Introduction to Accelerator Physics

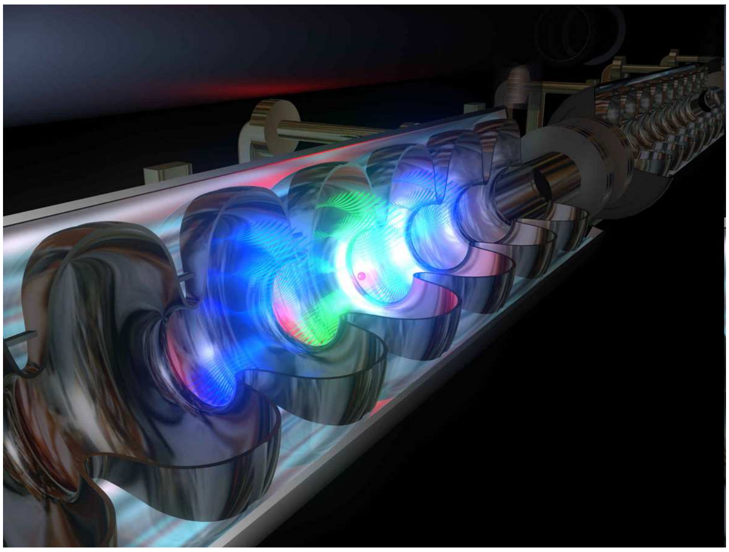
Part 3

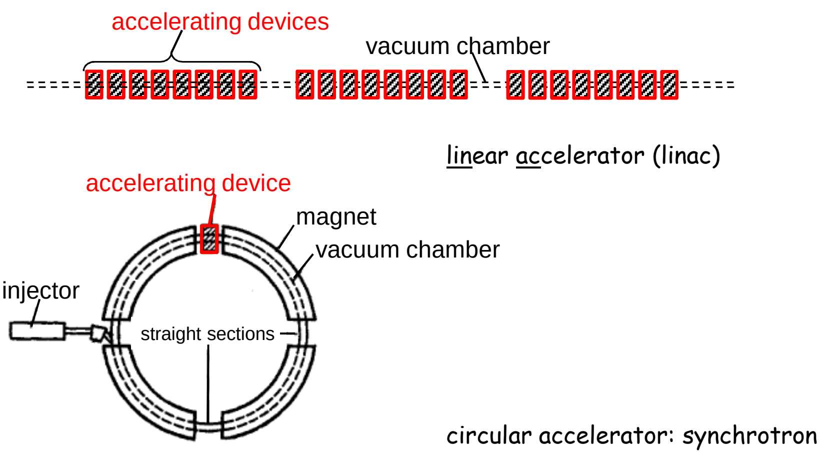
Pedro Castro / Accelerator Physics Group (MPY)
Hamburg, 23rd July 2019

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

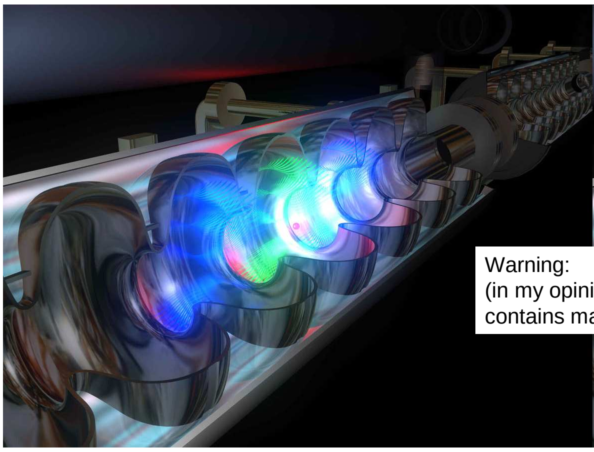
DESY.

How electromagnetic fields accelerate particles





MEDIA DATABASE. "Electron acceleration – a virtual simulation"



Warning:
(in my opinion) this movie
contains many physical inaccuracies

DESY→Press→Media database→European XFEL (with filter: media type=movies)

https://media.desy.de/DESYmediabank/?l=en#l=en&cid=3980&cname=European%20XFEL&f=2165&s=&p=&r=

YouTube: https://www.youtube.com/watch?v=FJO_DmM4q7M
search text: electron acceleration

Motion in electric and magnetic fields

Equation of motion under Lorentz Force

$$\frac{d\vec{p}}{dt} = \vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$$

momentum, charge, velocity — of the particle

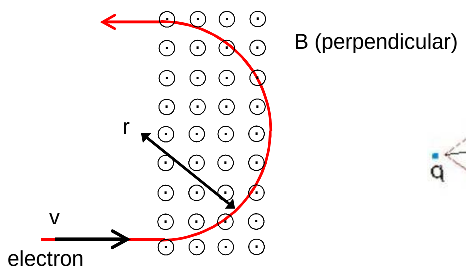
electric field

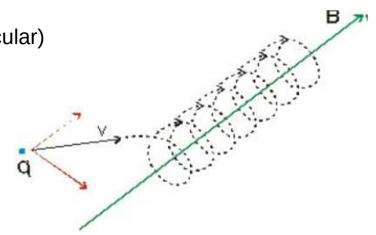
magnetic field

Motion in magnetic fields

if the electric field is zero ($\vec{E} = 0$), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B} \quad \rightarrow \quad \vec{F} \perp \vec{v}$$





Magnetic fields do not change the particles energy

Motion in magnetic fields

if the electric field is zero (E=0), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$

$$E^2 = \vec{p}^2 c^2 + E_0^2$$

energy-momentum relation in special relativity

total energy

momentum

energy at rest

Motion in magnetic fields

if the electric field is zero (E=0), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$

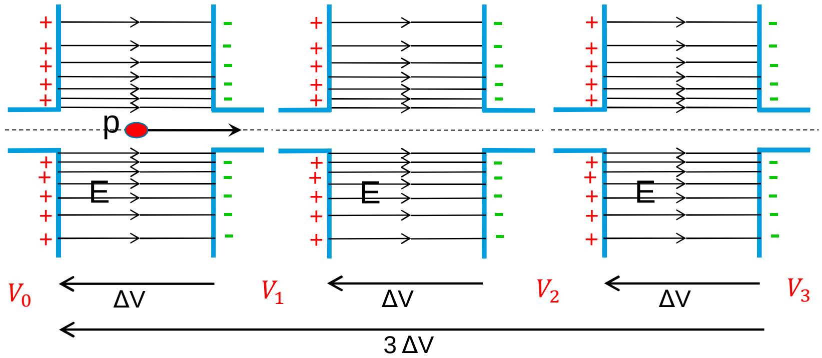
$$E^2 = \vec{p}^2 c^2 + E_0^2$$

$$E\frac{dE}{dt} = c^2 \vec{p}\frac{d\vec{p}}{dt} = c^2 q \vec{p}\left(\vec{v} \times \vec{B}\right) = c^2 q|\vec{p}|\left|\vec{v} \times \vec{B}\right| \cos\emptyset = 0$$

since $\vec{v} \times \vec{B} \perp \vec{v}$ ➔ $\emptyset = 90°$

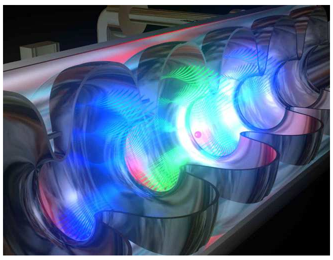
Magnetic fields <u>do not change</u> the particles energy, only electric fields do !

acceleration with DC electric fields

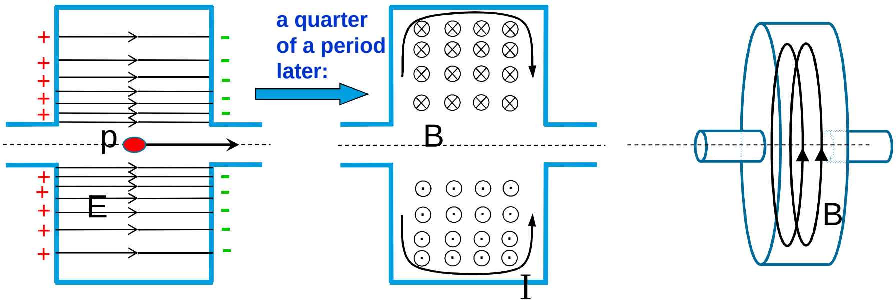


In general:

- Static magnetic fields → to guide (bend + focus) particle beams
- Static electric fields → accelerate particle beams (low energy)
- Radio-frequency EM fields → accelerate particle beams (high E)



RF cavity basics: a cylindrical cavity



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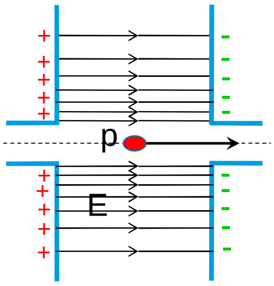
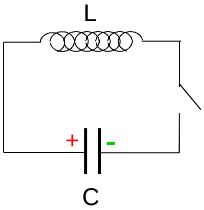


LC circuit (or resonant circuit) analogy:



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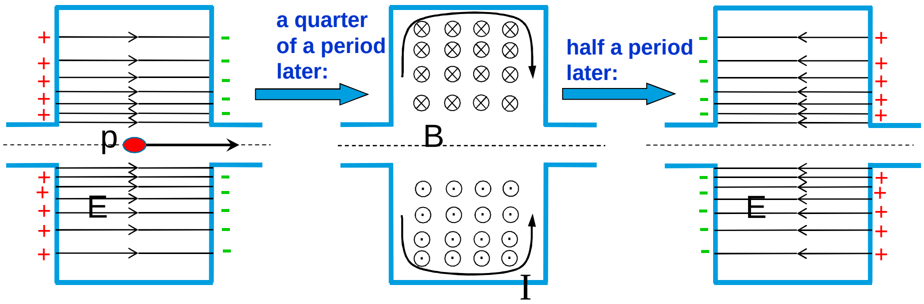

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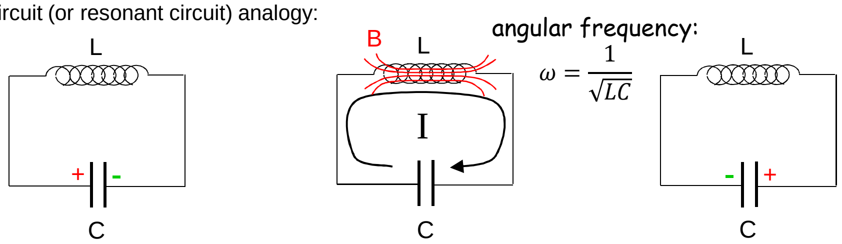
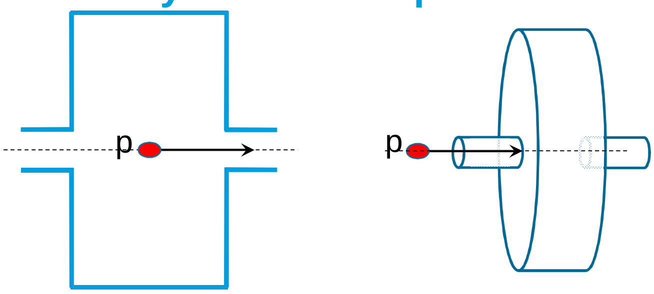
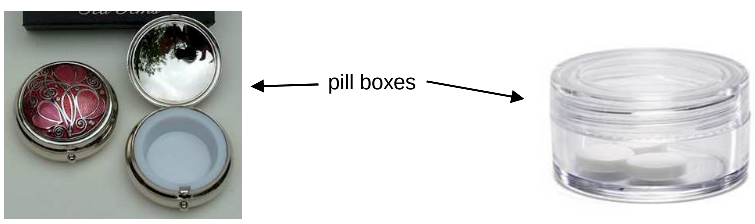


RF cavity basics: the pill box cavity





Maxwell's equations
(differential formulation in SI units)

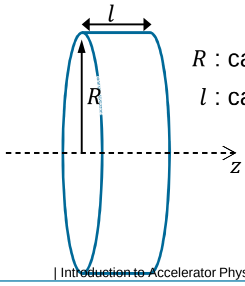
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

+ boundary conditions



R : cavity radius

l : cavity length

⇨ set of solutions with $B_z = 0$ (that is, $\vec{B}$ is transverse)

TM modes
(transverse magnetic modes)

~~set of solutions with $E_z = 0$ (that is, $\vec{E}$ is transverse)~~

~~TE modes~~
~~(transverse electric modes)~~

Maxwell's equations
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set of solutions with $B_z = 0$ (that is, $\vec{B}$ is transverse)

$$E_z = E_0 J_m\left(x_{mn}\frac{r}{R}\right)\cos m\theta \cos\left(\frac{p\pi}{l}z\right) e^{j\omega t}$$

$$E_r = -\frac{p\pi}{l}\frac{R}{x_{mn}} E_0 J'_m\left(x_{mn}\frac{r}{R}\right)\cos m\theta \sin\left(\frac{p\pi}{l}z\right) e^{j\omega t}$$

$$E_\theta = -\frac{p\pi}{l}\frac{mR^2}{x_{mn}{}^2 r} E_0 J_m\left(x_{mn}\frac{r}{R}\right)\sin m\theta \sin\left(\frac{p\pi}{l}z\right) e^{j\omega t}$$

$$B_z = 0$$

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indices:

$m = 0,1,2,\ldots$: number of full period variations in θ of the fields

$n = 1,2,\ldots$: number of zeros of the axial field component in $\vec{r}$

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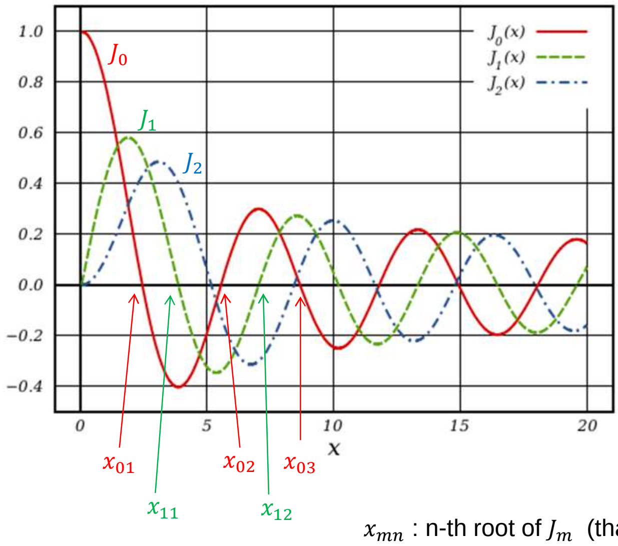
J_m : Bessel´s functions

J'_m : derivative of the Bessel´s functions

x_{mn} : n-th root of J_m (that is, $J_m(x_{mn}) = 0$)

angular frequency : $\omega = c\sqrt{\left(\frac{x_{mn}}{R}\right)^2 + \left(\frac{p\pi}{l}\right)^2}$

J_m : Bessel´s functions



m	x_{m1}	x_{m2}	x_{m3}
0	2.405	5.520	8.654
1	3.832	7.016	10.173
2	5.136	8.417	11.620

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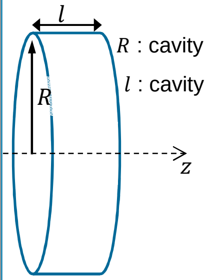
DESY. | Introduction to Accelerator Physics | Pedro Castro, 23rd July 2019 Page 20 boundary conditions



R : cavity radius

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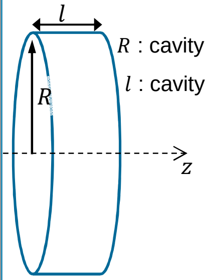
fundamental solution with $B_z = 0$ (that is, $\vec{B}$ is transverse)

$m = 0$: rotation symmetry of the fields

$n = 1$: no zeros of the axial field component in $\vec{r}$

$p = 0$: no variation in z of the fields

boundary conditions



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$$E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$

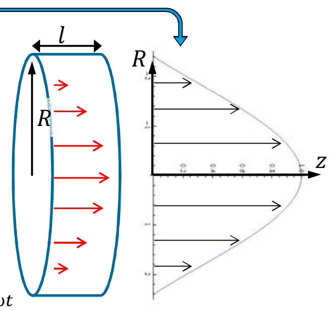
$$E_r = 0$$

$$E_\theta = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_\theta = j\omega \frac{R}{x_{01}c^2} E_0 J_1\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$



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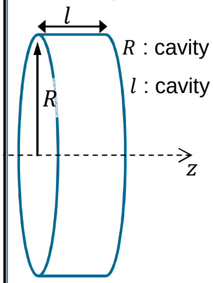
boundary conditions



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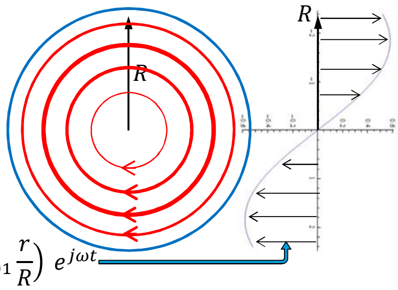

$m = 0$: rotation symmetry of the fields

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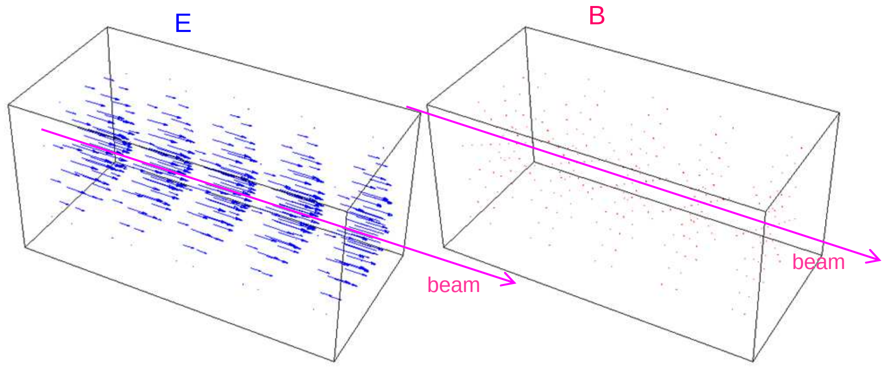
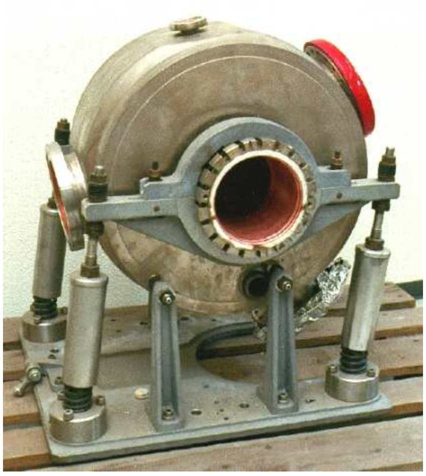
Pill box cavity: 3D visualisation of E and B



Examples of pill box cavities

DESY cavity (pill box)

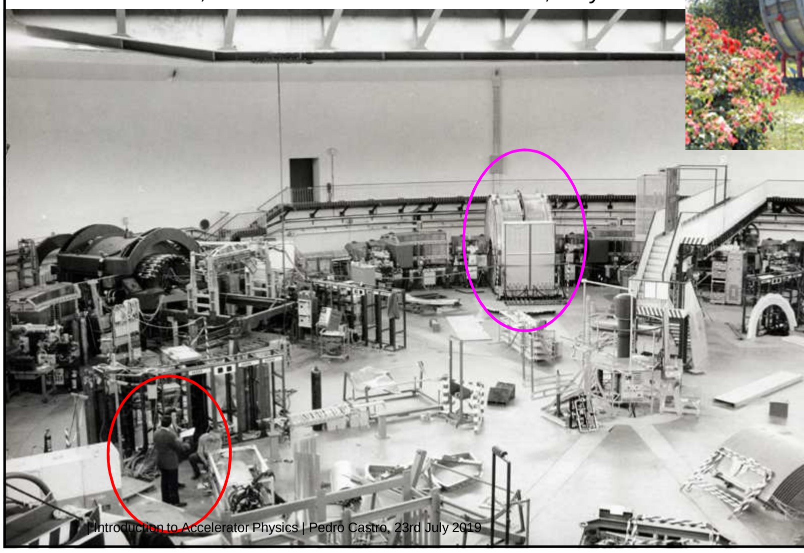


ADONE cavity 51 MHz (pill box)
Frascati lab, Italy

Examples of pill box cavities

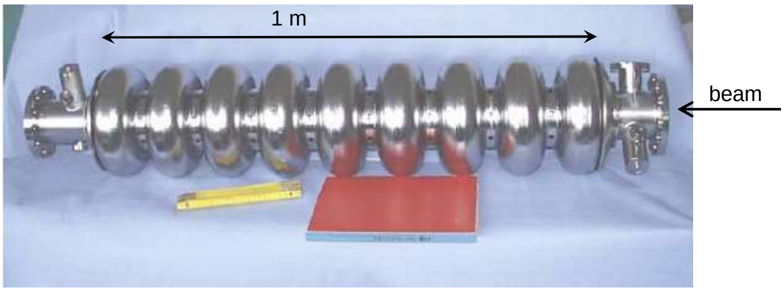
ADONE cavity 51 MHz (pill box)
Frascati lab, Italy

ADONE in 1963, Laboratori Nazionali di Frascati, Italy



Superconducting cavity used at DESY

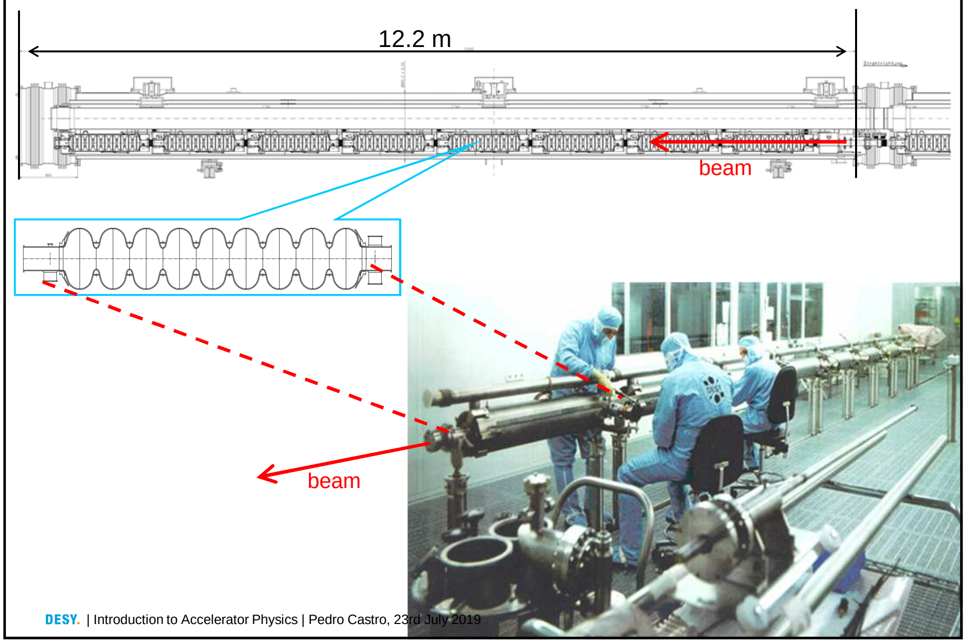
Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



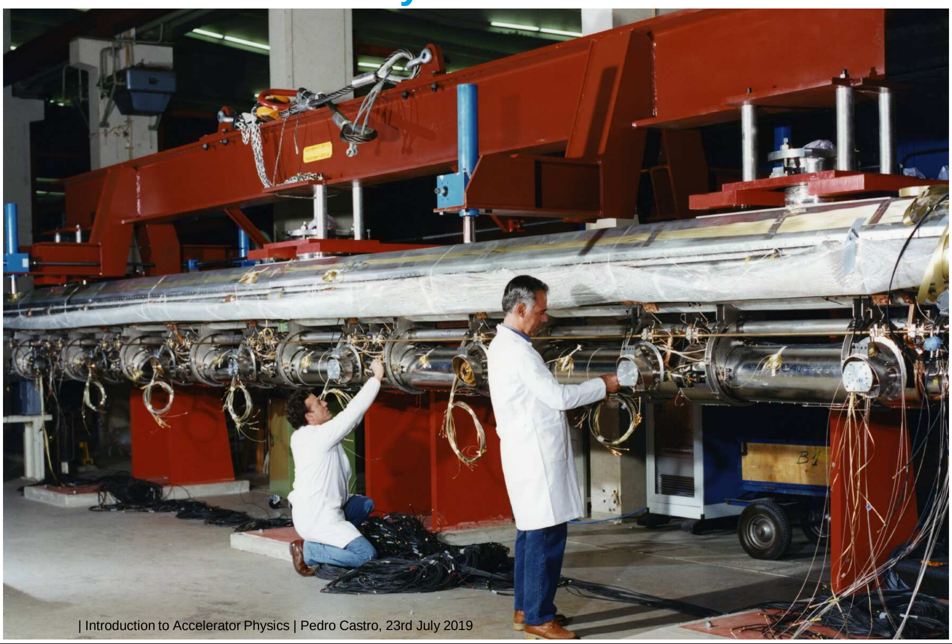
Free-electron LASer in Hamburg	0.3 km	DESY	2004- ?	e-	1.2 GeV
European X-ray Free-Electron Laser	3 km	DESY	2016- ?	e-	17.5 GeV
International Linear Collider	30 km	?	?	e-/e+	2x250 GeV

Cavities inside a cryostat

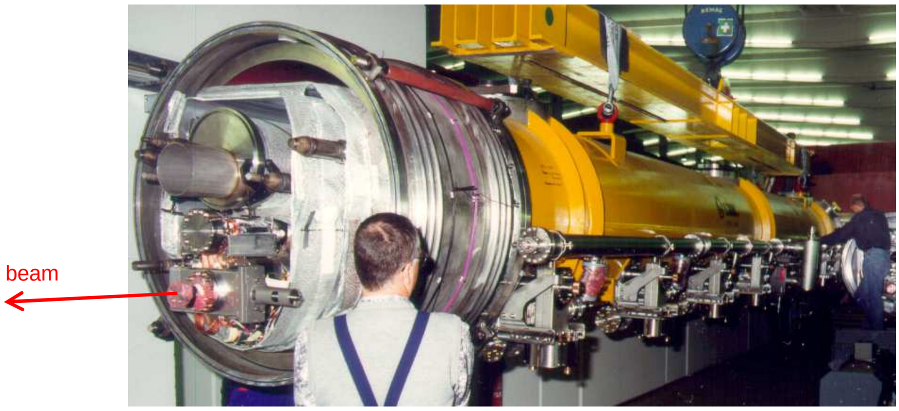
Number of cavities 8
Cavity length 1.038 m
Operating frequency 1.3 GHz
Operating temperature 2 K
Accelerating Gradient 23..35 MV/m



Cavities inside a cryostat

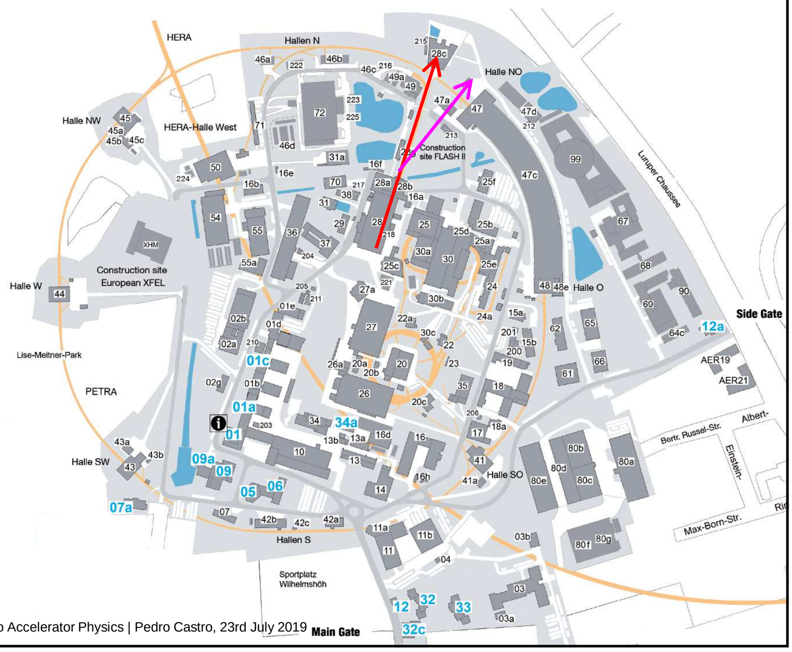


Cavities inside an accelerator module (cryostat)

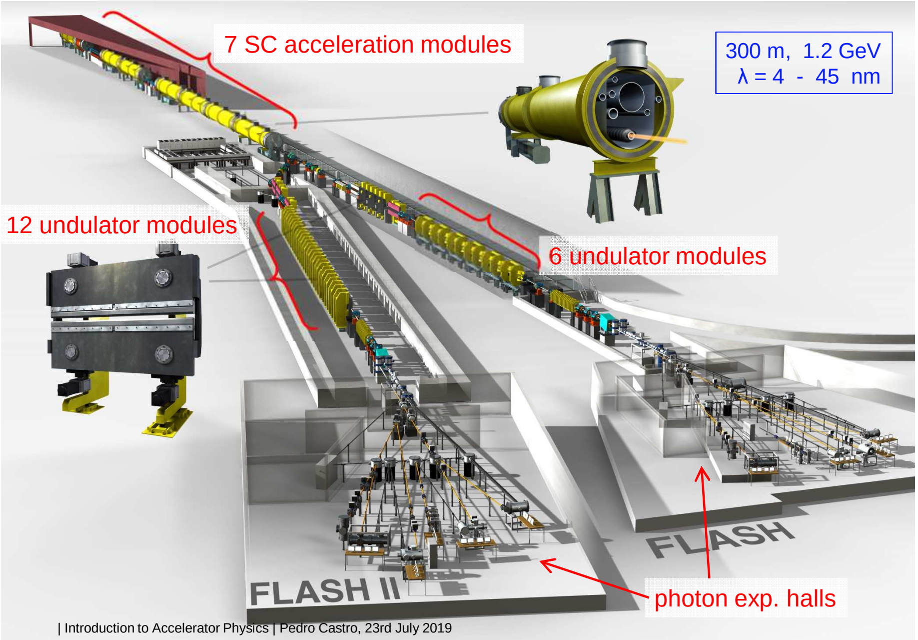


module installation in FLASH (2004)

Free-electron LASer in Hamburg (FLASH)



Free-electron LASer in Hamburg (FLASH)



100 accelerator modules (cryostats) in XFEL

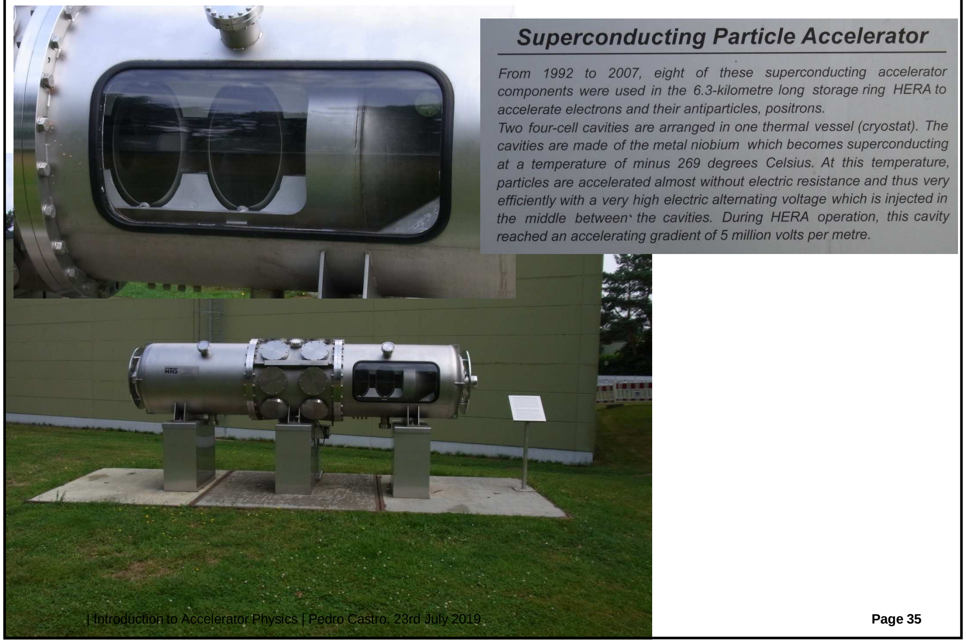
European X-ray Free-Electron Laser (XFEL)

(3 km, 17.5 GeV)

λ = 0.05 - 6 nm

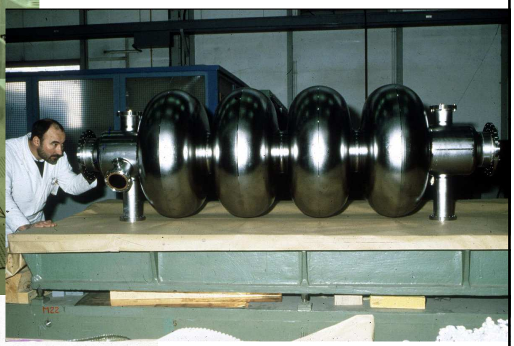
Superconducting cavities at HERA

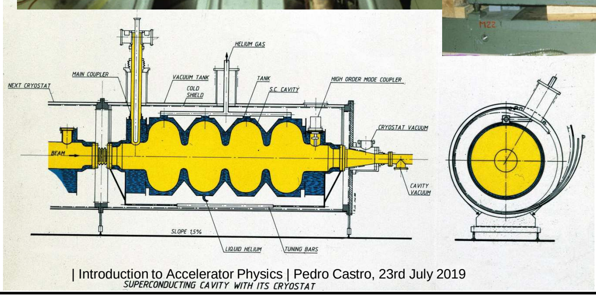
16 cavities
500 MHz



Superconducting cavities at LEP

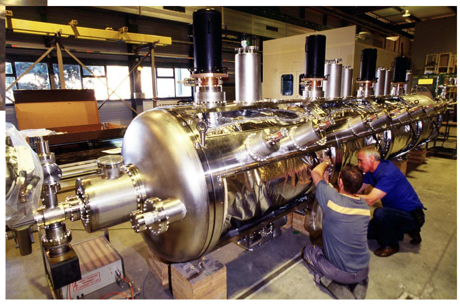
272 cavities
352 MHz

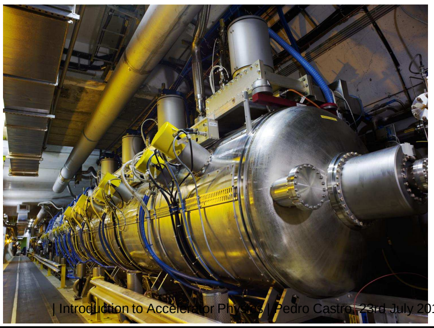




Superconducting cavities at LHC

16 cavities
400 MHz





Other accelerators using superconducting cavities

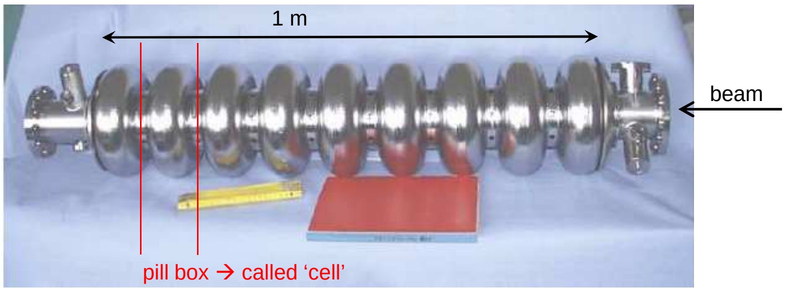
- 5 de-commissioned
- 11 in operation
- 4 in construction
- 9 in design phase

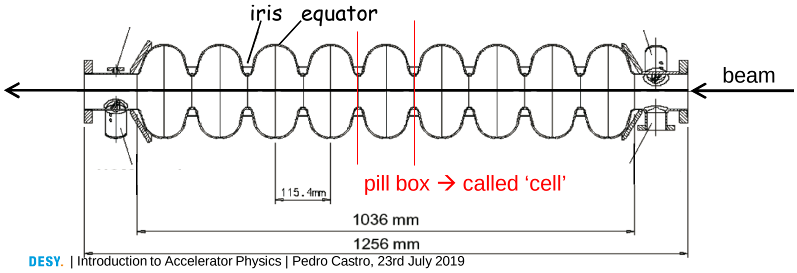
Total = 29

full list: http://tesla-new.desy.de/srf_accelerators

Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



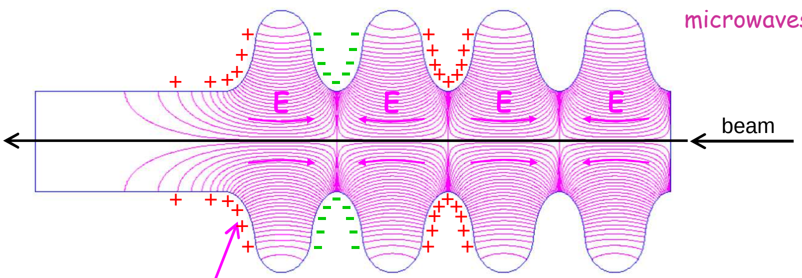


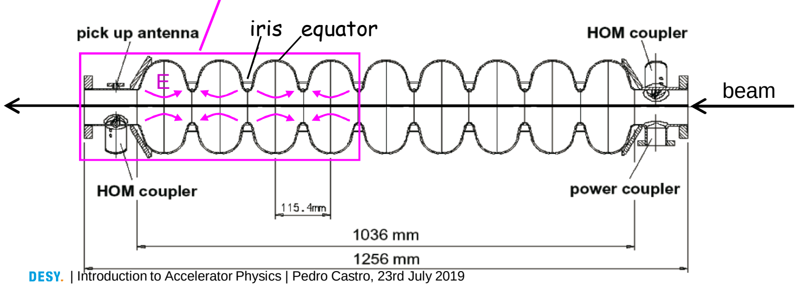
Accelerating field map

Simulation of the fundamental mode: electric field lines

$f_{RF} = 1.3 \text{ GHz}$

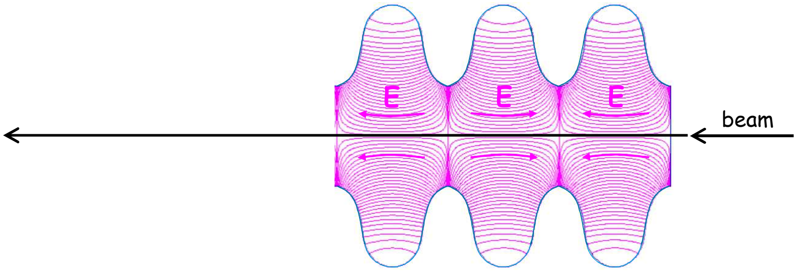
microwaves: (L-band)

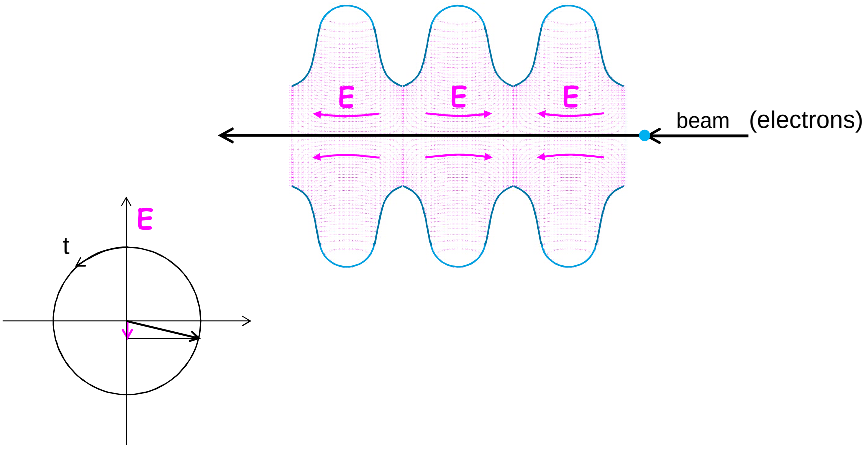


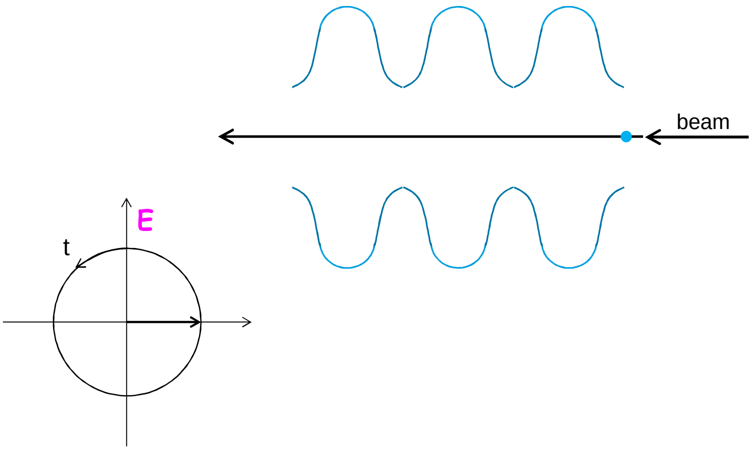


Is there a net acceleration?

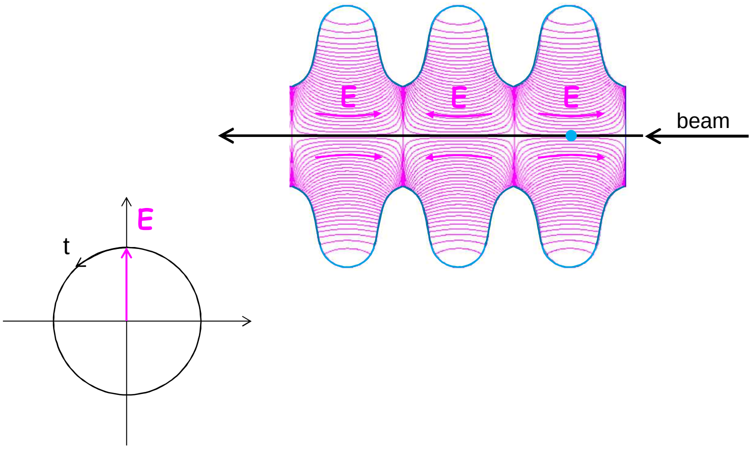
Simulation of the fundamental mode: electric field lines







beam
E
t



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beam
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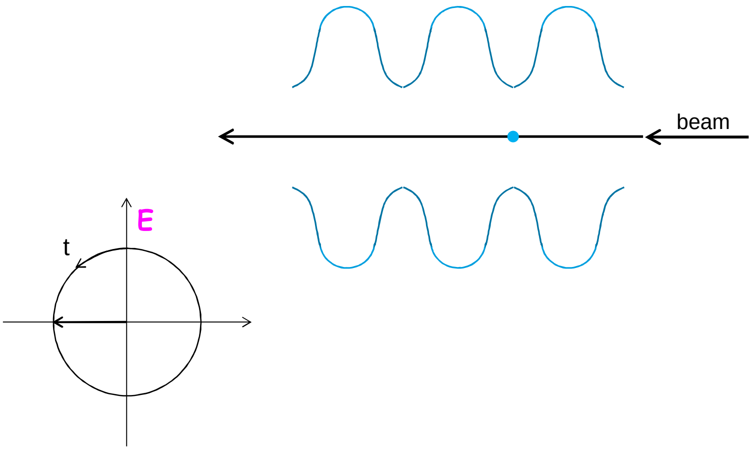
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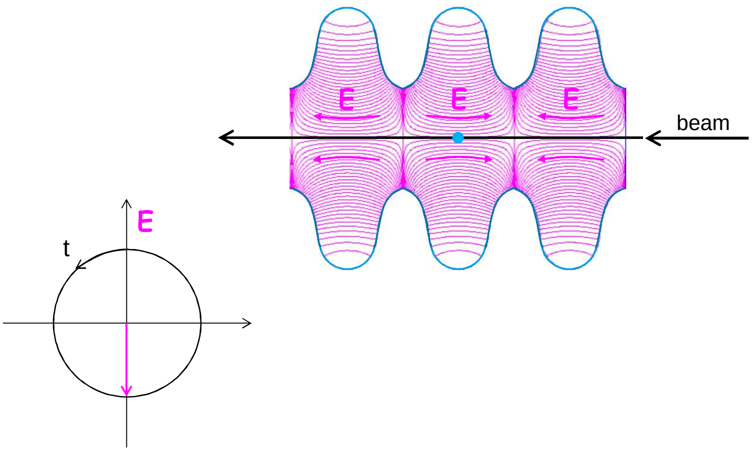
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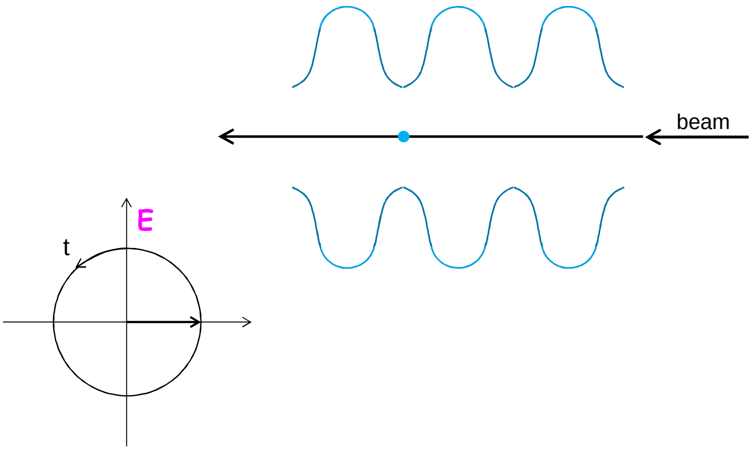
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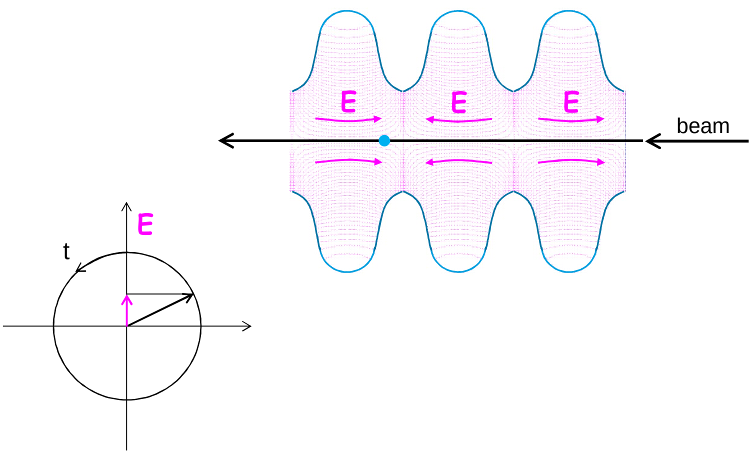
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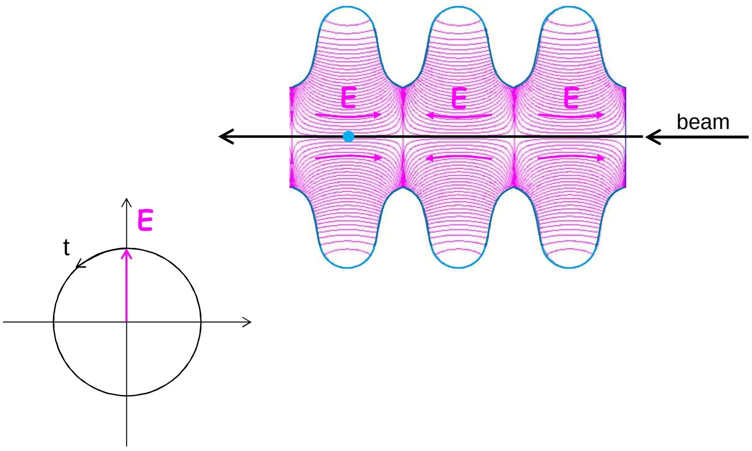
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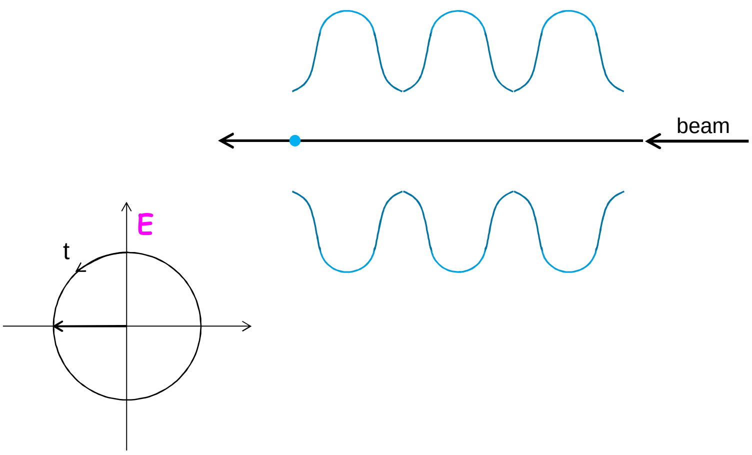
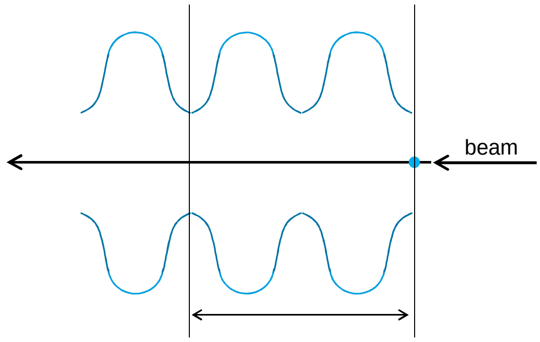

beam
E
t

Is there a net acceleration? ……………………………… timing is the key

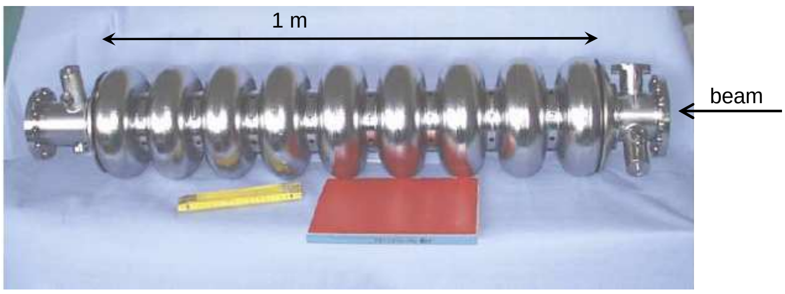
for electrons, $\beta \cong 1$



$$l = cT = \frac{c}{f} = \frac{3 \cdot 10^8}{1.3 \cdot 10^9} = 0.23\,m \quad \text{(2 cells)}$$

1 cavity (1.038 m) / 9 cells = 0.115 m

Superconducting cavity used at DESY



material: pure Niobium

operating temperature: 2 K

accelerating field gradient: up to 35 MV/m

Thermal conductivity

	$[W\ m^{-1}\ K^{-1}]$
water	0.56 – 0.61
copper (at 20 °C)	385 – 401
helium II	$> 10^5$

Frequently Asked Questions

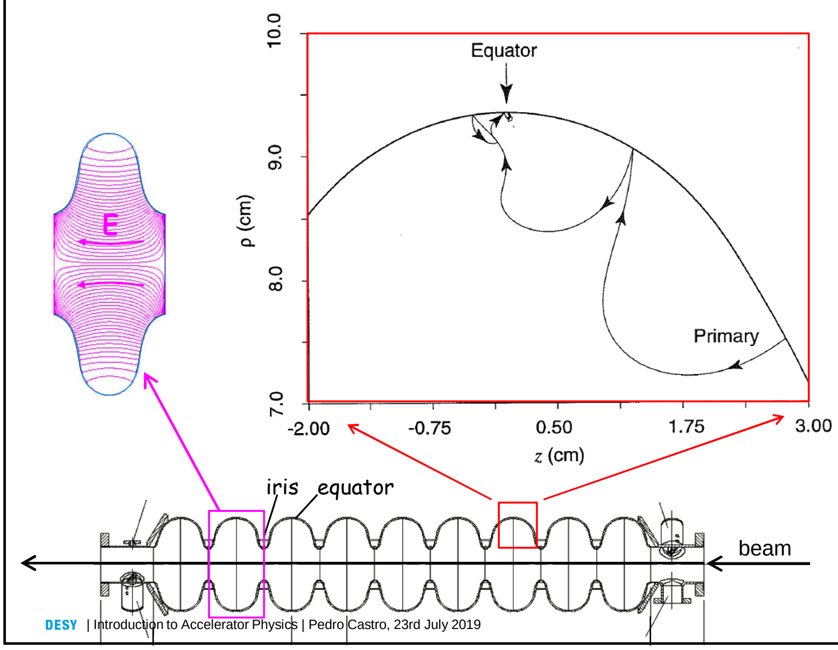
1) Why this shape?

2)

3)

4)

Multipacting mitigation in superconducting cavities



1) Why this shape? ……………………………………………… to reduce/avoid multipacting

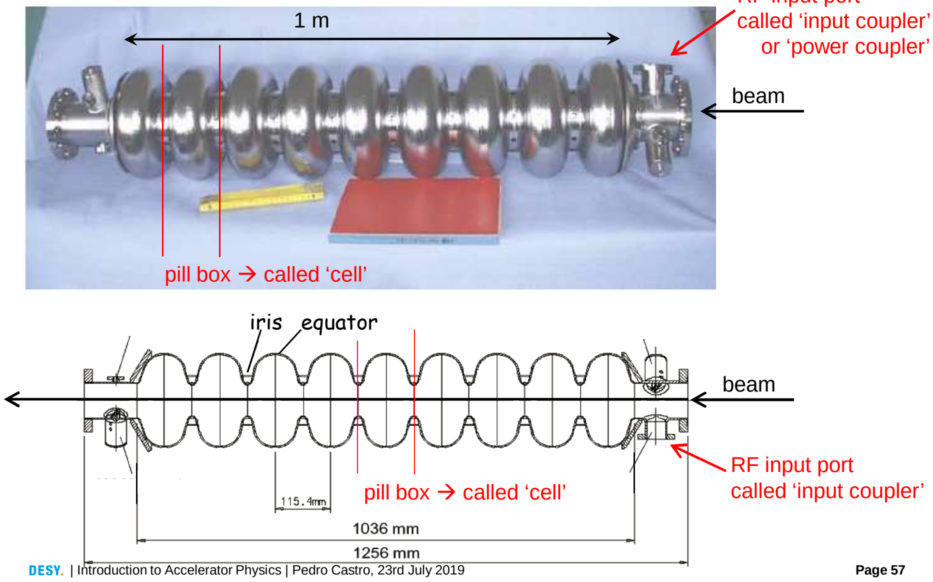
2) How to feed $\vec{E}$ in?

3)

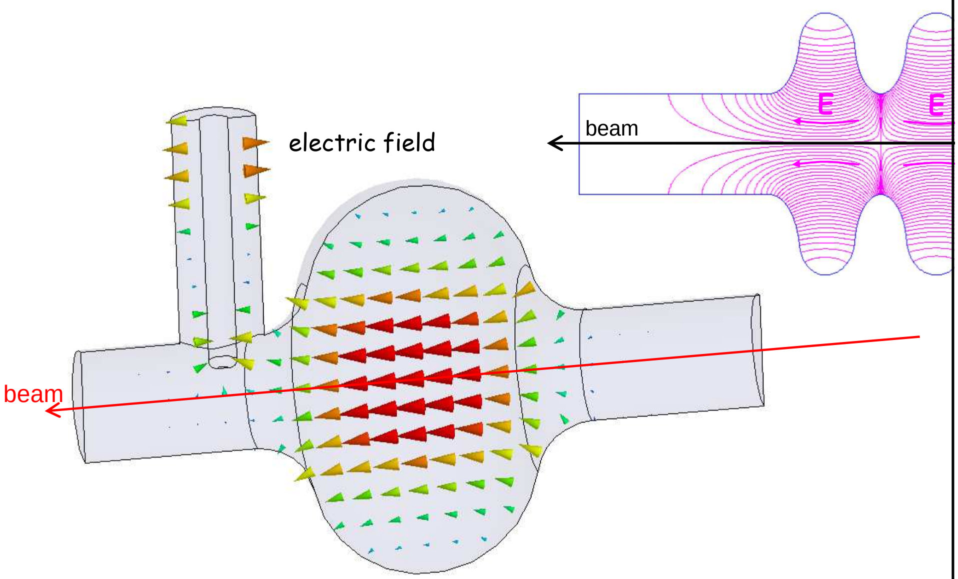
4)

Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



Fundamental mode coupler (input coupler)



1) Why this shape? ………………………………………… to reduce/avoid multipacting

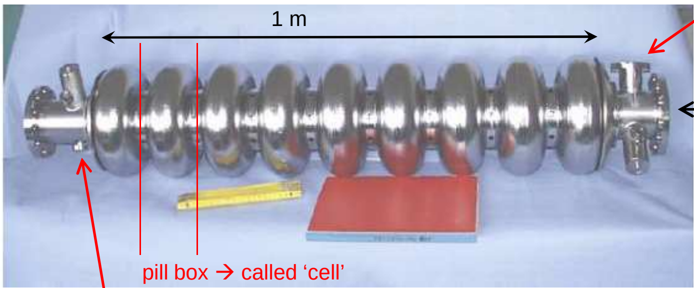
2) How to feed $\vec{E}$ in? ………………………………………… with input couplers

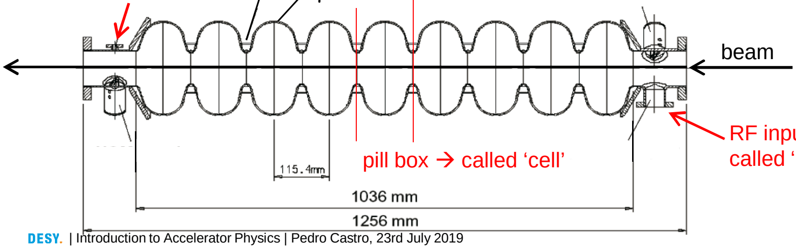
3) How to measure $\vec{E}$?

4)

Superconducting cavity used in FLASH and in XFEL

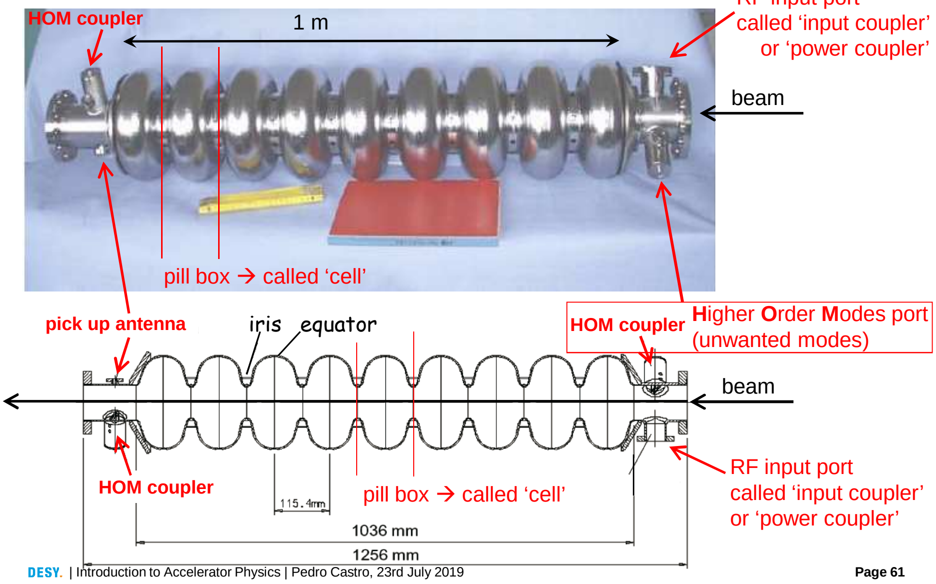
Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)





Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



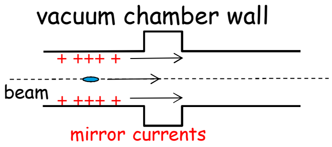
1) Why this shape? …………………………………………… to reduce/avoid multipacting

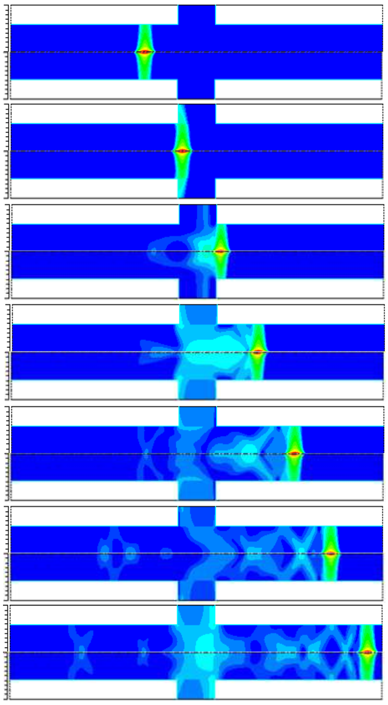
2) How to feed $\vec{E}$ in? ………………………………………… with input couplers

3) How to measure $\vec{E}$? ………………………………………… with pick up antennas

4) What are HOM couplers for?

Wakefields





1) Why this shape? ………………………………………… to reduce/avoid multipacting

2) How to feed $\vec{E}$ in? ……………………………………… with input couplers

3) How to measure $\vec{E}$? ……………………………………… with pick up antennas

4) What are HOM couplers for? …………………………… to reduce HOM (wakefields)

Summing-up of this part

Particle acceleration using radio-frequency fields:

basic cavity: pill box
- analogy to an LC circuit
- infinite number of solutions for $\vec{E}$ and $\vec{B}$
- eq. for the fundamental solution for $\vec{E}$ and $\vec{B}$

superconducting cavity
- multipacting mitigation
- RF couplers and antennas
- wakefields and HOMs
- FLASH and XFEL

Contact

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