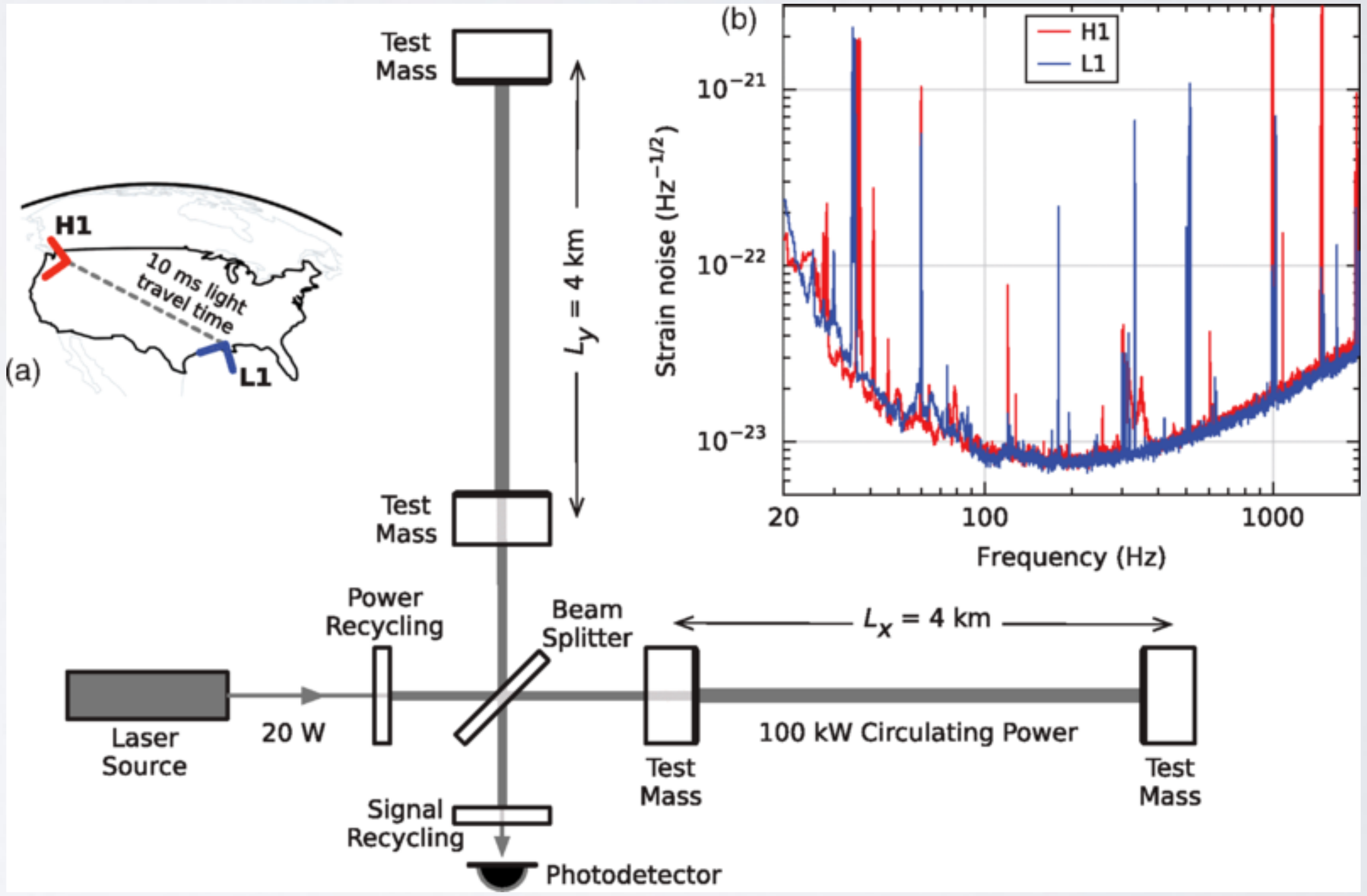
Verhagen *et al.* Nature **482**, 63 (2012)

OPTOMECHANICAL SYSTEMS

Gravitational wave detection

LIGO @ Hanford

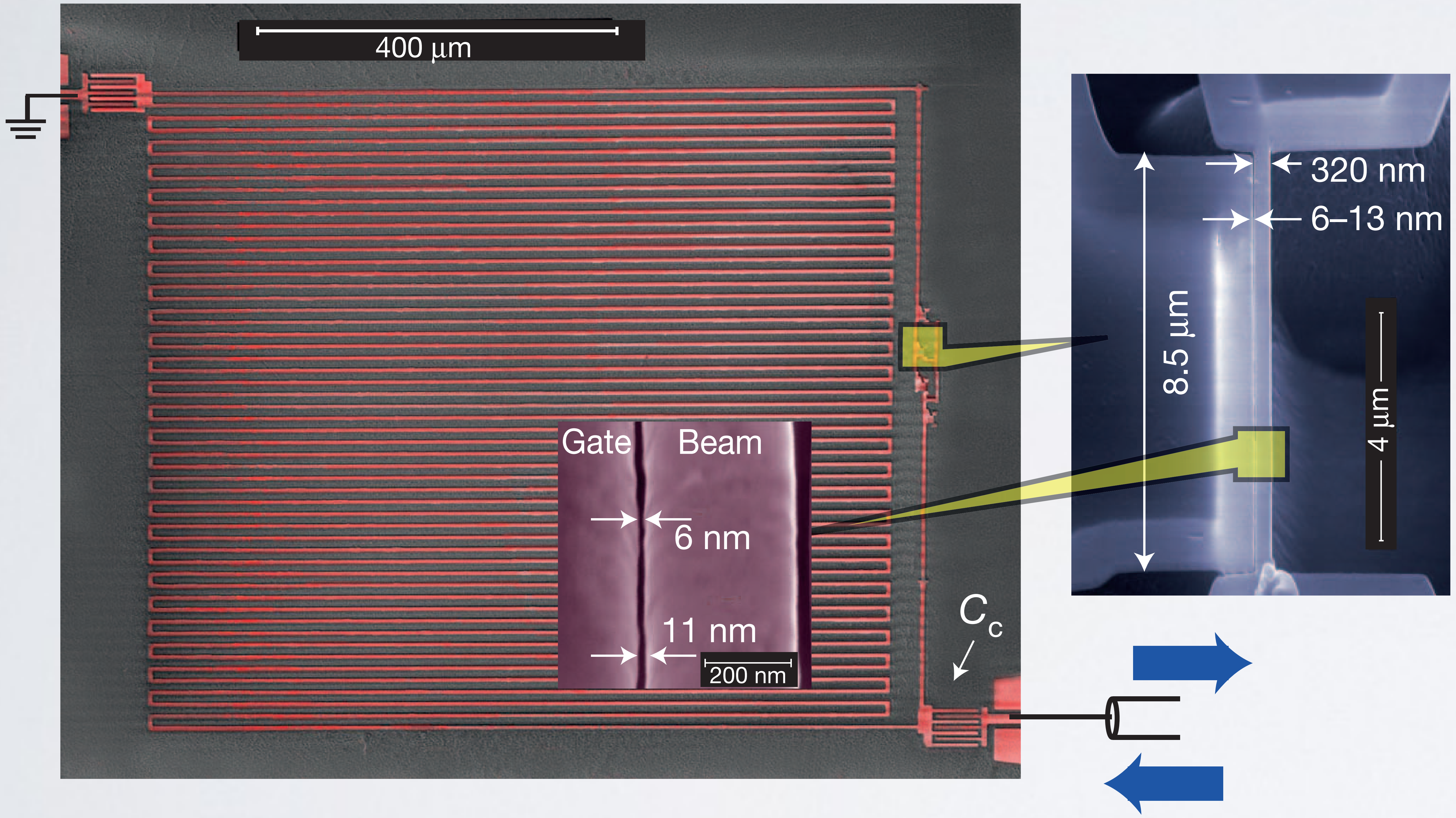




[1] B. P. Abbott, *et al.* , Phys. Rev. Lett. 116, 061102 (2016).

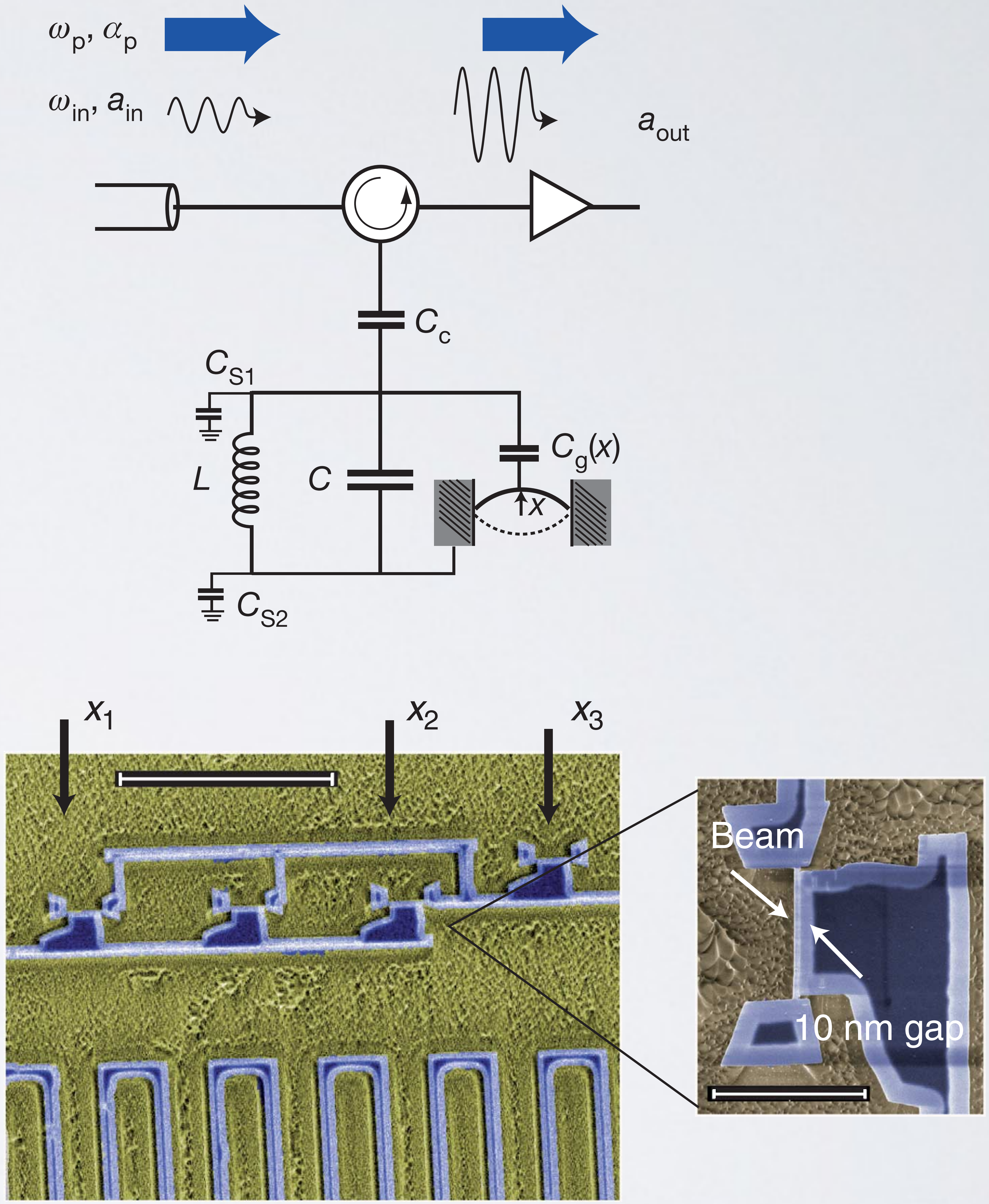
OPTOMECHANICAL SYSTEMS

Microwave domain



Massel *et al.* Nature **480**, 351(2011)

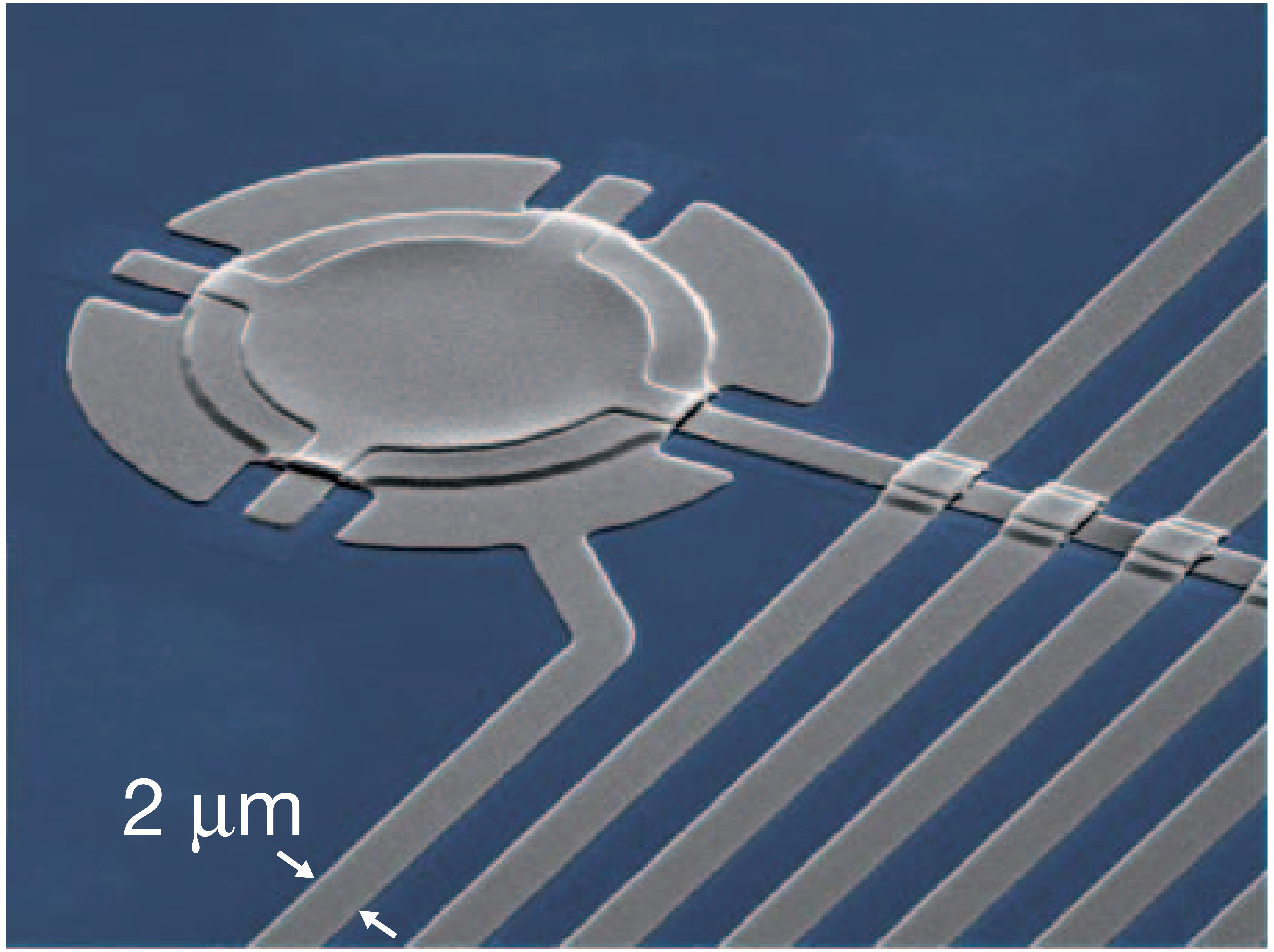
more about these later!



Massel *et al.* Nat. Comm, **3**, 987 (2012)

OPTOMECHANICAL SYSTEMS

Microwave domain

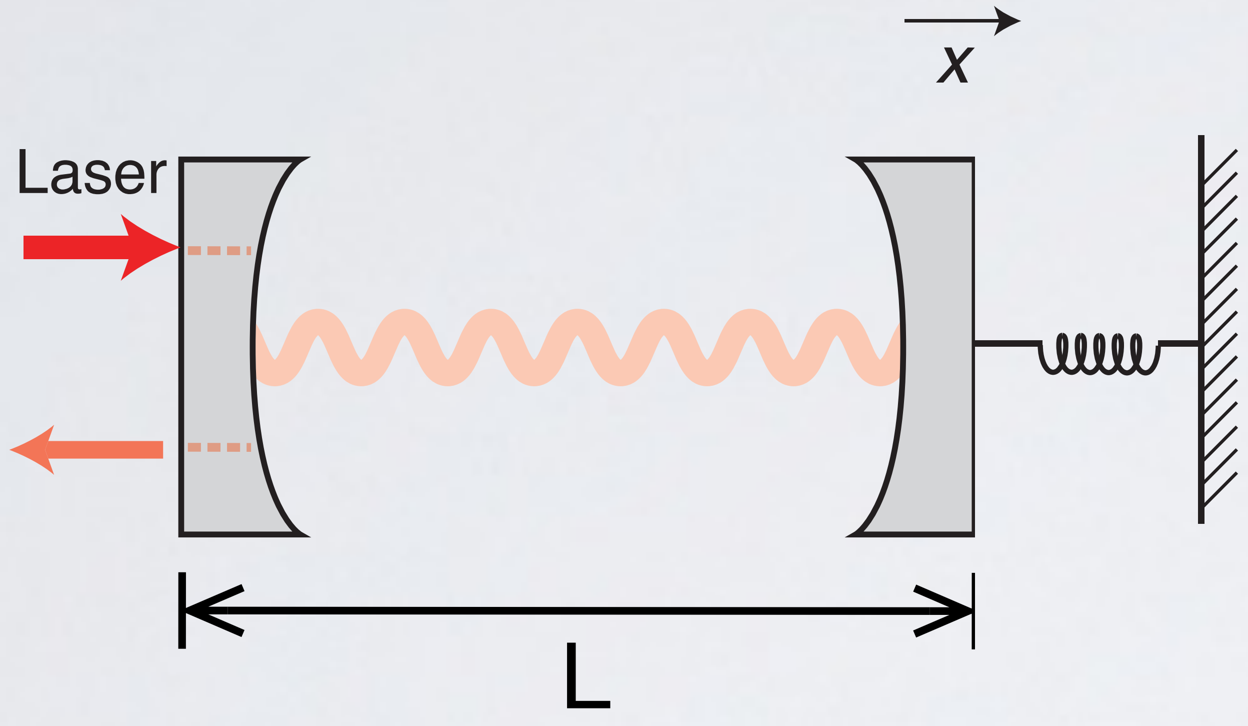


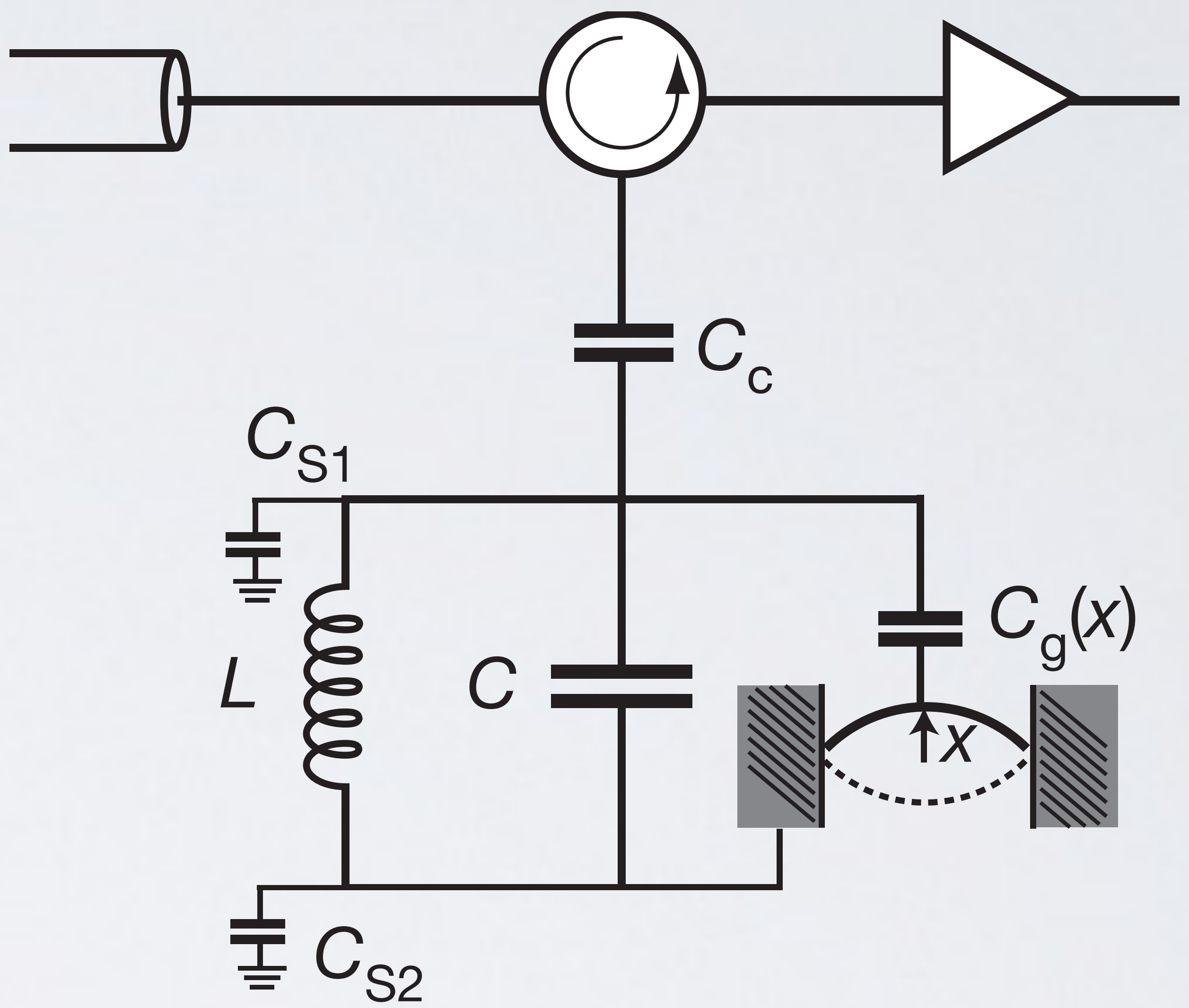
Teufel *et al.* Nature **475**, 359 (2011)

First to achieve ground-state cooling of the mechanical mode

OPTOMECHANICAL SYSTEMS

Microwave domain





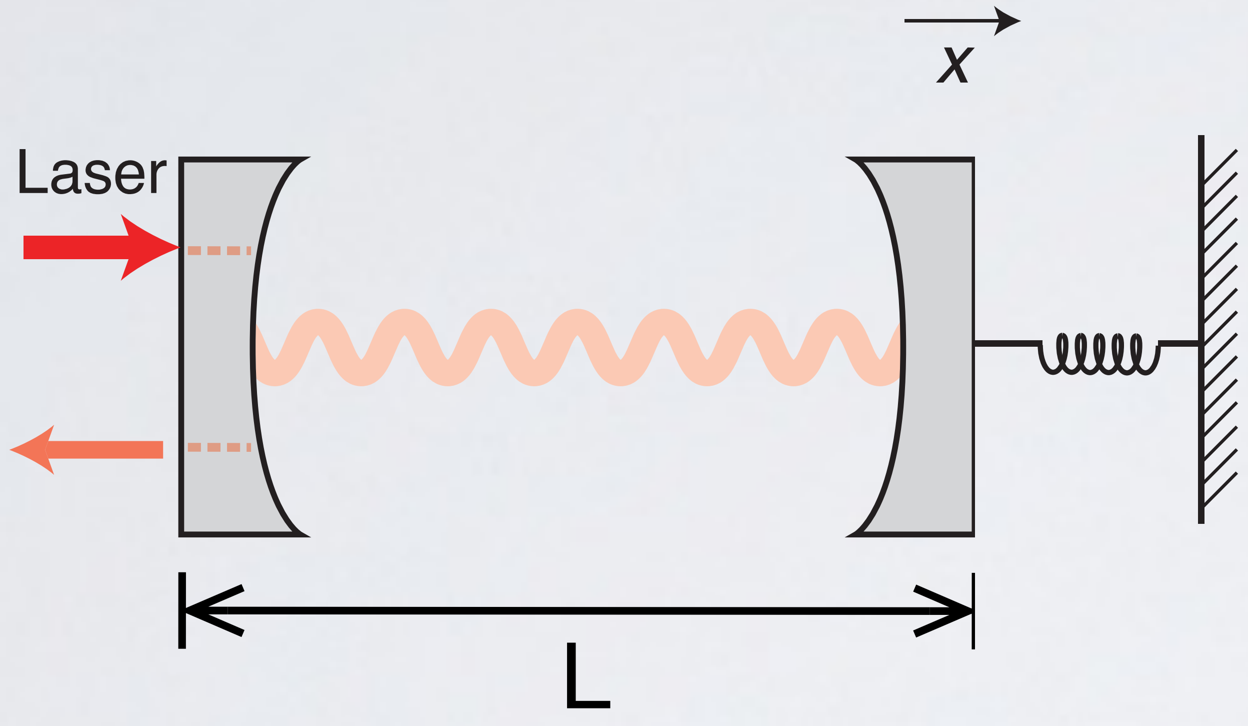
QM Hamiltonian

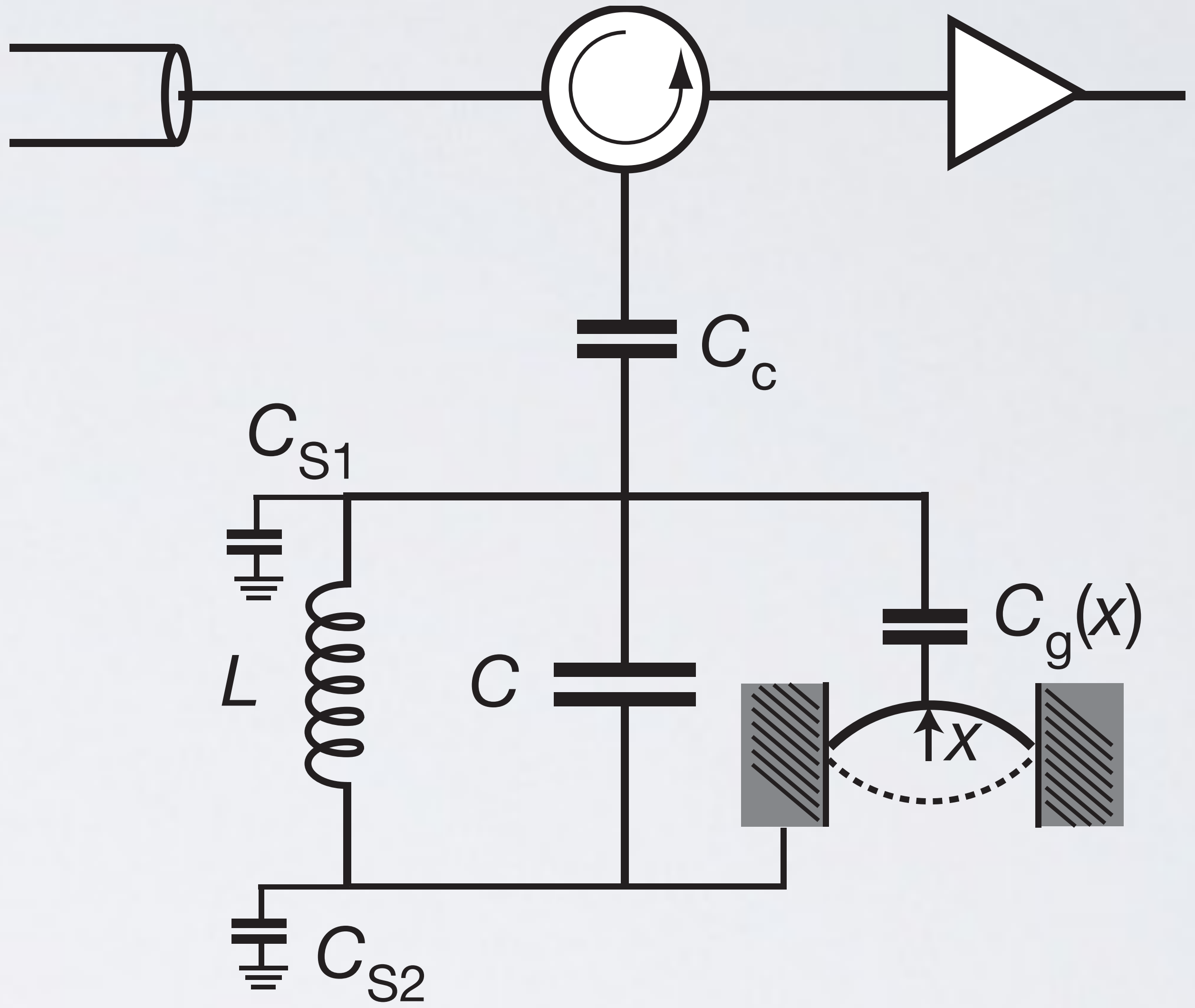
$$H = \omega_{\mathrm{c}} a^\dagger a + \omega_{\mathrm{m}} b^\dagger b + g\, a^\dagger a (b^\dagger + b)$$

$$\omega_c(\hat{x}) \simeq \omega_c + \left.\frac{\partial \omega_c}{\partial x}\right|_{x=0} \hat{x} + O(\hat{x}^2)$$

OPTOMECHANICAL SYSTEMS

Microwave domain





QM Hamiltonian

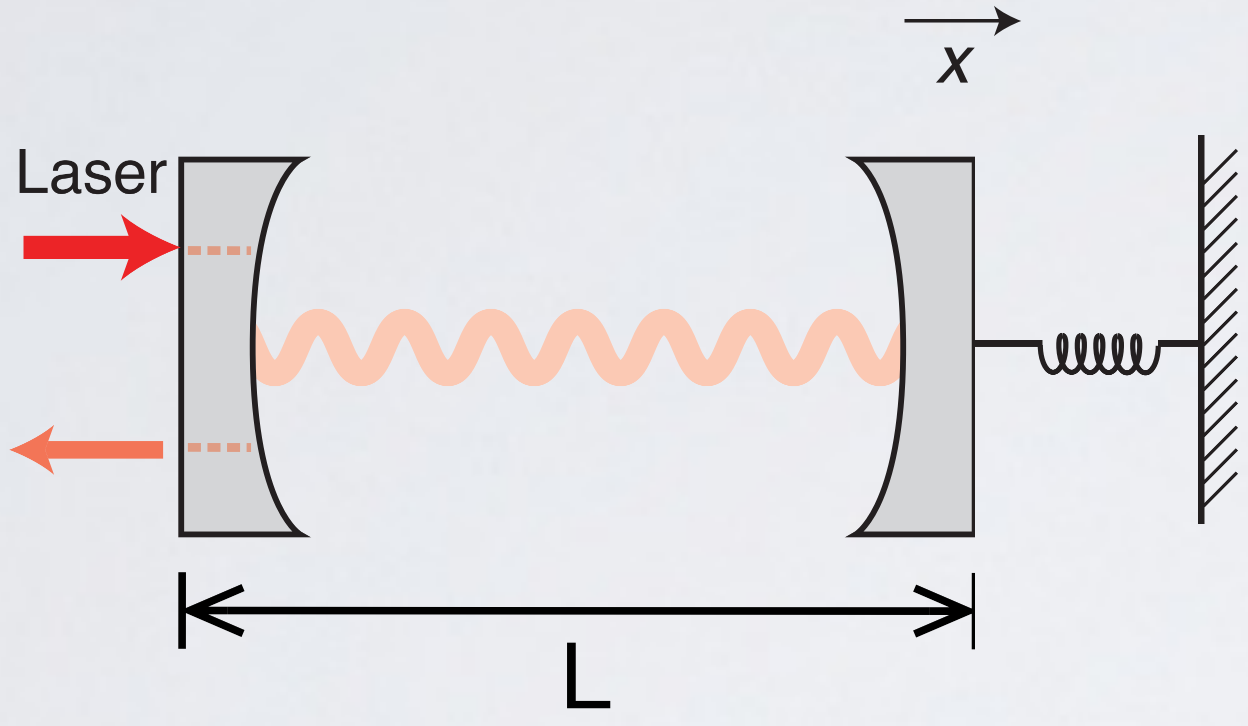
cavity

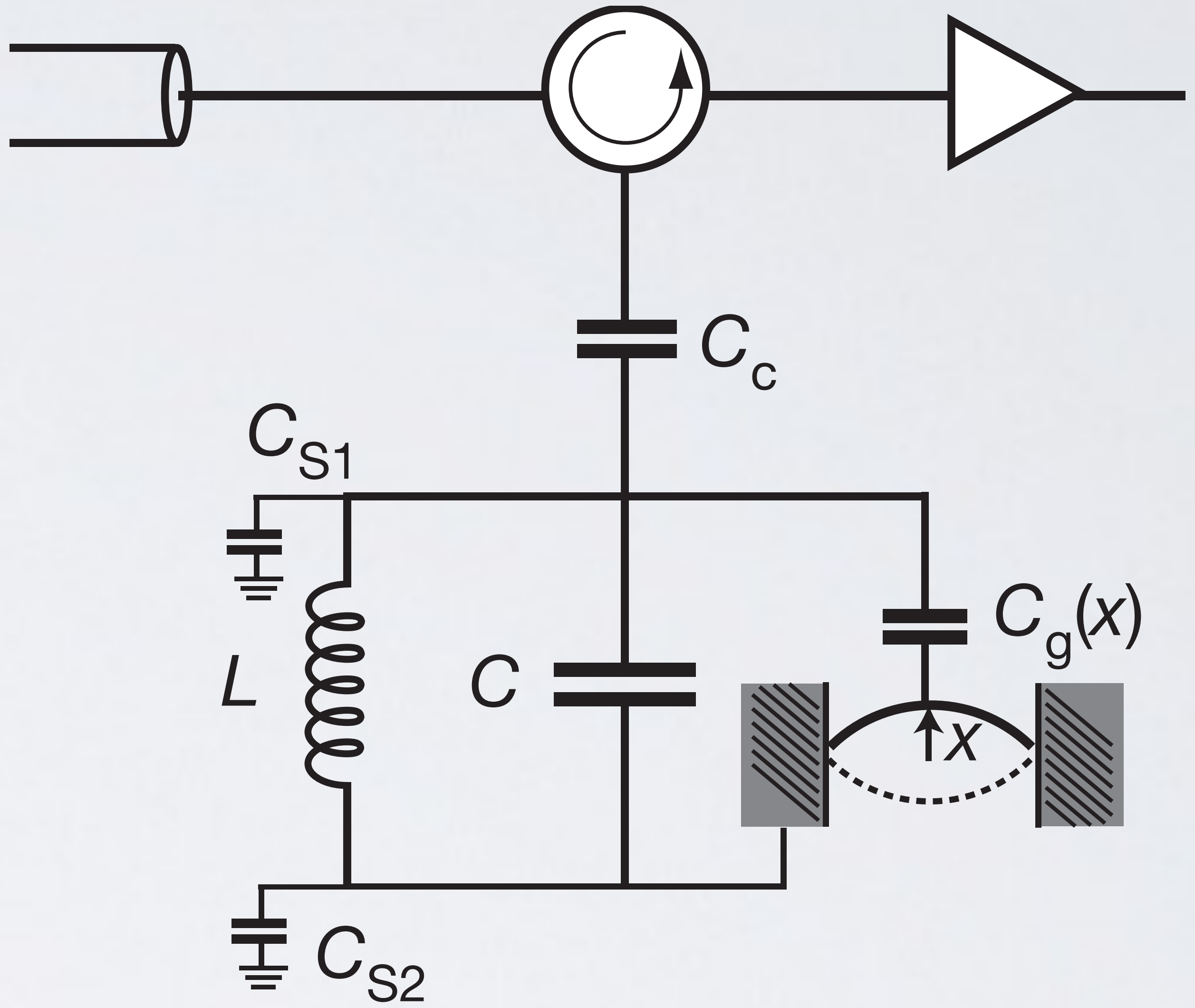
$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + g\, a^\dagger a (b^\dagger + b)$$

$$\omega_c(\hat{x}) \simeq \omega_c + \left.\frac{\partial \omega_c}{\partial x}\right|_{x=0} \hat{x} + O(\hat{x}^2)$$

OPTOMECHANICAL SYSTEMS

Microwave domain





QM Hamiltonian

cavity

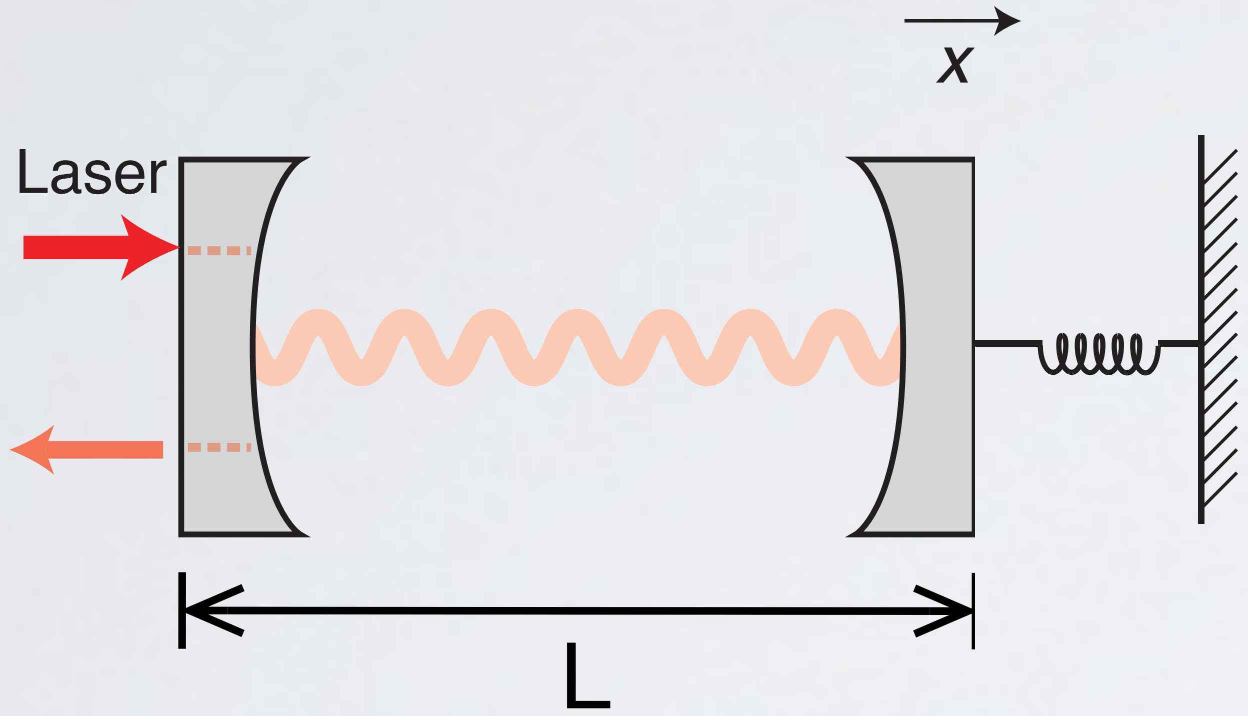
mechanics

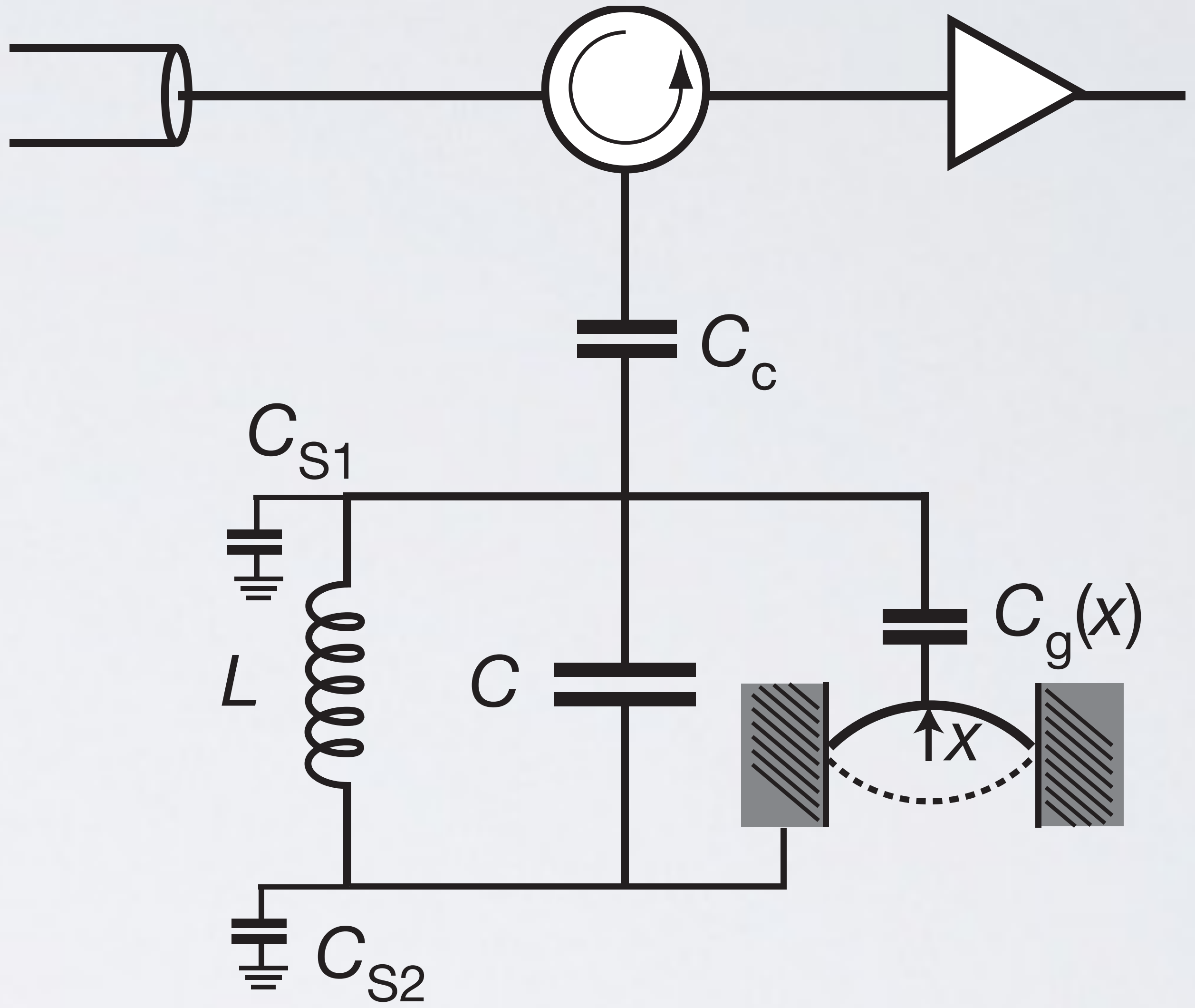
$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + g\, a^\dagger a (b^\dagger + b)$$

$$\omega_c(\hat{x}) \simeq \omega_c + \left.\frac{\partial \omega_c}{\partial x}\right|_{x=0} \hat{x} + O(\hat{x}^2)$$

OPTOMECHANICAL SYSTEMS

Microwave domain





QM Hamiltonian

cavity

mechanics

radiation-pressure coupling

$$H = \omega_\mathrm{c} a^\dagger a + \omega_\mathrm{m} b^\dagger b + g\, a^\dagger a (b^\dagger + b)$$

$$\omega_c(\hat{x}) \simeq \omega_c + \left.\frac{\partial \omega_c}{\partial x}\right|_{x=0} \hat{x} + O(\hat{x}^2)$$

QUANTUM LANGEVIN EQUATIONS

Let's write the EOMs for the fields a and b

$$\begin{cases} \dot{a}_t &= -i\omega_\mathrm{c} a_t - \frac{\kappa}{2}\, a_t - i g_0 a_t \left(b_t^\dagger + b_t\right) + \sqrt{\kappa} a_t^\mathrm{in} \\ \dot{b}_t &= -i\omega_\mathrm{m} b_t - \frac{\gamma}{2}\, b_t - i g_0 a_t^\dagger a_t + \sqrt{\gamma} b_t^\mathrm{in} \end{cases}$$

QUANTUM LANGEVIN EQUATIONS

Let's write the EOMs for the fields a and b

$$\begin{cases} \dot{a}_t &= -i\omega_{\rm c} a_t - \frac{\kappa}{2}\, a_t - i g_0 a_t \left(b_t^\dagger + b_t\right) + \sqrt{\kappa} a_t^{\rm in} \\ \dot{b}_t &= -i\omega_{\rm m} b_t - \frac{\gamma}{2}\, b_t - i g_0 a_t^\dagger a_t + \sqrt{\gamma} b_t^{\rm in} \end{cases}$$

Where do we go from here?

Impose a strong coherent drive to the "optical reservoir": its form determines most of the interesting results in the field of optomechanics of the last few years.

$$a_t^{\rm in} \to \alpha^{\rm in} \exp\left[-i\omega_{\rm p} t\right] + a_t^{\rm in}$$

QUANTUM LANGEVIN EQUATIONS

Let's write the EOMs for the fields a and b

$$\begin{cases} \dot{a}_t &= -i\omega_{\rm c} a_t - \frac{\kappa}{2}\, a_t - i g_0 a_t \left(b_t^\dagger + b_t \right) + \boxed{\sqrt{\kappa} a_t^{\rm in}} \\ \dot{b}_t &= -i\omega_{\rm m} b_t - \frac{\gamma}{2}\, b_t - i g_0 a_t^\dagger a_t + \sqrt{\gamma} b_t^{\rm in} \end{cases}$$

Where do we go from here?

Impose a strong coherent drive to the "optical reservoir": its form determines most of the interesting results in the field of optomechanics of the last few years.

$$a_t^{\rm in} \to \alpha^{\rm in} \exp\left[-i\omega_{\rm p} t\right] + a_t^{\rm in}$$

MECHANICAL COOLING AND AMPLIFICATION

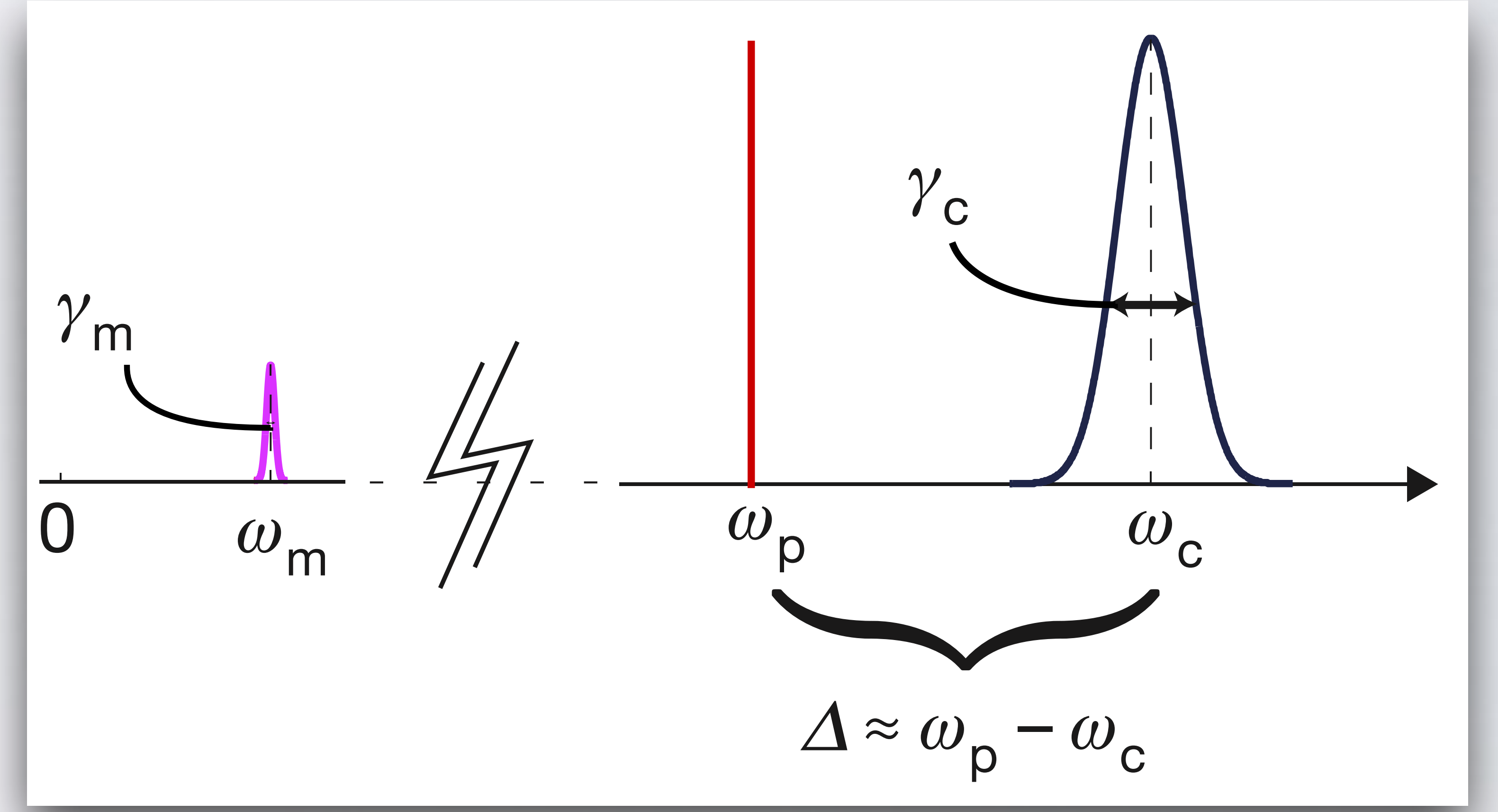
Input field: $a_t^{\mathrm{in}} \to \alpha^{\mathrm{in}} \exp\left[-i\omega_{\mathrm{p}} t\right] + a_t^{\mathrm{in}}$

MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\mathrm{in}} \to \alpha^{\mathrm{in}} \exp\left[-i\omega_{\mathrm{p}} t\right] + a_t^{\mathrm{in}}$

For $\omega_{\mathrm{p}} \simeq \omega_{\mathrm{c}} - \omega_{\mathrm{m}}$ (red-detuned case) the RWA gives, in the appropriate frame

$$\begin{cases} \dot{a}_t & = -\frac{\kappa}{2}\, a_t - i g_0 \alpha b_t + \sqrt{\kappa} a_t^{\mathrm{in}} \\ \dot{b}_t & = -\frac{\gamma}{2}\, b_t - i g_0 \alpha^* a_t + \sqrt{\gamma} b_t^{\mathrm{in}} \end{cases}$$

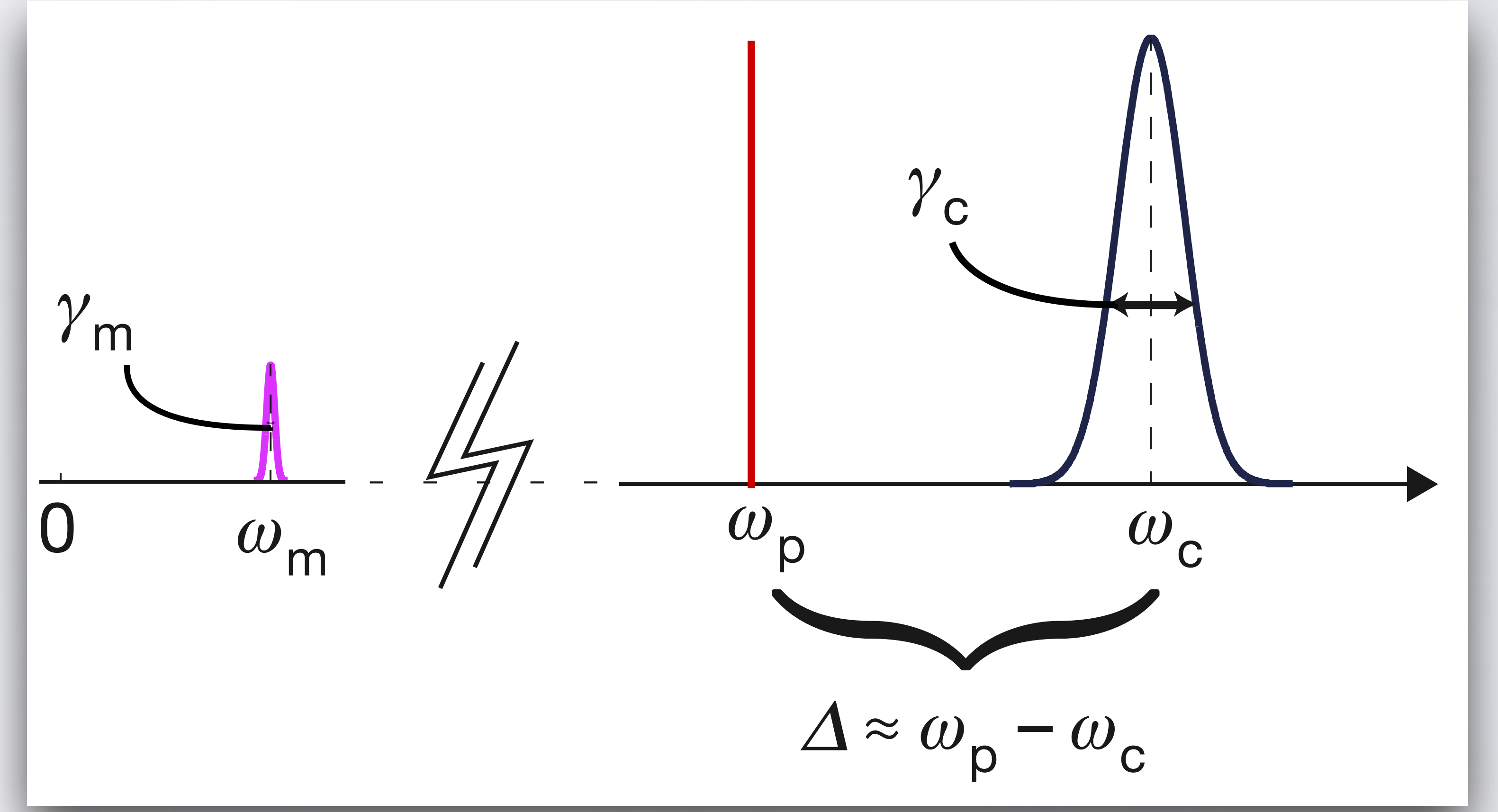


MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\rm in} \to \alpha^{\rm in} \exp\left[-i\omega_{\rm p} t\right] + a_t^{\rm in}$

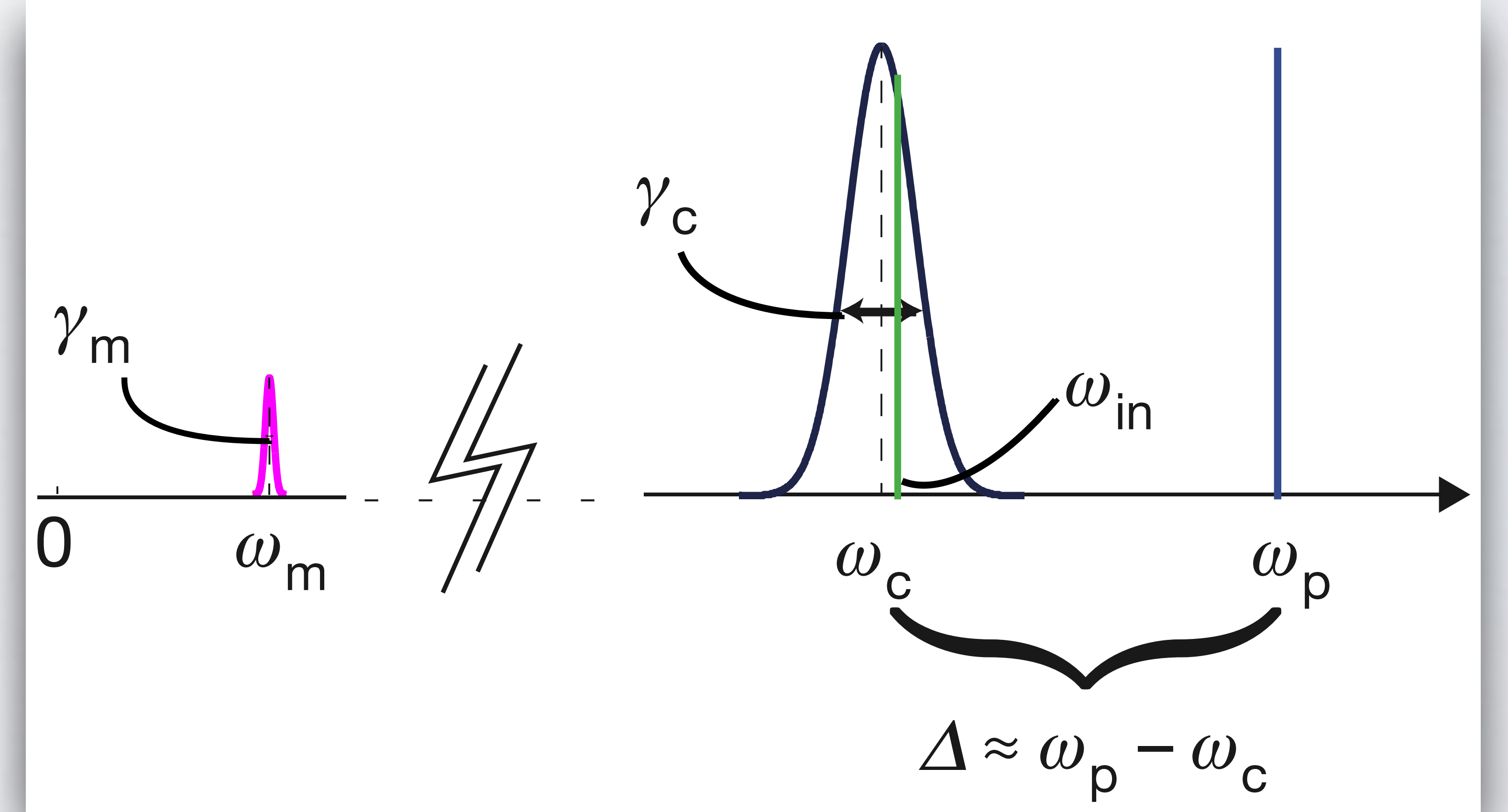
For $\omega_{\rm p} \simeq \omega_{\rm c} - \omega_{\rm m}$ (red-detuned case) the RWA gives, in the appropriate frame

$$\begin{cases} \dot{a}_t &= -\frac{\kappa}{2}\, a_t - i g_0 \alpha b_t + \sqrt{\kappa} a_t^{\rm in} \\ \dot{b}_t &= -\frac{\gamma}{2}\, b_t - i g_0 \alpha^* a_t + \sqrt{\gamma} b_t^{\rm in} \end{cases}$$



while for $\omega_{\rm p} \simeq \omega_{\rm c} + \omega_{\rm m}$ (blue-detuned case)

$$\begin{cases} \dot{a}_t &= -\frac{\kappa}{2}\, a_t - i g_0 \alpha b_t^\dagger + \sqrt{\kappa} a_t^{\rm in} \\ \dot{b}_t^\dagger &= -\frac{\gamma}{2}\, b_t^\dagger + i g_0 \alpha^* a_t + \sqrt{\gamma} b_t^{\rm in} \end{cases}$$



MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\mathrm{in}} \to \alpha^{\mathrm{in}} \exp\left[-i\omega_{\mathrm{p}} t\right] + a_t^{\mathrm{in}}$

Red-detuned case

$$\begin{cases} \dot{a}_t &= -\frac{\kappa}{2}\, a_t - i g_0 \alpha b_t + \sqrt{\kappa} a_t^{\mathrm{in}} \\ \dot{b}_t &= -\frac{\gamma}{2}\, b_t - i g_0 \alpha^* a_t + \sqrt{\gamma} b_t^{\mathrm{in}} \end{cases}$$

MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\mathrm{in}} \to \alpha^{\mathrm{in}} \exp\left[-i\omega_{\mathrm{p}} t\right] + a_t^{\mathrm{in}}$

Red-detuned case

$$\begin{cases} \dot{a}_t & = -\frac{\kappa}{2}\, a_t - i g_0 \alpha b_t + \sqrt{\kappa} a_t^{\mathrm{in}} \\ \dot{b}_t & = -\frac{\gamma}{2}\, b_t - i g_0 \alpha^* a_t + \sqrt{\gamma} b_t^{\mathrm{in}} \end{cases}$$

$$-i\omega a_\omega = -\frac{\kappa}{2} a_\omega - iG b_\omega + \sqrt{\kappa} a_{\mathrm{in},\omega}$$

$$-i\omega b_\omega = -\frac{\gamma}{2} b_\omega - iG a_\omega + \sqrt{\gamma} b_{\mathrm{in},\omega}$$

MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\text{in}} \to \alpha^{\text{in}} \exp\left[-i\omega_{\text{p}} t\right] + a_t^{\text{in}}$

Red-detuned case

$$\begin{cases} \dot{a}_t &= -\frac{\kappa}{2}\, a_t - i g_0 \alpha b_t + \sqrt{\kappa} a_t^{\text{in}} \\ \dot{b}_t &= -\frac{\gamma}{2}\, b_t - i g_0 \alpha^* a_t + \sqrt{\gamma} b_t^{\text{in}} \end{cases}$$

$$-i\omega a_\omega = -\frac{\kappa}{2} a_\omega - iG b_\omega + \sqrt{\kappa} a_{\text{in},\omega}$$

$$-i\omega b_\omega = -\frac{\gamma}{2} b_\omega - iG a_\omega + \sqrt{\gamma} b_{\text{in},\omega}$$

$$A^{-1} \begin{pmatrix} a_\omega \\ b_\omega \end{pmatrix} = \begin{pmatrix} \sqrt{\kappa} a_{\text{in},\omega} \\ \sqrt{\gamma} b_{\text{in},\omega} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \kappa/2 - i\omega & -iG \\ -iG & \gamma/2 - i\omega \end{pmatrix}$$

MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\text{in}} \to \alpha^{\text{in}} \exp\left[-i\omega_{\text{p}} t\right] + a_t^{\text{in}}$

Red-detuned case

$$\begin{cases} \dot{a}_t &= -\frac{\kappa}{2}\, a_t - i g_0 \alpha b_t + \sqrt{\kappa} a_t^{\text{in}} \\ \dot{b}_t &= -\frac{\gamma}{2}\, b_t - i g_0 \alpha^* a_t + \sqrt{\gamma} b_t^{\text{in}} \end{cases}$$

$$-i\omega a_\omega = -\frac{\kappa}{2} a_\omega - iG b_\omega + \sqrt{\kappa} a_{\text{in},\omega}$$

$$-i\omega b_\omega = -\frac{\gamma}{2} b_\omega - iG a_\omega + \sqrt{\gamma} b_{\text{in},\omega}$$

$$A^{-1} \begin{pmatrix} a_\omega \\ b_\omega \end{pmatrix} = \begin{pmatrix} \sqrt{\kappa} a_{\text{in},\omega} \\ \sqrt{\gamma} b_{\text{in},\omega} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \kappa/2 - i\omega & -iG \\ -iG & \gamma/2 - i\omega \end{pmatrix}$$

$$A = \frac{1}{1 + G^2 \chi_c \chi_m} \begin{pmatrix} \chi_c & -iG\chi_c\chi_m \\ -iG\chi_c\chi_m & \chi_m \end{pmatrix}$$

$$\chi_c = [\kappa/2 - i\omega]^{-1}$$

$$\chi_m = [\gamma/2 - i\omega]^{-1}$$

MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\rm in} \to \alpha^{\rm in} \exp\left[-i\omega_{\rm p} t\right] + a_t^{\rm in}$

Red-detuned case.

Considering that $\kappa \gg \gamma$, the I/O relation (for b_ω) simplifies (approximately) to

$$b_\omega \simeq \frac{\sqrt{\gamma}}{\gamma/2 - i\omega + 2G^2/\kappa} b_{{\rm in},\omega} - \frac{iG}{\frac{\kappa}{2}(\frac{\gamma}{2} - i\omega)} a_{{\rm in},\omega}$$

MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\text{in}} \to \alpha^{\text{in}} \exp[-i\omega_{\text{p}} t] + a_t^{\text{in}}$

Red-detuned case.

Considering that $\kappa \gg \gamma$, the I/O relation (for b_ω) simplifies (approximately) to

$$b_\omega \simeq \frac{\sqrt{\gamma}}{\gamma/2 - i\omega + 2G^2/\kappa} b_{\text{in},\omega} - \frac{iG}{\frac{\kappa}{2}\left(\frac{\gamma}{2} - i\omega\right)} a_{\text{in},\omega}$$

$$\gamma_{\text{eff}} = \gamma + \frac{4G^2}{\kappa} \longrightarrow \text{cooling}$$

MECHANICAL COOLING AND AMPLIFICATION

Input field: $a_t^{\text{in}} \to \alpha^{\text{in}} \exp\left[-i\omega_{\text{p}} t\right] + a_t^{\text{in}}$

Red-detuned case.

Considering that $\kappa \gg \gamma$, the I/O relation (for b_ω) simplifies (approximately) to

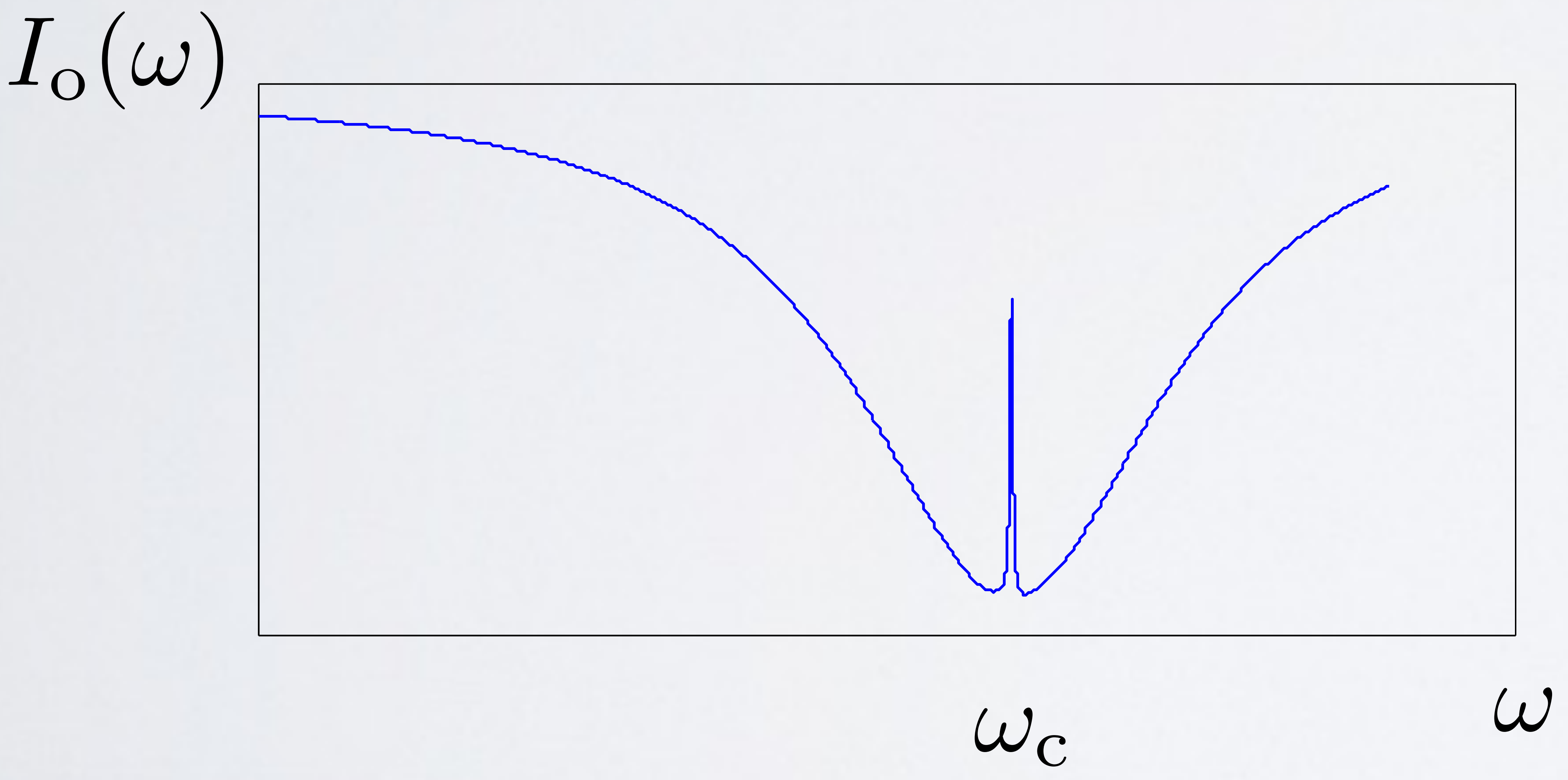
$$b_\omega \simeq \frac{\sqrt{\gamma}}{\gamma/2 - i\omega + 2G^2/\kappa} b_{\text{in},\omega} - \frac{iG}{\frac{\kappa}{2}\left(\frac{\gamma}{2} - i\omega\right)} a_{\text{in},\omega}$$

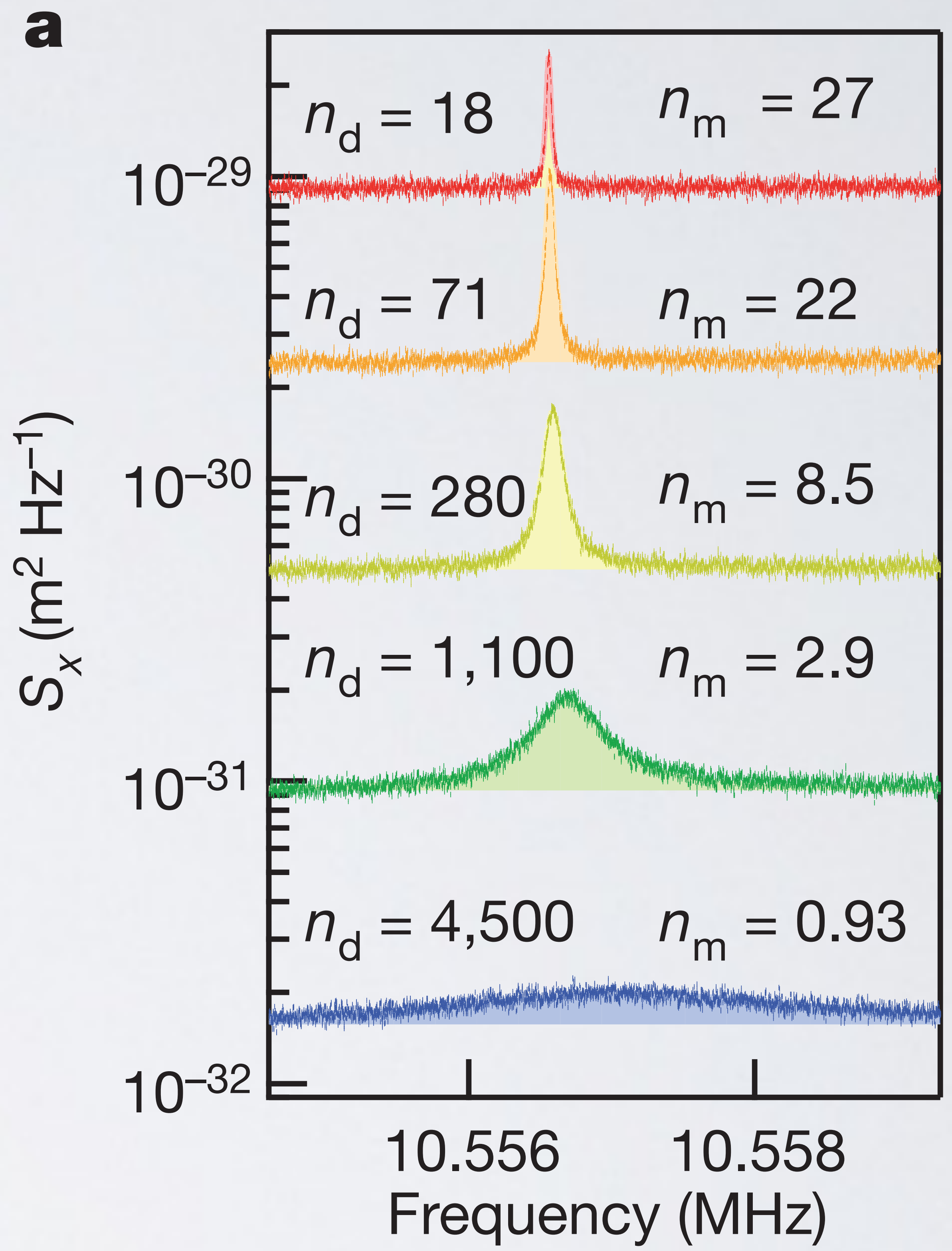
$$\gamma_{\text{eff}} = \gamma + \frac{4G^2}{\kappa}$$ → cooling

For the ampli case: $$\gamma_{\text{eff}} = \gamma - \frac{4G^2}{\kappa}$$

OPTOMECHANICAL SYSTEM

Sideband cooling

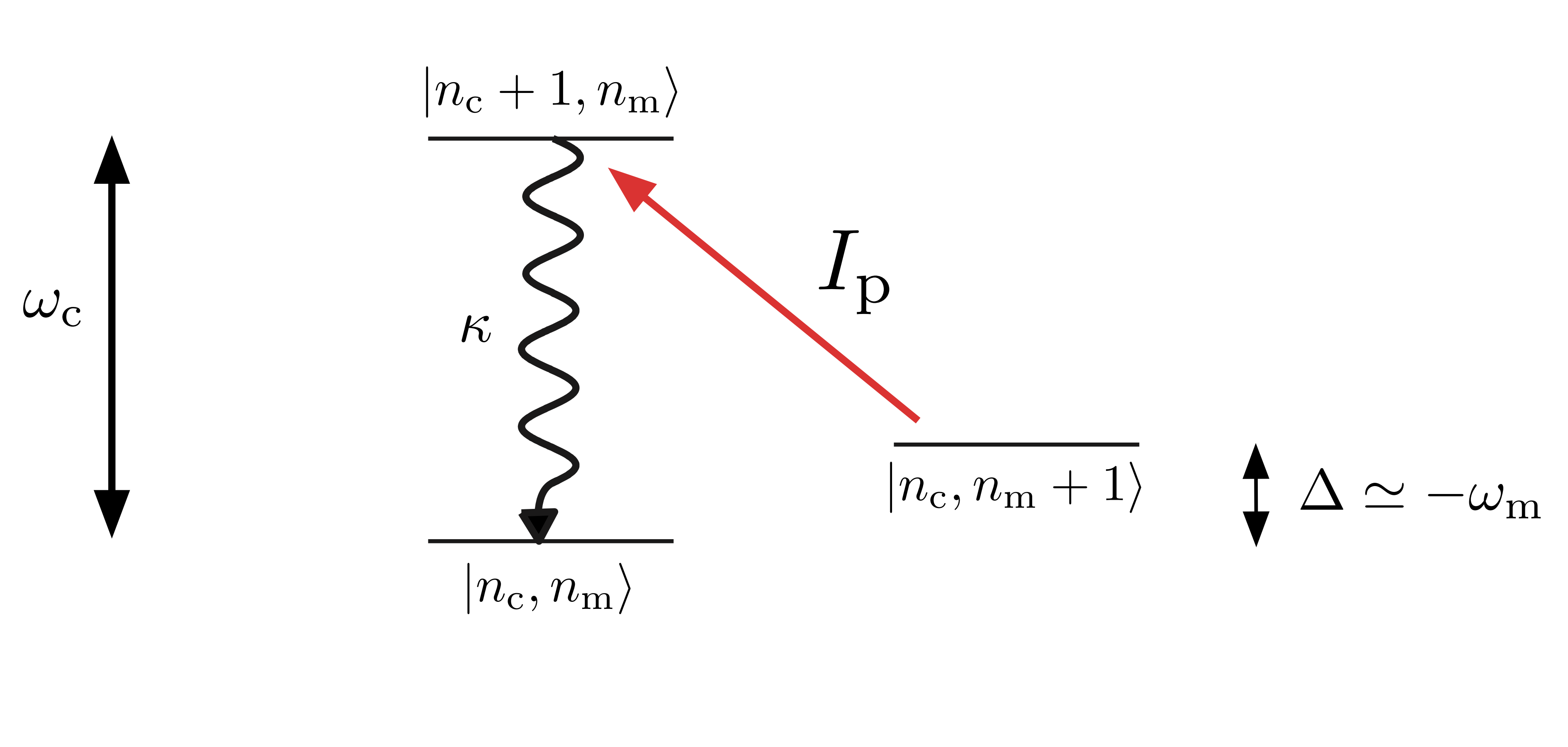


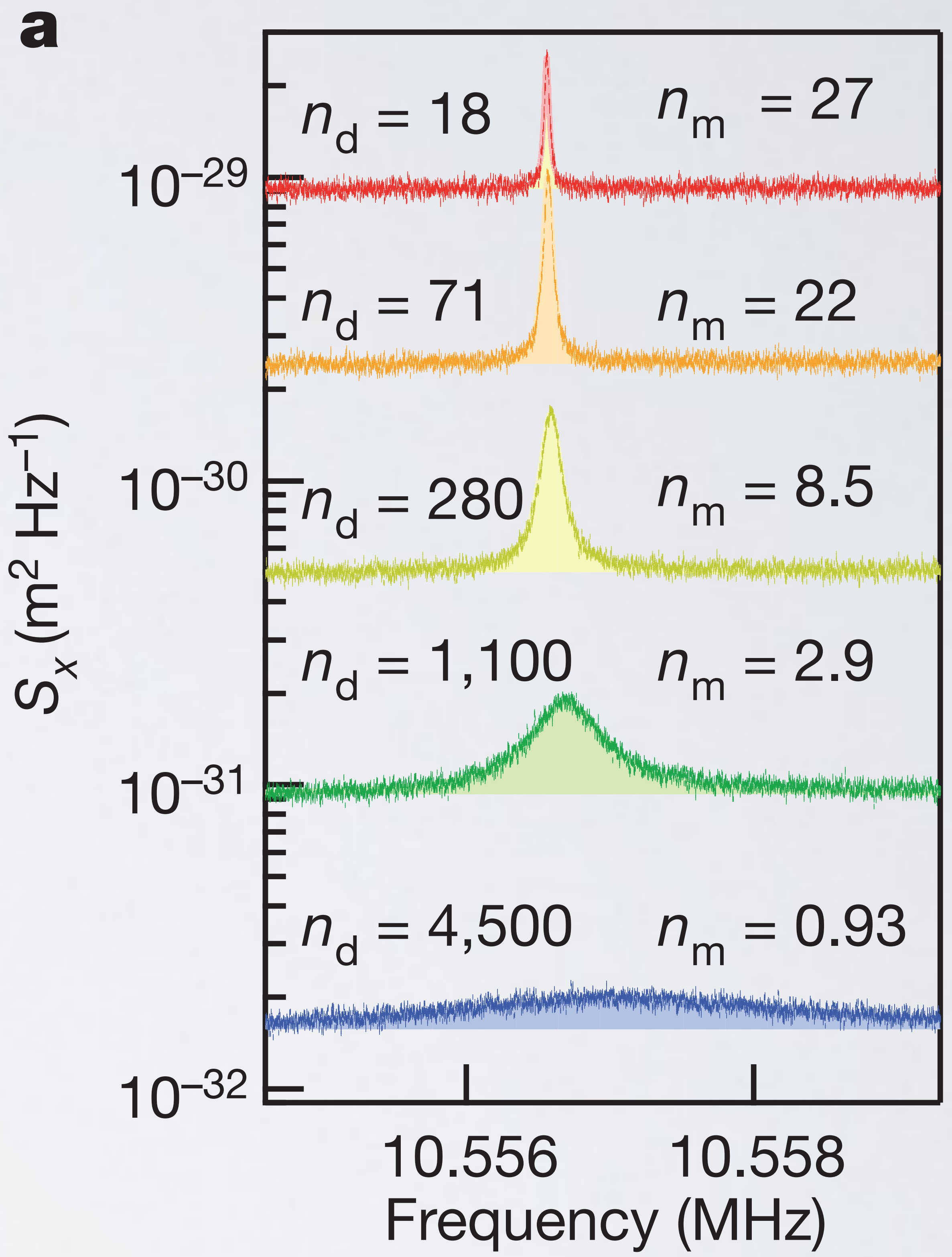


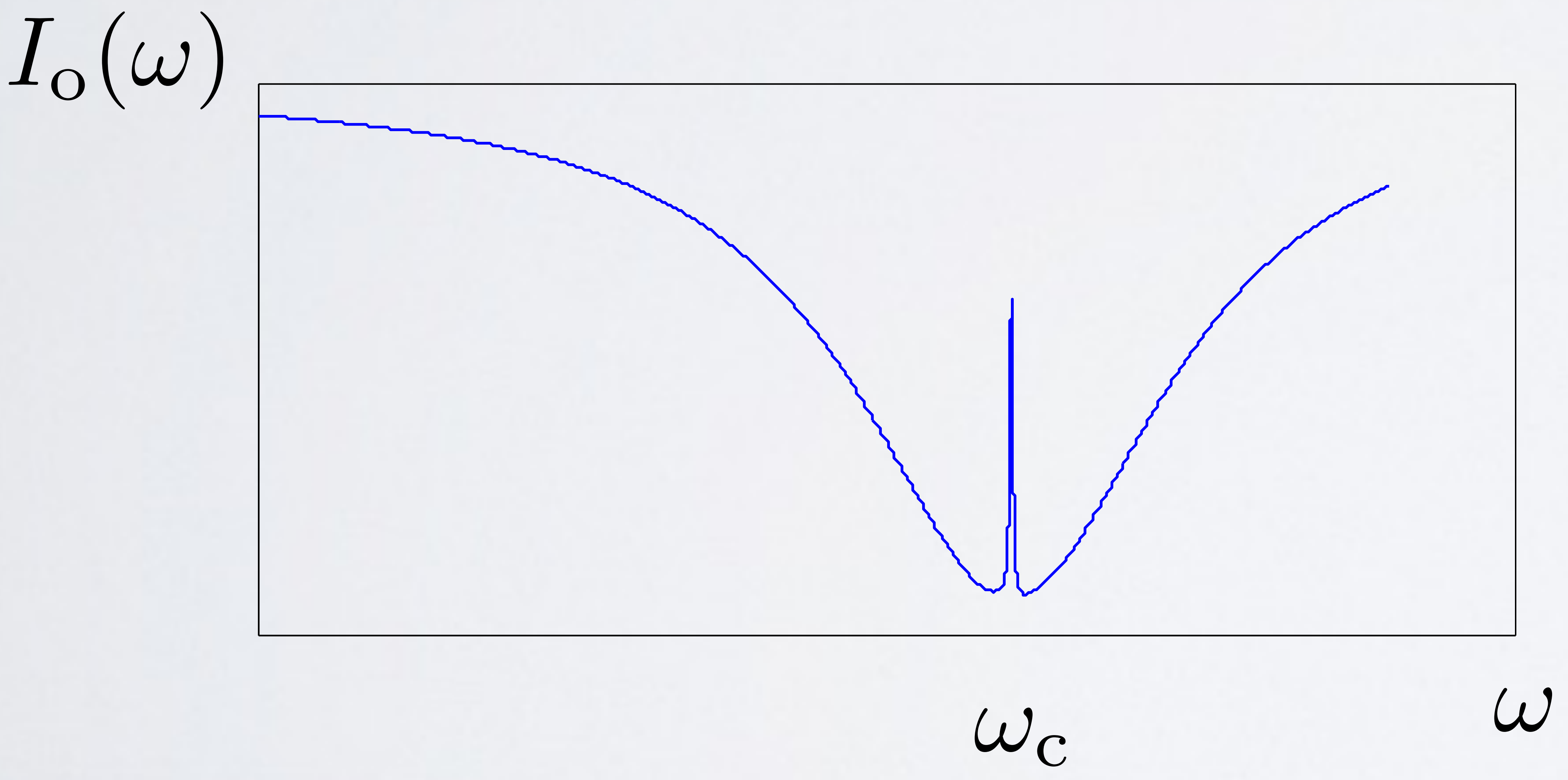
Teufel *et al.* Nature **475**, 359 (2011)

OPTOMECHANICAL SYSTEM

Sideband cooling



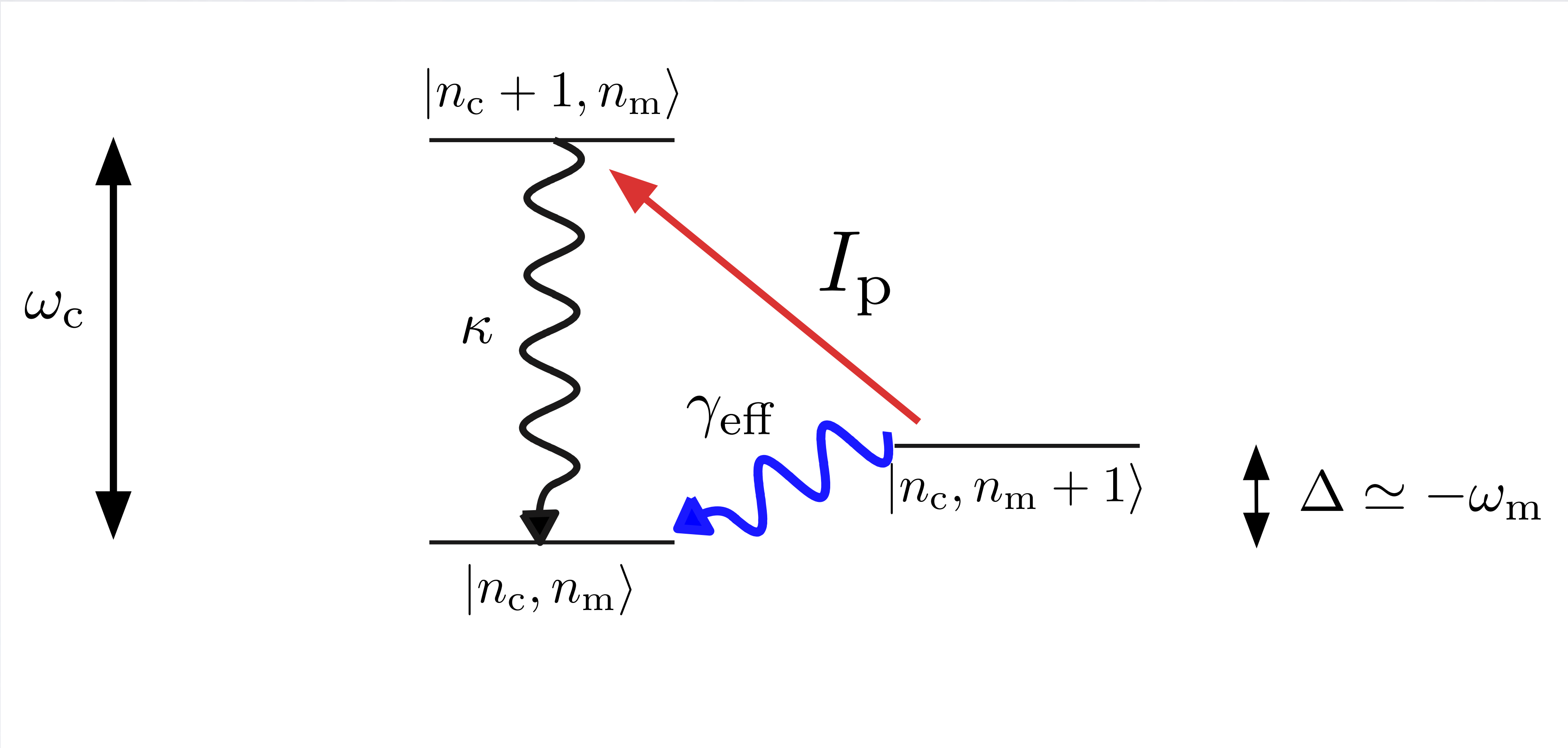


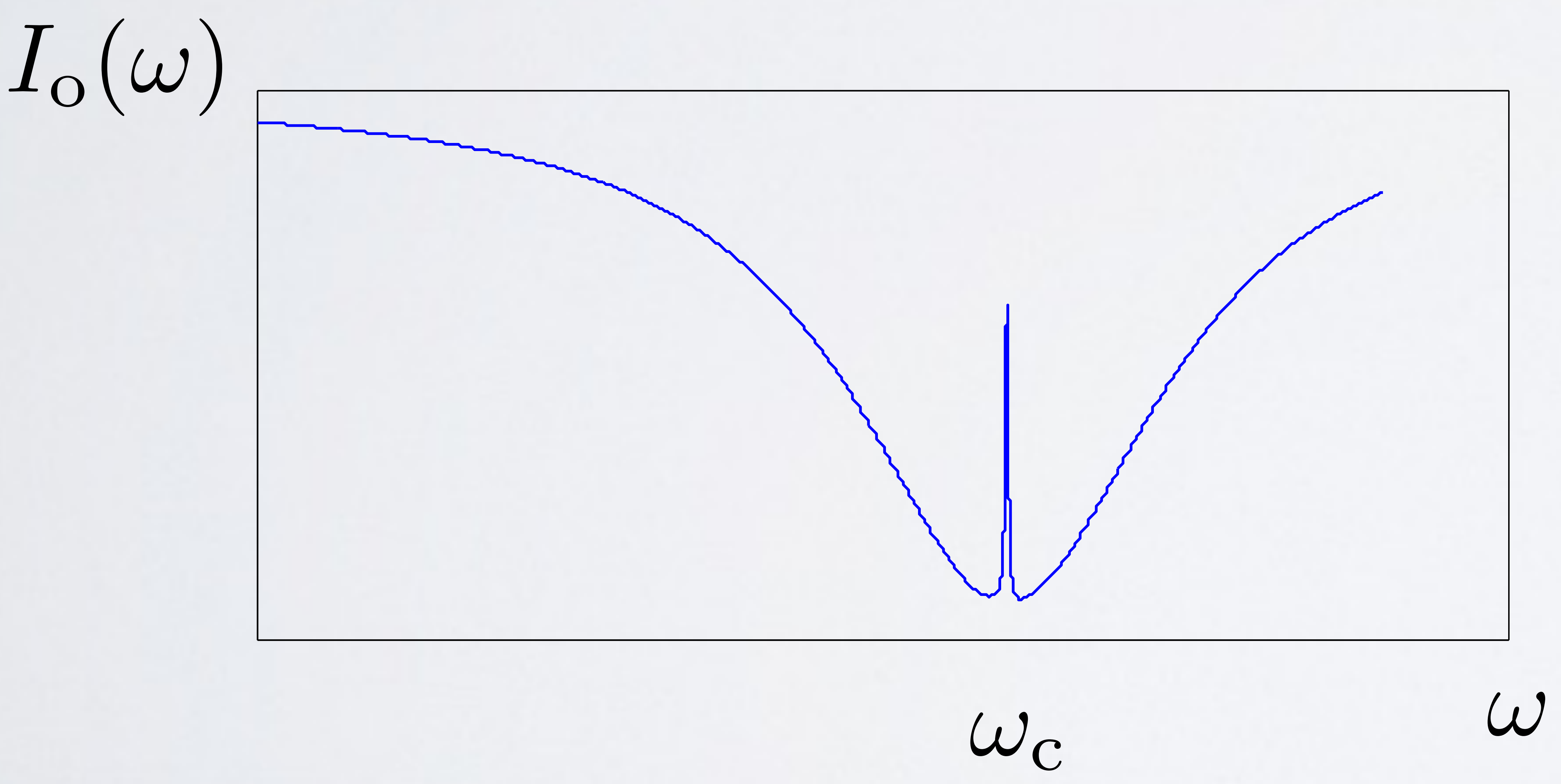


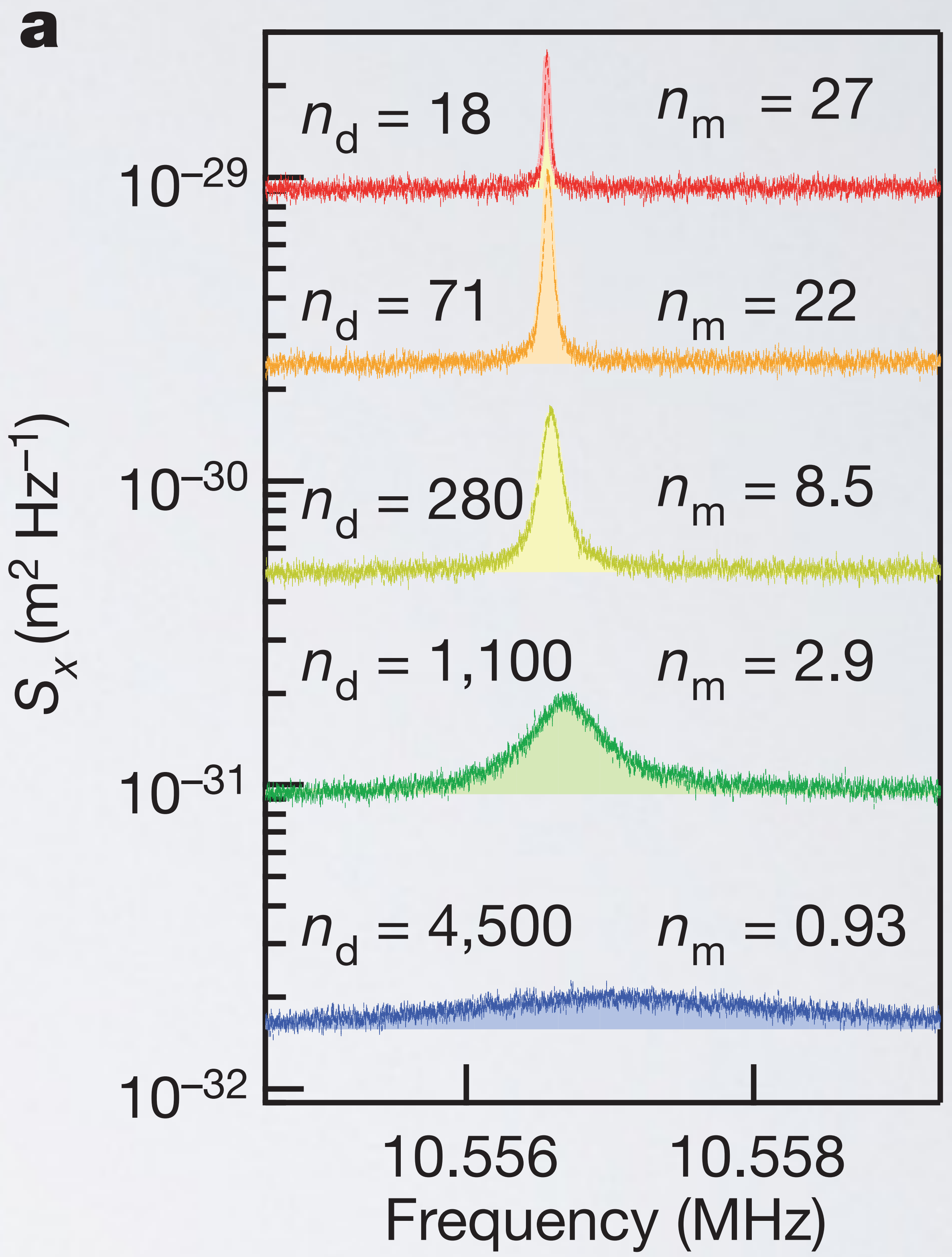
Teufel *et al.* Nature **475**, 359 (2011)

OPTOMECHANICAL SYSTEM

Sideband cooling



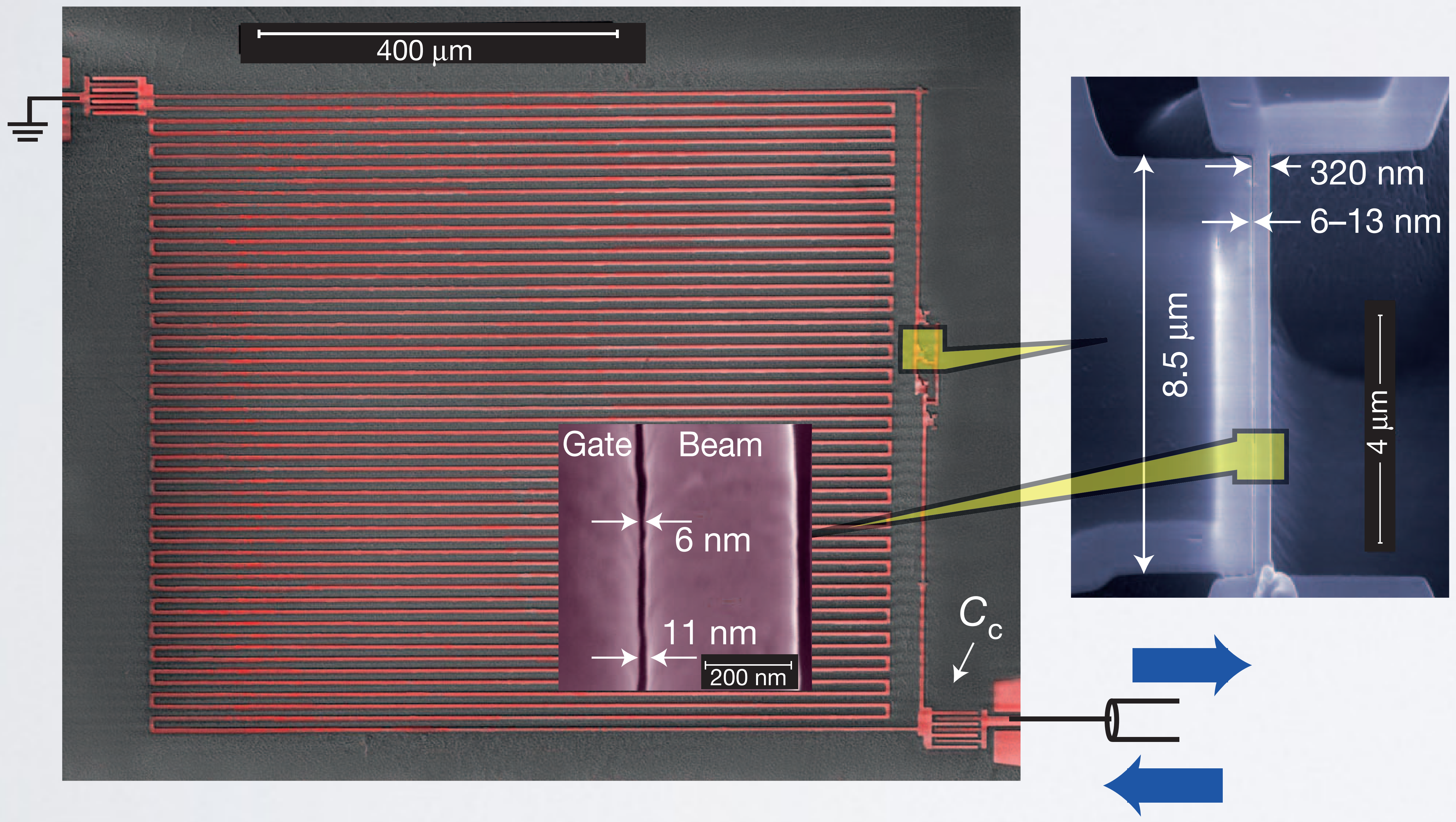




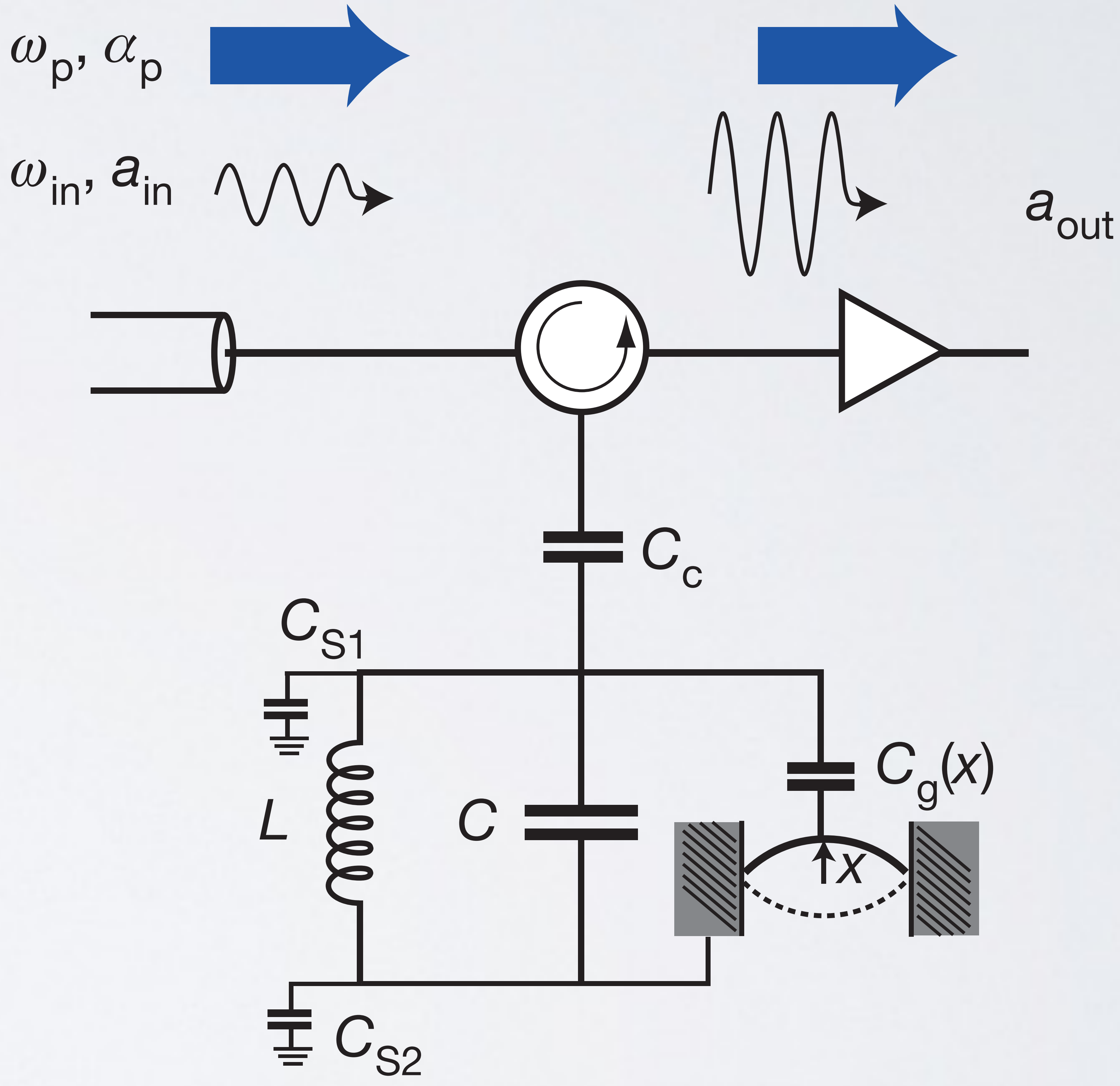
Teufel *et al.* Nature **475**, 359 (2011)

AMPLIFICATION

Meandering microwave strip



Lumped elements model

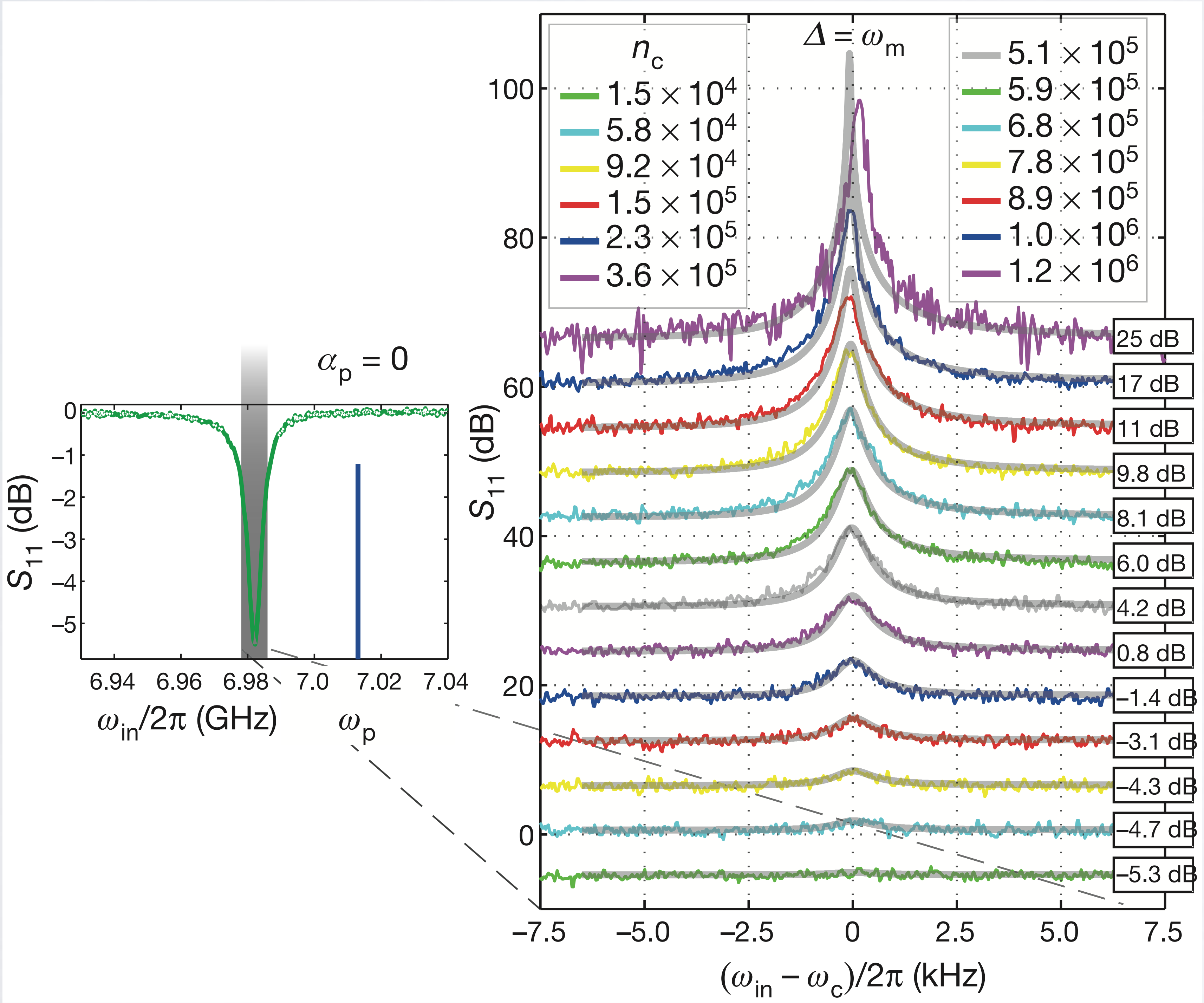


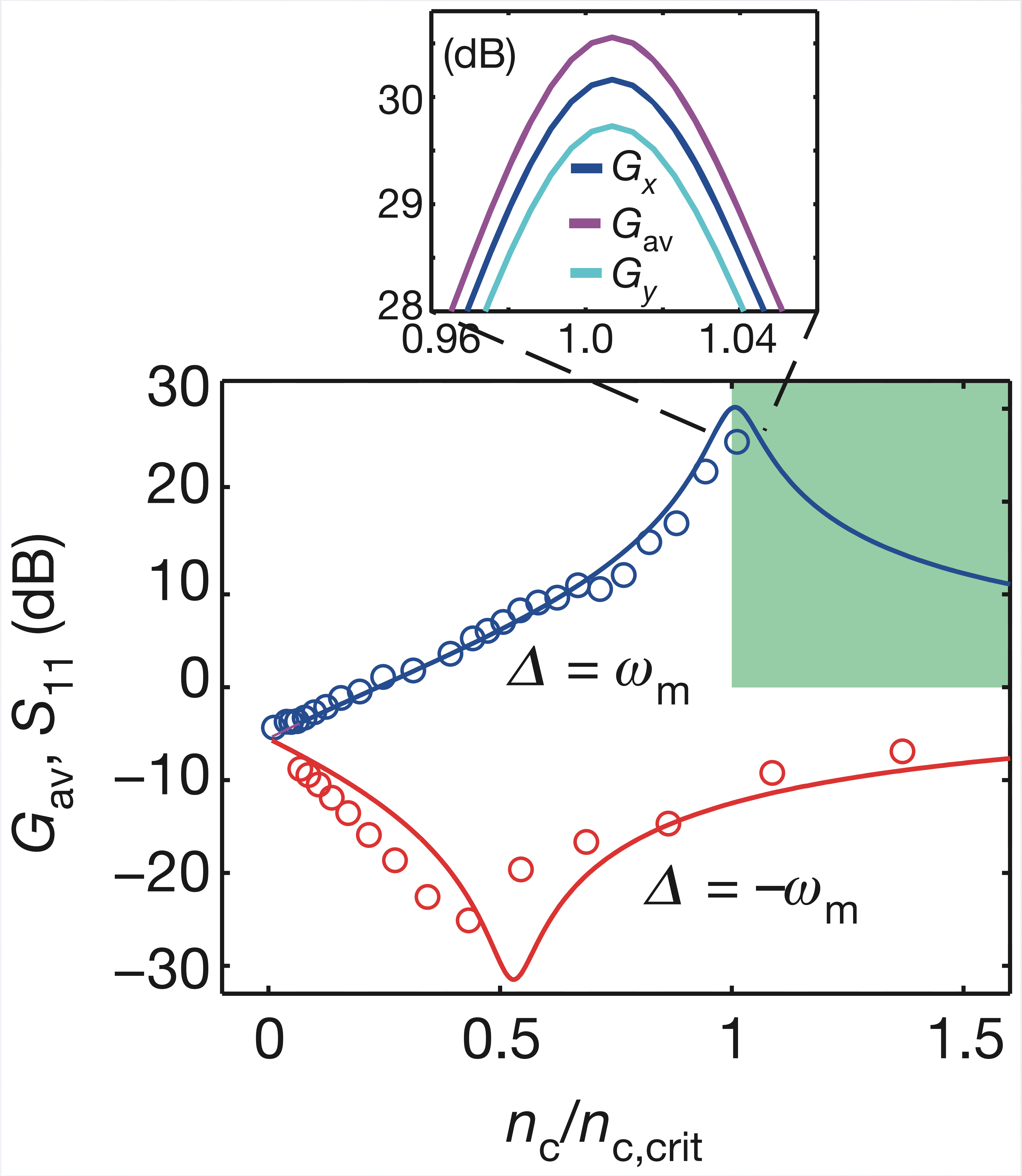
AMPLIFICATION

Signal

$$G_{\mathrm{av}} \propto \left(\frac{\gamma_{\mathrm{m}}}{\gamma_{\mathrm{eff}}}\right)^2 \quad \gamma_{\mathrm{eff}} = \gamma_{\mathrm{m}} \pm \Gamma I_{\mathrm{p}}$$

$$I_o = G_{\mathrm{av}} I_i$$

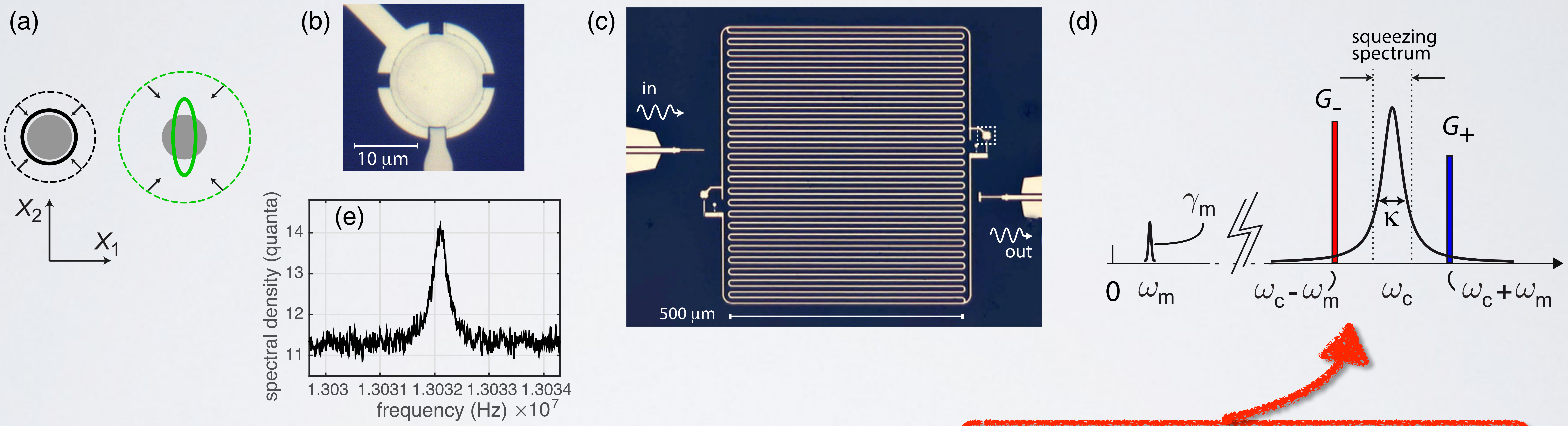




OPTOMECHANICAL SYSTEMS

Microwave domain

First example of squeezing below the SQL



Driving both with red and blue detuning

J. M. Pirkkalainen, *et al.* Phys. Rev. Lett. 115, 243601 (2015).

OPTOMECHANICAL SYSTEMS

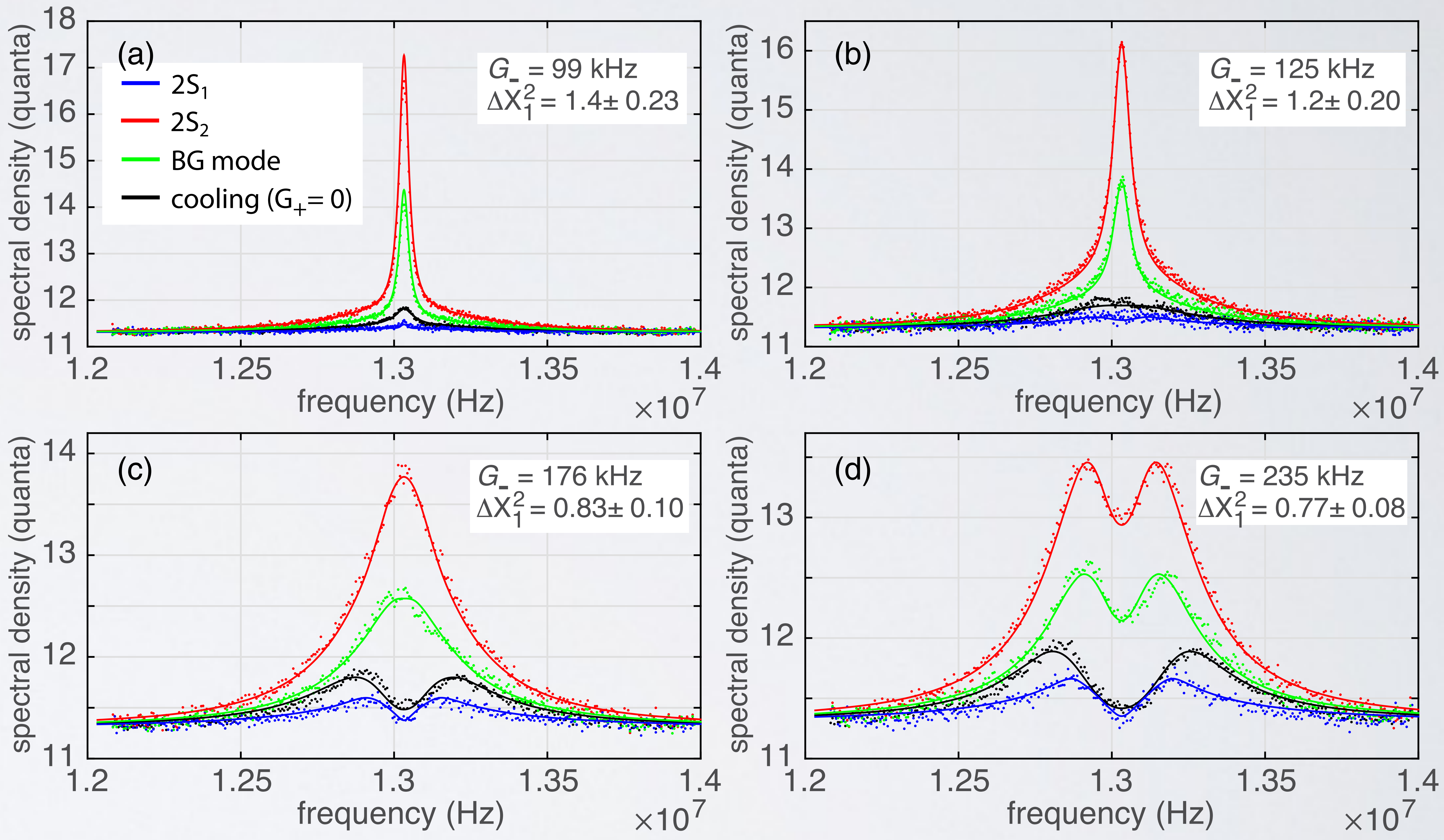
First example of squeezing below the SQL

Cooling of a Bogoliubov mode

$$\beta = ub + vb^\dagger$$

$$u = \frac{G_-}{\sqrt{G_-^2 - G_+^2}}$$

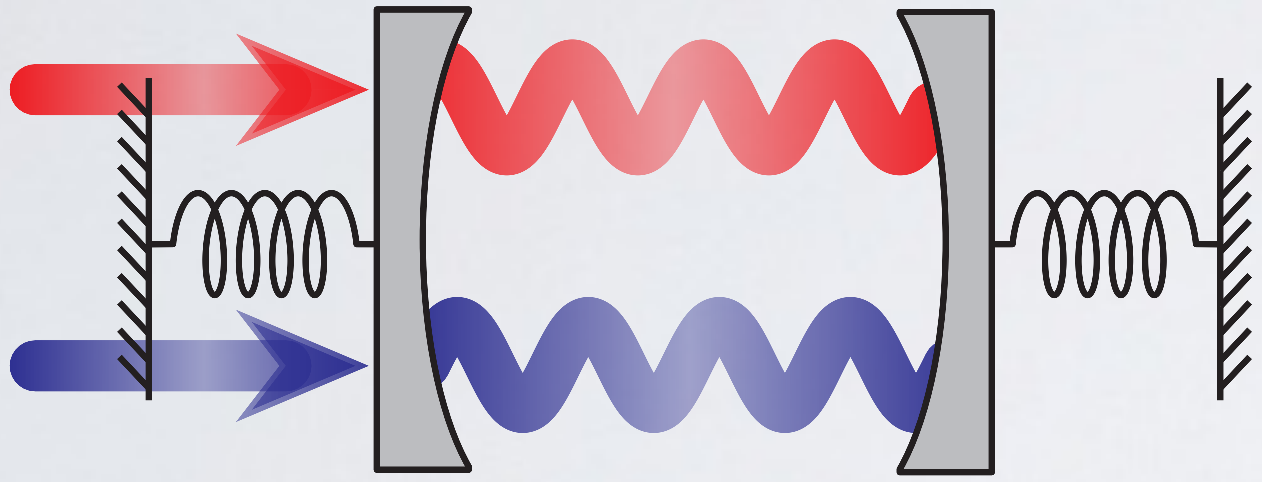
$$v = \frac{G_+}{\sqrt{G_-^2 - G_+^2}}$$



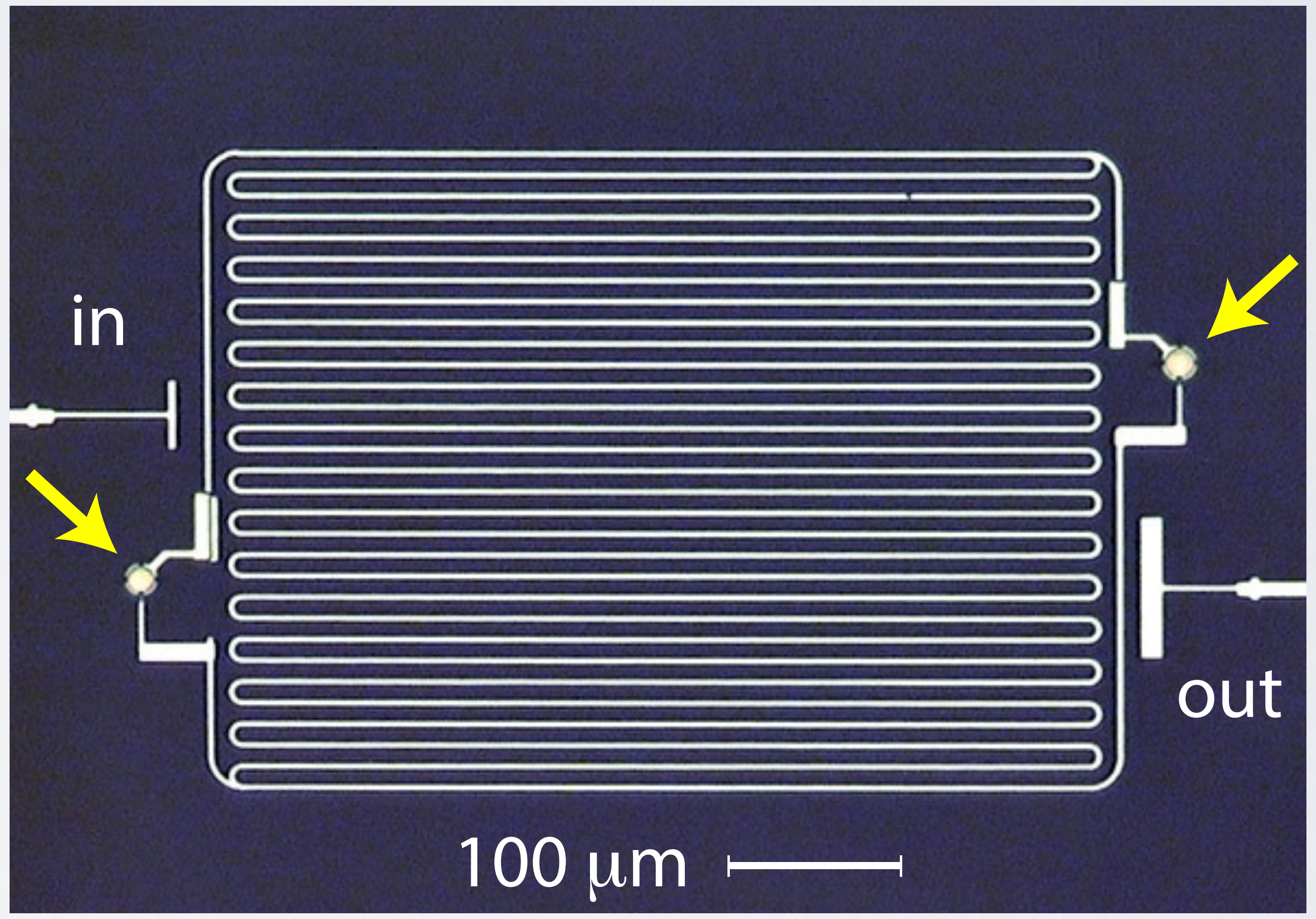
J. M. Pirkkalainen, *et al.* Phys. Rev. Lett. 115, 243601 (2015).

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS

Appropriately drive the cavity with two moveable mirrors



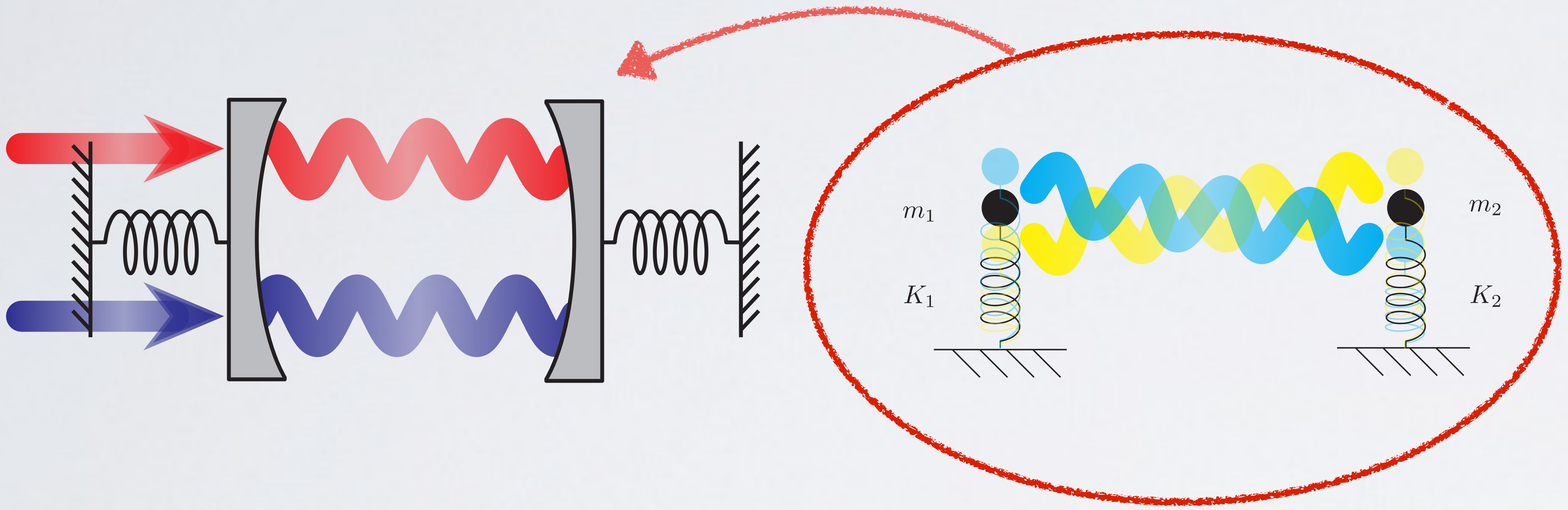
In the experimental setup, it's actually a microwave cavity with two compliant capacitors [1]



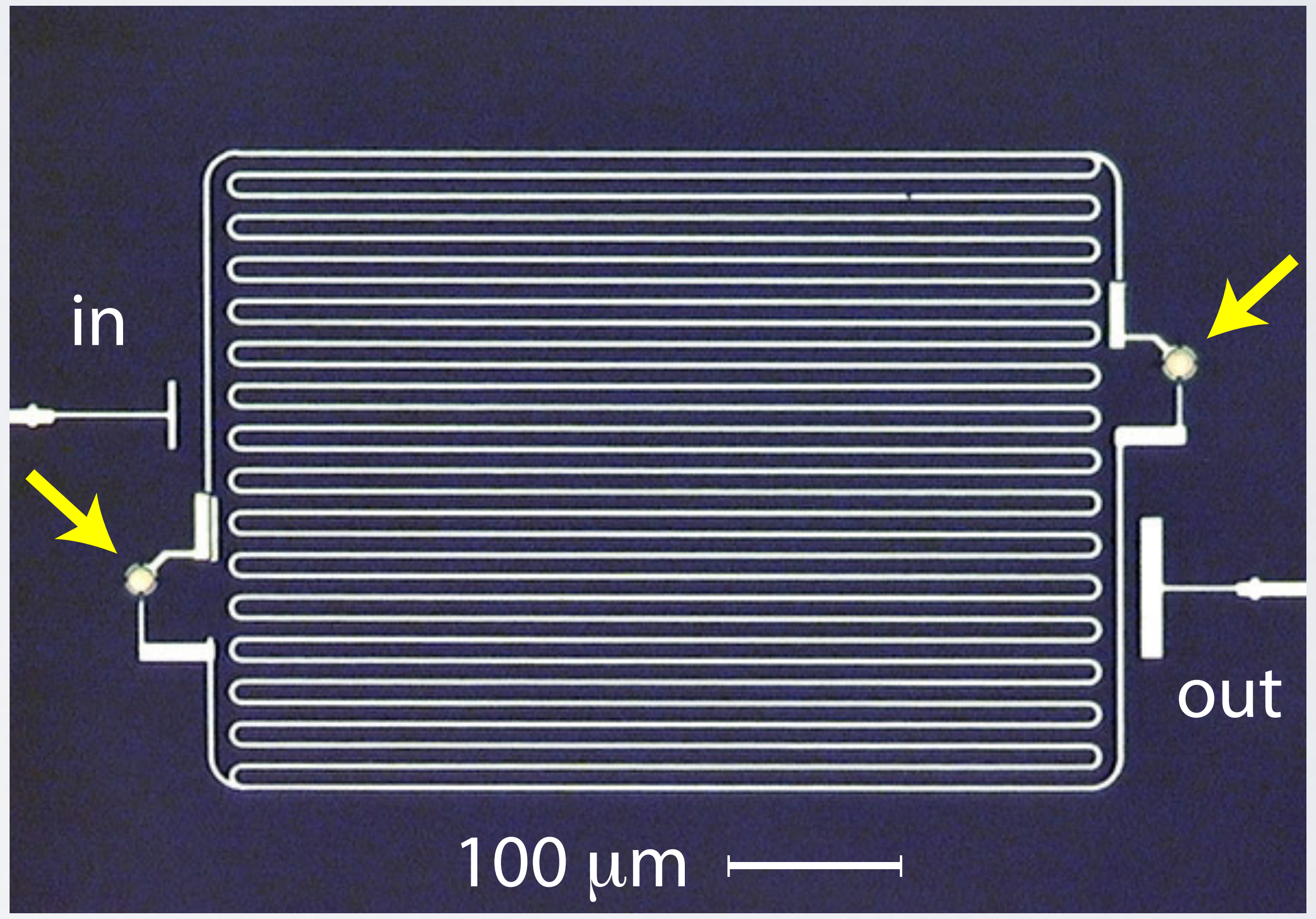
[1] C. F. Ockeloen-Korppi, *et al.* , Nature 556, 478 (2018).

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS

Appropriately drive the cavity with two moveable mirrors



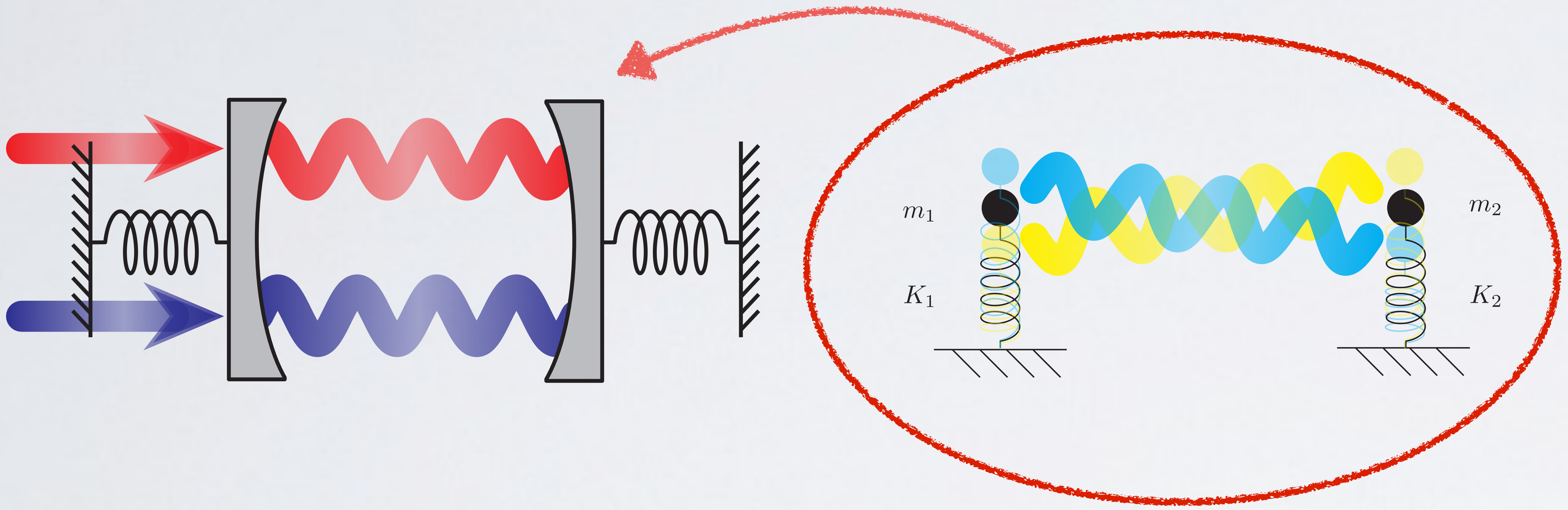
In the experimental setup, it's actually a microwave cavity with two compliant capacitors [1]



[1] C. F. Ockeloen-Korppi, *et al.* , Nature 556, 478 (2018).

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS

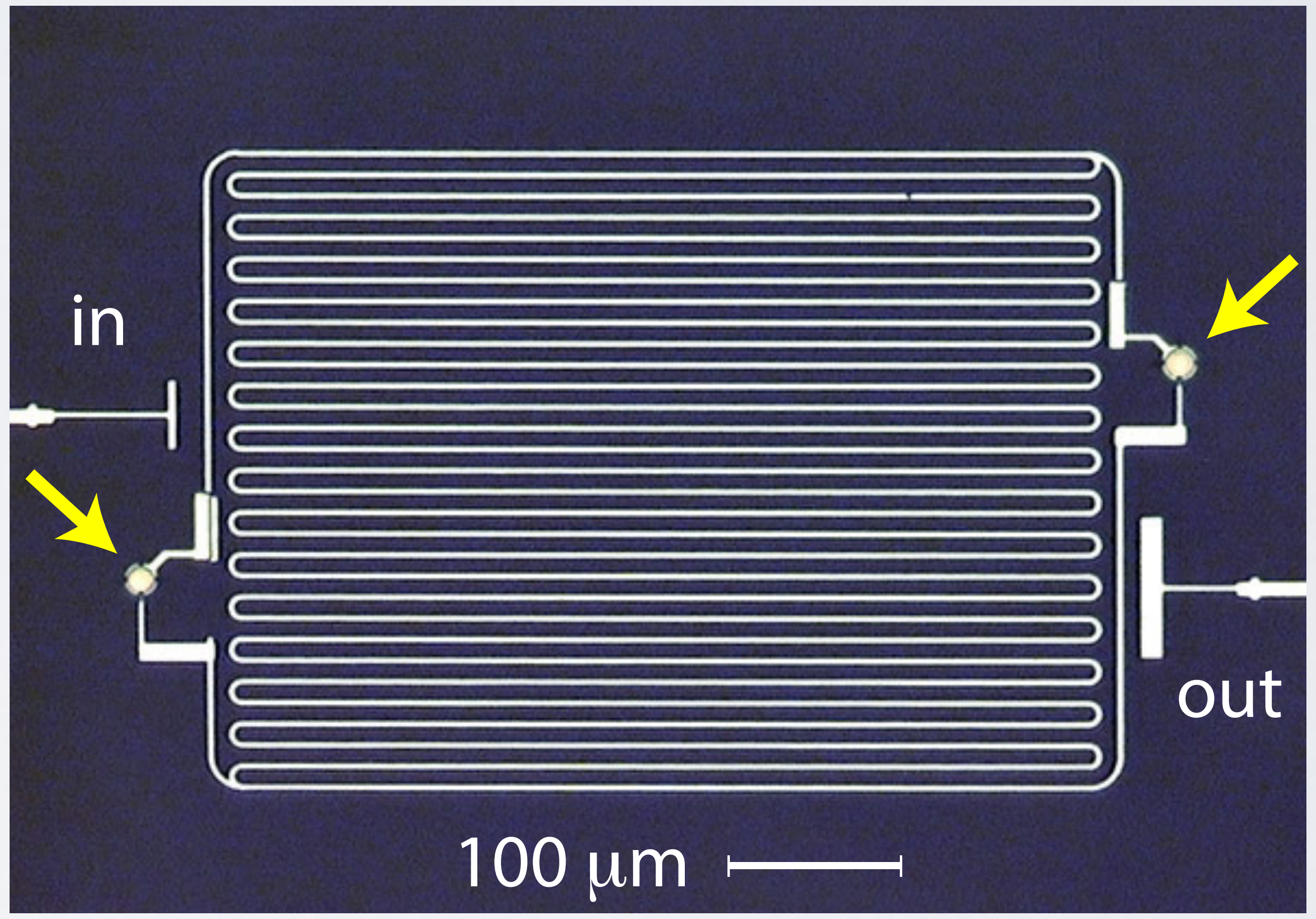
Appropriately drive the cavity with two moveable mirrors



radiation-pressure coupling

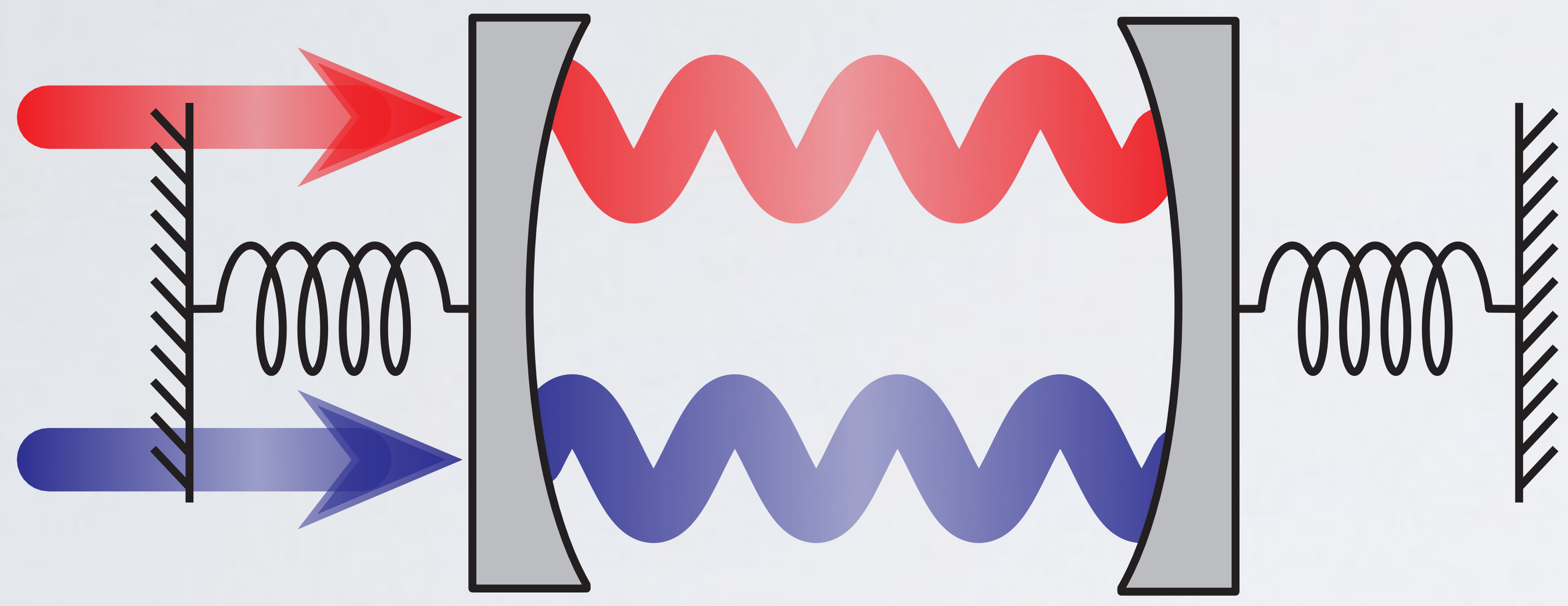
$$H = \omega_c a^\dagger a + g_1 \left(b_1 + b_1^\dagger\right) a^\dagger a + g_2 \left(b_2 + b_2^\dagger\right) a^\dagger a$$
$$+ \omega_1 b_1^\dagger b_1 + \omega_2 b_2^\dagger b_2 + H_{\rm drive}$$

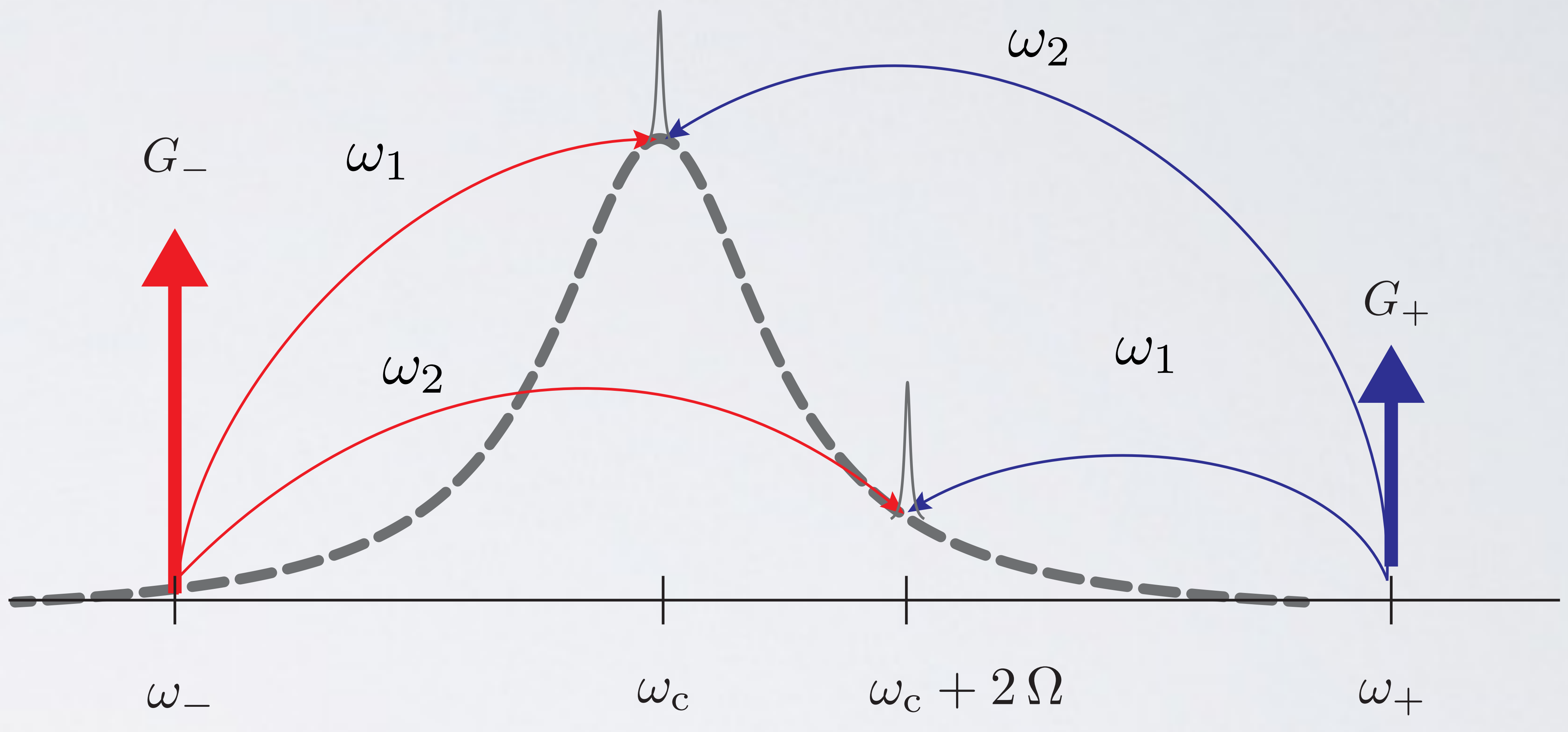
In the experimental setup, it's actually a microwave cavity with two compliant capacitors [1]



[1] C. F. Ockeloen-Korppi, *et al.*, Nature 556, 478 (2018).

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS





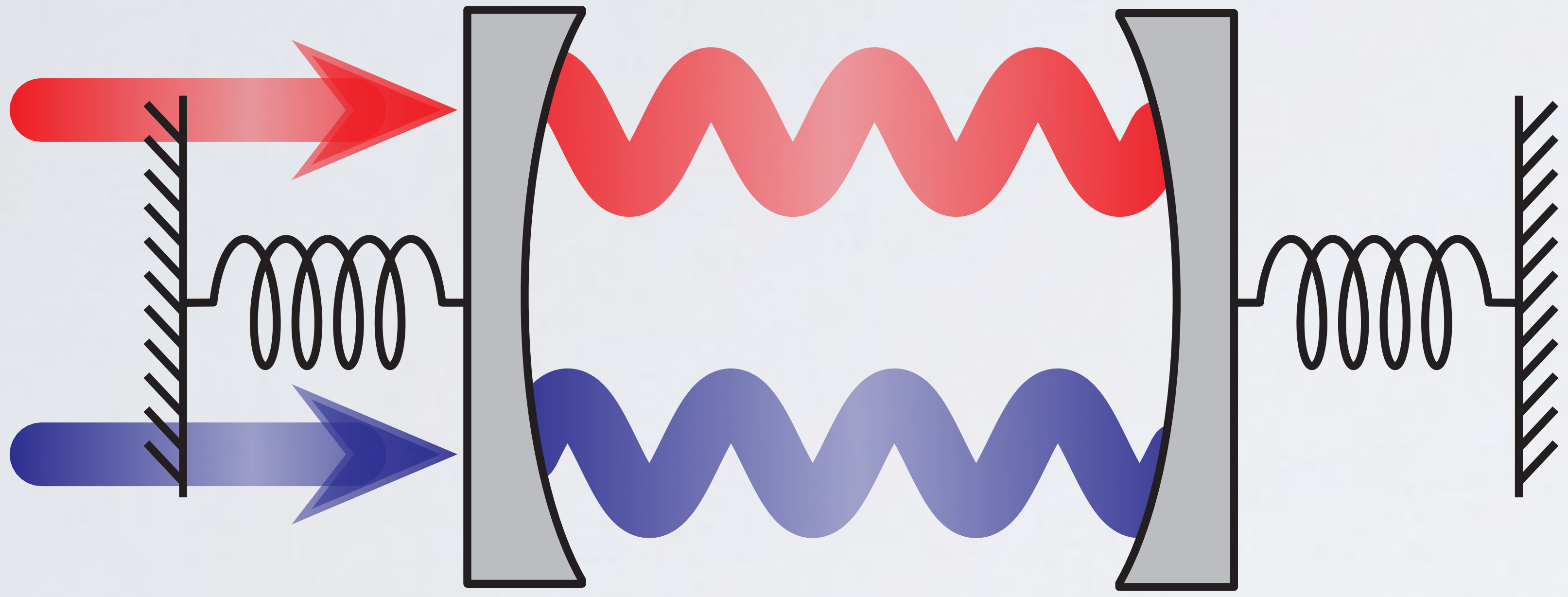
linearising around the driving tone (+ rotating frame, RWA)

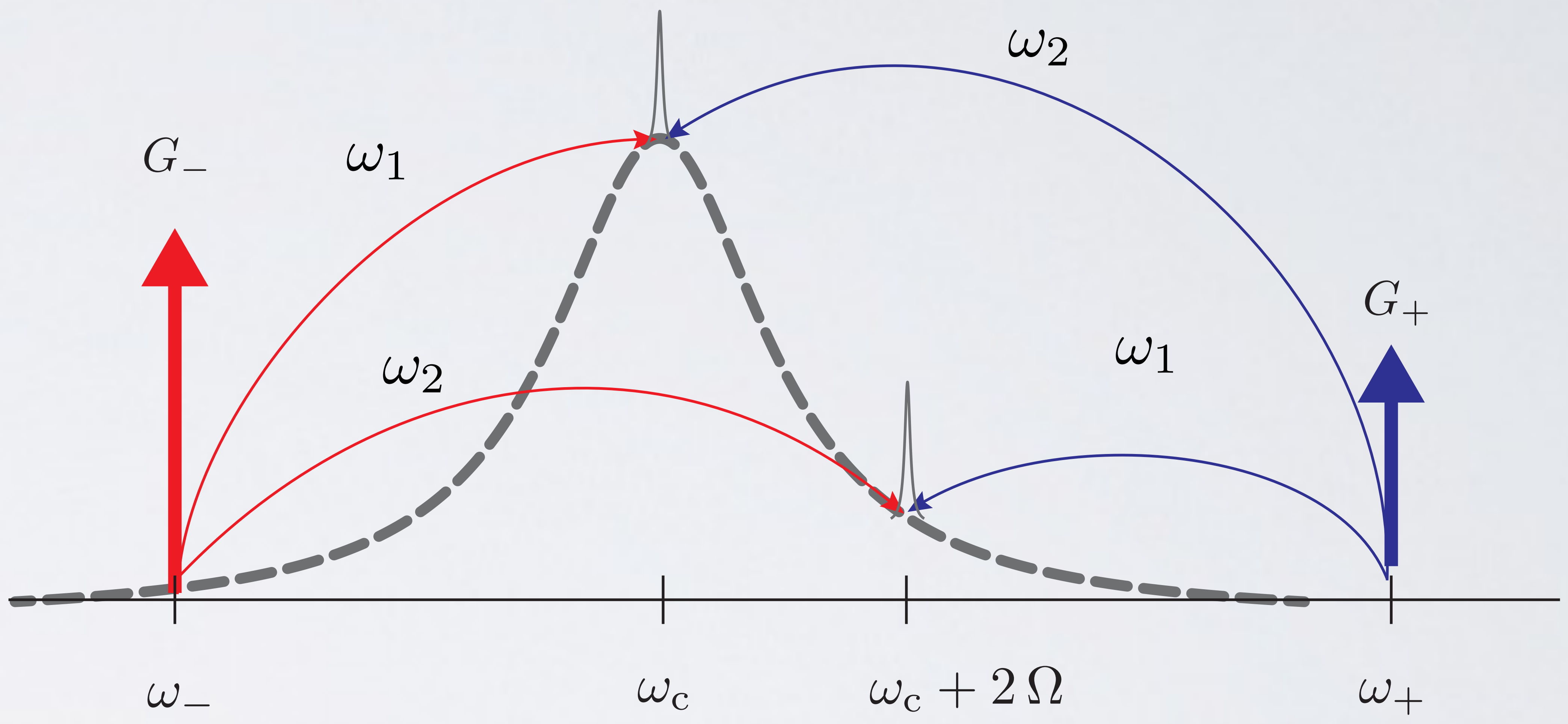
$$H = \omega_c a^\dagger a + g_1 \left(b_1 + b_1^\dagger\right) a^\dagger a + g_2 \left(b_2 + b_2^\dagger\right) a^\dagger a$$
$$+ \omega_1 b_1^\dagger b_1 + \omega_2 b_2^\dagger b_2 + H_{\text{drive}}$$

$$H_{\text{drive}} = \left(\mathcal{E}_+^* e^{i\omega_+ t} + \mathcal{E}_-^* e^{i\omega_- t}\right) a + \text{h.c.}$$

$$H_I = -\Omega\, a^\dagger a + \Omega \left(b_2^\dagger b_2 - b_1^\dagger b_1\right)$$
$$+ G_- a^\dagger \left(b_1 + b_2\right) + G_+ a^\dagger \left(b_1^\dagger + b_2^\dagger\right) + \text{h.c.}$$

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS





$$H = \omega_\mathrm{c} a^\dagger a + g_1 \left(b_1 + b_1^\dagger\right) a^\dagger a + g_2 \left(b_2 + b_2^\dagger\right) a^\dagger a$$
$$+ \omega_1 b_1^\dagger b_1 + \omega_2 b_2^\dagger b_2 + H_\mathrm{drive}$$

$$H_\mathrm{drive} = \left(\mathcal{E}_+^* e^{i\omega_+ t} + \mathcal{E}_-^* e^{i\omega_- t}\right) a + \mathrm{h.c.}$$

linearising around the driving tone (+ rotating frame, RWA)

$$H_\mathrm{I} = -\Omega\, a^\dagger a + \Omega \left(b_2^\dagger b_2 - b_1^\dagger b_1\right)$$
$$+ G_- a^\dagger \left(b_1 + b_2\right) + G_+ a^\dagger \left(b_1^\dagger + b_2^\dagger\right) + \mathrm{h.c.}$$

$G_\pm \propto \mathcal{E}_\pm$

$\Omega = \left(\omega_2 - \omega_1\right)/2$

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS

$$H_{\mathrm{I}} = -\Omega\, a^\dagger a + \Omega\left(b_2^\dagger b_2 - b_1^\dagger b_1\right) + G_- a^\dagger \left(b_1 + b_2\right) + G_+ a^\dagger \left(b_1^\dagger + b_2^\dagger\right) + \mathrm{h.c.}$$

$\longrightarrow$

$$H_{\mathrm{I}} = -\Omega\, a^\dagger a + \Omega\left(\beta_2^\dagger \beta_2 - \beta_1^\dagger \beta_1\right) + \mathcal{G}\left[a^\dagger \left(\beta_1 + \beta_2\right) + a\left(\beta_1^\dagger + \beta_2^\dagger\right)\right]$$

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS

$$H_{\mathrm{I}} = -\Omega\, a^\dagger a + \Omega \left(b_2^\dagger b_2 - b_1^\dagger b_1 \right) + G_- a^\dagger \left(b_1 + b_2 \right) + G_+ a^\dagger \left(b_1^\dagger + b_2^\dagger \right) + \mathrm{h.c.}$$

$$H_{\mathrm{I}} = -\Omega\, a^\dagger a + \Omega \left(\beta_2^\dagger \beta_2 - \beta_1^\dagger \beta_1 \right) + \mathcal{G} \left[a^\dagger \left(\beta_1 + \beta_2 \right) + a \left(\beta_1^\dagger + \beta_2^\dagger \right) \right]$$

$$\beta_1 = b_1 \cosh r + b_2^\dagger \sinh r$$

$$\beta_2 = b_2 \cosh r + b_1^\dagger \sinh r$$

$$\tanh r = G_- / G_+$$

$$\mathcal{G} = \sqrt{G_-^2 - G_+^2}$$

VIOLATION OF THE DUAN BOUND IN OPTOMECHANICS

$$H_{\mathrm{I}} = -\Omega\, a^\dagger a + \Omega\left(b_2^\dagger b_2 - b_1^\dagger b_1\right) + G_- a^\dagger\left(b_1 + b_2\right) + G_+ a^\dagger\left(b_1^\dagger + b_2^\dagger\right) + \mathrm{h.c.}$$

$$H_{\mathrm{I}} = -\Omega\, a^\dagger a + \Omega\left(\beta_2^\dagger \beta_2 - \beta_1^\dagger \beta_1\right) + \mathcal{G}\left[a^\dagger\left(\beta_1 + \beta_2\right) + a\left(\beta_1^\dagger + \beta_2^\dagger\right)\right]$$

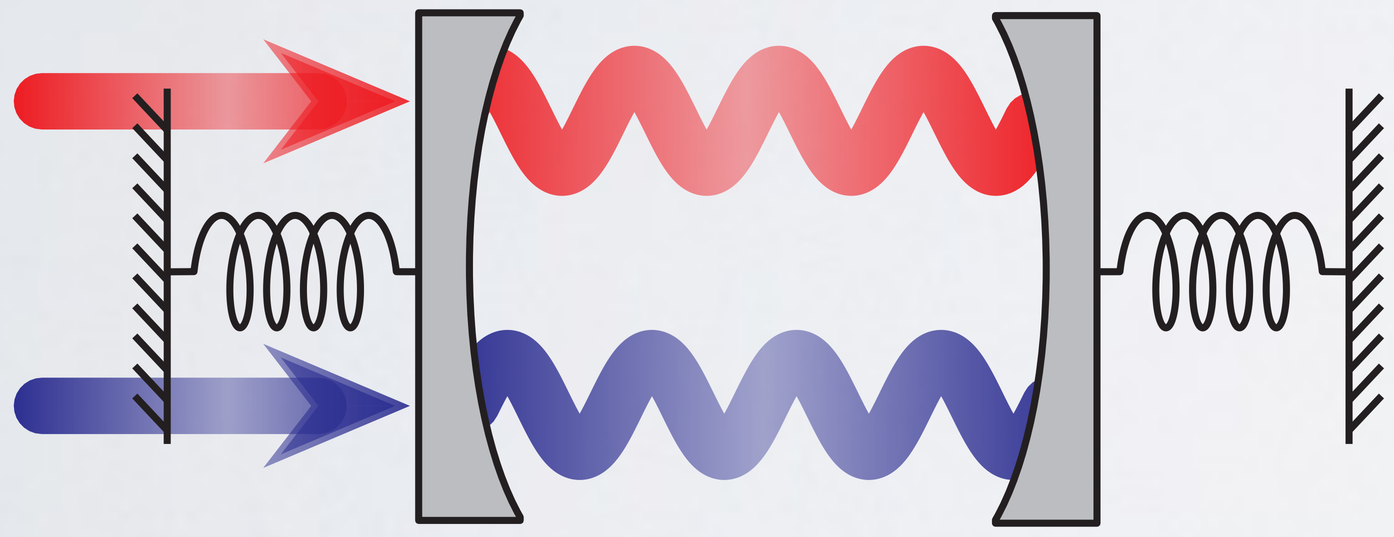
$$\beta_1 = b_1 \cosh r + b_2^\dagger \sinh r$$

$$\beta_2 = b_2 \cosh r + b_1^\dagger \sinh r$$

$$\tanh r = G_-/G_+$$

$$\mathcal{G} = \sqrt{G_-^2 - G_+^2}$$

2-mode squeezing



Cooling of the sum of Bogolyubov modes optically, sympathetic cooling of the difference of Bogolyubov modes.

2-MODE SQUEEZED STATES & ENTANGLEMENT

If a system is in a 2-mode squeezed (vacuum) state, then the 2 modes are entangled.

Squeezed vacuum
(vacuum for B. modes)

$$S_2(r) = \exp\left[r(b_1\, b_2 - b_1^\dagger\, b_2^\dagger)\right]$$

$$|\sigma_2\rangle = S_2(r)\,|0\rangle$$

$$\tanh r = G_- / G_+$$

2-MODE SQUEEZED STATES & ENTANGLEMENT

If a system is in a 2-mode squeezed (vacuum) state, then the 2 modes are entangled.

Squeezed vacuum
(vacuum for B. modes)

$$S_2(r) = \exp\left[r(b_1\, b_2 - b_1^\dagger\, b_2^\dagger)\right]$$

$$|\sigma_2\rangle = S_2(r)\,|0\rangle$$

$$\tanh r = G_-/G_+$$

Variances on the squeezed vacuum

$$\langle\sigma_2|\,\Delta X_\Sigma^2\,|\sigma_2\rangle = \langle\sigma_2|\,\Delta P_\Delta^2\,|\sigma_2\rangle = e^{-2r}$$

$$\langle\Delta X_\Sigma^2\rangle + \langle\Delta P_\Delta^2\rangle = 2e^{-2r}$$

2-MODE SQUEEZED STATES & ENTANGLEMENT

If a system is in a 2-mode squeezed (vacuum) state, then the 2 modes are entangled.

Squeezed vacuum (vacuum for B. modes)

$$S_2(r) = \exp\left[r(b_1\, b_2 - b_1^\dagger\, b_2^\dagger)\right]$$

$$|\sigma_2\rangle = S_2(r)\,|0\rangle$$

$$\tanh r = G_-/G_+$$

→

Variances on the squeezed vacuum

$$\langle\sigma_2|\,\Delta X_\Sigma^2\,|\sigma_2\rangle = \langle\sigma_2|\,\Delta P_\Delta^2\,|\sigma_2\rangle = e^{-2r}$$

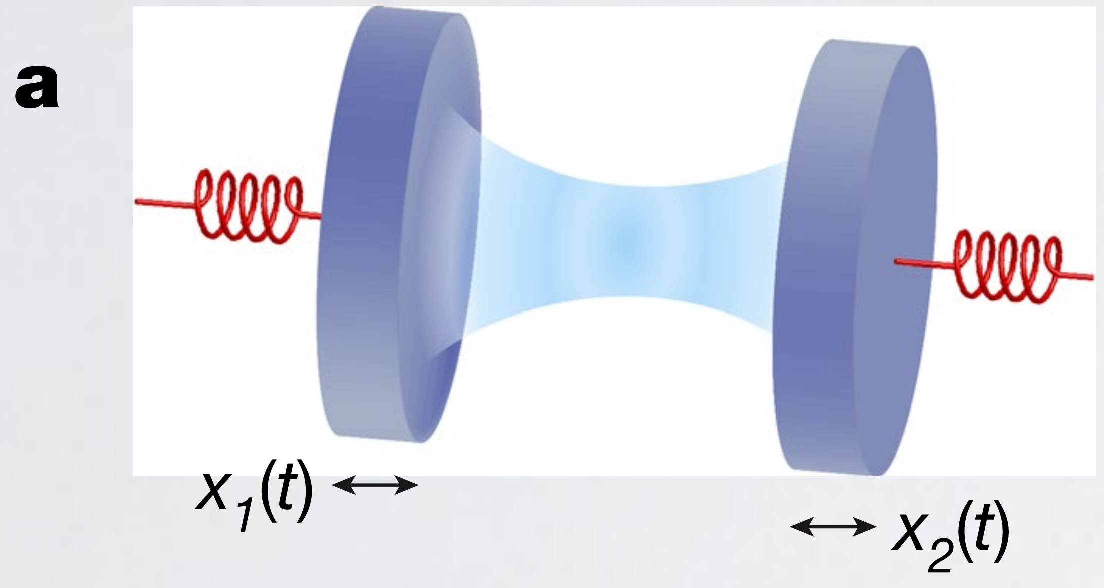
$$\langle\Delta X_\Sigma^2\rangle + \langle\Delta P_\Delta^2\rangle = 2e^{-2r}$$

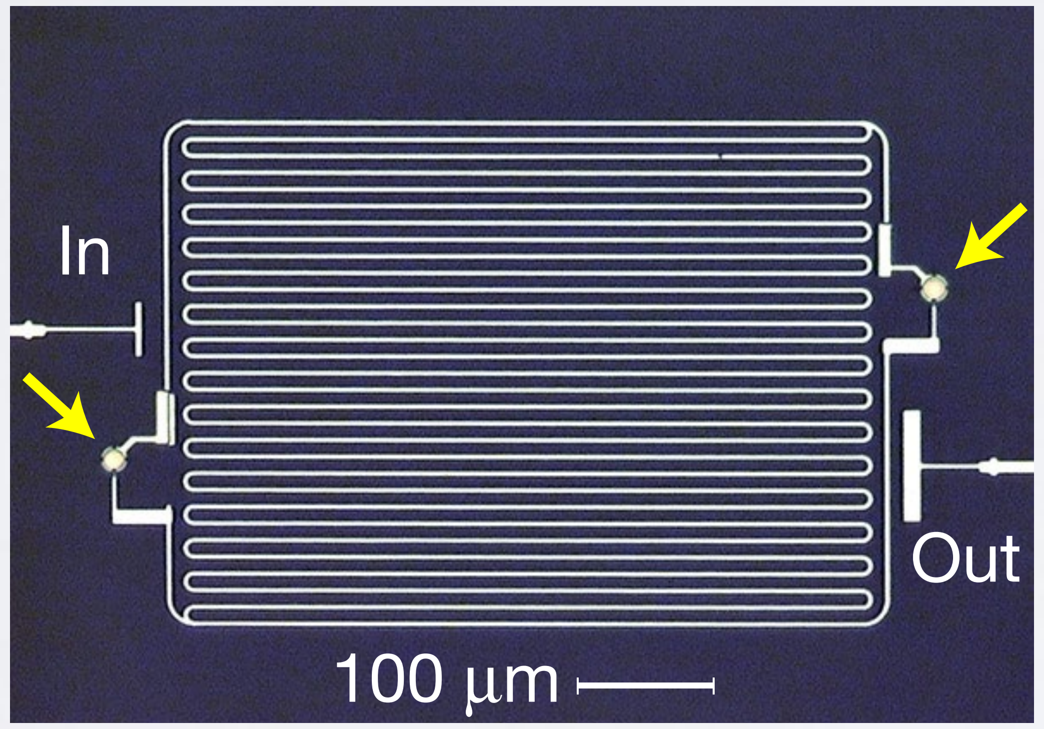
Possible violation of the Duan bound

OPTOMECHANICAL SYSTEMS

Microwave domain

First example of stationary entanglement between mechanical resonators



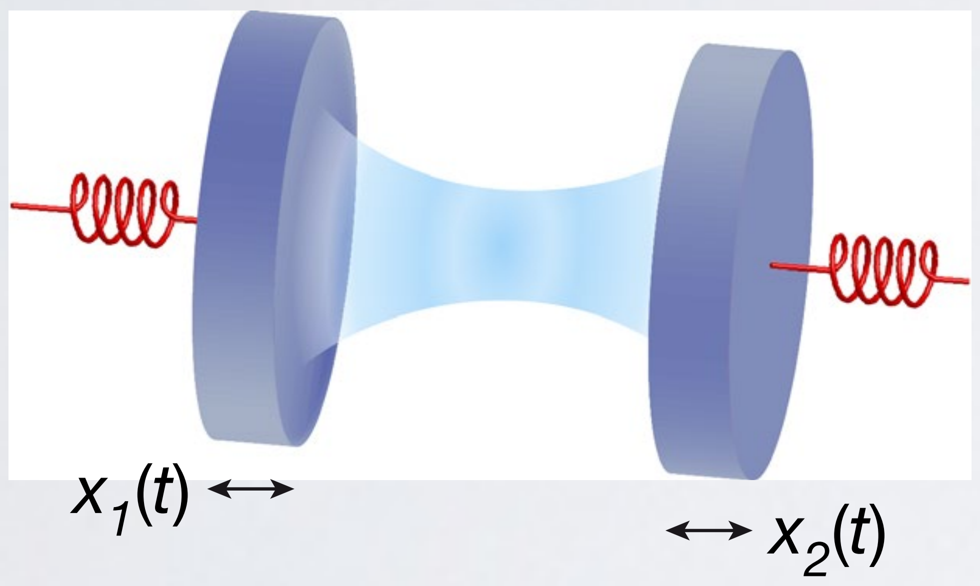


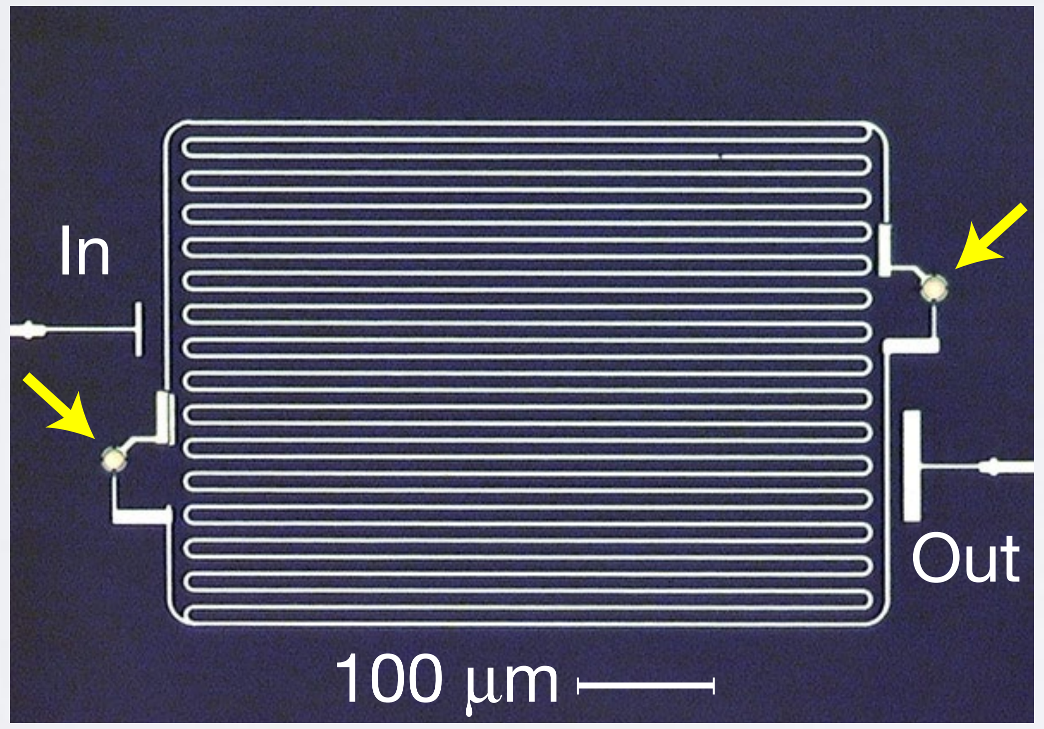
[1] C. F. Ockeloen-Korppi, *et al.* , Nature 556, 478 (2018).

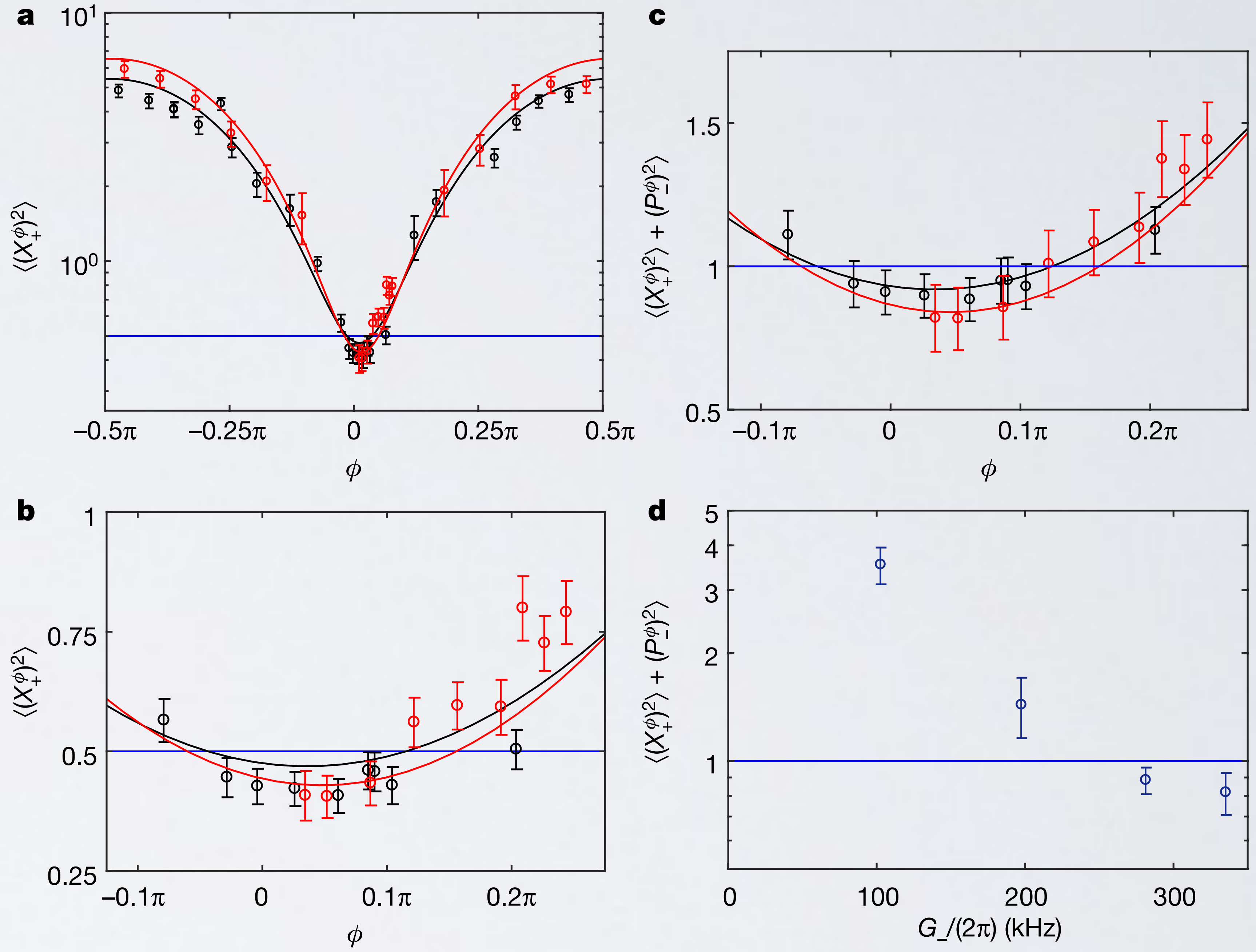
OPTOMECHANICAL SYSTEMS

Microwave domain

First example of stationary entanglement between mechanical resonators





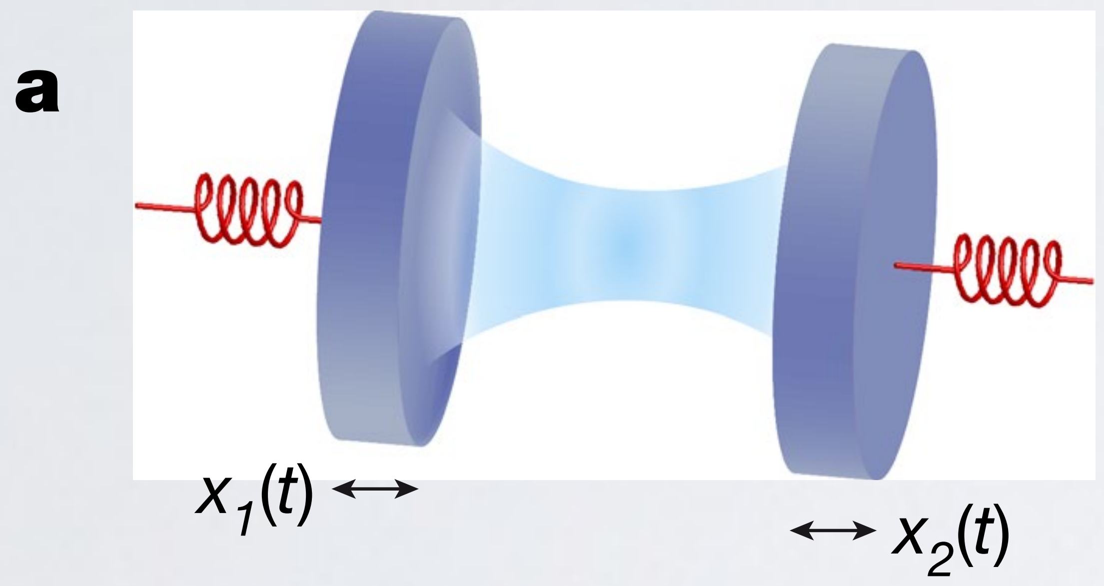


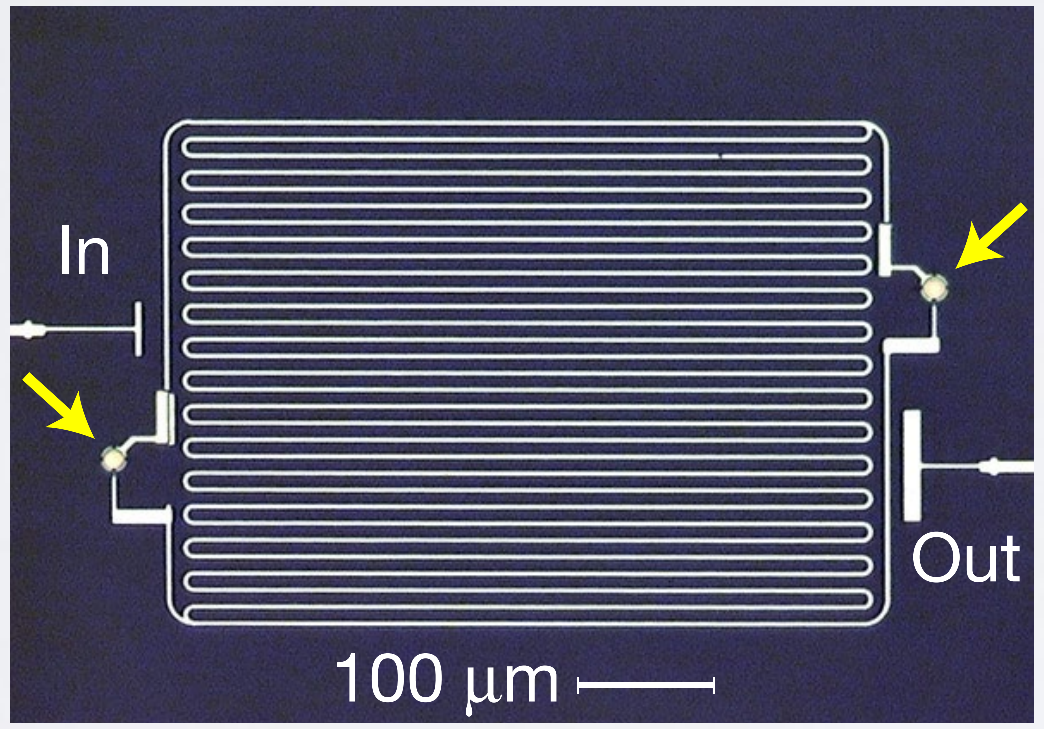
[1] C. F. Ockeloen-Korppi, *et al.* , Nature 556, 478 (2018).

OPTOMECHANICAL SYSTEMS

Microwave domain

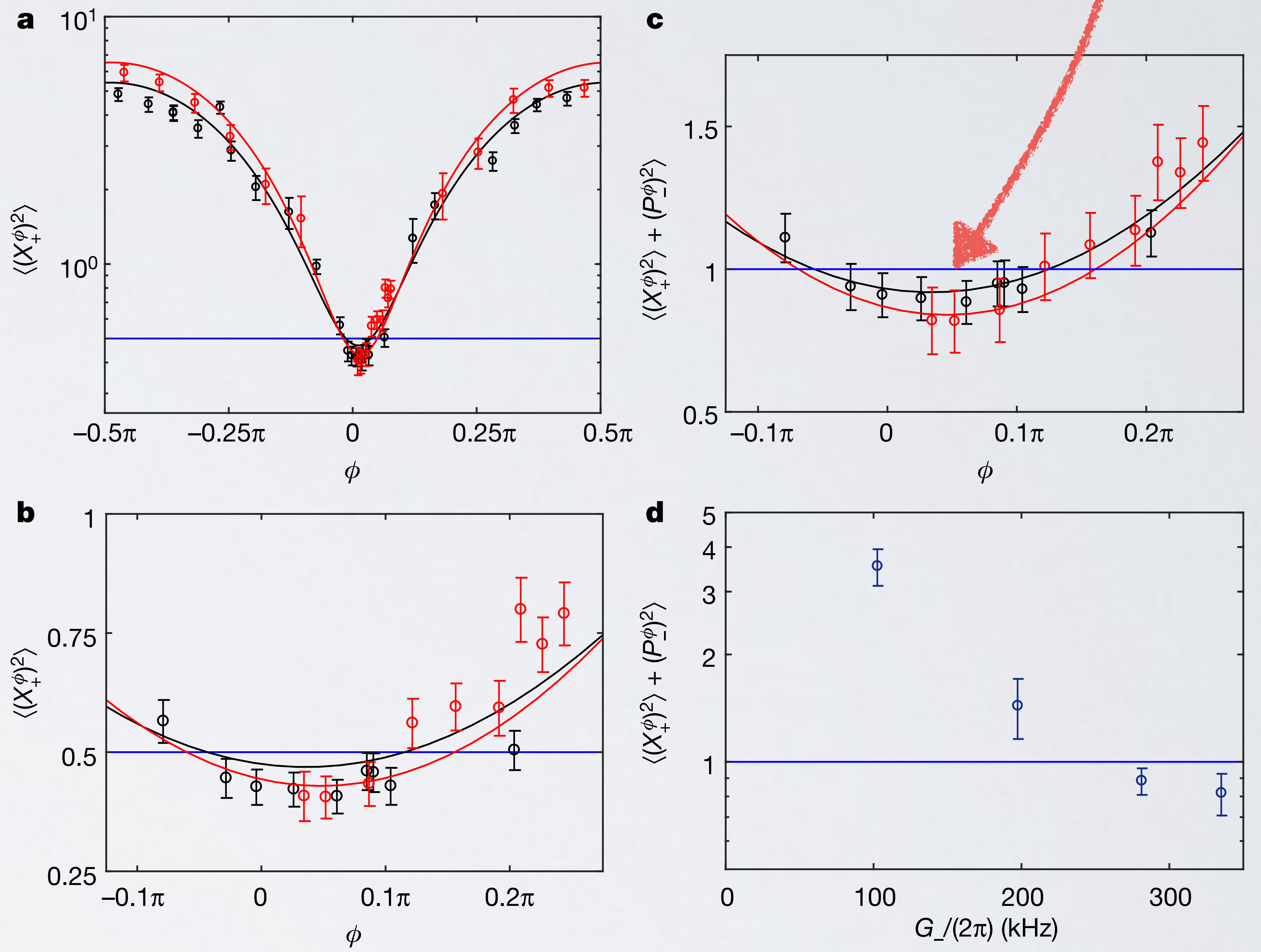
First example of stationary entanglement between mechanical resonators





$$\langle \Delta X_\Sigma^2 \rangle_{\rho_s} + \langle \Delta P_\Delta^2 \rangle_{\rho_s} \geq 1$$

entangled!



[1] C. F. Ockeloen-Korppi, *et al.*, Nature 556, 478 (2018).

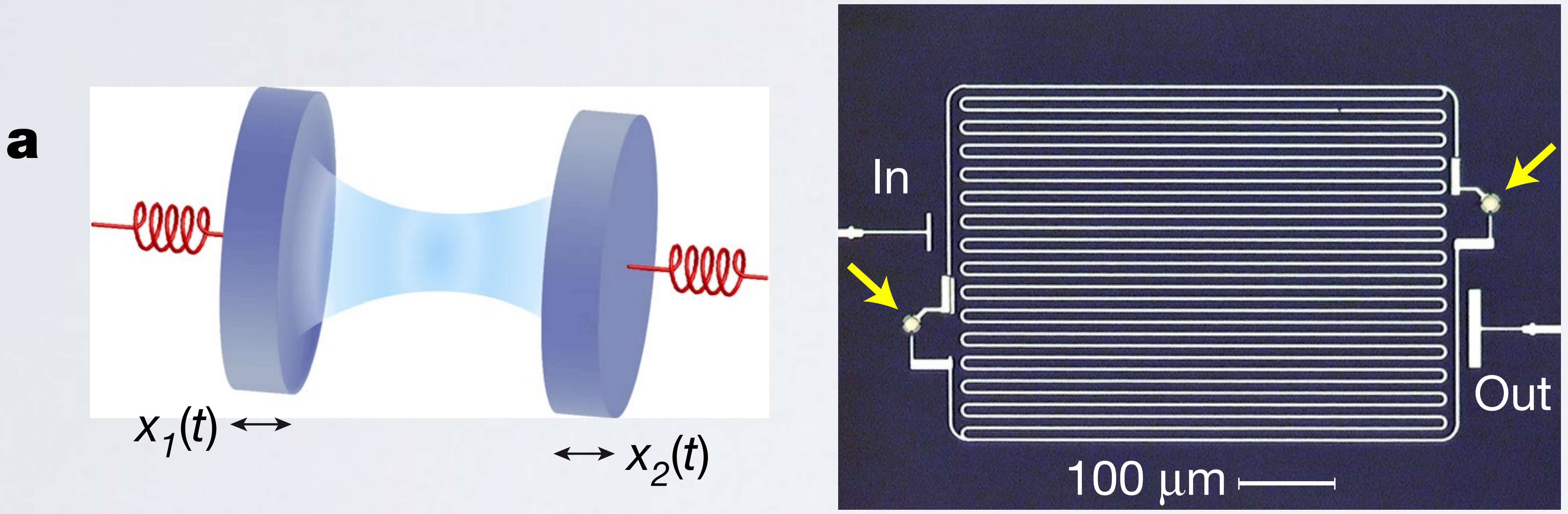
OPTOMECHANICAL SYSTEMS

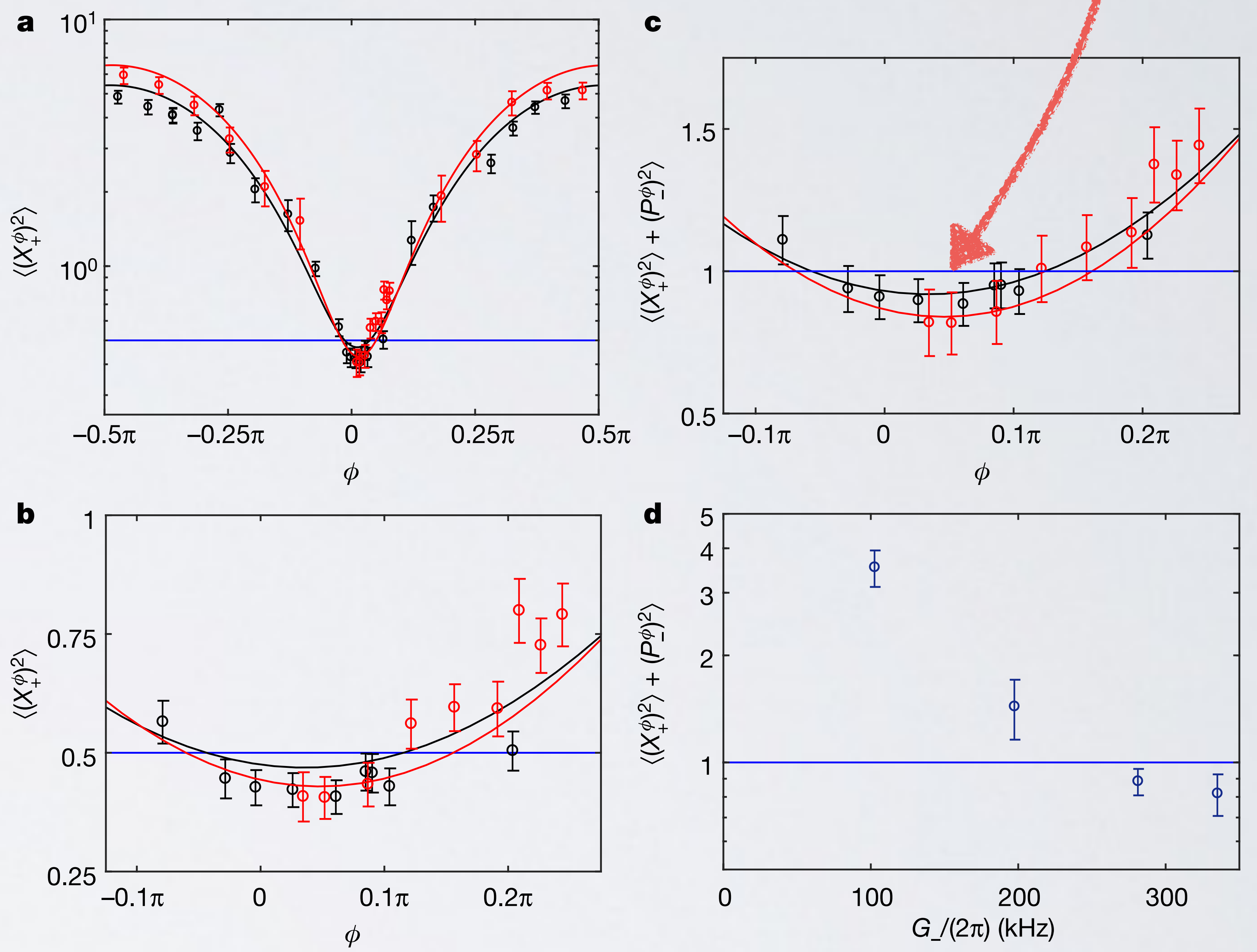
Microwave domain

First example of stationary entanglement between mechanical resonators

$$\langle \Delta X_\Sigma^2 \rangle_{\rho_s} + \langle \Delta P_\Delta^2 \rangle_{\rho_s} \geq 1$$

entangled!





$$X_\Sigma = (X_A + X_B)/\sqrt{2}$$
$$P_\Delta = (P_A - P_B)/\sqrt{2}$$
$$[X_A, P_B] = i\delta_{AB}$$

[1] C. F. Ockeloen-Korppi, *et al.*, Nature 556, 478 (2018).

SUMMARIZING

Nanometric scale is a "large scale" for quantum physicists

SUMMARIZING

Nanometric scale is a “large scale” for quantum physicists

Example 1: superconducting (charge) qubit

SUMMARIZING

Nanometric scale is a "large scale" for quantum physicists

Example 1: superconducting (charge) qubit

Quantization of a LC resonator

SUMMARIZING

Nanometric scale is a “large scale” for quantum physicists

Example 1: superconducting (charge) qubit

Quantization of a LC resonator

Josephson junction as a nonlinear inductance

SUMMARIZING

Nanometric scale is a "large scale" for quantum physicists

Example 1: superconducting (charge) qubit

Quantization of a LC resonator

Josephson junction as a nonlinear inductance

Example 2: optomechanical systems

SUMMARIZING

Nanometric scale is a “large scale” for quantum physicists

Example 1: superconducting (charge) qubit

Quantization of a LC resonator

Josephson junction as a nonlinear inductance

Example 2: optomechanical systems

Ground-state cooling

SUMMARIZING

Nanometric scale is a "large scale" for quantum physicists

Example 1: superconducting (charge) qubit

Quantization of a LC resonator

Josephson junction as a nonlinear inductance

Example 2: optomechanical systems

Ground-state cooling

Amplification

SUMMARIZING

Nanometric scale is a "large scale" for quantum physicists

Example 1: superconducting (charge) qubit

Quantization of a LC resonator

Josephson junction as a nonlinear inductance

Example 2: optomechanical systems

Ground-state cooling

Amplification

Squeezing & entanglement