ν-masses and leptogenesis in SUSY SO(10) model

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In collaboration with

M. K. Parida and Amitava Raychaudhuri

Plan

- Why do we need a low intermediate symmetry breaking scale?
- Difficulties to obtain a low intermediate scale in the minimal SUSY SO(10) model
- Different ways to achieve a low intermediate scale.
- Different type of Seesaw mechanism
- How can we explain Neutrino masses and Leptogenesis in an extended Double seesaw mechanism in SUSY SO(10) model
- Conclusion

Neutrino mass

- In the SM ν is massless. So, no ν_R in SM.

- Approx limit on neutrino mass is
$1.4 \times 10^{-3}\mathrm{eV}^2 < \Delta m^2_{\mathrm{atm}} (= m^2_{\nu_3} - m^2_{\nu_2}) < 3.7 \times 10^{-3}\mathrm{eV}^2$.
$5.4 \times 10^{-5}\mathrm{eV}^2 < \Delta m^2_{\mathrm{sol}} (= m^2_{\nu_2} - m^2_{\nu_1}) < 9.5 \times 10^{-5}\mathrm{eV}^2$.

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Seesaw:

$$M_\nu = \begin{pmatrix} \nu & \nu^c \end{pmatrix}_L \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}_L ,$$

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- So, the neutrino masses are: $m_{\nu_R} = M_R$, and $m_\nu = -\frac{m_D^2}{M_R}$

- Thus to have a mass $m_\nu \sim 10^{-2}\mathrm{eV}$ and $m_D \sim 1\mathrm{MeV}\ \ \mathrm{to}\ \ 1\mathrm{GeV}$ we need $m_R \sim 10^{(5-11)}\mathrm{GeV}$

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- Gravitino constraint forced us to take

$$M_R < 10^9 \mathrm{GeV}.$$

Unification in MSSM

$\boxed{particle <=> sparticle} \Rightarrow Supersymmetry$

- RGE in MSSM :

$$16\pi^2 E\frac{dg_i}{dE} = b_i g^3 + \Theta(E - M_s)(\tilde{b}_i - b_i)g^3 = \beta_{MSSM}(g)$$

- Solution :

$$\alpha_i{}^{-1}(E) = \alpha_i{}^{-1}(M_Z) - \frac{b_i}{2\pi} ln\frac{M_S}{M_Z} - \frac{(\tilde{b_i} - b_i)}{2\pi} ln\frac{E}{M_S}$$

where,

$$\begin{pmatrix} b_Y \\ b_{2L} \\ b_{3C} \end{pmatrix} = \begin{pmatrix} \frac{21}{5} \\ -3 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{b}_Y \\ \tilde{b}_{2L} \\ \tilde{b}_{3C} \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}$$

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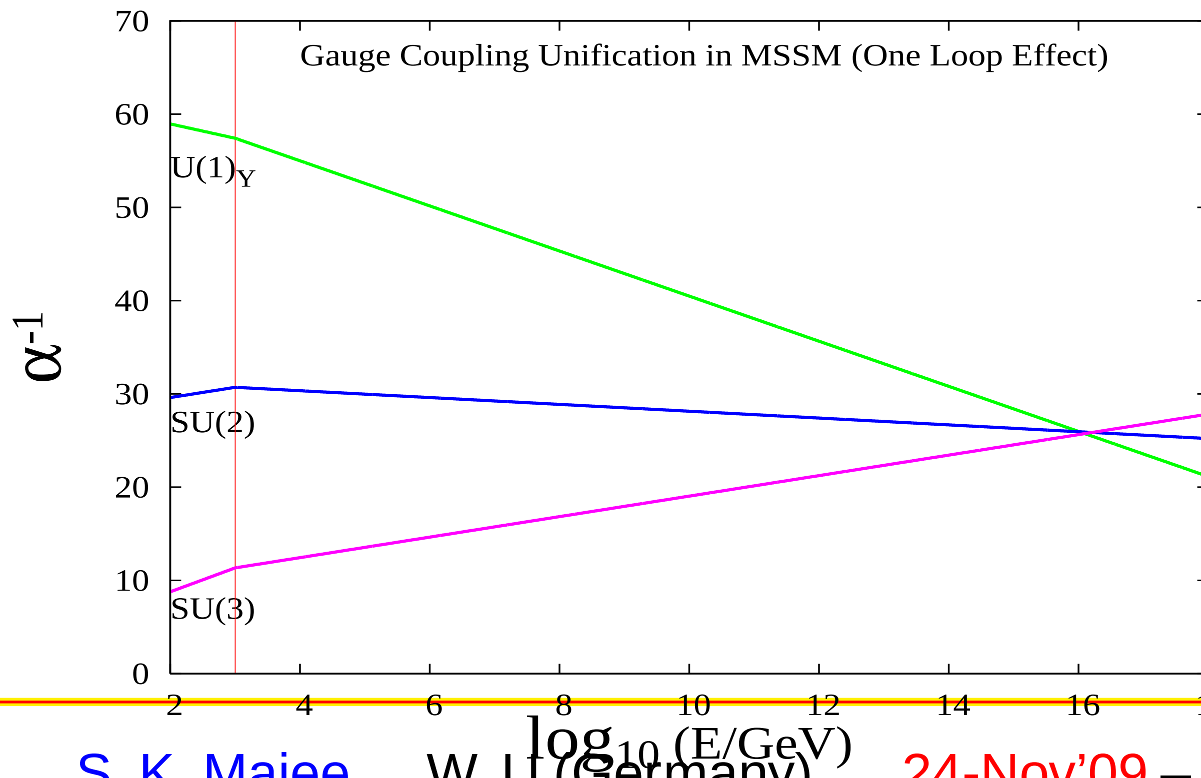
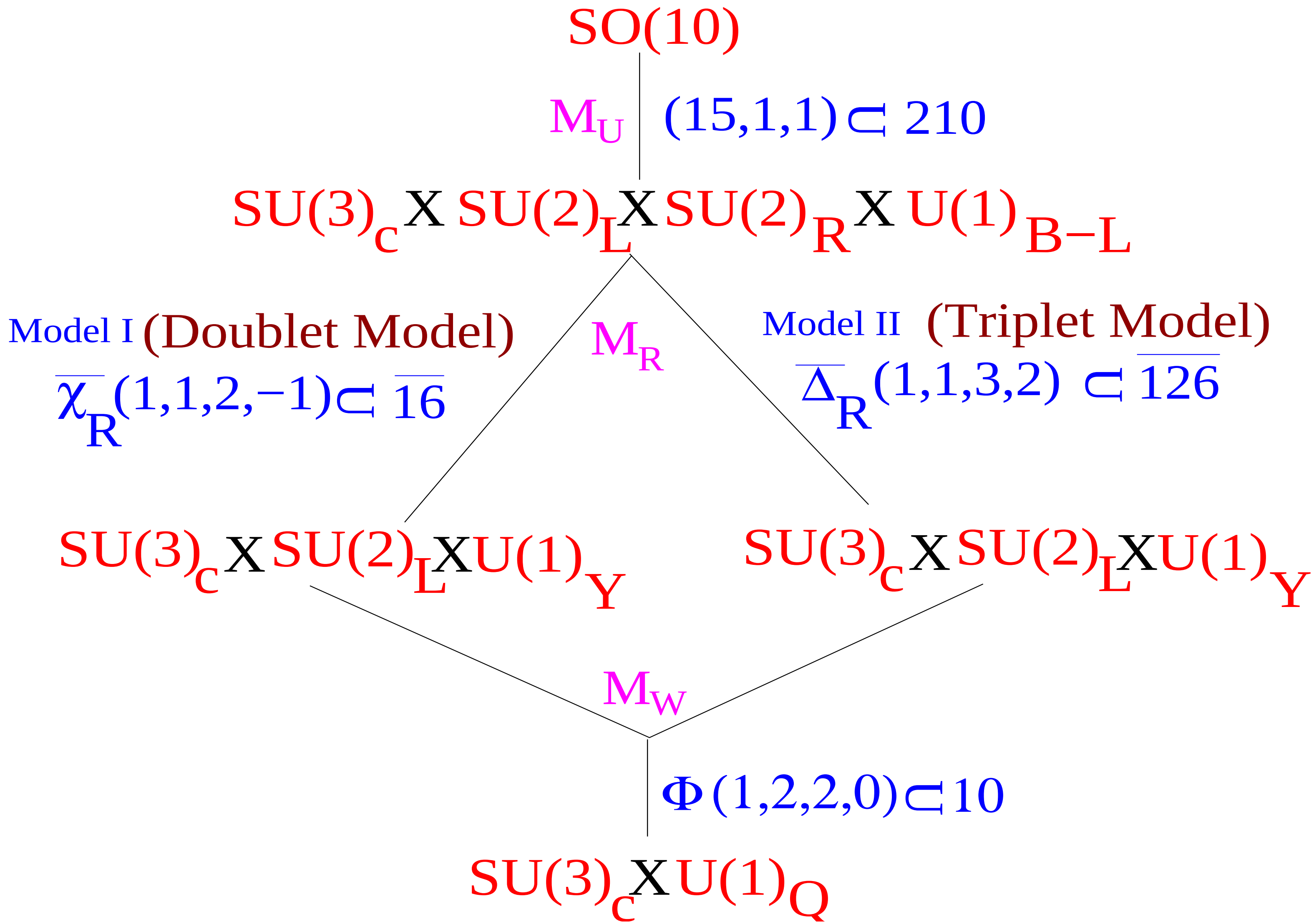


Flowchart : minimal SO(10) GUT



Minimal SO(10) model

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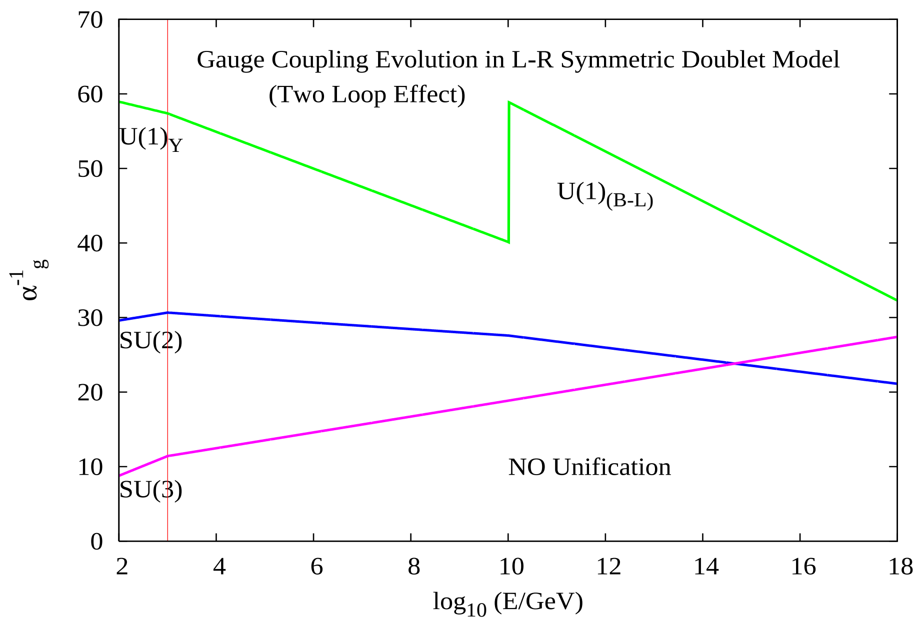
$$(b'_{B-L}, b'_{2L}, b'_{2R}, b'_{3C}) = (9, 2, 2, -3)$$



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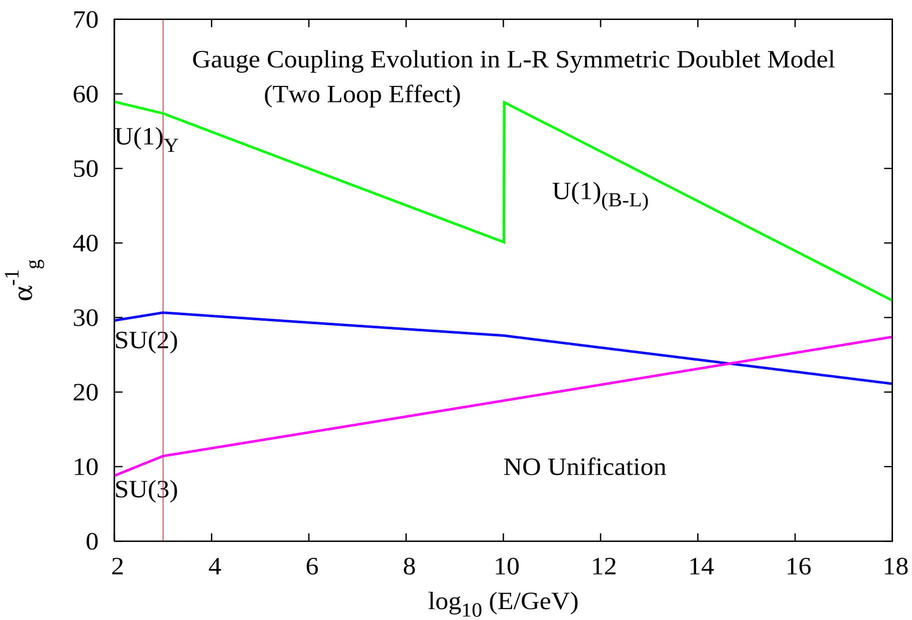
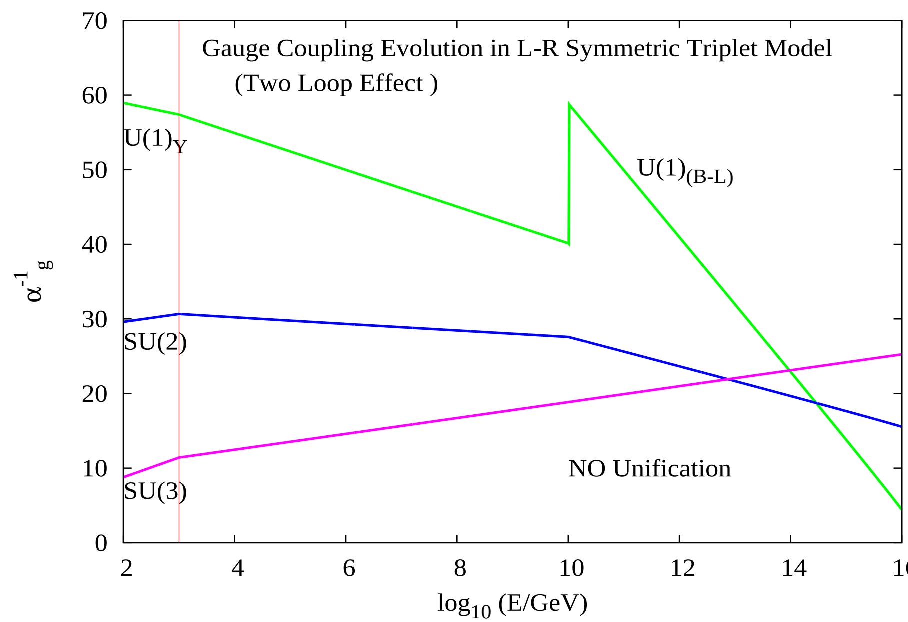
$(b'_{B-L}, b'_{2L}, b'_{2R}, b'_{3C}) = (9, 2, 2, -3)$ $\qquad$ $(b'_{B-L}, b'_{2L}, b'_{2R}, b'_{3C}) = (24, 5, 5, -3)$





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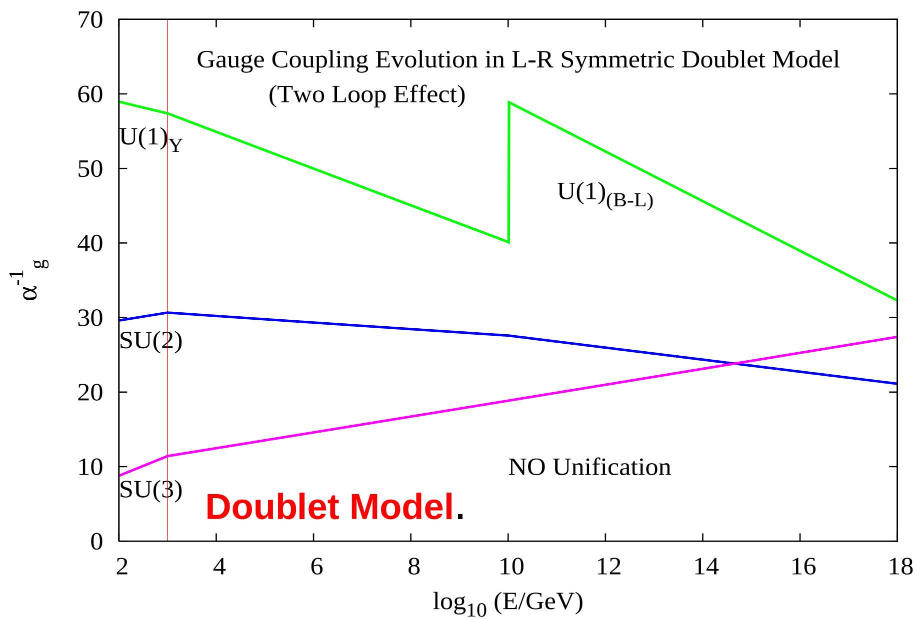

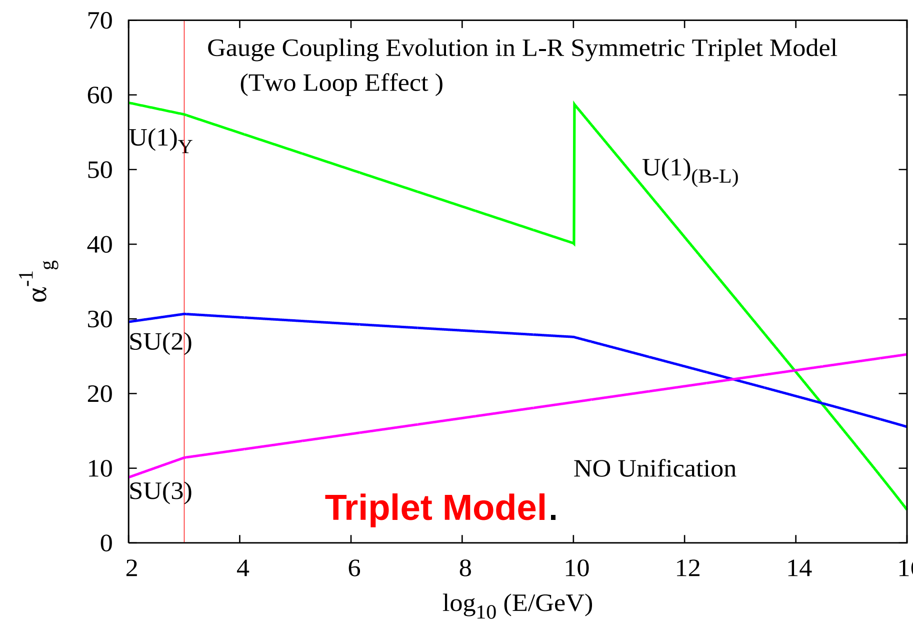

Using these one can have unification for large intermediate scale.

$M_R = 1.3 \times 10^{16}$ GeV, $\quad M_U = 2.9 \times 10^{16}$ GeV. in Model I, **No intermediate scale.**

$M_R = 7.9 \times 10^{15}$ GeV , $\quad M_U = 1.9 \times 10^{16}$ GeV in Model II.

for $\alpha_S(M_Z) = 0.1187, \alpha(M_Z) = 1/127.9, \sin^2\theta_W = 0.2312.$

Different ways to achieve low M_R

Additional light fields

$\sigma(3,1,1,4/3) \oplus \overline{\sigma}(\overline{3},1,1,-4/3) \subset \mathbf{45}, \mathbf{210}$,

$\eta(1,1,1,2) \oplus \overline{\eta}(1,1,1,-2) \subset \mathbf{120}$.

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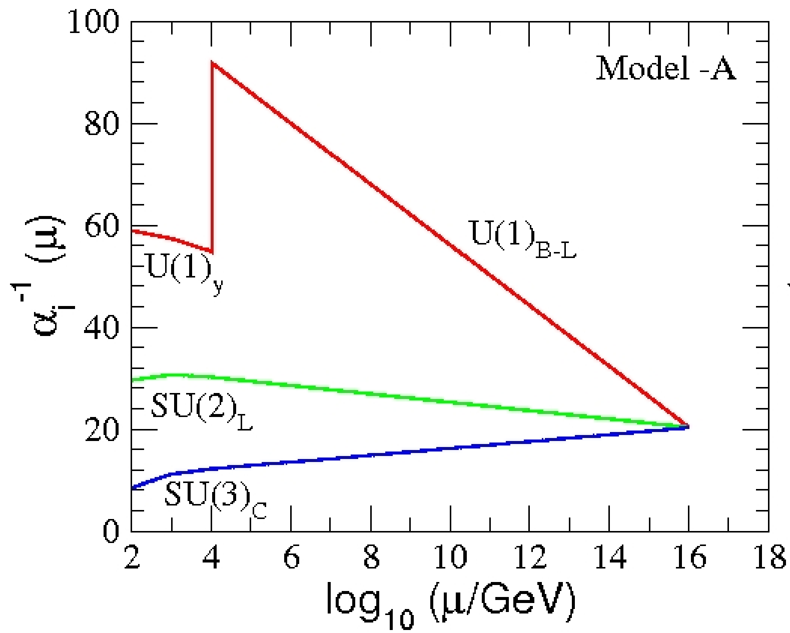

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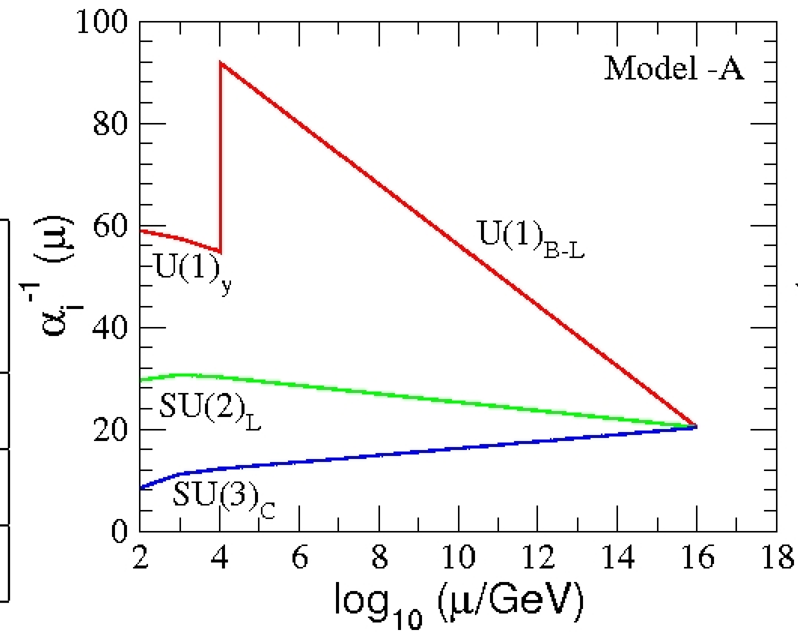

Threshold Correction

M_R GeV)	M_U (GeV)	$\frac{M_1}{M_U}$	$\frac{M_2}{M_U}$	$\frac{M_3}{M_U}$	α_G^{-1}
10^{11}	1.2×10^{19}	$(1.48)^{-1}$	1.48	$(1.48)^{-1}$	23.7
10^7	5×10^{16}	0.151	2.750	1.524	27.7
10^3	10^{19}	0.154	4.760	1.301	28.7

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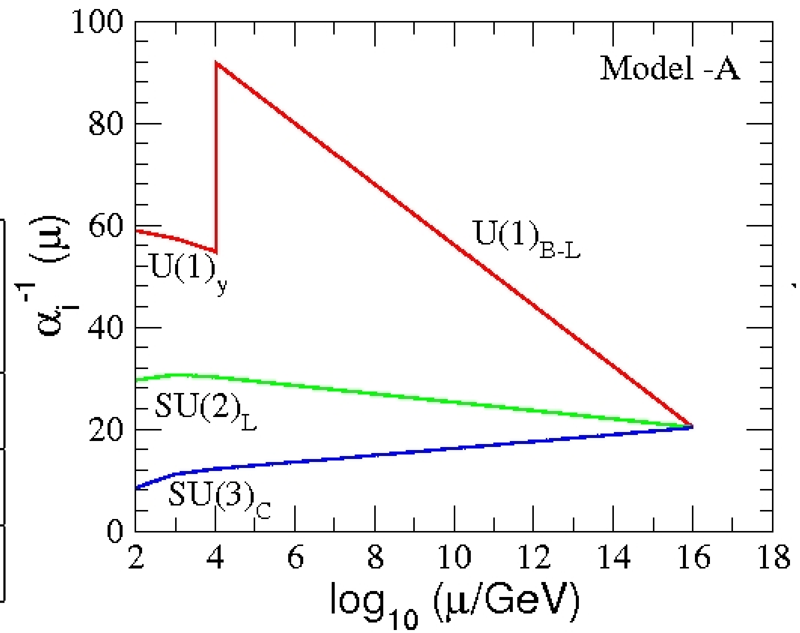

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Non-renormalizable interactions at the Planck scale

$$\mathcal{L}_{NRO} = -\frac{\eta_1}{2M_G} Tr\left(F_{\mu\nu}\Phi_{210}F^{\mu\nu}\right) - \frac{\eta_2}{2M_G} Tr\left(F_{\mu\nu}\Phi_{54}F^{\mu\nu}\right)$$

M_R (GeV)	M_U (GeV)	η_1	η_2	α_G^{-1}
10^9	3×10^{18}	0.305	0.96	25.00
10^6	8×10^{17}	2.728	4.77	25.32
10^5	3×10^{18}	0.671	1.34	25.32

Type-I seesaw

- Introduce a right-handed neutrino ν_R, singlet under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group.

  Minkowski; Yanagida; Mohapatra, Senjanovic; Gell-Mann,Ramon,Slansky; Shafi, Wetterich

- The terms relevant to the neutrino masses are given by

$$\mathcal{L} = \bar{L}_L Y_D \tilde{\Phi} \nu_R + \frac{1}{2} \bar{\nu}_R^c M_R \nu_R + h.c.$$

where,

$$\text{Higgs doublet} \;\; \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \equiv \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\eta) \end{pmatrix} \quad \text{and,} \;\; \tilde{\Phi} = i\tau_2 \Phi^*.$$

- The neutrino mass matrix is

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix},$$

where, $m_D = Y_D < \tilde{H} >= Y_D v.$

- So, the neutrino masses are: $m_{\nu_R} = M_R$, and $m_\nu = -\frac{m_D^2}{M_R}$

Type-II seesaw

- The model is centred on a very heavy Higgs SU(2) Triplet $\Delta \equiv (\Delta^{++}, \Delta^{+}, \Delta^{0})$ with hypercharge $Y = 2$

  Konetschny,Kummer; Shafi, Wetterich; Mohapatra, Senjanovic; Schechter, Valle; ...

- The Lagrangain

$$\mathcal{L} = \frac{1}{\sqrt{2}} \bar{L}_L^c Y_t \Delta L_L$$

- The potential is given by

$$V(\phi, \Delta) = -M^2 \Delta^\dagger \Delta - \mu^2 \phi^\dagger \phi + \lambda_\phi (\phi^\dagger \phi)^2 + \lambda_\Delta (\Delta^\dagger \Delta)^2 + \lambda_{\phi\Delta} (\phi^\dagger \phi)(\Delta^\dagger \Delta) + \mathbf{M} \phi^{\mathbf{T}} \mathbf{\Delta}^\dagger \phi$$

- Once, the neutral component "ϕ" gets vev, we can have

$$v_\Delta \sim \frac{v_\phi^2}{M} << v_\phi$$

- If $M >> v_\phi$, we have a very low vev, for example, if we take, $M = 10^{14}\mathrm{GeV}$, $v_\phi = 100\mathrm{GeV}$, we have $v_\Delta = 0.1\mathrm{eV}$

- The light neutrino mass, thus, can be obtained as well ($m_\nu \propto Y_t v_\Delta$).

Type-III seesaw

- A $SU(2)_L$ triplet lepton $\Sigma(1,3,0)$ is there in addition with the standard model particle.

  Foot, Lew, He and Joshi Z. Phys. **C44** 441, 1989

- The relevant term in the Lagrangian is

$$-\mathcal{L} = \frac{1}{2} Tr\left[\bar{\Sigma} M_\Sigma \Sigma^c + h.c\right] + \tilde{\Phi}^\dagger \bar{\Sigma} \sqrt{2} Y_\Sigma L_L + h.c$$

- The neutrino mass matrix in this scenario can be written, in the basis of left-handed neutrino and the neutral component of Σ, as

$$M_\nu = \begin{pmatrix} 0 & \sqrt{2} Y_\Sigma^T v/2 \\ \sqrt{2} Y_\Sigma v/2 & M_\Sigma/2 \end{pmatrix}.$$

Inverse seesaw or Double seesaw

- To implement this inverse seesaw, we need per generation, a sterile neutrino, S in addition with a right-handed neutrino, N and a left-handed neutrino ν.

  Mohapatra Phys. Rev. Lett**56** 561, 1986; Mohapatra and Valle

- Lagrangian

$$\mathcal{L}_{\text{mass}} = (\bar{\nu} m_D N + \bar{N} M_N S + h.c) + \mu S S$$

- In the basis ν, N, S, the neutrino mass matrix becomes

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_N \\ 0 & {M_N}^T & \mu \end{pmatrix}$$

- In the SO(10) model

$$W_Y = h^a_{ij} \mathbf{16}^i_F \mathbf{16}^j_F \mathbf{10}^a_H + f_{ijk} \mathbf{16}^i_F \mathbf{1}^j_F \bar{\mathbf{16}}^k_H + \mu_{ij} \mathbf{1}^i_F \mathbf{1}^j_F$$

- After $(B-L)$ breaking, the neutrino mass matrix above can be obtaned as $m_D = h v_u$, $M_N = f \bar{v}_R$.
- The lightest neutrino mass, for one generation, is

$$m_\nu \equiv \mu \left(\frac{h v_u}{f \bar{v}_R} \right)^2$$

Inverse seesaw or Double seesaw

- In case of three generations, after integrating out the heavy singlets, the effective light neutrino mass matrix can be written as

$$m_\nu = (m_D)^T \left[(M_N^{-1})^T \mu_s (M_N^{-1}) \right] (m_D)$$

- Two points to note:
  - we will have small m_ν if μ_s is small. The opposite to the type-I seesaw case. So, it is Inverse seesaw mechanism.
  - The term in the square-bracket, for one generation, can be written as

$$\frac{1}{(M_N)^T \frac{1}{\mu_s} (M_N)}$$

which itself in the form of seesaw mass term. So it is Double seesaw mechanism.

Extended Double seesaw

The uncharged fermions in this model, per generation, are the following:

- a left-handed neutrino ν
- a right-handed neutrino, N
- a sterile neutrino, S.

Higgs used in our model: $\mathbf{210}, \mathbf{54}$, $\mathbf{126} \oplus \overline{\mathbf{126}}$, $\mathbf{16} \oplus \overline{\mathbf{16}}$, and $\mathbf{10}$

For the three generation neutral fermion system, the mass matrix on which we focus is:

$$\mathcal{M}_\nu = (\,\nu \quad N^c \quad S\,)_L \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mathbf{M_N} & M_X \\ 0 & {M_X}^T & \mu \end{pmatrix} \begin{pmatrix} \nu^c \\ N \\ S \end{pmatrix}_R$$

Kang & Kim: Phys. Lett.**B646** 248, 2007

$M_X = F * x_R$ with $< \chi_R(1,2,1,-1) > = < \overline{\chi}_R(1,2,1,1) > = x_R$.

Mass matrix

- Consider the mechanism of D-parity breaking. $\Delta_R(1,3,1,-2) \oplus \overline{\Delta}_R(1,3,1,2)$, $\chi_R(1,2,1,-1) \oplus \overline{\chi}_R(1,2,1,1)$ can have much lower musses in compare to the left-handed counterpart.

- No Direct vev to RH-triplet. $\lambda \Delta_R \overline{\chi}_R \overline{\chi}_R$
  $\rightarrow$ An effective vev $< \Delta_R(1,3,1,-2) > \equiv v_R = \lambda \frac{x_R^2}{m_{\Delta_R}}$.

  $M_N \sim f < \Delta_R(1,3,1,-2) >= f \lambda \frac{x_R^2}{m_{\Delta_R}}$, where f is a typical Yukawa coupling of Majorana type.

- If m_{Δ_R} is around 1 TeV, which can be arranged by a tuning of the D-parity breaking term in the Lagrangian, the entries of M_N are $\mathcal{O}(10^{11})$ GeV.
  Without any loss of generality, M_N can be chosen to be diagonal.

- In the limit $M_N \gg M_X \gg \mu \gg m_D$ leads to:

$$m_\nu \sim -m_D \left[{M_X}^{-1} \mu ({M_X}^T)^{-1} \right] {m_D}^T,$$

$$M_S \sim \mu - \frac{M_X^2}{M_N}, \quad \text{and} \quad M \sim M_N + \frac{M_X^2}{M_N}.$$

Flow chart

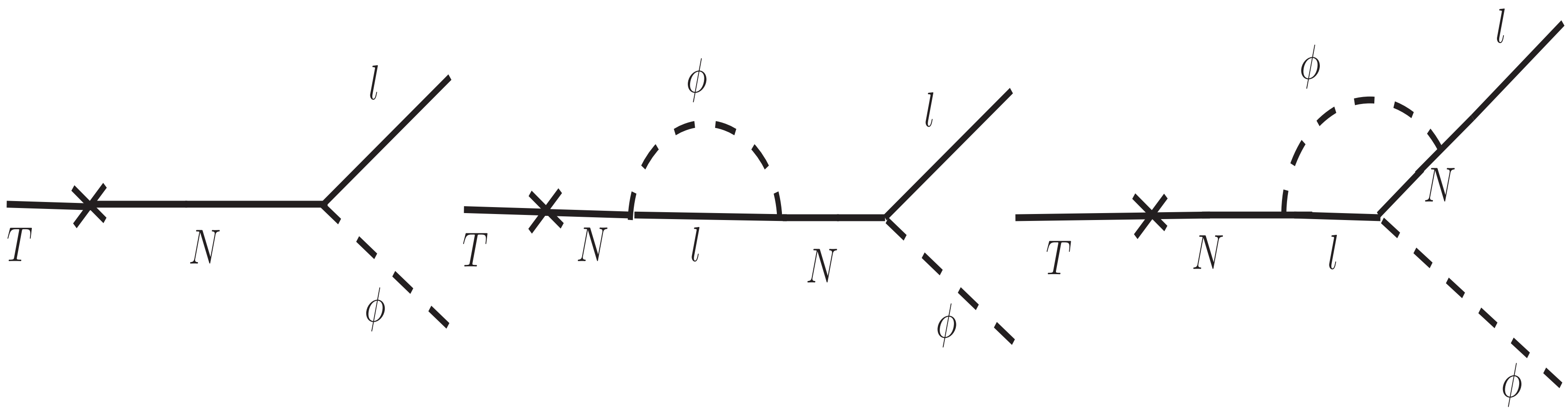
- The effective neutrino mass matrix, thus, we have

$$m_\nu \sim -m_D \left[{M_X}^{-1} \mu ({M_X}^T)^{-1} \right] {m_D}^T,$$

$$M_S \sim \mu - \frac{M_X^2}{M_N}, \quad \text{and} \quad M \sim M_N + \frac{M_X^2}{M_N}.$$

- In SO(10) GUT model, due to quark-lepton symmetry, we are considering the Dirac mass $m_D^{diag} = diag(m_u, m_c, m_t)$
- Using the CKM-matrix, we have the exact Dirac mass-matrix as $m_D = V_{CKM} * m_D^{diag} * V_{CKM}^\dagger$
- Considering the experimental values for Δm_{21}^2 and Δm_{32}^2 with, lightest one massless, and neutrino mixing angles θ_{23}, θ_{12}, and θ_{13} we are constrcting the light neutrino mass matrix.
- The diagonal mass matrix for M_X and M_N are choosen.
- Using these different matrices, we are constructing the μ mass matrix and hence it's eigenvalues. This is what is used in the formula to generate the lepton asymmetry.

Leptogenesis



$$\frac{dY_T}{dz} = -\left(Y_T - Y_T^{eq}\right)\left[\frac{\Gamma_D^T}{zH(z)} + \frac{\Gamma_s^T}{zH(z)}\right], \quad \frac{dY_L}{dz} = \epsilon_T \frac{\Gamma_D^T}{zH(z)}\left(Y_T - Y_T^{eq}\right) - \frac{\Gamma_W^\ell}{zH(z)} Y_L .$$

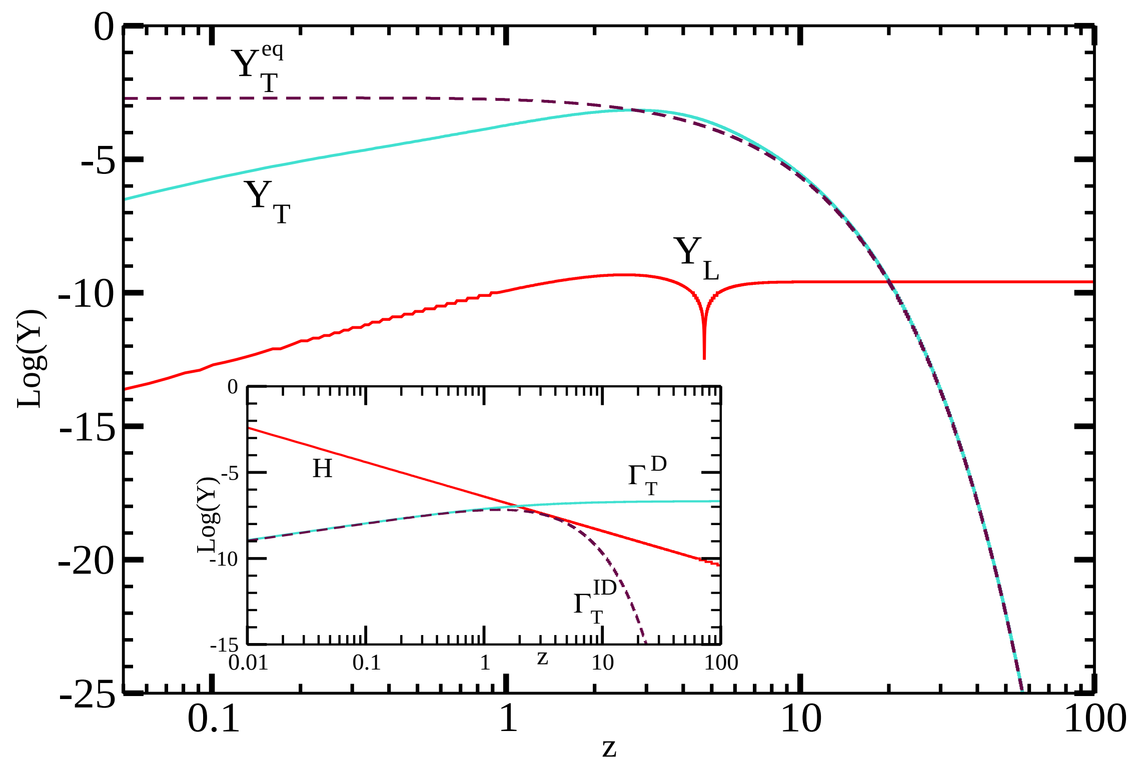
$Y_T = n_T/n_S$ and $Y_L = n_L/n_S$, where n_T, n_L and n_S are the number densities.

- The Hubble expansion rate $H(z)$, where $z = M_T/\mathrm{T}$, and the CP-violation parameter are given by

$$H(z) = \frac{H(M_T)}{z^2}, \quad H(M_T) = 1.67 g_*^{1/2} \frac{M_T^2}{M_{pl}}, \quad \epsilon_T = \frac{\Gamma(T \to l\phi) - \Gamma(T \to \bar{l}\phi^*)}{\Gamma(T \to l\phi) + \Gamma(T \to \bar{l}\phi^*)}.$$

Quantitative Analysis

- up-type quarks: $m_u = 0.0023, m_c = 1.2, m_t = 170.9$ (in GeV unit) $\rightarrow$ $m_D^{diag} = diag(m_u, m_c, m_t)$
- CKM-matrix : $sin\theta_{12} = 0.2272$, $sin\theta_{23} = 0.0422$, $sin\theta_{13} = 0.0039$, $\delta = 0.994$ radian. $m_D = V_{CKM} * m_D^{diag} * V_{CKM}^{\dagger}$
- M_X = diag (0.2, 0.3, 0.4)*x_R with $x_R = 6 \times 10^6$ GeV
- For m_ν: $\Delta m_{21}^2 = 8.0 \times 10^{-5} eV^2$, $\Delta m_{32}^2 = 2.5 \times 10^{-3} eV^2$, with, lightest one massless.
- Neutrino mixing angles $\theta_{23} = 45°$, $\theta_{12} = 32°$, $\theta_{13} = 7°$.
- $M_N = \mathrm{diag}(0.1, 0.5, 0.9) \times 10^{11}$ GeV



- No other CP-phase, except for CKM-mixing matrix for the quark sector.

Conclusions

- We have presented a SUSY $\mathbf{SO(10)}$-based model relying on a double see-saw mechanism which is
  - consistent with the known neutrino masses and mixing,
  - can lead to a correct lepton asymmetry via the decays of sterile, i.e., $\mathbf{SO(10)}$ singlet, neutrinos while remaining in concordance with the gravitino constraint.
- The intermediate scales are obtained through an RG analysis of the gauge coupling running and are consistent with a long-lived proton.
- This model will have a natural extension to an $\mathbf{E(6)}$-GUT wherein the matter multiplets and the singlet fields will constitute the fundamental **27** representation of the gauge group.
- An important outcome of the symmetry breaking is that out of the triplet Δ_R the components $\Delta_R^{\pm}$ and $Re\Delta_R^{\circ}$ are absorbed as longitudinal modes of the broken generators of $SU(2)_R \times U(1)_{B-L}$. The physical states are Δ_R^{++} and $Im\Delta_R^{\circ}$ and their superpartners. They will be within striking range of the LHC and the ILC with $m_\Delta \simeq 300$ GeV – 1 TeV.

Thank You !

Standard Model Higgs

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$$\lambda > 0$$

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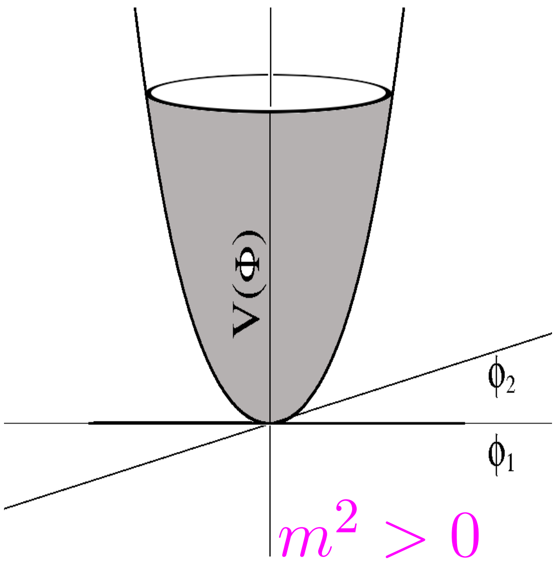

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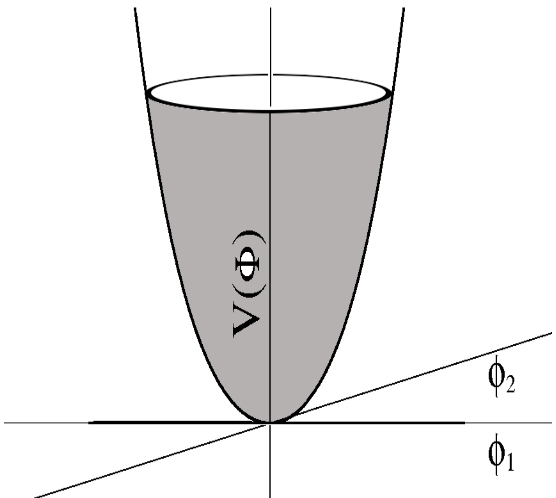

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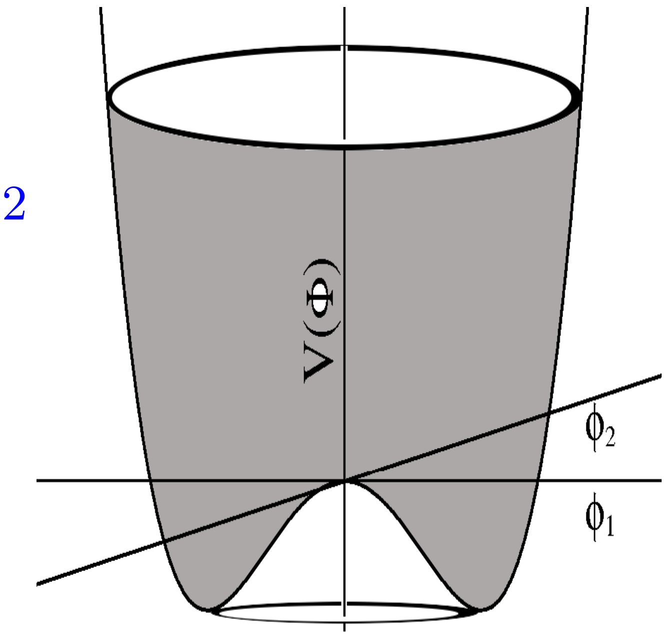

$m^2 < 0 \Rightarrow \langle 0 | \phi^0 | 0 \rangle = v$

$m_H = \sqrt{-2m^2}$, $v = \sqrt{\frac{-m^2}{\lambda}}$ = 246 GeV

Other components of $\Phi \Rightarrow$ *Goldstone bosons*

Larger Symmetry $\Rightarrow$ GUT

More symmetric $\Rightarrow$ more elegant and beautiful theory.

$SU(2)_L \times SU(2)_R \times SU(4)_C$, $SU(5)$, $SO(10)$ ...

SM-particles are

accommodated in $\mathbf{\bar{5}}$ and $\mathbf{10}$ of $SU(5)$.

$$SU(5) \quad \xrightarrow{\mathbf{\Phi_{24}}} \quad SU(3)_C \times SU(2)_L \times U(1)_Y$$

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$$16\pi^2 E \frac{dg_i}{dE} = b_i g_i^3 = \beta_{SM}(g)$$

$$\alpha_i{}^{-1}(E) = \alpha_i{}^{-1}(M_Z) - \frac{b_i}{2\pi} ln \frac{E}{M_Z}$$

$$\boxed{b_1 = \frac{41}{10}, \;\; b_2 = -\frac{19}{6}, \;\; b_3 = -7}$$

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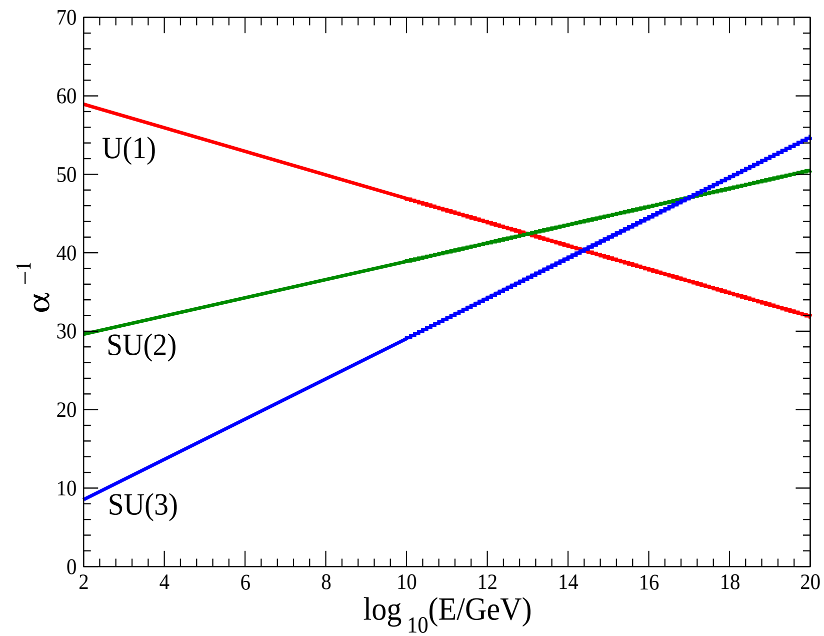

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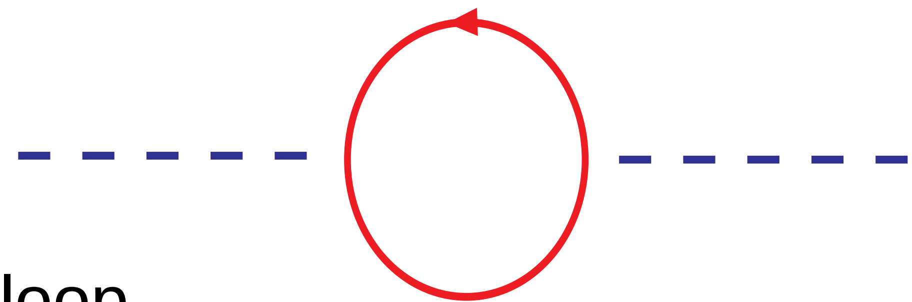
GUT scale $\sim 10^{15-16}$ GeV

Hierarchy problem

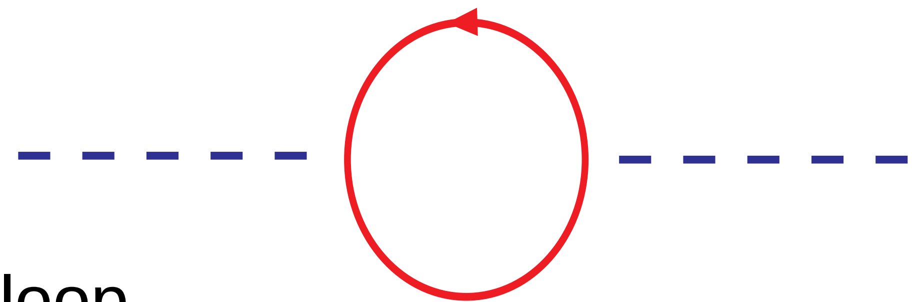


Due to femion loop

$$\Pi^f_{hh} = (-1)\int_0^\Lambda \frac{d^4k}{(2\pi)^4} Tr\left\{ (\frac{-i\lambda_f}{\sqrt{2}}) \frac{i}{\not{k}-m_f} (\frac{-i\lambda_f}{\sqrt{2}}) \frac{i}{\not{p}-\not{k}-m_f} \right\}$$
$$= -\frac{\lambda_f^2}{8\pi^2}\Lambda^2 + \ldots$$

- $m_H^2 = m_{H_0}^2 + \delta m_H^2$
- If $\Lambda \simeq 10^{16}$ GeV, required fine-tuning to 1 part in 10^{26}.

Hierarchy problem and SUSY



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- If $\Lambda \simeq 10^{16}\ \mathrm{GeV}$, required fine-tuning to 1 part in 10^{26}.
- Due to scalar loop : $\delta \mathrm{m}_H^2 = \frac{\lambda_S}{16\pi^2}\Lambda^2 - \ldots$

$$\boxed{\lambda_S = |\lambda_f|^2} \Rightarrow Supersymmetry$$

MSSM

- SUSY partners to SM particles :

$$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L \equiv (3,2,1/6), \quad \begin{matrix} \tilde{u}^c_L \equiv (3,1,2/3) \\ \tilde{d}^c_L \equiv (3,1,-1/3) \end{matrix} \quad \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_L \equiv (1,2,-1/2), \quad \tilde{e}^c_L \equiv (1,2,-1)$$

$$\begin{matrix} \tilde{G} \equiv (8,1,0) \\ \tilde{W} \equiv (1,3,0) \\ \tilde{B} \equiv (1,1,0) \end{matrix} \qquad \tilde{H} \equiv \begin{pmatrix} \tilde{\phi^+} \\ \tilde{\phi^0} \end{pmatrix} \equiv (1,2,1/2)$$

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- RGE in MSSM :

$$16\pi^2 E \frac{dg_i}{dE} = b_i g^3 + \Theta(E - M_s)(\tilde{b}_i - b_i) g^3 = \beta_{MSSM}(g)$$

- Solution :

$$\alpha_i{}^{-1}(E) = \alpha_i{}^{-1}(M_Z) - \frac{b_i}{2\pi} ln \frac{M_S}{M_Z} - \frac{(\tilde{b}_i - b_i)}{2\pi} ln \frac{E}{M_S}$$

where,

$$\begin{pmatrix} b_Y \\ b_{2L} \\ b_{3C} \end{pmatrix} = \begin{pmatrix} \frac{21}{5} \\ -3 \\ -7 \end{pmatrix} \quad \& \quad \begin{pmatrix} \tilde{b}_Y \\ \tilde{b}_{2L} \\ \tilde{b}_{3C} \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}.$$

RGE in L-R model

- The extra Higgs scalars at the scale M_R are
  $\chi_L(1,2,1,-1)+\chi_R(1,1,2,-1)+$ C.C in the doublet model and $\Delta_L(1,3,1,2)+\Delta_R(1,1,3,2)+$ C.C in the triplet model

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$$\begin{pmatrix} b'_{B-L} \\ b'_{2L} \\ b'_{2R} \\ b'_{3C} \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 2 \\ -3 \end{pmatrix} \text{ in doublet model} \qquad \begin{pmatrix} b'_{B-L} \\ b'_{2L} \\ b'_{2R} \\ b'_{3C} \end{pmatrix} = \begin{pmatrix} 24 \\ 5 \\ 5 \\ -3 \end{pmatrix} \text{ in triplet model.}$$

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Two-loop scenario:

$$16\pi^2 E\frac{dg_i}{dE} = b_i g_i^3 + b_{ij} g_j^2 g_i^3$$

$$\Rightarrow {\alpha_i}^{-1}(E) = {\alpha_i}^{-1}(M_Z) - \frac{b_i}{2\pi} ln\frac{E}{M_Z} - \frac{1}{4\pi}\sum_j \frac{b_{ij}}{a_j} \ln\frac{\alpha_j(M_U)}{\alpha_j(M_Z)}$$

LR symmetric model

- Leptons are :
$$\psi_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv (1,2,1,-1) \subset \mathbf{16}, \;\; \psi_R \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_R \equiv (1,1,2,-1) \subset \mathbf{16}.$$

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$$\mathcal{L}_Y = f\overline{\psi}_L\psi_R\phi + \tilde{f}\overline{\psi^c_L}\psi_L\bar{\Delta}_L + \tilde{f}\overline{\psi^c_R}\psi_R\bar{\Delta}_R, \quad \text{with} \quad \begin{matrix} \bar{\Delta}_R \equiv (1,1,3,2) \\ \bar{\Delta}_L \equiv (1,3,1,2) \end{matrix} \subset \overline{\mathbf{126}}$$

- So, neutrino mass matrix can be written as
$$M_\nu = \begin{pmatrix} \nu & \nu^c \end{pmatrix}_L \begin{pmatrix} m_L & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}_L ,$$
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- So, the neutrino masses are: $m_{\nu_R} = M_R$, and $m_\nu = m_L - \frac{m_D^2}{M_R} \sim -\frac{m_D^2}{M_R}$

- Thus to have a mass $m_\nu \sim 10^{-2}\mathrm{eV}$ and $m_D \sim 1\mathrm{MeV}$ to $1\mathrm{GeV}$ we need $m_R \sim 10^{(5-11)}\mathrm{GeV}$

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- To have leptogenesis, Gravitino constraint forced us to take
$$M_R < 10^9\mathrm{GeV}.$$

Inclusion of additional field

Model A

$$\sigma(3,1,1,4/3) \oplus \overline{\sigma}(\overline{3},1,1,-4/3) \subset \mathbf{45}, \mathbf{210},$$
$$\eta(1,1,1,2) \oplus \overline{\eta}(1,1,1,-2) \subset \mathbf{120}.$$

- The one- and two-loop coefficients including these fields are,

$$\begin{pmatrix} a'_{BL} \\ a'_{2L} \\ a'_{2R} \\ a'_{3C} \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \\ 2 \\ -2 \end{pmatrix} \qquad b'_{ij} = \begin{pmatrix} 241/6 & 27/2 & 27/2 & 88/3 \\ 9/2 & 32 & 3 & 24 \\ 9/2 & 3 & 32 & 24 \\ 11/3 & 9 & 9 & 76/3 \end{pmatrix}, \ i,j = BL, 2L, 2R,$$

Model A	$\mathrm{M_R}$ (GeV)	$\mathrm{M_U}$ (GeV)	α_G^{-1}
	10^9	1.15×10^{16}	22.22
	10^5	1.10×10^{16}	20.83
	10^4	10^{16}	20.40

Inclusion of additional field

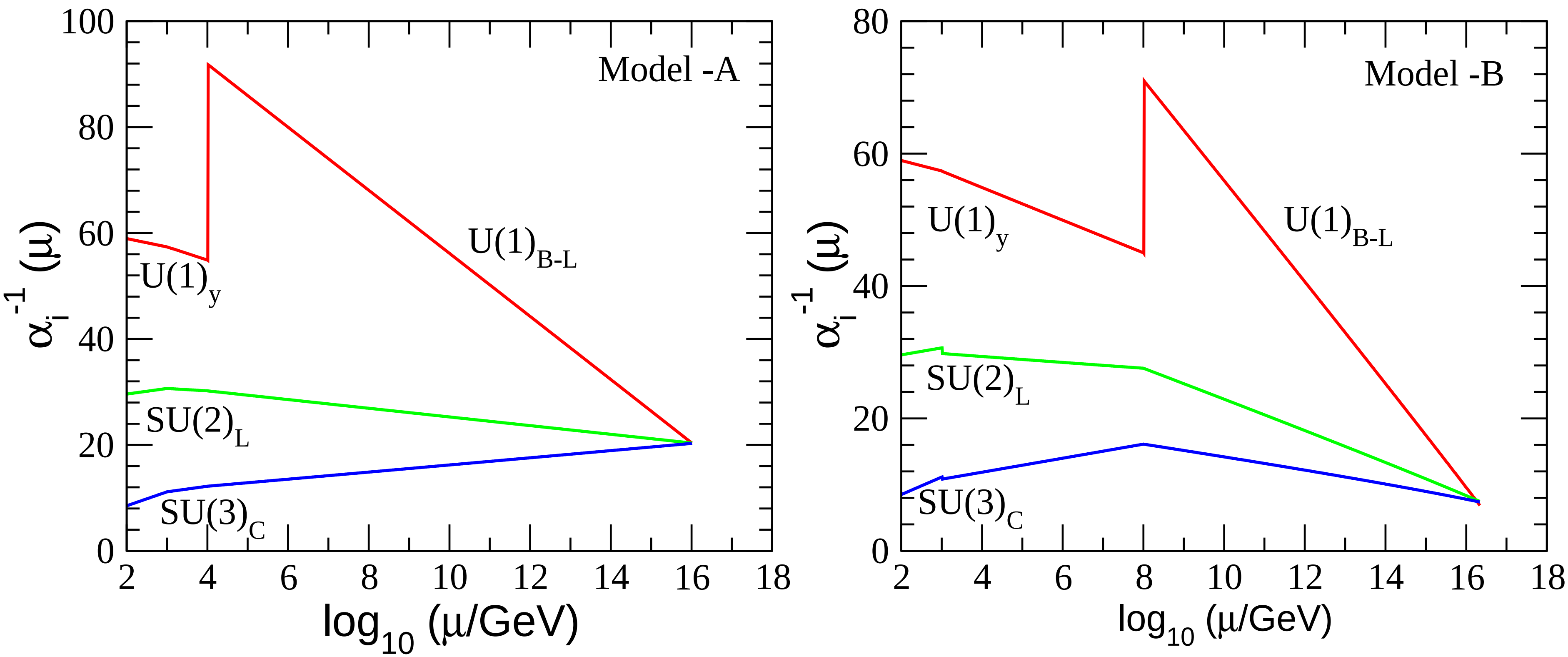
Model B

$$\begin{aligned} \xi(6,1,1,4/3) \oplus \overline{\xi}(\overline{6},1,1,-4/3,) &\subset \mathbf{54}, \\ \eta(1,1,1,2) \oplus \overline{\eta}(1,1,1,-2) &\subset \mathbf{120}, \\ \text{A pair of } C(1,2,2,0) &\subset \mathbf{10}, \mathbf{120}, \mathbf{126}, \\ D_L(1,3,1,0) \oplus D_R(1,1,3,0) &\subset \mathbf{45}, \mathbf{210}, \end{aligned}$$

- The one- and two-loop coefficients in this scenario are

$$\begin{pmatrix} a'_{BL} \\ a'_{2L} \\ a'_{2R} \\ a'_{3C} \end{pmatrix} = \begin{pmatrix} 20 \\ 6 \\ 6 \\ 2 \end{pmatrix}, b'_{ij} = \begin{pmatrix} 305/6 & 27/2 & 27/2 & 344/3 \\ 9/2 & 70 & 9 & 24 \\ 9/2 & 9 & 70 & 24 \\ 43/3 & 9 & 9 & 332/3 \end{pmatrix}, \quad i,j = BL, 2L, 2R, 3$$

Model B	$\mathrm{M_R}$ (GeV)	$\mathrm{M_U}$ (GeV)	α_G^{-1}
	10^9	1.82×10^{16}	7.58
	10^8	2.00×10^{16}	10.13



100
80
60
40
20
0
2
4
6
8
10
12
14
16
18
Model -A
U(1)B-L
U(1)y
SU(2)L
SU(3)C
αi-1 (μ)
log10 (μ/GeV)
80
60
40
20
0
Model -B
U(1)y
U(1)B-L
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Threshold correction

- RGE with threshold correction is given by

$$\frac{1}{\alpha_i(M_R)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi}\ln\frac{M_R}{M_Z} + \Delta_i,$$
$$\frac{1}{\alpha_i(M_U)} = \frac{1}{\alpha_i(M_R)} - \frac{b'_i}{2\pi}\ln\frac{M_U}{M_R} + \Delta'_i,$$

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- They are defined as

$$\Delta_i^{(S)} = \frac{1}{2\pi}\Sigma_\alpha b_i^\alpha \ln\frac{M^\alpha}{M_S} \equiv \frac{b_i}{2\pi}\ln\frac{M_i}{M_S}, \quad b_i = \Sigma_\alpha b_i^\alpha,$$
$$\Delta_i^{(R)} = \frac{1}{2\pi}\Sigma_\beta c_i^\beta \ln\frac{M^\beta}{M_R} \equiv \frac{b'_i}{2\pi}\ln\frac{M_i}{M_R}, \quad b'_i = \Sigma_\beta c_i^\beta,$$
$$\Delta'_i = \frac{1}{2\pi}\Sigma_\gamma d_i^\gamma \ln\frac{M^\gamma}{M_U} \equiv \frac{b''_i}{2\pi}\ln\frac{M_i}{M_U}, \quad b''_i = \Sigma_\gamma d_i^\gamma.$$

Threshold Corrections

Model-I (Doublet Model)

- The effect of the threshold correction to the intermediate scale and GUT scale one can obtain as

$$\Delta \ln \frac{M_R}{M_Z} = \frac{1}{7}\left[\frac{125}{3}\ln\frac{M_1}{M_U} - 106\ln\frac{M_2}{M_U} + \frac{196}{3}\ln\frac{M_3}{M_U}\right],$$

$$\Delta \ln \frac{M_U}{M_Z} = \frac{1}{7}\left[\frac{25}{3}\ln\frac{M_1}{M_U} + 53\ln\frac{M_2}{M_U} - \frac{196}{3}\ln\frac{M_3}{M_U}\right].$$

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M_R (GeV)	M_U (GeV)	$\frac{M_1}{M_U}$	$\frac{M_2}{M_U}$	$\frac{M_3}{M_U}$	α_G^{-1}
10^{11}	1.2×10^{19}	$(1.48)^{-1}$	1.48	$(1.48)^{-1}$	23.7
10^9	10^{18}	0.272	1.770	0.831	23.7
10^7	10^{18}	0.158	1.950	0.832	23.7
10^7	5×10^{16}	0.151	2.750	1.524	27.7
10^5	5×10^{18}	0.180	3.30	1.076	26.7
10^3	10^{19}	0.154	4.760	1.301	28.7

Threshold Corrections

Model-II (triplet model)

- The effect of the threshold correction to the intermediate scale and GUT scale one can obtain, in the triplet model, as

$$\Delta \ln \frac{M_R}{M_Z} = \left[\frac{202}{9} \ln \frac{M_1}{M_U} - 87 \ln \frac{M_2}{M_U} + \frac{1159}{18} \ln \frac{M_3}{M_U}\right],$$
$$\Delta \ln \frac{M_U}{M_Z} = \left[\frac{101}{9} \ln \frac{M_1}{M_U} - 29 \ln \frac{M_2}{M_U} + \frac{305}{18} \ln \frac{M_3}{M_U}\right].$$

M_R (GeV)	M_U (GeV)	$\frac{M_1}{M_U}$	$\frac{M_2}{M_U}$	$\frac{M_3}{M_U}$	α_G^{-1}
5×10^9	1.58×10^{16}	2.204	1.200	0.659	15.0
10^{10}	1.58×10^{16}	2.065	1.160	0.659	15.0
10^{11}	1.58×10^{16}	1.661	1.050	0.656	15.0

Gravity Effect

gravity induced non-renormalizable terms could change the usual field theoretic predictions of gauge coupling unification

$$\mathcal{L}_{NRO} = -\frac{\eta_1}{2M_G} Tr\left(F_{\mu\nu}\Phi_{210}F^{\mu\nu}\right) - \frac{\eta_2}{2M_G} Tr\left(F_{\mu\nu}\Phi_{54}F^{\mu\nu}\right).$$

effective gauge coupling constants at the unification point get changed due to these non-renormalizable terms.

$$\begin{aligned} \epsilon_{2L} &= \epsilon_{2R} = -\frac{3}{2}\epsilon_2,\ \epsilon_{3C} = \epsilon_2 - \epsilon_1,\ \ \epsilon_{BL} = 2\epsilon_1 + \epsilon_2, \\ \epsilon' &= \frac{4}{3}\epsilon_1 + \frac{5}{3}\epsilon_2,\ \ \epsilon'' = 4\epsilon_1 - 5\epsilon_2, \end{aligned}$$

where

$$\epsilon_1 = \frac{3\eta_1}{4}\frac{M_U}{M_G}\left[\frac{1}{4\pi\alpha_G}\right]^{\frac{1}{2}}, \qquad \epsilon_2 = \frac{3\eta_2}{4}\frac{M_U}{M_G}\left[\frac{1}{15\pi\alpha_G}\right]^{\frac{1}{2}}$$

Gravity Effect

Due to that we have the following corrections on the mass scales,

$$\left(\Delta \ln \frac{M_R}{M_Z}\right)_{gr} = -\frac{\pi}{7\alpha_G}\left[\epsilon_1 + 10\epsilon_2\right], \quad \text{and} \quad \left(\Delta \ln \frac{M_U}{M_Z}\right)_{gr} = \frac{\pi}{7\alpha_G}\left[5\epsilon_2 - 3\epsilon_1\right].$$

While, the change in the individual coupling constants near the GUT scale change are,

$$\Delta_{2L}^{(\mathrm{gr})} = \frac{3\epsilon_2}{2\alpha_G}, \quad \Delta_{BL}^{(\mathrm{gr})} = -\frac{(2\epsilon_1 + \epsilon_2)}{\alpha_G}, \quad \Delta_{3C}^{(\mathrm{gr})} = \frac{(\epsilon_1 - \epsilon_2)}{\alpha_G}.$$

M_R (GeV)	M_U (GeV)	η_1	η_2	α_G^{-1}
10^9	3.16×10^{18}	0.305	0.96	25.00
10^7	3.16×10^{18}	0.494	1.16	25.64
10^6	8×10^{17}	2.728	4.77	25.32
10^5	3.16×10^{18}	0.671	1.34	25.32

Bound on M_R in the triplet model

- In the minimal model, the RGE is given by

$$\frac{1}{\alpha_i(M_R)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi}\ln\frac{M_R}{M_Z},$$
$$\frac{1}{\alpha_i(M_U)} = \frac{1}{\alpha_i(M_R)} - \frac{b'_i}{2\pi}\ln\frac{M_U}{M_R},$$

Solving this we get,

$$\mu_0 = M_R \exp\left[\frac{2\pi}{a'_{BL}}\frac{1}{\alpha_{BL}(M_R)}\right],$$

here

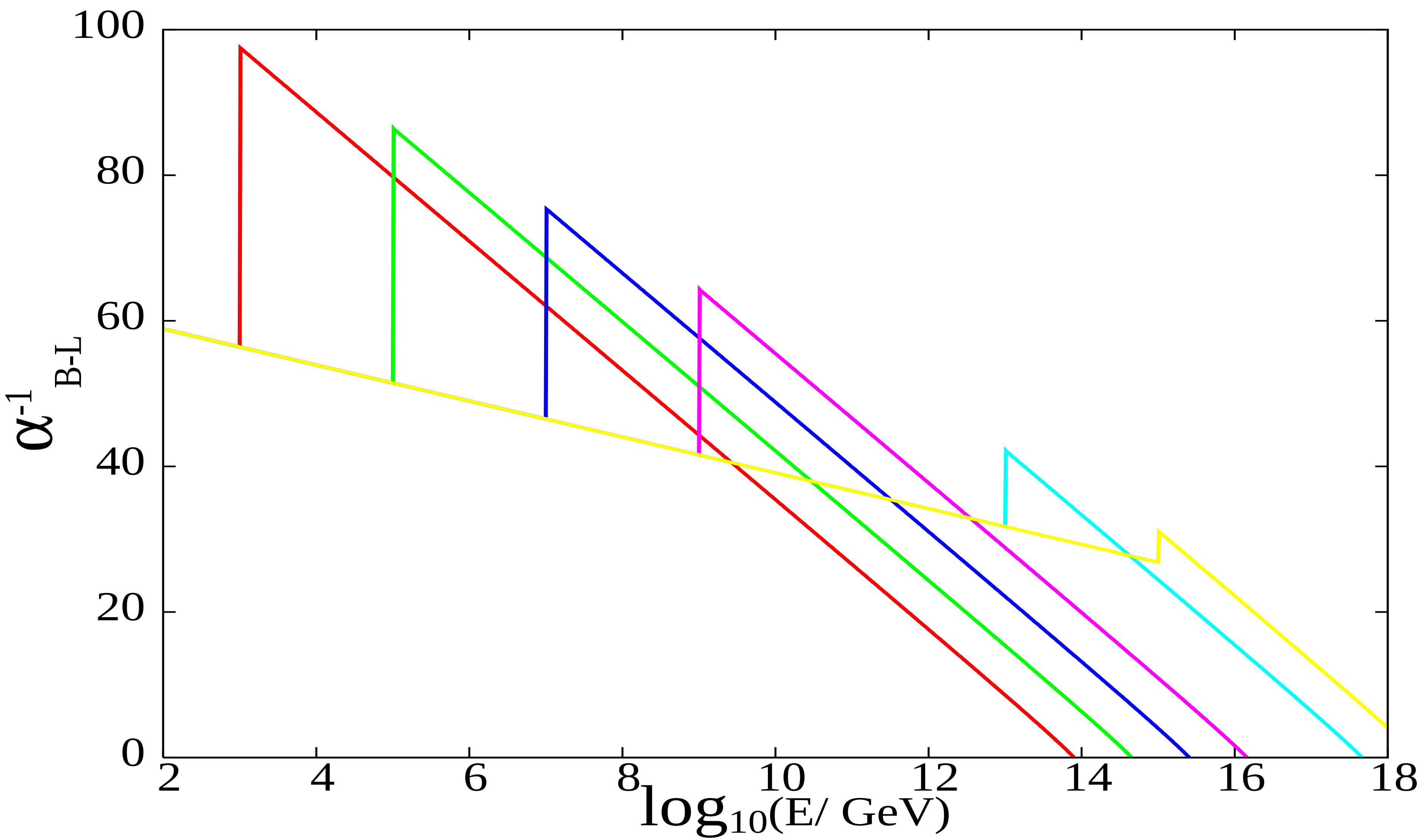
$$\frac{1}{\alpha_{BL}(M_R)} = \frac{5}{2}\left(\frac{1}{\alpha_Y(M_Z)} - \Theta_Y + \Delta_Y\right)$$
$$-\frac{3}{2}\left(\frac{1}{\alpha_{2L}(M_Z)} - \Theta_{2L} + \Delta_{2L}\right) - \frac{1}{4\pi}(5a_Y - 3a_{2L})\ln\frac{M_R}{M_Z}$$

Lower bound on M_R

M_R (GeV)	$\frac{1}{\alpha_{BL}(M_R)}$	μ_0 (GeV)
10^3	97.429	7.76×10^{13}
10^5	86.407	4.56×10^{14}
10^7	75.406	2.56×10^{15}
10^8	69.907	6.16×10^{15}
10^9	64.409	1.44×10^{16}
10^{10}	58.912	3.46×10^{16}
10^{11}	53.415	8.31×10^{16}

requirement of perturbativity in the triplet model leads to the conservative lower bond on the intermediate scale, $M_R > 10^9$ GeV.

Lower bound on M_R



Leptogensis - decay widths

In a relativistic frame the decay rate is given by

(-34) $$\Gamma_D^S = \frac{1}{8\pi v^2} M_S \left(\frac{K_1(z)}{K_2(z)}\right) \tilde{m}_3 \frac{F_{33}^2 v_R^2}{M_{33}} \sin^2\theta \,.$$

The Γ_s^S in Eq. (**??**) represents the processes which violate lepton number by one unit and is given by

(-33) $$\Gamma_S = 4\Gamma_{\phi,s}^S + 2\Gamma_{\phi,t}^S \,.$$

We have not included the SUSY processes. It is shown that upon inclusion of those processes the result doesn't change significantly.

(-32) $$\Gamma_W = \frac{1}{2}\Gamma_{ID}^S + 2\Gamma_{\phi,t}^l + 2\Gamma_{\phi,s}^l \left(\frac{Y_{N1}}{Y_{N1}^{eq}}\right) + 2\Gamma_{N1}^l + 2\Gamma_{N1,t}^l \,,$$

where $\Gamma_{ID}^S = (n_S^{eq}/n_l^{eq})\Gamma_D^S$. In Eqs. (-33) and (-32) the Γ's are defined as $\Gamma_i^x = (\gamma_i/n_x^{eq})$ where γ_i is the scattering density and are defined as

Leptogenesis - density matrices

$$\gamma_{\phi,s} = \frac{M_S^4 m_t^2}{64\pi^5 v^4 z}\tilde{m}_3 \frac{F_{33}^2 v_R^2}{M_{33}} \sin^2\theta \int_1^\infty dx_1 \sqrt{x_1} K_1(z\sqrt{x_1}) \left[1 - \frac{1}{x_1}\right]^2 \quad (-31)$$

$$\gamma_{\phi,t} = \frac{M_S^4 m_t^2}{128\pi^5 v^4 z}\tilde{m}_3 \frac{F_{33}^2 v_R^2}{M_{33}} \sin^2\theta \int_1^\infty dx_1 \sqrt{x_1} K_1(z\sqrt{x_1}) \left[1 - \frac{1}{x_1} + \frac{1}{x_1}\ln\left(\frac{x_1 - 1 + y}{y}\right)\right] \quad (-30)$$

where v is the vev of SM Higgs and $x_1 = \frac{s}{M_S^2}$, s being the Mandelstam variable, and $y = \frac{m_\phi^2}{M_s^2}$.

$$\gamma_{N1} = \frac{(F_{11}v_R)^3 M_S \tilde{m}_1^2}{128\pi^5 v^4 z}\left(\frac{F_{11}^2 v_R^2}{M_{11}}\right)^2 \int_0^\infty dx_2 \sqrt{x_2} K_1\left(z\sqrt{x_2}\frac{F_{11}v_R}{M_{11}}\right)$$

$$\left[1 + \frac{1}{D_1(x_2)} + \frac{x_2}{2D_1^2(x_2)}\left\{1 + \frac{1+x_2}{D_1(x_2)}\right\}\ln(1+x_2)\right] \quad (-29)$$

$$\gamma_{N1,t} = \frac{(F_{11}v_R)^3 M_S \tilde{m}_1^2}{128\pi^5 v^4 z}\left(\frac{F_{11}^2 v_R^2}{M_{11}}\right)^2 \int_0^\infty dx_2 \sqrt{x_2} K_1\left(z\sqrt{x_2}\frac{F_{11}v_R}{M_{11}}\right)\left[\frac{x_2}{x_2+1} + \frac{1}{x_2}\ln(x_2 \right. \quad (-28)$$

where $x_2 = \frac{s}{M_1^2}$ and

$$\frac{1}{D_1(x_2)} = \frac{x_2 - 1}{(x_2-1)^2 + \left(\ldots\right)} \quad (-27)$$

SM

- 1st gen. fermion members : $(SU(3)_C \times SU(2)_L \times U(1)_Y)$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv (3,2,1/6), \quad \begin{matrix} u_R \equiv (3,1,2/3) \\ d_R \equiv (3,1,-1/3) \end{matrix} \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv (1,2,-1/2), \quad l_R \equiv (1,2,-1)$$

- vector and scalar:

$$\begin{matrix} \vec{G}_\mu \equiv (8,1,0) \\ \vec{W}_\mu \equiv (1,3,0) \\ \vec{B}_\mu \equiv (1,1,0) \end{matrix} \qquad H \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \equiv (1,2,1/2)$$

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- RGE in SM :

$$16\pi^2 E \frac{dg_i}{dE} = b_i g^3 = \beta_{SM}(g) \quad \Rightarrow \quad \frac{d}{dlnE}\alpha_i{}^{-1}(E) = -\frac{b_i}{2\pi}$$

- Solution :

$$\alpha_i{}^{-1}(E) = \alpha_i{}^{-1}(M_Z) - \frac{b_i}{2\pi} ln \frac{E}{M_Z} \text{ with } \begin{pmatrix} b_Y \\ b_{2L} \\ b_{3C} \end{pmatrix} = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix}.$$