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# Dissipation in QFT and the Hot Big Bang

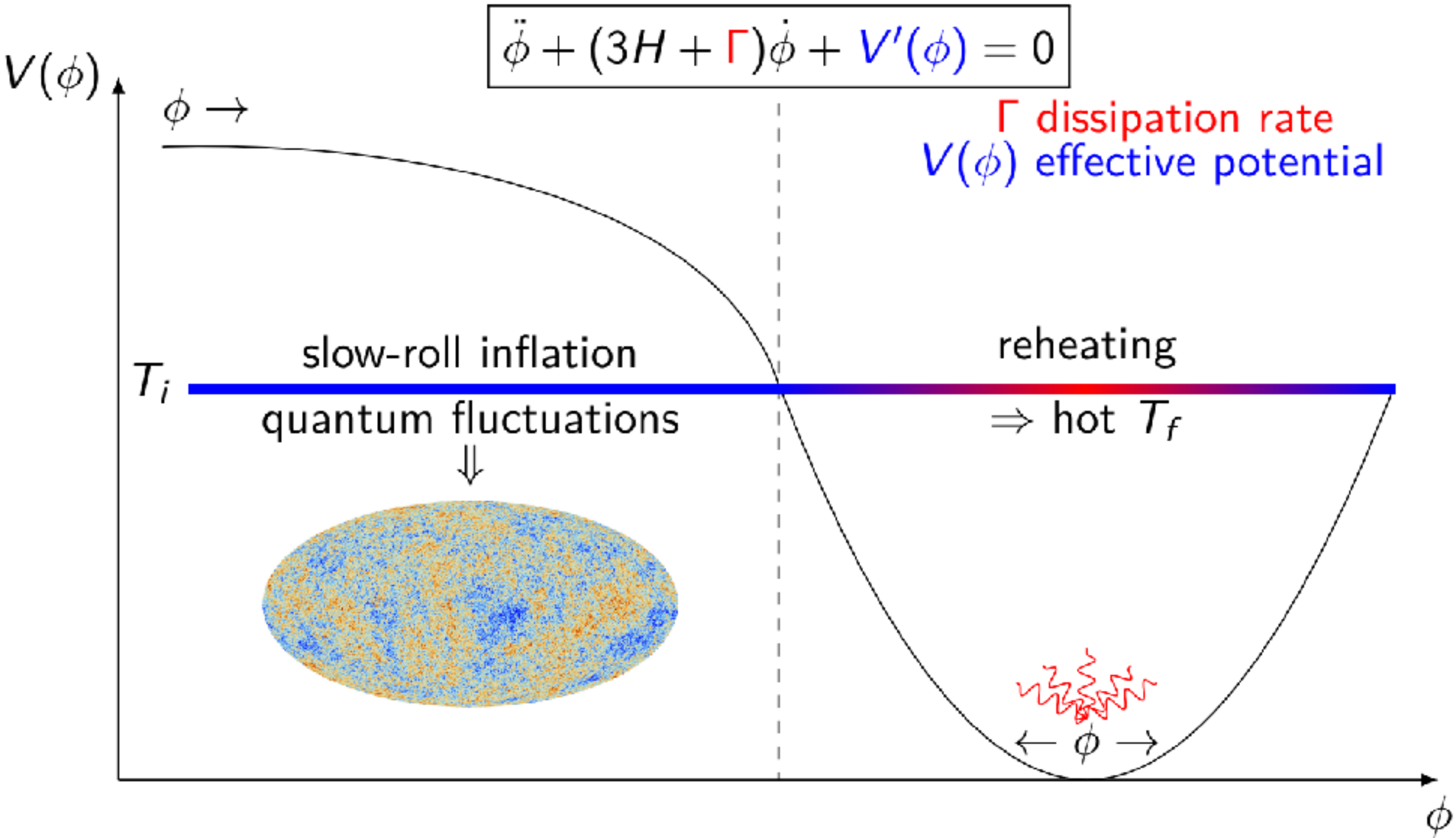
04.11.2019

DESY

Hamburg, Germany

work in collaboration with Jin U Kang, Gilles Buldgen, Ui Ri Mun, Jong Chol Kim

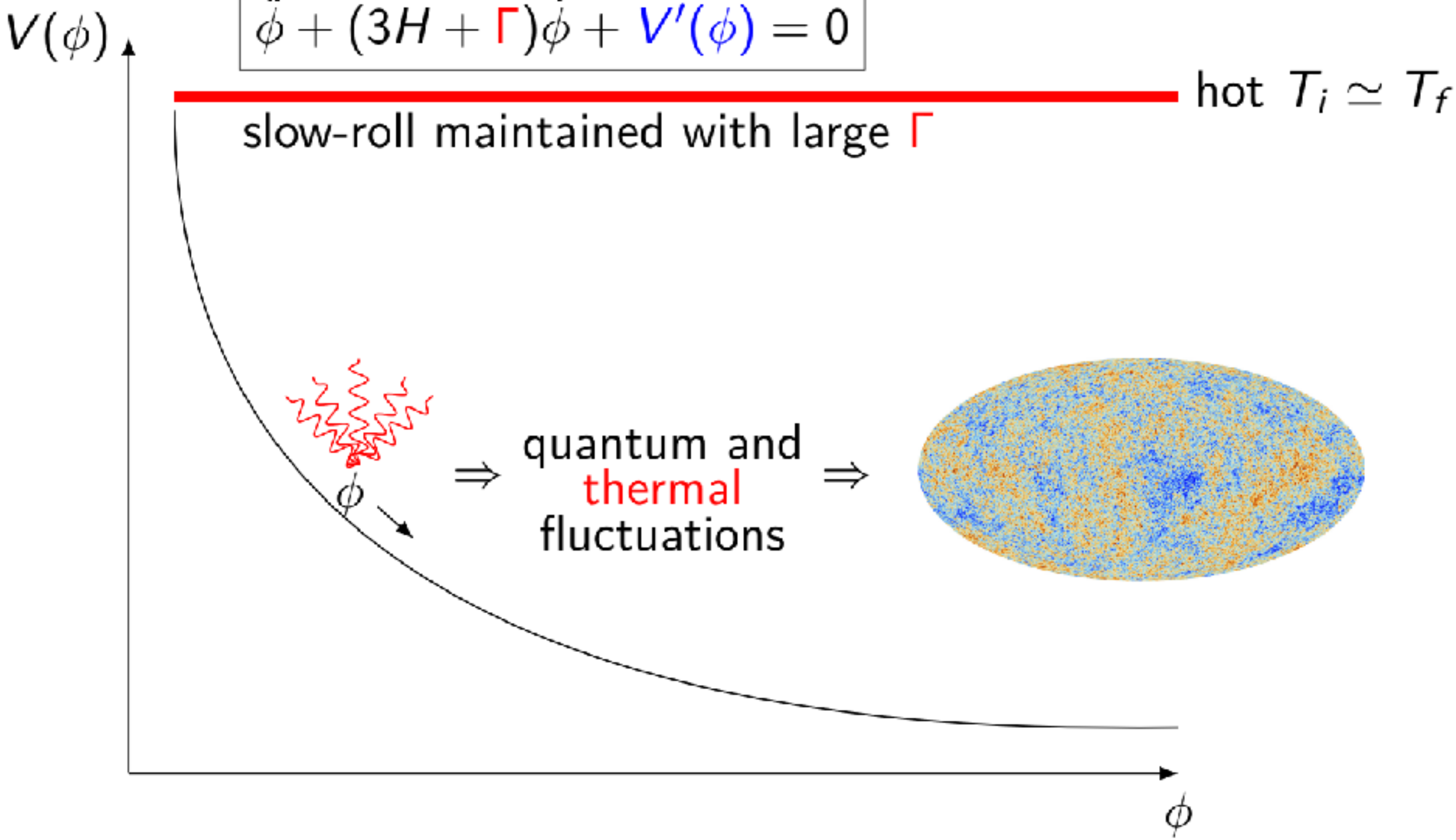
# Dissipation during/after inflation



# Dissipation during/after inflation

A.Berera Phys.Rev.Lett. 75 (1995) 3218-3221

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'(\phi) = 0$$



# This talk

- What is the rate of dissipation during inflation?

- How does the dissipation rate during reheating depend on microphysical parameters?

# This talk

- What is the rate of dissipation during inflation?

**Is warm inflation feasible?**

- How does the dissipation rate during reheating depend on microphysical parameters?

**Can one “measure'' the inflaton couplings in cosmological data?**

# Part I:
# Dissipation during inflation

# Previous computations

e.g. Bastero-Gil/Berera/Ramos 1207.0445

- Assumed validity of the equation

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + \partial_\phi V(\phi) = 0$$

- Computed the dissipation rate using the assumptions
  - Minkowski space propagators can be used
  - assume instantaneous equilibration in plasma
  - assume constant temperature
  - compute thermal corrections to quasiparticle properties in the ground state

- Usually do not systematically compute corrections to $V$

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**OK, we checked in 1504.04444**

- Computed the dissipation rate using the assumptions
  - Minkowski space propagators can be used **probably OK if T >> H**
  - assume instantaneous equilibration in plasma **probably OK if T >> H**
  - assume constant temperature **to be checked…**
  - compute thermal corrections to quasiparticle properties in the ground state **not OK**

- Usually do not systematically compute corrections to $V$ **not OK**

# Closed Time Path Formalism

We need equation of motion for the mean field

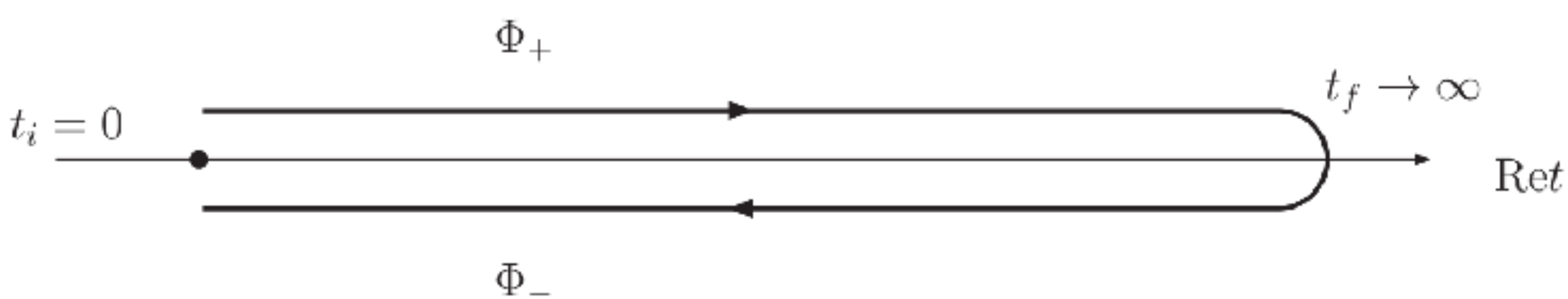
$$\varphi(x) \equiv \langle \phi(x) \rangle$$

and propagators

$$\Delta^-_\phi(x,y) \equiv \mathrm{i}\langle [\phi(x), \phi(y)\rangle$$

$$\Delta^+_\phi(x,y) \equiv \frac{1}{2}\langle \{\phi(x), \phi(y)\}\rangle - \varphi(x)\varphi(y)$$

They can be derived from the 2PI effective action on Schwinger-Keldysh contour



# 2PI Effective Action

The 2PI effective action on the Closed Time Path reads

$$\boldsymbol{\Gamma}[\varphi, \Delta] \equiv S[\varphi] + \boldsymbol{\Gamma}_{\mathrm{loop}}[\varphi, \Delta] = S[\varphi] + \boldsymbol{\Gamma}_1[\varphi, \Delta] + \boldsymbol{\Gamma}_2[\varphi, \Delta].$$

Up to one loop includes classical action and Coleman-Weinberg term

$$\boldsymbol{\Gamma}_1[\varphi, \Delta] = \frac{i}{2}\mathrm{Tr}\ln\left(\Delta^{-1}\right) + \frac{i}{2}\mathrm{Tr}\left(\Delta_0^{-1}(\varphi)\,\Delta\right)$$

with the "classical propagator"

$$i\Delta_{0,ab}^{-1}(x, y; \varphi) \equiv \left.\frac{\delta^2 S[\phi]}{\delta\phi(x)\delta\phi(y)}\right|_{\phi=\varphi}$$

and the "rest" made of diagrams with two or more loops $\boldsymbol{\Gamma}_2[\varphi, \Delta]$.

# The Model

We consider a scalar toy model with the classical

$$S[\phi,\chi] = \int_{\mathcal{C}} d^4x \left[ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_\phi^2}{2}\phi^2 - \frac{\lambda_\phi}{4!}\phi^4 \right.$$

$$\left. + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{m_\chi^2}{2}\chi^2 - \frac{h}{2}\phi^2\chi^2 - \mathcal{L}_{\chi\mathrm{int}} \right]$$

Now apply

$$\frac{\delta\Gamma[\varphi,\Delta]}{\delta\varphi_a(x)} = -J_a(x) - \int_z R_{ac}(x,z)\varphi_c(z) \quad \text{and} \quad \frac{\delta\Gamma[\varphi,\Delta]}{\delta\Delta_{ab}(x,y)} = -\frac{1}{2}R_{ab}(x,y)$$

and set "sources" $R$ and $J$ to zero.

# Kadanoff Baym Equations for Propagators

$$\begin{aligned}
\left(\partial_{t_1}^2 + \omega_a^2(t_1;\mathbf{p})\right)\Delta_{aa}^{-}(t_1,t_2;\mathbf{p}) &= -\int_{t_2}^{t_1} dt'\, \Pi_{aa}^{-}(t_1,t';\mathbf{p})\Delta_{aa}^{-}(t',t_2;\mathbf{p})\,, \\
\left(\partial_{t_1}^2 + \omega_a^2(t_1;\mathbf{p})\right)\Delta_{aa}^{+}(t_1,t_2;\mathbf{p}) &= -\int_{t_i}^{t_1} dt'\, \Pi_{aa}^{-}(t_1,t';\mathbf{p})\Delta_{aa}^{+}(t',t_2;\mathbf{p}) \\
&\quad + \int_{t_i}^{t_2} dt'\, \Pi_{aa}^{+}(t_1,t';\mathbf{p})\Delta_{aa}^{-}(t',t_2;\mathbf{p})\,.
\end{aligned}$$

where

$$\omega_a(t_1;\mathbf{p}) \equiv \sqrt{\mathbf{p}^2 + (M_a^{\mathrm{tree}}(t_1))^2 + \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, \Pi_{aa}^{0}(t_1,t_1;\mathbf{q})} \equiv \sqrt{\mathbf{p}^2 + M_a^2(t_1)}$$

with $M_a(t_1) \equiv \sqrt{(M_a^{\mathrm{tree}}(t_1))^2 + \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, \Pi_{aa}^{0}(t_1,t_1;\mathbf{q})}$.

# WKB Solutions

$$\Delta^-(t_1, t_2; \mathbf{p}) \simeq \frac{\sin\left(\int_{t_2}^{t_1} dt'\, \Omega_{t'}\right) e^{-\frac{1}{2}\left|\int_{t_2}^{t_1} dt' \Gamma_{t'}\right|}}{\sqrt{\Omega_{t_1}\, \Omega_{t_2}}},$$

$$\Delta^+(t_1, t_2; \mathbf{p}) = \frac{\cos\left(\int_{t_2}^{t_1} dt' \Omega_{t'}\right) e^{-\frac{1}{2}\left|\int_{t_2}^{t_1} dt' \Gamma_{t'}\right|}}{2\sqrt{\Omega_{t_1} \Omega_{t_2}}} \Big(1 + 2f(t_B)\Big),$$

MaD / Mendizabal / Weniger 1202.1301

**Quasiparticle properties are time dependent.**

Dispersion relation $\Omega_t = \mathrm{Re}\hat{\Omega}$ and width $\Gamma_t = -2\mathrm{Im}\hat{\Omega}$

are obtained from $0 = \hat{\Omega}^2 - \omega^2(t; \mathbf{p}) - \tilde{\Pi}^-(t, \hat{\Omega}; \mathbf{p}),$

**Occupation numbers evolve as** $\partial_t f(t) = -\Gamma_t (f(t) - \bar{f}(t))\,,$

# Equation of Motion for φ

$$\left(\Box_x + m_\phi^2 + \frac{\lambda_\phi}{6}\varphi(x)^2 + \frac{\lambda_\phi}{2}\Delta_\eta(x,x) + \frac{h}{2}\Delta_\chi(x,x)\right)\varphi(x) + \frac{\delta\Gamma_2[\varphi,\chi,\Delta_\phi,\Delta_\chi]}{\delta\varphi(x)} = 0$$


Analytic solutions can be found using the “slow roll” approximation

$$\varphi(t')^n \simeq \varphi(t)^n + n(t'-t)\dot{\varphi}(t)\varphi(t)^{n-1}.$$

For instance

$$\Delta^+(t,t;\mathbf{p}) \simeq \left(\frac{1+2f_B(\Omega_t)}{2\Omega_t}\right) + \left(\frac{\lambda\,\varphi(t)}{4T\,\Omega_t^2\Gamma_t(\cosh(\Omega_t/T)-1)}\right)\dot{\varphi}(t)$$

Time dependent frequencies create dissipation from “local” diagram!

# Main Results

$$\boxed{\ddot{\varphi}(t) + \Gamma_\varphi \dot{\varphi}(t) + \partial_\varphi \mathcal{V} = 0,}$$

2-loop results: Buldgen/MaD/Kang/Kim/Mun in preparation

$$\partial_\varphi \mathcal{V} = m_\phi^2 \varphi(t) + \frac{\lambda_\phi}{3!}\varphi(t)^3 + (\lambda_\varphi + h)\frac{T^2}{24}\varphi(t) + \partial_\phi \mathcal{V}_{\rm sun}$$

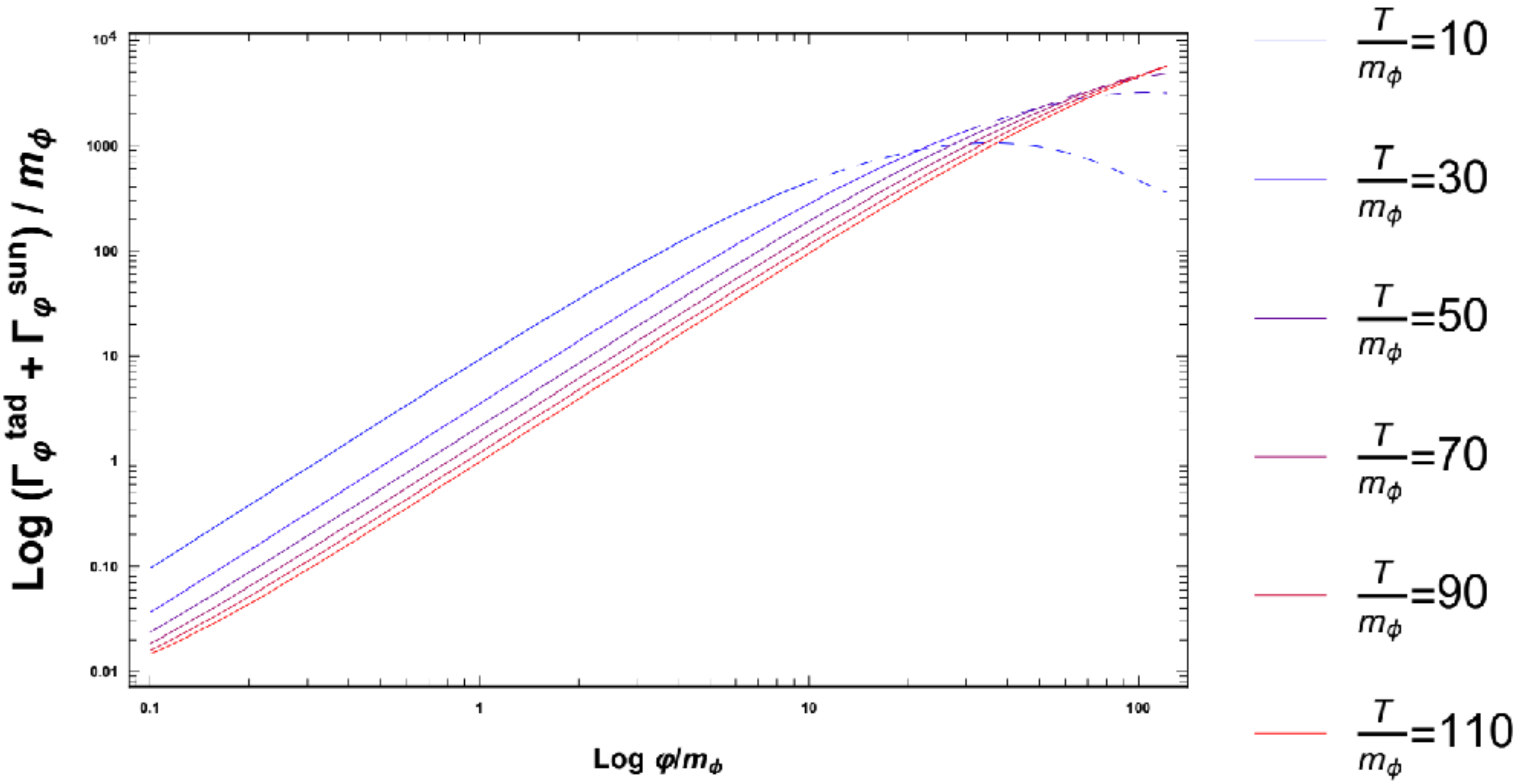
$$\Gamma_\varphi = \Gamma_\varphi^{\rm tad} + \Gamma_\varphi^{\rm sun}$$

$$\Gamma_\varphi^{\rm tad} = \frac{h^2}{(4\pi)^2}\frac{\varphi(t)^2}{T}\int \frac{p^2 dp}{\omega_\chi^2 \Gamma_\chi(\cosh(\omega_\chi/T) - 1)}$$

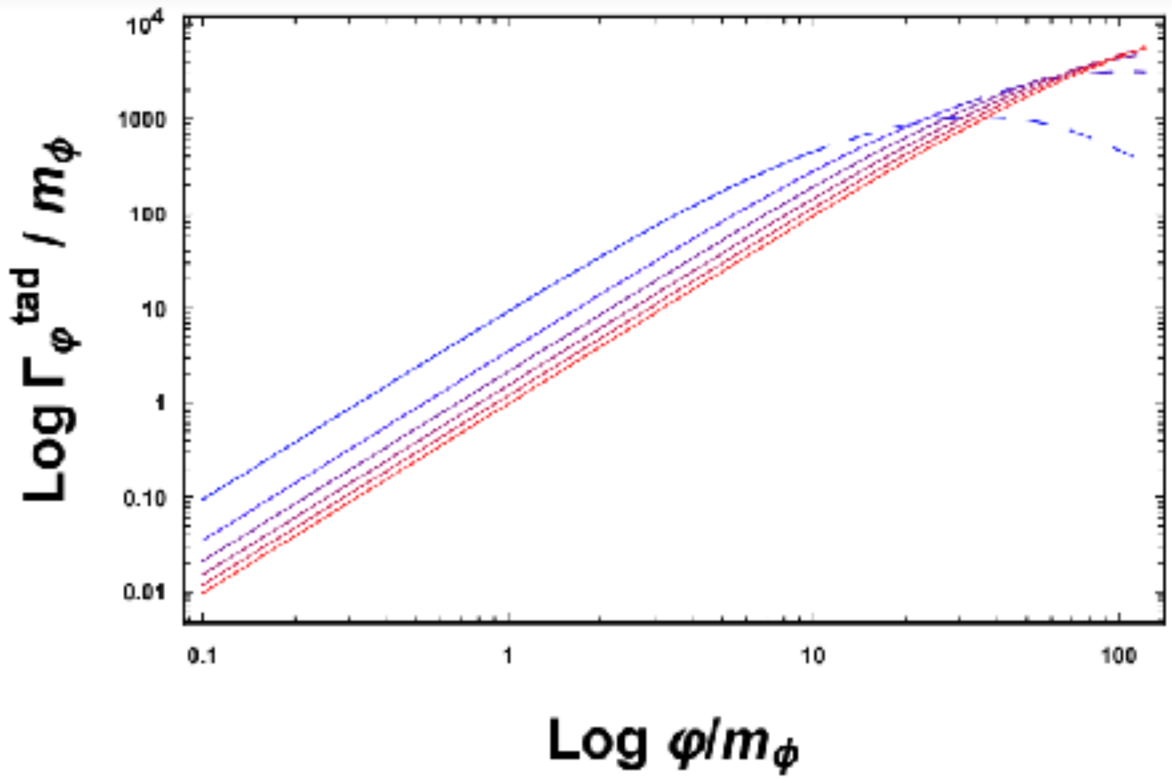
$$\Gamma_\varphi^{\rm sun} = \frac{h^2}{(4\pi)^3}\frac{T^2}{M_\phi}\log\frac{M_\phi}{M_\chi}$$

$$\omega_a^2 = p^2 + M_a^2[\varphi, T] = p^2 + m_a^2 + \frac{g_a}{2}\varphi(t)^2 + (\lambda_a + h)T^2/24$$

# Main Results

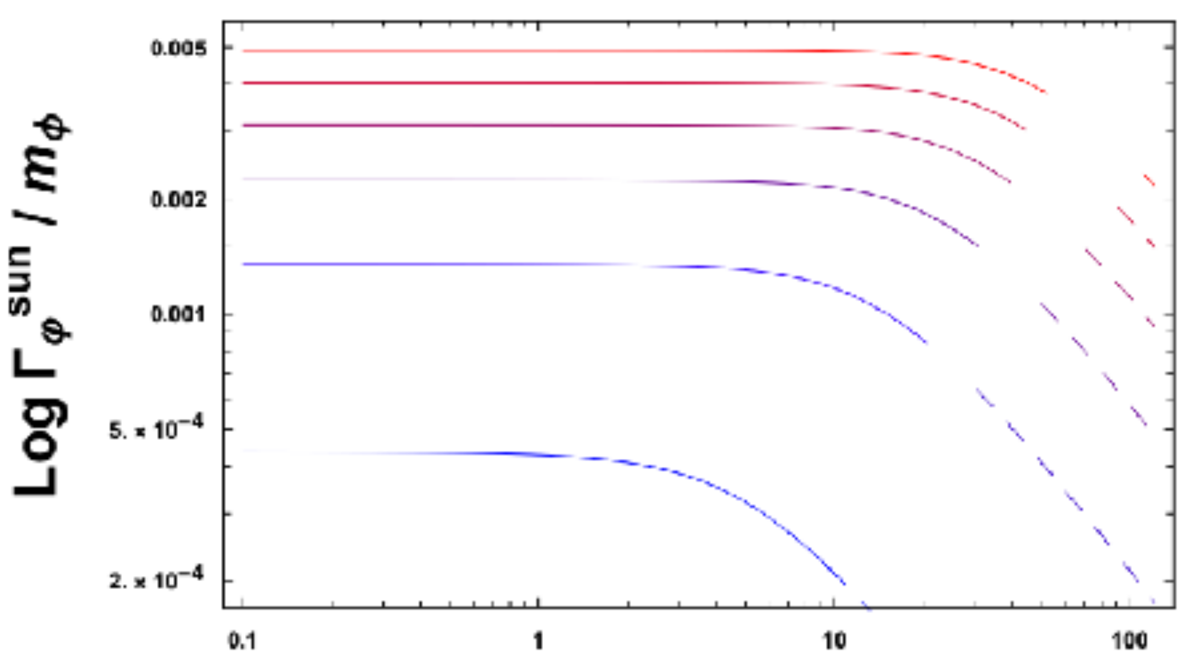


# Interpretation of $\Gamma$



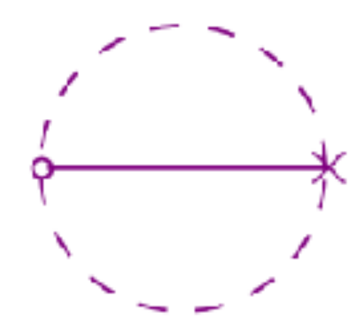

$$\Gamma_\varphi^{\rm tad} = \frac{h^2}{(4\pi)^2}\frac{\varphi(t)^2}{T}\int \frac{p^2\,dp}{\omega_\chi^2\Gamma_\chi(\cosh(\omega_\chi/T)-1)}$$

- scales like $\varphi^2/T$
- resonant enhancement suppressed by increasing $\Gamma_\chi$ width




$$\Gamma_\varphi^{\rm sun} = \frac{h^2}{(4\pi)^3}\frac{T^2}{M_\phi}\log\frac{M_\phi}{M_\chi}$$

- $\Gamma_\varphi^{\rm sun} \ll \Gamma_\varphi^{\rm tad}$
- scales like $T^2/M_\phi$



# Summary I

- **We refined the computation of *Γ* and *V* in several ways in a simple scalar model**

- **We find that the leading dissipation terms scales as *1/T*, while warm inflation usually requires it to grow with *T***

- **The results are not conclusive because the warm inflation literature used slightly more complicated models…**

- **…and it remains to be seen what effect the corrections have in those**

# Part II:
# Dissipation after inflation

# The Reheating Era

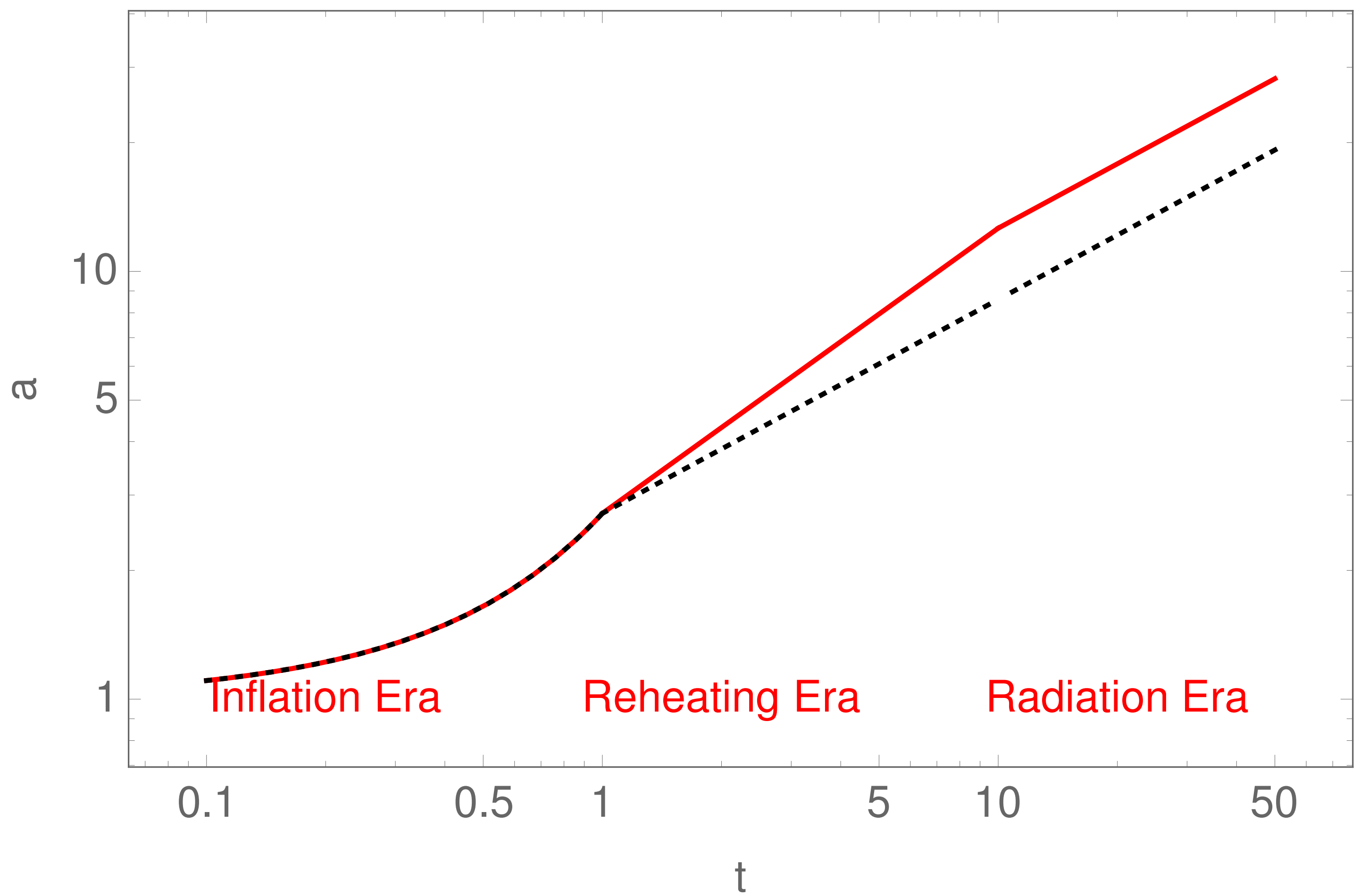
$$\ddot{\phi} + (3H + \Gamma_\varphi)\dot{\phi} + \partial_\phi V(\phi) = 0$$

- **inflation ends when kinetic energy is sizeable**

  ***w* > -1/3**

- **reheating ends when dissipation exceeds Hubble damping**

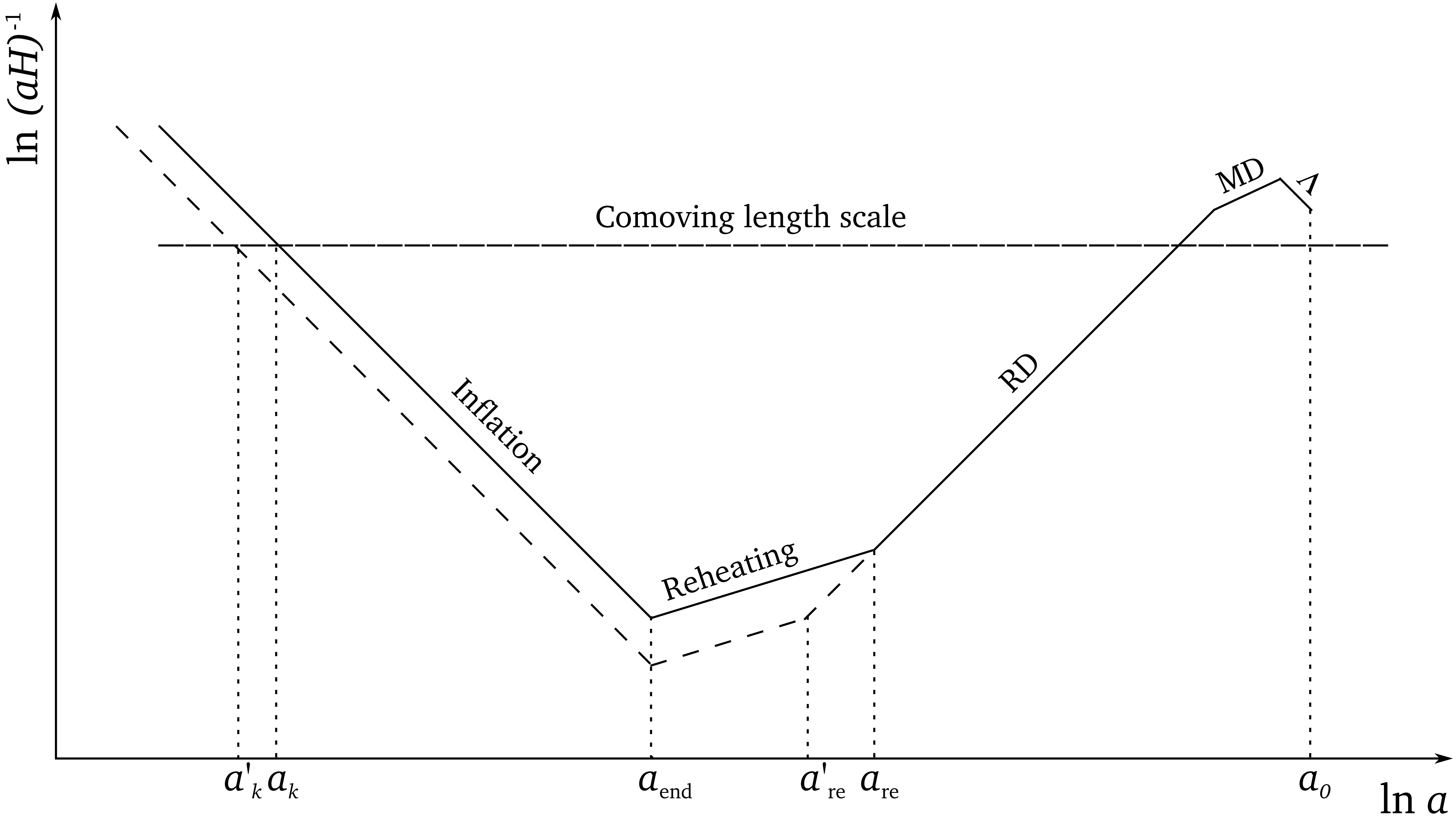
  ***Γ* = *H***




**In between: -1/3 < w < 1/3**

**⇒ affects expansion history and redshifting of CMB modes!**

# Effect on CMB Modes



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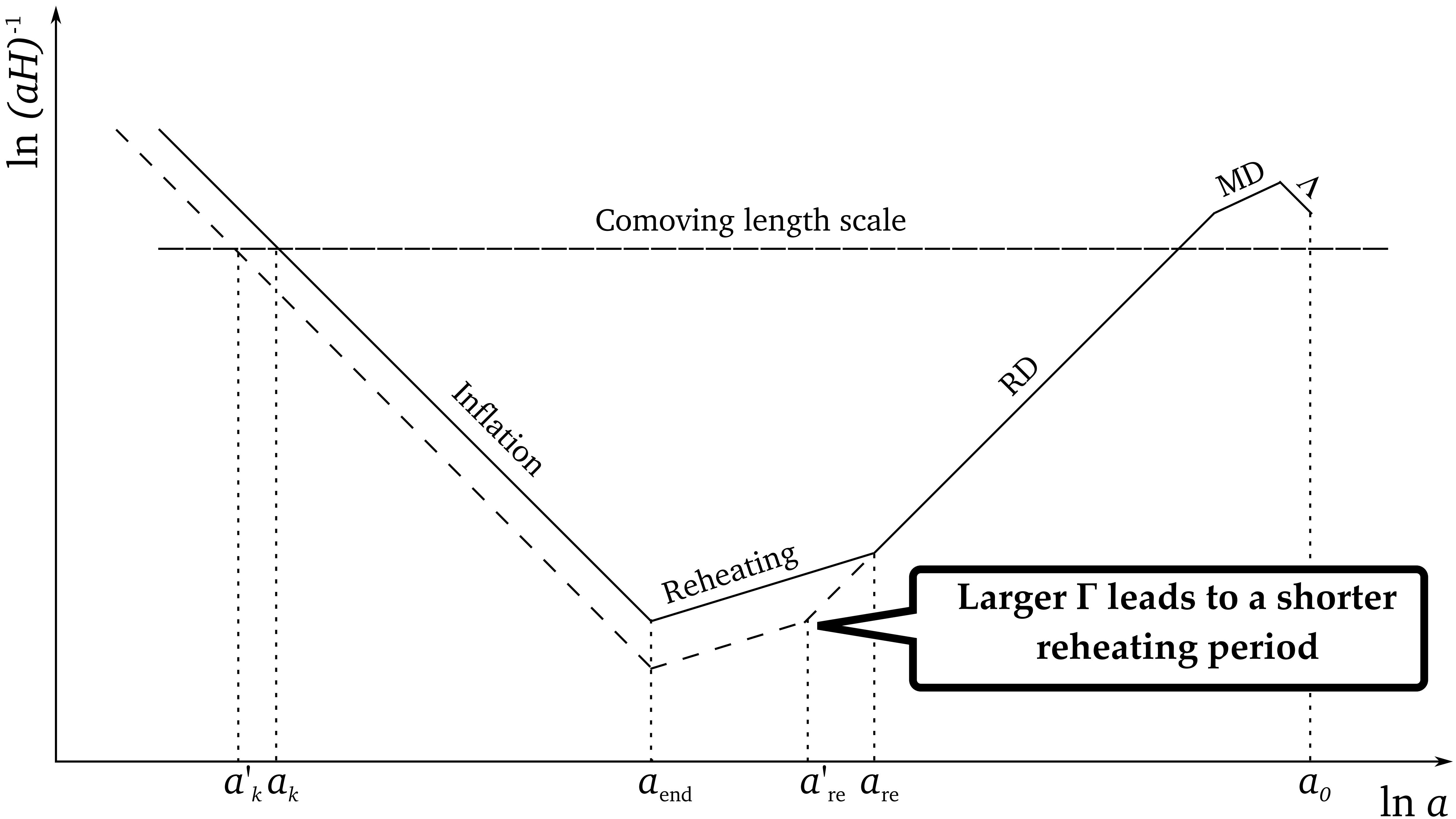

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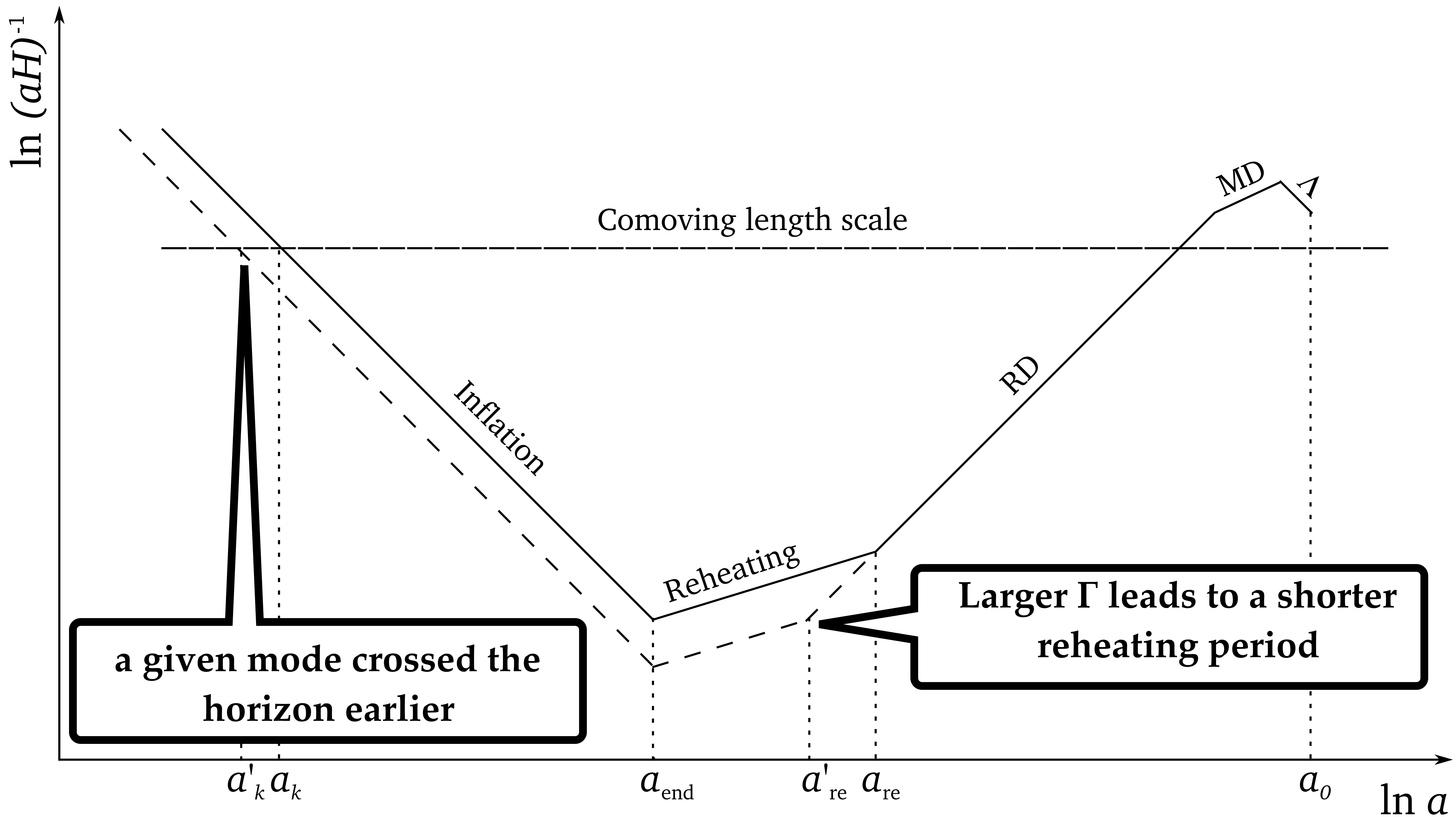
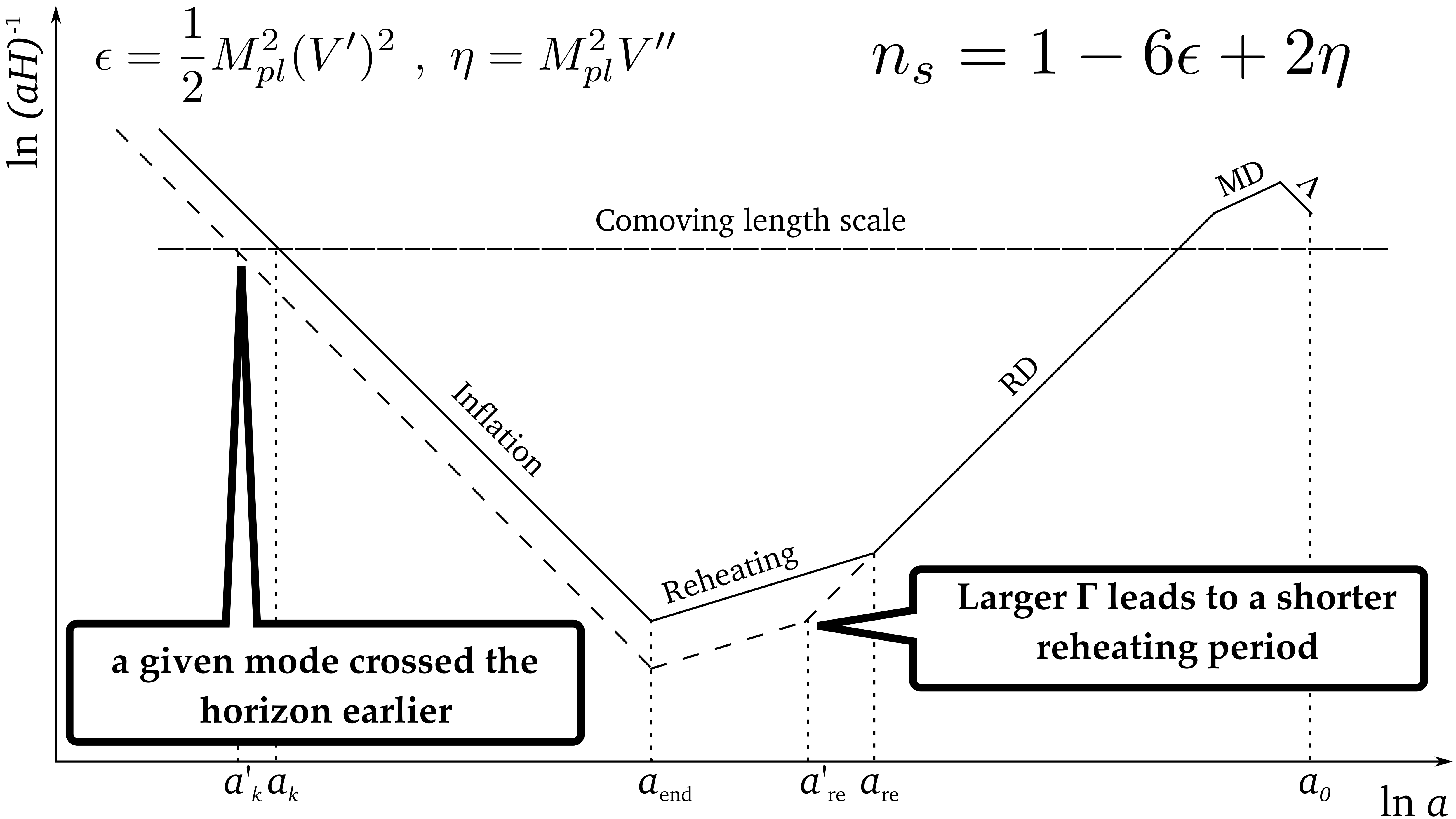


# Effect on CMB Modes

$$\epsilon = \frac{1}{2} M_{pl}^2 (V')^2 \;,\; \eta = M_{pl}^2 V''$$

$$n_s = 1 - 6\epsilon + 2\eta$$



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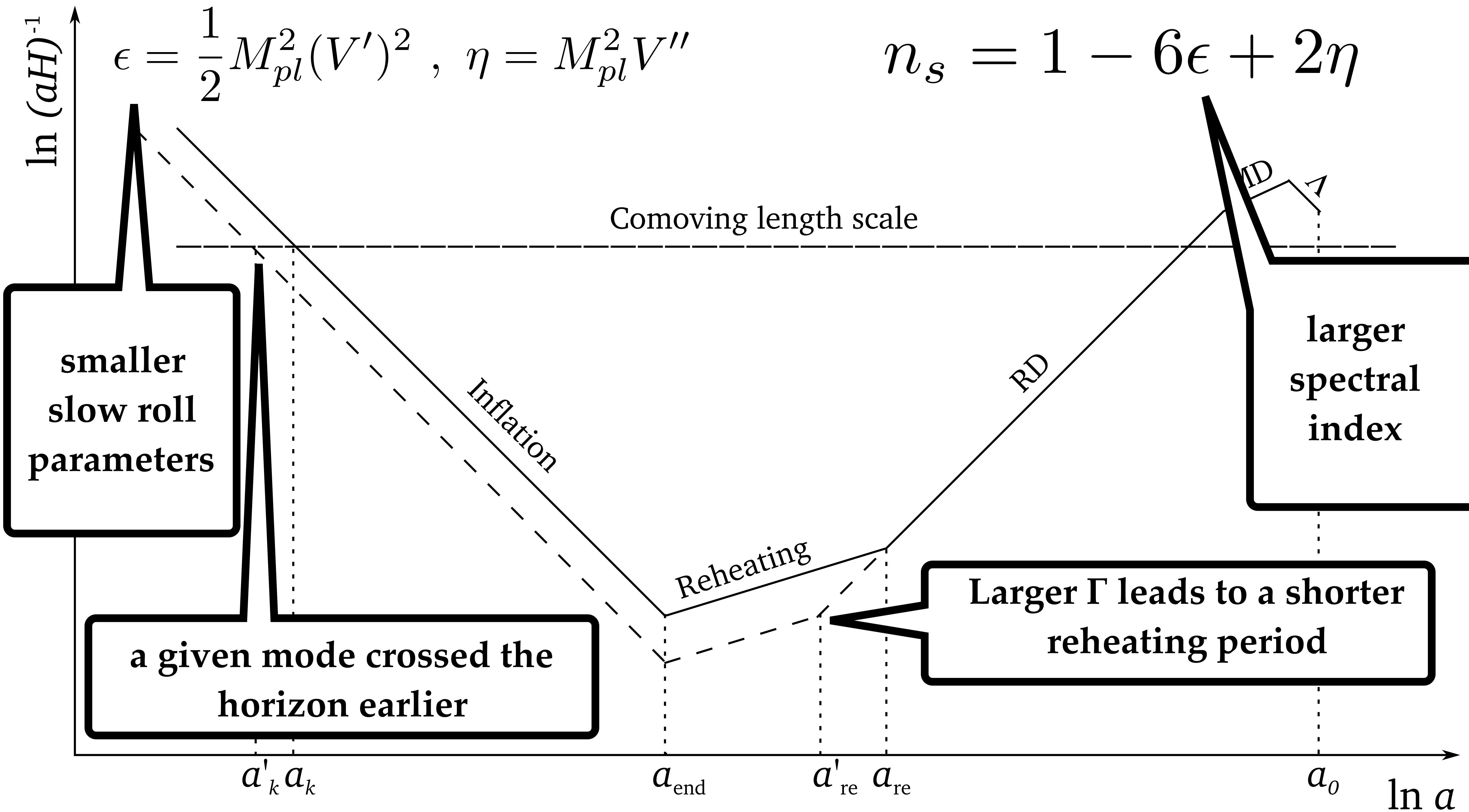

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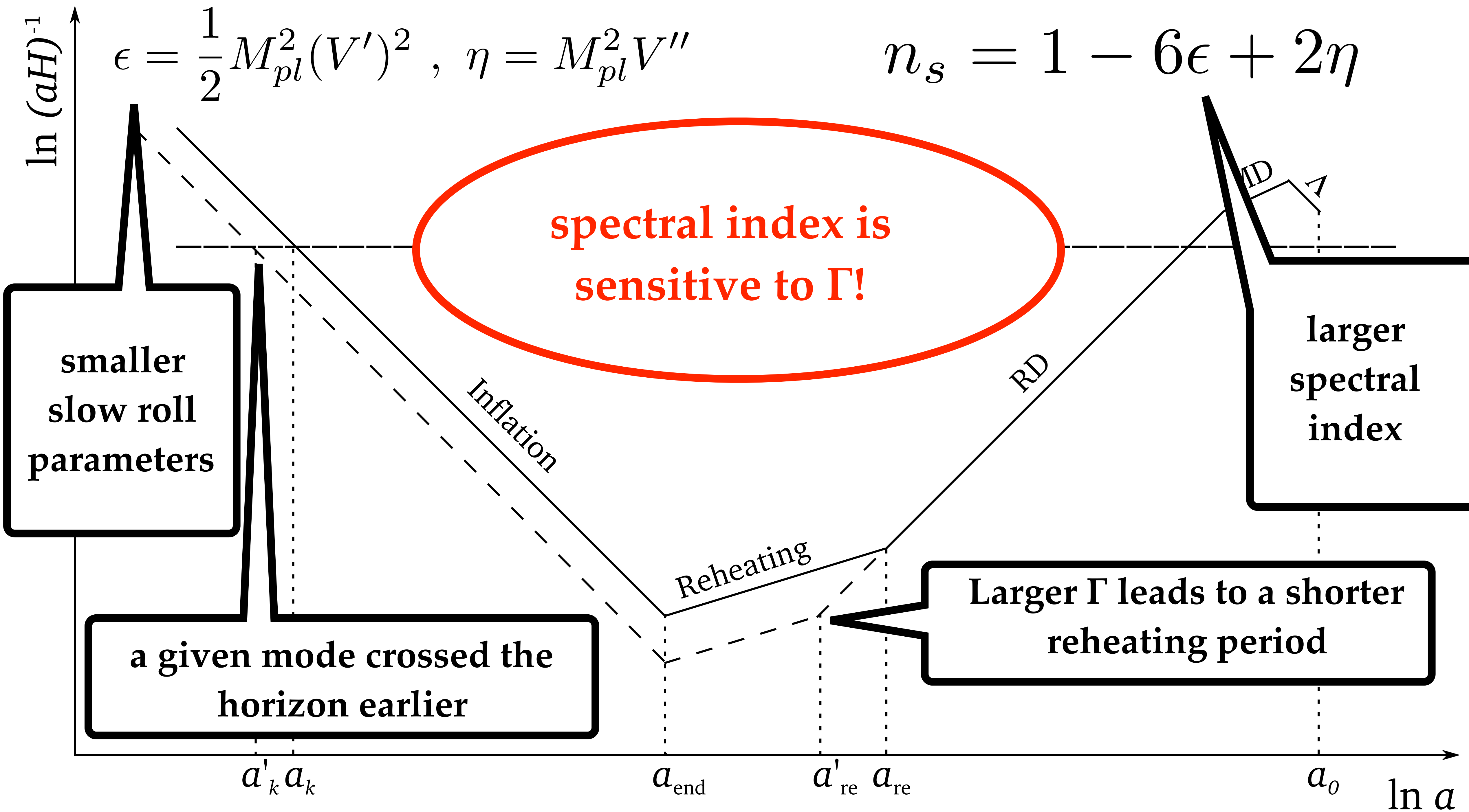


This is not really new…

see e.g. Kinney/Riotto 2006, Martin/Ringeval 2010.

…but one may ask

**Can one translate a "measurement" of $\Gamma$**

**into a "measurement" of microphysical parameters?**

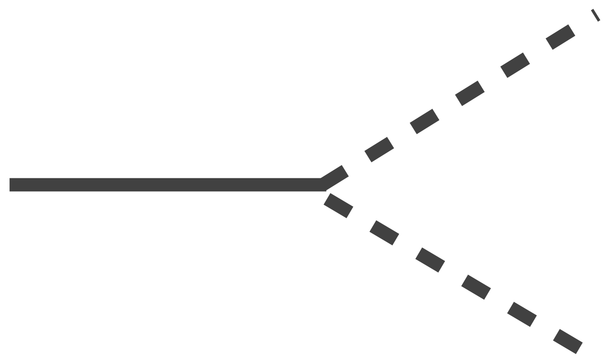
MaD 1511.03280

**$\Rightarrow$ gain information about embedding of inflation mechanism into a fundamental theory!**

# Inflaton Decay

Consider a simple scalar interaction $g\phi\chi^2$

In vacuum, the inflaton decays via $1 \rightarrow 2$ decays



$$\Gamma = \frac{g^2}{8\pi m_\phi}$$

But what about the feedback of the produced particles on $\Gamma$?

Feedback will lead to a very complicated relation between $g$ and $\Gamma$(t).

# Parametric Resonance

Mode equation for produced particles

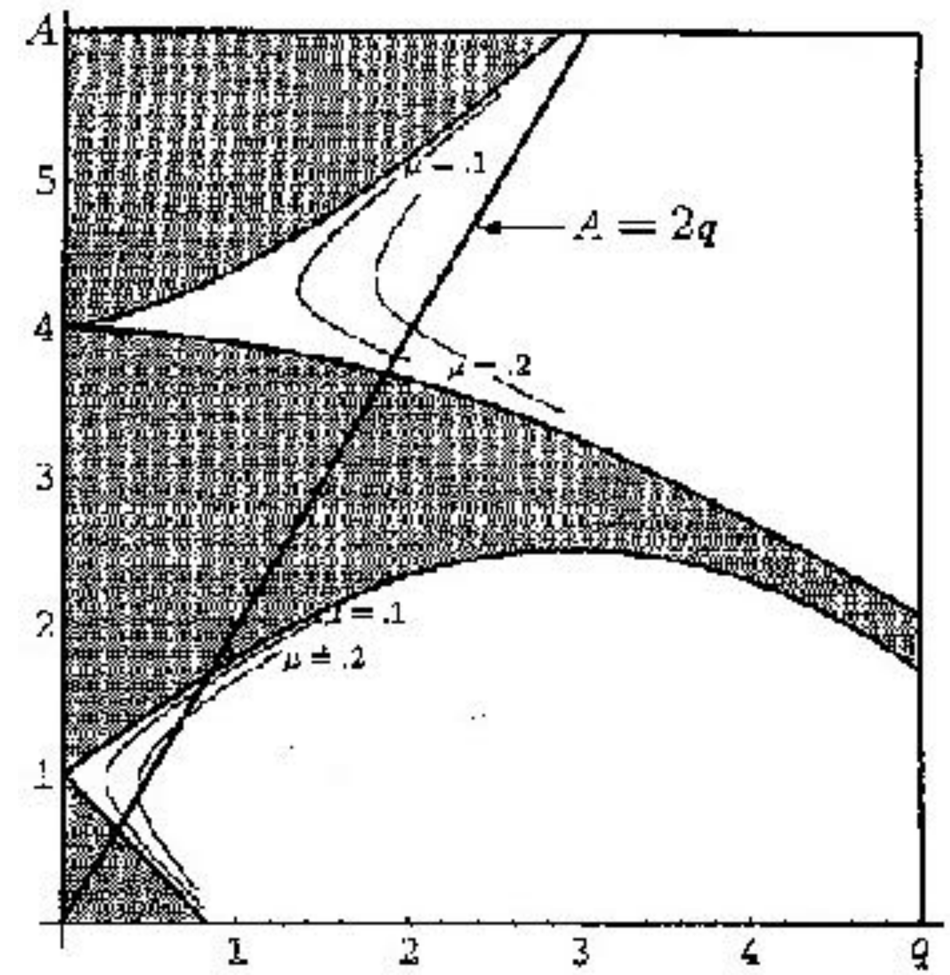
$$\ddot{\chi}_k(t) + \left[\mathbf{k}^2 + m_\chi^2 + g\varphi(t)\right]\chi_k(t) = 0$$

Can are rewritten as **Mathieu equation**

$$\chi_k''(z) + \left[A_k - 2q\cos(2z)\right]\chi_k(z) = 0$$

with $A_k = \frac{4\omega_k^2}{m_\phi^2}\ ,\ q = -2\frac{g}{m_\phi}\frac{\varphi_{\rm end}}{m_\phi}.$

Kofman/Linde/Starobinski



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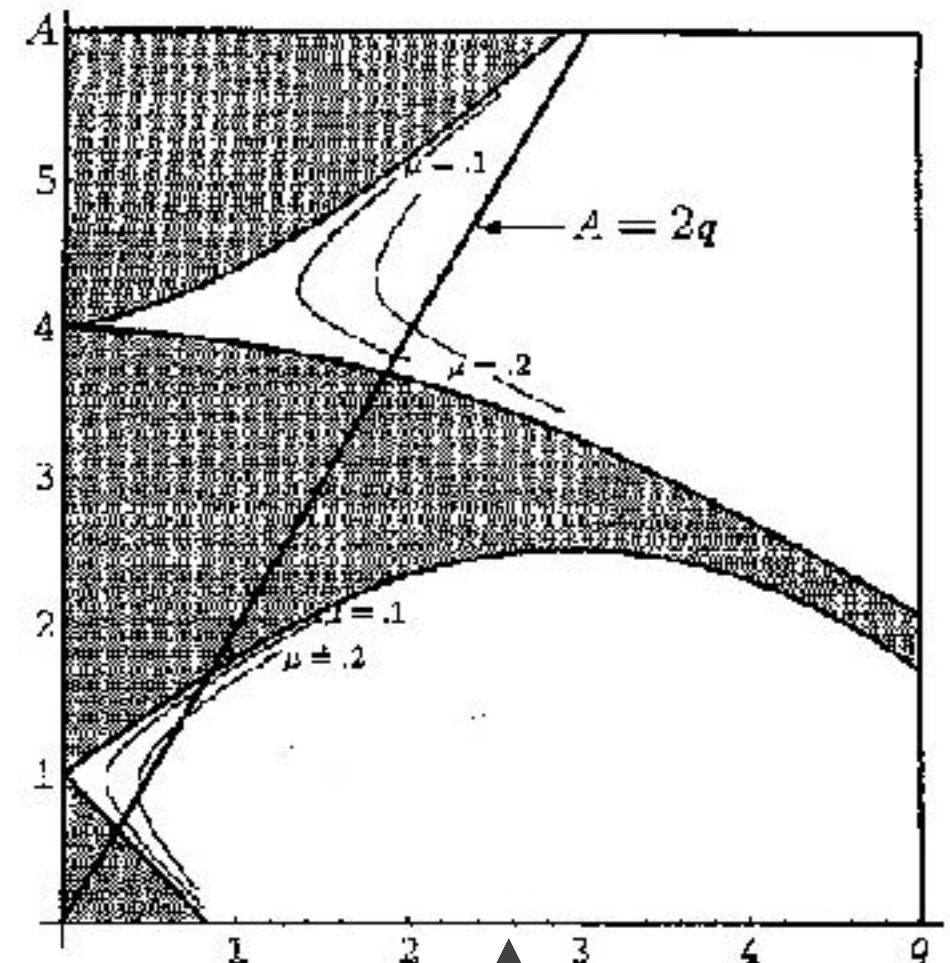
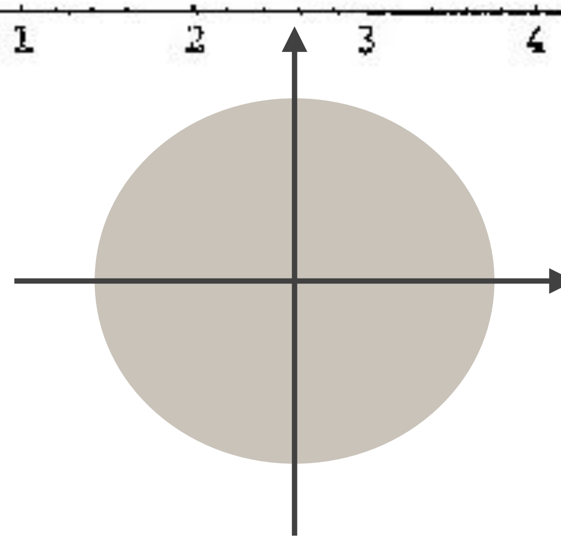



**"broad resonance" for *q* > *1*, i.e.** $\tilde{g} > m_\phi/\Phi$

non-perturbative production of particles
with momenta $k < (m_\phi^2 \tilde{g}\Phi)^{1/3}$



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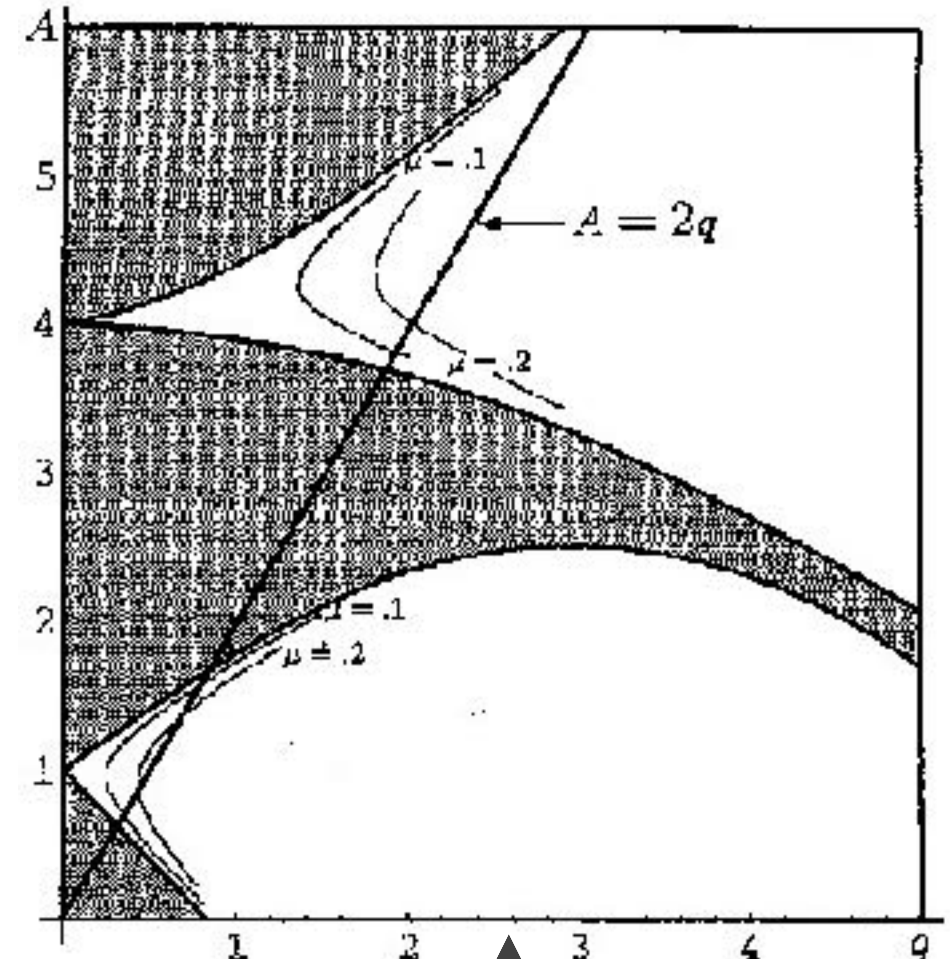
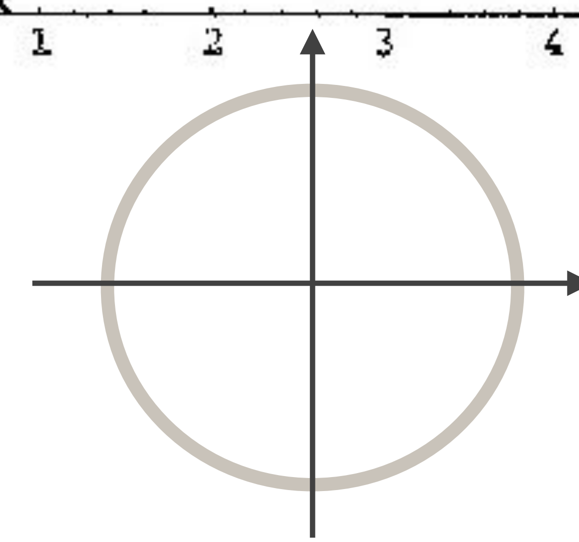



**"narrow resonance" for *q* < *1*, i.e.** $\tilde{g} < m_\phi/\Phi$

Bose-enhanced production of particles
with momenta $k = m_\phi/2$



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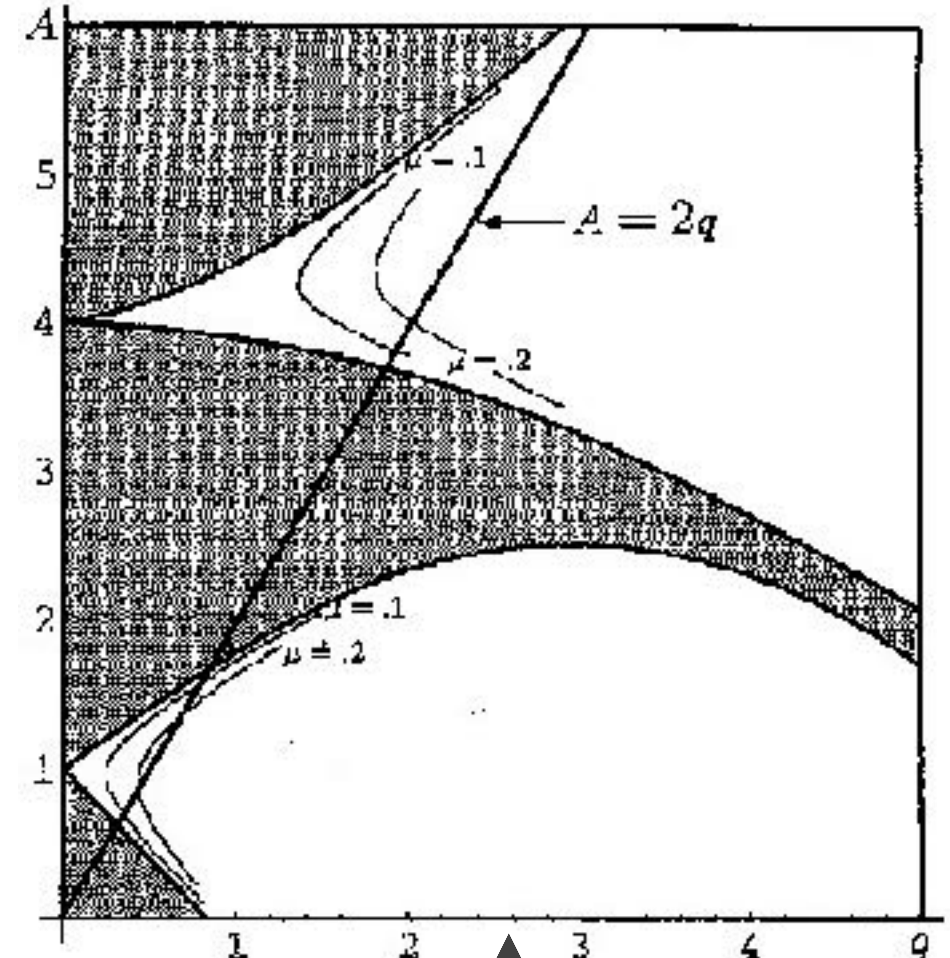


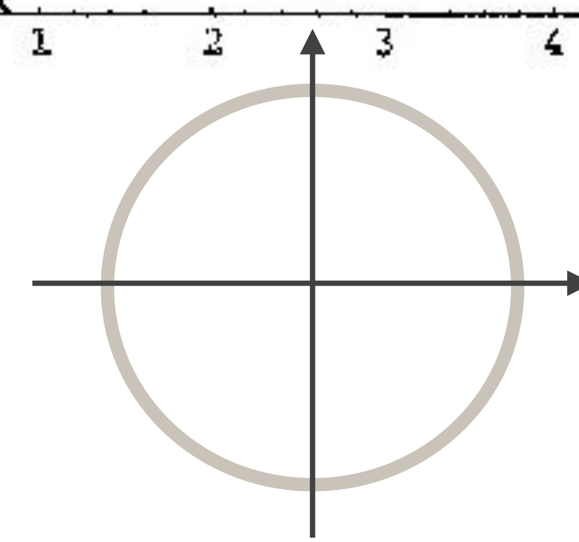

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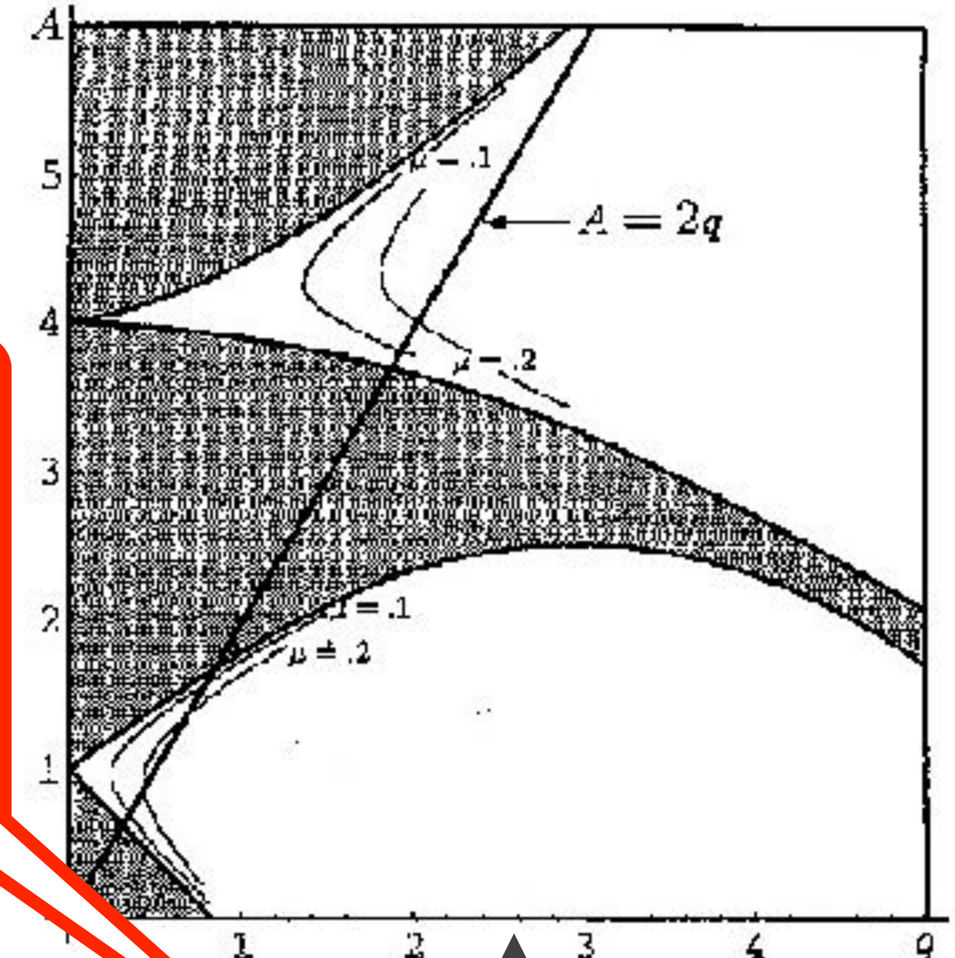
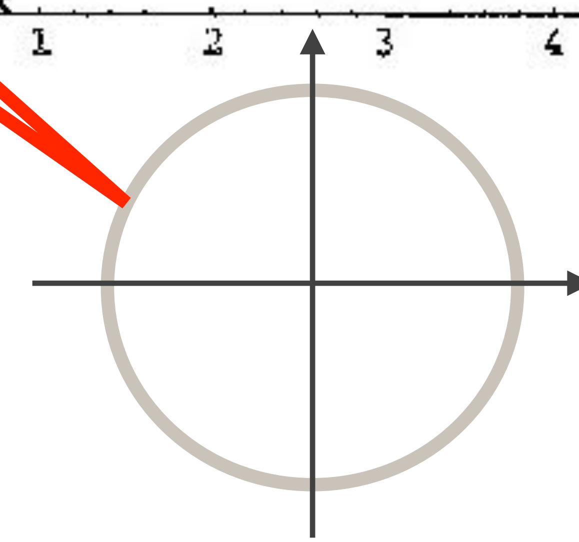



Can be avoided due to redshifting if $\Gamma < H$ or

$$\varphi_{\rm end}\frac{g}{m_\phi} < \sqrt{\frac{m_\phi}{6M_{pl}}}\mathcal{V}_{\rm end}^{1/4}$$

**"narrow resonance" for *q* < *1*, i.e.** $\tilde{g} < m_\phi/\Phi$

Bose-enhanced production of particles with momenta $k = m_\phi/2$



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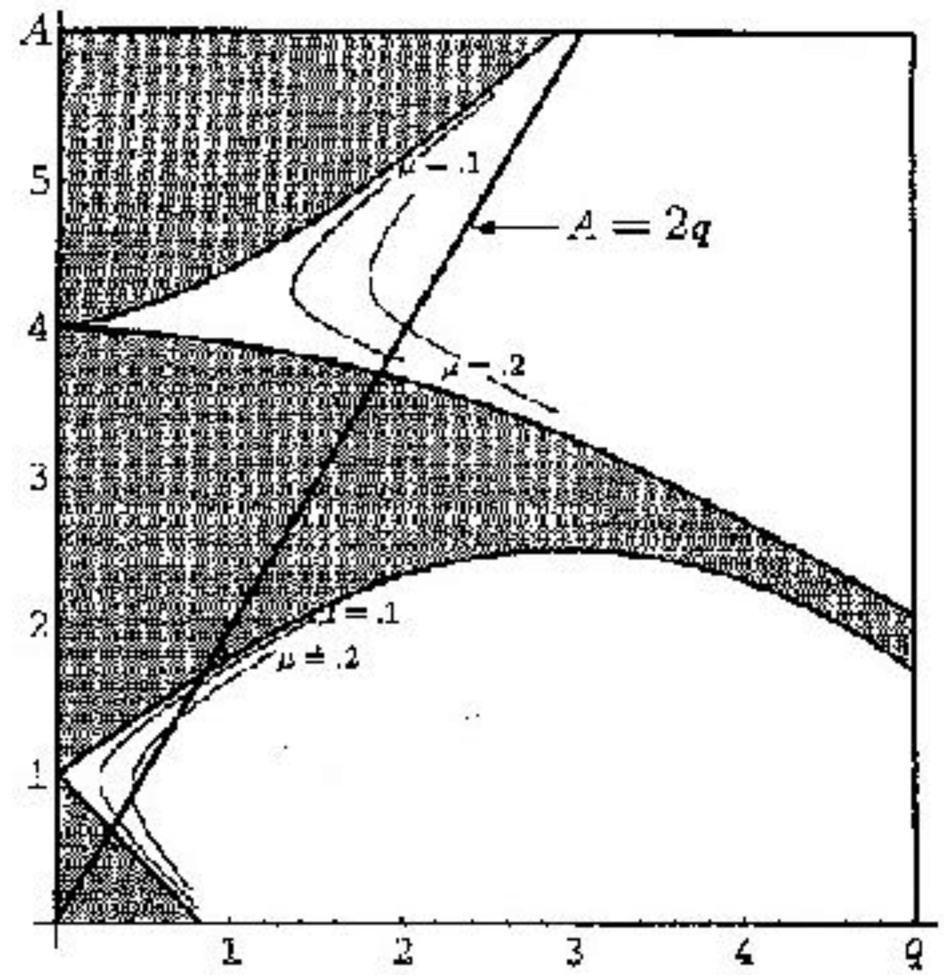


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with $A_k = \frac{4\omega_k^2}{m_\phi^2}$ , $q = -2\frac{g}{m_\phi}\frac{\varphi_{\rm end}}{m_\phi}$.

I) $q > 1$ "broad resonance" occurs

II) $q\, m > 3H$ "narrow resonance" dominates friction

III) $q^2 m > H$ resonance efficiently reheats universe ($\Gamma > H$)

# Range of Accessible Couplings

IV) Big Bang Nucleosynthesis requires T > 10 MeV when $\Gamma = H$.

Estimate reheating temperature

$$T_{\rm re} = \sqrt{\Gamma M_{pl}} \left(\frac{30}{\pi^2 g_*}\right)^{1/4}$$

This implies

$$\frac{g}{m_\phi} > \frac{T_{\rm BBN}}{\sqrt{m_\phi M_{pl}}} \pi \left(g_* \frac{64}{30}\right)^{1/4}$$

**The vacuum decay rate can be used to describe reheating if**

$$10^{-10}\sqrt{\frac{\rm GeV}{m_\phi}} < \frac{g}{m_\phi} < 10^{-19}\frac{m_\phi}{\rm GeV}$$

MaD 1903.09599

# General Considerations

**Interactions linear in ϕ**: Mathieu equation generally has the form

$$\chi_k''(z) + [A_k - 2q\cos(2z)]\,\chi_k(z) = 0$$

Previous results can e.g. be applied to

- Yukawa interactions $y\phi\bar{\psi}\psi$ with $\frac{g}{m_\phi} \to y.$
- axion like interactions $\alpha\phi\Lambda^{-1}F_{\mu\nu}\tilde{F}^{\mu\nu}$ with $\frac{g}{m_\phi} \to \alpha\sqrt{2}\frac{m_\phi}{\Lambda}$

**General rule: Inflaton coupling can be "measured" if**

***coupling < $m_\phi$ / $M_{pl}$*** MaD 1903.09599

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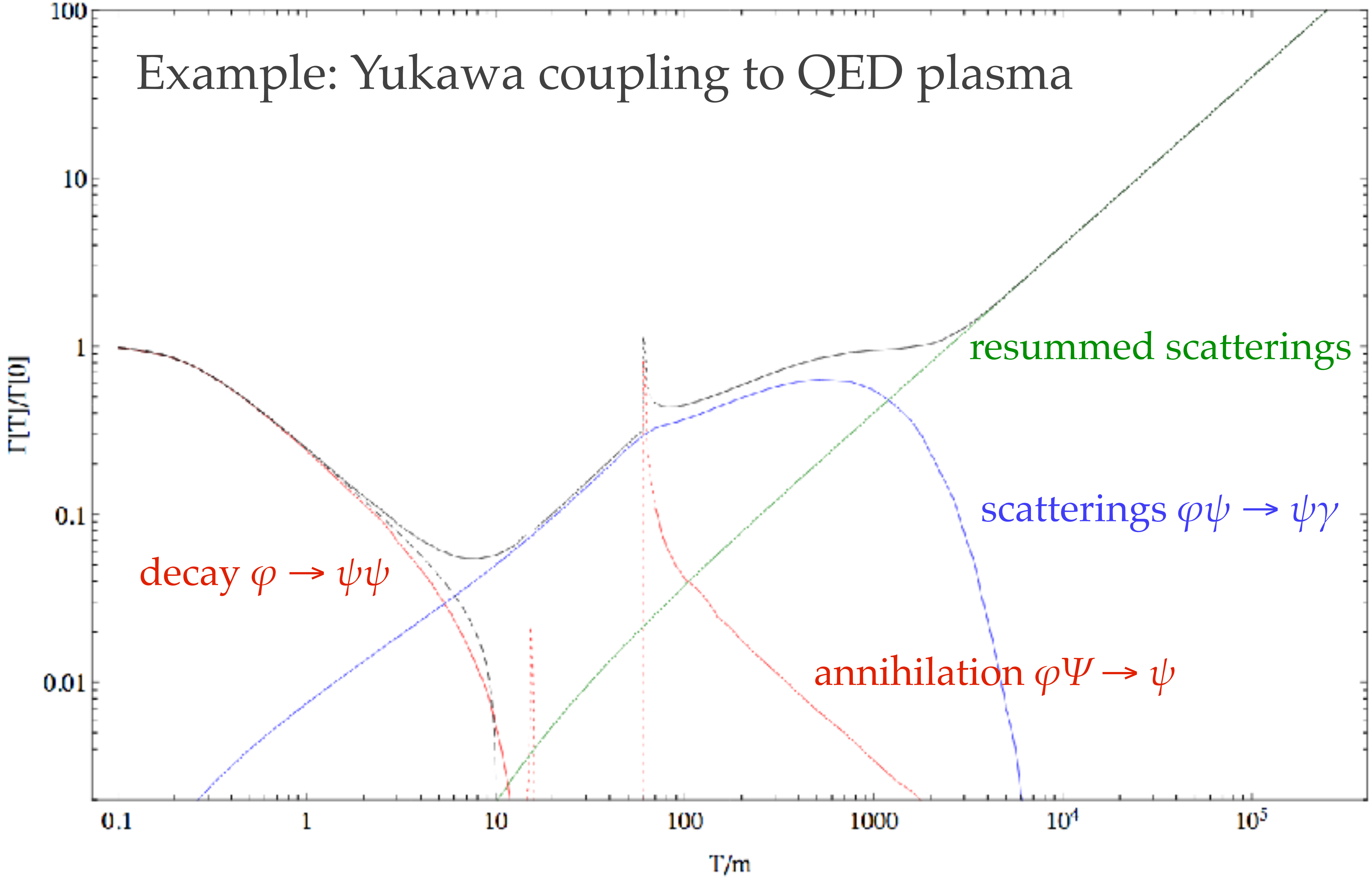
**Interactions involving higher powers in ϕ**:

$q$ involves higher powers of $\phi$, leading to stronger restrictions

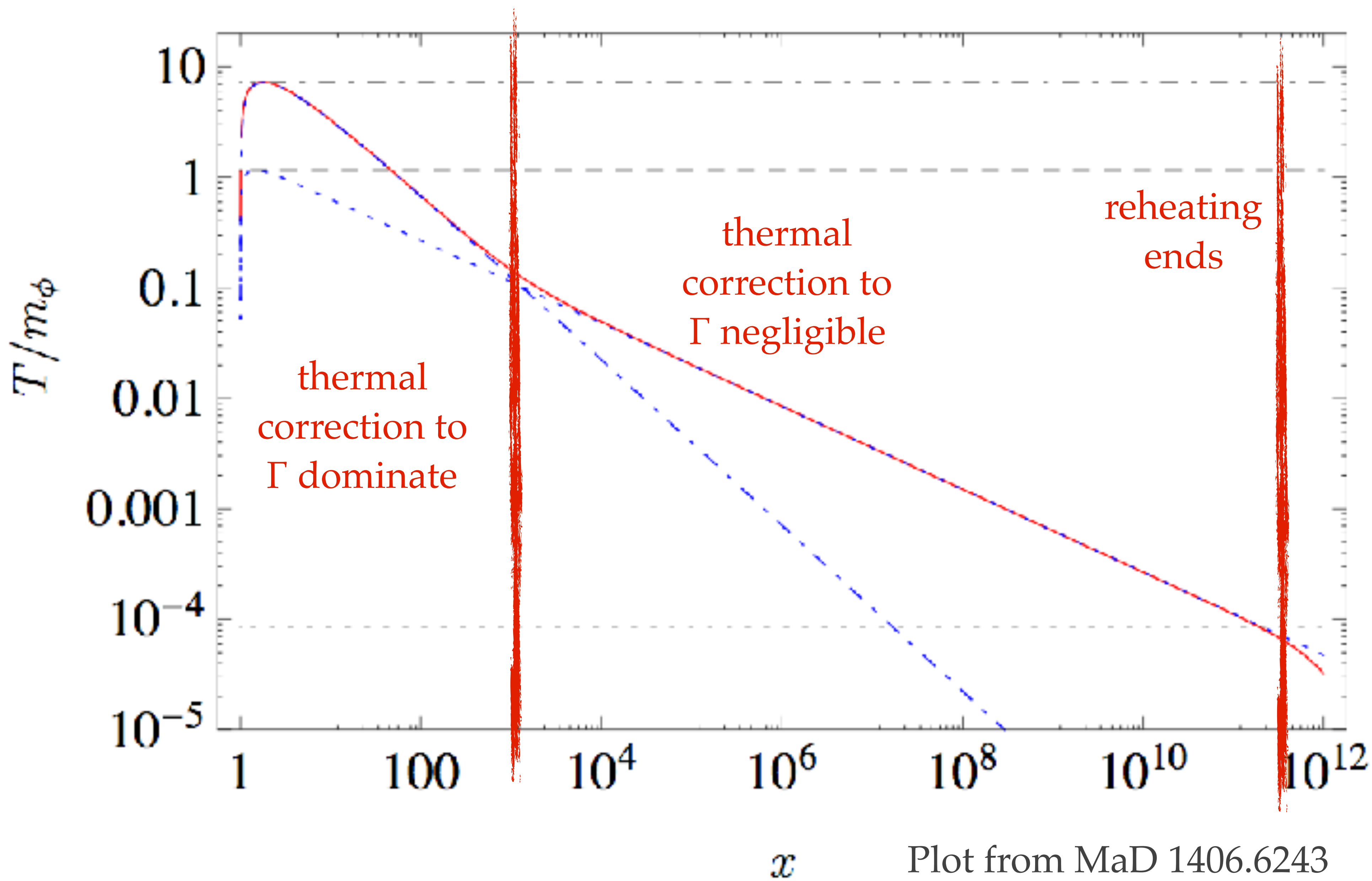
# Thermal Corrections?




MaD / Kang 1305.0267

# Thermal vs Expansion History

Thermal corrections modify the evolution of the temperature during reheating, but the effect on the expansion history is subdominant.




Plot from MaD 1406.6243

**No visible effect in CMB!**

MaD 1511.03280

# Example: α Attractor E Model

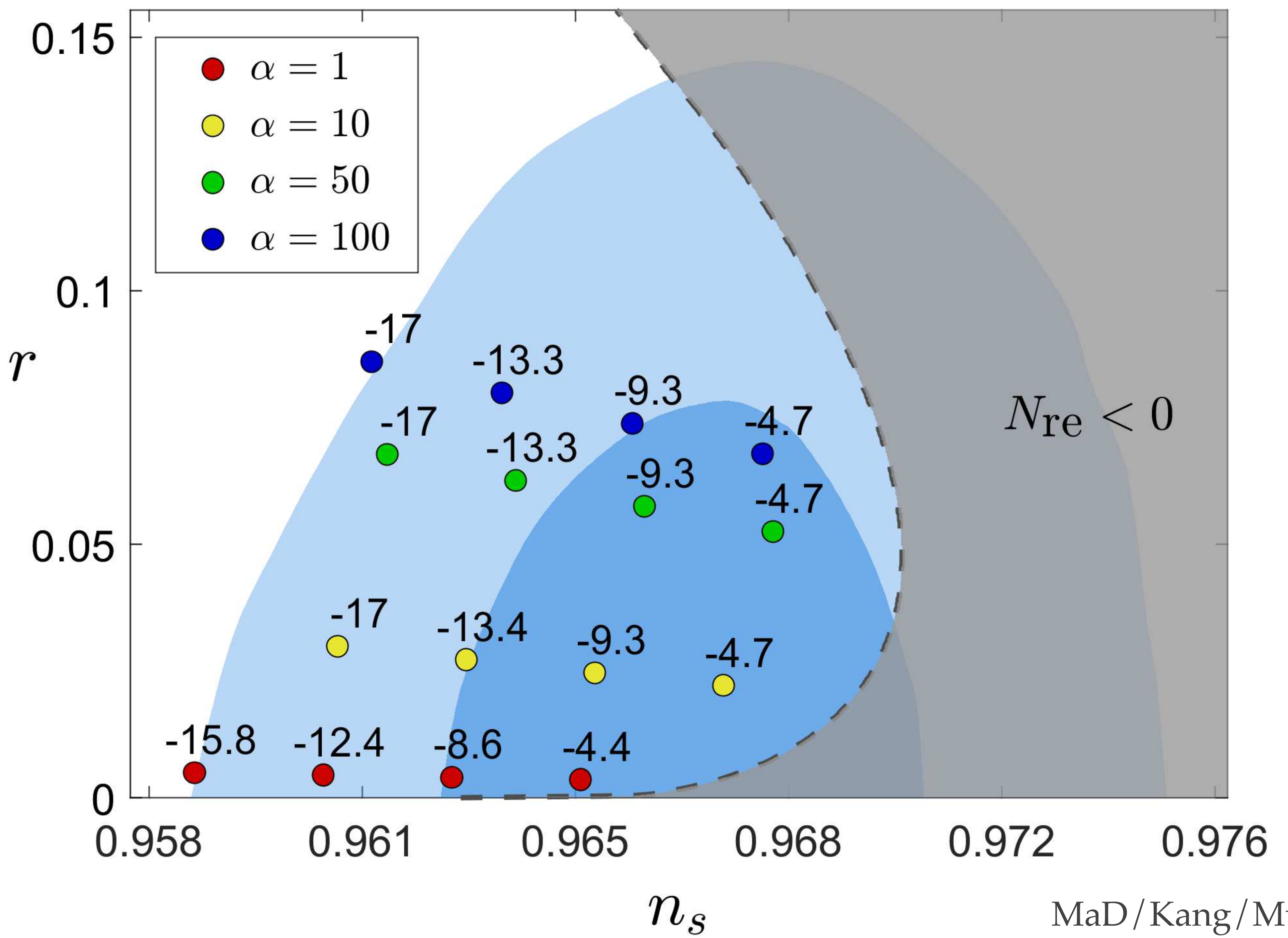
$$V = \Lambda^4 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_{pl}}}\right)^{2n}$$

Kallosh/Linde 2013 …

$$\text{unknowns} \ : \ (\Lambda, \alpha, n, g)$$
$$\text{observables} \ : \ (A_s, n_s, r)$$

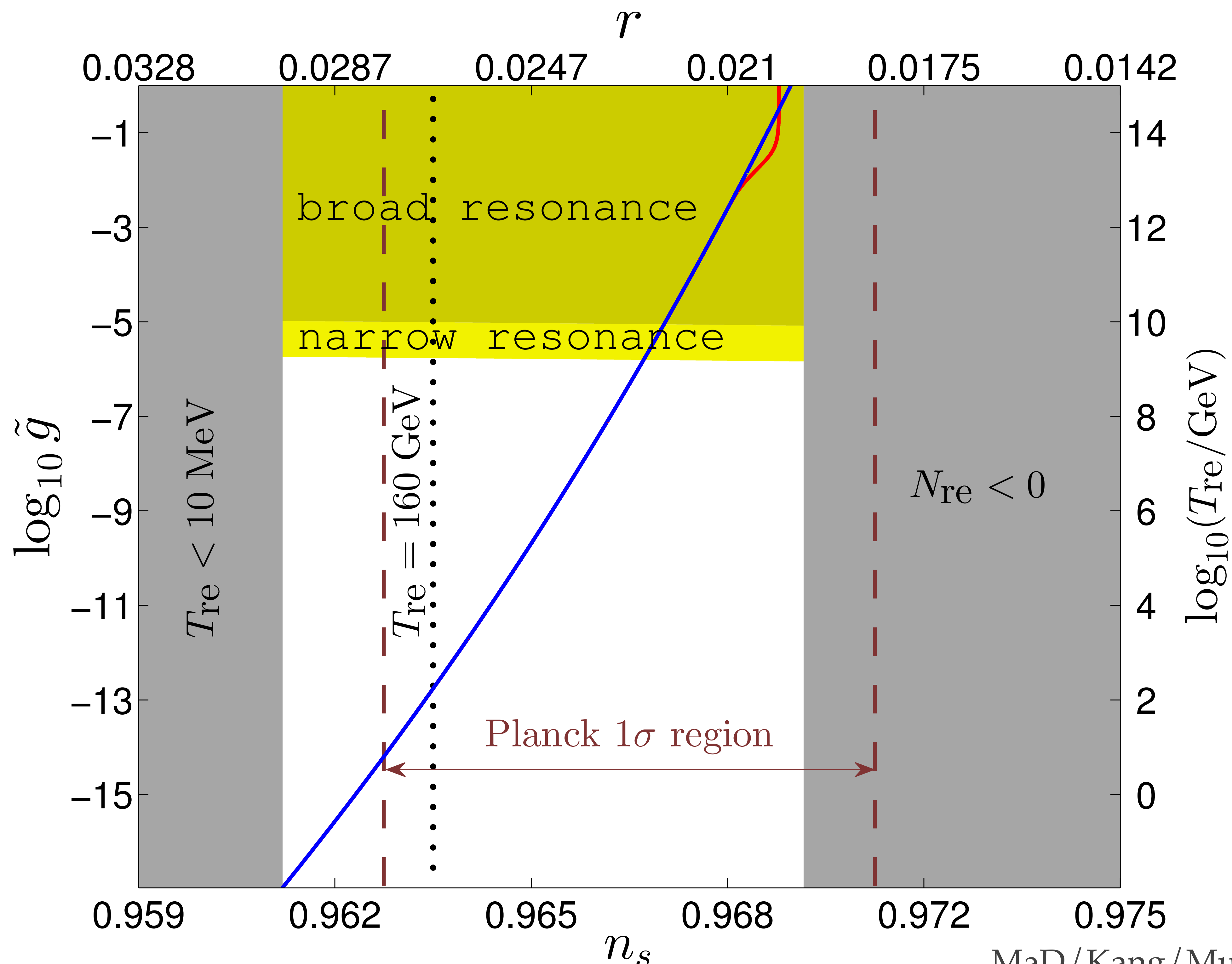
- **We fix *n=1* and study different values of *α***
- ***r* is uniquely determined by the spectral index**
- **reheating temperature is fixed by *g***

# Results for gφχ² Interaction




MaD/Kang/Mun 1708.01197

# Results for gφχ² Interaction

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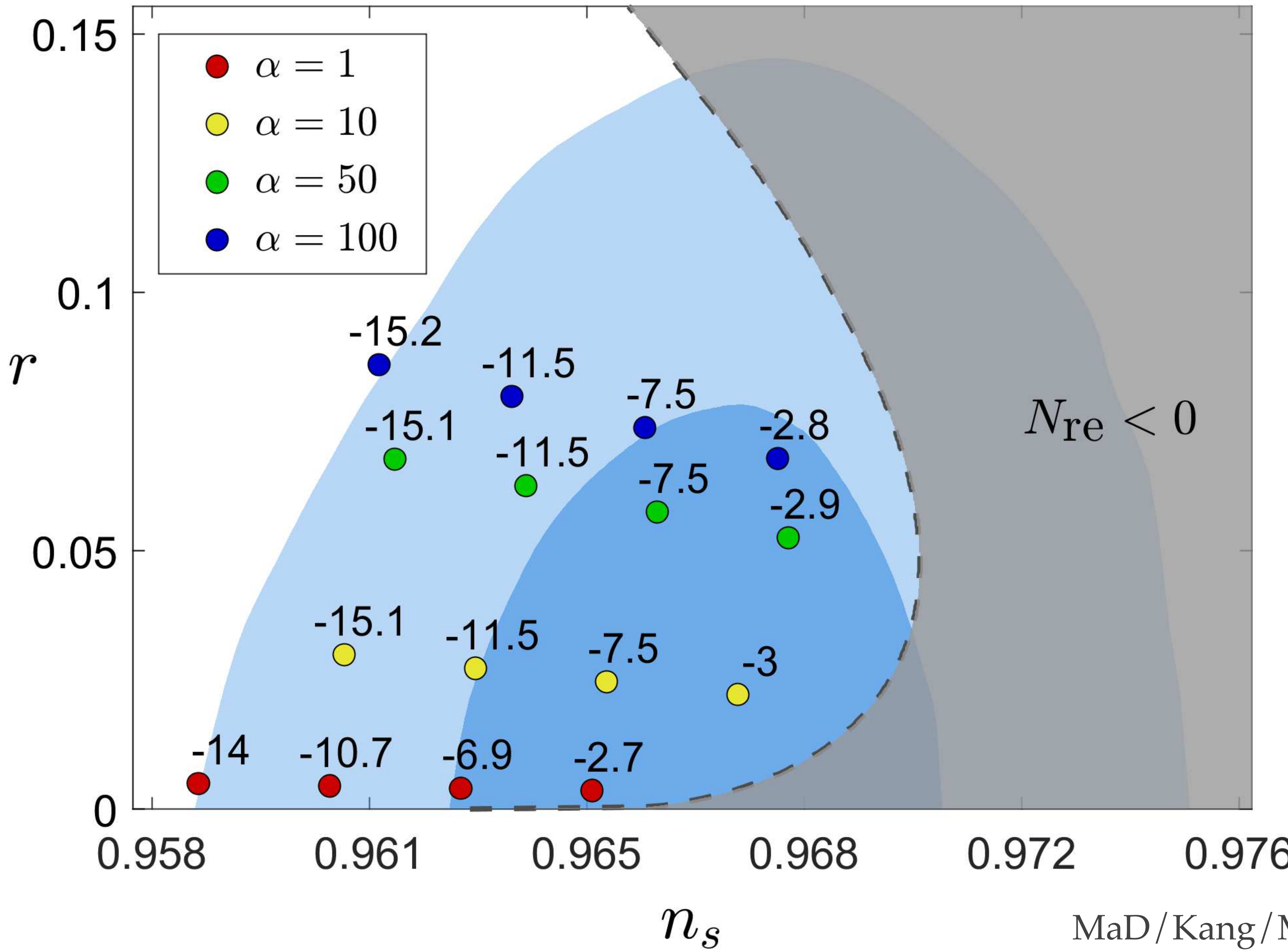
# Results for hφχ³ Interaction

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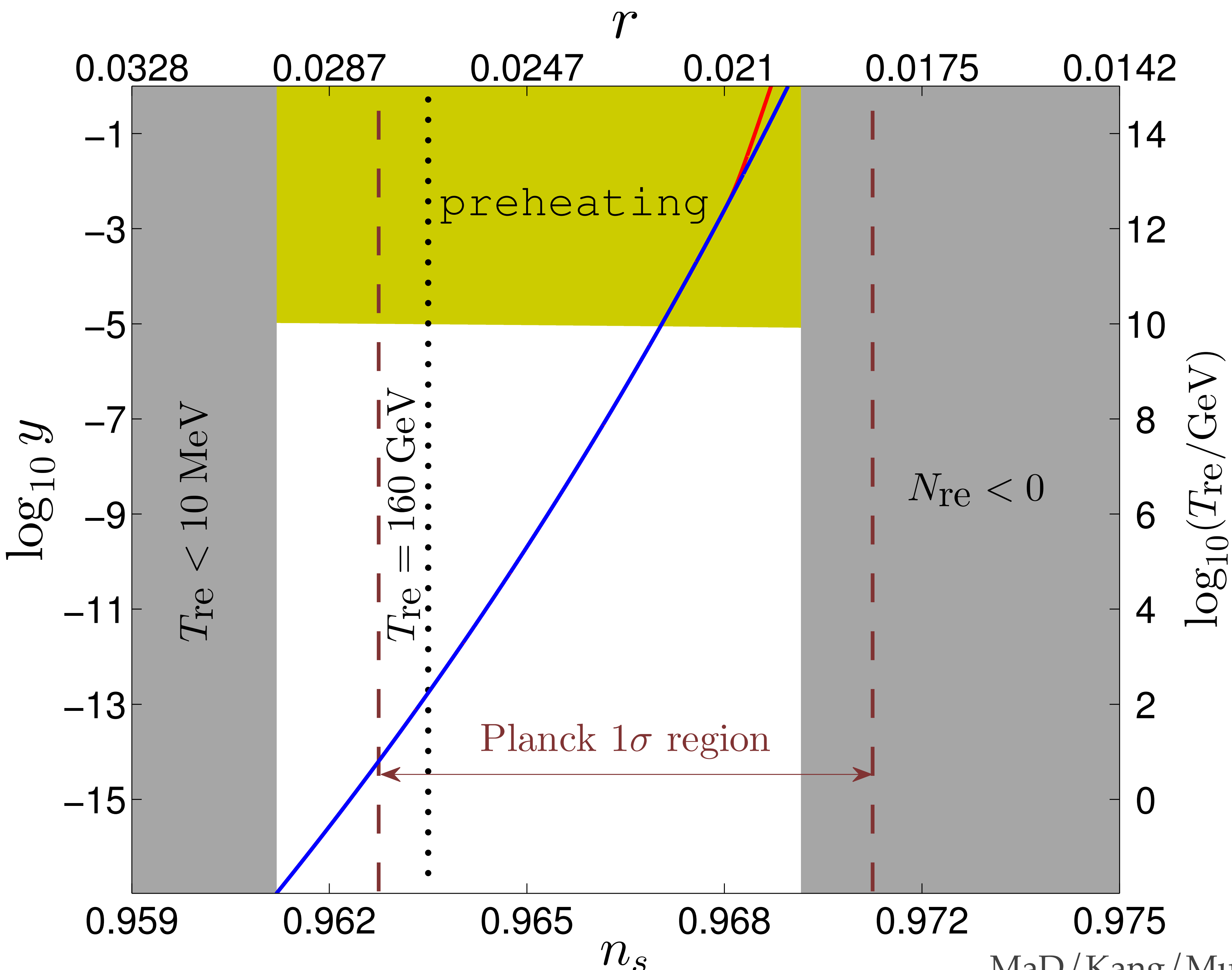
# Results for hφχ³ Interaction

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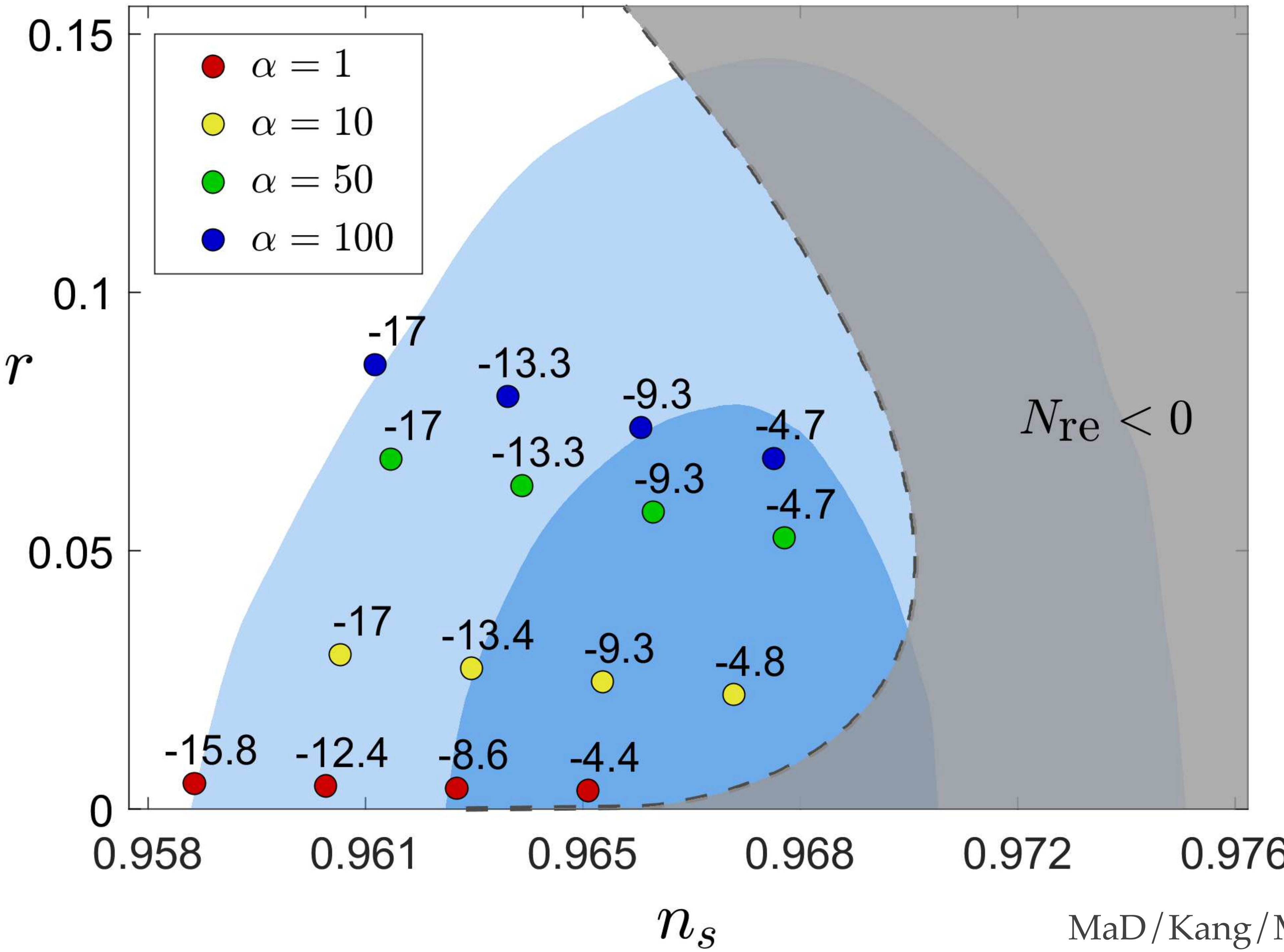
# Results for Yukawa Interaction

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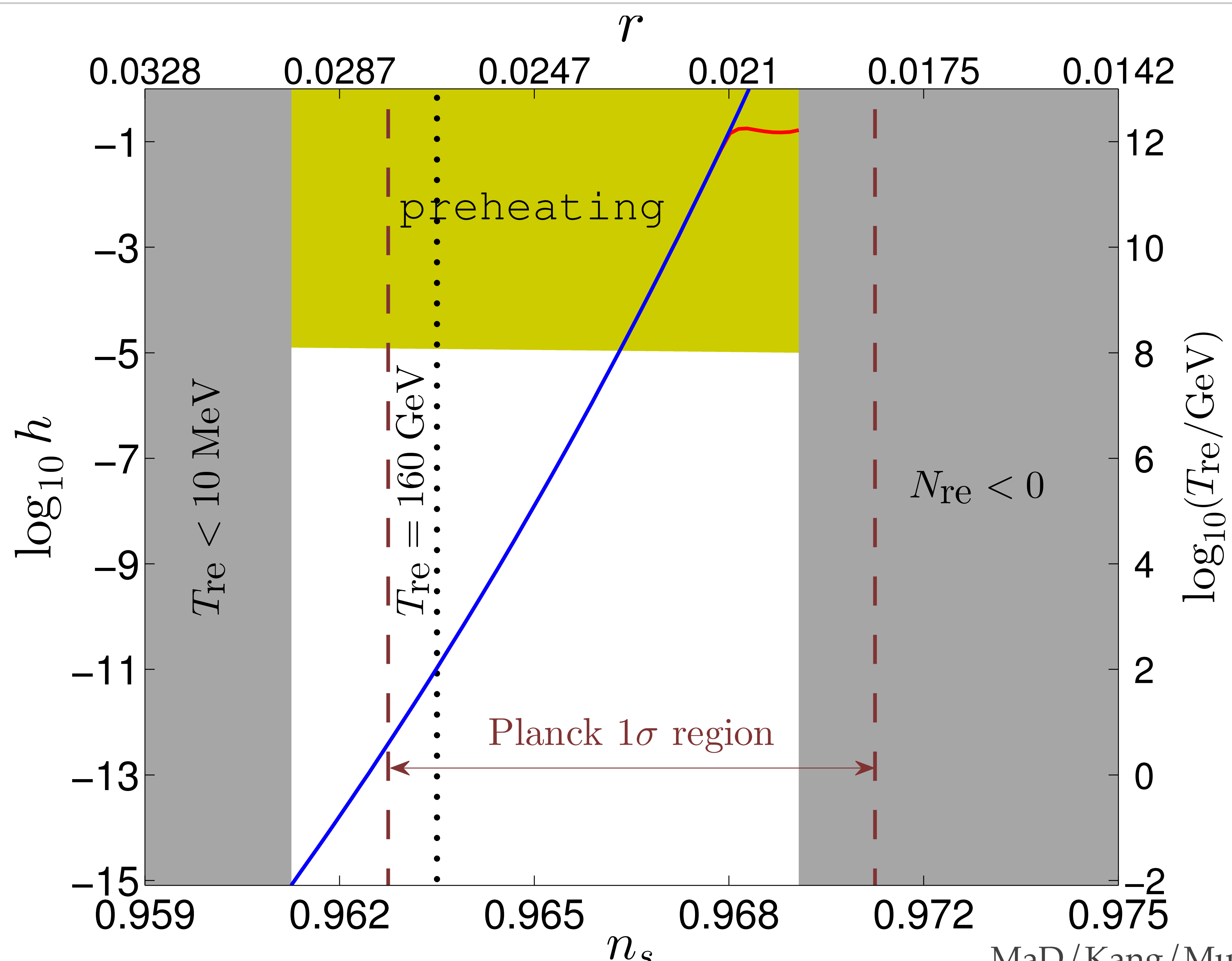
# Results for Yukawa Interaction




MaD/Kang/Mun 1708.01197

# Summary II

- **CMB data constrains inflaton decay rate $\Gamma$ in a given inflation model**

- **For couplings linear in $\phi$: constraint on $\Gamma$ can be translated in constraint on inflaton coupling if**
  *coupling constant* $< m_\phi / M_{pl}$

- **We have illustrated this for $\alpha$ attractor models**

- **Currently constraints are weak, but will improve with better measurements of the spectral index**