

$$|\rho^{\text{R}}_{\infty}) = \int_t^{\infty} d\tau \, \mathcal{H}_I(\tau) |\rho(t))$$