

# $B \rightarrow D^{(*)}$ form factors and $V_{cb}$ determination

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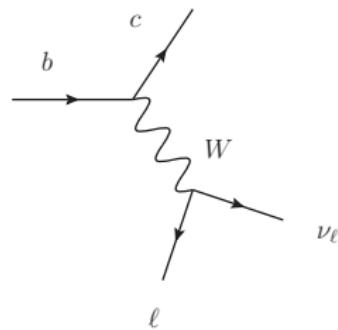
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“Physics at the Terascale”  
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# Importance of (semi-)leptonic decays

In the Standard Model

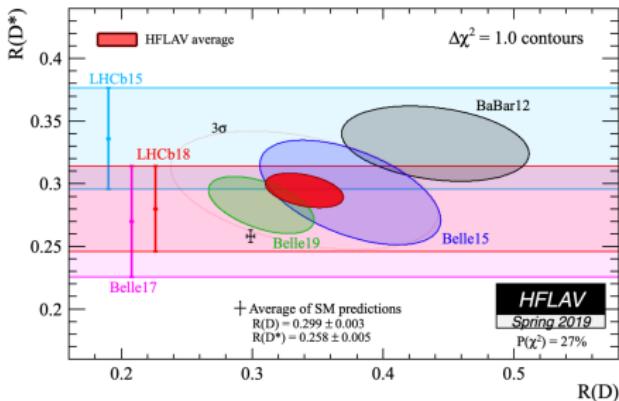
- Tree-level processes  $\sim G_F^2 |V_{ij}|^2 \mathcal{F}^2$
- They are perfect processes to extract  $|V_{ij}|$
- Branching ratios are usually large



Beyond the Standard Model

- Moderate NP can be seen if we understand the SM predictions precisely
- Can test structures  $\neq$  from SM ones (especially for heavy leptons)

# Motivation



- Average of BaBar, Belle, LHCb data yields  $3 - 3.5\sigma$  deviation with the SM

$NP \sim 10 - 15\%$  of a SM tree decay  $\Rightarrow$  huge effect

- If NP interfere with SM we need it to be 10%
- If NP doesn't interfere with SM, we need it to be a 40% effect
- Check SM predictions!  $\Leftarrow$  main topic of this talk
- NP analysis: rely on hadronic inputs

# The longstanding $V_{cb}$ puzzle

- The inclusive determination is extremely under control (up to  $\mathcal{O}(1/m_b^3, \alpha_2/m_b^2, \alpha_s^2)$ )

$$V_{cb}^{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

- The exclusive determination is less clear and depends on different data set/assumptions for the form factors
- Unfolded differential measurements made available by Belle: allows fits with different parametrisations for the form factors are possible
- Especially for  $B \rightarrow D^*$  decays, a tension  $\sim 2\sigma$  with the inclusive determination remains
- NP explanations are disfavoured

[Jung, Straub, '18]

# Hadronic Matrix Elements

$$\langle D | \bar{c} \Gamma_{\mu_1 \mu_2} b | \overline{B} \rangle = \sum_i S_{\mu_1 \mu_2}^i F_i(q^2)$$

$$\langle D^*(\lambda) | \bar{c} \Gamma_\mu b | \overline{B} \rangle = \sum_\lambda \sum_i \epsilon^\alpha(\lambda) S_{\alpha \mu}^i F_i(q^2)$$

**Form Factor:** scalar function which encodes the non-perturbative dynamics

- $B \rightarrow D$ : 2FF + 1 tensor for NP
  - vector current: 2 FF
  - tensor current: 1 FF
- $B \rightarrow D^*$ : 4FF + 3 tensor for NP
  - vector current: 1 FF
  - axial-vector current: 3 FF
  - tensor current: 3 FF

# Hadronic Matrix Elements: Lattice

Lattice: discretised space-time

- prediction for high  $q^2$
- unstable particles ( $D^*$ ) are problematic

State of the art:

- $B \rightarrow D$ : complete set of SM FFs calculated
- $B \rightarrow D^*$ : only few prediction at the zero recoil point
- preliminary results shown at Lattice 19

[HPQCD, 2015,  
Fermilab/MILC, 2015  
FLAG, 2016]

[Fermilab/MILC, 2014,  
HPQCD, 2017]

[Talk by A. Vaquero]

More theory inputs needed

# HQET in a nutshell

- QCD distinguishes among flavour only through masses
- $b \rightarrow c$ : the partonic transition involves only heavy quarks
- in the limit  $m_{b,c} \rightarrow \infty$  but  $m_c/m_b = \text{finite}$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\infty + \mathcal{O}(1/m_Q)$$

and  $\mathcal{L}_\infty$  is independent of the heavy quark masses

- HQET's spin-flavour symmetry relates the various form factors, with breaking between symmetry relations suppressed by powers of  $1/m_Q$

To leading power the form factors are all  
proportional to a single Isgur-Wise function  $\xi(w)$

- $\xi(w)$  is the same for any  $b \rightarrow c$  transitions involving  $B^{(*)}$  and  $D^{(*)}$ .

# The $B \rightarrow D^*$ case

How do we get informations on the  $B \rightarrow D^*$  form factors?

- HQET +  $\alpha_s$  and  $1/m_{b,c}$  corrections + data inputs from Belle

[Fajfer, Kamenik, Nisandzic, 2012]

- We can also use dispersive bounds to set constraints on the form factors

[Boyd, Grinstein, Lebed, '95]

Caprini, Lellouch, Neubert, '97]

- HQET + dispersive bounds + data

[Bigi, Gambino, Schacht, 2017]

Bernlochner, Ligeti, Papucci, Robinson, 2017]

Is there a way to parametrise form factors without using data?

# BGL vs CLN parametrisations

## CLN

[Caprini, Lellouch, Neubert, '97]

- Expansion of FFs using HQET
- $1/m_{b,c}$  corrections included
- Expansion of leading IW function up to 2nd order in  $(w - 1)$

## BGL

[Boyd, Grinstein, Lebed, '95]

- Based on analyticity of the form factors
- Expansion of FFs using the conformal variable  $z$
- Large number of free parameters

# Our approach

Working assumption:

- We expand the FFs using HQET
- We introduce a **consistent** power counting:  $\frac{\alpha_s}{\pi} \sim \frac{\Lambda_{QCD}}{2m_b} \sim \frac{\Lambda_{QCD}^2}{4m_c^2}$ 
  - full  $1/m_c^2$  terms **must** be introduced
  - available only partially
- We use the **full set** of unitary bounds for all the decays  $B^{(*)} \rightarrow D^{(*)}$

[Jung, Straub, '18]

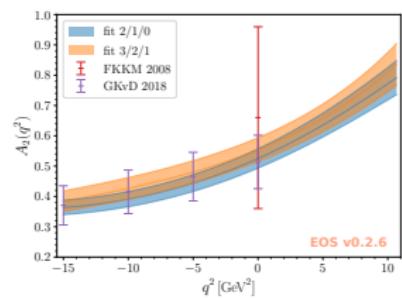
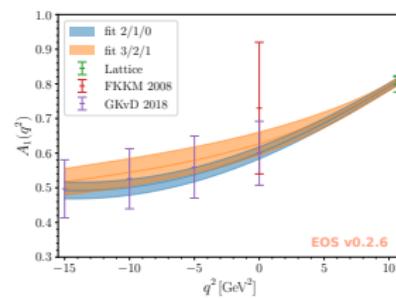
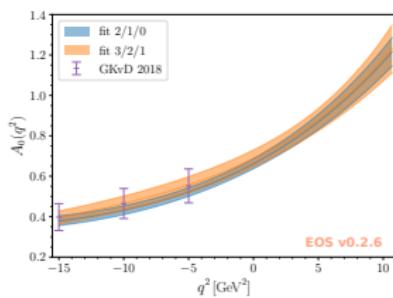
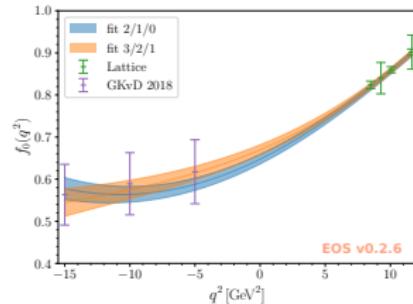
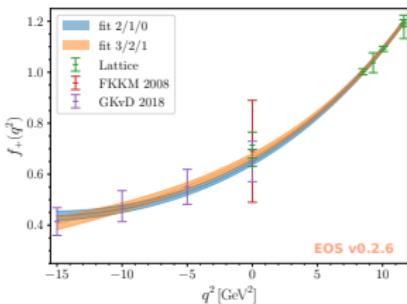
Inputs:

- Lattice points for  $B \rightarrow D$
- Zero-recoil lattice points for  $B \rightarrow D^*$
- QCD sum rules for subleading Isgur-Wise Functions
- Introduce new LCSR results

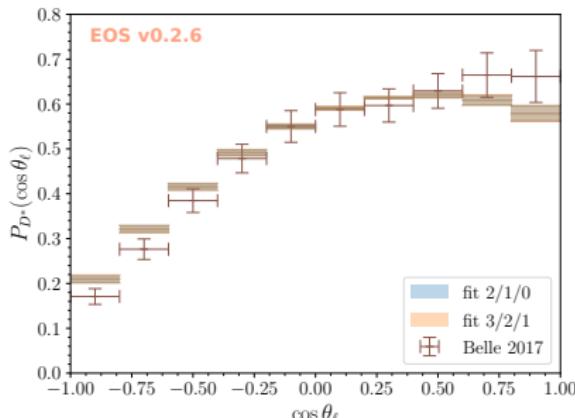
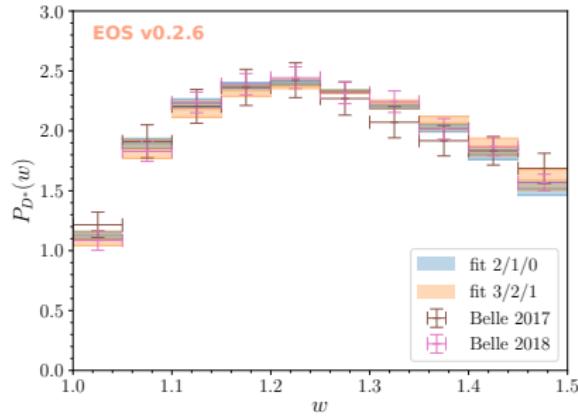
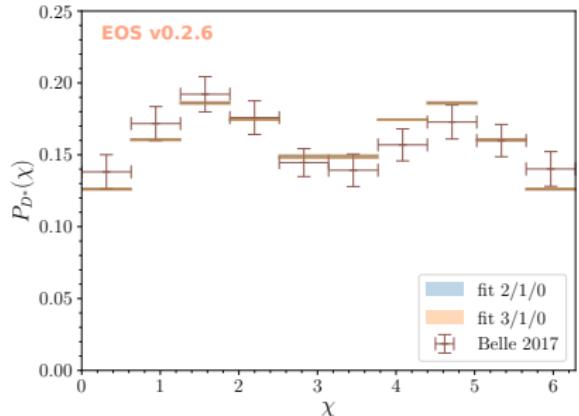
[Gubernari, Kokulu, van Dyk, 2018]

# Fit results for the form factors

[MB, Jung, van Dyk, 1907.xxxx]



# Comparison with kinematical distributions



good agreement with kinematical distributions

# Predictions in the 321 fit model

[MB, Jung, van Dyk, 1908.09398]

	HFLAV	Our predictions
$R_D$	$0.299 \pm 0.003$	$0.298 \pm 0.003$
$R_{D^*}$	$0.258 \pm 0.005$	$0.247 \pm 0.006$

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$V_{cb}^D$	$(39.18 \pm 1.00) \times 10^{-3}$	$(40.7 \pm 1.2) \times 10^{-3}$
$V_{cb}^{D^*}$	$(38.71 \pm 0.75) \times 10^{-3}$	$(39.3 \pm 1.7) \times 10^{-3}$

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1 $\sigma$  and 1.5 $\sigma$  from incl.  $V_{cb}$

$$V_{cb}^{incl.} = (42.00 \pm 0.64) \times 10^{-3}$$

- adding shape informations shifts slightly the central values and shrinks the errors

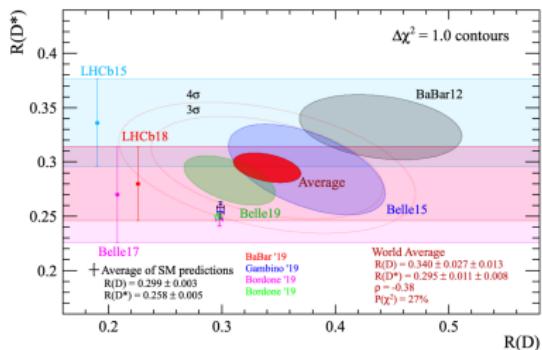
# Summary

Our approach is based on:

- a HQET based parametrisation of the  $B \rightarrow D$  and  $B \rightarrow D^*$  form factors
- power corrections up to  $\mathcal{O}(1/m_c^2)$
- inputs from lattice, QCD sum rules, update LCSR
- all set of unitarity constraints

We obtain the following:

- results are in agreement with kinematical distributions
- interesting predictions for phenomenological quantities



# Appendix

# HQET in a nutshell

- In HQET it is convenient to work with velocities instead of momenta
- Instead of  $q^2$  we use the dimensionless variable  $w = v_B \cdot v_{D^*}$
- When the  $B(b)$  decays such that the  $D^*(c)$  is at rest in the  $B(b)$  frame

$$v_B = v_{D^*} \quad \Rightarrow \quad w = 1$$

- The brown muck doesn't realise that anything changed
- At zero recoil, the leading IW function is normalized

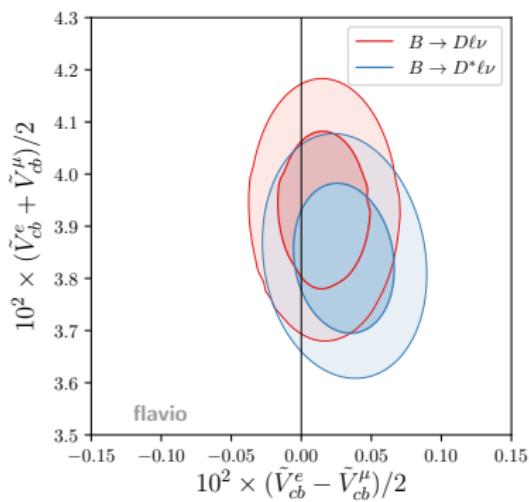
$$\xi(w = 1) = 1$$

- If we allow LFUV between  $\mu$  and electrons

$$\tilde{V}_{cb}^\ell = V_{cb}(1 + C_{V_L}^\ell)$$

- Fitting data from Babar and Belle

$$\frac{\tilde{V}_{cb}^e}{\tilde{V}_{cb}^\mu} = 1.011 \pm 0.012$$



$$\frac{1}{2}(\tilde{V}_{cb}^e + \tilde{V}_{cb}^\mu) = (3.87 \pm 0.09)\%$$
$$\frac{1}{2}(\tilde{V}_{cb}^e - \tilde{V}_{cb}^\mu) = (0.022 \pm 0.023)\%$$

# Motivation

## Anatomy of the ratios

$$\frac{d\Gamma_\tau}{dq^2} = \frac{d\Gamma_{\tau,1}}{dq^2} + \frac{d\Gamma_{\tau,2}}{dq^2}$$

$$\frac{d\Gamma_{\tau,1}}{dq^2} = \frac{d\Gamma}{dq^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(1 + \frac{m_\tau^2}{2q^2}\right)$$

$$\frac{d\Gamma_{\tau,2}}{dq^2} = \Gamma_0 \frac{m_\tau^2}{q^2} c_0$$

$$R_{D^{(*)}}^{\tau,1} = \frac{\int_{m_\tau^2}^{q_{\max}^2} dq^2 \frac{d\Gamma_{\tau,1}}{dq^2}}{\int_0^{q_{\max}^2} dq^2 \frac{d\Gamma}{dq^2}}$$

$$R_{D^{(*)}}^{\tau,2} = \frac{\int_{m_\tau^2}^{q_{\max}^2} dq^2 \frac{d\Gamma_{\tau,2}}{dq^2}}{\int_0^{q_{\max}^2} dq^2 \frac{d\Gamma}{dq^2}}$$

$$R_D^{\tau,1} = 0.176$$

$$R_D^{\tau,2} = 0.123$$

$$R_{D^*}^{\tau,1} = 0.232$$

$$R_{D^*}^{\tau,2} = 0.028$$

The contribution of  $R_{D^*}^{\tau,2}$  in the error budget is small

# The $z$ -expansion

We can map the variable  $w$  into the conformal variable  $z$ :

$$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Easier implementation of unitarity and analiticity
- The value of  $|z|$  is expected to be small  $\Rightarrow$  better convergence of the expansion
- We can also combine HQET and dispersive bounds

## The effect on $R_{D^{(*)}}$

$R_D$	$0.299 \pm 0.011$	1503.07237 (FNAL/MILC)
	$0.300 \pm 0.008$	1505.03925 (HPQCD)
	$0.299 \pm 0.003$	1703.05330
	$0.299 \pm 0.004$	1703.09977
$R_{D^{(*)}}$	$0.252 \pm 0.003$	1203.2654
	$0.257 \pm 0.003$	1703.05330
	$0.258^{0.010}_{-0.009}$	1707.09509
	$0.257 \pm 0.005$	1703.09977

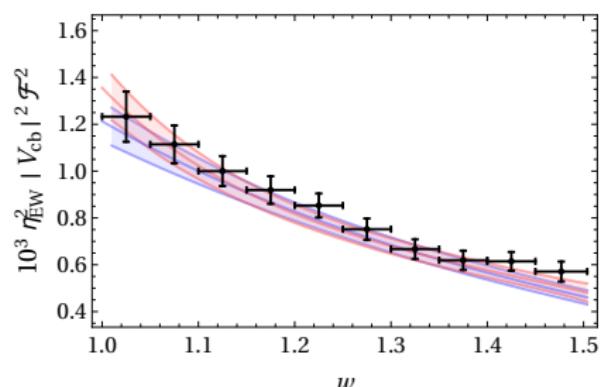
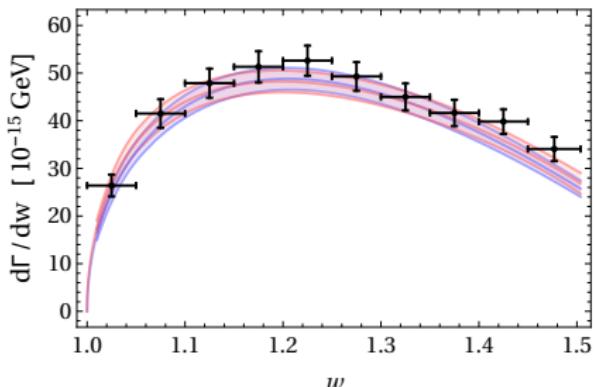
# BGL vs CLN

- Both BGL and CLN parametrisation of form factors rely on using unitarity arguments.

[Boyd, Grinstein, Lebed, '95]

Caprini, Neubert, Lellouch, '98]

- CLN relies on HQET.
- Unfolded distributions from Belle allowed to repeat an independent fit.



BGL has a more conservative error

Provides better agreement with inclusive  $V_{cb}$

# Motivation

If I assume  $\Lambda_{\text{NP}} \gg v$ : the SM gauge group is not broken up to  $\Lambda_{\text{NP}}$

I can use SMEFT and match it to the WET

$$C_{V_L}^{\ell\ell'} = -v^2 \frac{V_{ci}}{V_{cb}} C_{lq}^{(3)\ell\ell'i3} + v^2 \frac{V_{ci}}{V_{cb}} C_{\phi q}^{(3)i3} \delta_{\ell\ell'} \quad C_{V_R}^{\ell\ell'} = + \frac{v^2}{2} C_{\phi ud}^{23} \delta_{\ell\ell'}$$

$$C_{S_R}^{\ell\ell'} = - \frac{v^2}{2} \frac{V_{ci}}{V_{cb}} C_{ledq}^{\ell\ell'3i} \quad C_T^{\ell\ell'} = - \frac{v^2}{2} \frac{V_{ci}}{V_{cb}} C_{lequ}^{(3)\ell\ell'3i}$$

$$C_{S_L}^{\ell\ell'} = - \frac{v^2}{2} \frac{V_{ci}}{V_{cb}} C_{lequ}^{(1)\ell\ell'3i}$$

The WC  $C_{V_R}^{\ell\ell'}$  must be flavour universal and diagonal

The coefficients might be constrained by different flavour processes