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HEP-Theory | **Homework 2** | **Date: 26.07.2019**

Exercice 1: EM action for photons

The photon field strength is defined to be

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,,$$

where the 4-vector A^μ is defined from the scalar and vector potential $(\phi, \vec{A})$.

a) From the classical EM definition of the electric and magnetic fields from the scalar and vector potential

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \wedge \vec{A}$$

show that

$$\vec{E}_i = -F_{i0} \quad \vec{B}_i = -\frac{1}{2}\epsilon_{ijk}F_{jk}$$

b) Derive the expression of the gauge field Lagrangian density, $-\frac{1}{4\pi}F_{\mu\nu}F^{\mu\nu}$, in terms of the electric and magnetic fields and recognise the usual expression of the energy density stored in the electromagnetic fields.

c) There is another Lorentz-invariant Lagrangian density that one can construct from the electromagnetic field:

$$\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}.$$

Compute this Lagrangian density in terms of $\vec{E}$ and $\vec{B}$.

Exercice 2: Two-body decays

Using energy and momentum conservation, show that in a 2-body decay, $A \to B + C$, the energy and momentum of the daughter particles in the rest frame of the mother particle are given by

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A}c^2, \quad E_C = \frac{m_A^2 + m_C^2 - m_B^2}{2m_A}c^2$$

$$p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A}c$$

with

$$\lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C).$$

Exercice 3: Standard Model: interactions and conservation laws
Are the following decays permitted in the Standard Model? If not, why?

1. $n \to p\mu^-\bar{\nu}_\mu$
2. $\mu^- \to e^-e^-e^+$
3. $n \to p\nu_e\bar{\nu}_e$
4. $p \to e^+\pi^0$
5. $\pi^0 \to \gamma\gamma$
6. $\tau^- \to \mu^-\gamma$
7. $K^0 \to \mu^+e^-$
8. $\mu^- \to \pi^-\nu_\mu$
9. $\mu^- \to e^-\gamma$
10. $\mu^- \to e^-\nu_e\bar{\nu}_\mu$

Exercice 4: BSM proton decay
With the particle content of the SM, baryon number is an accidental symmetry when restricting to renormalisable interactions. What is the mass dimension of the interactions that can induce a decay of the proton? Given that the current experimental lower bound on the lifetime of the proton in 10^{34} years, find the lower bound on the scale of these interactions.