Signs for the onset of gluon saturation in exclusive photo-production of vector mesons at the LHC

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Outline

1. Motivation: the gluon at low x

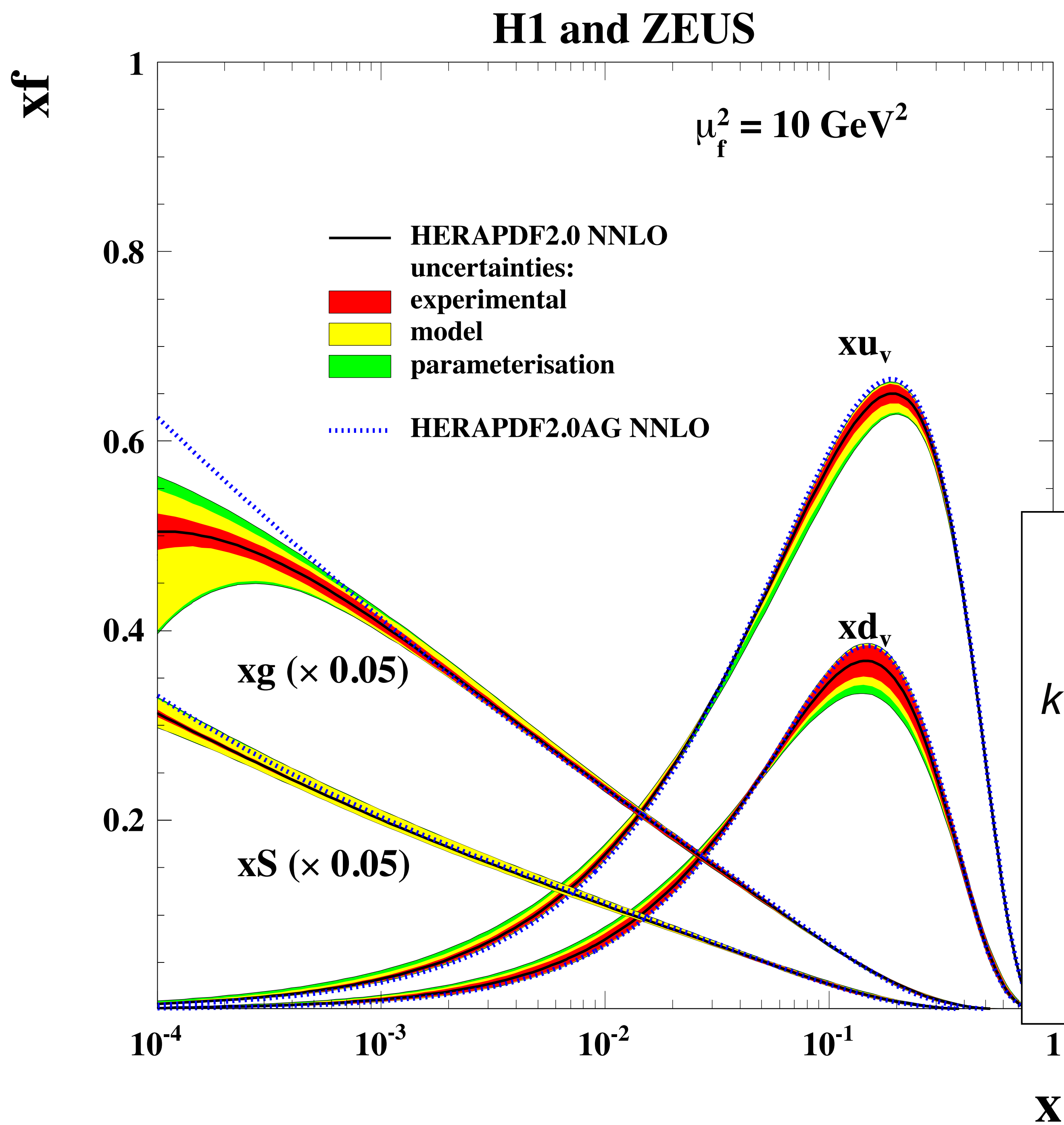
2. Exclusive photo-production of vector mesons at HERA and LHC

3. How strong is the growth of the low density gluon?

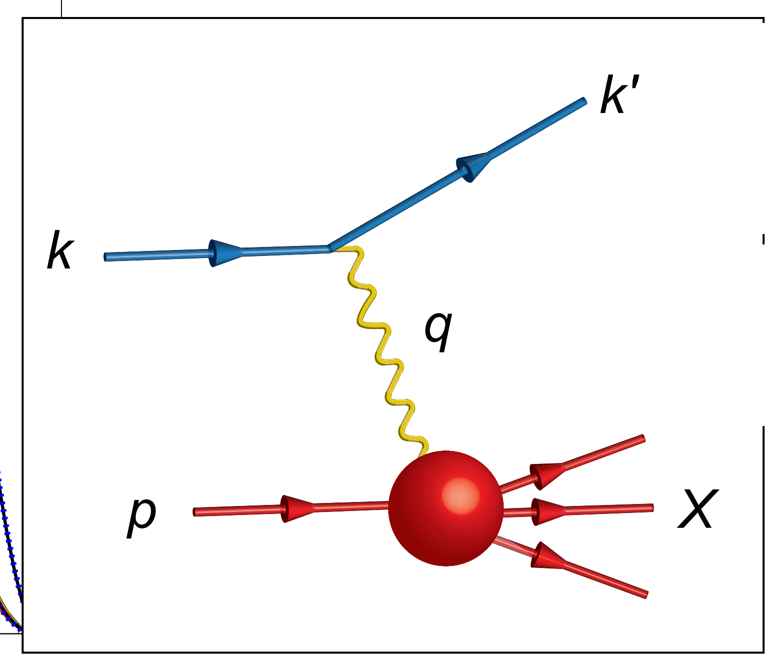
4. Conclusions

Motivation: the gluon at low x

Experimental fact:



power like growth of the gluon distribution at low x



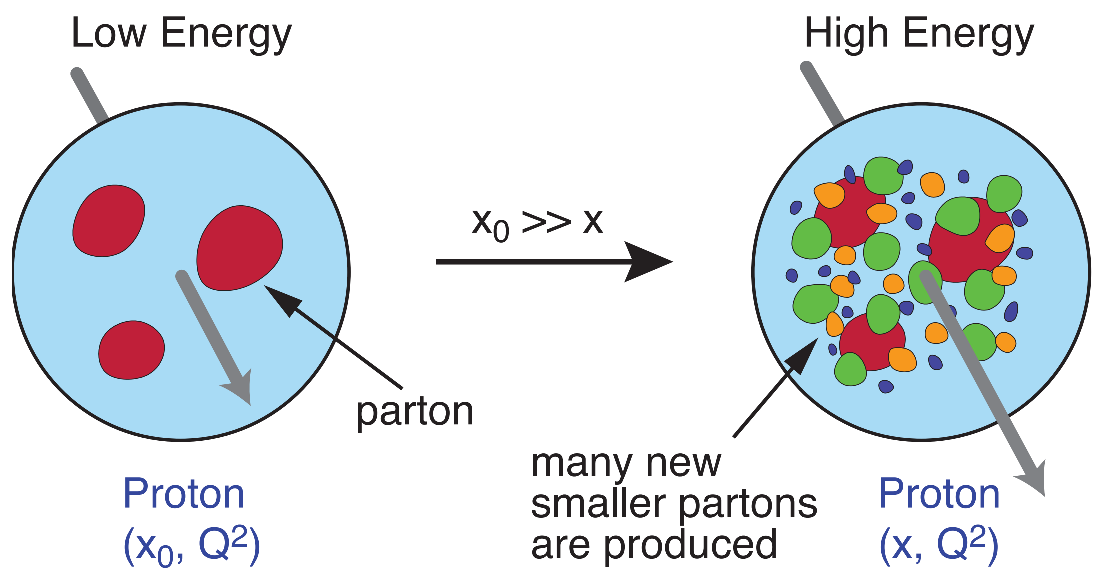
Photon virtuality $Q^2 = -q^2$

Bjorken $\boldsymbol{x} = \dfrac{Q^2}{2p \cdot q}$

[H1 and ZEUS collaborations; 1506.06042]

low x = high energy at fixed hard scale Q

underlying mechanism:

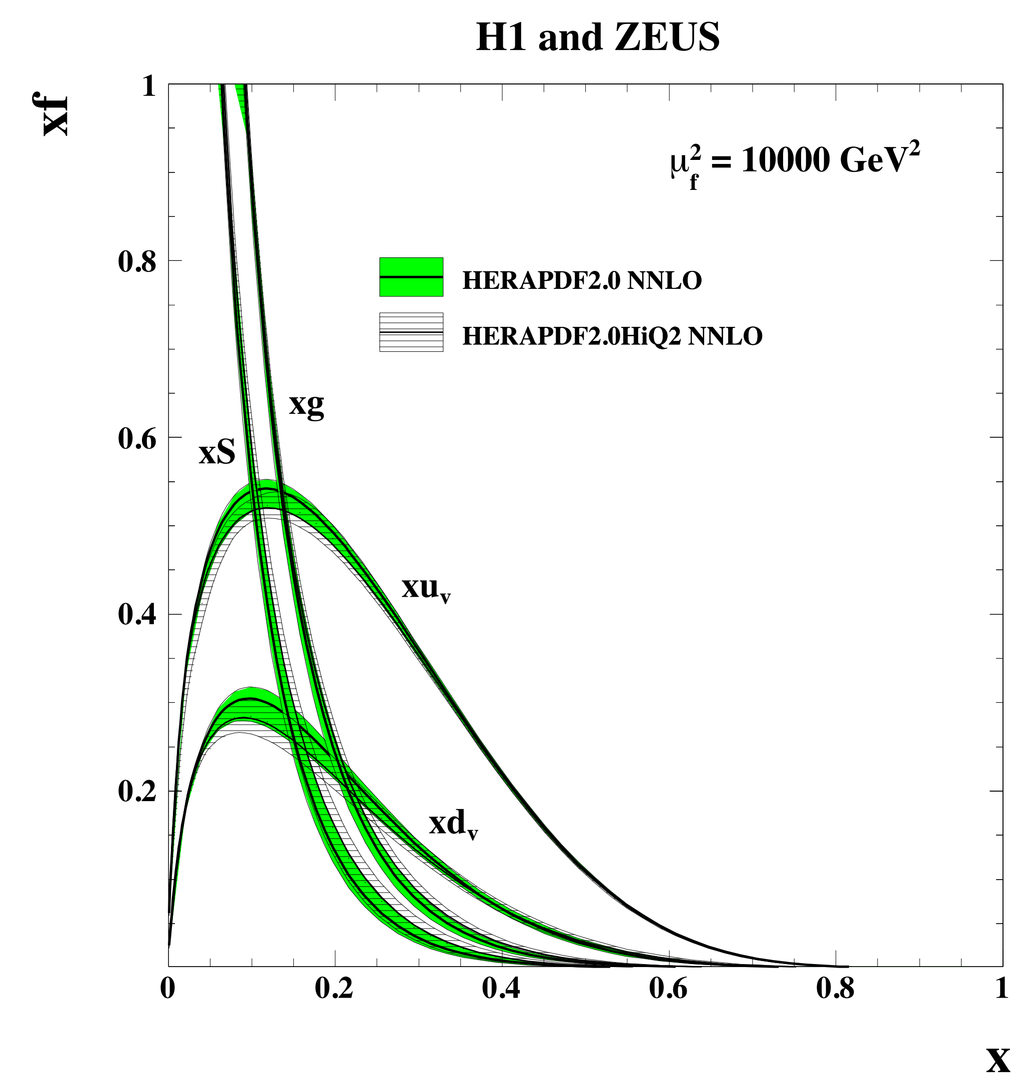


- proton made up of quanta that fluctuate in and out of existence

- at low x: fluctuations time dilated on scales of strong interactions

⇛ long lived gluons radiate further small x gluons

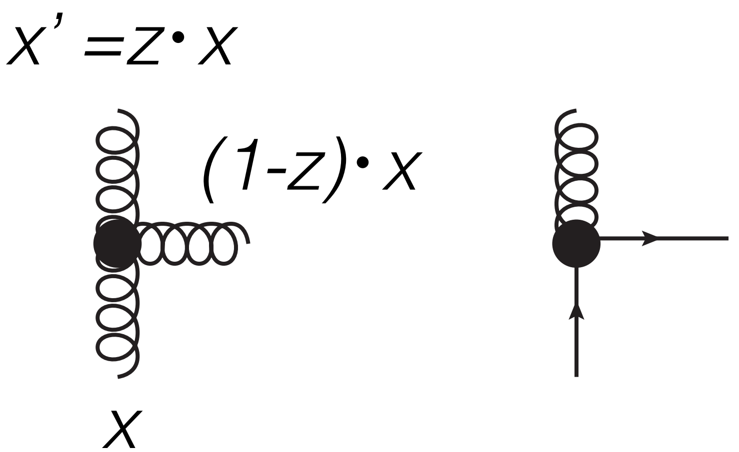
⇛ power-like rise of gluon and sea-quark distribution



Perturbative QCD:

Manifest in DGLAP evolution:

splitting functions enhance low x region



$P_{gg}(z)$, $P_{qg}(z)$ $\quad \sim \dfrac{\alpha_s}{z}$

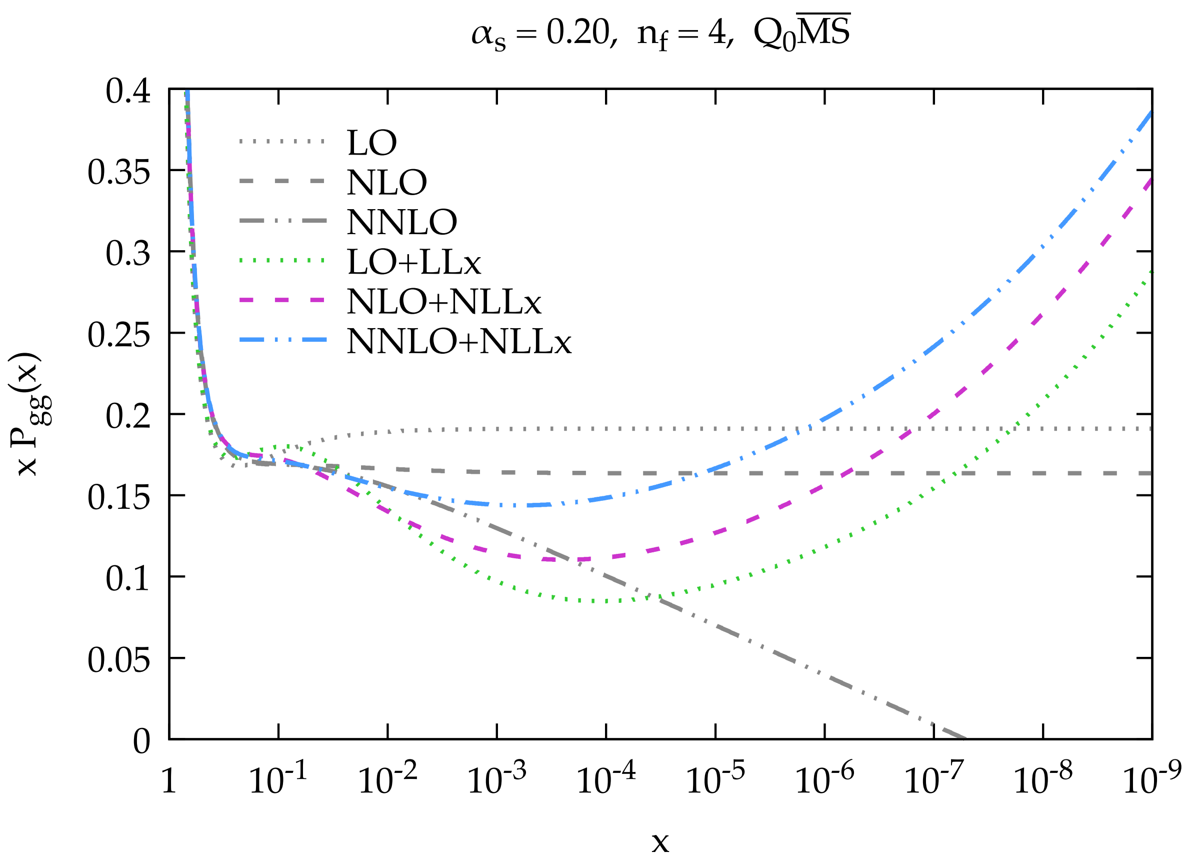
Systematic resummation of logarithmic enhanced terms to all orders: BFKL

$$\alpha_s^n \ln^n \frac{1}{x}, \qquad \alpha_s^{n+1} \ln^n \frac{1}{x}$$

LL NLL

[Fadin, Lipatov, Kuraev, PLB429 (1998) 127],
[Balitsky, Lipatov, Sov.J.Nucl.Phys. 28 (1978)]

[Fadin, Lipatov; hep-ph/ 9802290]
[Camici, Ciafaloni; hep/ph/9707390]

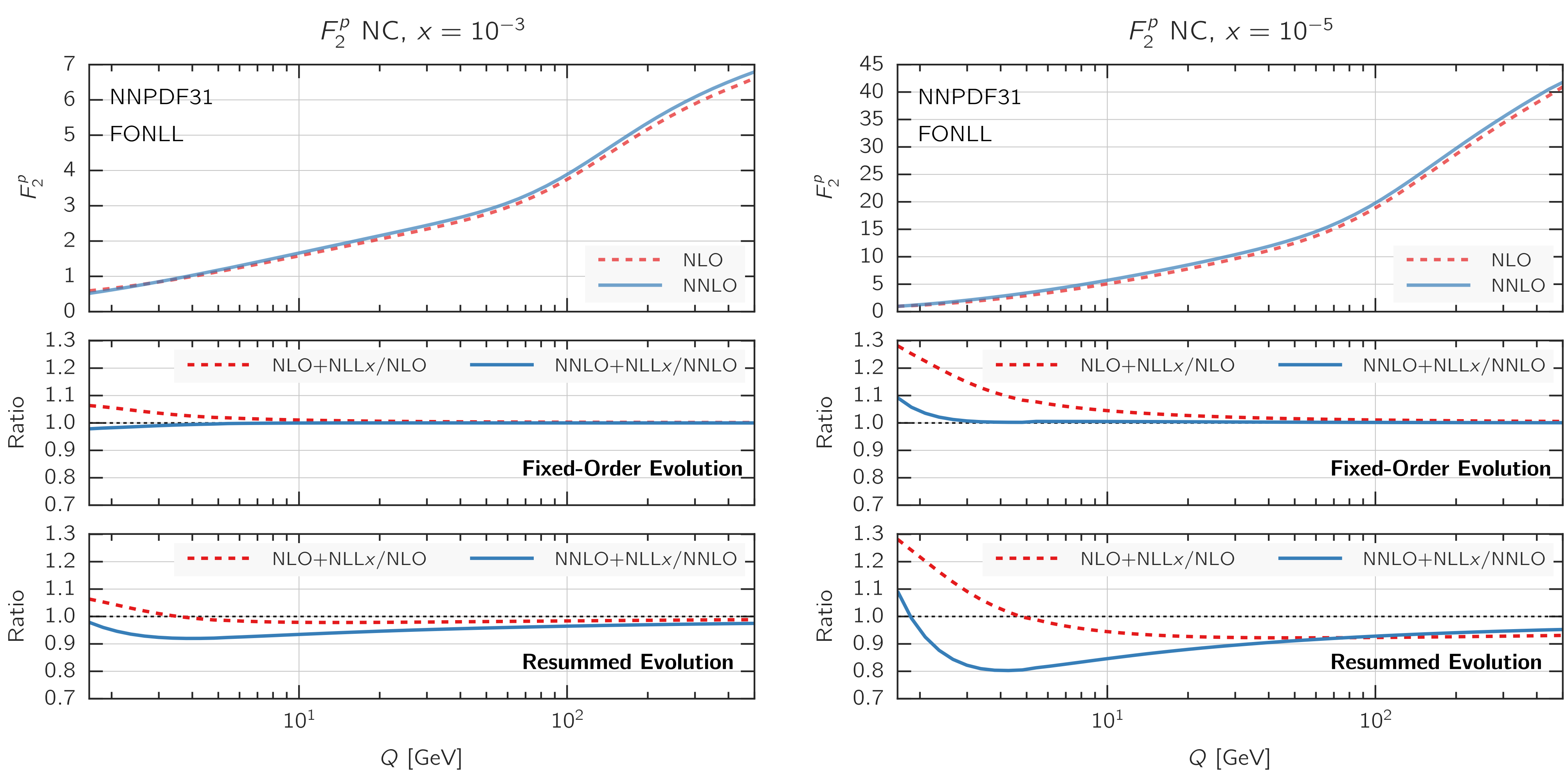


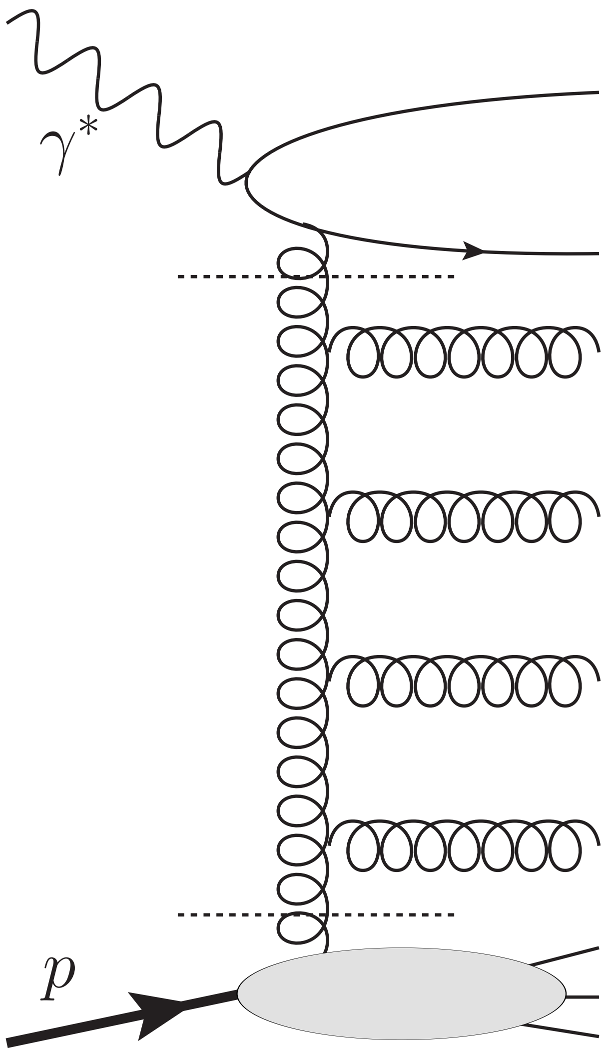
BFKL can be used to resum DGLAP evolution

[Catani, Hautmann; hep-ph/9405388]

realization in (recent) pdf fits:

[Ball, Bertone, Bonvini, Marzani, Rojo, Rottoli; 1710.05935]





direct application of high energy factorization:

- k_T dependent factorization of cross-sections (vs. momentum fractions)
- x-dependence from *perturbative* BFKL evolution (+ collinear resummation)
- initial conditions: model & fit k_T distribution at large x

$$F_2(x, Q^2) = \int_0^\infty d\boldsymbol{k}^2 \int_0^\infty \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \Phi_2\left(\frac{\boldsymbol{k}^2}{Q^2}\right) \mathcal{F}^{\text{DIS}}_{\text{BFKL}}(x, \boldsymbol{k}^2, \boldsymbol{q}^2) \Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)$$

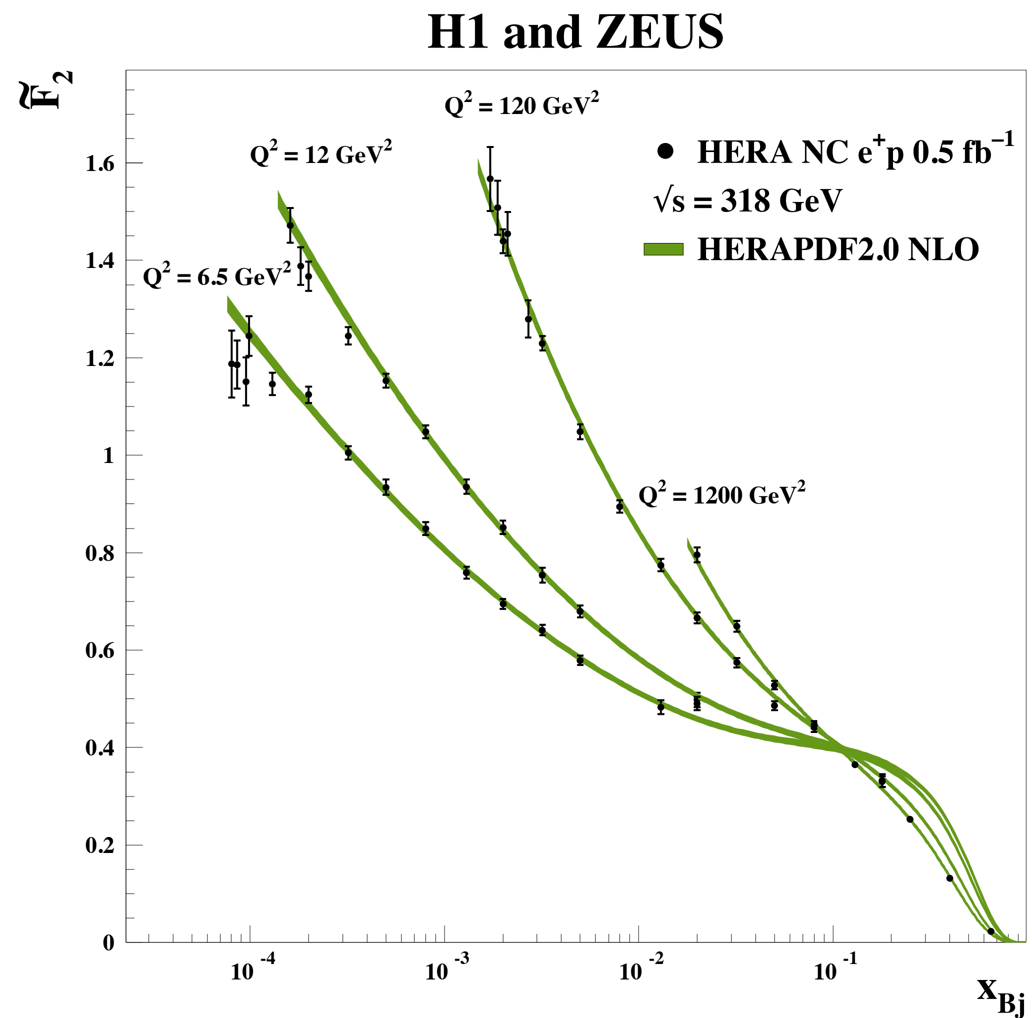
initial distribution e.g.

$$\Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}, \delta\right) = \frac{\mathcal{C}}{\pi\Gamma(\delta)}\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)^\delta e^{-\frac{\boldsymbol{q}^2}{Q_0^2}}$$

x-dependence

allows for fit of combined HERA data

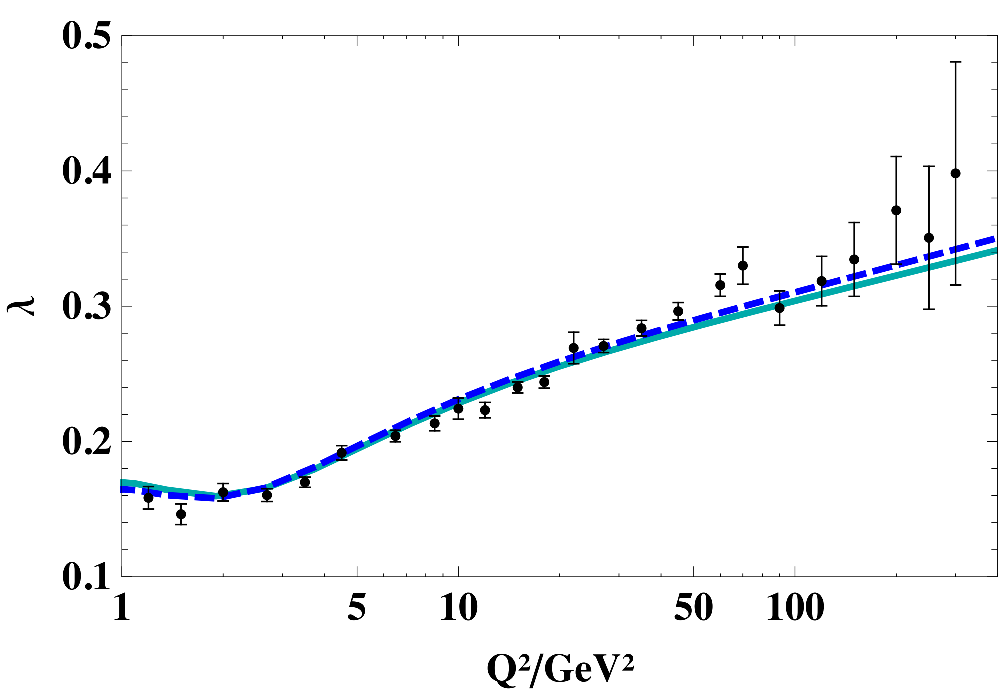
[MH, Salas, Sabio Vera; 1301.5283]



and describes Pomeron intercept $F_2 \sim x^{-\lambda}$

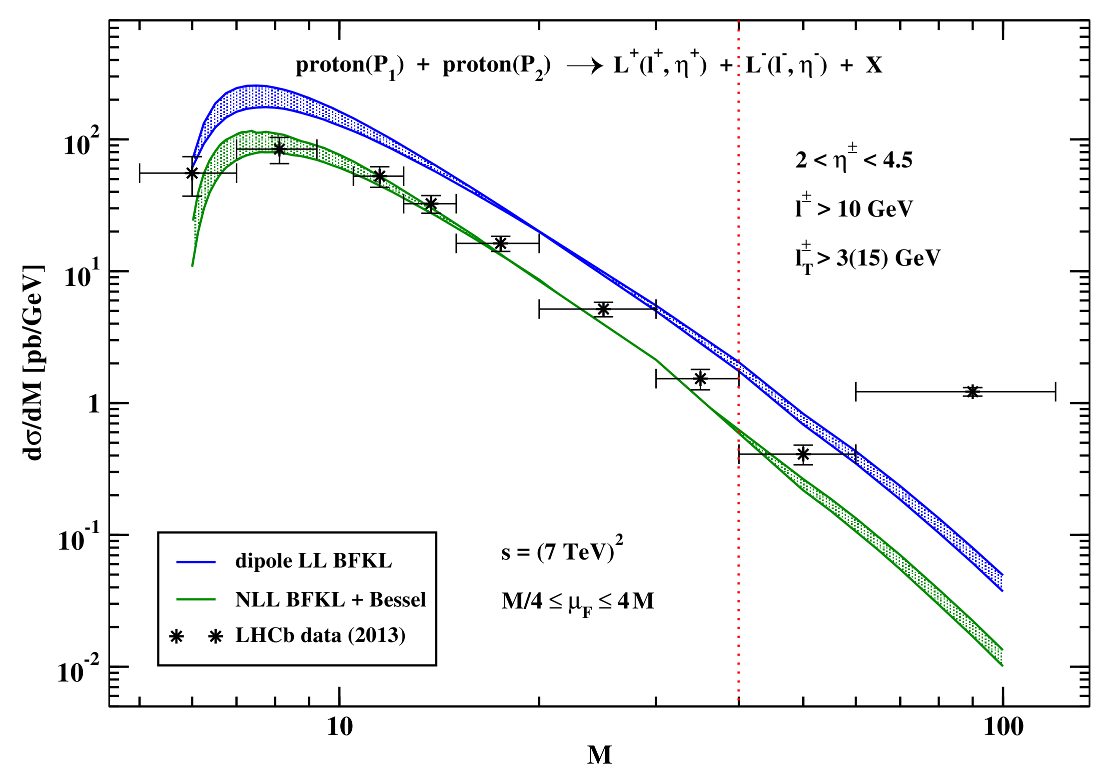
NLO BFKL descriptions:

[Kowalski, Lipatov, Ross, Watt; 1005.0355]
[MH, Sabio Vera, Salas; 1209.1353]



at LHC: *e.g.* DY

[Celiberto, Gordo Gómez, Sabio Vera; 1808.09511]



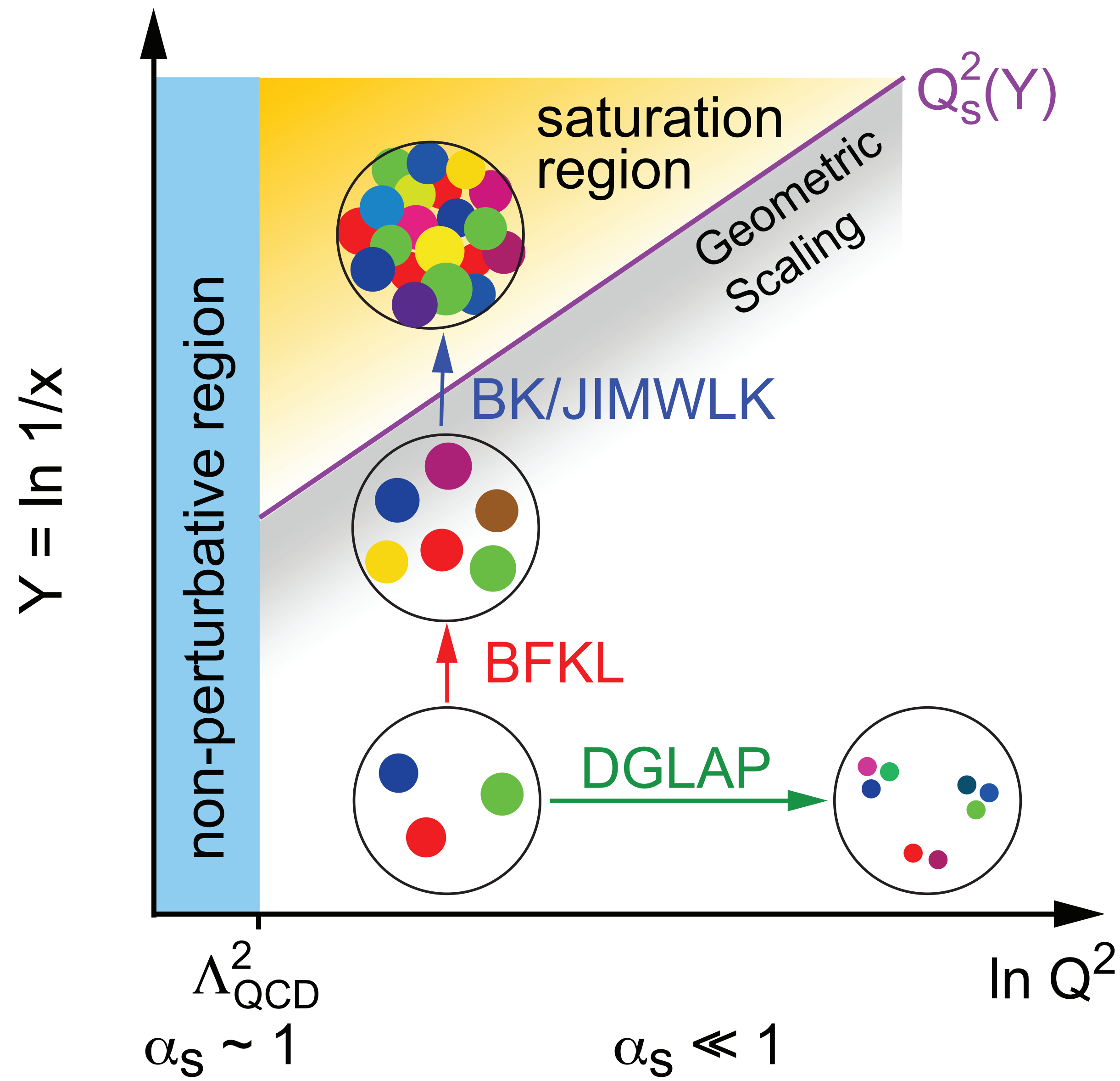
it seems we understand the growth of the gluon distribution at low x

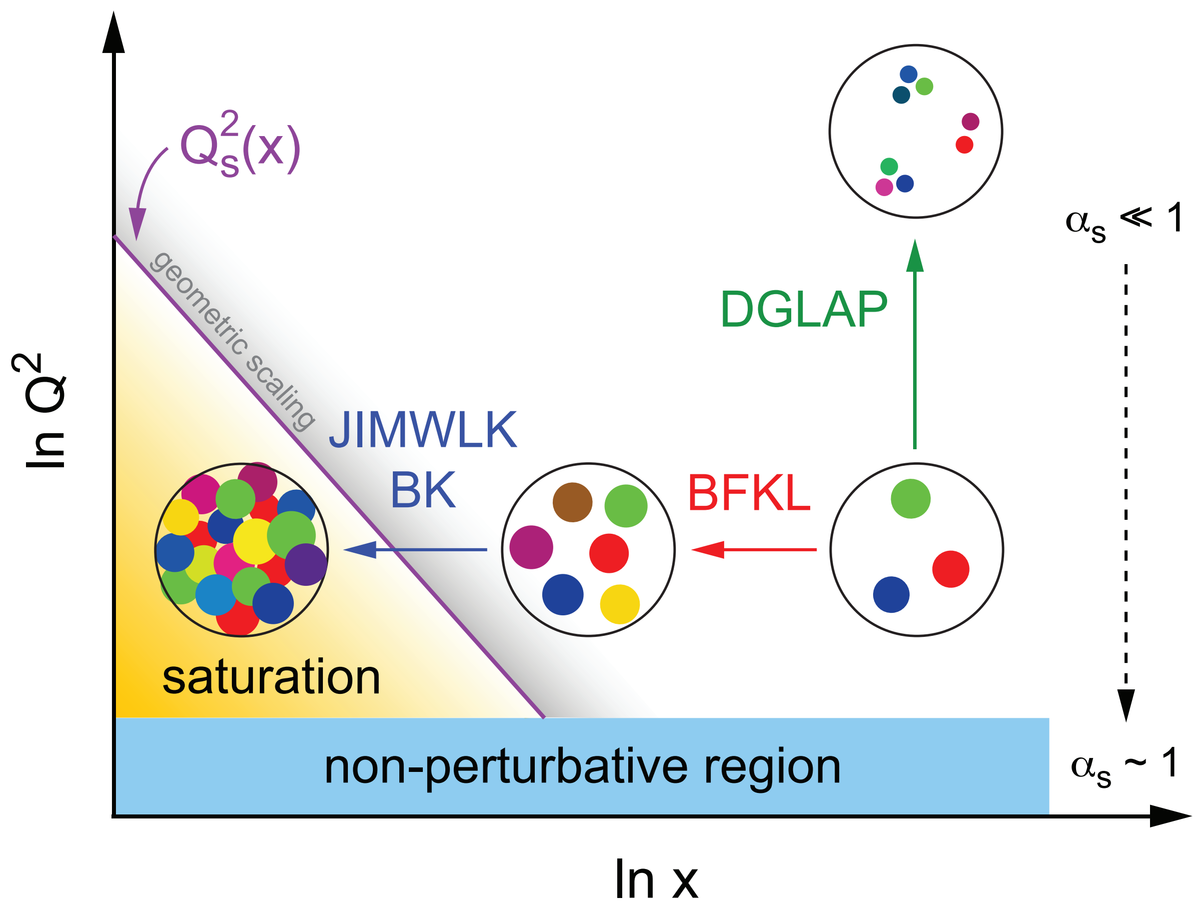
but there is one essential problem:

Saturation of gluon densities at low x

[Gribov, Levin & Ryskin Phys. Rept. 100 (1983)]
Color Glass Condensate effective theory: [McLerran, Venugopalan PRD 49 (1994) 3352]

- if continued forever, power like growth of gluon violates unitarity bounds
- power-like growth drives us eventually into region of high parton densities
- can show: high densities slow down/ stop growth of low x gluon: _saturation_



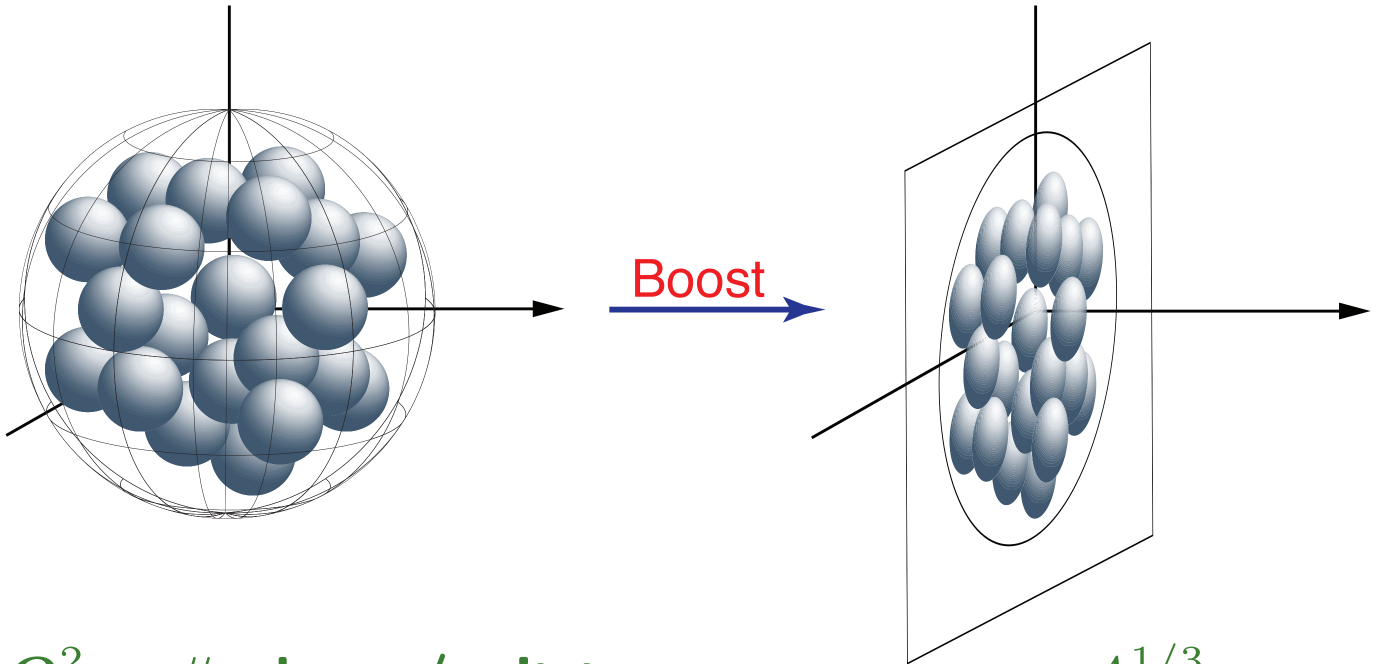


partons in theory: point-like particle

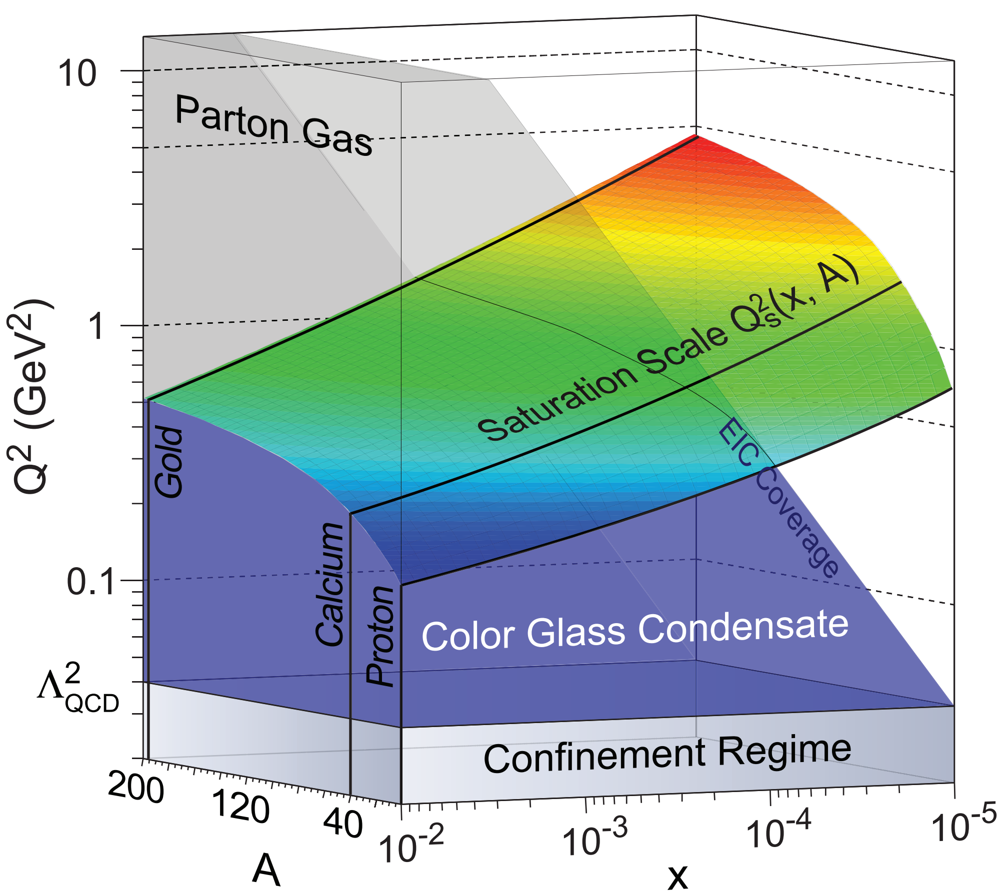
partons in DIS at hard scale Q: effective size 1/Q

- at some x « 1: partons will start to overlap → high density effects become relevant/‘recombination’
- implies: system characterized by an x-dependent <u>saturation scale</u> $Q_s(x)$
- grows with energy & can reach in principle perturbative values

$$Q_s \sim x^{-\Delta}$$



$Q_s^2 \sim \#$ **gluons/unit transverse area** $\sim A^{1/3}$



- of particular interest: collision which involve large nuclei
- densities (& therefore saturation scale) naturally enhanced
- but: difficult to verify effects directly (complexity!)

Phenomenological evidence: geometric scaling

saturation scale contains the entire x-dependence → proton structure functions depend only on a single parameter **τ** instead of x and Q^2

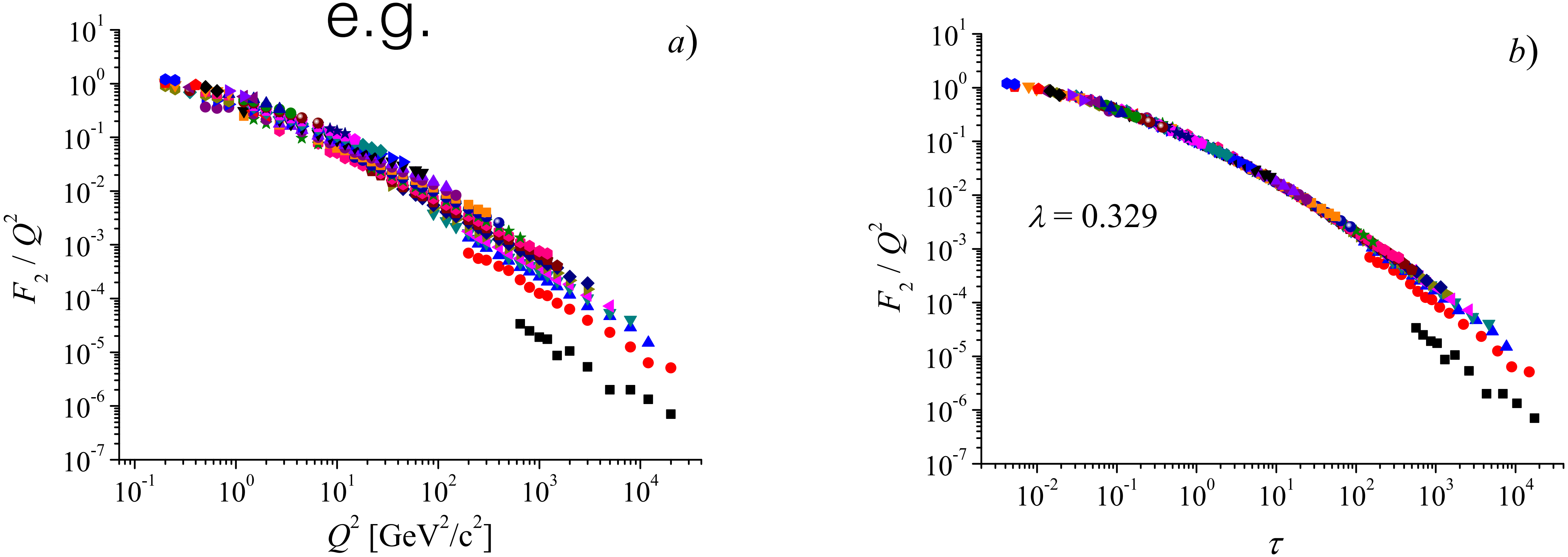
$$\sigma_{\gamma^* p}(x_i, Q^2) = \frac{1}{Q_0^2} F(\tau)$$

$$\tau = Q^2 / Q_s^2(x)$$

$$Q_s^2(x) = Q_0^2 (x/x_0)^{-\lambda}$$

[Praszalowicz, Stebel; JHEP 1303, 090 (2013); JHEP 1304, 169 (2013)]

e.g.



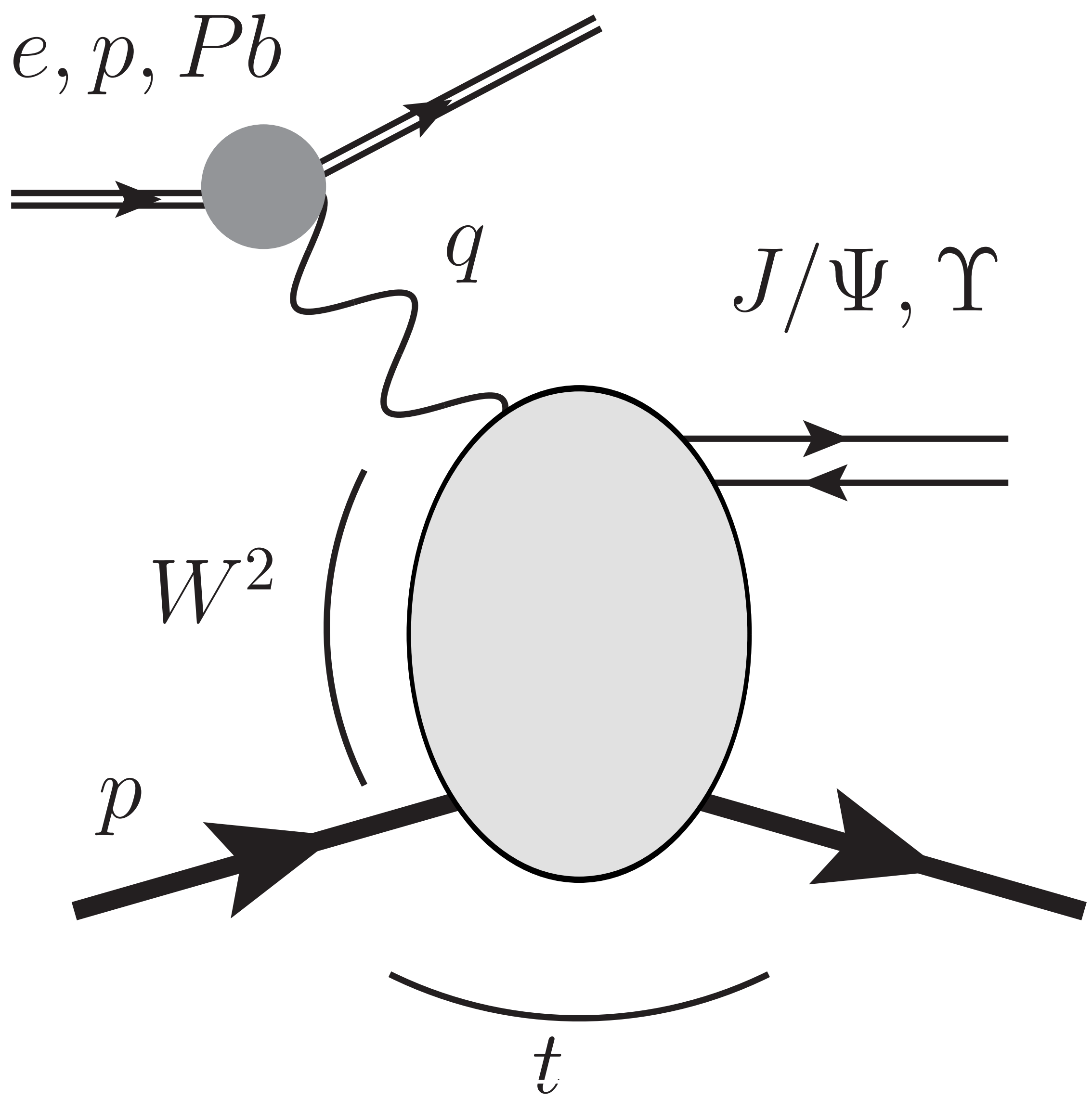
- emergence of saturation scale also found in numerical solution of non-linear evolution equations for the high density region (BK, JIMWLK)
- Can use those equations as well as saturation models to fit e.g. low x HERA data
- Can use those fits to describe data *e.g*. de-correlation of forward di-hadrons/dijets
- various successful application to heavy ion collisions and description of high multiplicity events

Question:

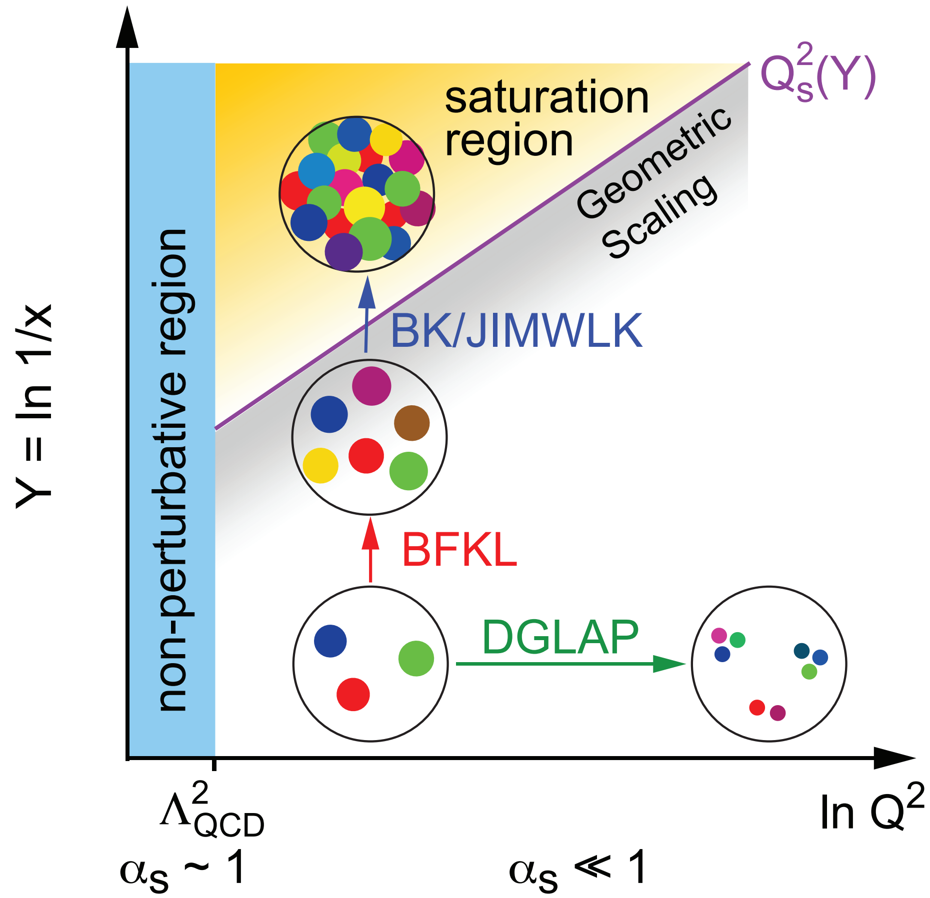
→ Can we see the emergence of saturation it in a more direct way? As a consequence of evolution?

Exclusive photo-production of vector mesons at HERA and LHC

A process to explore the low x gluon at the LHC: exclusive photo-production of J/Ψs



- hard scale: charm mass (small, but perturbative)
- reach up to $x \gtrsim .5 \cdot 10^{-6}$
- perturbative cross-check: ϒ (b-mass)
- measured at **LHC** (LHCb, ALICE, CMS) & **HERA** (H1, ZEUS)



our study:

- linear low x (BFKL) vs. non-linear low x (BK)
- failure of BFKL = sign for BK
  → high & saturated gluon

details:

BK evolution for dipole amplitude $N(x,r) \in [0,1]$ [related to gluon distribution]

kernel calculated in pQCD

non-linear term relevant for N~1 (=high density)

$$\frac{dN(x,r)}{d\ln\frac{1}{x}} = \int d^2\boldsymbol{r}_1 K(\boldsymbol{r},\boldsymbol{r}_1)\left[N(x,r_1)+N(x,r_2)-N(x,r) - N(x,r_1)N(x,r_2)\right]$$

linear BFKL evolution = subset of complete BK linear low x evolution as benchmark →requires precision

use: HSS NLO BFKL fit [MH, Salas, Sabio Vera; 1301.5283]

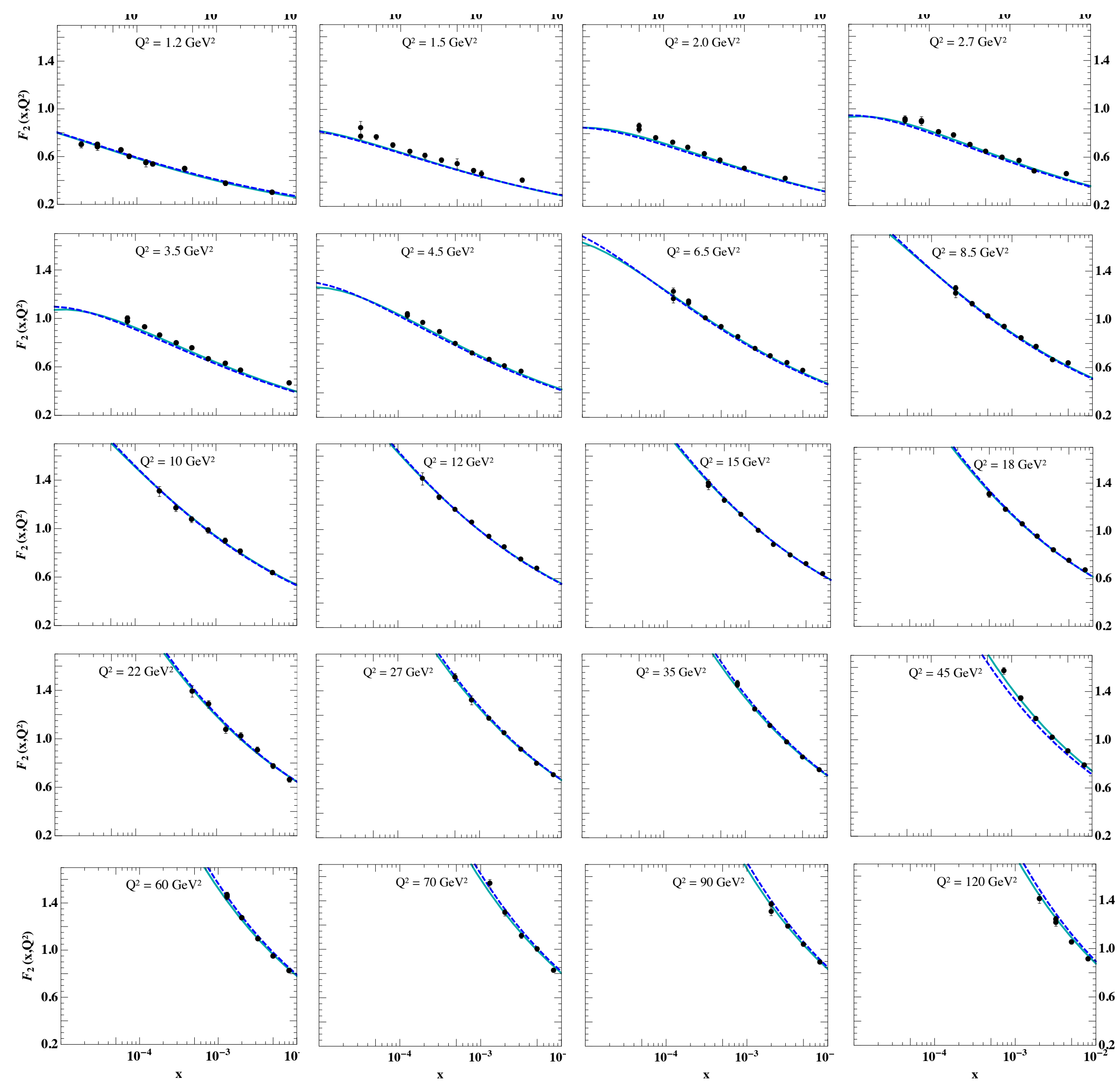
- uses NLO BFKL kernel

  [Fadin, Lipatov; PLB 429 (1998) 127]

  + resummation of collinear logarithms

- initial kT distribution from fit to combined HERA data

  [H1 & ZEUS collab. 0911.0884]



$$G\left(x,\boldsymbol{k}^2,M\right) = \frac{1}{\boldsymbol{k}^2}\int\limits_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{d\gamma}{2\pi i}\;\hat{g}\left(x,\frac{M^2}{Q_0^2},\frac{\overline{M}^2}{M^2},\gamma\right)\;\left(\frac{\boldsymbol{k}^2}{Q_0^2}\right)^{\gamma}$$

re-introduce two scales: hard scale of process (M) and scale of running coupling ($\overline{M}$)

$\hat{g}$: operator in γ space!

$$\hat{g}\left(x,\frac{M^2}{Q_0^2},\frac{\overline{M}^2}{M^2},\gamma\right) = \frac{\mathcal{C}\cdot\Gamma(\delta-\gamma)}{\pi\Gamma(\delta)}\;\cdot\;\left(\frac{1}{x}\right)^{\chi\left(\gamma,\frac{\overline{M}^2}{M^2}\right)}\;\cdot$$
$$\left\{1+\frac{\bar{\alpha}_s^2\beta_0\chi_0\left(\gamma\right)}{8N_c}\log\left(\frac{1}{x}\right)\left[-\psi\left(\delta-\gamma\right)+\log\frac{M^2}{Q_0^2}-\partial_\gamma\right]\right\},$$

resummed NLO BFKL eigenvalue with optimal scale setting ($\to$ modifies $\chi_1(\gamma)$):

$$\chi\left(\gamma,\frac{\overline{M}^2}{M^2}\right) = \bar{\alpha}_s\chi_0\left(\gamma\right)+\bar{\alpha}_s^2\tilde{\chi}_1\left(\gamma\right)-\frac{1}{2}\bar{\alpha}_s^2\chi_0'\left(\gamma\right)\chi_0\left(\gamma\right)$$
$$+\chi_{RG}(\bar{\alpha}_s,\gamma,\tilde{a},\tilde{b})-\frac{\bar{\alpha}_s^2\beta_0}{8N_c}\chi_0(\gamma)\log\frac{\overline{M}^2}{M^2}.$$

$$\chi_{RG}(\bar{\alpha}_s, \gamma, a, b) = \bar{\alpha}_s(1 + a\bar{\alpha}_s)\left(\psi(\gamma) - \psi(\gamma - b\bar{\alpha}_s)\right) - \frac{\bar{\alpha}_s^2}{2}\psi''(1-\gamma) - \frac{b\bar{\alpha}_s^2 \cdot \pi^2}{\sin^2(\pi\gamma)}$$

$$+ \frac{1}{2}\sum_{m=0}^{\infty}\left(\gamma - 1 - m + b\bar{\alpha}_s - \frac{2\bar{\alpha}_s(1+a\bar{\alpha}_s)}{1-\gamma+m} + \sqrt{(\gamma - 1 - m + b\bar{\alpha}_s)^2 + 4\bar{\alpha}_s(1+a\bar{\alpha}_s)}\right)$$

resums (anti-) collinear 'logs' ($=\gamma$-poles) of $\bar{\alpha}_s\chi_0(\gamma) + \bar{\alpha}_s\chi_1(\gamma) - \frac{1}{2}\bar{\alpha}_s^2\chi_0'(\gamma)\,\chi_0(\gamma)$
[Salam; hep-ph/9806482], [Sabio Vera; hep-ph/0505128]

optimal scale setting $\rightarrow$ γ-dependent running coupling

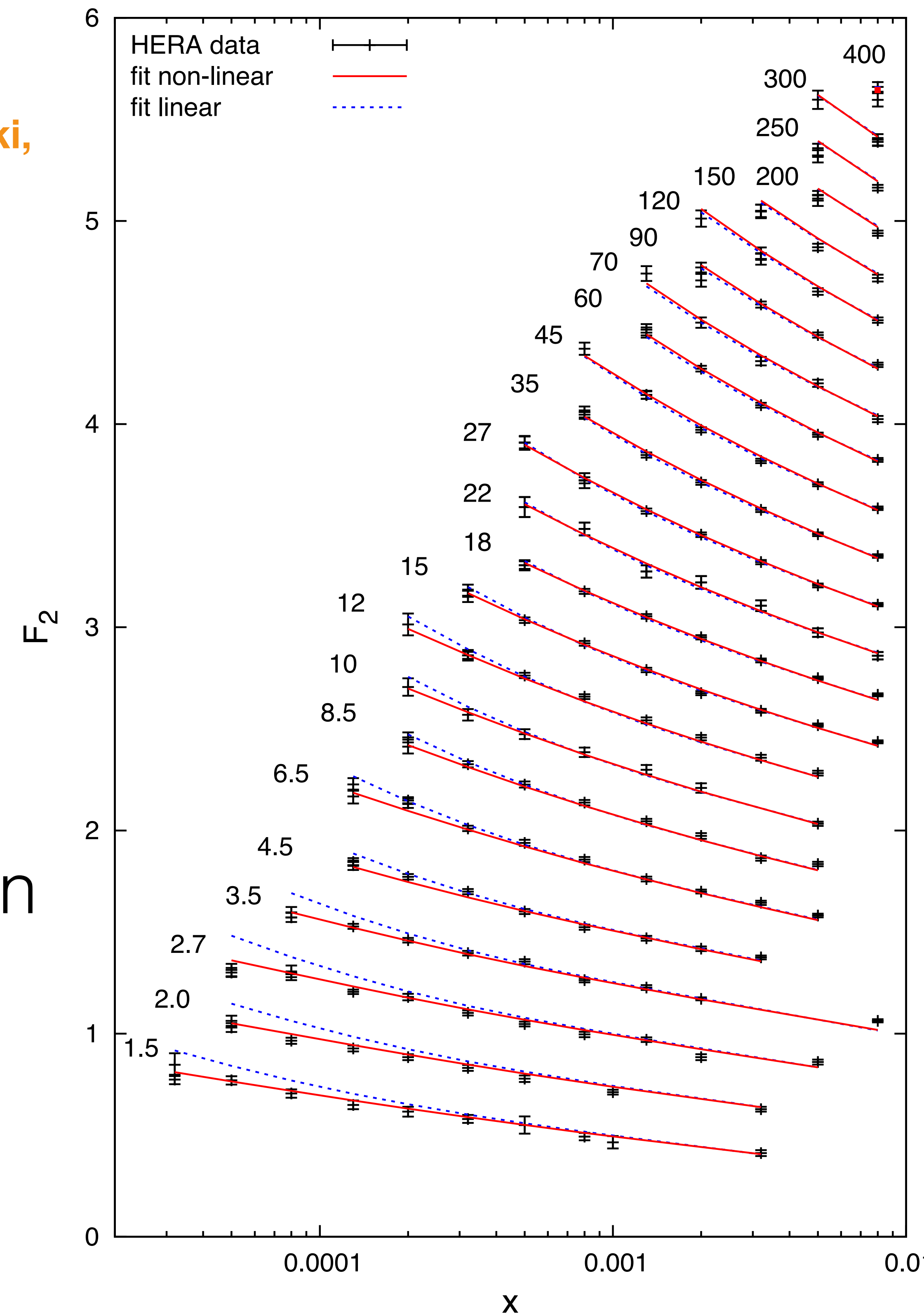
$$\bar{\alpha}_s\left(\overline{M}\cdot Q_0, \gamma\right) = \frac{4N_c}{\beta_0\left[\log\left(\frac{\overline{M}\cdot Q_0}{\Lambda^2}\right) + \frac{1}{2}\chi_0(\gamma) - \frac{5}{3} + 2\left(1 + \frac{2}{3}Y\right)\right]},$$

also use parametrization of running coupling in the infra-red [Webber; hep-ph/9805484]

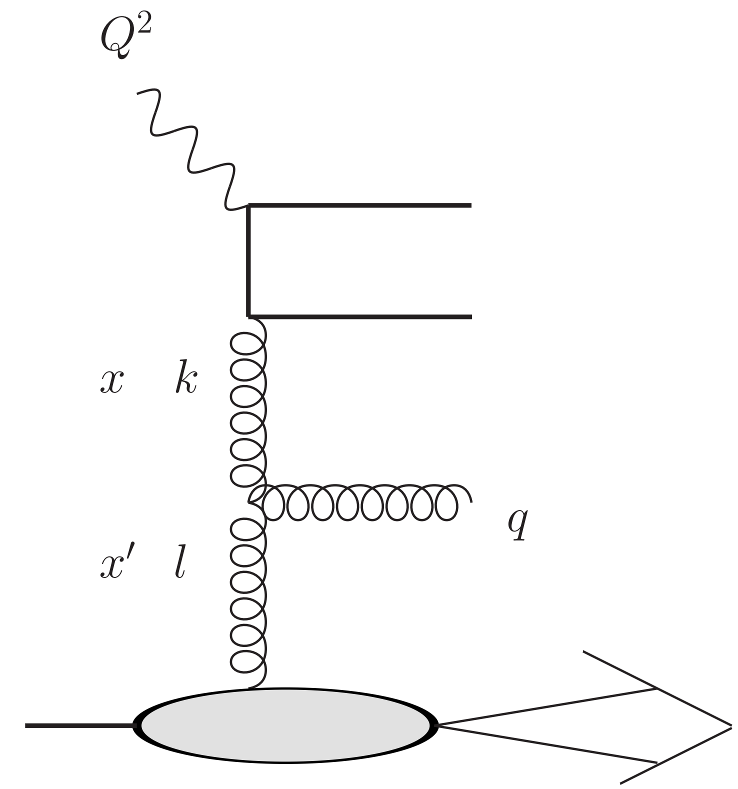
$$\alpha_s\left(\mu^2\right) = \frac{4\pi}{\beta_0 \ln\frac{\mu^2}{\Lambda^2}} + f\left(\frac{\mu^2}{\Lambda^2}\right), \quad f\left(\frac{\mu^2}{\Lambda^2}\right) = \frac{4\pi}{\beta_0}\,\frac{125\left(1 + 4\frac{\mu^2}{\Lambda^2}\right)}{\left(1 - \frac{\mu^2}{\Lambda^2}\right)\left(4 + \frac{\mu^2}{\Lambda^2}\right)^4},$$

gluon with non-linear terms: KS gluon [Kutak, Sapeta; 1205.5035]

- based on unified (leading order) DGLAP+BFKL framework [Kwiecínski, Martin, Stasto, PRD 56(1997) 3991]
- combined with leading order BK evolution [Kutak, Kwiecinski;hep-ph/0303209] [Kutak, Stasto; hep-ph/0408117]
- initial conditions: fit to combined HERA data [H1 & ZEUS collab. 0911.0884]
- both non-linear and linear version available (= non-linearity switched off)



the BK evolution equation matched to leading order DGLAP underlying the KS-fit

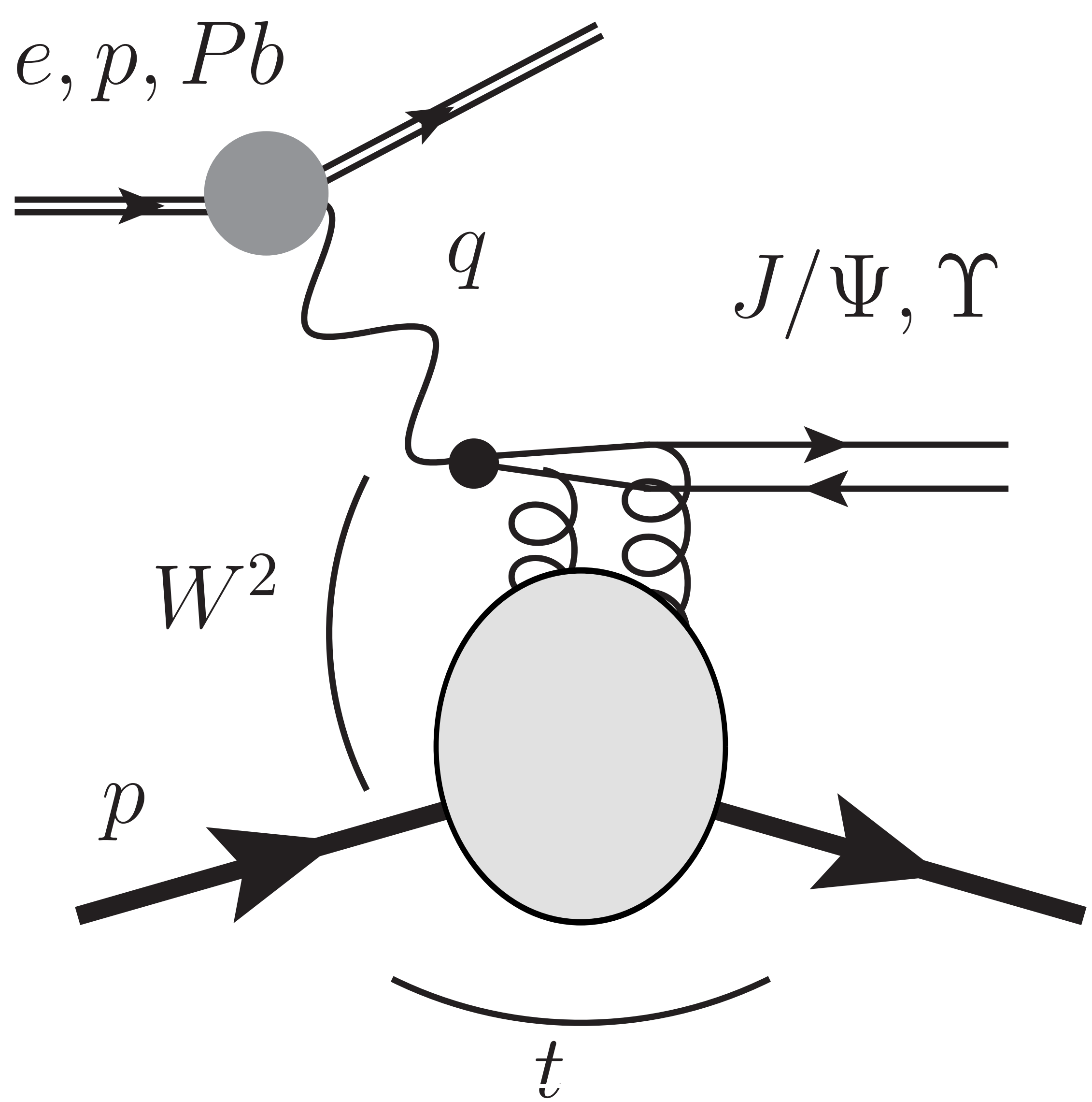


initial conditions

$$
\begin{aligned}
\phi_p(x,k^2) \; = \; & \phi_p^{(0)}(x,k^2) \\
& + \frac{\alpha_s(k^2) N_c}{\pi} \int_x^1 \frac{dz}{z} \int_{k_0^2}^{\infty} \frac{dl^2}{l^2} \left\{ \frac{l^2 \phi_p(\frac{x}{z},l^2)\, \theta(\frac{k^2}{z}-l^2) \, - \, k^2 \phi_p(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2 \phi_p(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}} \right\} \\
& + \frac{\alpha_s(k^2)}{2\pi k^2} \int_x^1 dz \left[\left(P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_0^2}^{k^2} dl^2 \, \phi_p\left(\frac{x}{z},l^2\right) + z P_{gq}(z) \Sigma\left(\frac{x}{z},k^2\right) \right] \\
& - \frac{2\alpha_s^2(k^2)}{R^2} \left[\left(\int_{k^2}^{\infty} \frac{dl^2}{l^2} \phi_p(x,l^2) \right)^2 + \phi_p(x,k^2) \int_{k^2}^{\infty} \frac{dl^2}{l^2} \ln\left(\frac{l^2}{k^2}\right) \phi_p(x,l^2) \right] ,
\end{aligned}
$$

R: radius of proton from fit

description of the exclusive photo-production cross-section requires



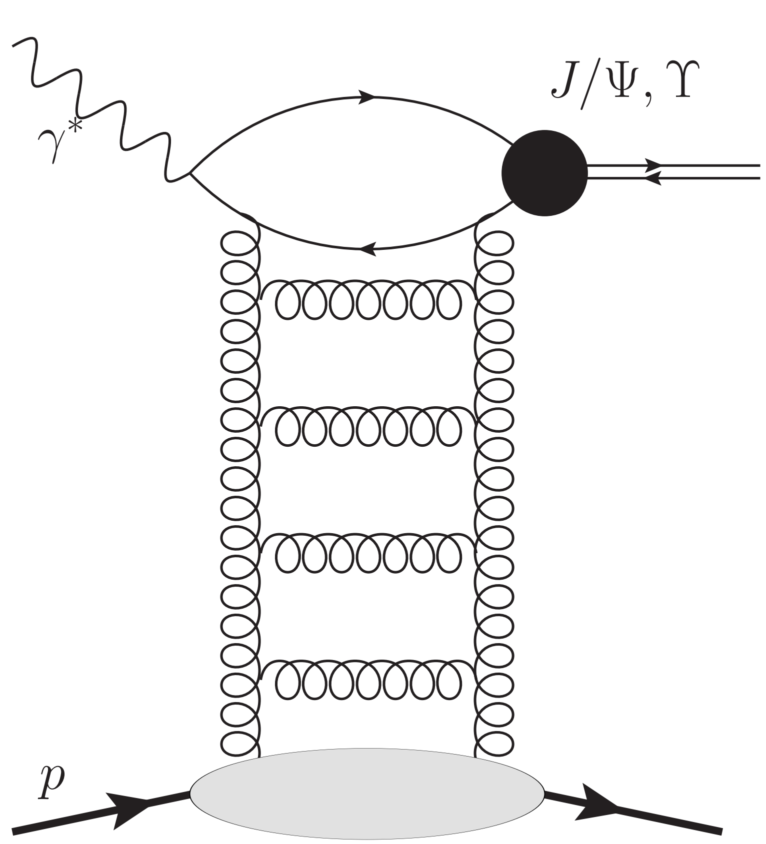
✓ gluon distribution in the proton

❌ coupling of gluon to photon-VM system

❌ take into account diffractive nature of the process

diffractive nature of process: Need to relate 2 pictures

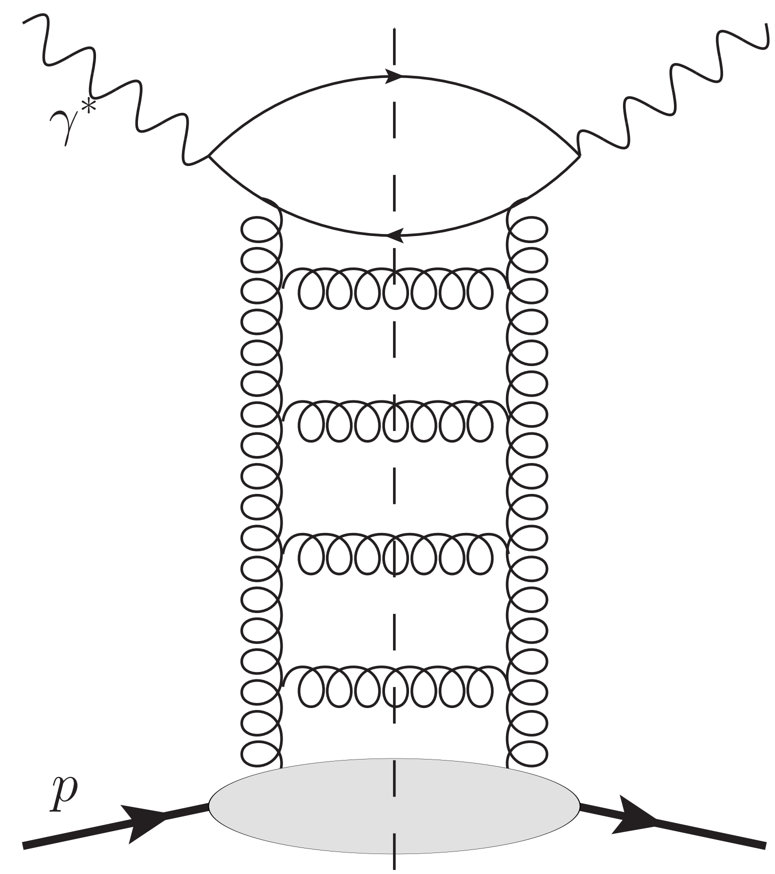
a) exclusive photo-production of vector mesons:



'uncut' Pomeron: diffractive/elastic scattering (amplitude level)

$$\mathcal{A}(s,t)$$

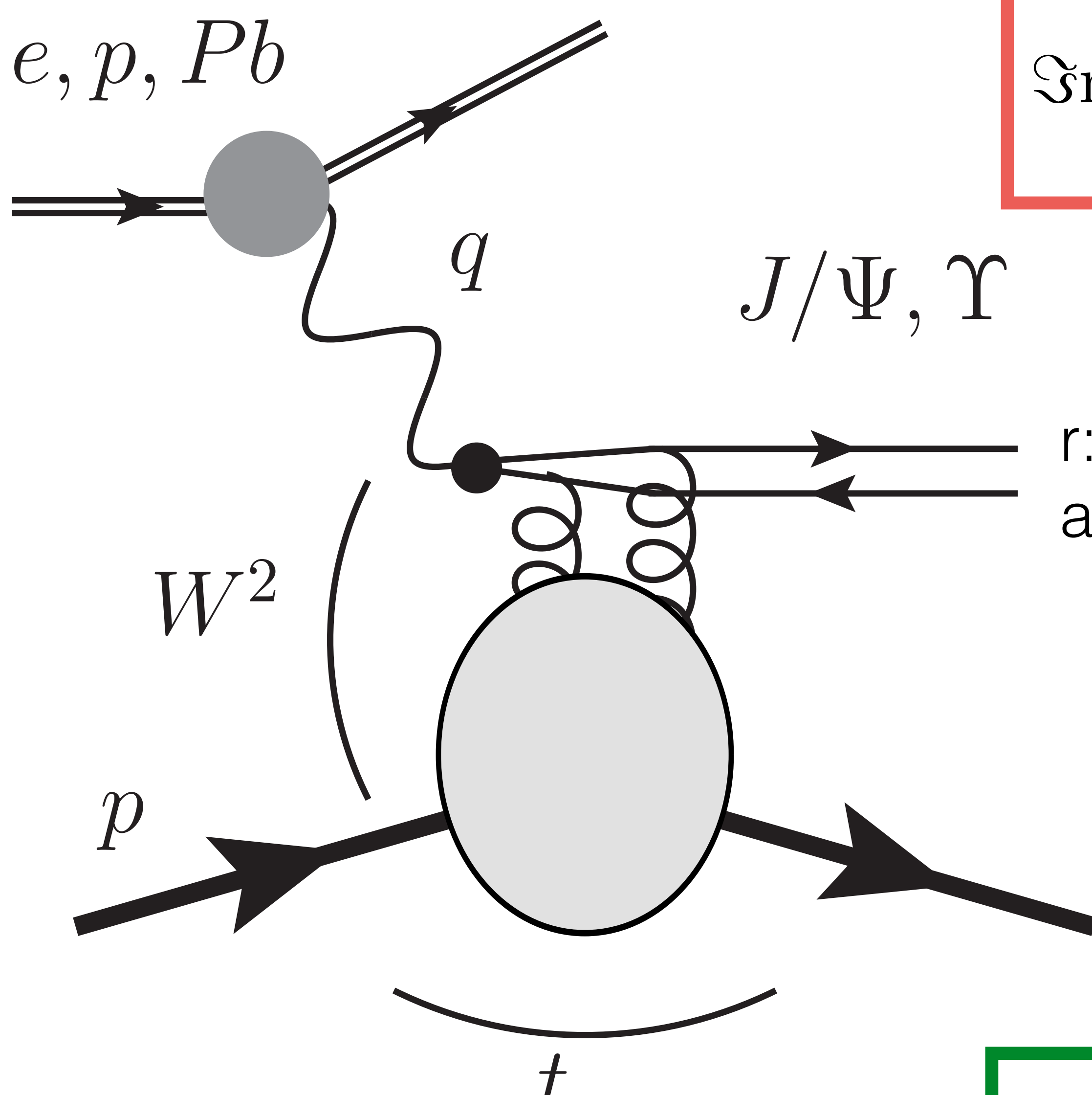
b) proton structure functions:



'cut' Pomeron: high multiplicity events (total X-sec.)

$$\sigma_{\text{tot}} = \frac{1}{s}\Im\text{m}\mathcal{A}(s,t=0)$$

Possible: imaginary part of scattering amplitude in forward limit t=0



$$\Im\mathrm{m}\mathcal{A}^{\gamma p \to V p}(x, t=0) = \int_0^\infty dr W(r) \sigma_{q\bar{q}}(x, r)$$

r: transverse size of quark-antiquark dipole

dipole cross-section from unintegrated gluon $\mathscr{F}$=**object of interest**

inclusive gluon → only calculate imaginary part of scattering amplitude at t=0

$$\sigma_{q\bar{q}}(x, r) = \frac{4\pi}{N_c} \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2} \left(1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right) \alpha_s \mathcal{F}(x, \boldsymbol{k}^2)$$

the transition

photon → quark-antiquark dipole → vector meson

$$W(r) = 2\pi r \int_0^1 \frac{dz}{4\pi} \, (\Psi_V^* \Psi)_T(r,z)$$

$$(\Psi_V^* \Psi)_T \, (r,z) = \frac{\hat{e}_f e N_c}{\pi z(1-z)} \left\{ m_f^2 K_0(\epsilon r) \phi_T(r,z) - \left[z^2 + (1-z)^2 \right] \epsilon K_1(\epsilon r) \partial_r \phi_T(r,z) \right\}$$

boosted Gaussian scalar wave function using Brodsky-Huang-Lepage prescription

$$\phi_{T,L}^{1s}(r,z) = \mathcal{N}_{T,L} z(1-z) \exp\left(-\frac{m_f^2 \mathcal{R}_{1s}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_{1s}^2} + \frac{m_f^2 \mathcal{R}_{1s}^2}{2} \right)$$

parameters from normalization and decay width [Armesto, Rezaeian; 1402.4831] (J/Ψ) and [Goncalves, Moreira,Navarra;1408.1344] (Υ)

Meson	m_f/GeV	$\mathcal{N}_T$	$\mathcal{R}^2$/GeV^{-2}	M_V/GeV
J/ψ	$m_c = 1.4$	0.596	2.45	3.097
Υ	$m_b = 4.2$	0.481	0.57	9.460

how to relate the imaginary part of the scattering amplitude to experiment?

(standard procedure for this kind of study)

a) analytic properties of scattering amplitude → real part

$$\mathcal{A}^{\gamma p \to Vp}(x,t=0) = \left(i + \tan\frac{\lambda(x)\pi}{2}\right) \cdot \Im\mathrm{m}\mathcal{A}^{\gamma p \to Vp}(x,t=0)$$

with intercept $\lambda(x) = \frac{d\ln \Im\mathrm{m}\mathcal{A}(x,t)}{d\ln 1/x}$

b) differential Xsection at t=0:

$$\frac{d\sigma}{dt}(\gamma p \to Vp)\bigg|_{t=0} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma p \to Vp}(W^2, t=0)\right|^2$$

c) from experiment: $\frac{d\sigma}{dt}(\gamma p \to Vp) = e^{-B_D(W)\cdot|t|} \cdot \frac{d\sigma}{dt}(\gamma p \to Vp)\Big|_{t=0}$

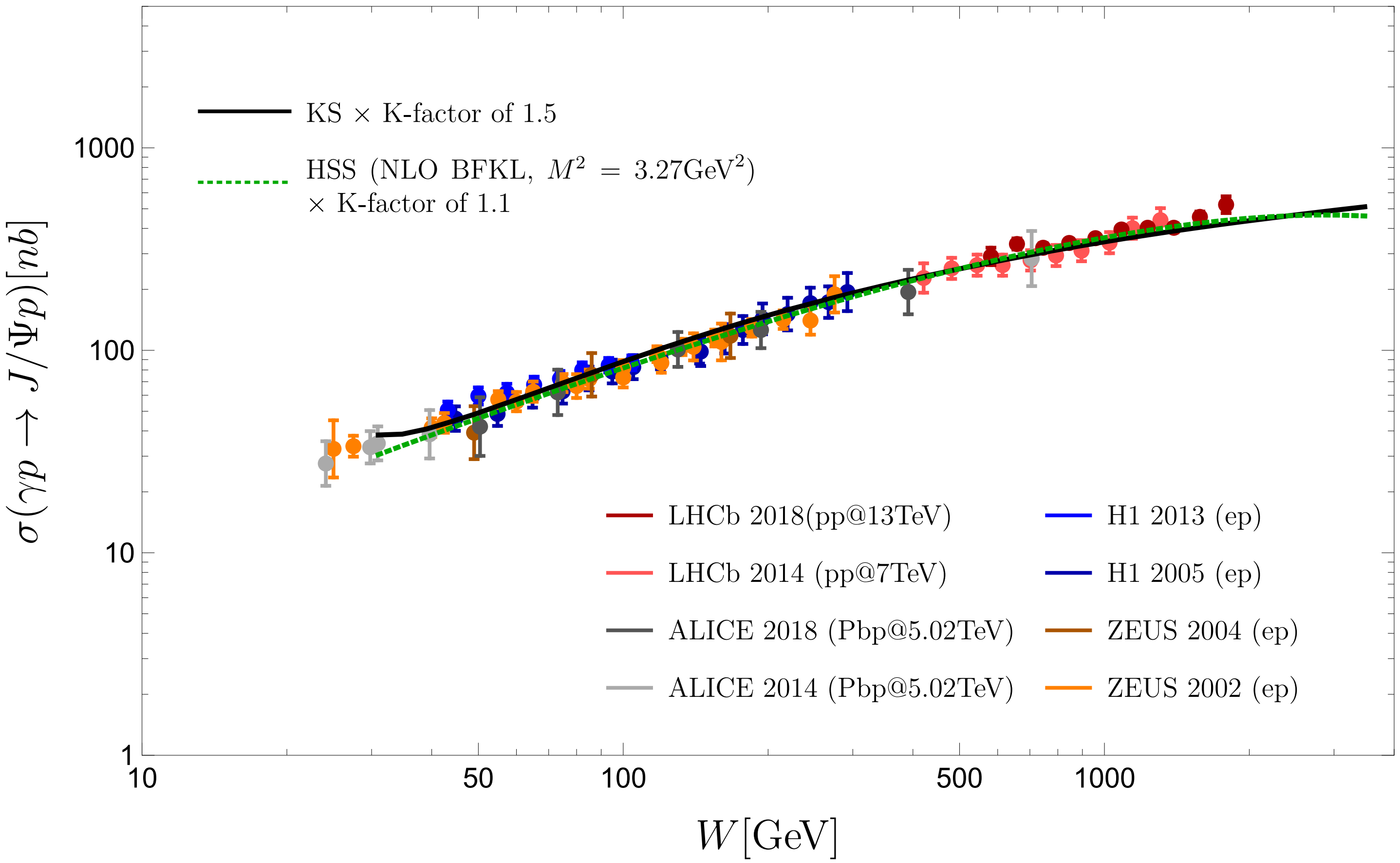
$$\sigma^{\gamma p \to Vp}(W^2) = \frac{1}{B_D(W)}\frac{d\sigma}{dt}(\gamma p \to Vp)\bigg|_{t=0}$$

extracted from data

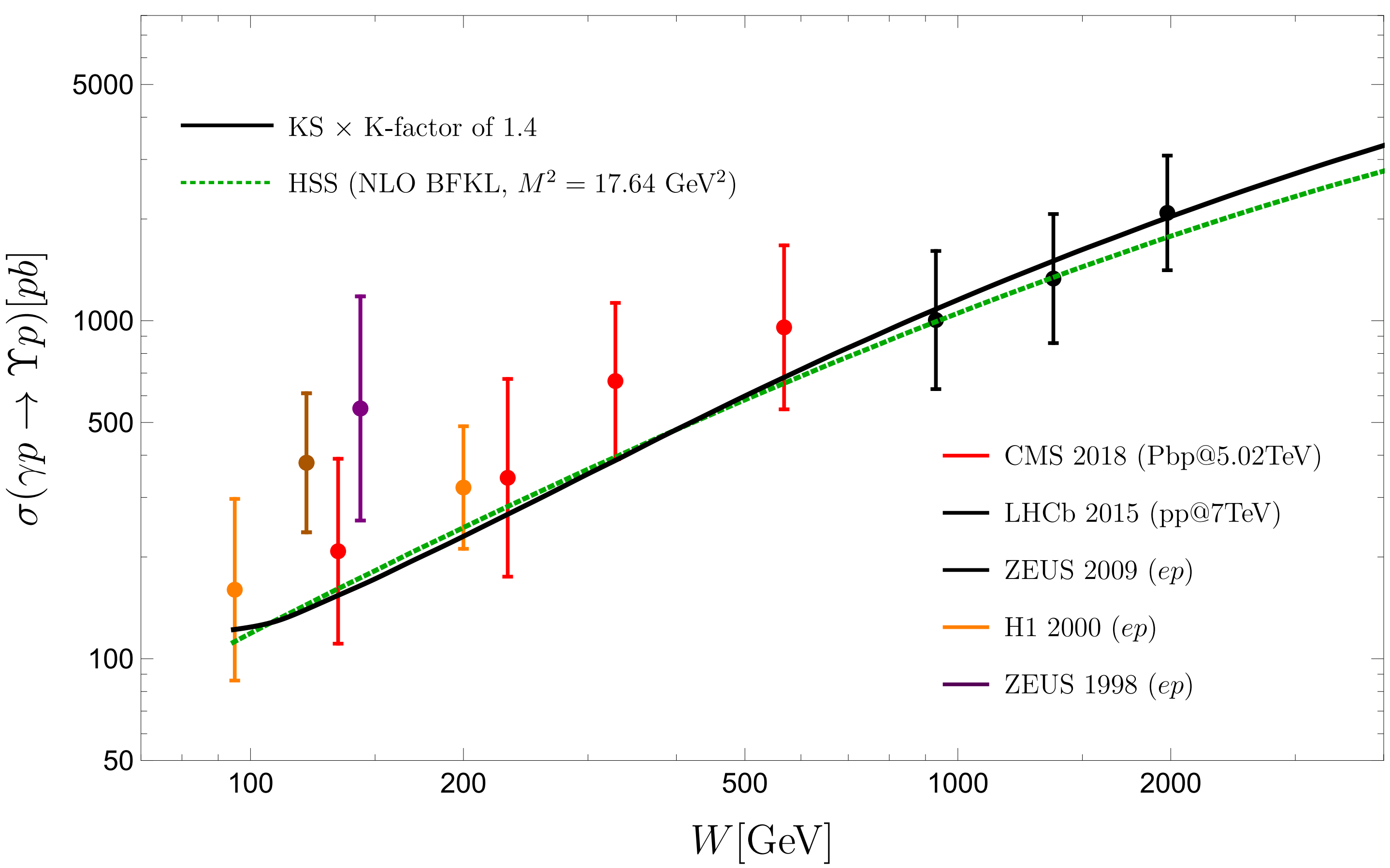
weak energy dependence from slope parameter

$$B_D(W) = \left[b_0 + 4\alpha' \ln\frac{W}{W_0}\right]\,\mathrm{GeV}^{-2}.$$

Results I



Υ as perturbative control:



- leading order wave function → don't control normalization (scale of α_s)

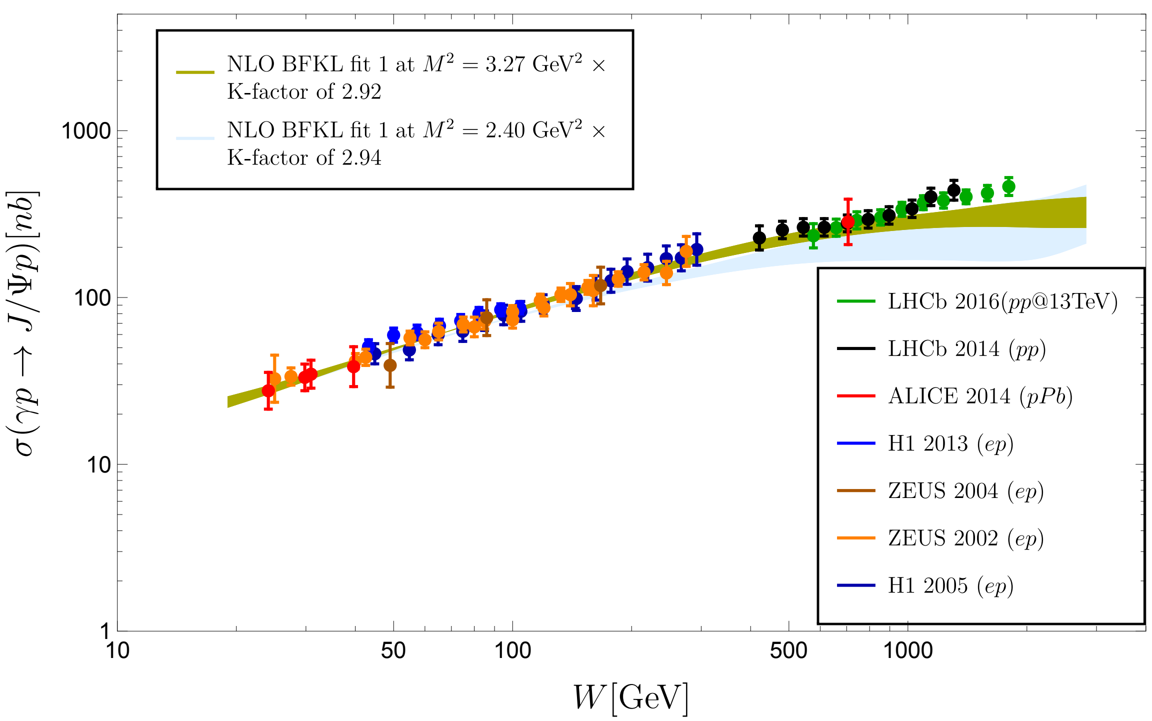
$$\Im\mathrm{m}\mathcal{A}^{\gamma p \to Vp} \sim \alpha_s(\mu^2)$$
$$\Rightarrow \sigma^{\gamma p \to Vp} \sim \alpha_s^2(\mu^2)$$

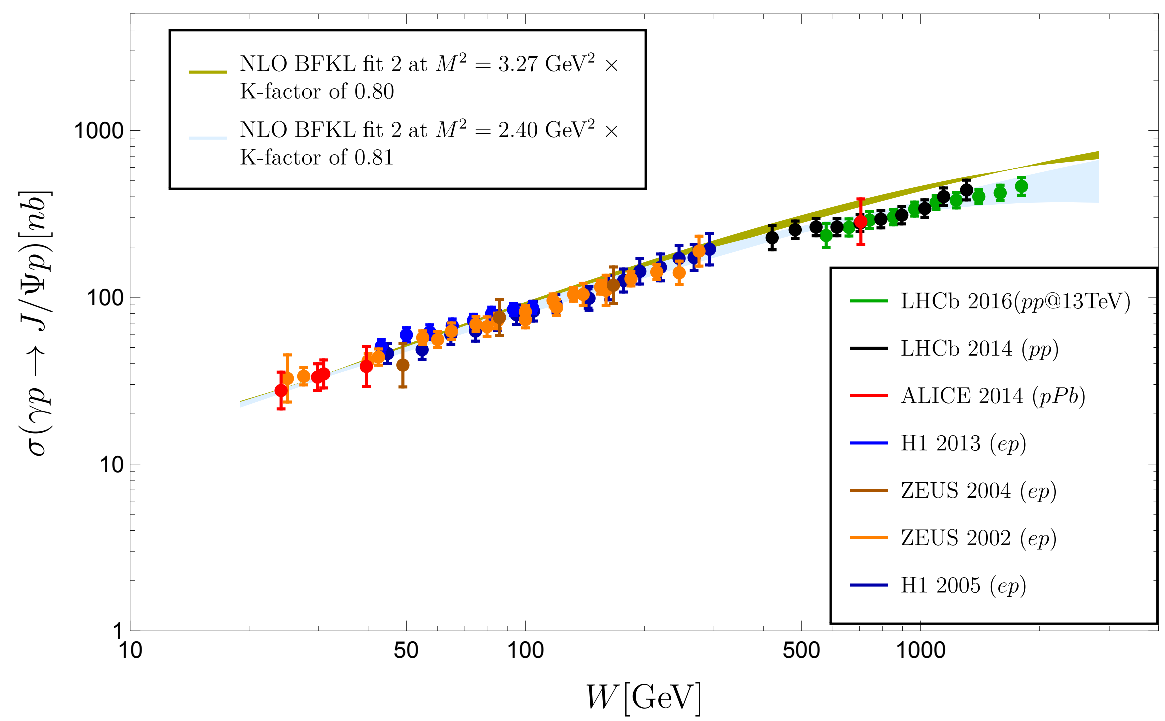
- standard scale choices for dipole cross-sections (~external scales)→ very good description of energy dependence with both HSS and KS gluon
- premature (?) conclusion: non-linear dynamics is absent

How strong is the growth of the low density gluon?

A first hint from a previous BFKL study

[Bautista, MH, Fernandez-Tellez;1607.05203]





error band: variation of renormalization scale
→ in general pretty small = stability
…but error blows up for highest energies

does it mean something?

Inspection of the HSS dipole

after Fourier transform still 2 terms:

$$\sigma_{q\bar{q}}^{(\mathrm{HSS})}(x,r) = \alpha_s \hat{\sigma}_{q\bar{q}}^{(\mathrm{HSS})}(x,r).$$

$$\hat{\sigma}_{q\bar{q}}^{(\mathrm{HSS})}(x,r) = \hat{\sigma}_{q\bar{q}}^{(\mathrm{dom.})}(x,r) + \hat{\sigma}_{q\bar{q}}^{(\mathrm{corr.})}(x,r),$$

running coupling corrections which do not exponentiate = a perturbative correction

$$\hat{\sigma}_{q\bar{q}}^{(\mathrm{dom})}(x,r,M^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4}{r^2 Q_0^2}\right)^{\gamma} \frac{\bar{\alpha}_s(M\cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma, Q_0, \delta, r) \left(\frac{1}{x}\right)^{\chi(\gamma, M^2)}$$

$$\hat{\sigma}_{q\bar{q}}^{(\mathrm{corr.})}(x,r,M^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4}{r^2 Q_0^2}\right)^{\gamma} \frac{\bar{\alpha}_s(M\cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma, Q_0, \delta, r) \left(\frac{1}{x}\right)^{\chi(\gamma, M^2)}$$

$$\times \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{8N_c} \log\left(\frac{1}{x}\right) \left[-\psi(\delta-\gamma) + \log\frac{M^2 r^2}{4} - \frac{1}{1-\gamma} - \psi(2-\gamma) - \psi(\gamma)\right]$$

NLO BFKL kernel (BLM scale setting) + coll. resummation

$$\chi(\gamma, M^2) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \tilde{\chi}_1(\gamma) - \frac{1}{2}\bar{\alpha}_s^2 \chi_0'(\gamma)\chi_0(\gamma) + \chi_{\mathrm{RG}}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b}).$$

What is the problem?

2nd term = running coupling correction

$$\sigma_{q\bar{q}}^{(\mathrm{HSS})}(x,r) = \alpha_s \hat{\sigma}_{q\bar{q}}^{(\mathrm{HSS})}(x,r).$$

$$\hat{\sigma}_{q\bar{q}}^{(\mathrm{HSS})}(x,r) = \hat{\sigma}_{q\bar{q}}^{(\mathrm{dom.})}(x,r) + \hat{\sigma}_{q\bar{q}}^{(\mathrm{corr.})}(x,r),$$

running coupling corrections which do not exponentiate = a perturbative correction

negative
+ enhanced by log(1/x)

$$\hat{\sigma}_{q\bar{q}}^{(\mathrm{corr.})}(x,r) = -\alpha_s^2 \ln\left(\frac{1}{x}\right) \hat{\sigma}_{q\bar{q}}^{(1)}(x,r)$$

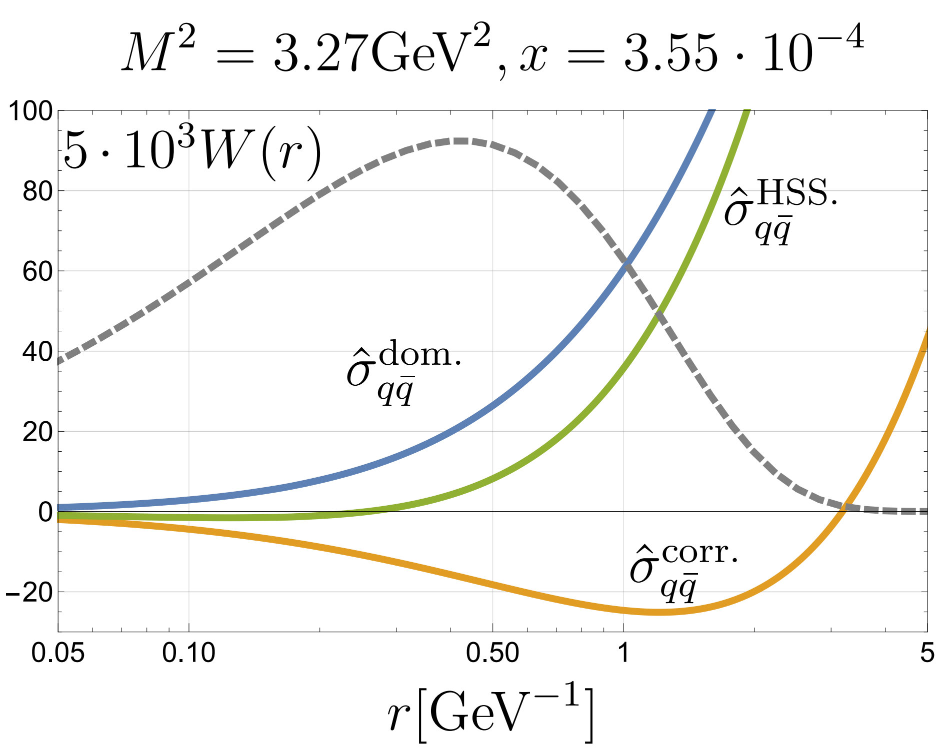
→ at some (maybe very very small x), this will eventually dominate the leading term!

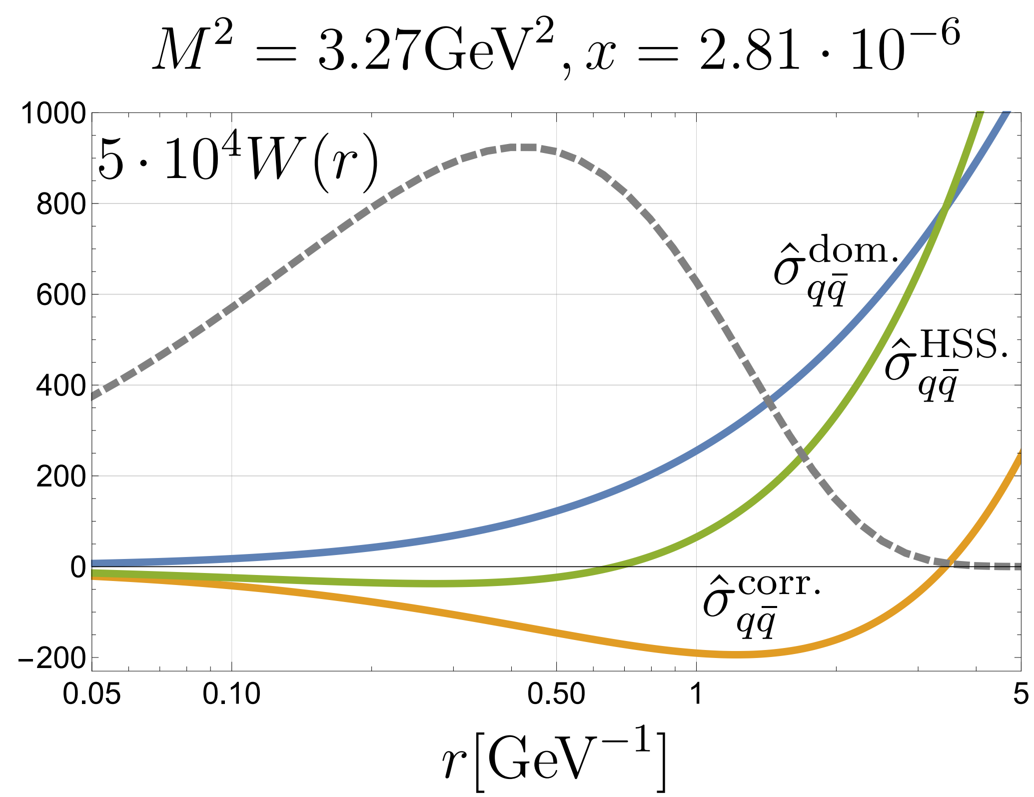
recall:

$$\Im\mathrm{m}\mathcal{A}^{\gamma p\to Vp}(x,t=0)=\int_0^\infty dr W(r)\sigma_{q\bar{q}}(x,r)$$

compare dominant vs. correction & how they add up

turns out

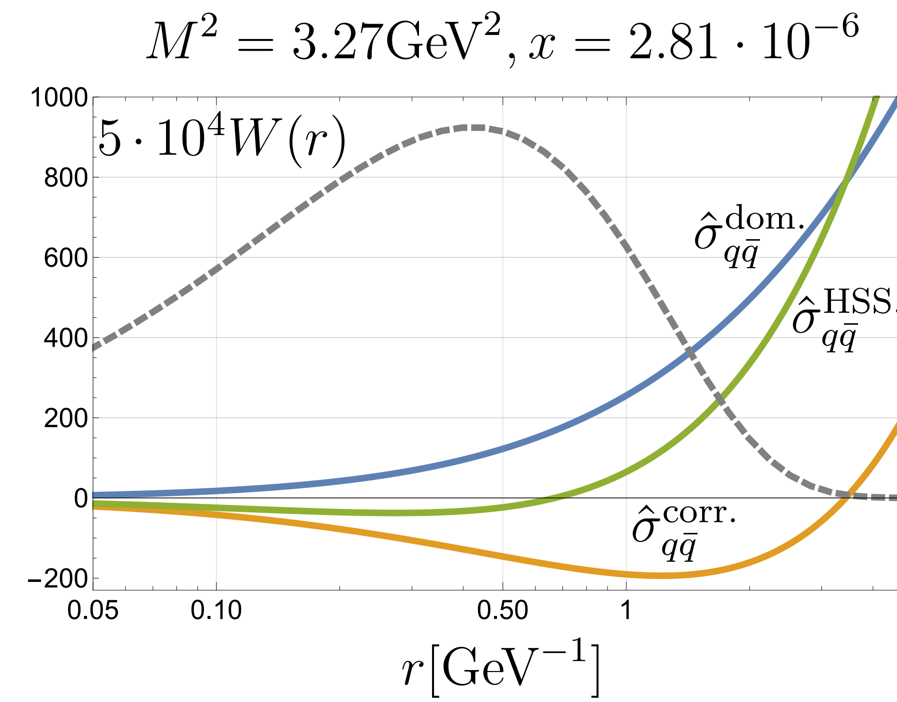




the correction is small for HERA kinematics → not a problem for the initial fit

…. but the correction dominates for small dipole sizes *r* at the smallest x values reached

how to cure it?



recall that the 2nd terms is a running coupling correction:

$$\hat{\sigma}_{q\bar{q}}^{(\text{corr.})}(x, r, M^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4}{r^2 Q_0^2}\right)^{\gamma} \frac{\bar{\alpha}_s(M \cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma, Q_0, \delta, r) \left(\frac{1}{x}\right)^{\chi(\gamma, M^2)}$$

$$\times \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{8N_c} \log\left(\frac{1}{x}\right) \left[-\psi(\delta - \gamma) + \log\frac{M^2 r^2}{4} - \frac{1}{1-\gamma} - \psi(2-\gamma) - \psi(\gamma)\right]$$

problem arises for small dipole sizes

the troublemaker

natural solution to such problem: chose *r*-dependent running coupling scale = the inverse transverse size of the dipole

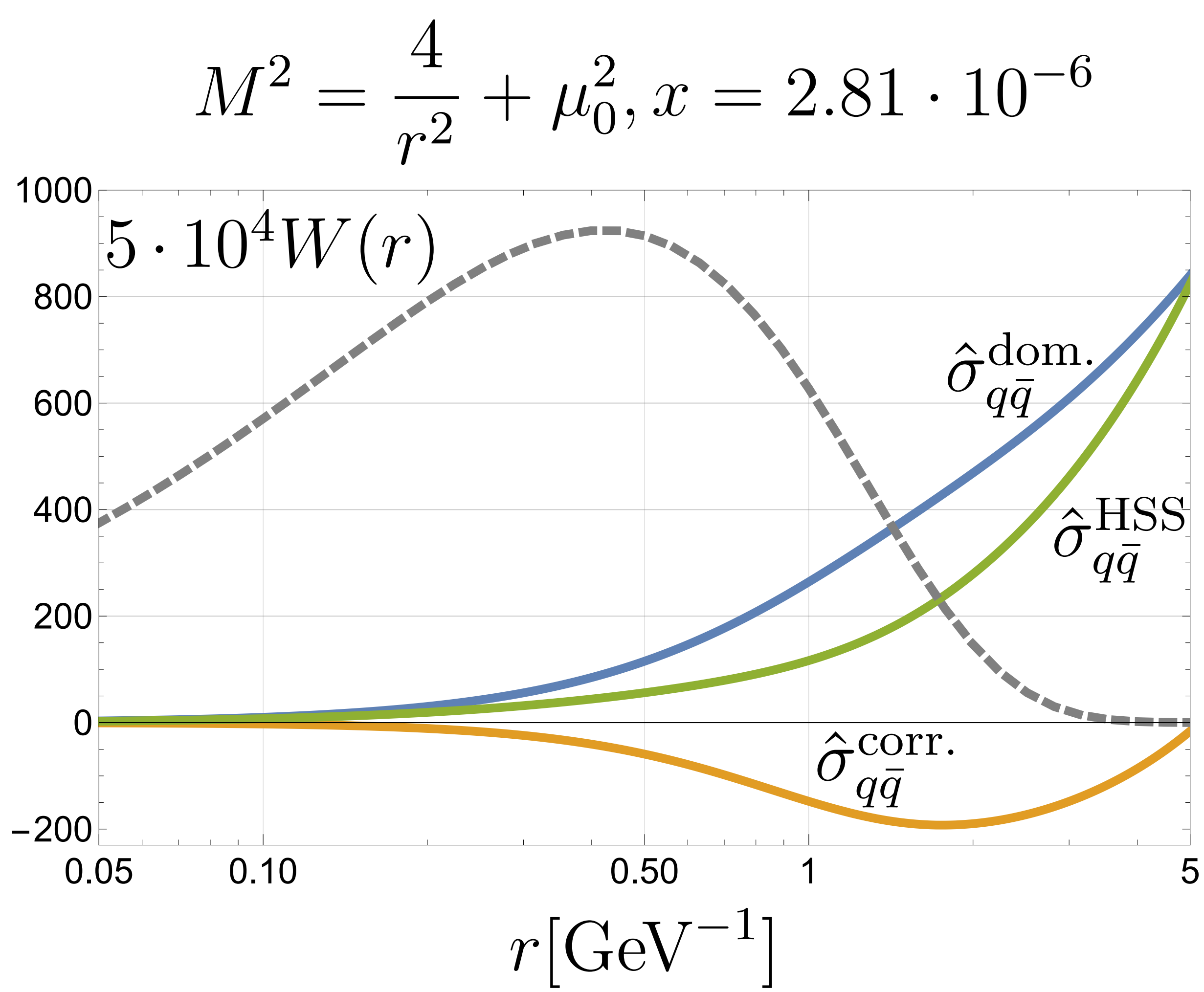
$$\boxed{M^2 = \frac{4}{r^2} + \mu_0^2 \text{ with } \mu_0^2 = 1.51 \text{ GeV}^2}$$

this has been used already:

scale choice used in IPsat dipole model **[Bartels, Golec-Biernat, Kowalksi, hep-ph/0203258];**

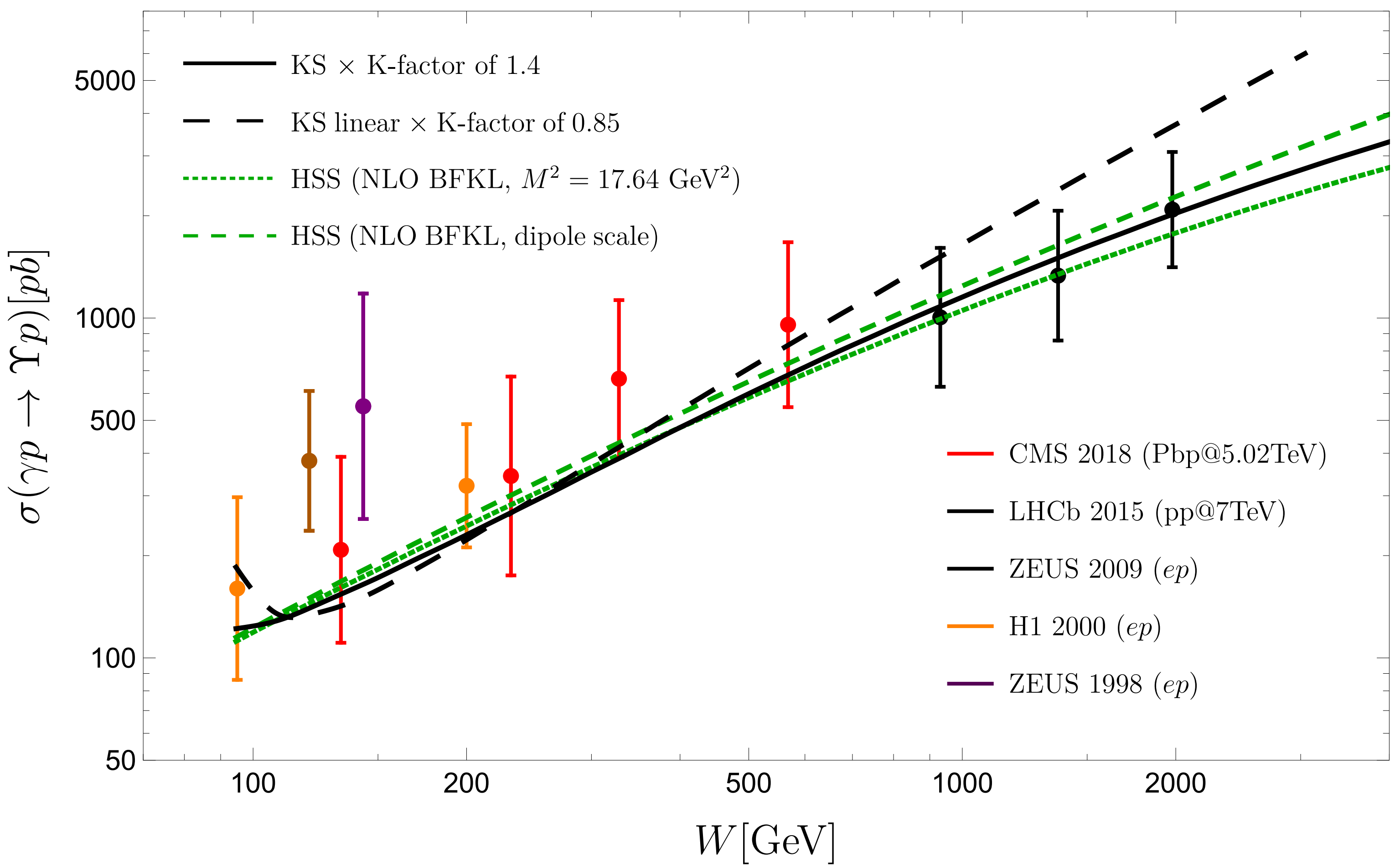
fit: **[Rezaeian, Siddikov, Van de Klundert, Venugopalan; 1212.2974]**

resum large log into running coupling
→ stabilize perturbative expansion



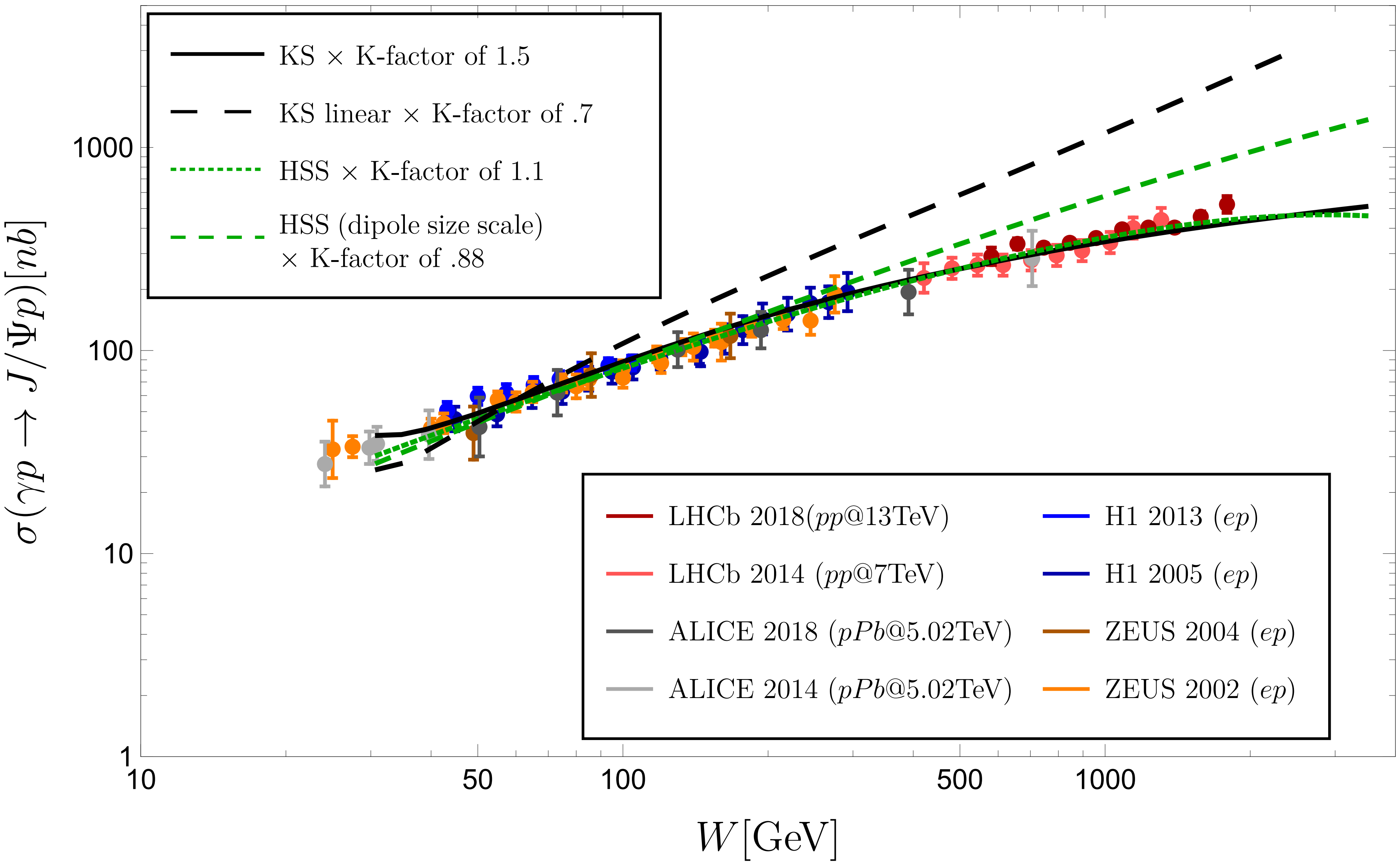
What does it imply for our description of data?

a small variation for the perturbative check = the ϒ



the growth is slightly stronger

essentially a scale variation around the b-mass

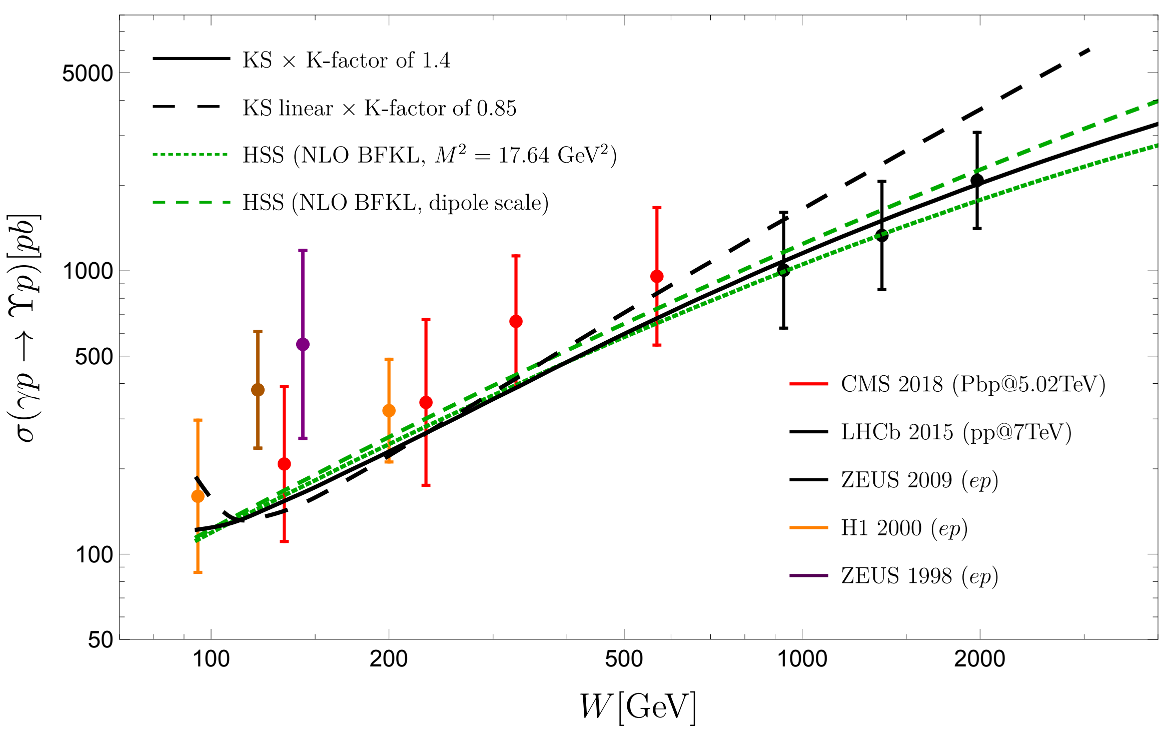


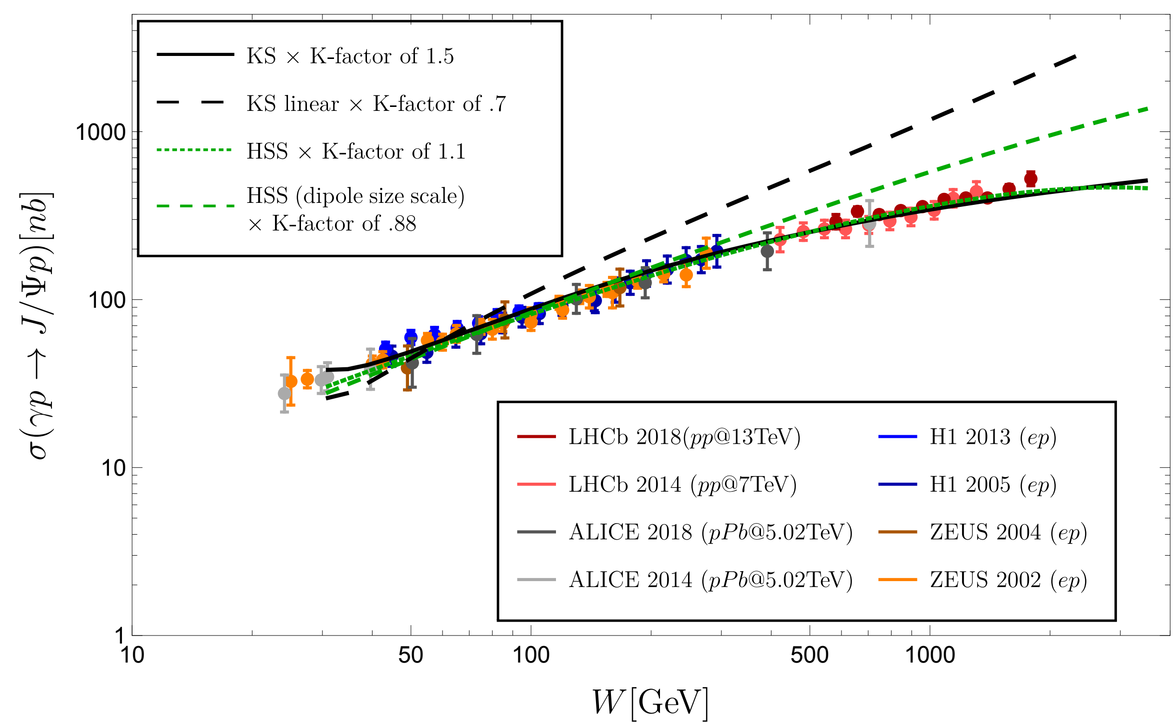
- The J/ψ is still well described in the HERA region ($W < 400$GeV)→ expected since the correction was small for such x-values

- The (running coupling resummed) NLO BFKL overshoots however now the data→ classical signal for gluon saturation!

Do we see something no related to a non-linear effect?

→ turn off non-linear terms in the KS gluon
(originally a separate fit with certain IR cut-off)





linear KS gluon → growth too strong for both ϒ & J/ψ

→ non-linear terms are essential for KS description of data

most direct evidence for gluon saturation seen so far?

What have we seen so far?

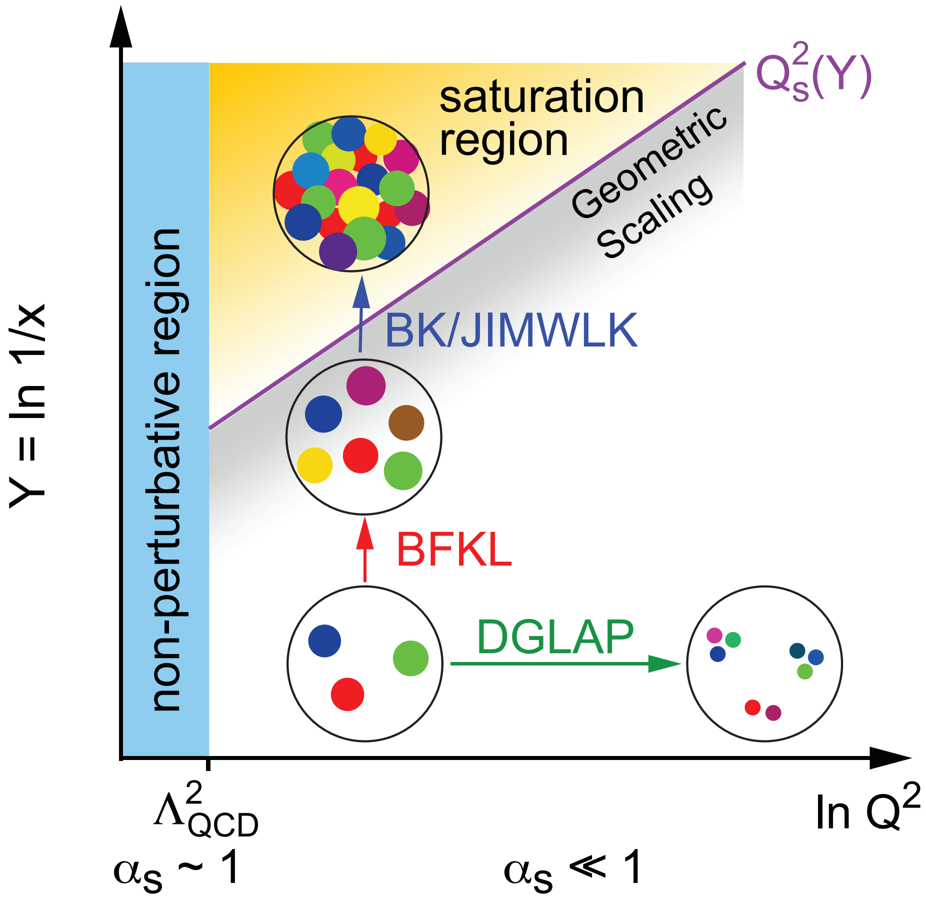
- NLO BFKL (linear evolution) only describes data if (negative) perturbative corrections is larger than the leading term (= breakdown of expansion)

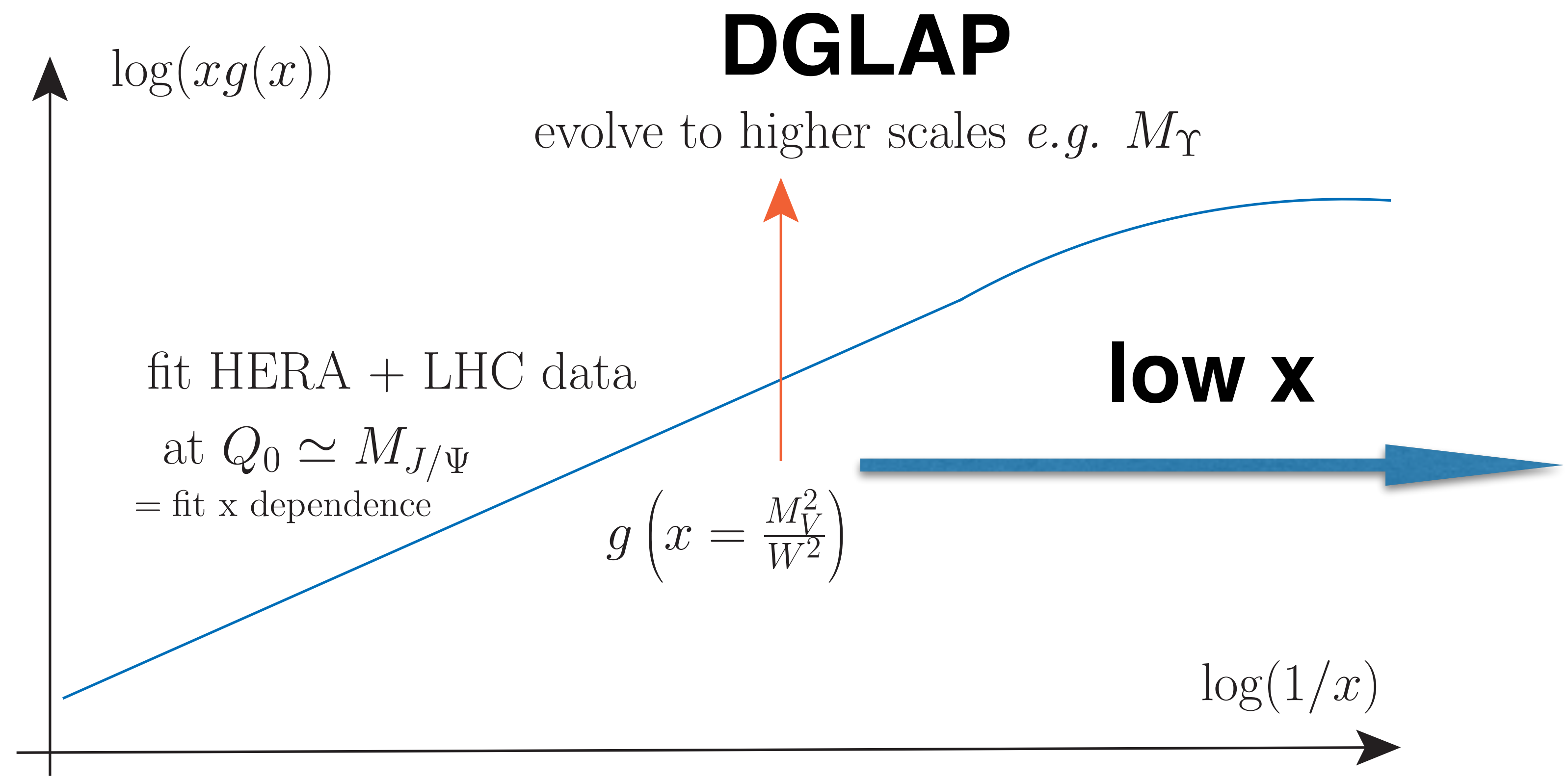
- Tame size of correction → description of Υ and J/ψ in HERA region, growth too strong for J/ψ at LHC

- non-linear KS gluon describes data & non-linear terms essential

→ What about other approaches?

schematic vs. reality



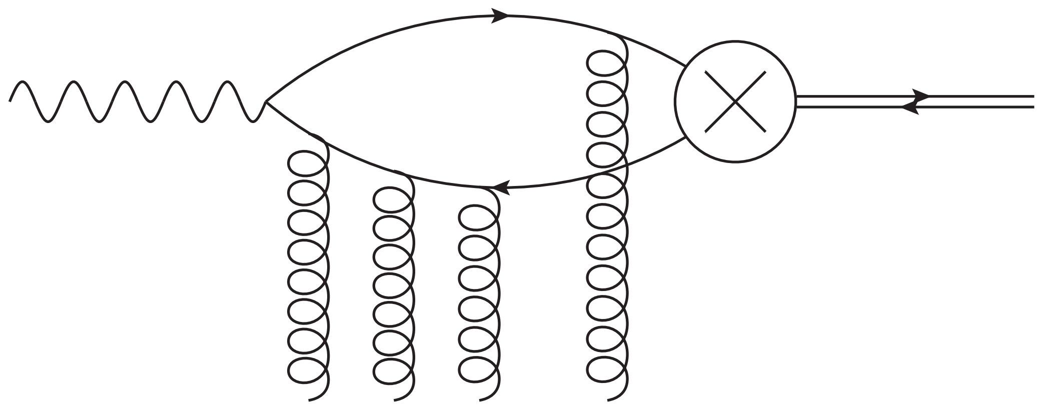


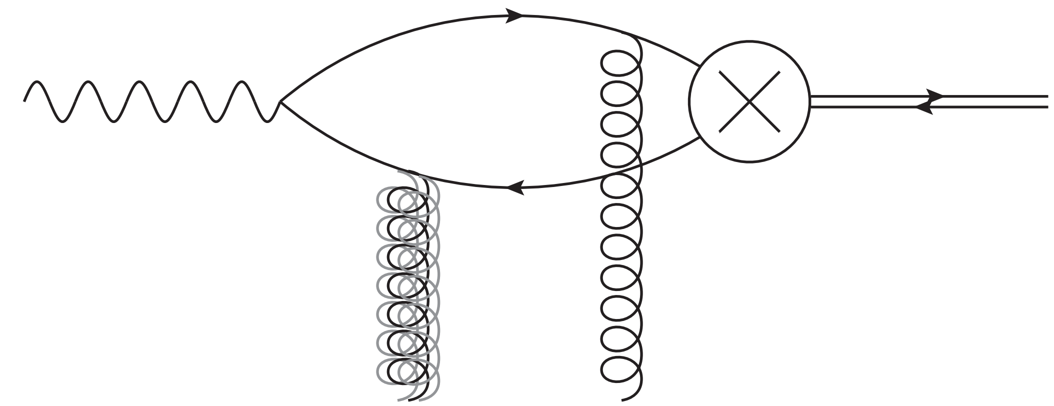
DGLAP:

- fit x-dependence + evolve from J/Ψ (2.4 GeV^2) to Υ (22.4 GeV^2)
- DGLAP shifts large x input (low scales) to low x (high scales)
  + higher twist dies away fast in evolution

→ constrain pdfs, but don't learn about saturation (easily overseen)

reason: for dipole cross-section many gluons couple to dipole as if few





[Bartels, Wusthoff, Z. Phys. C 66 (1995)]

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943]

[MH; 1802.06755]

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

$$\sigma_{q\bar{q}}(x, r) = \int d^2\boldsymbol{b}\, \frac{\left\langle 1 - V\left(\boldsymbol{b} + \frac{\boldsymbol{r}}{2}\right) V^\dagger\left(\boldsymbol{b} - \frac{\boldsymbol{r}}{2}\right)\right\rangle_x}{N_c}$$

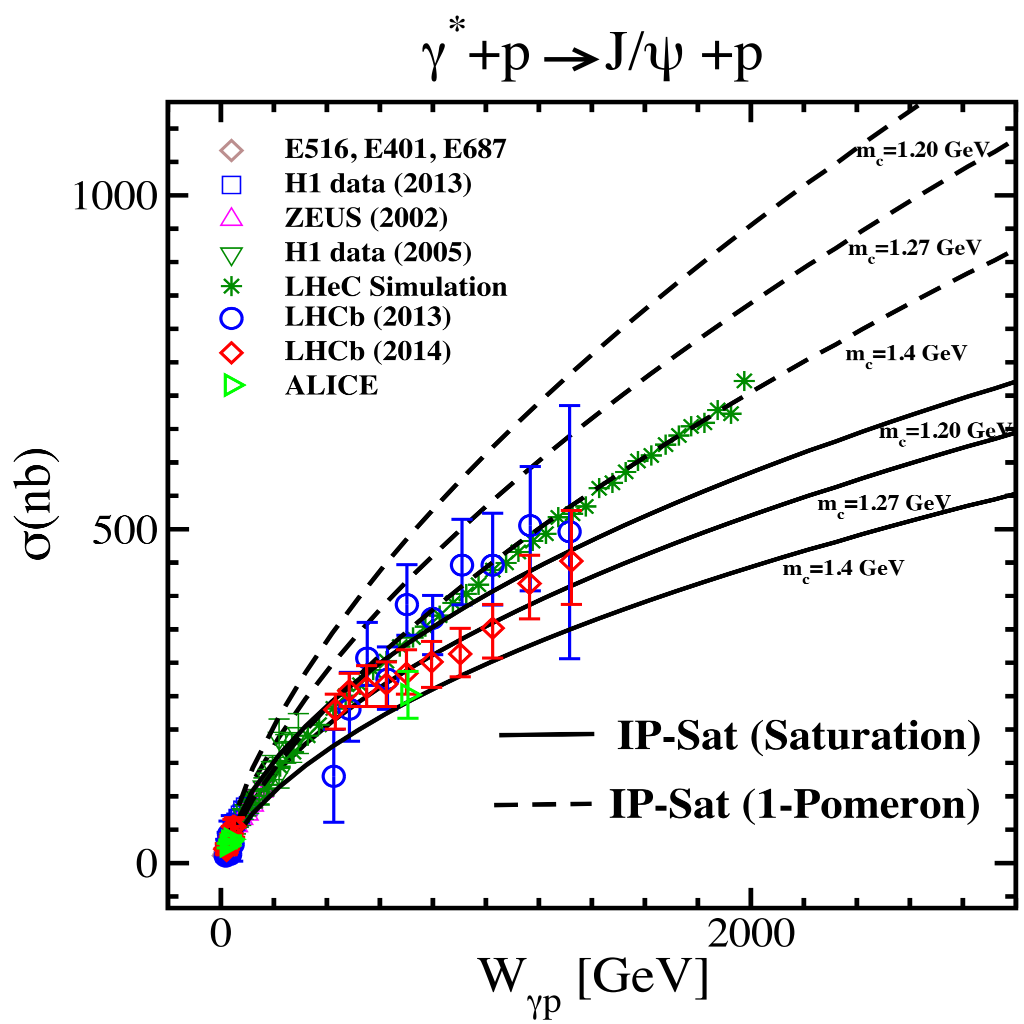
$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$

formal: dipole Xsection from 2 Wilson line correlator

high density effects entirely from low x evolution → for DGLAP fitted to data!
+ high density effects in evolution twist suppressed (large N_c)

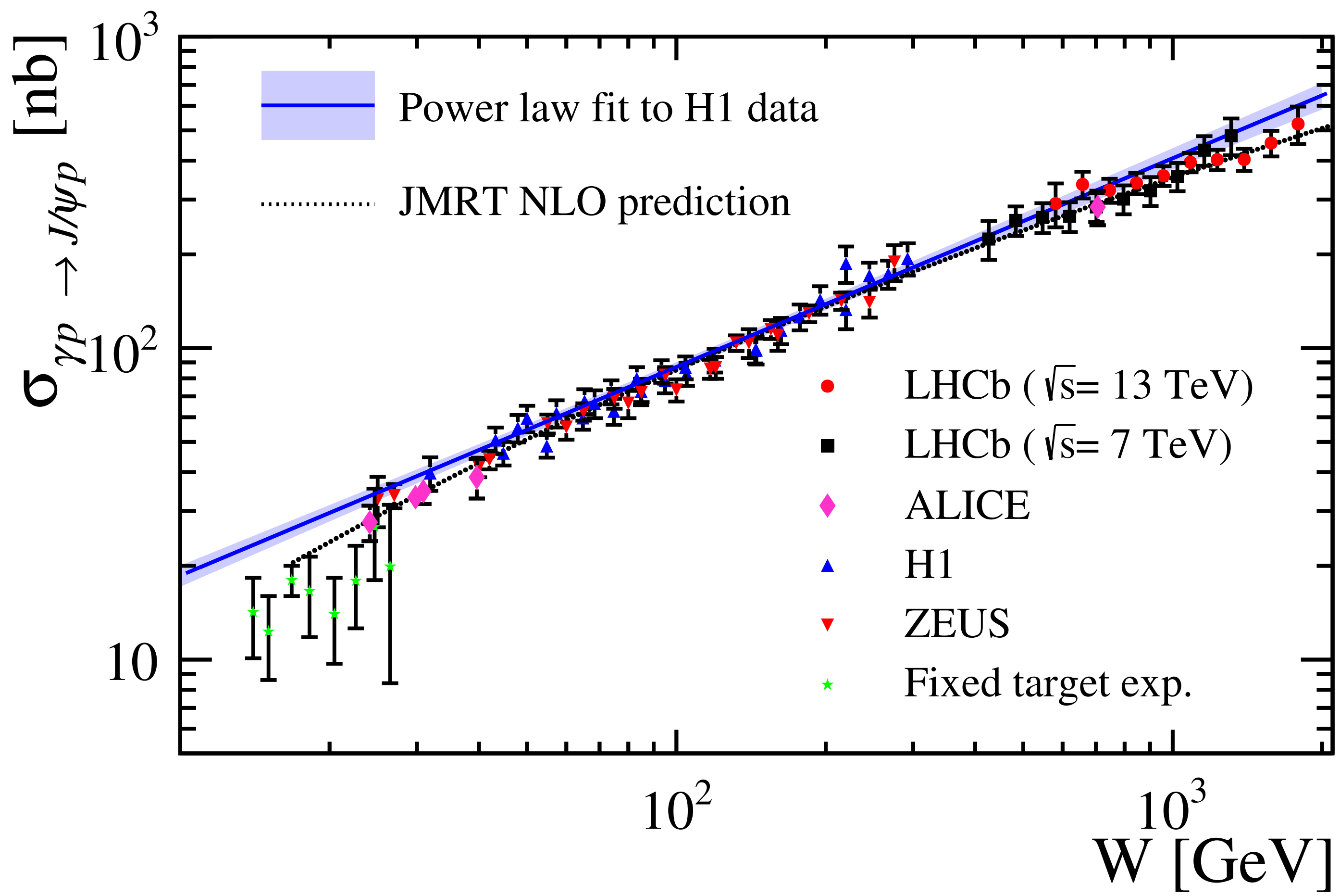
[Bartels, Kutak; 0710.3060]

Is this seen also by someone else?



log of IPsat model

[Armesto, Rezaeian; 1402.4831]



deviation of H1 power law fit

[LHCb coll.; 1806.04076]

our study: QCD low x evolution

Possible limitations

- NLO accuracy for both non-linear evolution, wave functions for VM production + DIS fit highly desirable
- extraction of γp an own challenge (gap survival factors etc.)→ how well do we control the errors?

↔

for this observable = this is how the onset of gluon saturation would like

→ need to complete picture with more observable & higher theoretical accuracy;

- at an EIC: control photon virtuality (→ hard scale) = more precise comparison to DGLAP evolution

- saturation should manifest itself in many ways in particular the pT spectrum should reveal the existence of a saturation scale→ dijets, 3 jets etc. = search for observables for which high density is more directly visible

- understanding higher order (=NLO) corrections and role of soft logarithms is just beginning → progress is needed

Summary and Conclusion

- NLO BFKL (linear evolution) only describes data if (negative) perturbative corrections is larger than the leading term (= breakdown of expansion)
- Tame size of correction → description of Υ and J/ψ in HERA region, growth too strong for J/ψ at LHC
- non-linear KS gluon describes data & non-linear terms essential

→ so far: most direct evidence for gluon saturation

Back up

HSS gluon = 2 terms

$$\hat{\sigma}^{(\mathrm{HSS})}_{q\bar{q}}(x,r) = \hat{\sigma}^{(\mathrm{dom.})}_{q\bar{q}}(x,r) + \hat{\sigma}^{(\mathrm{corr.})}_{q\bar{q}}(x,r).$$

$$\hat{\sigma}^{(\mathrm{dom})}_{q\bar{q}}(x,r,M^2) = \int\limits_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i}\left(\frac{4}{r^2Q_0^2}\right)^{\gamma}\frac{\bar{\alpha}_s(M\cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma,Q_0,\delta,r)\left(\frac{1}{x}\right)^{\chi(\gamma,M^2)}$$

$$\hat{\sigma}^{(\mathrm{corr.})}_{q\bar{q}}(x,r,M^2) = \int\limits_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i}\left(\frac{4}{r^2Q_0^2}\right)^{\gamma}\frac{\bar{\alpha}_s(M\cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma,Q_0,\delta,r)\left(\frac{1}{x}\right)^{\chi(\gamma,M^2)}$$
$$\times\frac{\bar{\alpha}_s^2\beta_0\chi_0(\gamma)}{8N_c}\log\left(\frac{1}{x}\right)\left[-\psi(\delta-\gamma)+\log\frac{M^2r^2}{4}-\frac{1}{1-\gamma}-\psi(2-\gamma)-\psi(\gamma)\right]$$

core element:

NLO BFKL eigenvalue with collinear resummation (‘RG’)

$$\chi(\gamma,M^2) = \bar{\alpha}_s\chi_0(\gamma)+\bar{\alpha}_s^2\tilde{\chi}_1(\gamma)-\frac{1}{2}\bar{\alpha}_s^2\chi_0'(\gamma)\chi_0(\gamma)+\chi_{\mathrm{RG}}(\bar{\alpha}_s,\gamma,\tilde{a},\tilde{b}).$$

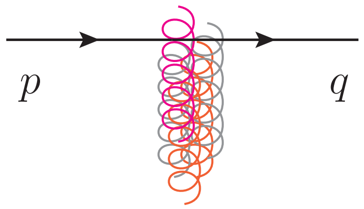
proton & dipole impact factors

$$f(\gamma,Q_0,\delta,r) = \frac{r^2\cdot\pi\Gamma(\gamma)\Gamma(\delta-\gamma)}{N_c(1-\gamma)\Gamma(2-\gamma)\Gamma(\delta)}$$

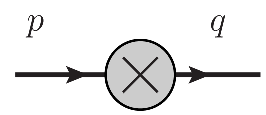
High density effects in theory description at low x

use light-cone gauge, with $k^- = n^+ \cdot k$, $(n^+)^2 = 0$, $n^+ \sim$ target momentum

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

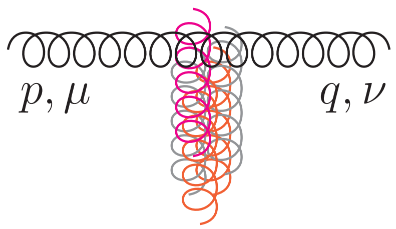


$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$

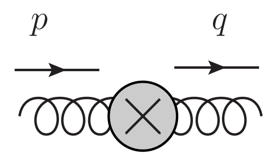


$$\tilde{S}_F^{(0)}(q)$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



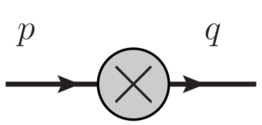
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

interaction with the background field:

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$

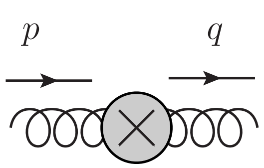


$$= \tau_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\,\not{n}$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{ \theta(p^+)\left[V_{ij}(\boldsymbol{z}) - 1_{ij}\right] - \theta(-p^+)\left[V_{ij}^\dagger(\boldsymbol{z}) - 1_{ij}\right] \right\}$$

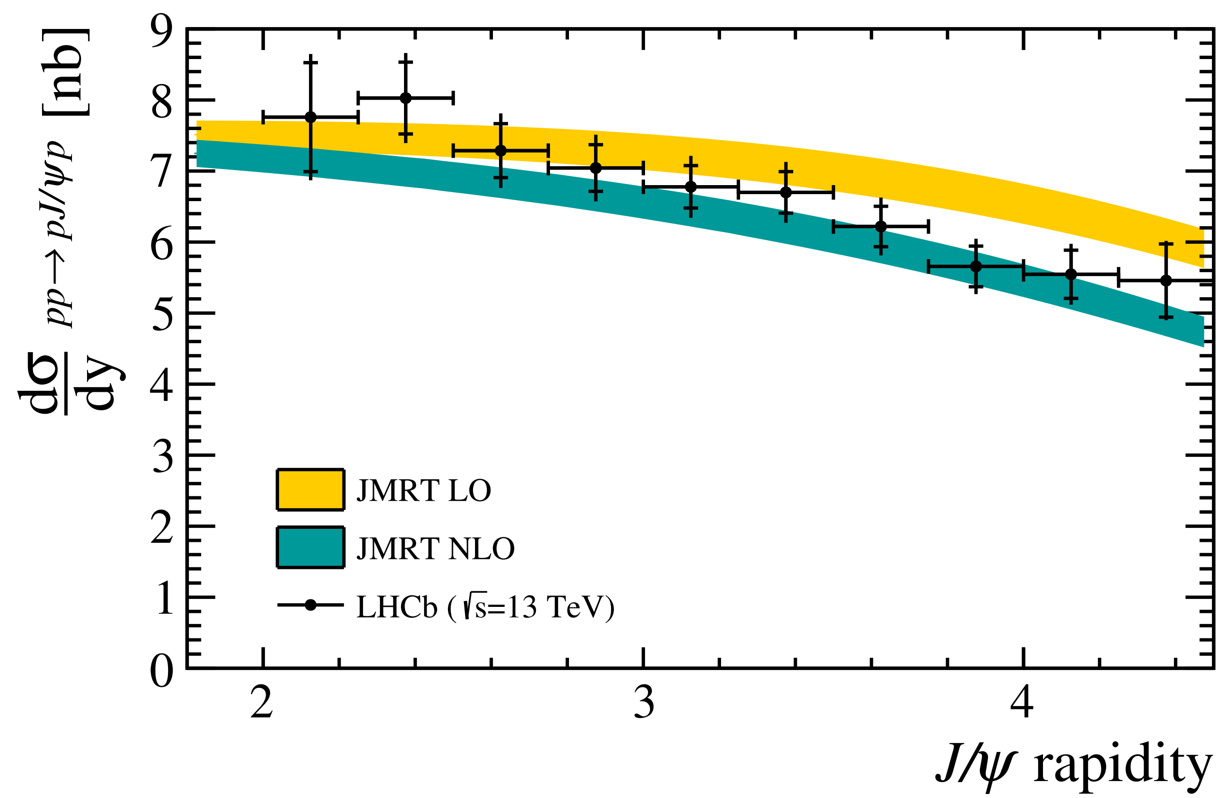
strong background field resummed into path ordered exponentials (Wilson lines)

$$A^-(x^+, x_t) = \delta(x^+)\alpha(x_t)$$

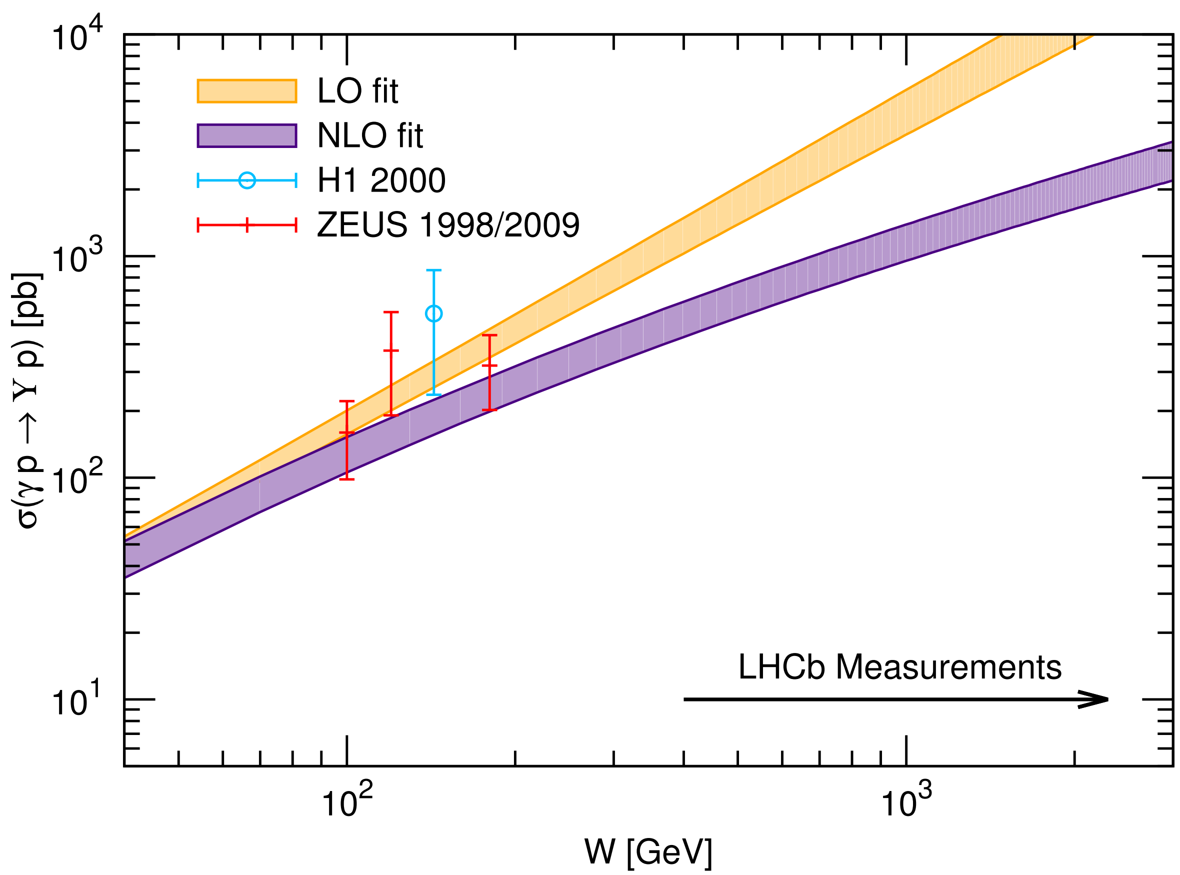


$$= \tau_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\ (-2p^+)$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{ \theta(p^+)\left[U^{ab}(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left(U^{ab}\right)^\dagger(\boldsymbol{z}) - 1\right] \right\}$$



[LHCb coll.; 1806.04076]



[Jones, Martin, Ryskin, Teubner; 1307.7099]