QCD for Collider Physics
Part 3

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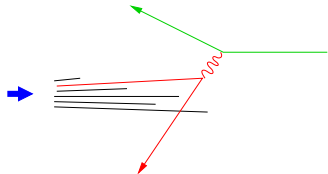
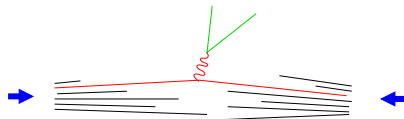DESY Summer Student Programme 2019, Hamburg

The parton model

▶ describe deep inelastic scattering, Drell-Yan process, etc.

- fast-moving hadron
  $\approx$ set of free partons $(q, \bar{q}, g)$ with low transverse momenta
- physical cross section
  $=$ cross section for partonic process $(\gamma^* q \to q,\ q\bar{q} \to \gamma^*)$
  $\times$ parton densities

Deep inelastic scattering (DIS): $\ell p \to \ell X$ Drell-Yan: $pp \to \ell^+ \ell^- X$

Nobel prize 1990 for
Friedman, Kendall, Taylor

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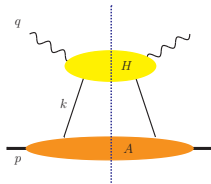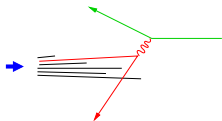 $\times$ parton densities

Factorisation

▶ implement and correct parton-model ideas in QCD

- conditions and limitations of validity
  kinematics, processes, observables
- corrections: partons interact
  α_s small at large scales $\rightsquigarrow$ perturbation theory
- define parton densities in field theory
  derive their general properties
  make contact with non-perturbative methods

Factorisation: physics idea and technical implementation

- ▶ idea: separation of physics at different scales
  - high scales: quark-gluon interactions
    ⤳ compute in perturbation theory
  - low scale: proton → quarks, antiquarks, gluons
    ⤳ parton densities

- ▶ requires hard momentum scale in process
 large photon virtuality $Q^2 = -q^2$ in DIS

Factorisation: physics idea and technical implementation
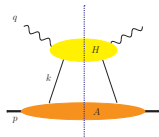

▶ implementation: separate process into

- "hard" subgraph H with particles far off-shell
  compute in perturbation theory
- "collinear" subgraph A with particles moving along proton
  turn into definition of parton density

Collinear expansion

▶ graph gives $\int d^4k\, H(k)A(k)$; simplify further

▶ light-cone coordinates $\rightsquigarrow$ blackboard
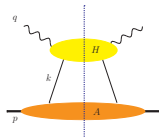
Collinear expansion

▶ graph gives $\int d^4k\, H(k) A(k)$; simplify further

▶ in hard graph neglect small components of external lines
  ⤳ Taylor expansion

$$H(k^+, k^-, k_T) = H(k^+, 0, 0) + \text{corrections}$$

 ⤳ loop integration greatly simplifies:

$$\int d^4k\, H(k)\, A(k) \approx \int dk^+\, H(k^+, 0, 0) \int dk^-\, d^2k_T\, A(k^+, k^-, k_T)$$

▶ in hard scattering treat incoming/outgoing partons as
  exactly collinear $(k_T = 0)$ and on-shell $(k^- = 0)$

▶ in collin. matrix element integrate over k_T and virtuality
  ⤳ collinear (or k_T integrated) parton densities
    only depend on $k^+ = x p^+$

further subtleties related with spin of partons, not discussed here

Definition of parton distributions
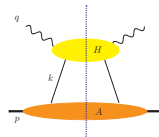
▶ matrix elements of quark/gluon operators

$$f_q(x) = \int \frac{dz^-}{2\pi}\, e^{ixp^+ z^-} \left\langle p \left| \bar{\psi}(0) \tfrac{1}{2} \gamma^+ \psi(z) \right| p \right\rangle \Big|_{z^+=0,\, z_T=0}$$

$\psi(z) =$ quark field operator: annihilates quark

$\bar{\psi}(0) =$ conjugate field operator: creates quark

$\tfrac{1}{2}\gamma^+$ sums over quark spin

$\int \frac{dz^-}{2\pi}\, e^{ixp^+ z^-}$ projects on quarks with $k^+ = xp^+$

▶ analogous definitions for polarised quarks, antiquarks, gluons

▶ analysis of factorisation used Feynman graphs
but here provide non-perturbative definition

further subtleties related with choice of gauge, not discussed here

Factorisation for pp collisions

▶ example: Drell-Yan process $pp \to \gamma^* + X \to \mu^+\mu^- + X$
where $X =$ any number of hadrons

▶ one parton distribution for each proton $\times$ hard scattering
$\rightsquigarrow$ deceptively simple physical picture
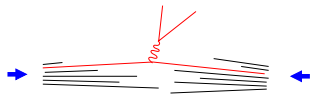
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$\rightsquigarrow$ deceptively simple physical picture

▶ "spectator" interactions produce additional particles
which are also part of unobserved system X ("underlying event")

▶ need not calculate this thanks to unitarity
as long as cross section/observable sufficiently inclusive

▶ but must calculate/model if want more detail on the final state

More complicated final states

▶ production of W, Z or other colourless particle (Higgs, etc)
same treatment as Drell-Yan

▶ jet production in ep or pp: hard scale provided by p_T

▶ heavy quark production: hard scale is m_c, m_b, m_t

Importance of factorisation concept

▶ describe processes for study of electroweak and BSM physics, e.g.

  • W mass measurement
  • determination of Higgs boson properties
  • signal and background in new physics searches

▶ determine parton densities as a tool to make predictions
and to learn about proton structure

  • requires many processes to disentangle quark flavors and gluons

A closer look at one-loop corrections

▶ example: DIS
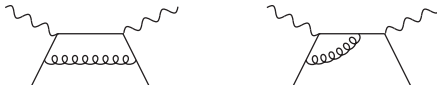
▶ UV divergences removed by standard renormalisation

▶ soft divergences cancel in sum over graphs

▶ collinear div. do not cancel, have integrals

$$\int\limits_{0} \frac{dk_T^2}{k_T^2}$$

what went wrong?

▶ hard graph should not contain internal collinear lines
   collinear graph should not contain hard lines

▶ must not double count $\rightsquigarrow$ factorisation scale μ
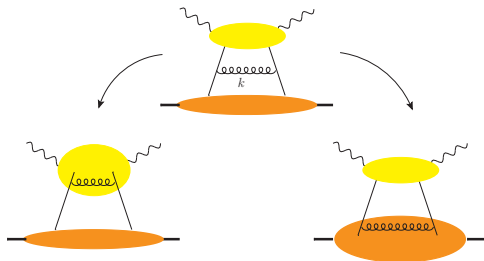
▶ with cutoff: take $k_T > \mu$ take $k_T < \mu$
   $1/\mu \sim$ transverse resolution

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   collinear graph should not contain hard lines

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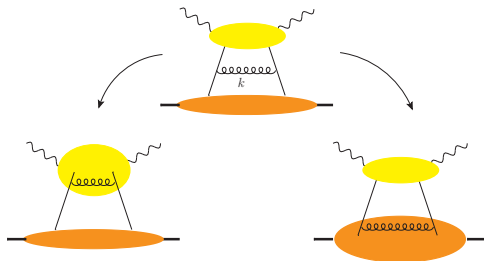
▶ with cutoff: take $k_T > \mu$ take $k_T < \mu$
   $1/\mu \sim$ transverse resolution

▶ in dim. reg.:
   subtract collinear divergence   subtract ultraviolet div.

The evolution equations

▶ DGLAP equations

$$\frac{d}{d\log\mu^2}\,f(x,\mu) = \int_x^1 \frac{dx'}{x'}\,P\Big(\frac{x}{x'}\Big)\,f(x',\mu) \,=\, \big(P\otimes f(\mu)\big)(x)$$

▶ $P =$ splitting functions
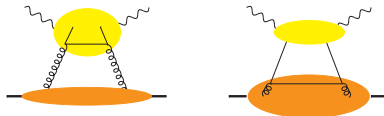
- have perturbative expansion

  $$P(x) = \alpha_s(\mu)\,P^{(0)}(x) + \alpha_s^2(\mu)\,P^{(1)}(x) + \alpha_s^3(\mu)\,P^{(2)}(x)\ldots$$

  known to $\mathcal{O}(\alpha_s^3)$, in part to $\mathcal{O}(\alpha_s^4)$ Moch, Vermaseren, Vogt

- contains terms $\propto \delta(1-x)$ from virtual corrections

▶ quark and gluon densities mix under evolution:

▶ matrix evolution equation

$$\frac{d}{d\log \mu^2} f_i(x,\mu) = \sum_{j=q,\bar{q},g} \big(P_{ij} \otimes f_j(\mu)\big)(x) \qquad (i,j = q,\bar{q},g)$$

more transitions
possible at higher
orders in α_s

▶ parton content of proton depends on resolution scale μ

Factorisation formula

▶ example: $p + p \to H + X$

$$\sigma(p + p \to H + X) = \sum_{i,j=q,\bar{q},g} \int dx_i \, dx_j \, f_i(x_i, \mu_F) \, f_j(x_j, \mu_F)$$

$$\times \, \hat{\sigma}_{ij}\big(x_i, x_j, \alpha_s(\mu_R), \mu_R, \mu_F, m_H\big) + \mathcal{O}\bigg(\frac{\Lambda^2}{m_H^4}\bigg)$$

- $\hat{\sigma}_{ij}$ = cross section for hard scattering $i + j \to H + X$
  m_H provides hard scale
- μ_R = renormalisation scale, μ_F = factorisation scale
  may take different or equal
- μ_F dependence in C and in f cancels up to higher orders in α_s
  similar discussion as for μ_R dependence
- accuracy: α_s expansion and power corrections $\mathcal{O}(\Lambda^2/m_H^2)$
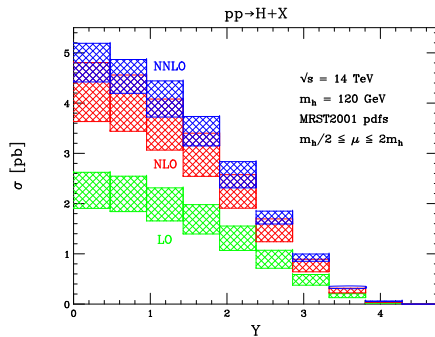
▶ can make σ and $\hat{\sigma}$ differential in kinematic variables, e.g. p_T of H

Scale dependence

examples: rapidity distributions in Z/γ^* and in Higgs production
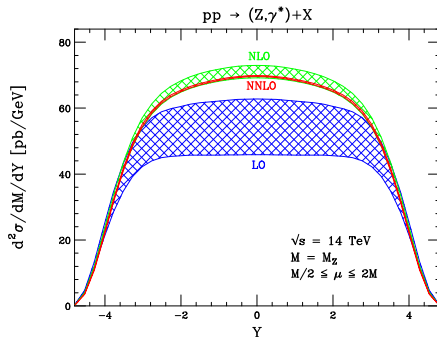
Anastasiou, Dixon, Melnikov, Petriello, hep-ph/0312266   Anastasiou, Melnikov, Petriello, hep-ph/0501130

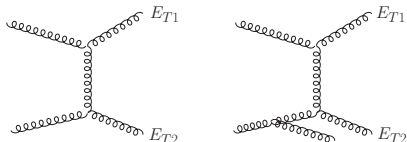$\mu_F = \mu_R = \mu$ varied within factor 1/2 to 2

LO, NLO, and higher

▶ instead of varying scale(s) may estimate higher orders by comparing N^nLO result with N^{n-1}LO

▶ caveat: comparison NLO vs. LO may not be representative for
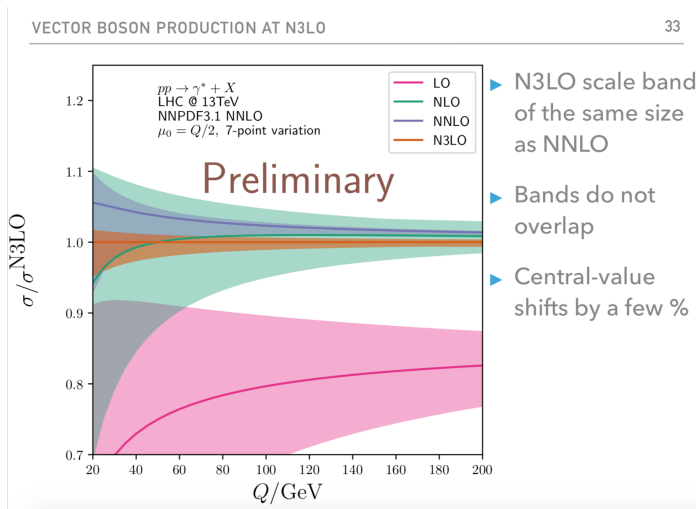situation at higher orders

often have especially large step from LO to NLO

  ▶ certain types of contribution may first appear at NLO
  e.g. terms with gluon density $g(x)$ in DIS, $pp \to Z + X$, etc.

  ▶ final state at LO may be too restrictive

  e.g. in $\dfrac{d\sigma}{dE_{T1}\, dE_{T2}}$ for dijet production

Just appeared: Drell-Yan at N^3LO

VECTOR BOSON PRODUCTION AT N3LO   33

$pp \to \gamma^* + X$
LHC @ 13TeV
NNPDF3.1 NNLO
$\mu_0 = Q/2$, 7-point variation

Preliminary

— LO
— NLO
— NNLO
— N3LO

σ/σ^{N3LO}

Q/GeV

▶ N3LO scale band of the same size as NNLO

▶ Bands do not overlap

▶ Central-value shifts by a few %

F. Dulat, talk given at QCD@LHC, 19 July 2019

Summary of Part 3

▶ implements ideas of parton model in QCD

- perturbative corrections (NLO, NNLO, . . .)
- field theoretical def. of parton densities
  ⤳ bridge to non-perturbative QCD

▶ valid for sufficiently inclusive observables
and up to power corrections in Λ/Q or $(\Lambda/Q)^2$
which are in general not calculable

▶ must in a consistent way

- remove collinear kinematic region in hard scattering
- remove hard kinematic region in parton densities
  ↔ UV renormalisation

procedure introduces factorisation scale μ_F

- separates "collinear" from "hard", "object" from "probe"