Black holes, holography and microstates

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Black holes in General Relativity and Quantum Theory

Black holes, as solution of Einstein theory, have many properties. Some can be now observed and measured in astrophysical black holes, other, especially quantum ones, are still speculative:

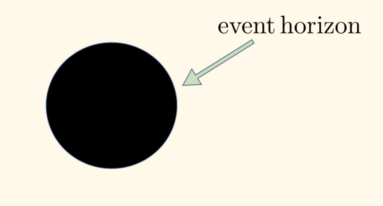
	certain	*quantum*
• Energy and angular momentum	✓	
• Gravitational waves	✓	
• Radiation		✓
• Heat		✓

Black hole thermodynamics

It is an old result that there is striking similarity between black hole mechanics and thermodynamics [Bekenstein;Hawking 70's]

$$dM = \frac{1}{8\pi G}\kappa dA \qquad \Longleftrightarrow \qquad dE = TdS$$

where A is the area of the horizon.

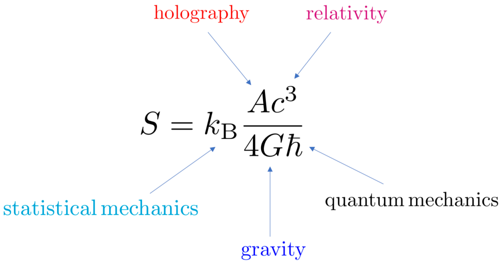


The black hole has a temperature and an entropy

$$T = \frac{\hbar\kappa}{2\pi} \qquad\qquad S = \frac{A}{4G\hbar}$$

Black hole thermodynamics

The Bekenstein-Hawking formula is quite remarkable



$$S = k_{\mathrm{B}} \frac{Ac^3}{4G\hbar}$$

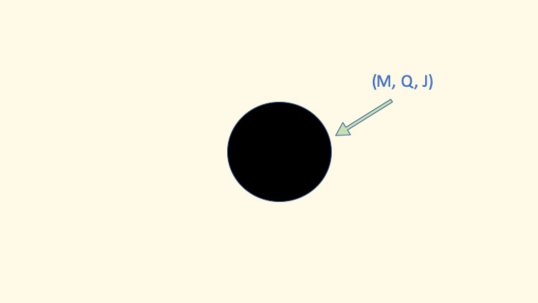
merging statistical mechanics, quantum physics and gravity

Black holes in effective theories

In general theories of quantum gravity, like string theory, a black hole is a solution of an effective theory of gravity, generalized electro-magnetic fields and scalars

$$\mathcal{L} = \sqrt{g}(R + g_{IJ}(\phi_i)F^I_{\mu\nu}F^{J\,\mu\nu} + h_{ij}(\phi)D_\mu\phi^i D^\mu\phi^j + V(\phi))$$

and are sources of mass, charges and angular momentum.



Statistical mechanics

The entropy is a function of the mass, charges and angular momentum only and, using Boltzmann relation, points to an integer degeneracy of states

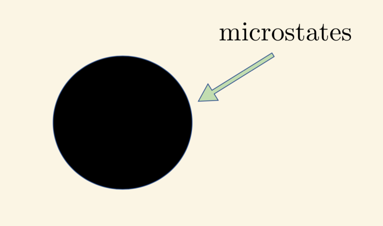
$$S(M, Q, J) = k_{\mathrm{B}} \log d(M, Q, J)$$

Extremal black holes have $T = 0$ and the entropy is just the number of ground states of the system.

To explain the microscopical origin of this degeneracy is the challenge for any theory of quantum gravity.

Holography

The Bekenstein-Hawking formula suggests that the microstates are localized at the horizon

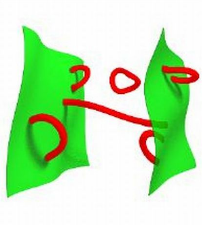


Holographic principle: a volume of space in quantum gravity can be described just in terms of boundary degrees of freedom [t'Hooft; Susskind 94]

Asymptotically flat black holes

One of the success of string theory is the microscopic counting of micro-states for a class of asymptotically flat black holes entropy
[Vafa-Strominger'96]

- The black holes are realized by putting together D-branes, extended objects that have gauge theories on the world-volume
- The entropy is obtained by counting states in the corresponding gauge theory



Asymptotically AdS black holes

No similar result for AdS black holes in $d \geq 4$ was known until very recently. But AdS should be simpler and related to holography:

- A gravity theory in AdS_{d+1} is the dual description of a CFT_d [Maldacena'97]

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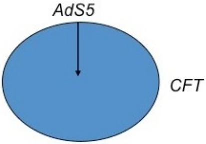
Concrete realization of the HOLOGRAPHIC PRINCIPLE: number of d.o.f of quantum gravity in a region increases with area (and not volume)



$$dr^2 + e^{2A(r)} dx_\mu^2$$

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  [Maldacena'97]

It is QUANTITATIVE: it computes explicitely mass spectrum of mesons and glueballs and Green functions.

$$\text{operators } O \qquad \Longrightarrow \qquad \text{gravity fields } h$$

$$(T_{\mu\nu}, J_\mu, \cdots) \qquad \Longrightarrow \qquad (g_{\mu\nu}, A_\mu, \cdots)$$

$$\left\langle e^{\int hO} \right\rangle_{QFT} = e^{S_{AdS_5}(\hat{h})}$$

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The entropy should be related to the counting of states in the dual CFT. People tried hard in the past: long standing problem.

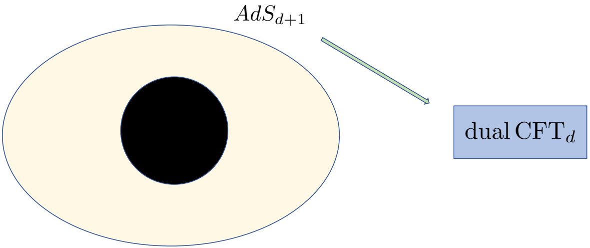
For AdS_4 and AdS_5 black holes this problem can be solved by using localization techniques that allow to evaluate exact quantities in supersymmetric gauge theories.

Outline

- **I: Field Theory Perspective and localization**
- **II: Microscoping counting for AdS Black Holes**
- **III: Overview**

PART I : Field Theory Perspective and localization

Dual Field Theory Perspective



black holes with
electric charges Q_a
angular momentum J_i

states in the CFT
with charge Q_a and spin J_i

supersymmetric black holes $\Longrightarrow$ $\begin{cases} T = 0 \\ M = \mu_a Q_a + \nu_i J_i \end{cases}$

Dual Field Theory Perspective

All information encoded in a gran-canonical partition function

$$Z(\Delta_a, \omega_i) = \mathrm{Tr}\left(e^{i(\Delta_a \mathcal{Q}_a + \omega_i \mathcal{J}_i)} e^{-\beta H}\right) \underset{T=0}{=} \sum_{Q_a, J_i} e^{S(Q_a, J_i)} e^{i(\Delta_a Q_a + \omega_i J_i)}$$

The entropy $S(Q_a, J_i) = \log$ number of states can be extracted as

$$e^{S(Q_a, J_i)} = \int d\Delta \int d\omega Z(\Delta_a, \omega_i) e^{-i(\Delta_a Q_a + \omega_i J_i)}$$

or, in the limit of large charges, by a saddle point, a Legendre Transform

$$S_{BH}(Q_a, J_i) \equiv \mathcal{I}(\Delta, \omega) = \log Z(\Delta_a, \omega_i) - i(\Delta_a Q_a + \omega_i J_i)\Big|_{\frac{d\mathcal{I}}{d\Delta} = \frac{d\mathcal{I}}{d\omega} = 0}$$

Dual Field Theory Perspective

The partition function become exactly computable in the supersymmetric case

- $Z^{\text{susy}}(\Delta_a, \omega_i) = \text{Tr}\left((-1)^F e^{i(\Delta_a \mathcal{Q}_a + \omega_i \mathcal{J}_i)} e^{-\beta H}\right)$
- cancellation between massive boson and fermions (Witten index)
- sum over supersymmetric ground states $H = 0$;

What's about $(-1)^F$? assume no cancellation between boson and fermions. True in the limit of large charges.

Localization

Exact quantities in supersymmetric theories with a charge $Q^2 = 0$ can be obtained by a saddle point approximation

$$Z = \int e^{-S} = \int e^{-S+t\{Q,V\}} \underset{t\gg 1}{=} e^{-\bar{S}|_{class}} \times \frac{\det_{fermions}}{\det_{bosons}}$$

$$\partial_t Z = \int \{Q, V\} e^{-S+t\{Q,V\}} = 0$$

Very old idea that has become very concrete recently, with the computation of partition functions on spheres and other manifolds supporting supersymmetry.

Localization

Localization ideas apply to path integral of Euclidean supersymmetric theories

- Compact space provides IR cut-off, making path integral well defined
- Localization reduces it to a finite dimensional integral, a matrix model

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- Compact space provides IR cut-off, making path integral well defined
- Localization reduces it to a finite dimensional integral, a matrix model

$$\int \prod_{i=1}^{N_1} du_i \prod_{j=1}^{N_2} dv_j \frac{\prod_{i<j} \sinh^2 \frac{u_i-u_j}{2} \sinh^2 \frac{v_i-v_j}{2}}{\prod_{i<j} \cosh^2 \frac{u_i-v_j}{2}} e^{\frac{ik}{4\pi}\left(\sum u_i^2 - \sum v_j^2\right)}$$

ABJM, 3d Chern-Simon theories, [Kapustin,Willet,Yakoov;Drukker,Marino,Putrov]

Localization

Carried out recently in many cases

- many papers on topological theories
- S^2, T^2
- S^3, $S^3/\mathbb{Z}_k$, $S^2 \times S^1$, Seifert manifolds
- S^4, $S^4/\mathbb{Z}_k$, $S^3 \times S^1$, ellipsoids
- S^5, $S^4 \times S^1$, Sasaki-Einstein manifolds

with addition of boundaries, codimension-2 operators, . . .

Pestun 07; Kapustin,Willet,Yakoov; Kim; Jafferis; Hama,Hosomichi,Lee, too many to count them all $\cdots$

Localization

In all cases, it reduces to a finite-dimensional matrix model on gauge variables, possibly summed over different topological sectors

$$Z_M(y) = \sum_{\mathfrak{m}} \int_C dx\, Z_{\text{int}}(x, y; \mathfrak{m})$$

with different integrands and integration contours.

When backgrounds for flavor symmetries are introduced, $Z_M(y)$ becomes an interesting and complicated function of y which can be used to test dualities

- Sphere partition function, Kapustin-Willet-Yakoov; $\cdots$
- Superconformal index, Spironov-Vartanov; Gadde,Rastelli,Razamat,Yan; $\cdots$
- Topologically twisted index, Benini,AZ; Closset-Kim; $\cdots$

PART II : Microscopic Counting for AdS black holes

Example I: Static black holes in $\mathrm{AdS}_4 \times S^7$

Black holes in M theory on $\mathrm{AdS}_4 \times S^7$: [Cacciatori, Klemm 08; Dall'Agata, Gnecchi; Hristov, Vandoren 10; Katmadas; Halmagyi 14; Hristov, Katmadas, Toldo 18]

- four electric $\mathfrak{q}_a$ and magnetic $\mathfrak{p}_a$ charges under $U(1)^4 \subset SO(8)$; only six independent parameters
- supersymmetry preserved with a topological twist
- entropy goes like $O(N^{3/2})$ and is a complicated function

$$S_{\mathrm{BH}}(\mathfrak{p}_a, \mathfrak{q}_a) \sim \sqrt{I_4(\Gamma, \Gamma, G, G) \pm \sqrt{I_4(\Gamma, \Gamma, G, G)^2 - 64 I_4(\Gamma) I_4(G)}}$$

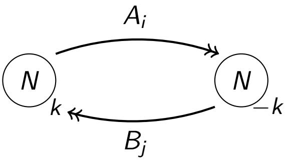
I_4 symplectic quartic invariant

$\Gamma = (\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4, \mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4)$ [Halmagyi 13]

$G = (0, 0, 0, 0, g, g, g, g)$

The dual field theory

The dual field theory to $\mathrm{AdS}_4 \times S^7$ is known: is the ABJM theory with gauge group $U(N) \times U(N)$



$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

Maximally supersymmetric Chern-Simons gauge theory in three dimensions

ABJM twisted index

Luckily enough, the topologically twisted partition function for ABJM can be evaluated using localization

$$Z^{\text{susy}}_{S^2\times S^1} = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\widetilde{\mathfrak{m}}\in\mathbb{Z}^N} \int \prod_{i=1}^{N} \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i}\, x_i^{k\mathfrak{m}_i}\, \tilde{x}_i^{-k\widetilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^{N} \left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right) \times$$

$$\times \prod_{i,j=1}^{N} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}\, y_1}}{1-\frac{x_i}{\tilde{x}_j}\, y_1}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{p}_1+1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}\, y_2}}{1-\frac{x_i}{\tilde{x}_j}\, y_2}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{p}_2+1}$$

$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}\, y_3}}{1-\frac{\tilde{x}_j}{x_i}\, y_3}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_3+1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}\, y_4}}{1-\frac{\tilde{x}_j}{x_i}\, y_4}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_4+1}$$

$$\prod_a y_a = 1\,, \qquad \sum \mathfrak{p}_a = 2$$

and solved in the large N limit. There is no cancellation between bosons and fermions and $\log Z = O(N^{3/2})$. [Benini-AZ; Benini-Hristov-AZ]

QFT/Gravity comparison

- dyonic static $AdS_4 \times S^7$ black holes:

Localization (topologically twisted index): [Benini,Hristov,AZ 05]

$$S(\mathfrak{p}_a, \mathfrak{q}_a) = \log Z(\Delta_a, \mathfrak{p}_a) - \sum_a i\Delta_a \mathfrak{q}_a \Big|_{crit} = \sum\nolimits_a i\mathfrak{p}_a \frac{\partial \mathcal{W}}{\partial \Delta_a} - i\Delta_a \mathfrak{q}_a \Big|_{crit}$$

twisted superpotential $\mathcal{W}_{\mathrm{on-shell}} = \frac{2}{3} i N^{3/2} \sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}$

$\sum_{a=1}^4 \Delta_a = 2\pi \qquad \mathrm{Re}\Delta_a \in [0, 2\pi]$

QFT computation = attractor mechanism in $\mathcal{N} = 2$ gauged supergravity:
[Ferrara-Kallosh-Strominger 96; Dall'Agata-Gnecchi 10]

$$S_{\mathrm{BH}}(\mathfrak{p}_a, \mathfrak{q}_a) = \log Z(X_a, \mathfrak{p}_a) - \sum_a iX_a \mathfrak{q}_a \Big|_{crit} = \sum\nolimits_a i\mathfrak{p}_a \frac{\partial \mathcal{F}}{\partial X_a} - iX_a \mathfrak{q}_a \Big|_{crit}$$

gauged supergravity prepotential $\mathcal{F} \sim \sqrt{X_1X_2X_3X_4}$

$\sum X_a = 2\pi$ horizon scalar fields

Example II: Rotating black holes in $\mathrm{AdS}_5 \times S^5$

Most famous BPS examples are asymptotic to $\mathrm{AdS}_5 \times S^5$

two angular momenta J_1, J_2 in AdS_5 $\qquad U(1)^2 \subset SO(4) \subset SO(2,4)$

three electric charges Q_I in S^5 $\qquad U(1)^3 \subset SO(6)$

with a constraint $F(J_i, Q_I) = 0$. They must rotate and preserves two supercharges.

$$S_{\rm BH} = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 - 2c(J_1 + J_2)} \qquad c = \frac{N^2-1}{4}$$

[Gutowski-Reall 04; Chong, Cvetic, Lu, Pope 05; Kunduri, Lucietti, Reall; Kim, Lee, 06]

The microstates correspond to states of given angular momentum and electric charge in $\mathcal{N} = 4$ SYM.

Entropy function for AdS_5 black holes

BPS entropy function [Hosseini,Hristov,AZ 17; Cabo-Bizet, Cassani, Martelli, Murthy 18]

$$\mathcal{S}_{\mathrm{BH}}(Q_I, J_i) = -i\pi(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} - 2\pi i\left(\sum_{I=1}^{3} Q_I\Delta_I + \sum_{i=1}^{2} J_i\omega_i\right)\Big|_{\bar{\Delta}_I,\bar{\omega}_i}$$

with $\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = \pm 1$

The critical values $\bar{\Delta}_I, \bar{\omega}_i$ are complex but, quite remarkably, the extremum is a *real* function of the black hole charges.

Field Theory Comparison

Entropy scales like $O(N^2)$ for $Q_I, J_i \sim N^2$. The superconformal index

$$\mathrm{Tr}(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{2\pi i(\Delta_I Q_I + \omega_i J_i)} = \oint \frac{dz_i}{2\pi i z_i} \prod_{1\le i<j\le N} \frac{\prod_{k=1}^3 \Gamma_e(y_k (z_i/z_j)^{\pm 1}; p, q)}{\Gamma_e((z_i/z_j)^{\pm 1}; p, q)}$$

- For real fugacities: $\log Z = O(1)$. Large cancellations between bosons and fermions. Long standing puzzle [Kinney, Maldacena, Minwalla, Raju 05]
- For complex fugacities (like the ones in sugra) is consistent with

$$\log Z(\Delta, \omega) \sim -i\pi(N^2 - 1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} \qquad \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = \pm 1$$

[Cardy limit $\omega_i \ll 1$: Choi, Kim, Kim, Nahmgoong]

[Modified index/partition function: Cabo-Bizet, Cassani, Martelli, Murthy]

[Large N and $J_1 = J_2$: Benini, Milan 18; Cabo-Bizet, Murthy 19]

PART III : Overview

Some universality?

We can embed BPS black holes in all maximally supersymmetric $\mathrm{AdS}_{d\geq 4}$ backgrounds

• M theory on $\mathrm{AdS}_4 \times S^7$		ABJM theory
• type IIB on $\mathrm{AdS}_5 \times S^5$	$\Longrightarrow$	$\mathcal{N} = 4$ SYM
• massive IIA on $\mathrm{AdS}_6 \times_W S^4$		5d UV fixed point
• M theory on $\mathrm{AdS}_7 \times S^4$		$(2,0)$ theory

Entropy controlled by anomalies in even dimensions and sphere partition functions in odd dimensions

electrically charged and rotating BH

$AdS_4 \times S^7$	$\mathcal{F}(\Delta_a) = \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$ $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) = \frac{4\sqrt{2}N^{3/2}}{3} \frac{\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}}{\omega_1}$ $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \omega_1 = 2\pi$
$AdS_5 \times S^5$	$\mathcal{F}(\Delta_a) = \Delta_1\Delta_2\Delta_3$ $\Delta_1 + \Delta_2 + \Delta_3 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) = -i\frac{N^2}{2} \frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2}$ $\Delta_1 + \Delta_2 + \Delta_3 + \omega_1 + \omega_2 = 2\pi$
$AdS_6 \times_W S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^{3/2}$ $\Delta_1 + \Delta_2 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) \sim N^{5/2} \frac{(\Delta_1\Delta_2)^{3/2}}{\omega_1\omega_2}$ $\Delta_1 + \Delta_2 + \omega_1 + \omega_2 = 2\pi$
$AdS_7 \times S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^2$ $\Delta_1 + \Delta_2 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) = -i\frac{N^3}{24} \frac{(\Delta_1\Delta_2)^2}{\omega_1\omega_2\omega_3}$ $\Delta_1 + \Delta_2 + \omega_1 + \omega_2 + \omega_3 = 2\pi$

[Disclaimer: normalizations and signs for sake of exposition]

magnetically charged BH and black strings

$AdS_4 \times S^7$	$\mathcal{F}(\Delta_a) = \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$ $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 2$	$\log \mathcal{Z} = -\frac{2\sqrt{2}N^{3/2}}{3} \sum_{a=1}^4 \mathfrak{p}_a \frac{\partial \mathcal{F}(\Delta)}{\partial \Delta_a}$ $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 2\pi$
$AdS_5 \times S^5$	$\mathcal{F}(\Delta_a) = \Delta_1\Delta_2\Delta_3$ $\Delta_1 + \Delta_2 + \Delta_3 = 2$	$\log \mathcal{Z} = -\frac{N^2}{2\beta} \sum_{a=1}^3 \mathfrak{p}_a \frac{\partial \mathcal{F}(\Delta)}{\partial \Delta_a}$ $\Delta_1 + \Delta_2 + \Delta_3 = 2\pi$
$AdS_6 \times_W S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^{3/2}$ $\Delta_1 + \Delta_2 = 2$	$\log \mathcal{Z} \sim N^{5/2} \sum_{a,b=1}^2 \mathfrak{p}_a \tilde{\mathfrak{p}}_b \frac{\partial^2 \mathcal{F}(\Delta)}{\partial \Delta_a \partial \Delta_b}$ $\Delta_1 + \Delta_2 = 2\pi$
$AdS_7 \times S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^2$ $\Delta_1 + \Delta_2 = 2$	$\log \mathcal{Z} \sim \frac{N^3}{\beta} \sum_{a,b=1}^2 \mathfrak{p}_a \tilde{\mathfrak{p}}_b \frac{\partial^2 \mathcal{F}(\Delta)}{\partial \Delta_a \partial \Delta_b}$ $\Delta_1 + \Delta_2 = 2\pi$

[Note: AdS_5 and AdS_7 refer to black strings in Cardy limit]

Conclusions

The main message of this talk is that there is still a lot of interesting physics in AdS black holes, involving general relativity, quantum field theory and holography.

A long way to go

- finite N matrix models $\Longrightarrow$ quantum black holes
- extremal non-supersymmetric and near-BPS black holes

Huge literature about asymptotically flat BH (remarkable precision tests including higher derivatives). The story about AdS BH is just begun.

Thank you for the attention !