

Back to the future:

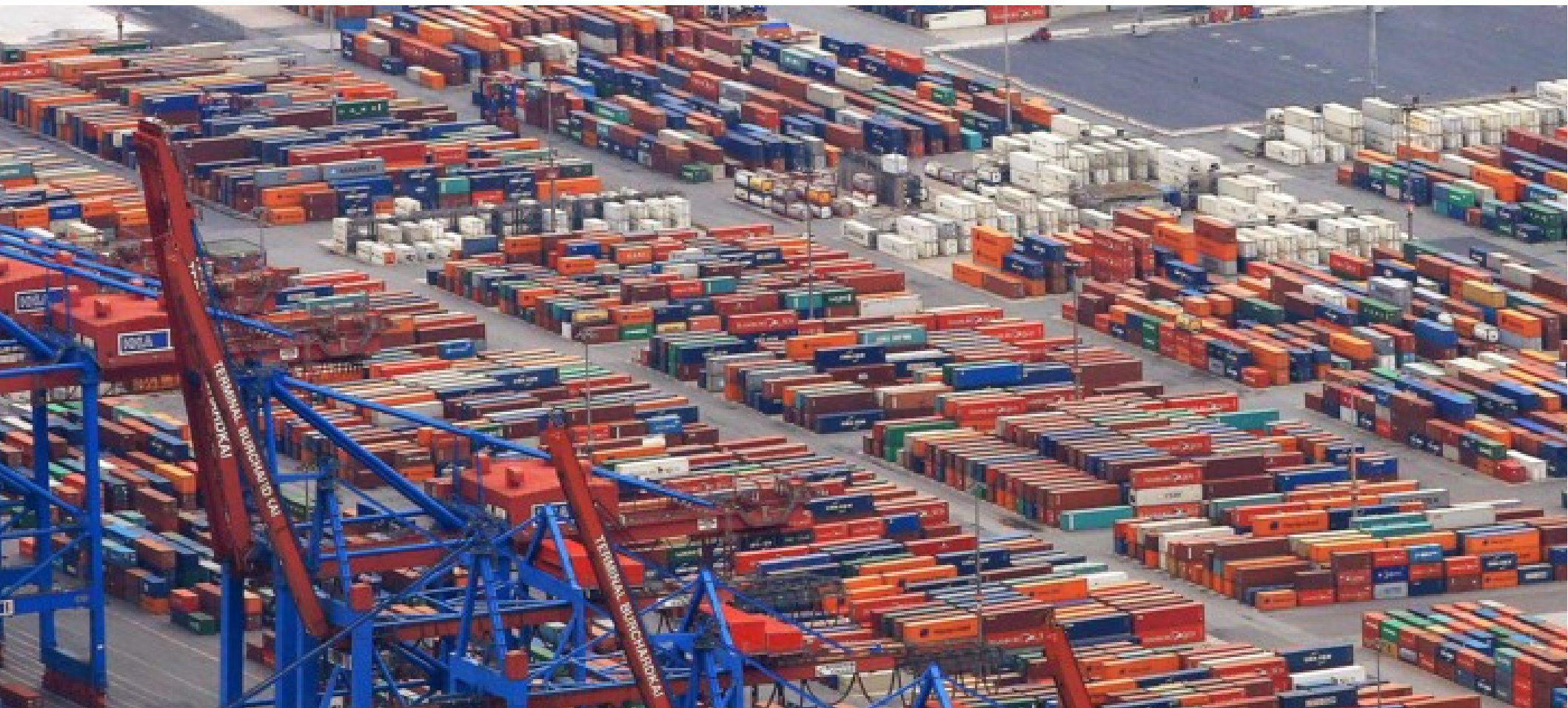
**NNLO QCD fits to HERA jets**

ZEUS-prel-19-001

H1prelim-19-041

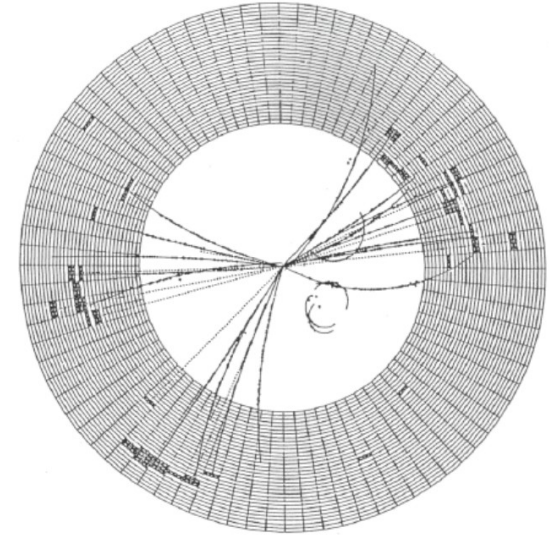
H1prelim-19-013

K. Wichmann on behalf of H1 and ZEUS Collaborations



# 40 years of jet production @ DESY

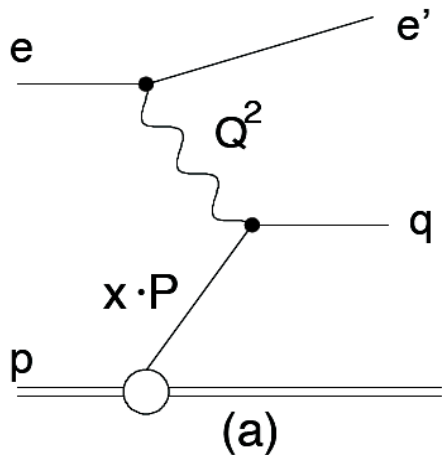
## Jets at PETRA, 1979



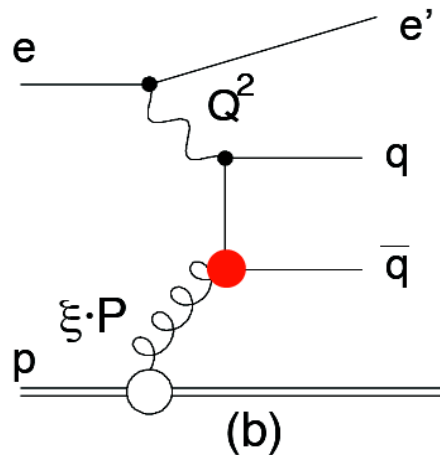
At HERA direct information on gluon distribution and  $\alpha_s$  comes from jet production

→ Possible simultaneous determination of parton densities and  $\alpha_s$

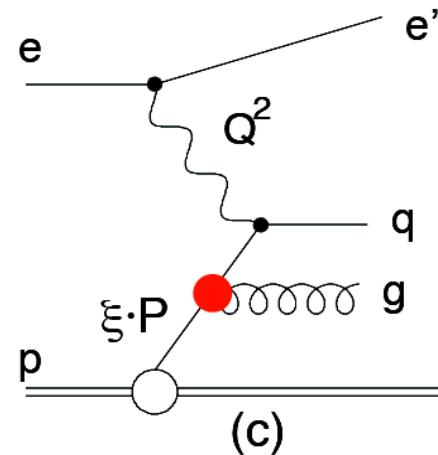
## Jets at HERA



elweak coupling



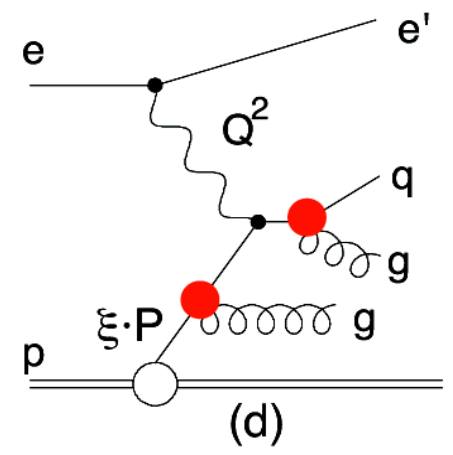
$\propto \alpha_s$



$\propto \alpha_s$

dijets

\*\*\* EUMS (GeV) \*\*\* P10T 35.788 PTRANS 29.964 PLONG -15.788 CHARGE -2  
TOTAL CLUSTER ENERGY 15.169 PHOTON ENERGY 4.693 NR OF PHOTONS 11



$\propto \alpha_s^2$

trijets

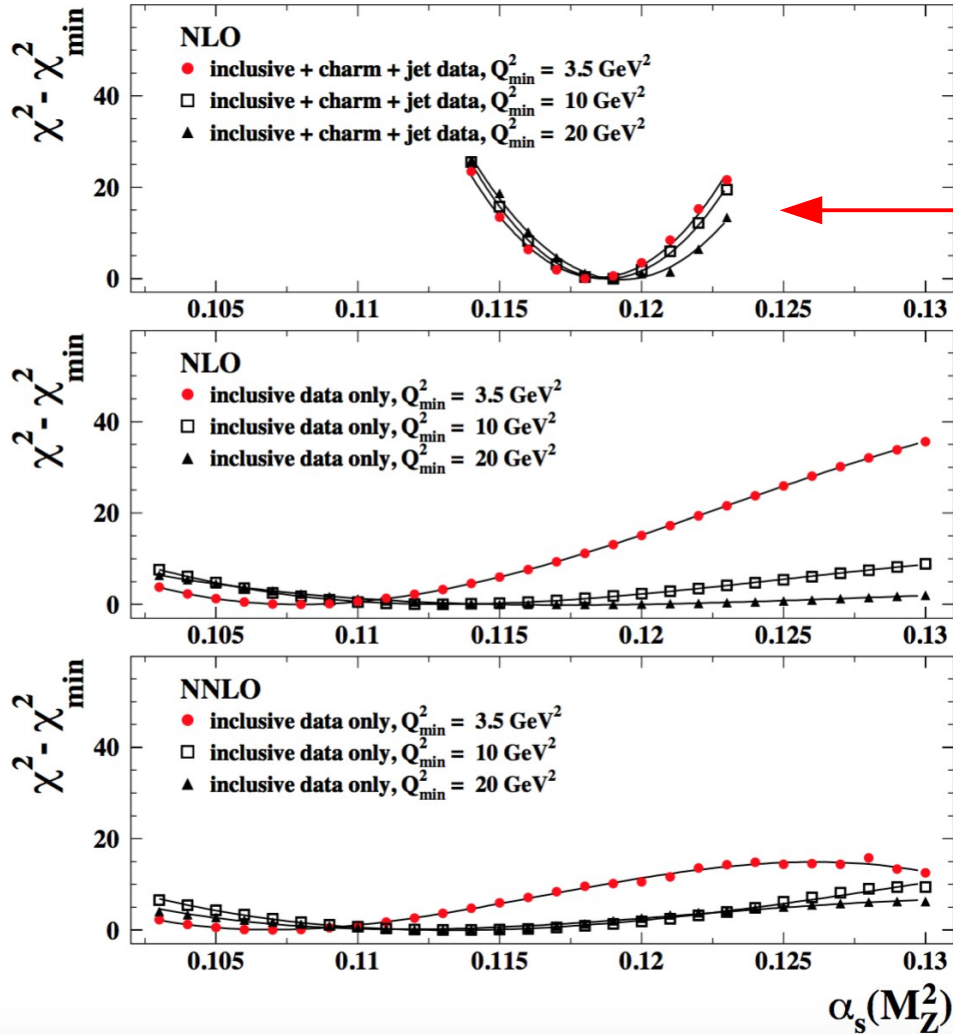
# New NNLO calculations for HERA jet production available now

- Possible simultaneous determination of parton densities and  $\alpha_s$  at NNLO
- Possible simultaneous fit to diffractive inclusive and dijet data and determination of NNLO diffractive parton densities

# Simultaneous determination of parton densities and $\alpha_s$ at NNLO

# Why study jets in DIS @ HERA?

## H1 and ZEUS



- HERA inclusive data carry little information on  $\alpha_s$
- Jet data sensitive to  $\alpha_s$
- So far NLO available

New NNLO calculations for HERA ep jet production available now

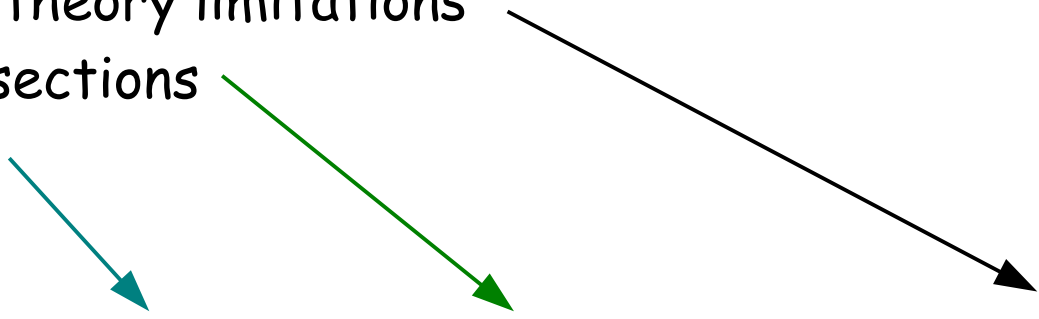
- Implemented in FastNLO and APPLEGRID → fast cross section calculation possible

→ Possible simultaneous determination of parton densities and  $\alpha_s$  at NNLO



# HERA jet data used in PDF fit

- Inclusive jets and **dijets**
- Some data points excluded due theory limitations
- Absolute and **normalised** cross sections
- **Low- $Q^2$**  and high- $Q^2$  production
- HERAI and HERAII



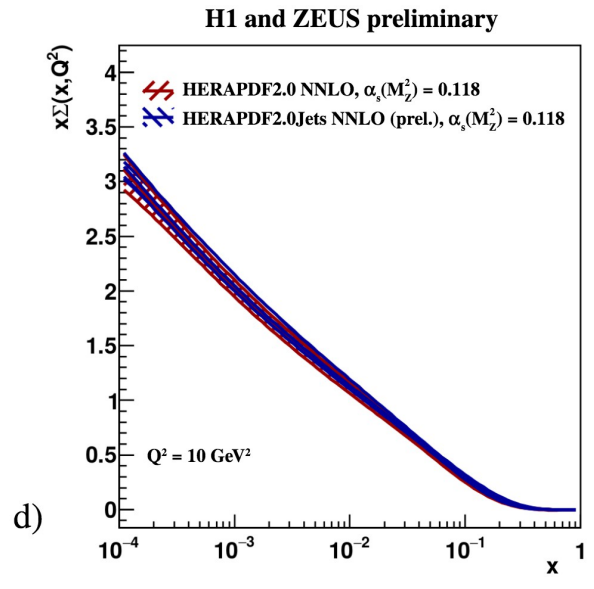
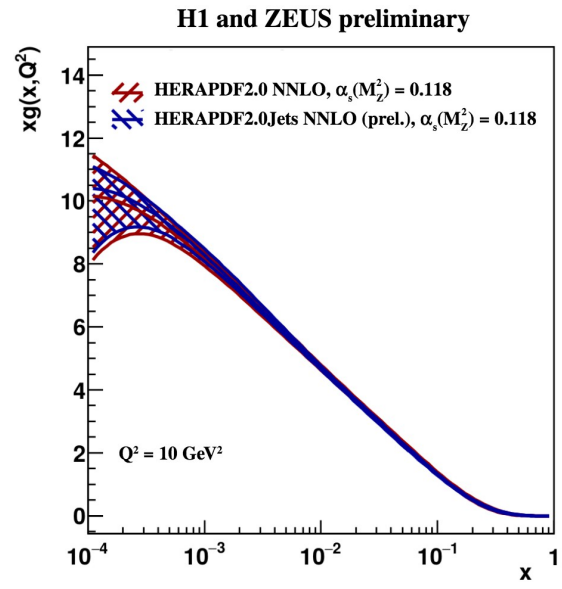
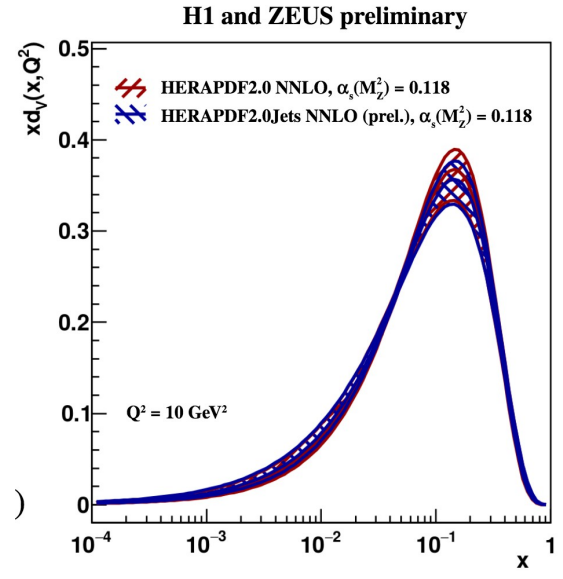
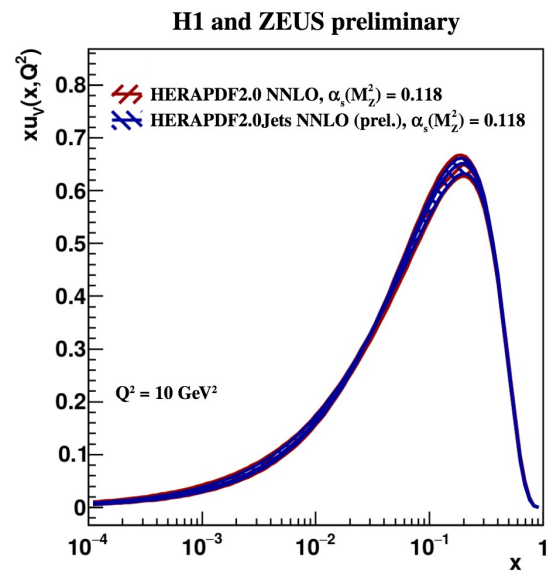
Data Set	taken		$Q^2$ [GeV <sup>2</sup> ] range		$\mathcal{L}$ pb <sup>-1</sup>	$e^+ / e^-$	norma- lised	all points	used points
	from	to	from	to					
H1 HERA I <b>normalised</b> jets	1999	2000	150	15000	65.4	$e^+ p$	yes	24	24
H1 HERA I jets at <b>low <math>Q^2</math></b>	1999	2000	5	100	43.5	$e^+ p$	no	28	16
H1 <b>normalised</b> inclusive jets at high $Q^2$	2003	2007	150	15000	351	$e^+ p / e^- p$	yes	30	24
H1 <b>normalised</b> <b>dijets</b> at high $Q^2$	2003	2007	150	15000	351	$e^+ p / e^- p$	yes	24	24
H1 <b>normalised</b> inclusive jets at <b>low <math>Q^2</math></b>	2005	2007	5.5	80	290	$e^+ p / e^- p$	yes	48	32
H1 <b>normalised</b> <b>dijets</b> at <b>low <math>Q^2</math></b>	2005	2007	5.5	80	290	$e^+ p / e^- p$	yes	48	32
ZEUS inclusive jets	1996	1997	125	10000	38.6	$e^+ p$	no	30	30
ZEUS <b>dijets</b>	1998 – 2000 & 2004 – 2007		125	20000	374	$e^+ p / e^- p$	no	22	16

- Possibilities for PDF fit with jet data
  - With fixed  $\alpha_s$
  - With free  $\alpha_s$  or doing  $\alpha_s$  scan  $\rightarrow$   **$\alpha_s$  value**



# Comparison to HERAPDF2.0

HERAPDF2.0Jets NNLO (prel.),  $\alpha_s(M_Z) = 0.118$





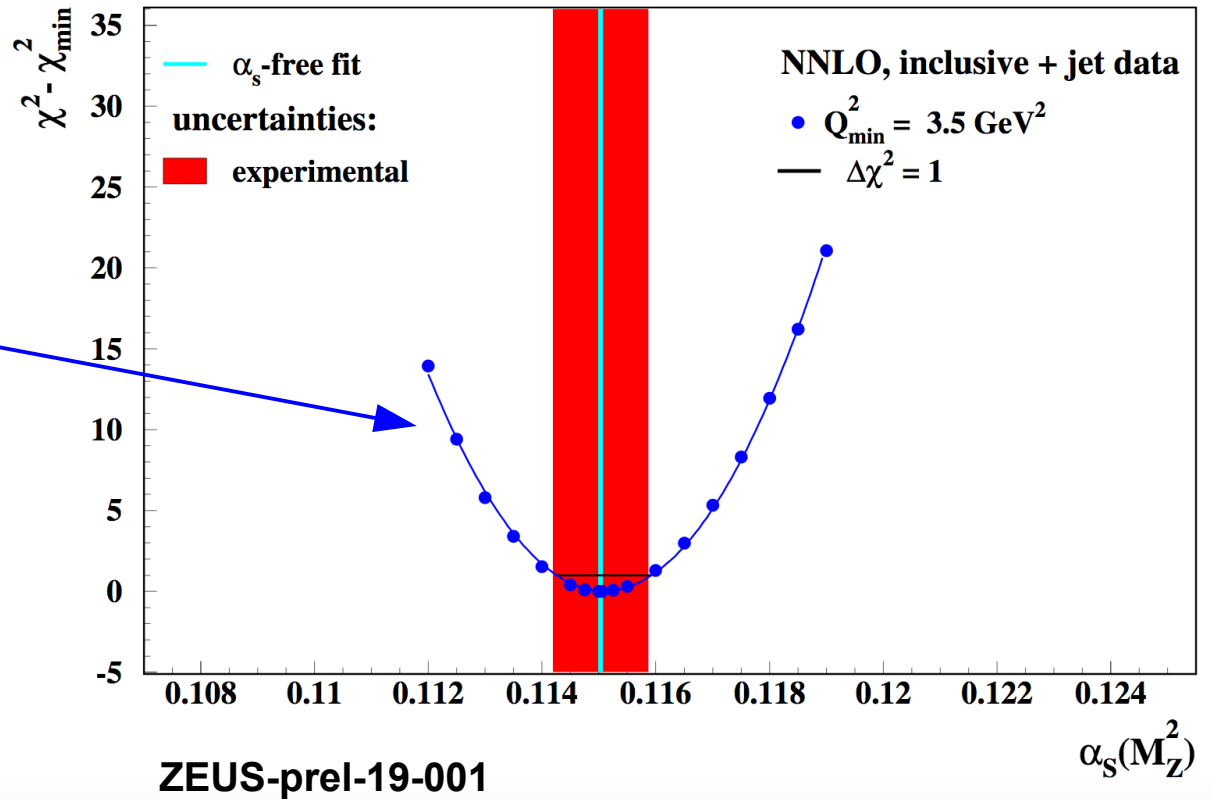
# $\alpha_s$ @ NNLO from HERA jets

H1 and ZEUS preliminary

- Two ways of estimating  $\alpha_s$  @NNLO using HERA jet data

- $\alpha_s$ -scan
- simultaneous fit of PDFs and  $\alpha_s$

- Both methods give the same result



$$\alpha_s(M_Z^2) = 0.1150 \pm 0.0008(\text{exp})$$

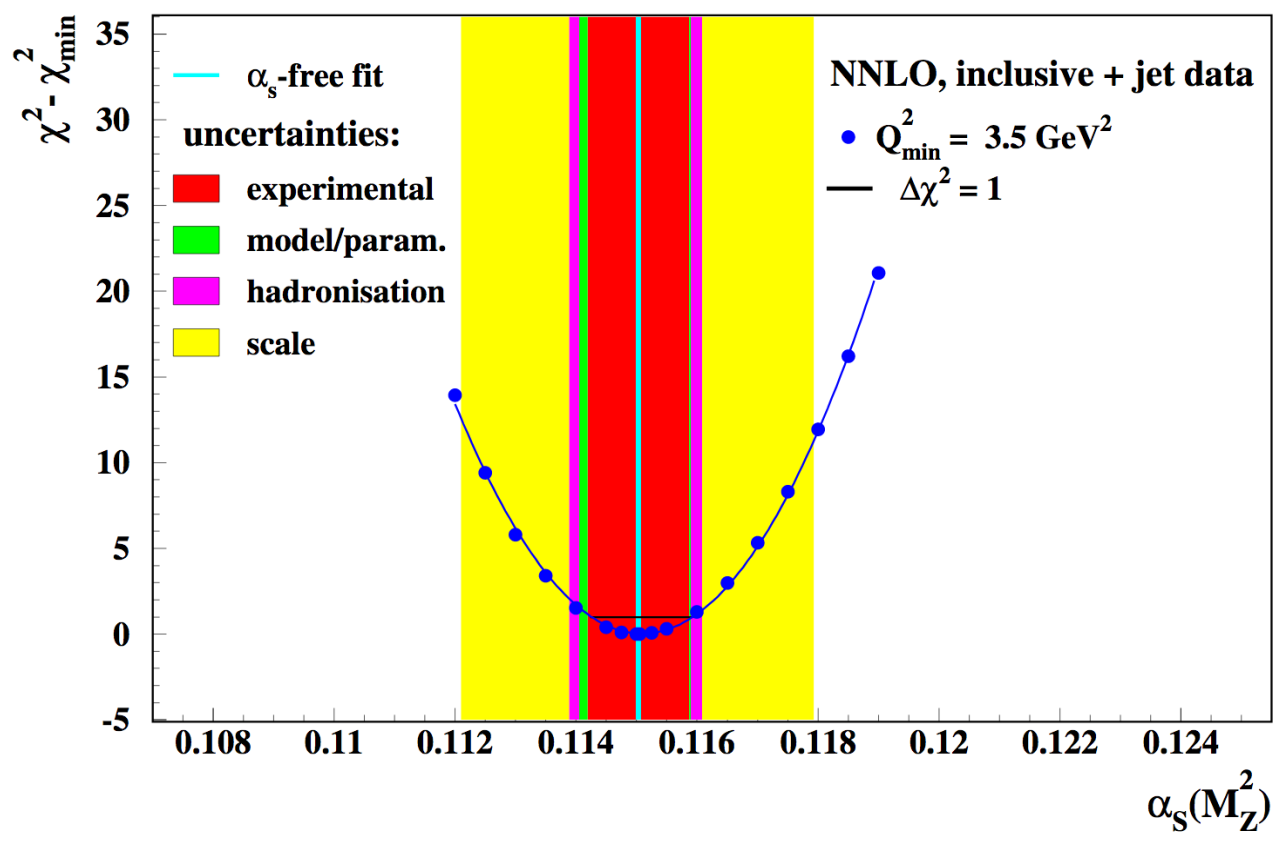




# Full uncertainties

- Experimental, model, parametrisation and hadronisation uncertainties
- In fits with free  $\alpha_s(M_Z)$  **scale uncertainty** important
  - factorisation and renormalisation scales varied both separately and simultaneously by a factor of two and taking maximal positive and negative deviations (assumed to be 50% correlated and 50% uncorrelated)

H1 and ZEUS preliminary





# Comparison to other HERAPDF2.0 fits

- NNLO fits with and without jets of similar quality
  - $\chi^2/\text{d.o.f} = 1.203$  for free  $\alpha_s(M_Z)$  fit with 1328 degrees of freedom
  - $\chi^2/\text{d.o.f} = 1.205$  for HERAPDF2.0NNLO with only 1131 degrees of freedom
- NLO and NNLO results for  $\alpha_s(M_Z)$  consistent within experimental uncertainties
  - **Scale uncertainties reduced**
    - as expected for NNLO calculations

## HERAPDF2.0Jets NNLO (prel.), free $\alpha_s(M_Z)$

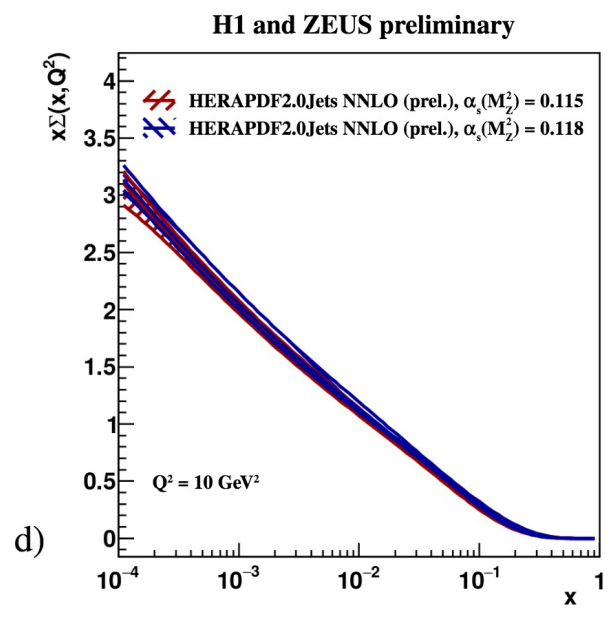
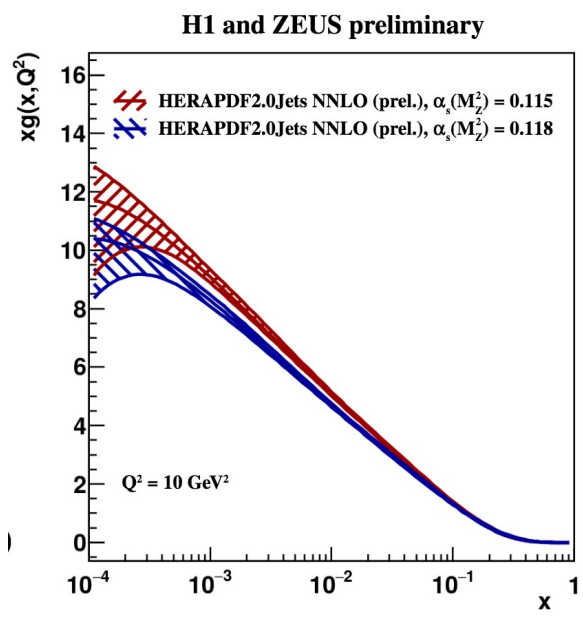
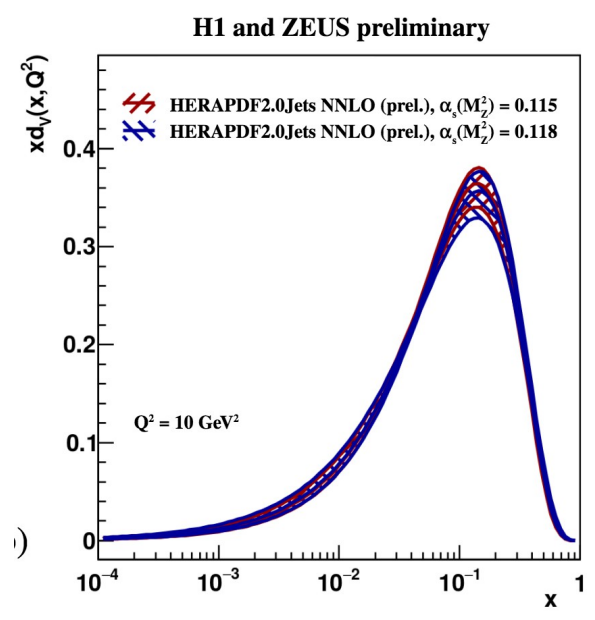
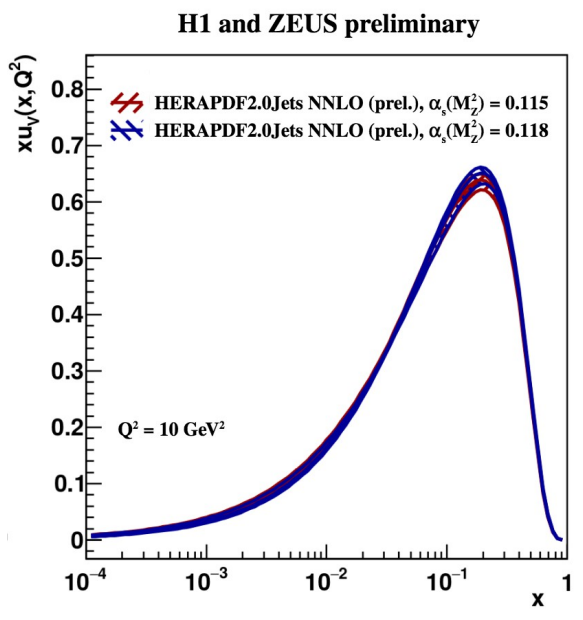
$$\alpha_s(M_Z^2) = 0.1150 \pm 0.0008(\text{exp})_{-0.0005}^{+0.0002}(\text{model/parameterisation}) \pm 0.0006(\text{hadronisation}) \pm 0.0027(\text{scale}) .$$

## HERAPDF2.0Jets NLO

$$\alpha_s(M_Z^2) = 0.1183 \pm 0.0009(\text{exp}) \pm 0.0005(\text{model/parameterisation}) \pm 0.0012(\text{hadronisation}) \begin{matrix} +0.0037 \\ -0.0030 \end{matrix}(\text{scale}) .$$



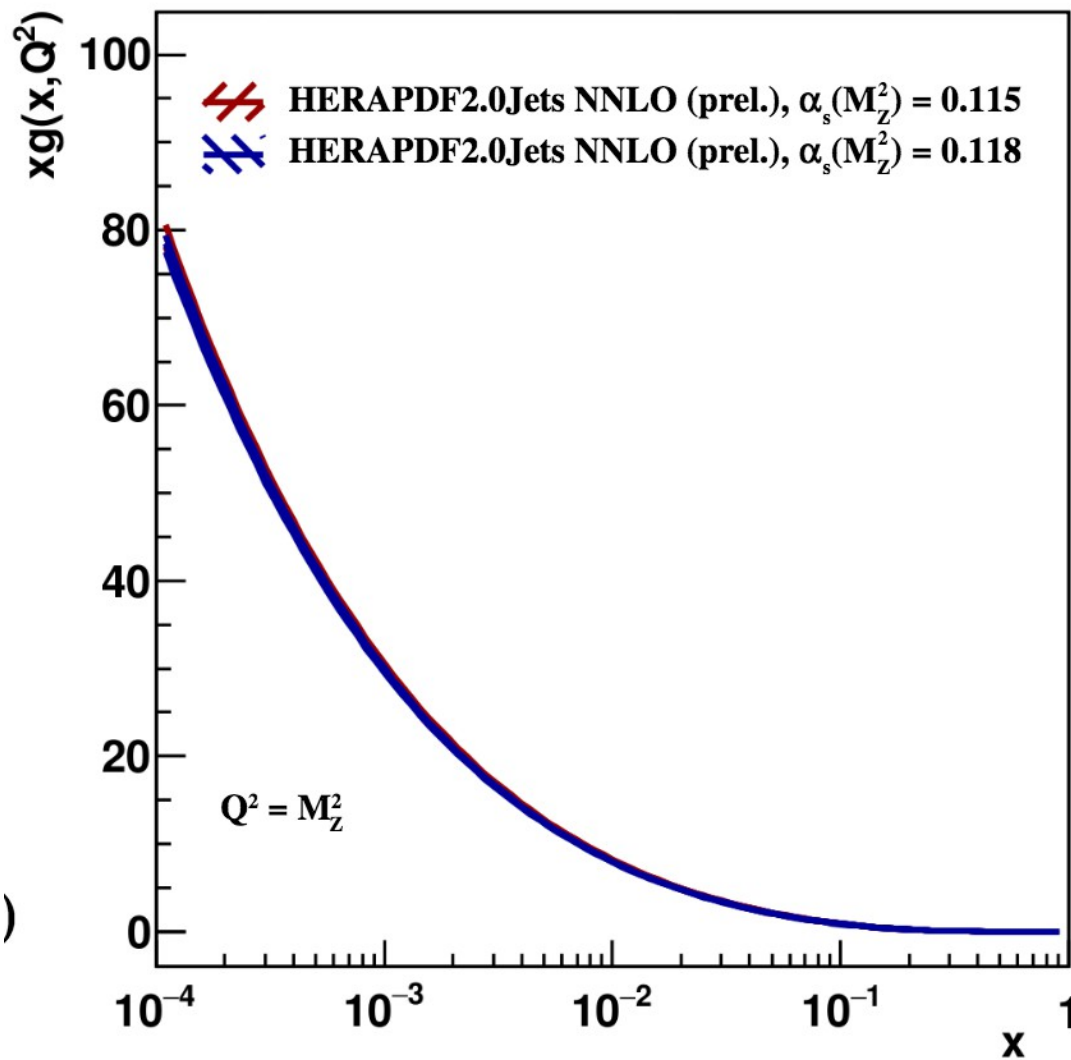
# Comparison between $\alpha_s = 0.115$ and $0.118$



# Gluon at scale of $M_Z^2$ very similar



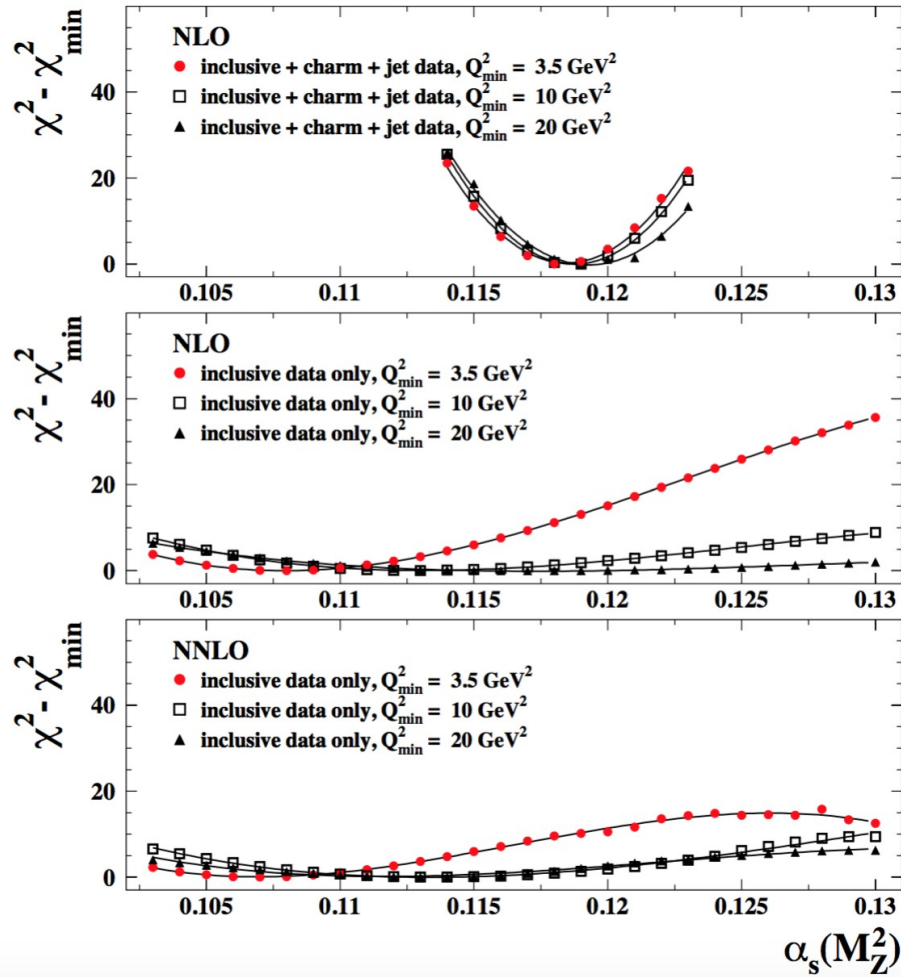
H1 and ZEUS preliminary



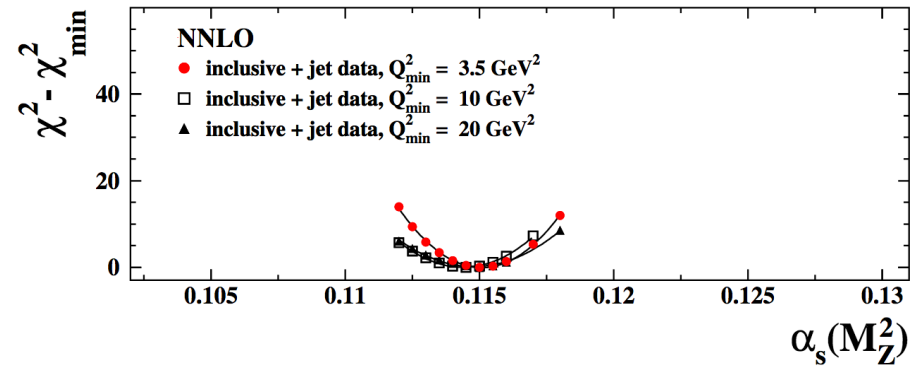


# Finally a full picture of jets@HERA

## H1 and ZEUS



## H1 and ZEUS preliminary



- Just as at NLO the jet data constrain  $\alpha_s(M_Z)$
- Similar level of accuracy at NNLO and NLO
- $\alpha_s(M_Z)$  clearly lower at NNLO

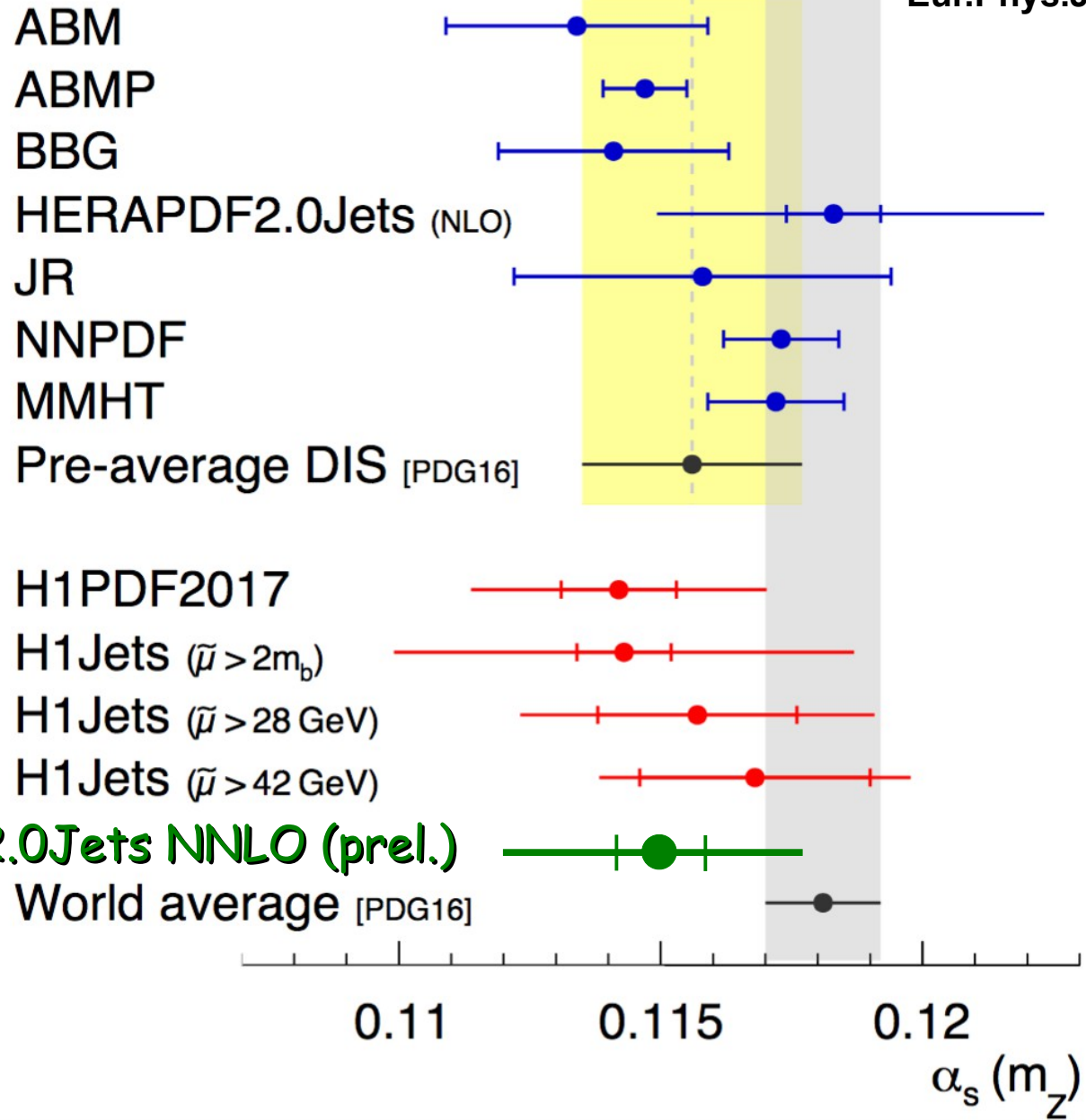


# Comparison to other NNLO results

$\alpha_s$  determinations in NNLO

H1 and NNLOJET

Eur.Phys.J.C77 (2017), 791

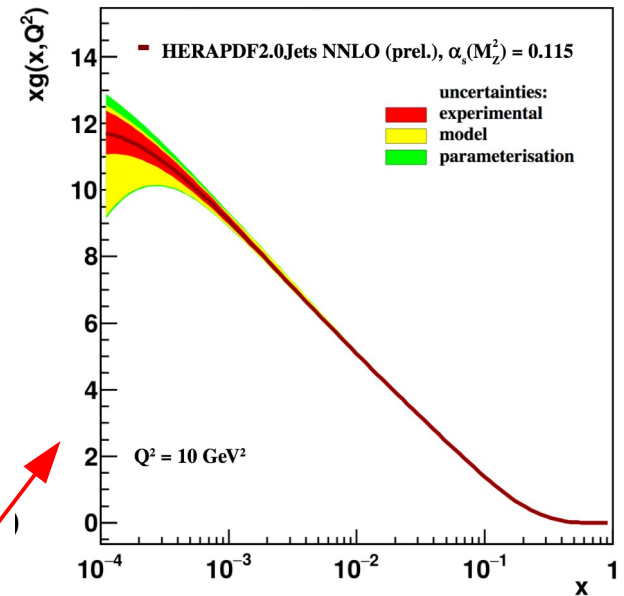


HERAPDF2.0Jets NNLO (prel.)



# Summary & conclusions

- HERAPDF2.0 family completed  
→ NNLO fit including jet data performed
- Two new PDF sets  
→ HERAPDF2.0Jets NNLO  $\alpha_s(M_Z) = 0.118$  → PDG  
→ HERAPDF2.0Jets NNLO (prel.),  $\alpha_s(M_Z) = 0.115$  → value favoured by our fit
- Jet data allow us to constrain  $\alpha_s(M_Z)$



$$\alpha_s(M_Z^2) = 0.1150 \pm 0.0008(\text{exp})_{-0.0005}^{+0.0002}(\text{model/parameterisation}) \pm 0.0006(\text{hadronisation}) \pm 0.0027(\text{scale}) .$$

- Compared to NLO result  $\alpha_s(M_Z^2) = 0.1183 \pm 0.0009(\text{exp}) \pm 0.0005(\text{model/parameterisation}) \pm 0.0012(\text{hadronisation})_{-0.0030}^{+0.0037}(\text{scale}) .$

Systematic shift downwards at NNLO and reduction of scale uncertainty

H1prelim-19-013  
April 2019

**A determination of diffractive parton distribution functions from H1 inclusive diffractive deep-inelastic scattering data and H1 diffractive dijet cross section data in next-to-next-to-leading order QCD**

H1 Collaboration





# Diffractive production in ep

In diffractive events the beam proton stays intact or dissociates into low mass hadronic system Y

**DIS variables:**

$$Q^2 = -(k - k')^2 \quad y = \frac{p \cdot q}{p \cdot k}$$

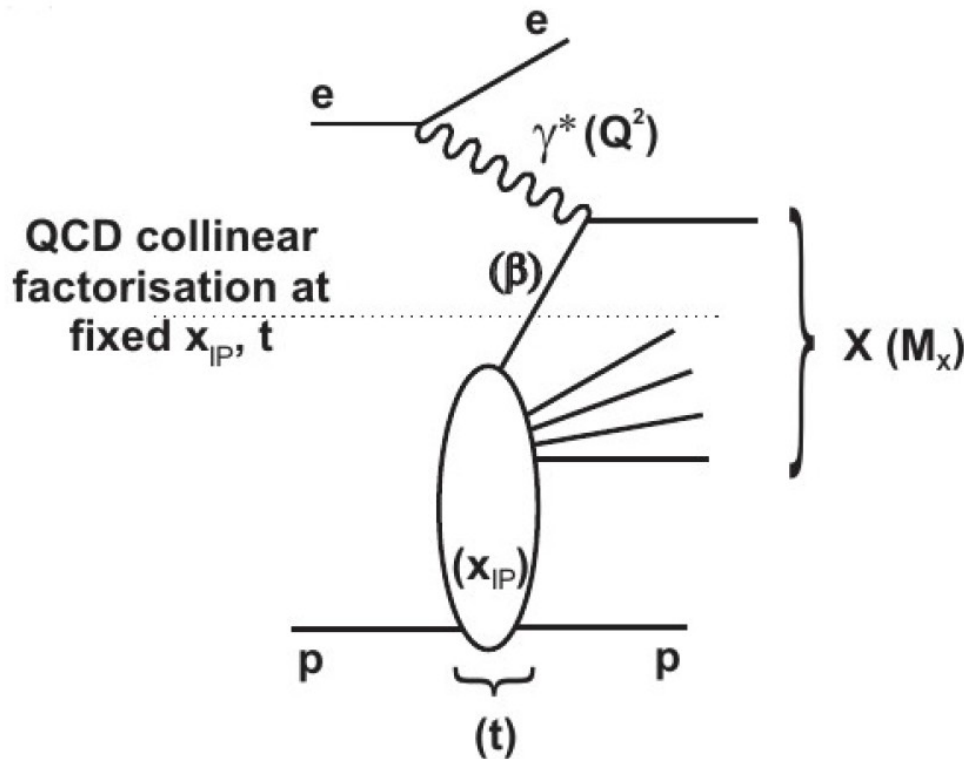
**Diffractive variables:**

$$x_{\mathbb{P}} = 1 - \frac{E'_p}{E_p} \quad t = (p - p')^2$$

$$\text{Mass: } M_X^2 = Q^2 \left( \frac{1}{\beta} - 1 \right)$$

At LO: The momentum fraction entering the hard subprocess with respect to the diffractive exchange

$$\beta = \frac{x_{Bj}}{x_{\mathbb{P}}} = \frac{Q^2}{syx_{\mathbb{P}}}$$



At HERA about 10% of low-x events are diffractive

# DDIS and diffractive jet data in DPDF fit

## Inclusive DDIS data:

Data set	$\sqrt{s}$	int. $\mathcal{L}$	DIS kinematic
[ref.]	[GeV]	[ $\text{pb}^{-1}$ ]	range
H1comb-LRG	319	336.6	$8.5 < Q^2 < 1600 \text{ GeV}^2$
H1-LowE-252	252	5.2	$8.5 < Q^2 < 44 \text{ GeV}^2$
H1-LowE-225	225	8.5	$8.5 < Q^2 < 44 \text{ GeV}^2$

*~40 times higher luminosity*

**Eur.Phys.J. C72 (2012) 2074**  
**[arXiv:1203.4495]**

**+ data at lower energies**  
**225, 252 GeV**

**Used first time in DPDF**

## The jet data:

New data sample		
2005-2007	920 + 27.6	290 $\text{pb}^{-1}$
Previously published		
1999-2000	920 + 27.5	51.5 $\text{pb}^{-1}$

*~6 times higher luminosity*

**JHEP 1503 (2015) 092**  
**[arXiv:1412.0928]:**

**With proper treatment of correlations between bins**

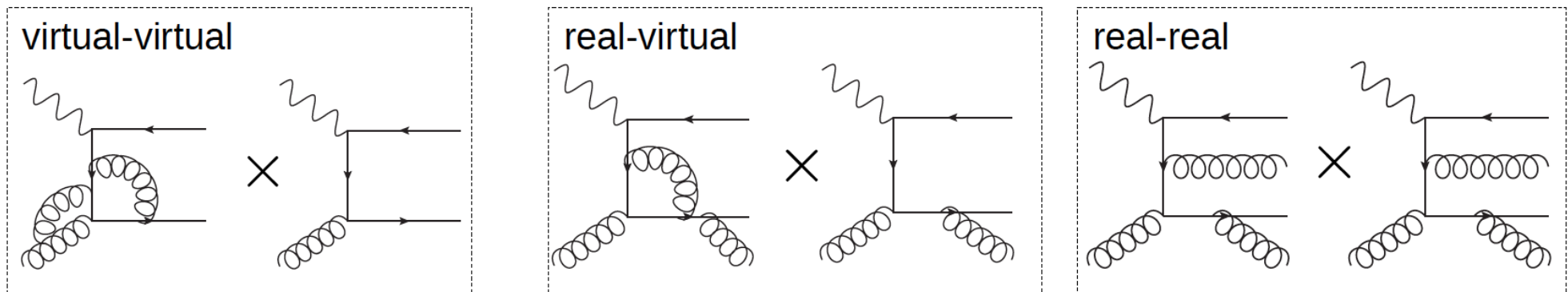
**Double differential in  $Q^2$  and jet  $p_T$**

# Details of QCD analysis

## Theory

- NNLO accuracy for both inclusive and jet production
- Using FONLL-C GM-VFNS (by APFEL) for inclusive production,  
→ default QCD scale for inc. production:  $\mu_R^2 = \mu_F^2 = Q^2$
- Using NNLOJET (massless quarks) + fastNLO for dijets,  
→ default QCD scale for dijets:  $\mu_R^2 = \mu_F^2 = Q^2 + \langle p_T^{*jets} \rangle^2$
- Scale unc. by simultaneous (for all processes)  
 $\mu_F = \mu_R \times 2, \times 0.5$  variation

Examples of  $\alpha_S^3$  diagrams contributing to dijet production



9

Fit performed using **Alpos** framework <https://indico.desy.de/indico/event/22011/session/7/contribution/23>

# Fitted parameters and model uncertainties

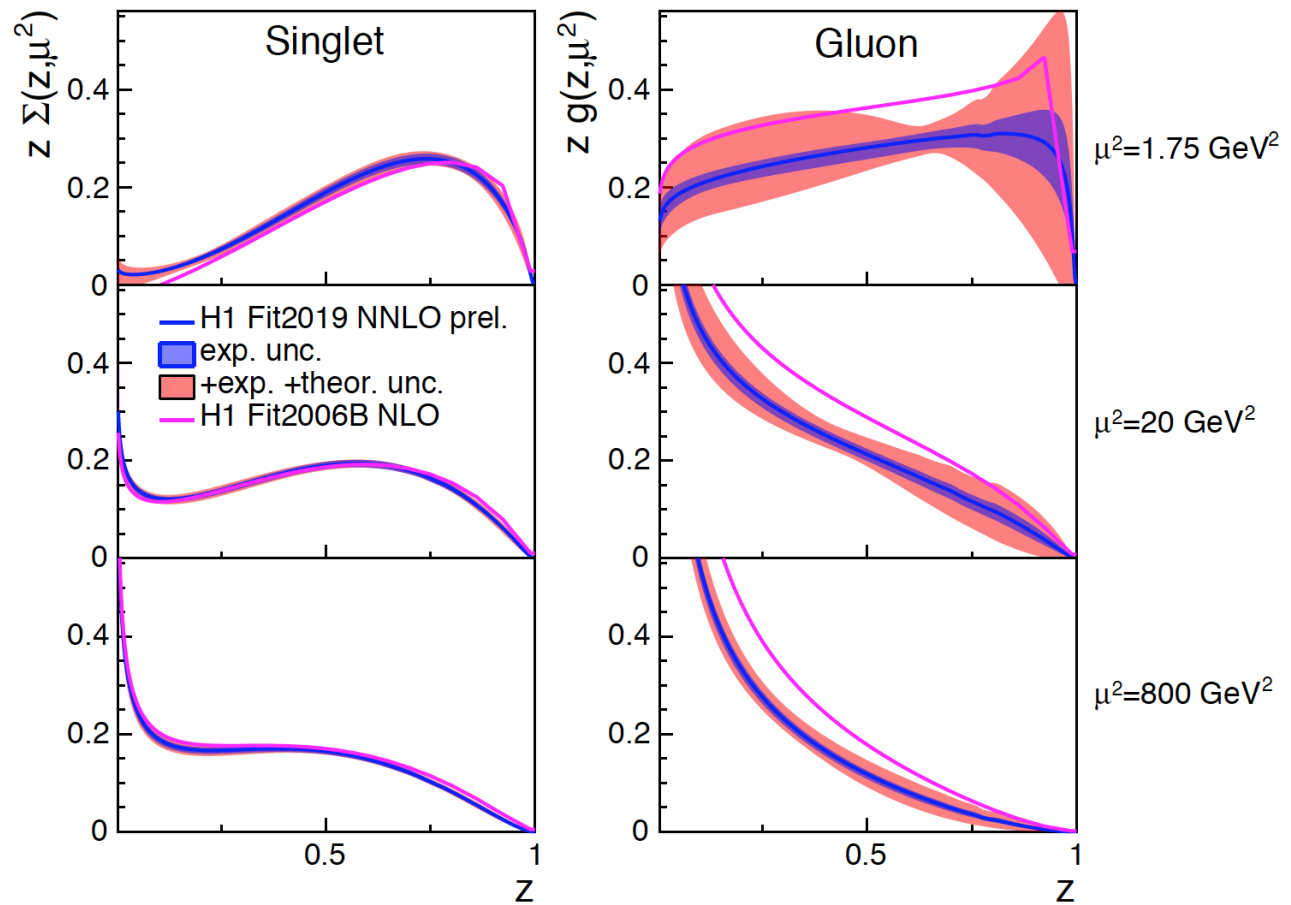
- Fixed params. mostly identical with H1 2006 & 2007 fits

	Parameter	Value	Source
Pomeron slope	$\alpha'_{\mathbb{P}}$	$0.04^{+0.08}_{-0.06} \text{ GeV}^{-2}$	H1 FPS HII [arXiv:1010.1476]
Pomeron B-slope	$B_{\mathbb{P}}^0$	$5.73^{+0.84}_{-0.93} \text{ GeV}^{-2}$	H1 FPS HII [arXiv:1010.1476]
Reggeon intercept	$\alpha_{\mathbb{R}}(0)$	$0.5 \pm 0.1$	H1 LRG HI [hep-ex/9708016]
Reggeon slope	$\alpha'_{\mathbb{R}}$	$0.3^{+0.6}_{-0.3} \text{ GeV}^{-2}$	H1 FPS HI [hep-ex/0606003]
Reggeon B-slope	$B_{\mathbb{R}}^0$	$1.6^{+0.4}_{-1.6} \text{ GeV}^{-2}$	H1 FPS HI [hep-ex/0606003]
charm mass	$m_c$	$1.4 \pm 0.2 \text{ GeV}$	PDG2004
bottom mass	$m_b$	$4.5 \pm 0.5 \text{ GeV}$	PDG2004
strong coupling	$\alpha_S(M_Z^2)$	$0.118 \pm 0.002$	PDG2004
starting scale of ev.	$\mu_0$	$1.15^{+0.24}_{-0.15} \text{ GeV}$	

- Parameters  $\alpha'_{\mathbb{P}}, B_{\mathbb{P}}$  (  $\alpha'_{\mathbb{P}}, B_{\mathbb{P}}$  ) strongly anti-correlated  
 $\rightarrow$  Varied simultaneously as (up,down) & (down,up)
- The QCD scale varied by a factor of 2  
 (dominant unc. together with  $\mu_0$  variation)
- 8 parameters fitted:** 6 of pomeron PDF +  $\alpha_{\mathbb{P}}(0)$  &  $n_{\mathbb{R}}$

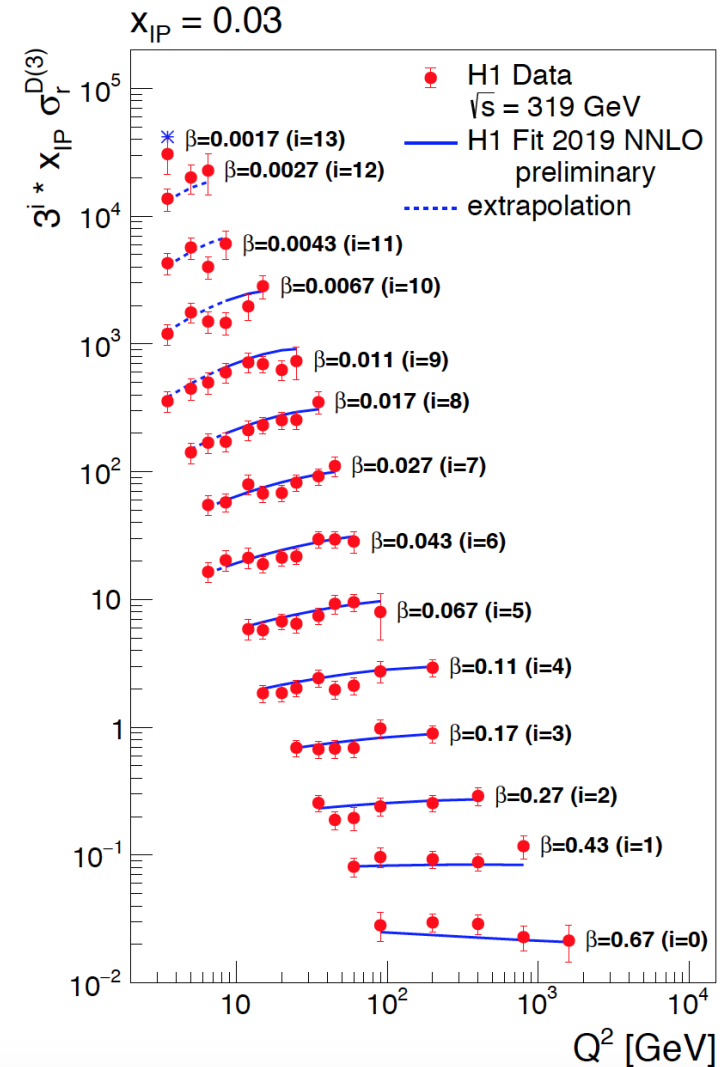
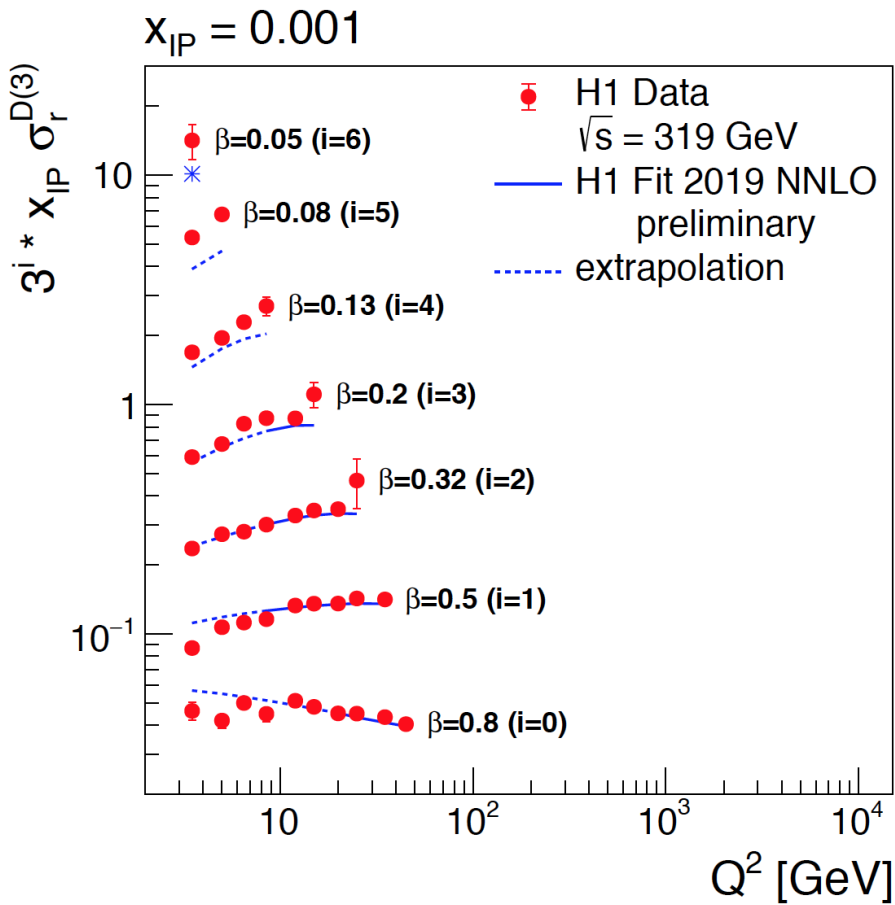
# DPDF Comparison (H1 Fit2019 NNLO vs H1 Fit2006B NLO)

- Old and new DPDFs in different QCD order & flavour scheme
- **comparison problematic**
- Quark single component comparable for both fits
- Gluon component of the newer fit  $\sim 25\%$  lower



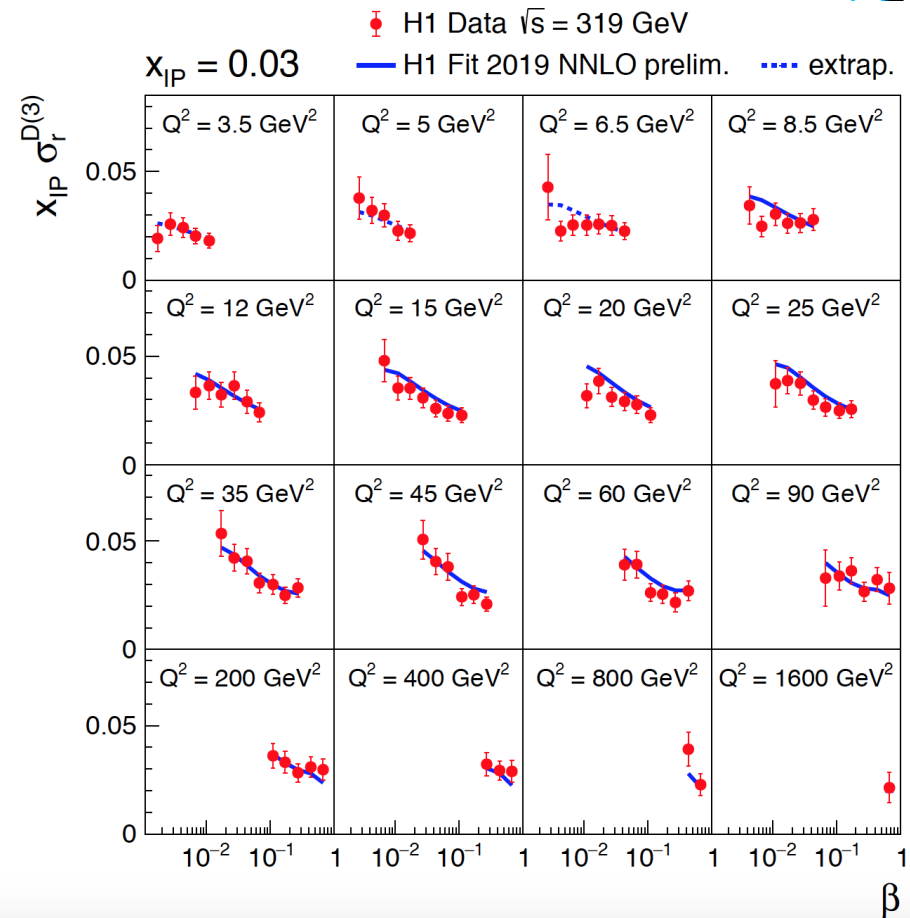
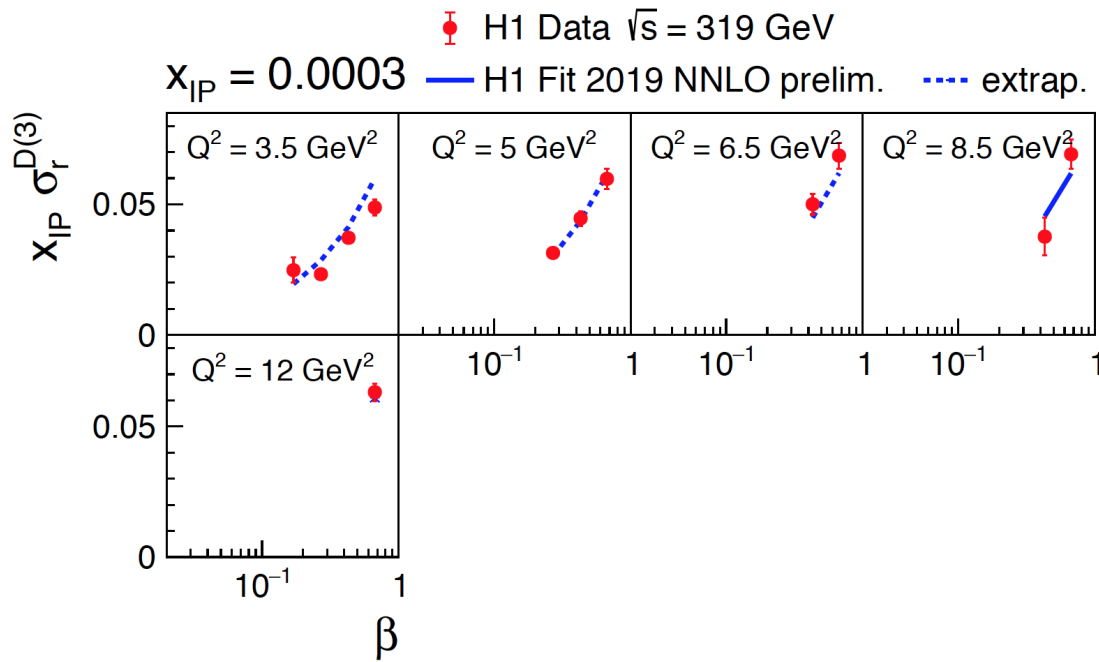
# Fitted data - Inclusive Sample - $Q^2$ dependence

- Good description of fitted combined H1 HERA I+HERA-II data for  $x_{IP} = 0.0003, 0.001, 0.003, 0.01, 0.03$
- Description in "extrapolated" region  $Q^2 < 8.5$  sometimes worse



# Fitted data - Inclusive Sample - $\beta$ dependence

- Data well described by NNLO QCD predictions over wide range of  $x_{IP}$  and  $\beta$ 
  - At LO  $\beta$  is the momentum fraction of parton entering hard process wrt pomeron (argument of DPDF)

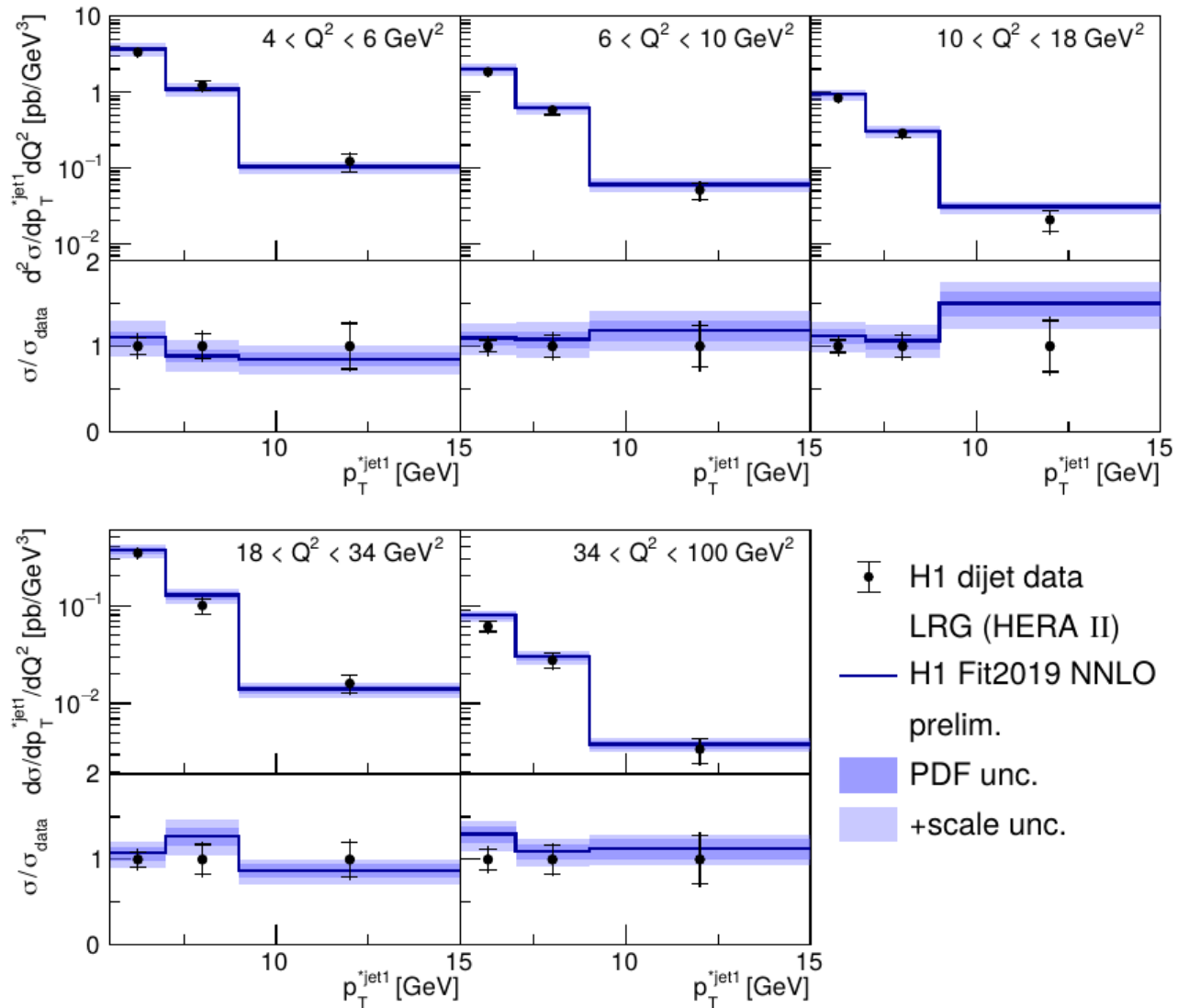


# Fitted data - diffractive jets

- Currently only the 2D  $p_T^{\text{jet}1}$  vs  $Q^2$  H1 HERA-II cross sections fitted
- Shown PDF & scale uncertainty of fit

- Good fit quality

$$\chi^2/\text{dof} = 12/15$$





# Testing QCD collinear factorisation

# Collinear QCD factorisation theorem in hard diffraction

- For diffractive events with a **hard scale** (e.g  $Q^2$  or jets  $p_T$ )
- Factorization of the diffractive cross section into **process independent DPDFs** and **partonic cross sections**

$$d\sigma(ep \rightarrow epX) = \sum_i f_i^D(x, Q^2, x_{IP}, t) \otimes d\sigma^{ie}(x, Q^2)$$

- For diffractive processes (including dijets) with high enough  $Q^2$  factorization proven by Collins within perturbative QCD, for low  $Q^2$  factorization breaking suggested

## Factorization of Hard Processes in QCD

John C. Collins (IIT, Chicago & SUNY, Stony Brook), Davison E. Soper (Oregon U.), George F. Sterman (SUNY, Stony Brook). May 30, 1989. 91 pp.  
Published in *Adv.Ser.Direct.High Energy Phys.* 5 (1989) 1-91  
ITP-SB-89-31

DOI: [10.1142/9789814503266\\_0001](https://doi.org/10.1142/9789814503266_0001)

e-Print: [hep-ph/0409313](https://arxiv.org/abs/hep-ph/0409313) | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[ADS Abstract Service](#)

[Detailed record](#) - [Cited by 812 records](#) 500+

## Proof of factorization for diffractive hard scattering

John C. Collins (Penn State U.). Sep 1997. 12 pp.  
Published in *Phys.Rev.* D57 (1998) 3051-3056, Erratum: *Phys.Rev.* D61 (2000) 019902  
PSU-TH-189

DOI: [10.1103/PhysRevD.57.3051](https://doi.org/10.1103/PhysRevD.57.3051), [10.1103/PhysRevD.61.019902](https://doi.org/10.1103/PhysRevD.61.019902)

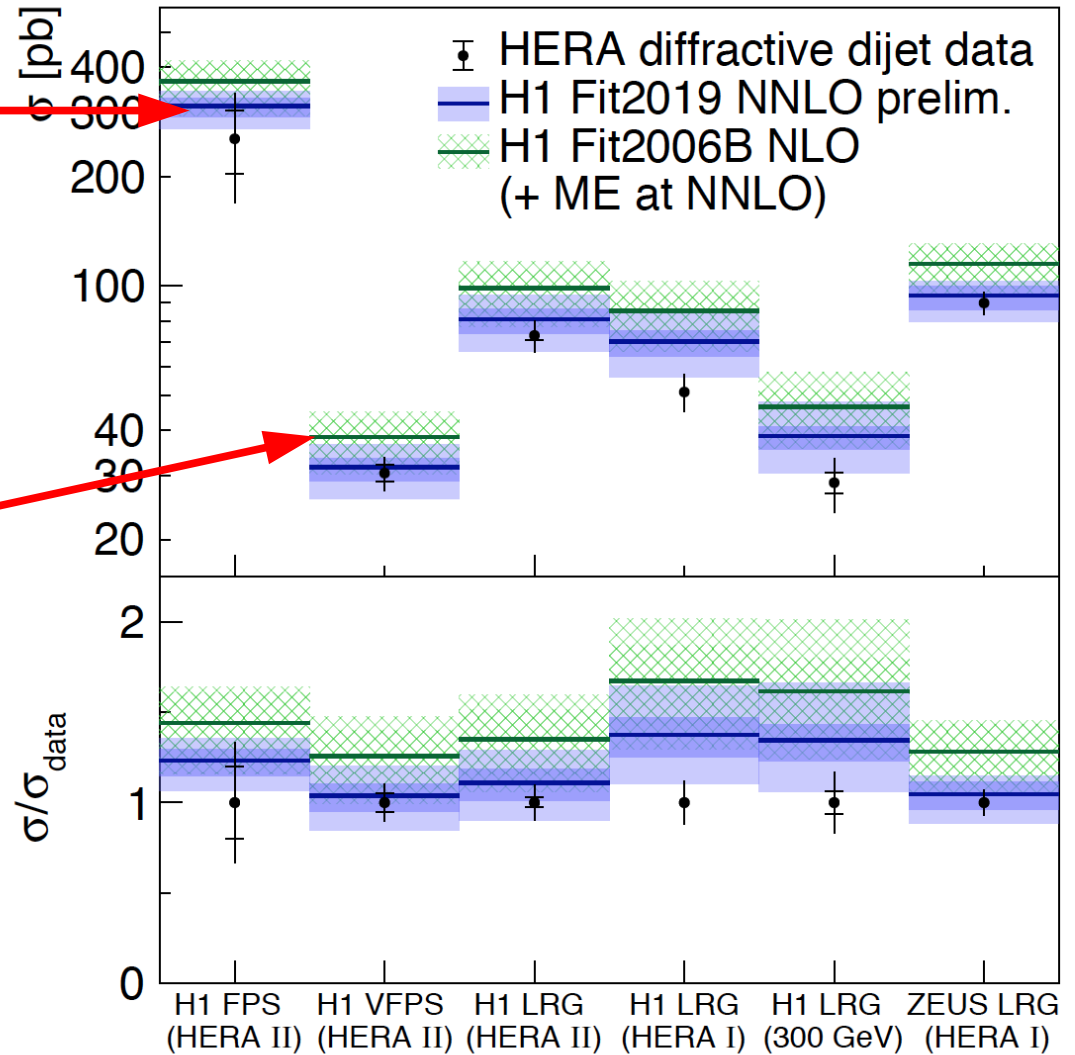
e-Print: [hep-ph/9709499](https://arxiv.org/abs/hep-ph/9709499) | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)  
[ADS Abstract Service](#); [OSTI.gov Server](#)

[Detailed record](#) - [Cited by 404 records](#) 250+

# NNLO compared to HERA dijets

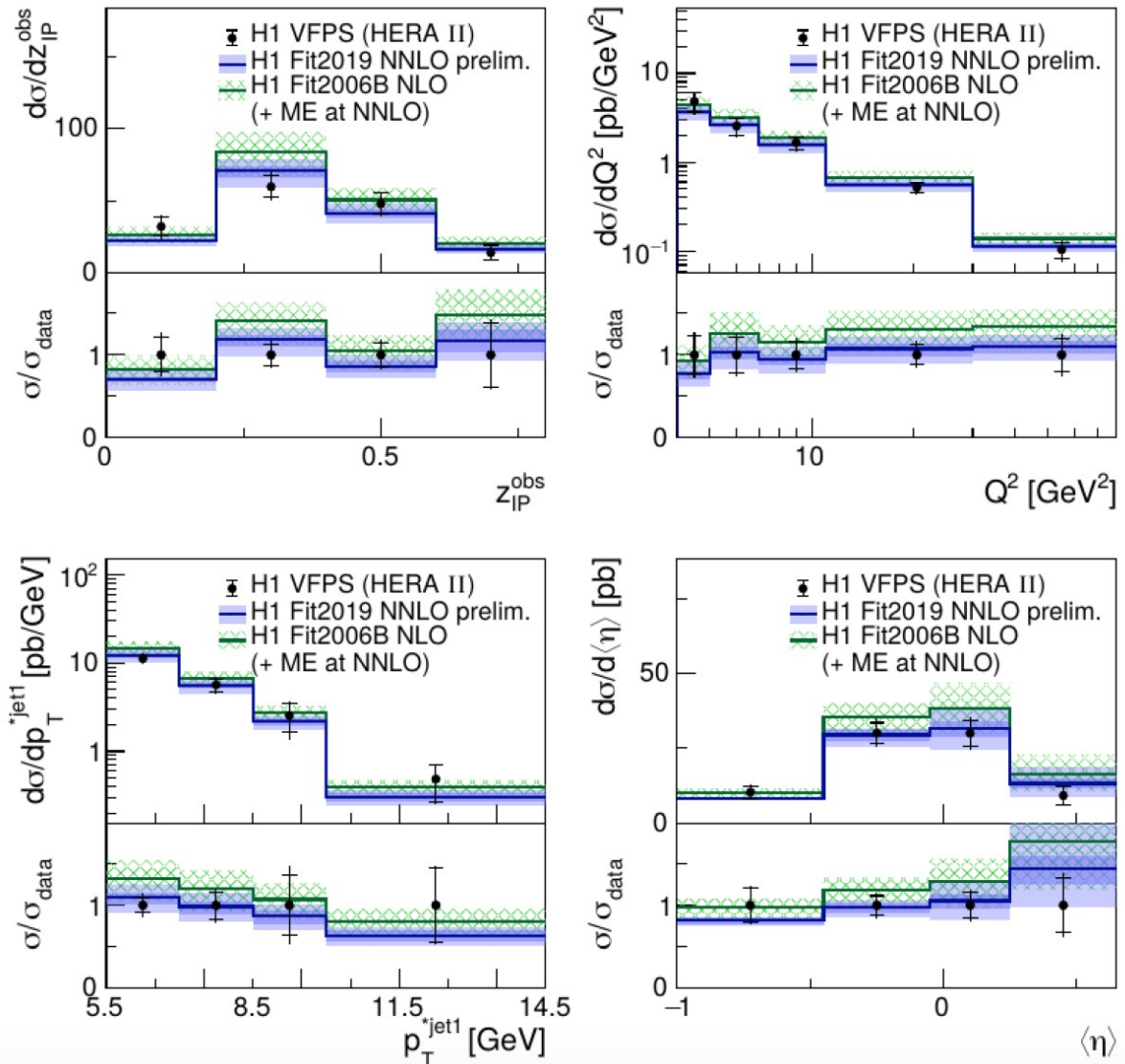
- H1 Fit2019 NNLO  
 → describes well the H1 HERA-II data + ZEUS HERA-I  
 → H1 HERA-I data slightly below
- H1 Fit2006B NLO with NNLO ME overestimates all cross sections



In addition to total cross sections we analyzed 39 single-differential and 4 double-differential distributions

# NNLO compared to H1 jets (VFPS)

- Data based on Very Forward Proton Spectrometer (VFPS) do not contain any proton dissociation and are in many ways systematically independent from LRG-based data
- Good description of kinematic variables  $z_{IP}$ ,  $Q^2$ ,  $p_T^{jet1}$ ,  $\langle \eta \rangle$

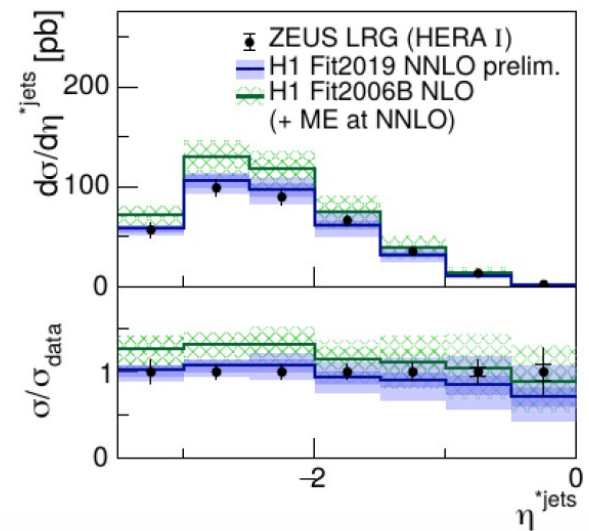
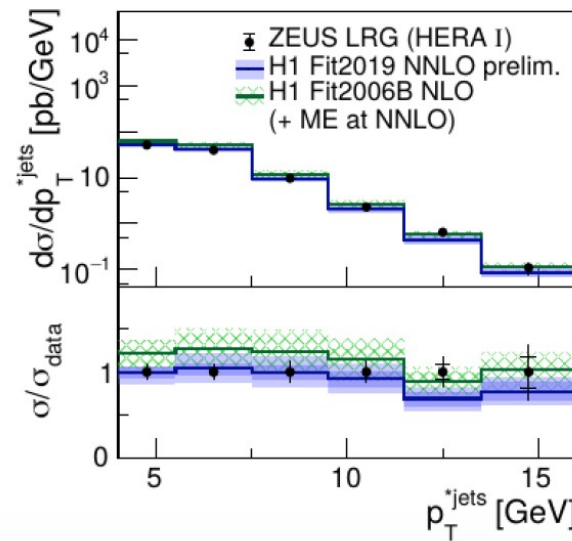
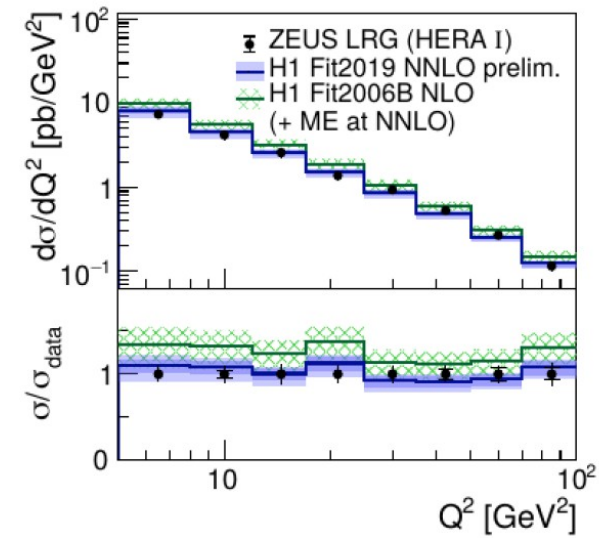
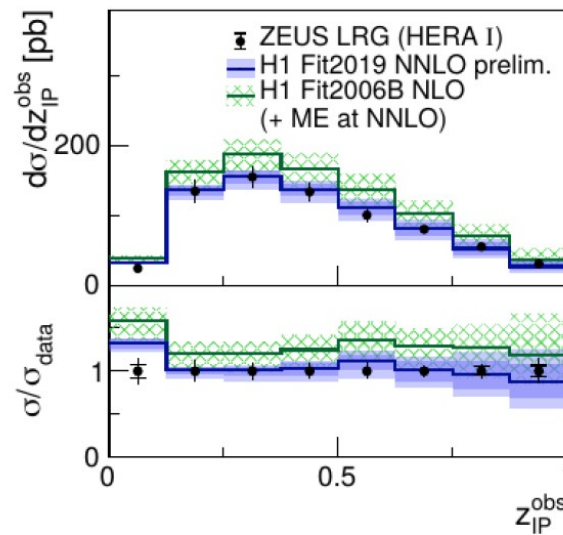


# NNLO compared to ZEUS LRG dijets

- Good agreement of H1 Fit2019 NNLO predictions with ZEUS dijet data [arXiv:0708.1415]

At LO the  $z_{IP}^{obs}$  directly related to the pomeron momentum fraction entering ME

$$z_{IP}^{obs} = \frac{Q^2 + M_{12}^2}{Q^2 + M_X^2}$$

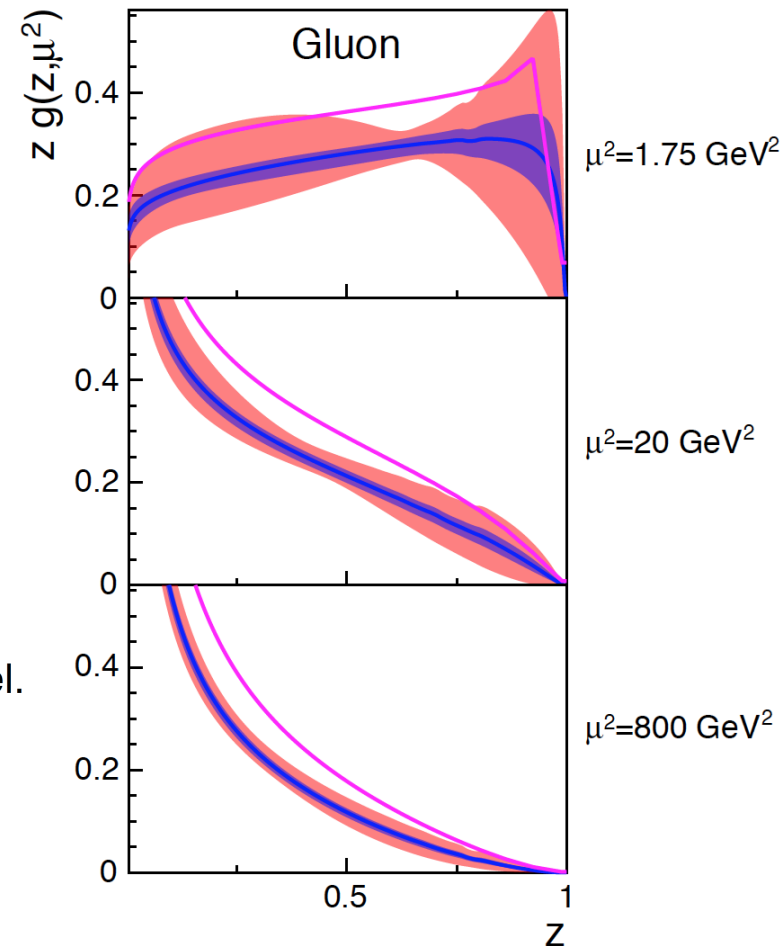


Factorisation in diffractive DDIS up to NNLO established

# Summary and conclusions

- First combined fit to inclusive+jet DDIS data at NNLO
- NNLO DPDF has lower gluon contribution compared to NLO
- Jet data compatible with new inclusive data (for both NLO and NNLO)

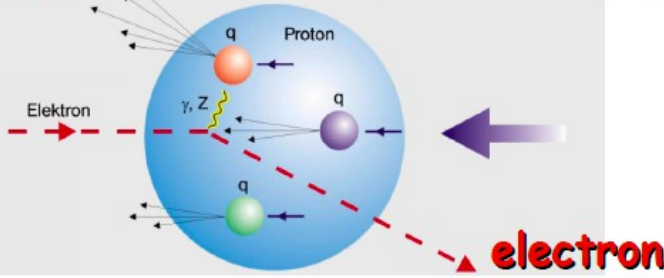
— H1 Fit2019 NNLO prel.  
 ■ exp. unc.  
 ■ +exp. +theor. unc.  
 — H1 Fit2006B NLO



Factorisation in diffractive DDIS up to NNLO established

# Additional information

# HERA combined inclusive DIS

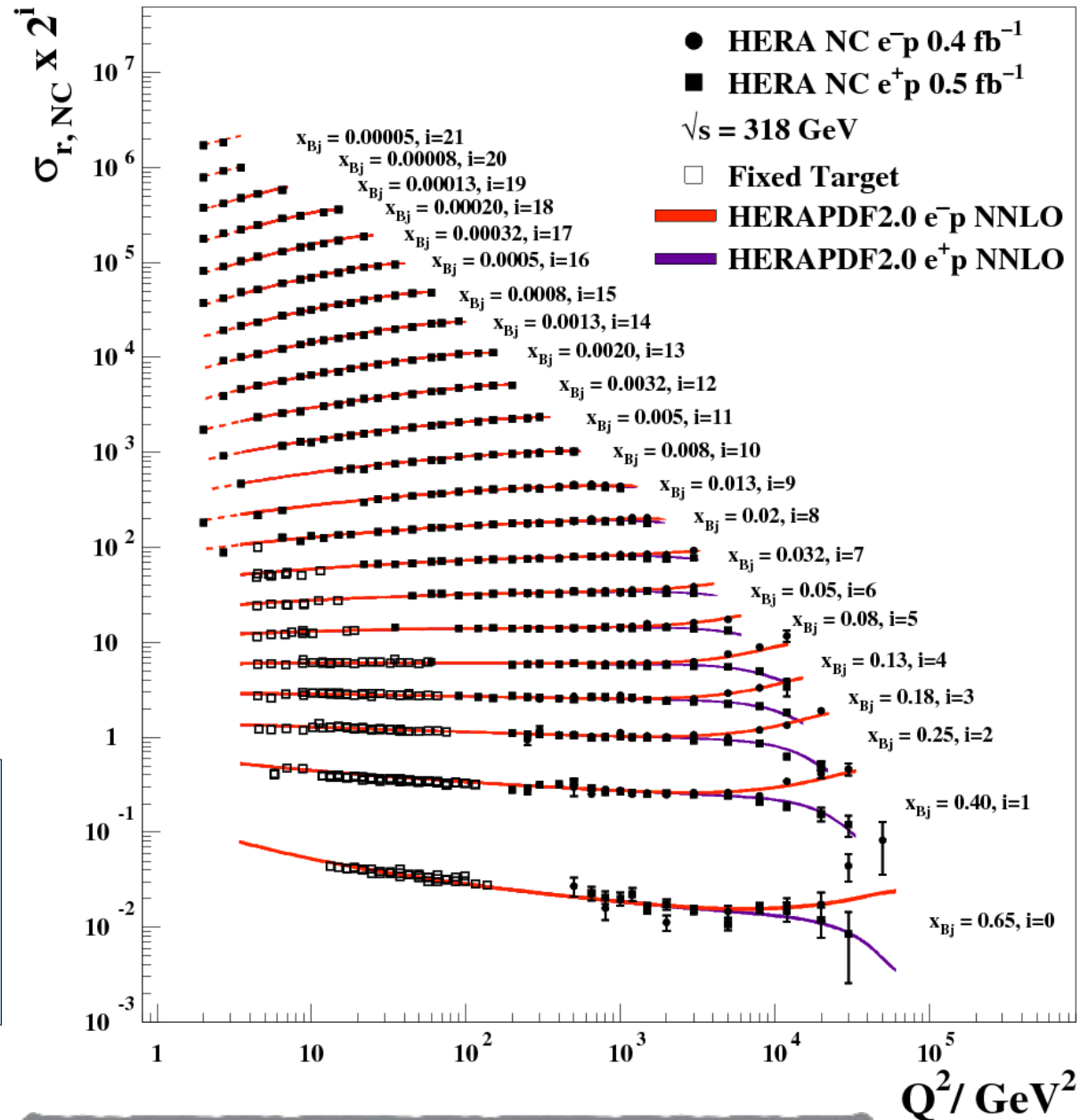


HERA combined DIS data are core of every modern PDF extraction

- 2927 data points combined to 1307
- impressive precision

HERAPDF approach uses ONLY HERA data in global QCD fit

## H1 and ZEUS





# HERAPDF2.0 parameterisation

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

$$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g},$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2),$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}},$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x),$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}.$$

- Additional constrains

- $A_{u_v}, A_{d_v}, A_g$ : constrained by the quark-number sum rules and momentum sum rule

- $B_{\bar{U}} = B_{\bar{D}}$ :

- $x\bar{s} = f_s x\bar{D}$  at starting scale,  $f_s = 0.4$

# PDF uncertainties

## HERAPDF experimental, model and parameterisation uncertainties

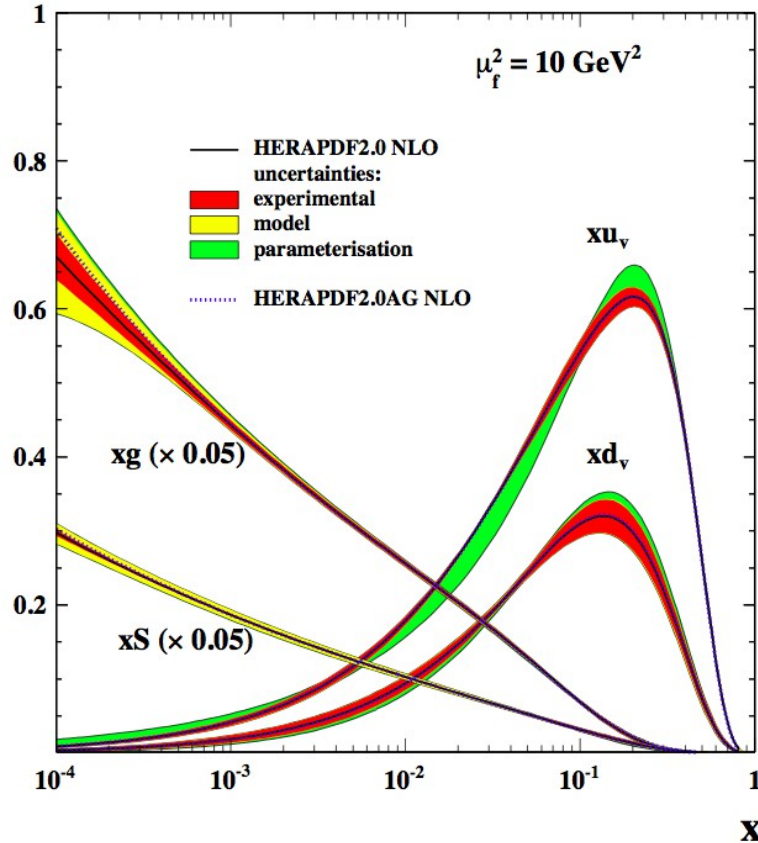
### Experimental uncertainties:

- Hessian method
- Conventional  $\Delta\chi^2 = 1 \Rightarrow 68\% \text{ CL}$

Variation	Standard Value	Lower Limit	Upper Limit
$Q_{\min}^2$ [GeV <sup>2</sup> ]	3.5	2.5	5.0
$Q_{\min}^2$ [GeV <sup>2</sup> ] HiQ2	10.0	7.5	12.5
$M_c$ (NLO) [GeV]	1.47	1.41	1.53
$M_c$ (NNLO) [GeV]	1.43	1.37	1.49
$M_b$ [GeV]	4.5	4.25	4.75
$f_s$	0.4	0.3	0.5
$\mu_{f_0}$ [GeV]	1.9	1.6	2.2

Adding D and E parameters to each PDF

H1 and ZEUS

 $\mu_f^2 = 10 \text{ GeV}^2$ 


Model uncertainties  
- variations added in quadrature

Parameterisation uncertainties  
- largest deviation

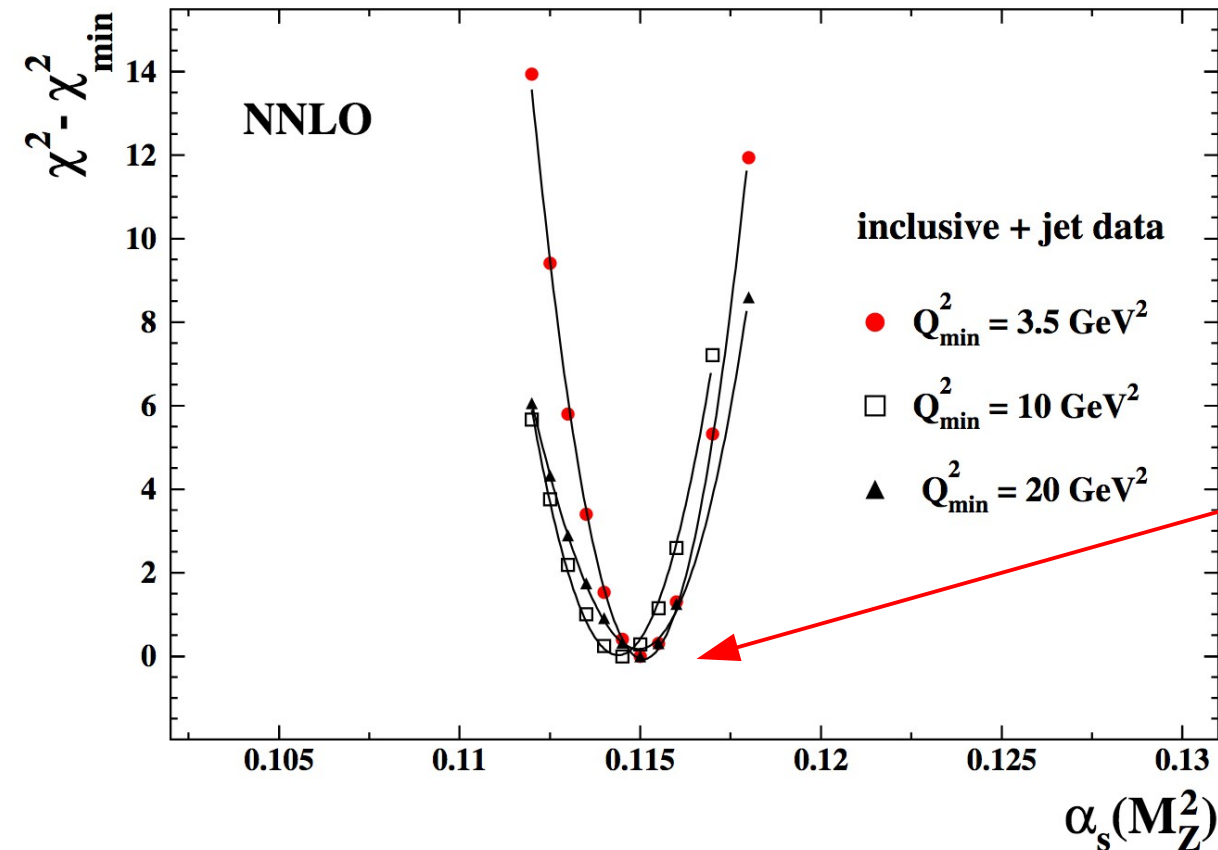
When jets included - also hadronisation uncertainty  
→ offsetting corrections given for each jet data set

# Scans with harder $Q^2$ cuts

- HERA data at low  $x$  and  $Q^2$  may be subject to need for  $\ln(1/x)$  resummation or higher twist effects

→  $\chi^2$  scans performed with harder  $Q^2$  cuts

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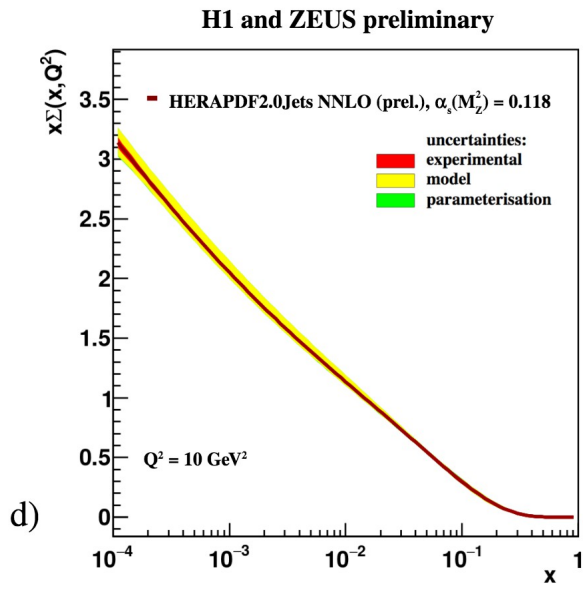
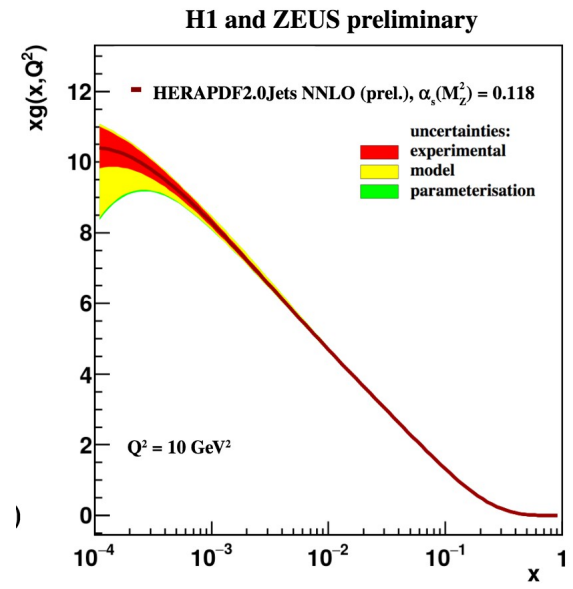
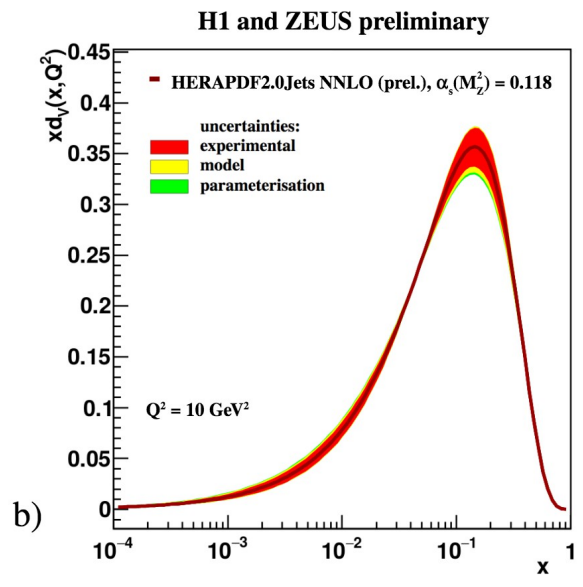
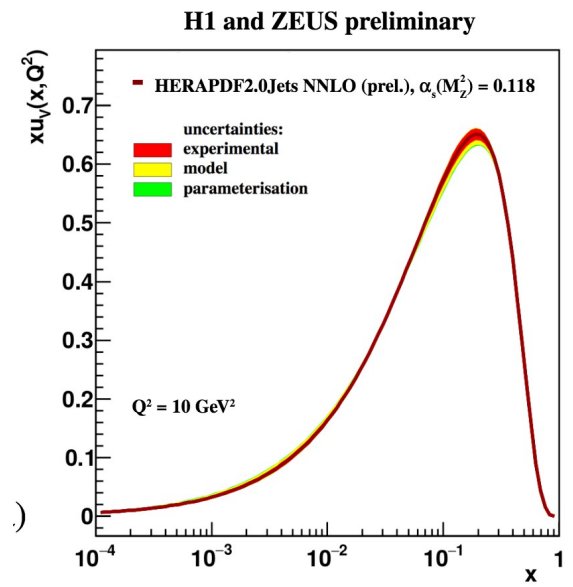


$Q^2$  cuts do not result in any significant change to the value of  $\alpha_s(M_Z)$

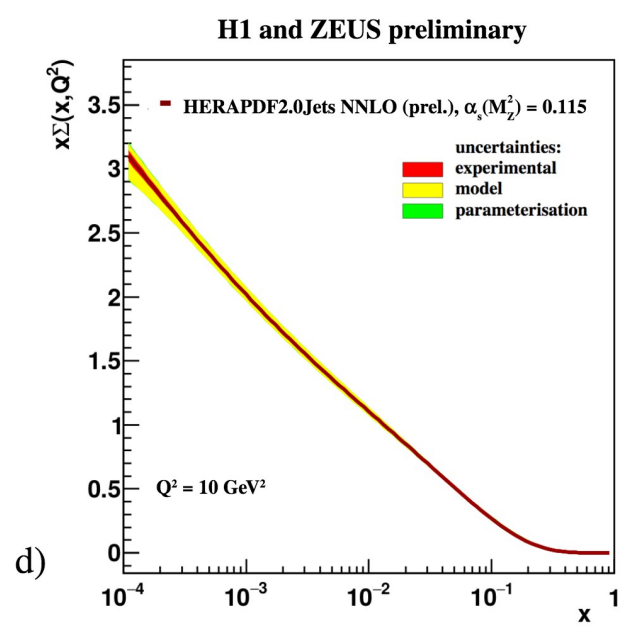
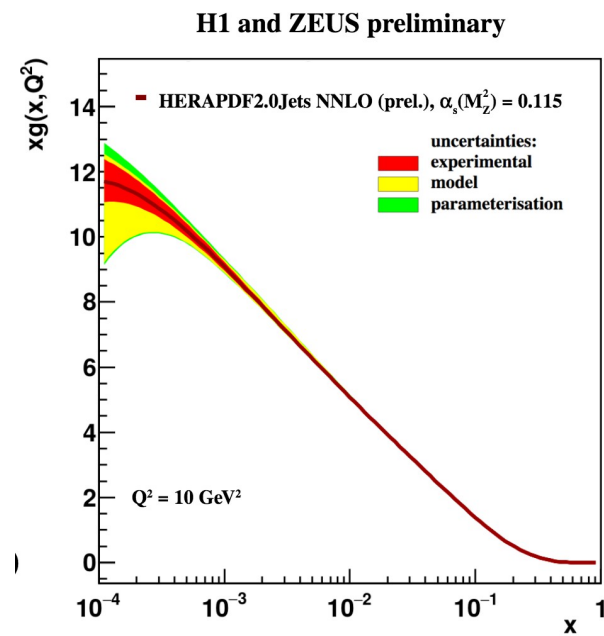
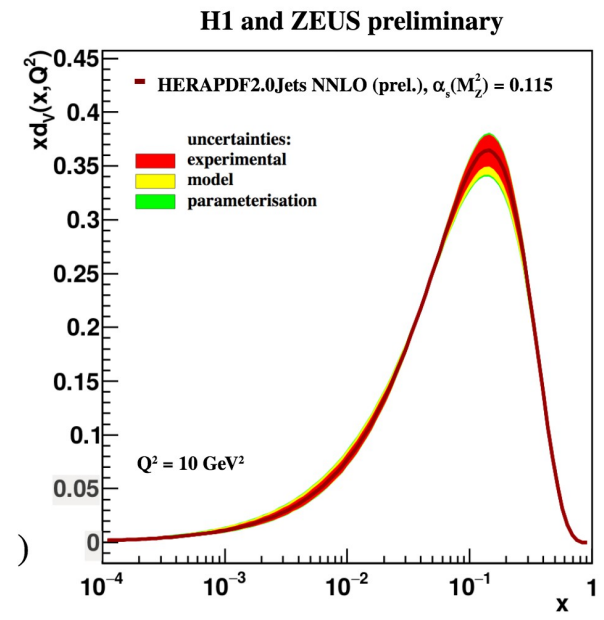
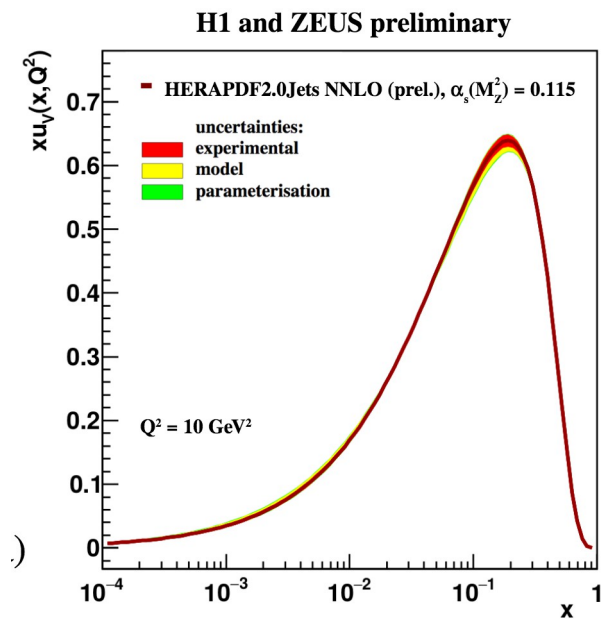


Let's first look at PDFs with  $\alpha_s = 0.118$ , as for HERAPDF2.0

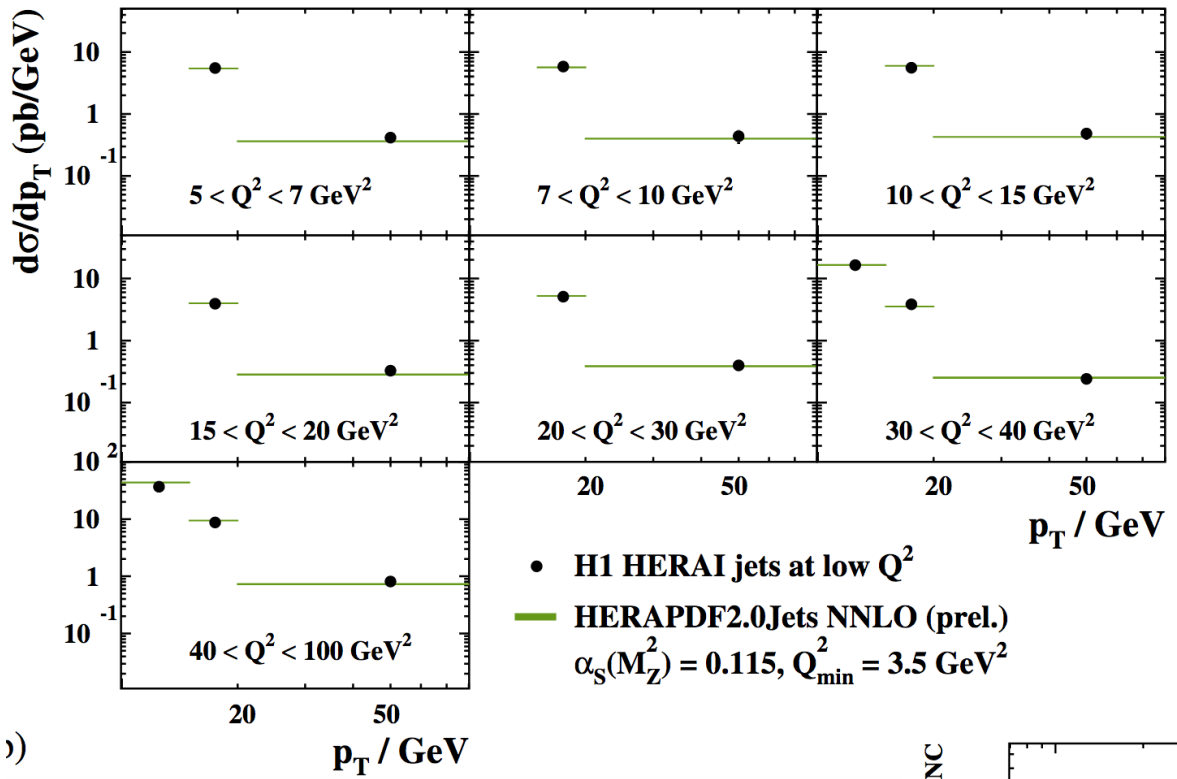
# HERAPDF2.0 Jets NNLO (prel.), $\alpha_s(M_Z) = 0.118$



# Let's look at PDF with $\alpha_s = 0.118$

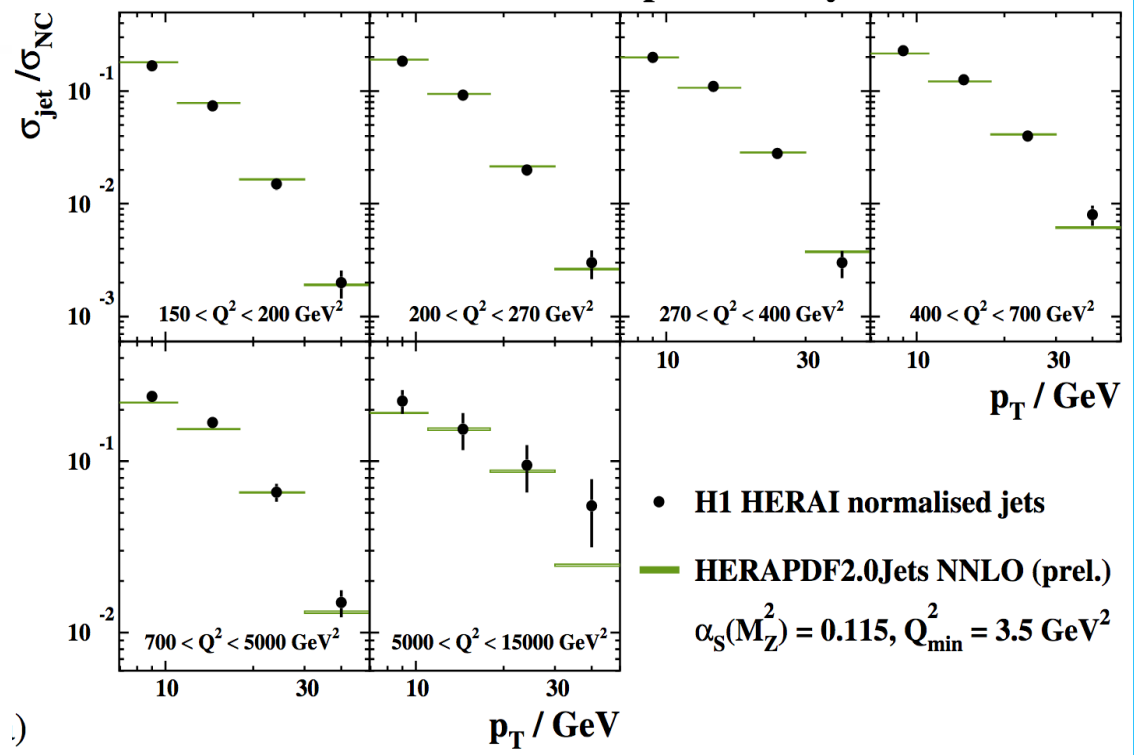


### H1 and ZEUS preliminary



Comparison of theory predictions to H1 HERA I inclusive jets @ low and high  $Q^2$   
 → good agreement

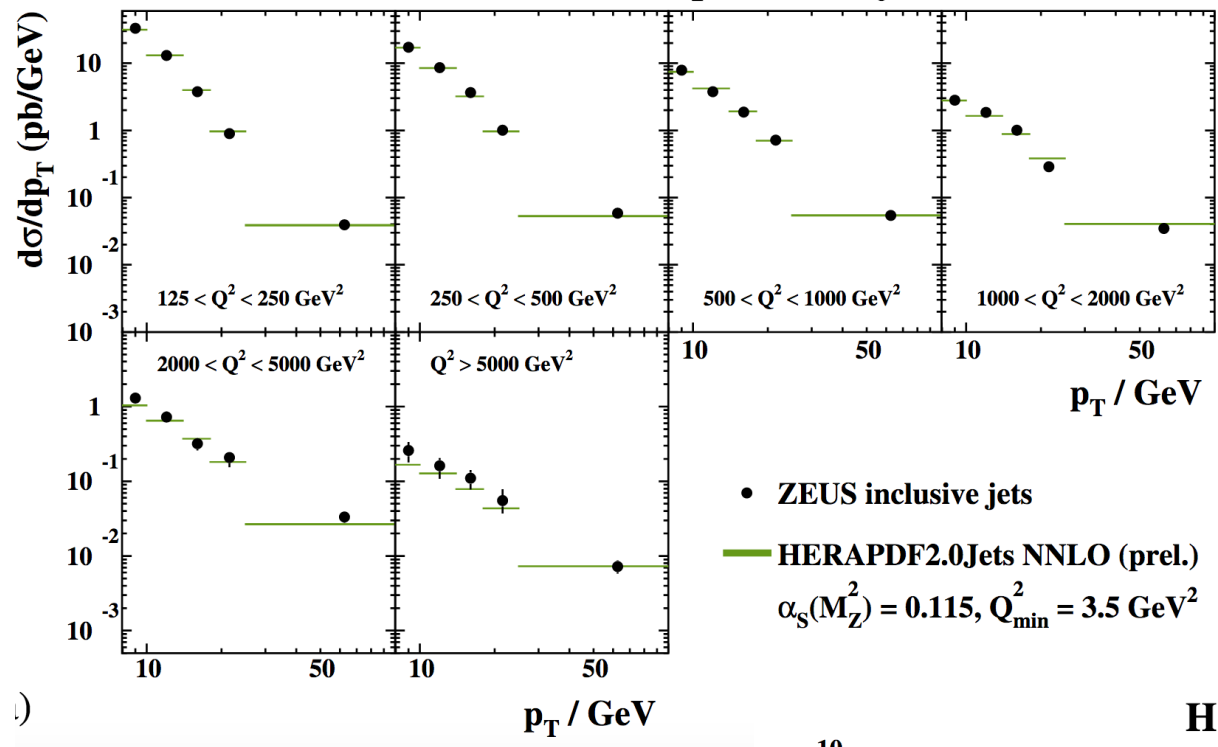
### H1 and ZEUS preliminary



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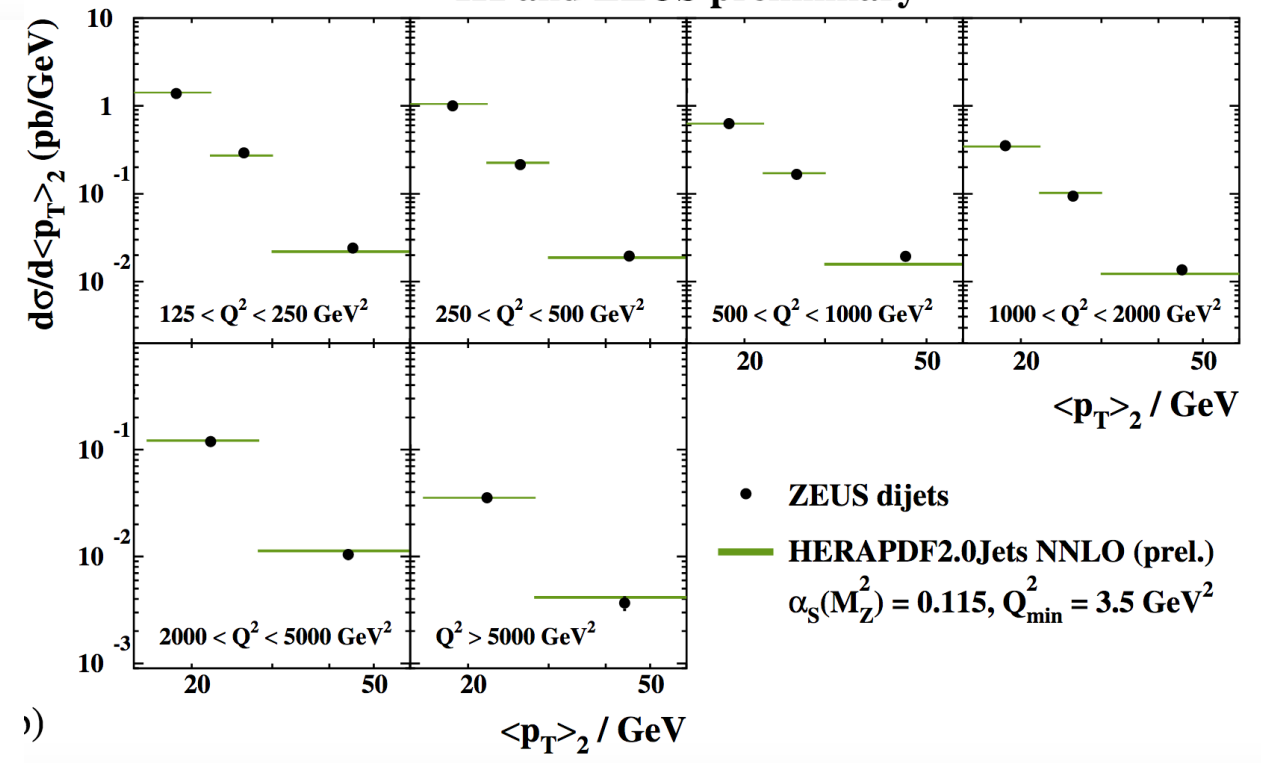
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### H1 and ZEUS preliminary



Comparison of theory predictions to ZEUS HERA I inclusive jets and dijets  
 → good agreement

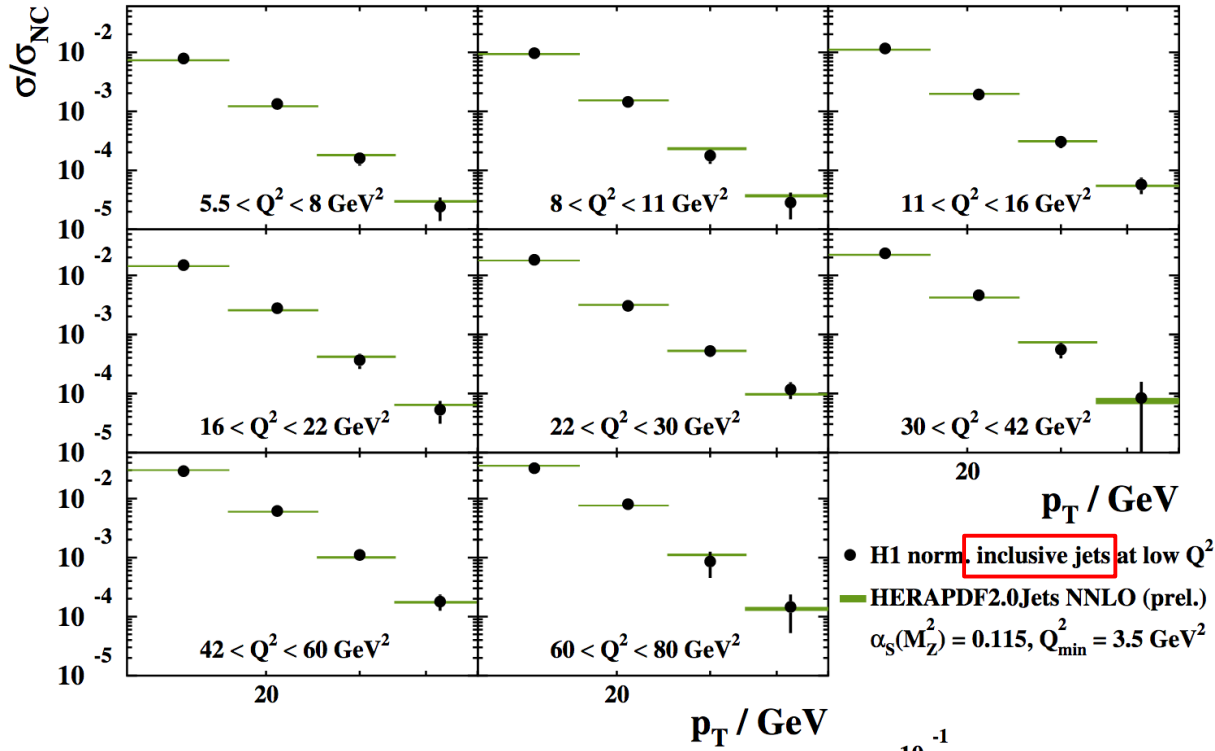
### H1 and ZEUS preliminary



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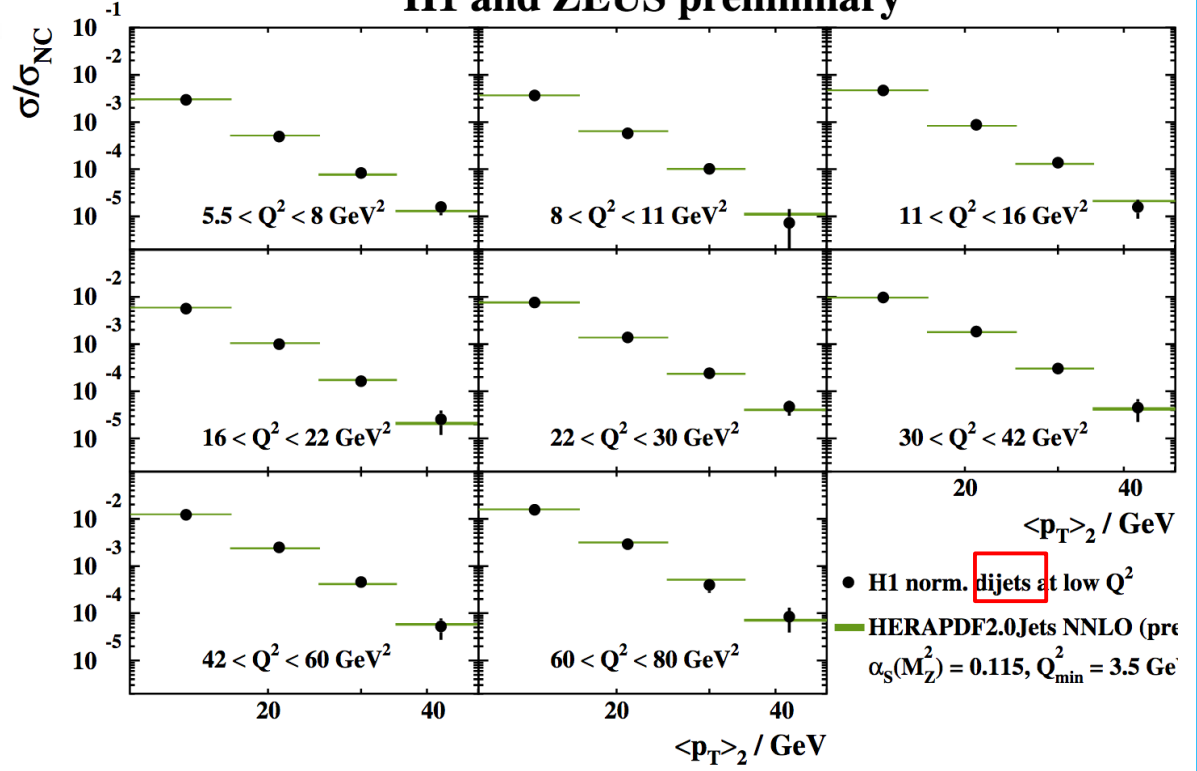
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### H1 and ZEUS preliminary



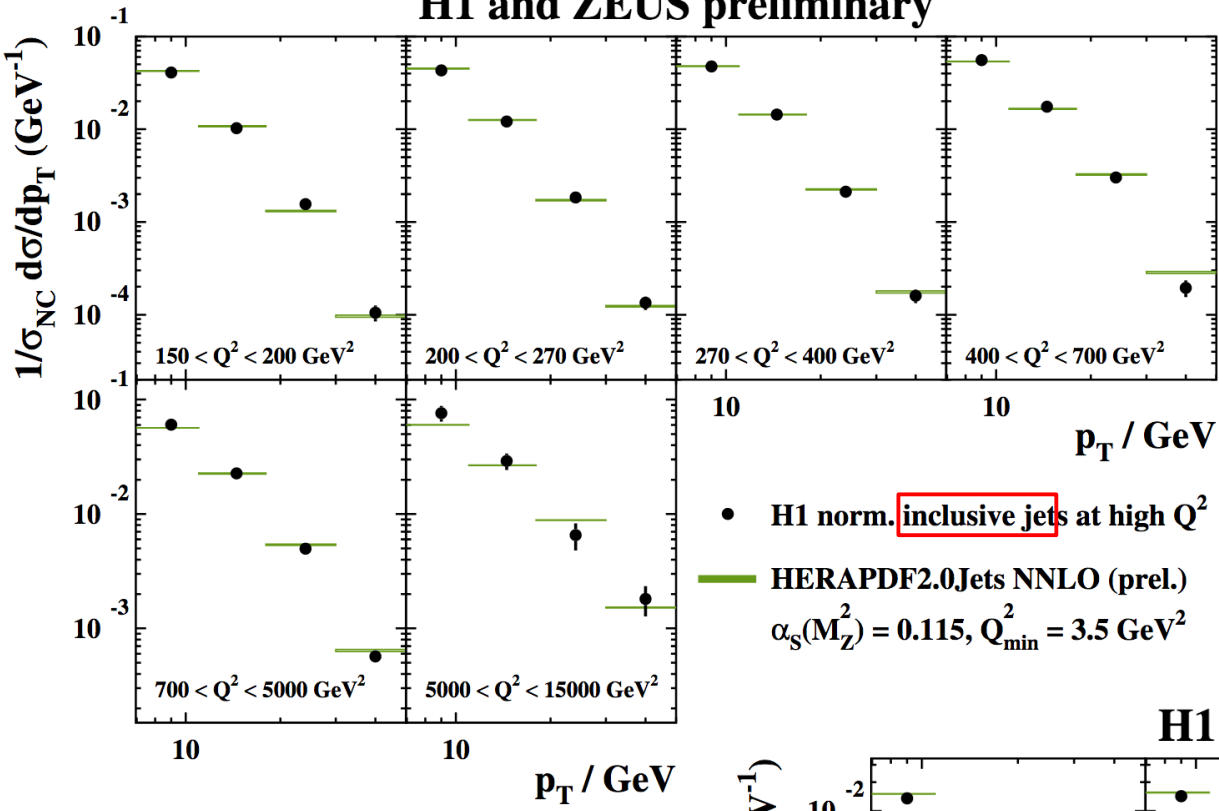
Comparison of theory predictions to H1 HERA II normalised jets @ low  $Q^2$   
 → good agreement

### H1 and ZEUS preliminary



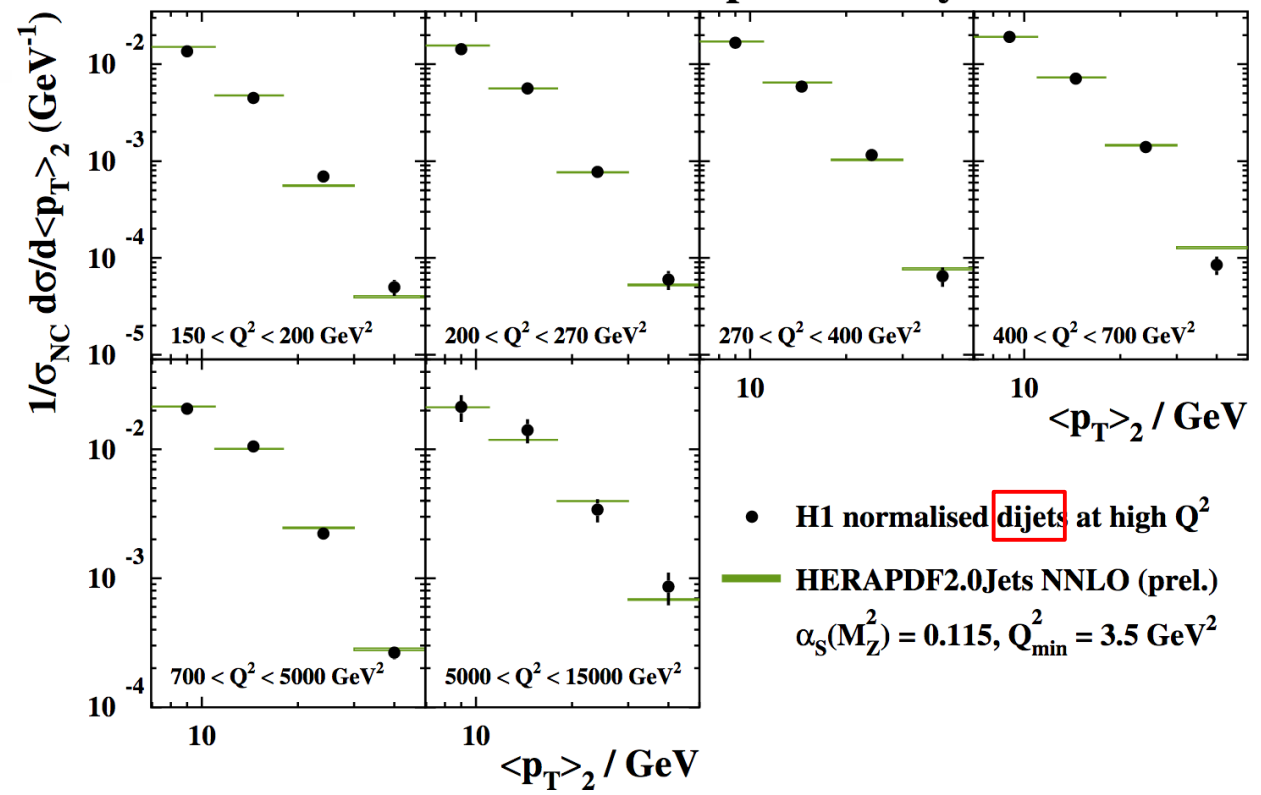


### H1 and ZEUS preliminary



Comparison of theory predictions to H1 HERA II normalised jets @ high  $Q^2$   
 → good agreement

### H1 and ZEUS preliminary



# Collinear QCD factorization in inclusive DDIS

$$\alpha_{em} \stackrel{\text{def}}{=} \frac{1}{137}$$

- The reduced diffractive cross section:

$$\frac{d^3\sigma^{ep \rightarrow eXY}}{dQ^2 d\beta dx_{\mathbb{P}}} = \frac{4\pi\alpha_{em}^2}{\beta Q^4} \left(1 - y + \frac{y^2}{2}\right) \underbrace{\left(F_2 - \frac{y^2}{1 + (1-y)^2} F_L\right)}_{\sigma_r^{D(3)}(\beta, Q^2, x_{\mathbb{P}})}$$

- Regge factorization ansatz

$$F_{2/L}^{D(3)}(\beta, Q^2, x_{\mathbb{P}}) = f_{\mathbb{P}/p}(x_{\mathbb{P}}) F_{2/L}^{\mathbb{P}}(\beta, Q^2) + n_{\mathbb{R}} \underbrace{f_{\mathbb{R}/p}(x_{\mathbb{P}}) F_{2/L}^{\mathbb{R}}(\beta, Q^2)}_{\text{Fixed}}$$

$$F_{2/L}^{\mathbb{P}}(\beta, Q^2) = C_{2/L}^i(\beta/z, Q^2, \mu^2) \otimes f_{i/\mathbb{P}}(z, \mu^2)$$

Up to NNLO

Standard DIS  
coef. functions

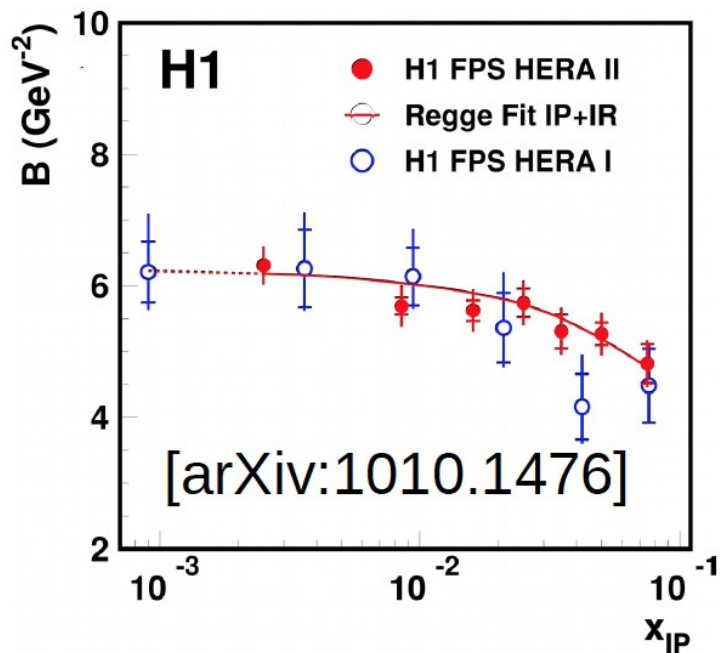
Obeys DGLAP

**Both coef. functions and DGLAP evolution depend on  $\alpha_s$  and  $m_c, m_b$**

# Flux Parametrization

- Param. inspired by Regge theory (Streng and Berger):

$$f_{IP/p}(x_{IP}, t) \propto \left( \frac{1}{x_{IP}} \right)^{2[\alpha_{IP}(0) + \alpha'_{IP} t] - 1} e^{B_{IP}^0 t} \quad \Rightarrow \quad \frac{d\sigma}{dt} \propto e^{-B|t|}$$



B-slope dependence:

$$B = B_{IP}^0 + 2\alpha'_{IP} \left( \log \frac{1}{x_{IP}} \right)$$

$$\alpha'_{IP} = 0.04_{-0.06}^{+0.08} \text{ GeV}^{-2}$$

$$B_{IP}^0 = 5.73_{-0.93}^{+0.84} \text{ GeV}^{-2}$$

Uncertainties anti-correlated

- t-integrated version:

$$f_{IP/p}(x_{IP}) \propto \left( \frac{1}{x_{IP}} \right)^{2\alpha_{IP}(0) - 1} \frac{1}{1 + 2 \frac{\alpha'_{IP}}{B_{IP}^0} \log \frac{1}{x_{IP}}} \doteq \left( \frac{1}{x_{IP}} \right)^{2\alpha_{IP}(0) - 1 - 2 \frac{\alpha'_{IP}}{B_{IP}^0}}$$

Annotations:  $2\alpha_{IP}(0) - 1 \approx -1.2$  (Fixed),  $2 \frac{\alpha'_{IP}}{B_{IP}^0} \approx -0.01$  (Fitted)