Broken Lines refinement

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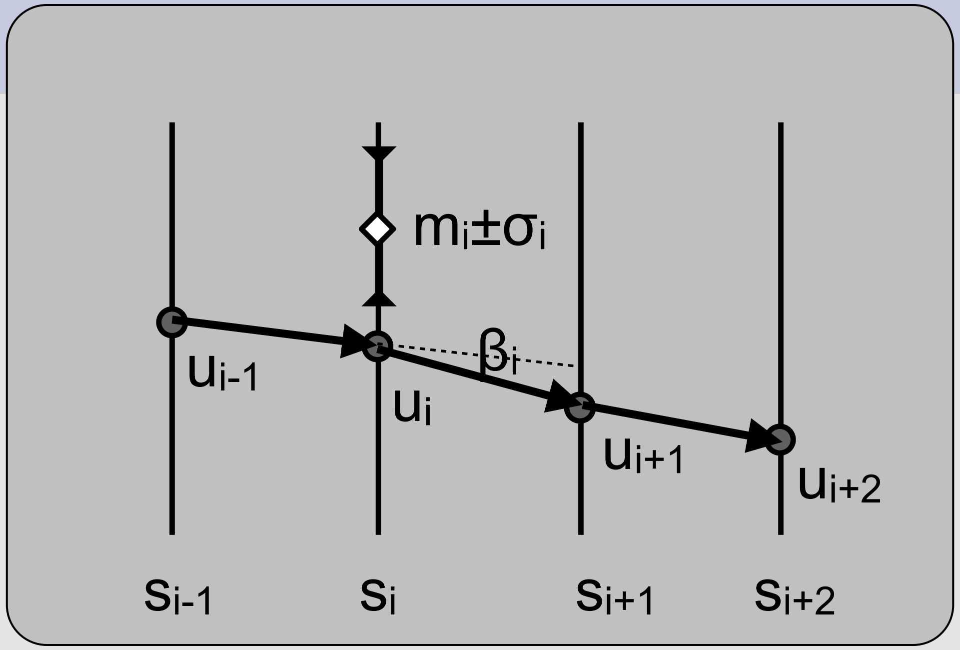
Reminder

- Broken Lines trajectory
  - Defined by offsets $u_i(s_i)$
  - Connected by kinks β_i and curvature
  - Variance V_β from multiple scattering
  - Definition of offsets should reflect material distribution (centers of gravity for thin scatterer approximation)



$$\beta_i = \left[u_{i-1}\delta_{i-1} - u_i\left(\delta_{i-1}+\delta_i\right) + u_{i+1}\delta_{i+1}\right] - \tfrac{1}{2}\left(\Delta s_{i-1} + \Delta s_i\right)\kappa$$

$$\Delta s_i = s_{i+1} - s_i, \quad \delta_i = 1/\Delta s_i$$

sign of κ changed, now compatible with q/p

Thin vs thick scatterer

- Thin scatterer θ_i^2 at s_i
  - Covariance matrix for (slope, offset)

$$\mathbf{V}_i(s_i) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \theta_i^2$$

  - Propagated to s

$$\mathbf{V}_i(s) = \begin{pmatrix} 1 & s-s_i \\ s-s_i & (s-s_i)^2 \end{pmatrix} \theta_i^2$$

- Thick scatterer
  - Sum of many thin scatters $\mathbf{V}(s) = \sum_i \mathbf{V}_i(s)$
  - Use weighted mean s_{cog} and variance Δs^2 of s_i

$$s_{cog} = \frac{1}{\theta^2} \sum_i s_i \theta_i^2 \qquad \Delta s^2 = \frac{1}{\theta^2} \sum_i \left(s_{cog} - s_i\right)^2 \theta_i^2 \qquad \Rightarrow V(s_{cog}) = \begin{pmatrix} 1 & 0 \\ 0 & \Delta s^2 \end{pmatrix} \theta^2 \qquad \left(\theta^2 = \sum_i \theta_i^2\right)$$

  - At large distance ($(s-s_{cog})^2 \gg \Delta s^2$) like thin scat. at center of gravity

$$\mathbf{V}(s) \approx \begin{pmatrix} 1 & s-s_{cog} \\ s-s_{cog} & (s-s_{cog})^2 \end{pmatrix} \theta^2$$

Material vs measurement (I)

- Simple case
  - Material and measurement at same place (e.g. Si sensor)
- Material u_i without measurement
  - Example
    - Z offset for sensor with only Rφ measurement
  - No difference for broken line

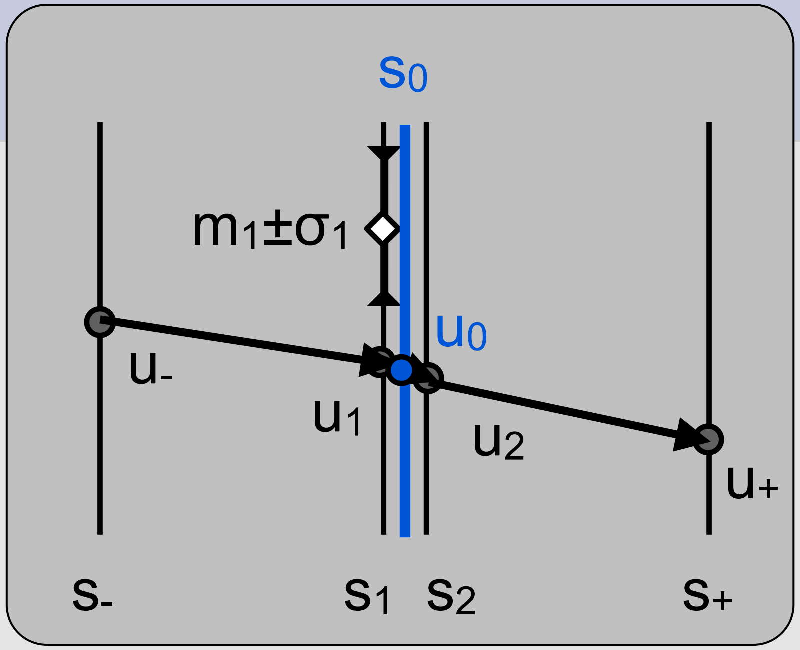
Material vs measurement (II)

- Measurement u_i without material
  - Example
    - Material COG of double layer is between measurements
  - No variance V_β at u_i
    - u_i isn't free (fit) parameter anymore
  - β_i equation can be solved for u_i

$$u_i = u_{i-1}\frac{s_{i+1}-s_i}{s_{i+1}-s_{i-1}} + u_{i+1}\frac{s_i - s_{i-1}}{s_{i+1}-s_{i-1}} + \tfrac{1}{2}(s_i - s_{i-1})(s_i - s_{i+1})\kappa$$

  - Curvature from (Kalman) fit usually correct
    - Correction $\kappa \approx 0 \Rightarrow$ linear interpolation sufficient
  - new Applied for combined layers in coarse broken lines

Double layers



- Combine (s_1, s_2) to one layer (s_0)
  - Assume same material $V_{\beta 1} = V_{\beta 2}$
  - $s_0 = (s_1+s_2)/2$, $V_{\beta 0}=V_{\beta 1}+V_{\beta 2}$
  - Broken line described by (u_-, u_0, u_+), $V_{\beta 0}$ (and κ)
- Offsets u_1, u_2 at measurements
  - Linear interpolation of (u_-, u_0, u_+)

$$u_1 = u_0 \frac{s_1 - s_-}{s_0 - s_-} + u_- \frac{s_0 - s_1}{s_0 - s_-}$$

$$u_2 = u_0 \frac{s_+ - s_2}{s_+ - s_0} + u_+ \frac{s_2 - s_0}{s_+ - s_0}$$

- Robust
  - Works even for $s_1=s_2$
    - Broken line with separate layers would fail $(1/(s_1-s_2)\,!)$