Neutrino masses and mixings from non-abelian discrete symmetries

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Neutrino masses and Lepton flavor violation at the LHC

Würzburg, 24-25 Nov

Introduction

Neutrino physics is entering a "precision era"

Recent data

from Gonzalez-Garcia&Maltoni, Phys.Rept.460:1-129,2008

Solar sector

$$\Delta m^2_{21} = 7.67^{+0.67}_{-0.61} \times 10^{-5}\, eV^2$$

$$\theta_{12} = 34.5^{+4.8}_{-4.0}$$

Atmospheric sector

$$\Delta m^2_{31} = 2.46^{+0.47}_{-0.42} \times 10^{-3}\, eV^2$$

$$\theta_{23} = 42.3^{+11.3}_{-7.7}$$

What we still do not know

$$\theta_{13} \quad - \quad \delta_{CP}$$

$$\theta_{23} = 45^o\, ?$$

mass hierarchy

Introduction

The data seem to be described by:

$$\sin\theta_{13} = 0$$
$$\tan^2\theta_{23} = 1$$
$$\tan^2\theta_{12} = 1/2$$

Very good first order approximation, called Tri-Bimaximal mixing (TBM)

The crucial question is:

Is TBM real ?

YES

One can invoke discrete symmetry groups which naturally give TBM at lowest order:
D4, A4, S3, S4 ....

NO

Agreement with data is accidental: many models fit the data and TBM does not play any fundamental role

Introduction

We follow here the approach **YES**

See talks by Majee and Seidl for models of type NO

General characteristics and predictions

The leading order (LO) results is TBM

Having specified the field content, the corrections (NLO) to TBM mixing arise from higher dimensional effective operators

→ All mixing angles receive corrections of the same order of magnitude, unless very special dynamical tricks are used

↓

The scale of the corrections is fixed from the solar angles (most contrained angle)

Introduction

$$\sin^2(2\theta_{12})^{exp} - \sin^2(2\theta_{12})^{TBM} \sim O(\lambda_C^2) \sim 0.04$$

note that λ_C^2 is a convenient hierarchy parameter not only for quarks but also in the charged lepton sector

$(m_\mu/m_\tau) \sim 0.06 \sim \lambda_C^2$

$(m_e/\ m_\tau) \sim 0.005 \sim \lambda_C^{3-4}$

Relevant prediction: $\theta_{13} \sim O(\lambda_C^2)$

within the sensitivity of the experiments which are now in preparation

Important: exp data do not exclude $\theta_{13} \sim O(\lambda_C)$

If so, once could argue that TBM is just an accidental coincidence

The neutrino mixing matrix

We know from oscillation experiments that neutrinos are massive and are at least 3:
mass and flavor bases are different

The mixing matrix connects these two bases,

like the CKM for quarks

$$\nu_\alpha = U_{\alpha i} \nu_i$$

Flavour eigenstates

Mass eigenstates

What is its origin?

After EW symmetry breaking you get mass matrices

$$\lambda^l_{\alpha\beta} \qquad \lambda^\nu_{\alpha\beta}$$

leptonic mass matrix: no real and no symmetric

neutrino mass matrix: no real but symmetric

The neutrino mixing matrix

To obtain the mass eigenvalues one needs to diagonalize both mass matrices

$(\lambda^l)^+ \lambda^l$ is diagonalized by an unitary U_l

λ^ν is diagonalized by an unitary U_ν

Interaction lagrangian in the lepton sector

$$\bar{l}_L \gamma_\mu \nu_L \Rightarrow (U^+_l U_\nu) \bar{l}_L \gamma_\mu \nu_L$$

$$U_{PMNS} = U^+_l U_\nu$$

PMNS=Pontecorvo-Maki-Nakagawa-Sakata

neutrino mixing matrix

The neutrino mixing matrix

$$U_{PMNS} = U^{+}_{l} U_{\nu}$$

PMNS=Pontecorvo-Maki-Nakagawa-Sakata

Relevant feature: both charged and neutral leptons contribute to the neutrino mixing matrix !!!

Experimental data are very well described with the following ansatz:

$$U_{PMNS} = U_{TBM} + O(\lambda_C^2)\, corrections$$

$$U_{PMNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta) R_{12}(\theta_{12})$$

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

The model building

Large interest in recent years to find models which reproduce, at least al LO, the TBM behaviour of the data (no GUT models in the list...)

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And many others...

TBM and discrete symmetries

The question is: why discrete non-abelian groups work so well

Let us first consider the case in which the data are exactly explained in terms of TBM

In the basis where charged leptons are diagonal

$$U_l = I \quad \Rightarrow \quad U_{PMNS} = U_\nu = U_{TBM}$$

Since U_ν diagonalizes the neutrino mass matrix we can infer the most general structure of λ^ν

$$\lambda^\nu = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

This is a symmetric, 2-3 symmetric matrix with $a_{11} + a_{12} = a_{22} + a_{23}$

TBM and discrete symmetries

Then if you want to get a TBM neutrino mixing you must be able to produce this kind of mass matrix

Let us take to group S4 to illustrate how these models generally work

1 - S4 is the group of permutations of 4 objects → 24 objects

2 – as usual, to generate all the group elements we need to identify " generators of the group" and their action

these are called S and T

↓

One possible "representation": $S^2=T^3=1$ and $S\,T^2S=T$

3 – they act as follows: (1234) → (2341) under S

(1234) → (2314) under T

TBM and discrete symmetries

Now we are done!

The 24 elements are obtained considering all possible permutations of 1234. They belong to 5 conjugate classes...

4 – the # of irreducible representations = # of conjugate classes

S4 has 5 irreducible representations

- two singlets 1_1 and 1_2 → useful for SM singlets
- one doublet 2 → useful for quarks
- two triplets 3_1 and 3_2 → useful for leptons

5 – a model is built when one specifies the field content and assign them to representation of the group

6 – all the interactions must respect the SM as well as the S4 symmetries

TBM and discrete symmetries

A representation of the group

"T-diagonal representation"

1_1 : S=1 T=1

1_2 : S=-1 T=1

2 : $S=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $T=\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$ where $\omega = e^{2\pi i/3}$

3_1 : $S=\frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}$ $T=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$

3_2 : S → -S T → T

Table of multiplication

$$1_1 \otimes any = any$$

$$1_2 \otimes 1_2 = 1_1 \quad 1_2 \otimes 2 = 2 \quad 1_2 \otimes 3_j = 3_j$$

$$2 \otimes 2 = 1_1 + 1_2 + 2 \quad 2 \otimes 3_i = 3_1 + 3_2$$

$$3_i \otimes 3_i = 1_1 + 2 + 3_1 + 3_2 \quad 3_1 \otimes 3_2 = 1_2 + 2 + 3_1 + 3_2$$

TBM and discrete symmetries

The TBM is derived considering that:

S4 is a symmetry of the Nature at a very high energy scale Λ

the symmetry is spontaneously broken by a set of scalar multiplets Φ (FLAVONS) with VEV aligned in some particular directions

The preserved subgroups have to be different in the charged and neutrino sectors otherwise

$$U_l = U_\nu \quad \Rightarrow \quad U_{PMNS} = I$$

↓

Different symmetry breaking patterns allowed !!!

Symmetry breaking

In the neutrino sector:

Choose the vevs of flavon fields as $\varphi_S \approx \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

This choice breaks T but preserves S: $T\varphi_S \neq \varphi_S \qquad S\varphi_S = \varphi_S$

This also preserves the element $TSTS^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

The preserved group after symmetry breaking is $Z_2 X Z_2$

↓

$$S\lambda^\nu S = \lambda^\nu \qquad (TSTS^2)\lambda^\nu(TSTS^2) = \lambda^\nu$$

The most general neutrino mass matrix diagonalized by TBM is left invariant by the generators of this subgroup of S4 !!!

Symmetry breaking

In the charged lepton sector:

Choose the vevs of flavon fields as $$\varphi_T \approx \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

The group S4 is completely broken $T\,\varphi_T \neq \varphi_T \qquad S\,\varphi_T \neq \varphi_T$

Having obtained TBM in the neutrino sector, U_l must be the identity

⇓

$(\lambda^l)^+ \lambda^l$ must be a diagonal matrix and, at the same time, we need to mantain the hierarchy between the masses of charged leptons

This is achieved observing that:

hierarchy: ϕ_T, ϕ_T^2/Λ and ϕ_T^3/Λ^2 preserve different subgroups (not of S4) with invariant mass matrices with only 1 column different from zero

diagonal form: Z_N eliminates unwanted couplings

A model realization: S4 x Z5

Standard Model fields + right-handed neutrinos

$$S_4: \quad \nu^c = \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix} \sim 3_1 \qquad l = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \end{pmatrix} \sim 3_1 \qquad e^c \sim 1_2 \qquad \mu^c \sim 1_1 \qquad \tau^c \sim 1_1$$

$$Z_5: \quad \omega^2 \qquad 1 \qquad \omega^3 \qquad \omega^2 \qquad \omega$$

Higgs fields

$$h_u \sim (1_{1,} \, \omega^2) \qquad h_d \sim (1_{1,} \, 1)$$

A model realization: S4 x Z5

Symmetry breaking sector

Charged leptons:

$$\varphi_T \sim (3_{1,}\,\omega^4) \qquad \eta \sim (2,\omega^4)$$

Neutrino sector:

$$\varphi_S \sim (3_{1,}\,\omega) \qquad \Delta \sim (2,\omega) \qquad \xi \sim (1_{1,}\,\omega)$$

remember

$$\langle \varphi_T \rangle = v_T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \langle \varphi_S \rangle = v_S \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

and also

$$\langle \eta \rangle = v_\eta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \langle \Delta \rangle = v_\Delta \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \langle \xi \rangle = u$$

A model realization: S4 x Z5

Most general lagrangians invariant under SM x S4 x Z5

Charged leptons:

τ entry

$$L = \frac{y_\tau}{\Lambda} \tau^c (l \varphi_T) h_d$$

μ entries

$$\frac{y_{\mu 1}}{\Lambda^2} \mu^c (l \varphi_T \varphi_T) h_d + \frac{y_{\mu 2}}{\Lambda^2} \mu^c (l \varphi_T \eta) h_d$$

electron

$$\begin{cases} \frac{y_{e1}}{\Lambda^3} e^c [l \varphi_T (\varphi_T \varphi_T)_2] h_d + \frac{y_{e2}}{\Lambda^3} e^c [l \varphi_T (\varphi_T \varphi_T)_{3_1}] h_d \\ \frac{y_{e3}}{\Lambda^3} e^c [l \eta (\varphi_T \varphi_T)_{3_1}] h_d + \frac{y_{e4}}{\Lambda^3} e^c [l \varphi_T (\eta \eta)_2] h_d \end{cases}$$

A model realization: S4 x Z5

Most general lagrangians invariant under SM x S4 x Z5

Charged leptons mass matrix:

$$\lambda^l = v_d \frac{v_T}{\Lambda} diag\left(\underbrace{\frac{1}{\Lambda^2} f\left(y_{e_i}\right)}_{m_e}, \underbrace{\frac{1}{\Lambda} g\left(y_{\mu_i}\right)}_{m_\mu}, \underbrace{y_\tau}_{m_\tau} \right)$$

$$f = \left(y_{e_1} + 2\, y_{e_1}\right) v_T^2 - 2\, y_{e_3} v_T v_\eta + y_{e_4} v_\eta^2$$

$$g = 2\, y_{\mu_1} v_T + y_{\mu_2} v_\eta$$

Assuming all y coefficients of O(1) $\longrightarrow$ $\frac{v_T}{\Lambda} \sim \frac{v_\eta}{\Lambda} \sim \lambda_C^2$

A model realization: S4 x Z5

Most general lagrangians invariant under SM x S4 x Z5

Neutrino sector:

we assume that neutrino masses are generated by the See-Saw mechanism

$$\lambda^{\nu} = -m_D^T m_M^{-1} m_D$$

Then we need both Majorana and Dirac mass matrices

$$L_{\nu} = \frac{1}{\Lambda} \nu^c \, l \, h_u \left(y_{\nu_1} \varphi_S + y_{\nu_2} \Delta + y_{\nu_3} \xi \right)$$

Dirac part

$$+ \nu^c \nu^c h_u \left(b \varphi_S + c \Delta + a \xi \right)$$

Majorana sector

A model realization: S4 x Z5

After symmetry breaking we get a light neutrino mass matrix,
diagonalized by TBM
with the following eigenvalues

$$m_1 = -\left(\frac{v_u}{\Lambda}\right)^2 \frac{(3\, y_{\nu_1} v_S - y_{\nu_2} v_\Delta + y_{\nu_3} u)^2}{a u + 3 b v_s - c v_\Delta} \qquad m_2 = -\left(\frac{v_u}{\Lambda}\right)^2 \frac{(2\, y_{\nu_2} v_\Delta + y_{\nu_3} u)^2}{a u + 2 c v_\Delta} \qquad m_3 = \left(\frac{v_u}{\Lambda}\right)^2 \frac{(3\, y_{\nu_1} v_S + y_{\nu_2} v_\Delta - y_{\nu_3} u)^2}{a u - 3 b v_s - c v_\Delta}$$

Masses depend on six complex Yukawa parameters
No sum rules exist among them (unlike A4)

↓

The model is less predictive but more manageable

easier to tune mass differences and recover the standard neutrino phenomenology (more about that later)

Important: Vacuum alignment

Any serious flavour model must derive the vacuum expectation values of the fields from general principles

To achieve this aim, one introduces new Standard Model singlet fields, DRIVING FIELDS with well defined transformation properties under S4 x Z5

- we build the most general superpotential W allowed by the symmetries of the theory and derive the scalar potential in the usual way

$$V = \Sigma \left| \frac{\partial W}{\partial \phi_i} \right|^2 + m_i^2 \left| \phi_i^2 \right| + \ldots$$

ϕ_i are driving fields components

Since m_i are expected to be smaller than the mass scales in W, we can neglect this term and then working in the SUSY limit

$$V = 0 \quad \Rightarrow \quad \left| \frac{\partial W}{\partial \phi_i} \right| = 0$$

We get a set of equations for the flavon field components; the solution of the system of equations is the VEV configuration

A bit of phenomenology

The parameter space is quite large: neutrino masses depend on 6 complex Yukawa parameters as well as 3 complex vevs and 1 large scale Λ

⇓

The model is not really predictive: we can easily account for any experimental data coming from neutrino oscillations and almost any value for Σm_i and m_{ee} are allowed

More interesting is to study the phenomenological consequences of the model assuming more *natural* conditions:

1 - all Yukawa are O(1) (natural values – no fine-tuning)
2 – the vevs are all of the same order of magnitude

The parameters of the model are then restricted requiring that (3 σ)

$$\Delta m^2_{sol} > 0$$

$$|\Delta m^2_{atm}| = 2.41 \pm 0.34 \times 10^{-3}\, eV^2$$

$$r = \frac{\Delta m^2_{sol}}{|\Delta m^2_{atm}|} = 0.032 \pm 0.006$$

A bit of phenomenology

Three interesting observables:

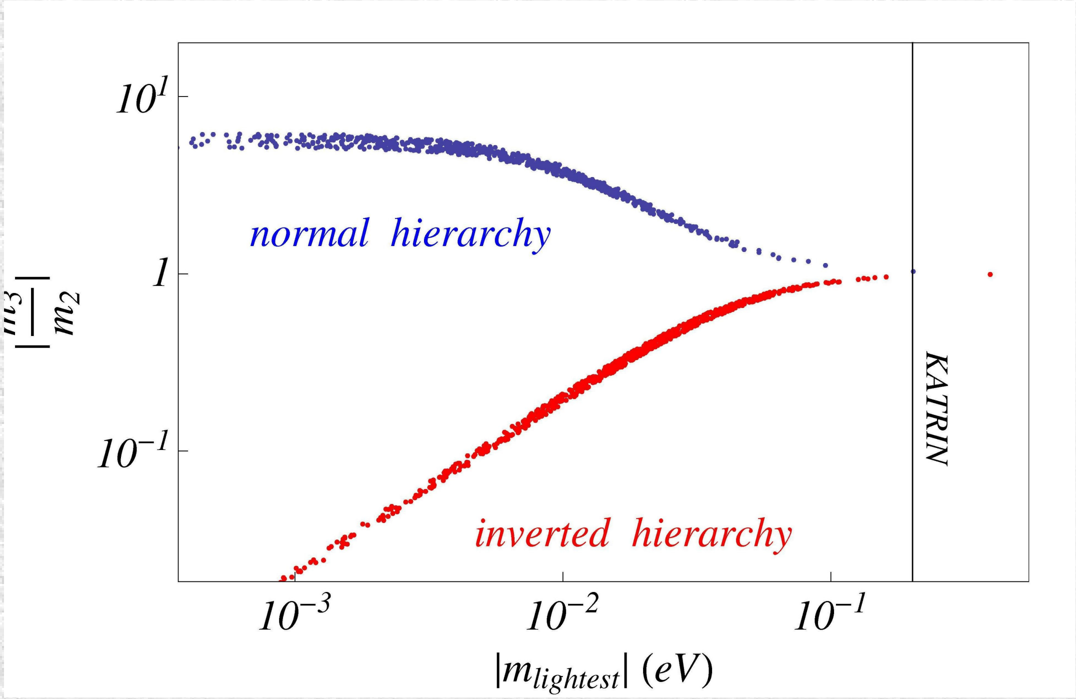
$$\left|\frac{m_3}{m_2}\right| \quad vs \quad \left|m_{lightest}\right|$$

useful to see which mass hierarchy is allowed

Normal hierarchy (NI): $m_1 < m_2 < m_3$

Inverted hierarchy (NI): $m_3 < m_1 < m_2$

KATRIN sensitivity on m ~ 0.2 eV

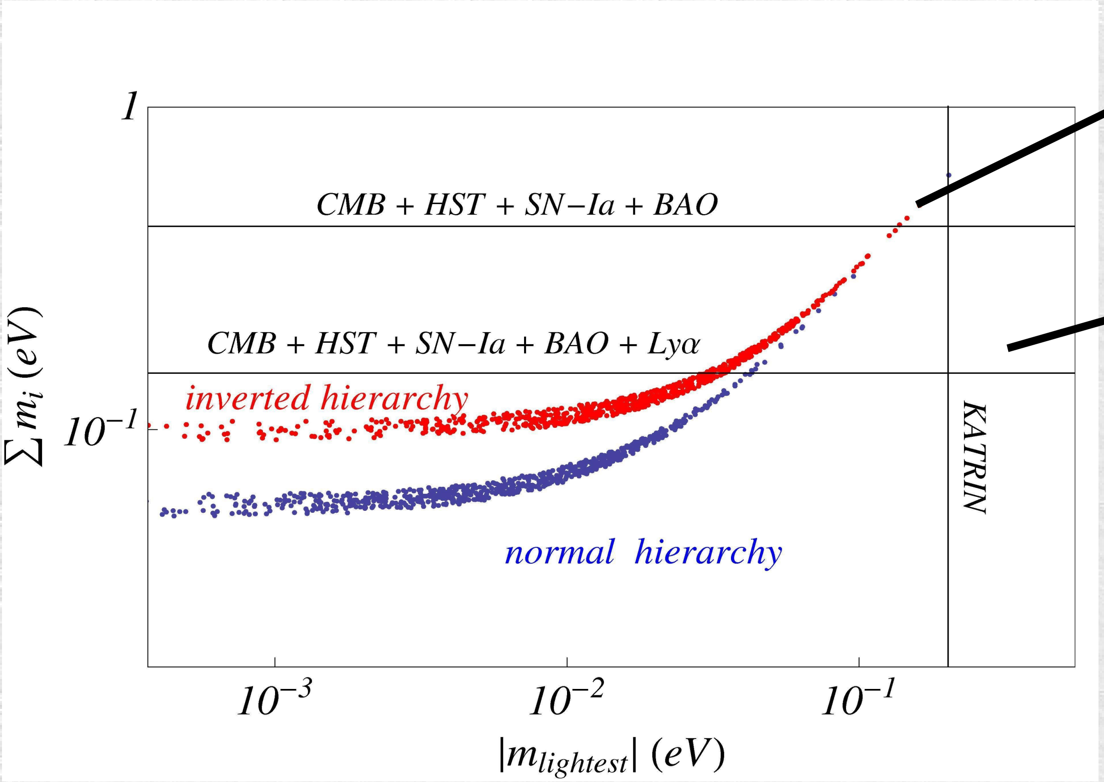


Degenerate spectrum disfavoured

Large hierarchies allowed for very small $m_{lightest}$

A bit of phenomenology

$\sum m_i$



0.6 eV from
WMAP+ACBAR+VSA+CBI+BOOMERANG

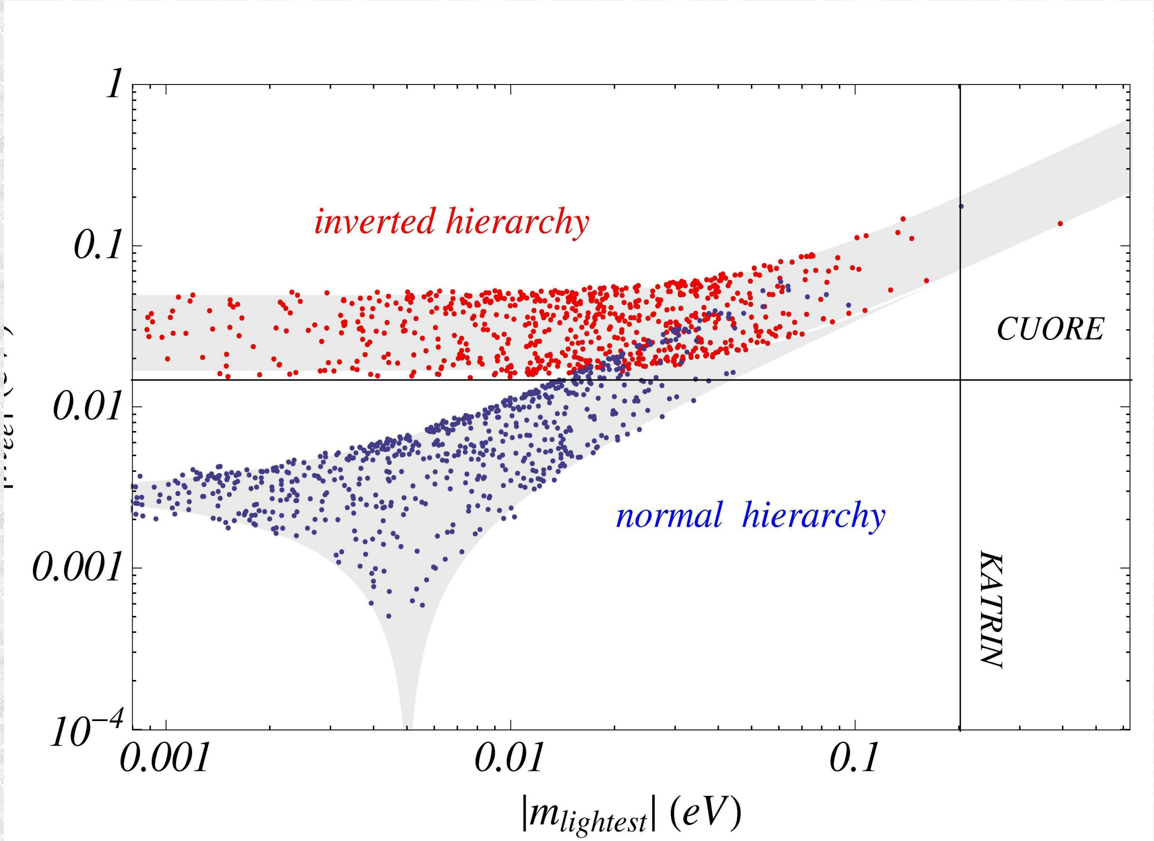
0.19 eV from the previous ones + Lyα

Σ m_i too similar to be distinguished using
the current cosmological information on the sum of the neutrino masses

A bit of phenomenology

$$|m_{ee}| = |m_1 U_{e1}^2 + m_2 U_{e2}^2|$$

in the limit of exact TBM



Many points in the region $m_{lightest}$ >~0.01eV where the two hierarchies can be distinguished

For NH, larger $|m_{ee}|$ are favoured

For IH, almost any allowed $|m_{ee}|$ can be obtained

<u>Salient feature</u>: degenerate spectrum is disfavoured

NLO corrections

They are important because TBM is only an approximate description of the neutrino mixing matrix

NLO correction could affect any sector of the theory

Charged lepton and neutrino masses are modified by

Corrections to the vacuum alignment

$$\langle \Phi \rangle \to \langle \Phi \rangle^{LO} + \delta\Phi \quad \text{where} \quad \delta\Phi \sim \frac{\langle \Phi \rangle^{LO}}{\Lambda}$$

Higher order operators (HO) of O(1/Λ) compared to the LO lagrangian

$$L^{tot} = L^{LO}\left(\langle \Phi \rangle^{LO}\right) + L^{LO}\left(\delta\Phi\right) + L^{HO}\left(\langle \Phi \rangle^{LO}\right)$$

NLO corrections

In the charged lepton sector

These corrections give a new mass matrix whose entries are as follows

$$\lambda^l \sim \begin{pmatrix} \lambda_C^4 & \lambda_C^4 & \lambda_C^4 \\ \lambda_C^4 & \lambda_C^2 & \lambda_C^4 \\ \lambda_C^2 & \lambda_C^2 & 1 \end{pmatrix}$$

$(\lambda^l)^+ \lambda^l$ is diagonalized by an unitary U_l different from the identity !

$$U_l \sim \begin{pmatrix} 1 & \lambda_C^2 & \lambda_C^2 \\ \lambda_C^2 & 1 & \lambda_C^2 \\ \lambda_C^2 & \lambda_C^2 & 1 \end{pmatrix}$$

This is already enough to generate deviation from TBM

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} + O(\lambda_C^2) & \frac{1}{\sqrt{3}} + O(\lambda_C^2) & O(\lambda_C^2) \\ \frac{-1}{\sqrt{6}} + O(\lambda_C^2) & \frac{1}{\sqrt{3}} + O(\lambda_C^2) & -\frac{1}{\sqrt{2}} + O(\lambda_C^2) \\ \frac{-1}{\sqrt{6}} + O(\lambda_C^2) & \frac{1}{\sqrt{3}} + O(\lambda_C^2) & \frac{1}{\sqrt{2}} + O(\lambda_C^2) \end{pmatrix}$$

remember that

$U_{PMNS} = U^+_l\, U_\nu = U^+_l\, U_{TBM}$

NLO corrections

Also in the neutrino sector the mass matrix is not diagonalized by TBM

We get U_l with elements of the firsts and second column corrected at $O(\lambda_C^2)$, the last one is identical to TBM
(this needs NNLO corrections --- it is a model dependent feature)

FINAL RESULTS

$$\sin\theta_{13} = |U_{e3}| \sim O(\lambda_C^2)$$

$$\sin\theta_{12} = \frac{|U_{e2}|}{\sqrt{1-|U_{e3}^2|}} \sim \frac{1}{\sqrt{3}} + O(\lambda_C^2)$$

$$\sin\theta_{23} = \frac{|U_{\mu 3}|}{\sqrt{1-|U_{e3}^2|}} \sim \frac{1}{\sqrt{2}} + O(\lambda_C^2)$$

Conclusions

- TBM is a good description of neutrino experimental data

- Models for neutrino masses and mixing based on non-abelian discrete symmetries are quite good in reproducing this pattern

- We presented an example based on S4, able to give NLO corrections to TBM compatible with the data

- Both normal and inverted hierarchies are allowed by the model. However, the degenerate spectrum is strongly disfavoured

Backup slides

Conjugate classes for S4

Given x,y in A, x ~ y if exists g of S4 so that y=g x

C1: 1

C2: S^2, TS^2T^2, $S^2TS^2T^2$

C3: T, T^2, S^2T, S^2T^2, $STST^2$, STS, TS^2, T^2S^2

C4: ST^2, T^2S, TST, $TSTS^2$, STS^2, S^2TS

C5: S, TST^2, ST, TS, S^3, S^3T^2

The TBM mixing matrix vs data

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_{\exp} = \begin{pmatrix} 0.79-0.88 & 0.47-0.61 & <0.2 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix}$$

Compatibility with leptogenesis

Just an estimate....

The asymmetry parameters are defined as follows:

$$\epsilon_i = \frac{1}{8\pi(\tilde{Y}\tilde{Y}^+)_{ii}} \Sigma_{j\neq i} \Im(\tilde{Y}\tilde{Y}^+)^2_{ij} f\left(\frac{|M_j^2|}{|M_i^2|}\right)$$

"hat" matrices are in the basis where the Majorana mass matrix is diagonal

For susy theories:

$$f(x) = -\sqrt{x}\left[\frac{2}{x-1} + \log\left(\frac{1+x}{x}\right)\right]$$

At the NLO, in the basis where charged leptons are diagonal:

$$v_u Y = \left(m_D + v_u \frac{\epsilon}{\epsilon'} \epsilon^2 y_{\nu_1} S\right)$$

LO result (m_D)

3X3 matrix of coefficients of O(1) (S)

Compatibility with leptogenesis

In the basis where Majorana mass matrix is diagonal:

$$v_u \tilde{Y} = v_u \Omega^T Y_\nu$$

At the leading order $\Omega = U_{TMB}$ and we get

$$\tilde{Y}\tilde{Y}^+ = \Omega^T Y_\nu Y_\nu^+ \Omega^* = diagonal\ matrix \Rightarrow \epsilon_i = 0$$

NLO are crucial

$$\delta(\tilde{Y}\tilde{Y}^+) = \Omega^T \delta(Y_\nu Y_\nu^+)\Omega^*$$

$$\Downarrow$$

$$\epsilon_1 \sim \epsilon_2 \sim \left(\frac{\epsilon}{\epsilon'}\right)\epsilon^4 \sim \lambda_C^8 \sim 6\times 10^{-6}$$

$$\epsilon_3 = 0$$

Symmetry breaking

In the charged lepton sector (2):

We see that:

– φ_T preserves the subgroup (external to S4) generated by $T_1 = \begin{pmatrix} \omega & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$

and the most general mass matrix invariant under T_1 is $m_1 = \begin{pmatrix} 0 & 0 & X \\ 0 & 0 & X \\ 0 & 0 & X \end{pmatrix}$

– $\frac{\varphi_T^2}{\Lambda} \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ preserves the subgroup generated by $T_2 = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and the most general mass matrix invariant under T_2 is $m_2 = \begin{pmatrix} 0 & X & 0 \\ 0 & X & 0 \\ 0 & X & 0 \end{pmatrix}$

– $\frac{\varphi_T^3}{\Lambda^2} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ preserves the generator T and the conserved mass matrix is $m_3 = \begin{pmatrix} X & 0 & 0 \\ X & 0 & 0 \\ X & 0 & 0 \end{pmatrix}$

Symmetry breaking

Then the low energy theory automatically produces hierarchical lepton mass matrix !

In particular:

$$\frac{m_\mu}{m_\tau} \sim \frac{\left(\frac{\varphi_T^2}{\Lambda}\right)}{\varphi_T} \sim \frac{\varphi_T}{\Lambda} = 0.06 \sim \lambda_C^2$$

And also m_e/m_μ is correctly reproduced but

$$\lambda^l \sim \begin{pmatrix} \lambda_C^4 & \lambda_C^2 & 1 \\ \lambda_C^4 & \lambda_C^2 & 1 \\ \lambda_C^4 & \lambda_C^2 & 1 \end{pmatrix}$$

clearly non diagonal

Introduction of additional abelian symmetries to suppress the unwanted couplings

Vacuum alignment

The alignment procedure works as follows

1- we introduce these *driving fields* with well defined transformation properties under S4 x Z5

$$\varphi_0^S \sim (3_{1,} \omega^3) \quad \varphi_0^T \sim (3_{1,} \omega^2) \quad \Delta_0 \sim (2, \omega^3) \quad \rho_0 \sim (1, \omega^2)$$

2- we build the most general superpotential allowed by the symmetries of the theory

Almost two separated sectors

$$\begin{aligned} W = & g_1(\varphi_0^S \varphi_S \varphi_S) + g_2(\varphi_0^S \varphi_S \xi) + g_3(\varphi_0^S \varphi_S \Delta) + g_4(\Delta_0 \Delta \Delta) \\ & + g_5(\Delta_0 \varphi_S \varphi_S) + g_6(\Delta_0 \Delta \xi) \\ & + h_1(\varphi_0^T \varphi_T \varphi_T) + h_2(\varphi_0^T \eta \varphi_T) + r_1 \rho_0(\varphi_T \varphi_T) + r_2 \rho_0(\eta \eta) \end{aligned}$$

3- the scalar potential is generally given by

$$V = \Sigma \left| \frac{\partial W}{\partial \phi_i} \right|^2 + m_i^2 \left| \phi_i^2 \right| + \ldots$$

ϕ_i = generic scalar field

m_i = soft masses

Vacuum alignment

Since m_i are expected to be smaller than the mass scales in W, we can neglect this term and then working in the SUSY limit

(we can account for soft breaking effects subsequently)

$$V = 0 \quad \Rightarrow \quad \left| \frac{\partial W}{\partial \phi_i} \right| = 0$$

4- We get a set of equations for the flavon fields deriving W with respect to the components of the driving fields and set them = 0

Vacuum alignment

charged lepton sector

$$\frac{\partial W}{\partial \varphi_{01}^{T}} = 2h_1(\varphi_{T_1}^2 - \varphi_{T_2}\varphi_{T_3}) + h_2(\eta_1\varphi_{T_2} + \eta_2\varphi_{T_3}) = 0$$

$$\frac{\partial W}{\partial \varphi_{02}^{T}} = 2h_1(\varphi_{T_2}^2 - \varphi_{T_1}\varphi_{T_3}) + h_2(\eta_1\varphi_{T_1} + \eta_2\varphi_{T_2}) = 0$$

$$\frac{\partial W}{\partial \varphi_{03}^{T}} = 2h_1(\varphi_{T_3}^2 - \varphi_{T_1}\varphi_{T_2}) + h_2(\eta_1\varphi_{T_3} + \eta_2\varphi_{T_1}) = 0$$

$$\frac{\partial W}{\partial \rho_0} = r_1(\varphi_{T_1}^2 + 2\varphi_{T_2}\varphi_{T_3}) + 2r_2\eta_1\eta_2 = 0$$

$$\langle \varphi_T \rangle = v_T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \langle \eta \rangle = v_\eta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad v_\eta = -2\left(\frac{h_1}{h_2}\right) v_T$$

Vacuum alignment

neutrino sector

$$\frac{\partial W}{\partial \varphi_{01}^{S}} = 2g_1(\varphi_{S_1}^2 - \varphi_{S_2}\varphi_{S_3}) + g_2\xi\varphi_{S_1} + g_3(\Delta_1\varphi_{S_2} + \Delta_2\varphi_{S_3}) = 0$$

$$\frac{\partial W}{\partial \varphi_{02}^{S}} = 2g_1(\varphi_{S_2}^2 - \varphi_{S_1}\varphi_{S_3}) + g_2\xi\varphi_{S_3} + g_3(\Delta_1\varphi_{S_1} + \Delta_2\varphi_{S_2}) = 0$$

$$\frac{\partial W}{\partial \varphi_{03}^{S}} = 2g_1(\varphi_{S_3}^2 - \varphi_{S_1}\varphi_{S_2}) + g_2\xi\varphi_{S_2} + g_3(\Delta_1\varphi_{S_3} + \Delta_2\varphi_{S_1}) = 0$$

$$\frac{\partial W}{\partial \Delta_{01}} = g_4\Delta_1^2 + g_5(\varphi_{S_3}^2 + 2\varphi_{S_1}\varphi_{S_2}) + g_6\xi\Delta_2 = 0$$

$$\frac{\partial W}{\partial \Delta_{02}} = g_4\Delta_2^2 + g_5(\varphi_{S_2}^2 + 2\varphi_{S_1}\varphi_{S_3}) + g_6\xi\Delta_1 = 0$$

$$\langle \varphi_S \rangle = v_S \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \Delta \rangle = v_\Delta \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle \xi \rangle = u \qquad v_\Delta = \frac{-g_2 u}{2g_3} \qquad v_S^2 = \frac{-2g_2 g_3 g_6 - g_2^2 g_4}{12 g_5 g_3^2}$$