

$$\begin{aligned}
& |\rho(t')\rangle = |\rho(t)\rangle \\
& + \int_t^{t'} d\tau \, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)] |\rho(t)\rangle \\
& + \int_t^{t'} d\tau_2 \, [\mathcal{H}_I(\tau_2) - \mathcal{V}_I(\tau_2)] \int_t^{\tau_2} d\tau_1 \, [\mathcal{H}_I(\tau_1) - \mathcal{V}_I(\tau_1)] |\rho(t)\rangle \\
& + \dots \\
& = \underbrace{\mathbb{T} \exp \left\{ \int_t^{t'} d\tau \, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)] \right\}}_{\text{}} |\rho(t)\rangle
\end{aligned}$$