Solving SUSY Gauge Theories

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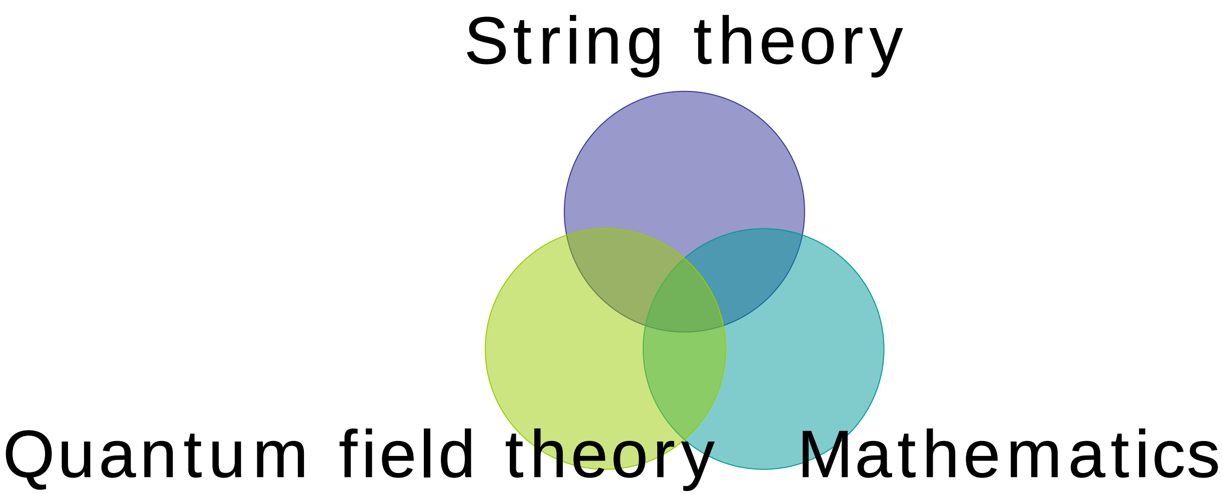
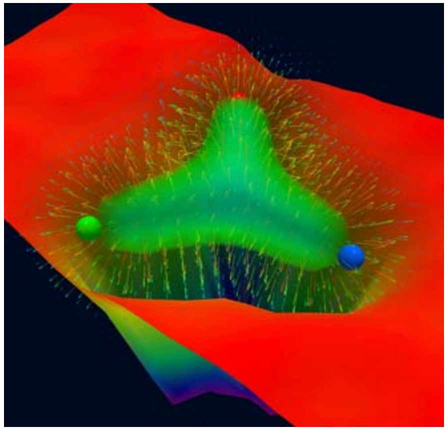
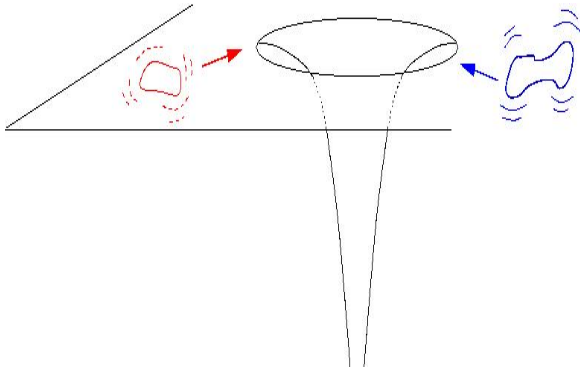




… brought together by foundational questions such as

- How do quarks and gluons behave at strong coupling?

- How do strings and quantum fields behave on curved space-times?



- United in the search for new theoretical tools –

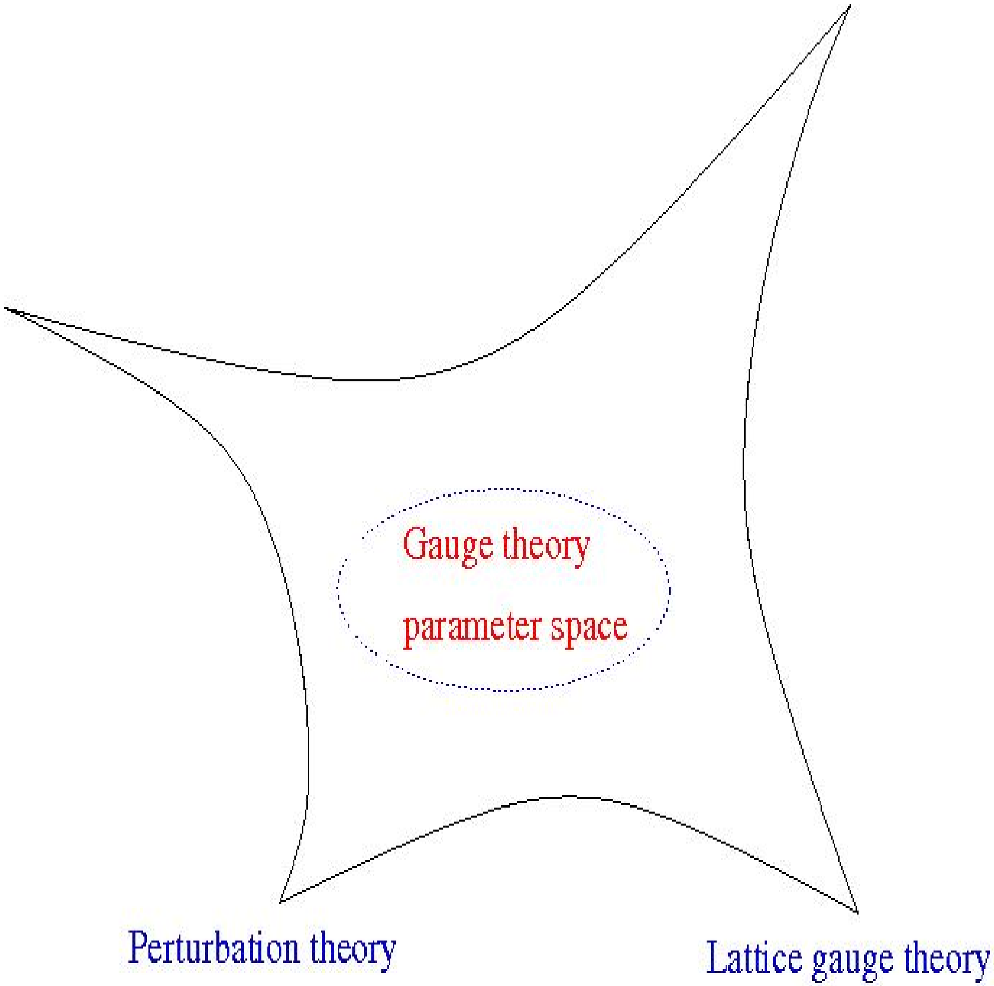
  Geometry, Integrability, Representation theory …..

- United under the roof of the Center for Mathematical Physics (ZMP)

Gauge theory

- Various approximate descriptions:
- Perturbation theory
- Lattice gauge theory
- ….

But no complete picture on full parameter space



Electric-magnetic duality and confinement

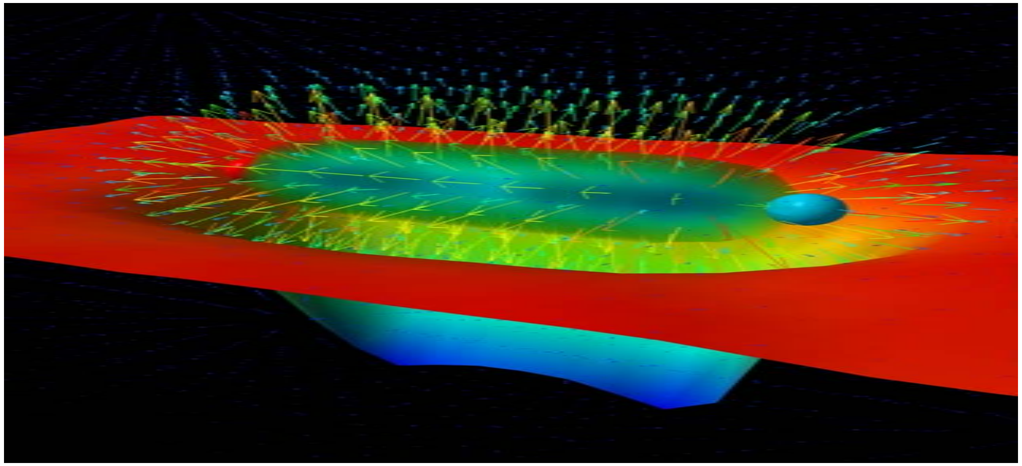
How to understand confinement?

Proposed explanations based on electric-magnetic duality:

Weak coupling: electrically charged particles light

Strong coupling: magnetic monopoles light -> particles of effective theory. Condensations of monopoles -> Confinement.

Conjecture (Polyakov): crossover caused by instantons

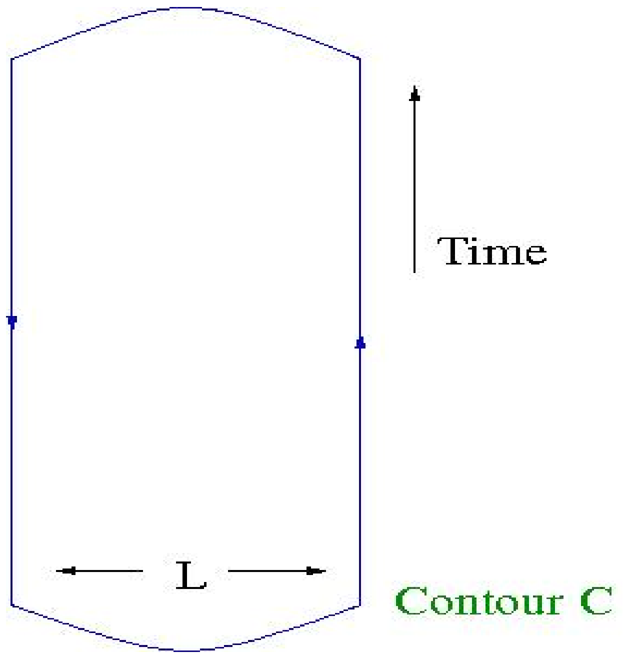


Wilson loops

Important observable in gauge theories:

$$W(C) = \mathrm{tr}_R \left[\mathcal{P} \exp \left(\oint A_\mu dx^\mu \right) \right]$$

Measure of potential energy between static probe quarks



Gives criterion for confinement: Area law

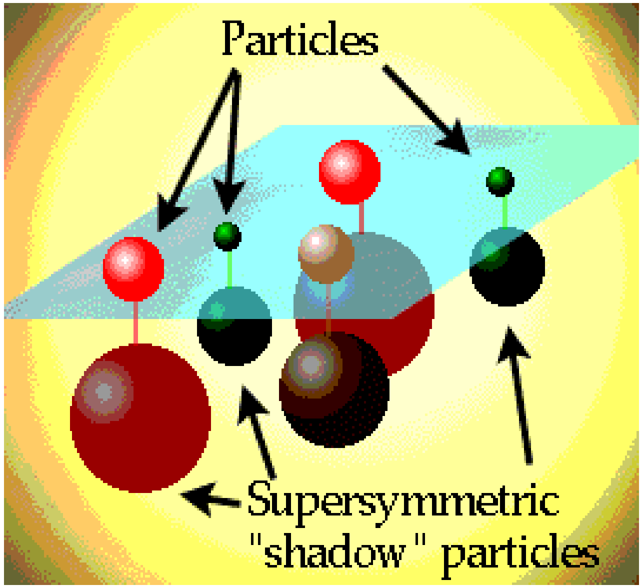
$$W(C) \sim \exp(-\kappa L T)$$

SUSY gauge theory

SUSY: Symmetry which ensures fine balance between bosonic and fermionic degrees of freedom.

SUSY suppresses certain quantum corrections !

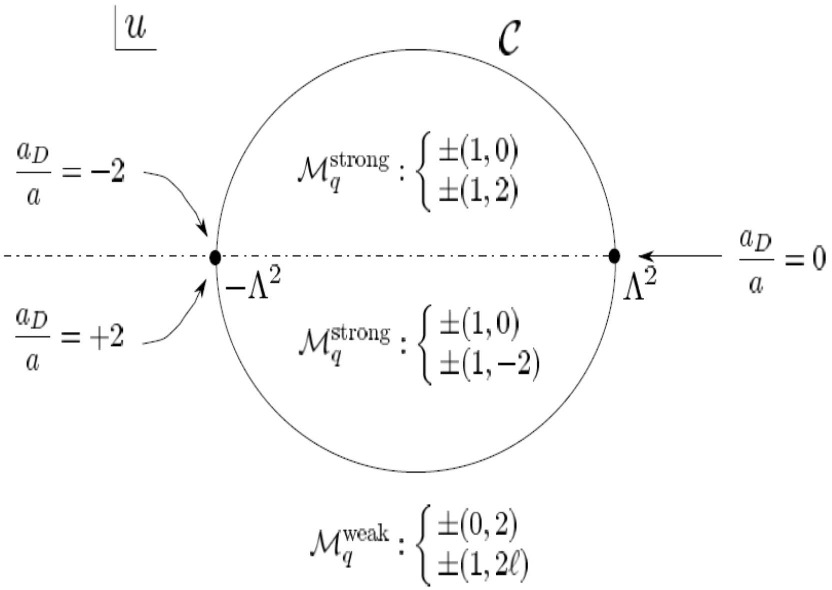
- May have complete control over low-energy physics!



Seiberg-Witten theory I

Complete control over **low-energy** physics of SUSY gauge theory:

Spectrum, masses and interactions of light particles



Seiberg-Witten theory II

Order parameter u
(scale parameter)
VEV of Higgs field

$$u(a) \;=\; \mathrm{Tr}\,\phi^2 \;=\; 2a^2$$

Low energy effective
theory completely
characterized by function
F(a) “Prepotential”

$$F(a) = \frac{1}{2}\tau_0 a^2 + \frac{i}{\pi} a^2 \log\left(\frac{a^2}{\Lambda^2}\right) + a^2 \sum_{n=1}^{\infty} F_n \left(\frac{a^2}{\Lambda^2}\right)^{4n}$$

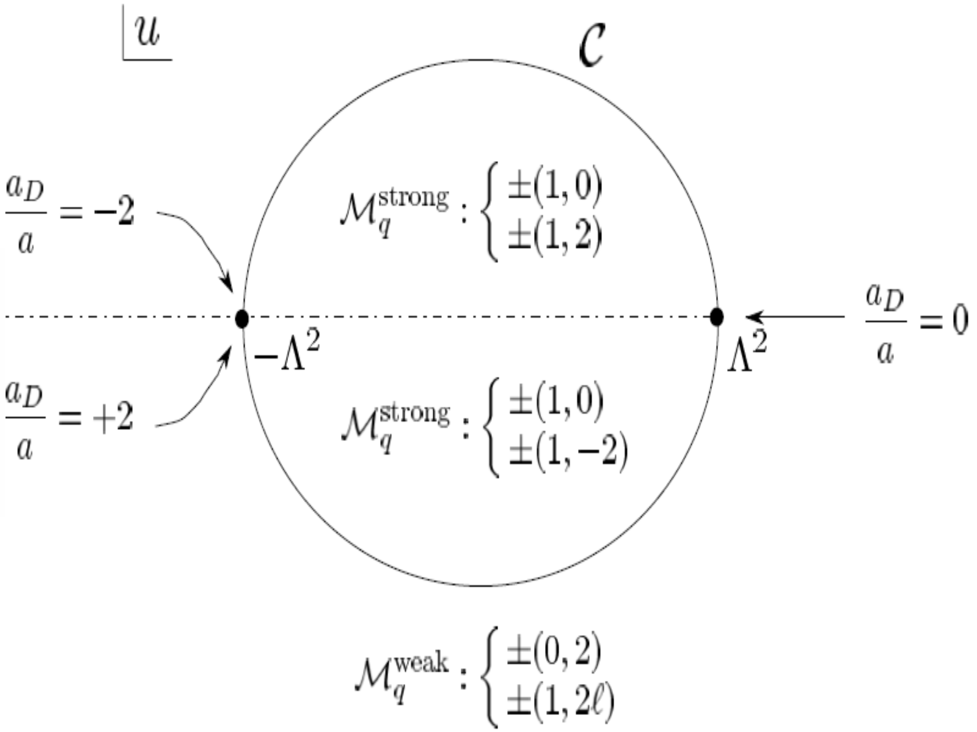
Seiberg-Witten theory III

Exhibits electric – magnetic duality:

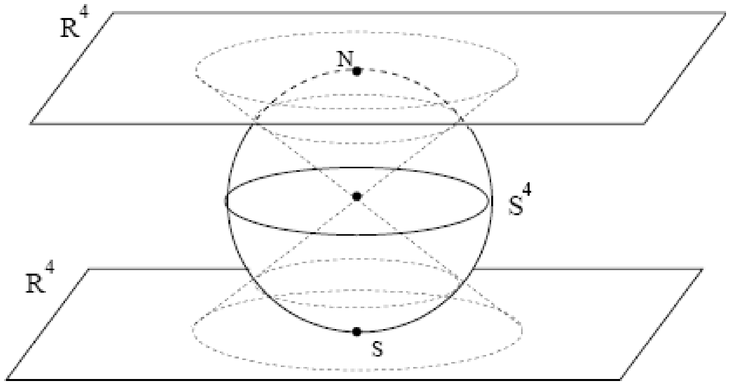
Summing over instantons

$$F(a) = \frac{1}{2}\tau_0 a^2 + \frac{i}{\pi} a^2 \log\left(\frac{a^2}{\Lambda^2}\right) + a^2 \sum_{n=1}^{\infty} F_n \left(\frac{a^2}{\Lambda^2}\right)^{4n}$$

- crossover into regime where magnetic monopoles get light !



Beyond Seiberg-Witten theory I



Consider expectation value of Wilson loops on four-sphere.

- Highly nontrivial function W(q) of UV coupling q

$$W(q) \equiv \langle \mathcal{L}_C \rangle =$$

$$= \int da\, a^2\, 2\cos(2\pi r a)\, |\mathcal{F}_a(q)|^2$$

$$\mathcal{F}_a(q) = q^{a^2+c} \sum_{n=0}^{\infty} f_a^{(n)} q^n$$

Pestun: Formula for W(q)
(sum of instanton contributions $f_a^{(n)}$,
calculated by Nekrasov)

Problem: Sum the series, demonstrate cross-over to strong-coupling regime ?

Beyond Seiberg-Witten theory II

Key observation:

(Alday, Gaiotto,Tachikawa)

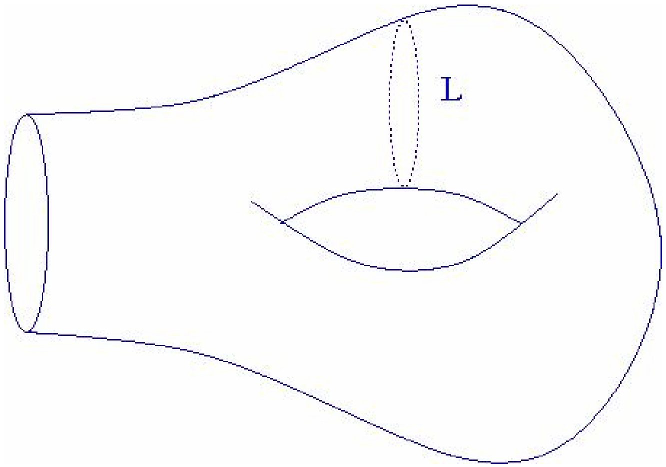
Expression for $\mathcal{F}_a(q)$ is holomorphic part of expectation values in Liouville theory.

$$\mathcal{L} = \frac{1}{4\pi}(\partial_a \phi)^2 + \mu e^{2b\phi}$$

$$\langle\, e^{2\beta\phi}\,\rangle_{\mathrm{Torus}} = tr_{\mathcal{H}}(e^{2\beta\phi} q^{H}) = \int da\, a^2 |\mathcal{F}_a(q)|^2$$

SUSY Gauge theory vs. Liouville theory

Conjecture (Polyakov):
Liouville theory = quantum theory of Riemann surfaces



This conjecture has been made precise and proven (J.T.).

Consequence:
$W(q)$ is expectation value of geodesic length operator:

$$W(q) \equiv \langle \, \mathcal{L}_C \, \rangle = \langle \, q \, | \, L \, | \, q \, \rangle$$

Beyond Seiberg-Witten theory III

Our knowledge of Liouville theory, together with

$$W(q) \equiv \langle \mathcal{L}_C \rangle = \langle q \,|\, L \,|\, q \rangle$$

allows us to describe cross-over to strong coupling (large q).

Main lessons:

a) There exists expansion at large q, generated by the dual Lagrangian description,

b) Wilson loop turns into ‘t Hooft loop, as expected.

Conclusion:

Classical + modern Mathematics

(Classical and quantum Riemann surfaces):

Highly relevant for low-energy physics of SUSY gauge theories!