Cross-sections et al.

Bake, boil or steam?
How to prepare a cross-section

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Ingredients

- Introduction
- Cross-section definition
- Counting signal and background events
- Tools for the job:
  - ABCD (matrix) method
  - Tag and probe
- Acceptance, efficiency and purity
- Binning and migration
- Luminosity
- Factorisation
- PDFs
- Systematic uncertainties

Introduction

- Me:
  - Experimental particle physicist
  - Worked on several e^+e^- machines and experiments:
    - PETRA (TASSO), DORIS (Crystal Ball), CESR (CLEO), LEP (L3)
  - ep collider HERA (ZEUS) – from 1996
  - pp collider LHC (ATLAS) – from 2006
- Examples from ZEUS and ATLAS

Cross-section

- A measure of the number of collisions
- Often measured as a function of angle and energy of target particles
- Also as a function of angle and energy of decay products
- Theory gives you matrix elements
- Use Fermi golden rule to calculate expected cross-section

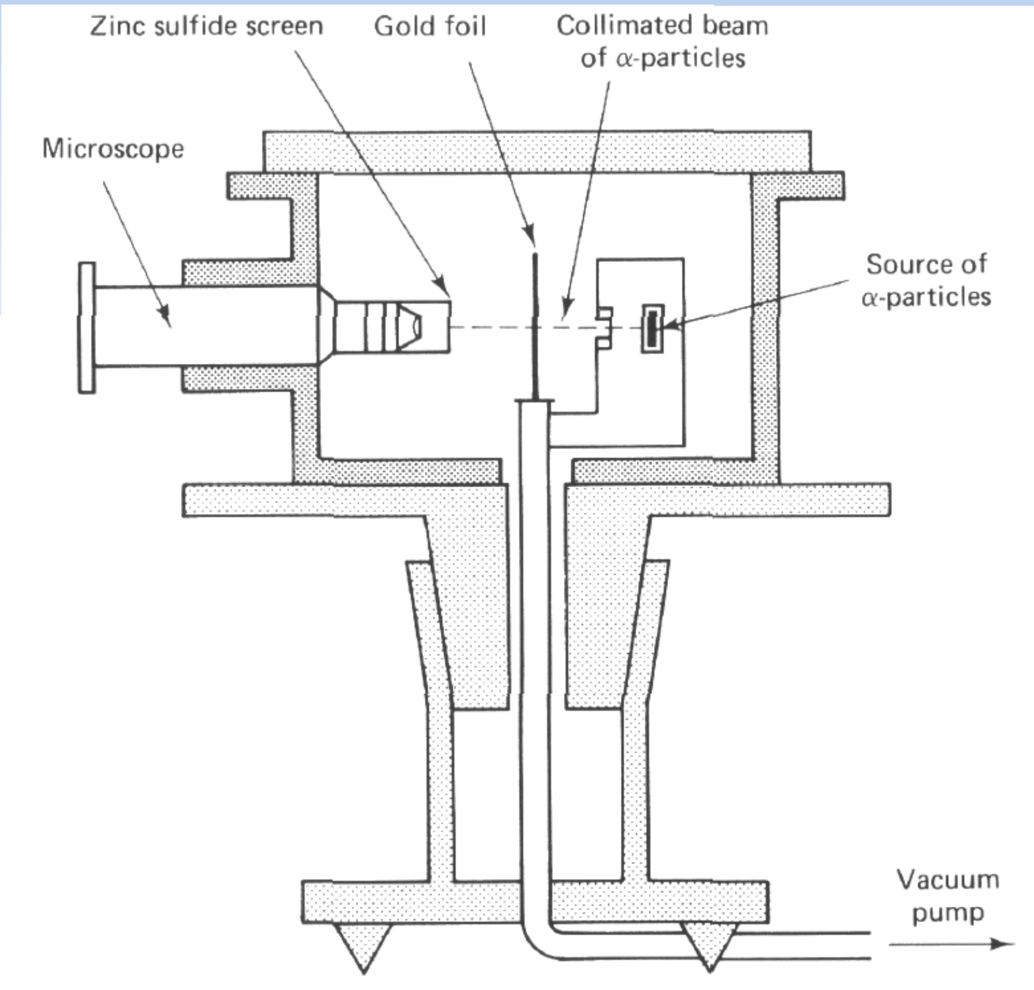
$$\text{Transition rate} = \frac{2\pi}{\hbar} |M^2| \times \text{ phase space}$$

Cross-section

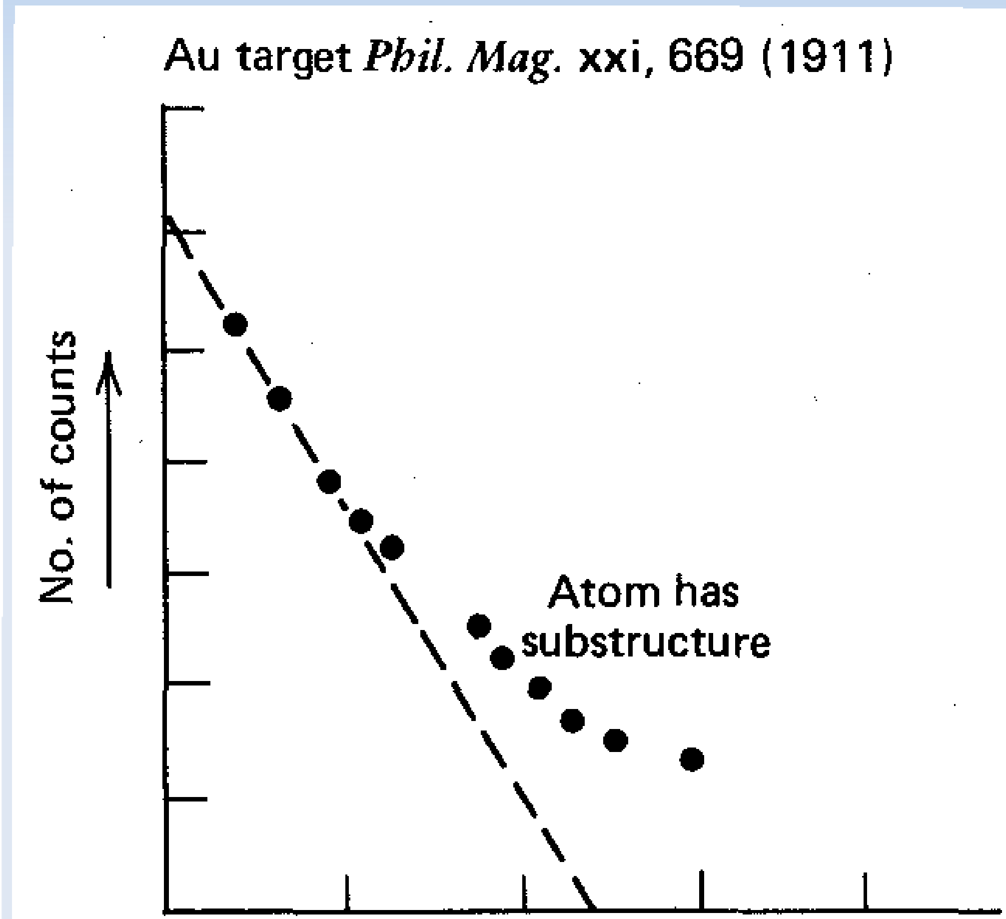
- Elementary interactions are not deterministic
- You can only know the probability of a collision and of producing a particular final state
- Experiment measures number of times particular interaction (with particular values of parameters) occurs
- Repeating experiment (collision) many times allows one to extract a probability distribution

Rutherford scattering

- The first “modern” scattering experiment



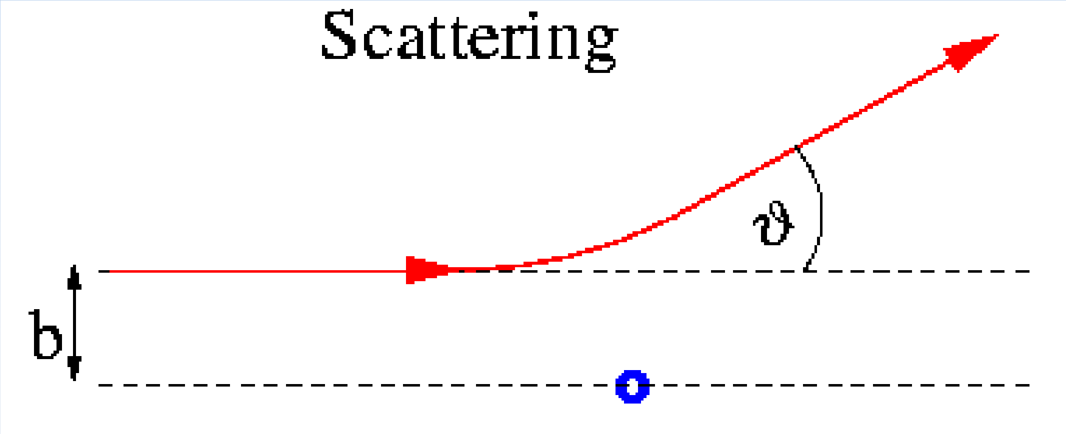
Rutherford, Geiger, Marsden 1909

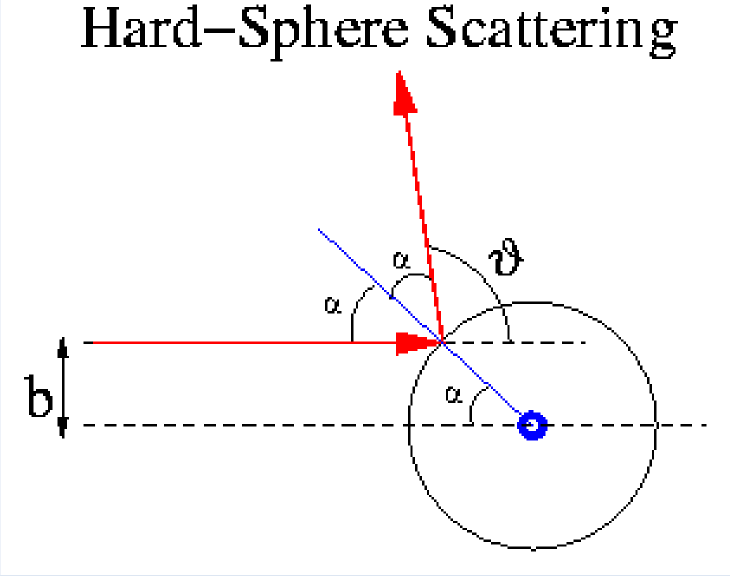


A first cross-section: Rutherford

- Scattering angle depends on impact parameter, b
- Calculation for hard-sphere (billiard-ball) scattering straightforward

$$d\sigma = D(\theta)\, d\Omega$$





$$\frac{d\sigma}{d\Omega} = \frac{R^2}{4} \quad \Longrightarrow \quad \sigma_{tot} = \int \left(\frac{d\sigma}{d\Omega} \right) d\Omega = \pi R^2$$

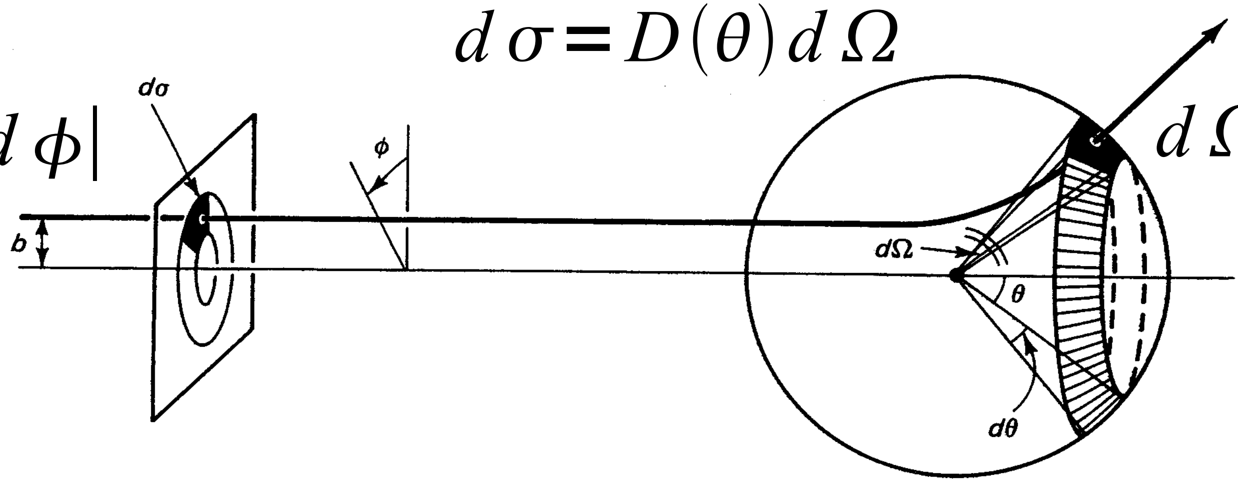
A first cross-section: Rutherford

- Calculation in Coulomb field more work:

$$d\sigma = D(\theta)\, d\Omega$$

$$d\sigma = |b\, db\, d\phi|$$

$$d\Omega = |\sin\theta\, d\theta\, d\phi|$$



$$V(r) = \frac{zZe^2}{r}$$

$$b = \frac{zZe^2}{2E_{kin}} \cot\left(\frac{\theta}{2}\right)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{4E_{kin}}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

Cross-section in experiment

- Experimental definition

$$\sigma = \frac{\dot{N}}{L\,\epsilon}$$

- In practice

$$\sigma = \frac{N}{\epsilon \int L\,dt}$$

- Luminosity is measure of possible collision rate

- Efficiency often has several components:
  - Trigger
  - Detector geometry
  - Reconstruction
- Error on cross-section
  - Statistical error
  - Efficiency error
  - Luminosity error

Counting events

- Signal
  - Absolute statistical error = $\sqrt{N}$
  - Relative statistical error = $1/\sqrt{N}$
- With background

$$\sigma = \frac{N_{sig}}{\epsilon \int L\,dt} = \frac{N_{tot} - N_{bkg}}{\epsilon \int L\,dt}$$

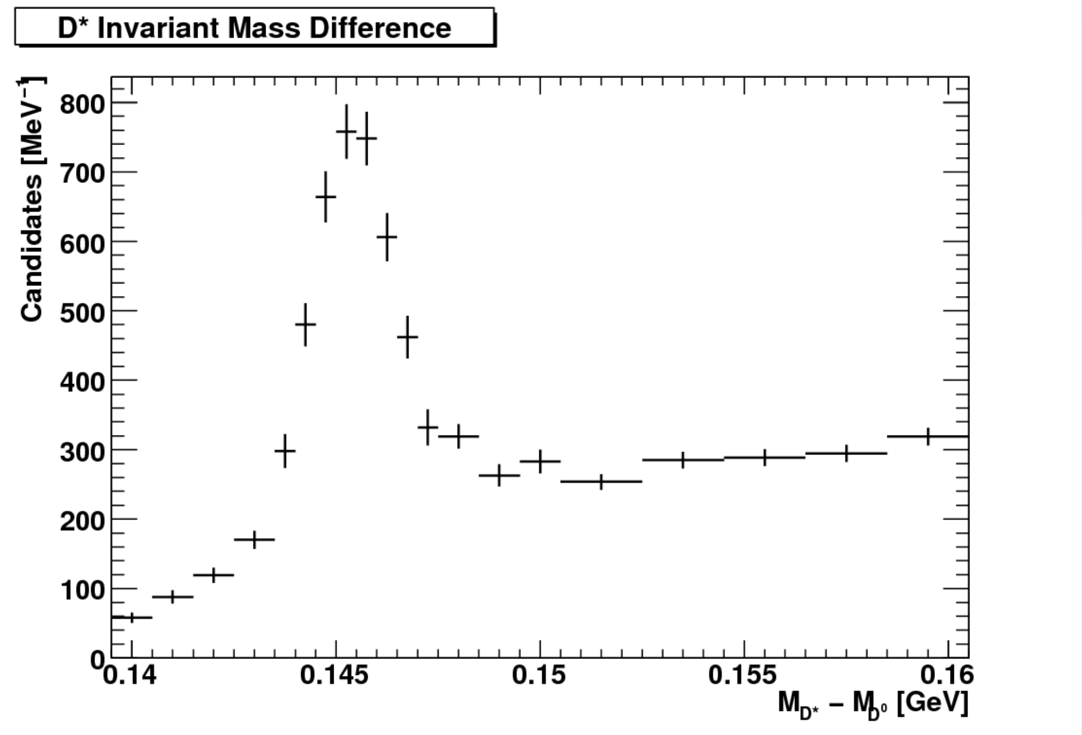
- Simple subtraction
  - Statistical error = $\sqrt{(N_{tot} + N_{bkg})}$
- Can we do better?
  - Subtraction or fitting?

Example 1: D* decay

- D^{*+} is an excited charm meson, m=2007 MeV
- Decays to $D^0 + \pi^+$, m = 1865 + 140 = 2005 MeV
- D^0 can decay to $K^- \pi^+$ (Br = 3.9%)
- Small mass difference means π^- follows D* direction and has low momentum π_s (slow)
- Reconstruct Kππ invariant mass
- Reconstruct Kπ invariant mass
- Take m(Kππ) - m(Kπ)

D* decay (ZEUS)

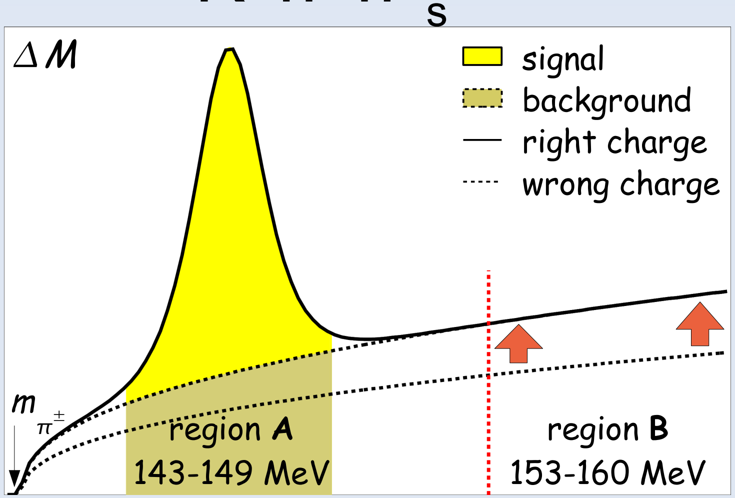
- Clear peak around 146 MeV seen
- Often called “golden” decay of D*
- How many D* are there in the peak?



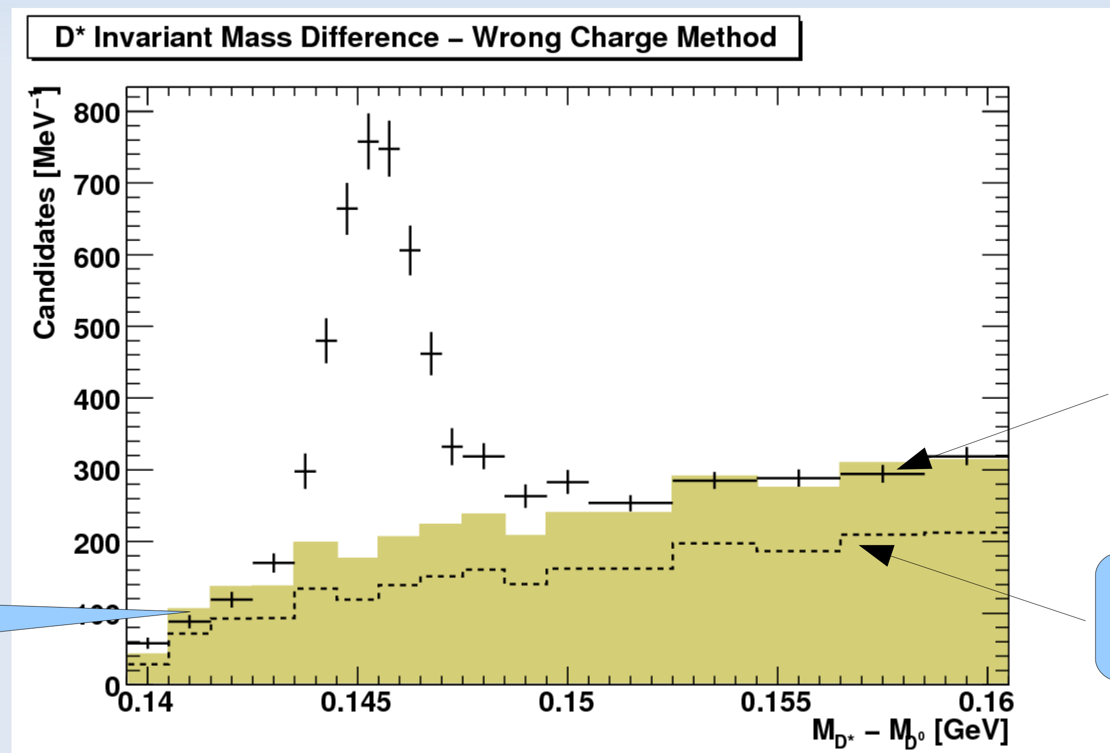
m(Kππ) – m(Kπ) [GeV]

D* decay (ZEUS)

- Expected charges (RC):
  - $K^- \pi^+ \pi^+_s$
- Wrong charge (WC) combination:
  - $K^- \pi^- \pi^+_s$
- Use WC combinations as background estimate
- Use region above peak to determine scaling factor



Is the wrong charge shape a good description of the background?

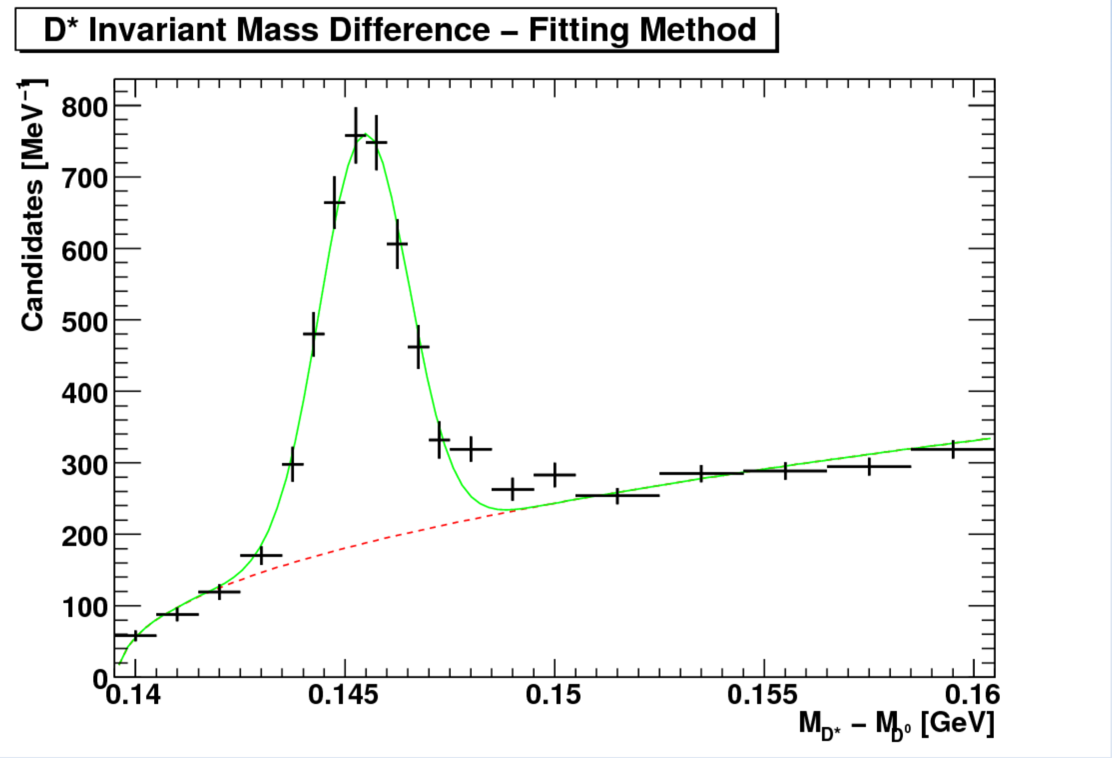


Scaled WC bkg

Unscaled WC bkg

D* decay (ZEUS)

- Fit signal and background
  - Gaussian
  - Polynomial (Chebyshev)
- Needs good description of shape
- Use same method for data and MC

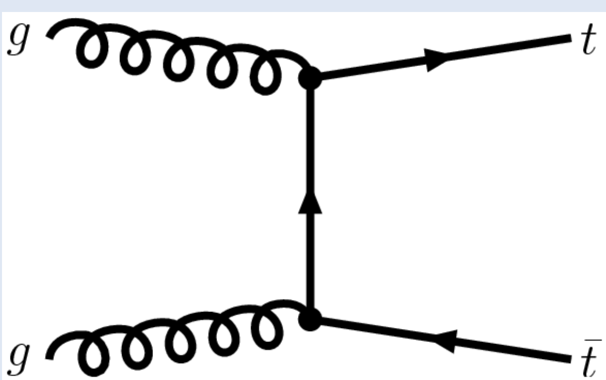


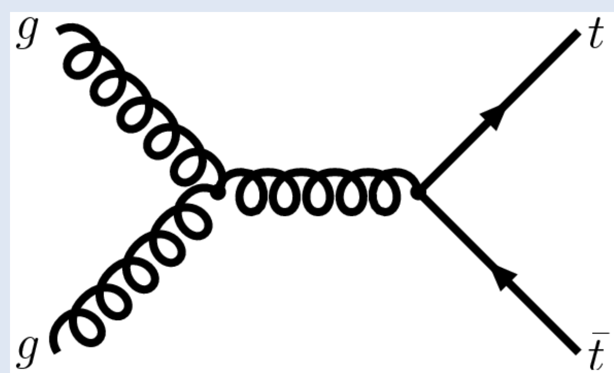
D* decay (ZEUS)

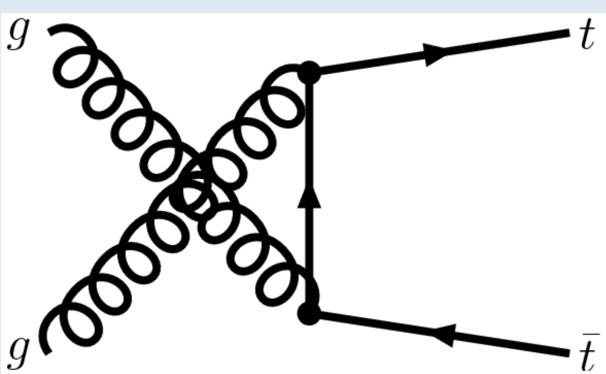
- Wrong charge method is simple and often works quite well
  - Error on signal are larger due to direct subtraction
  - What region do we define as “signal”?
  - Do data and MC have the same width?
- Fit method requires appropriate function
  - Error on signal smaller, as whole spectrum used to fix background
  - Easier to cope with different resolutions in data and MC

Example 2: Top quarks in pp collisions

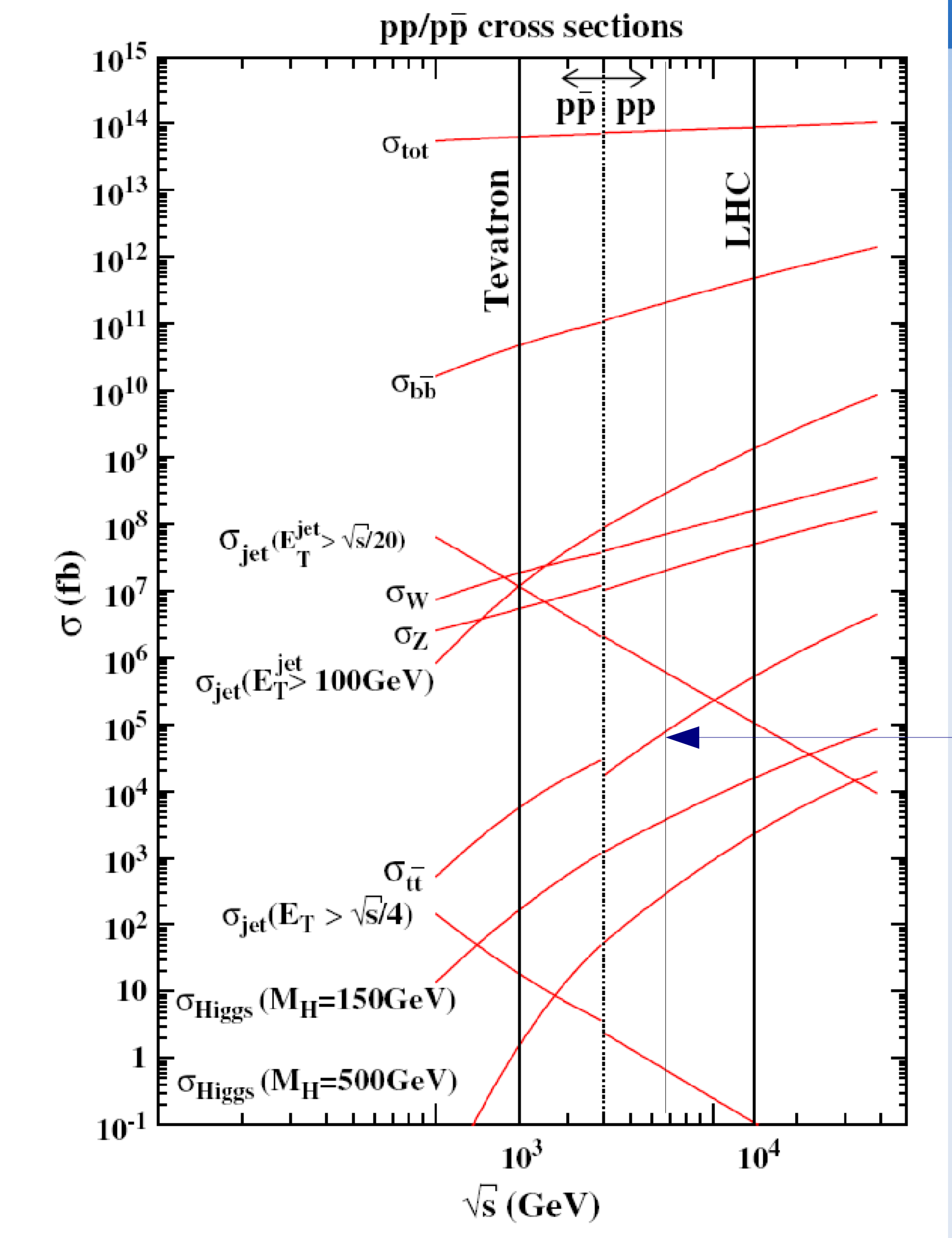
- Physics to be discussed tomorrow
- Here concentrate on inputs for cross-section measurement
- At LHC gluon-gluon fusion is main production mechanism







Expected cross-section



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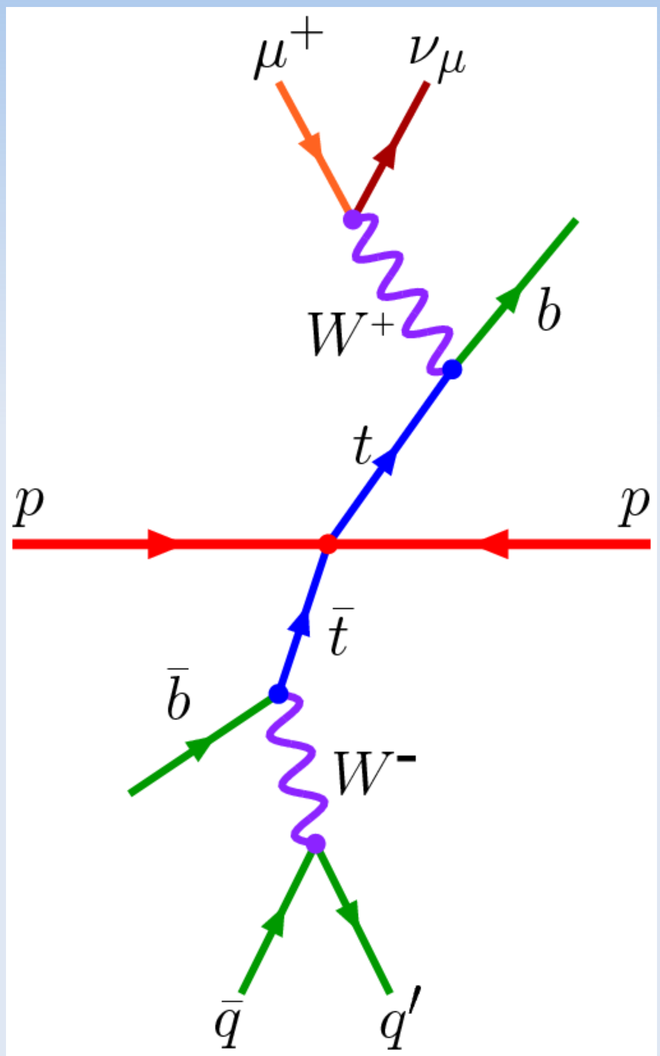
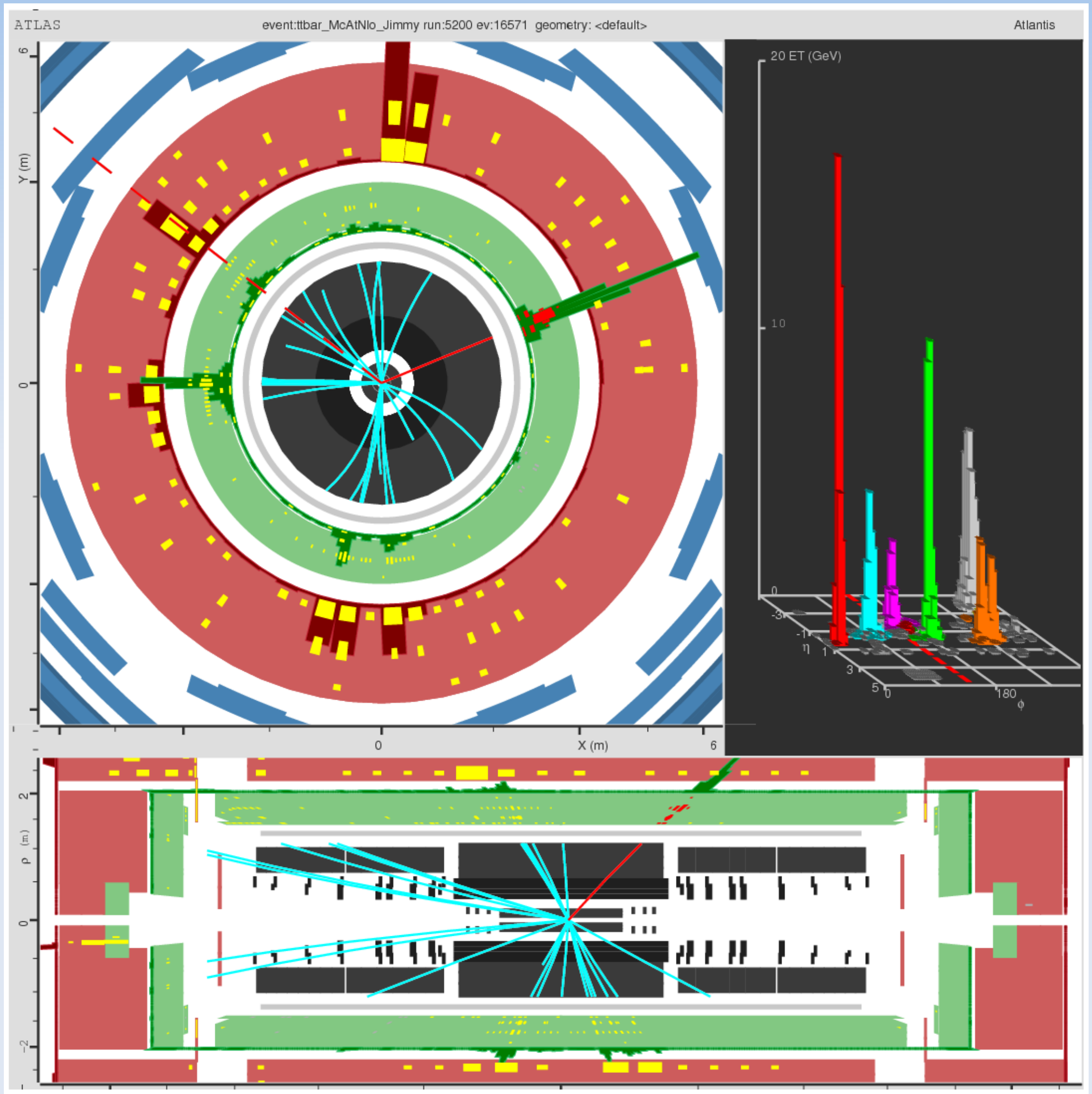
Top quark selection

- $t\bar{t}$ decay in pp collisions
  - B(t→blv + t→blv)   = 11%
  - B(t→blv + t→bqq)   = 45%
  - B(t→bqq + t→bqq) = 45%
- For illustration use lepton + jets channel
  - N.B. Leptons usually means e, μ;
  - Includes τ→μ,e, but not other τ decays



$t\bar{t}$ event

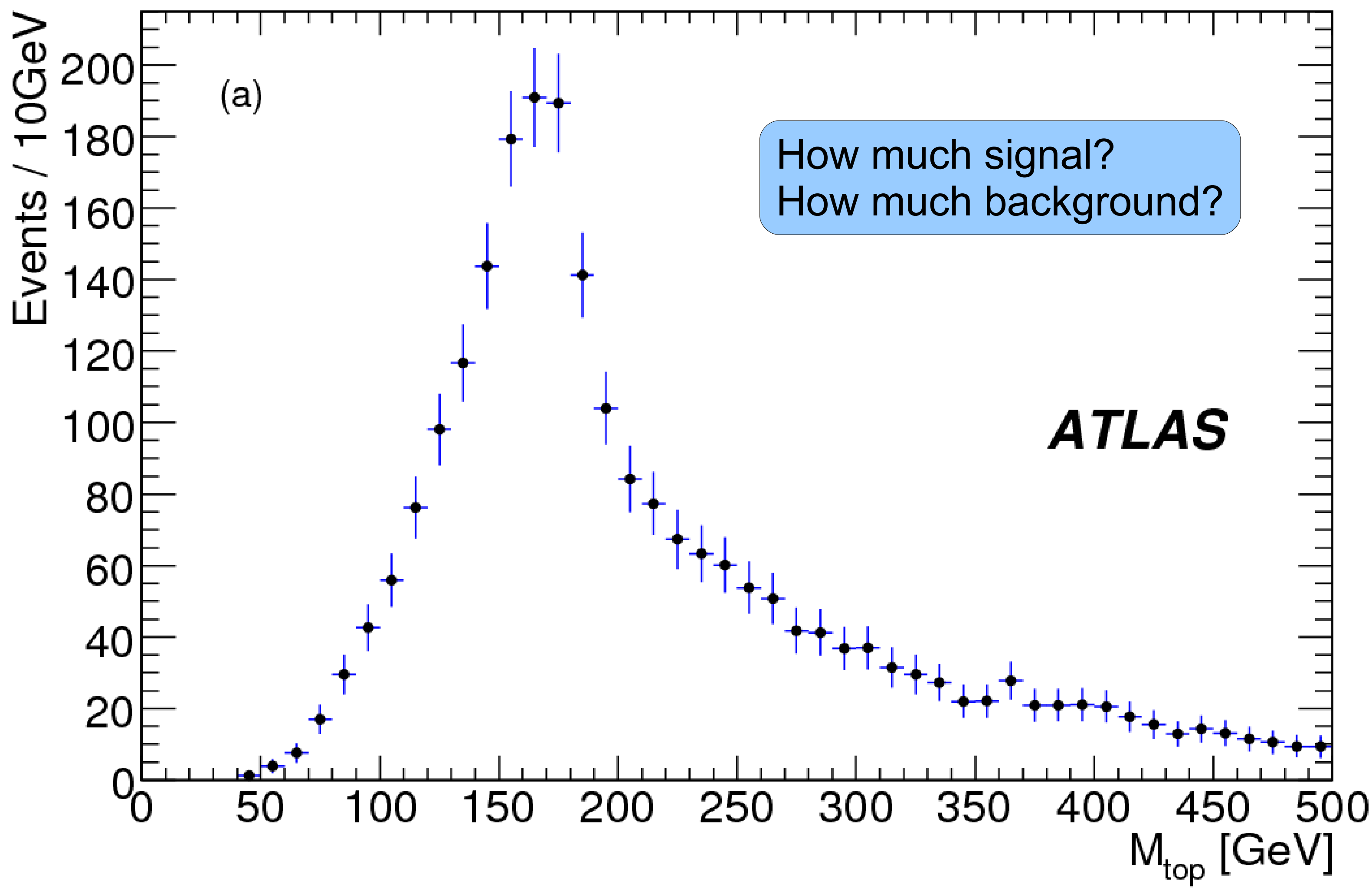


ATLAS
event:ttbar_McAtNlo_Jimmy run:5200 ev:16571 geometry: <default>
Atlantis
Y (m)
X (m)
20 ET (GeV)
10
η
φ
180
ρ (m)

Top quark selection

- Lepton + Jets selection
  - Select event with at least 4 jets (p_T > 20-40 GeV)
  - One and only one high p_T lepton
  - Missing transverse energy
- 3 jets from 1 top, 1 from other top
  - Select combination with highest p_T
    - Not very efficient - 30-40% correct assignment
  - Calculate invariant mass of 3 jets

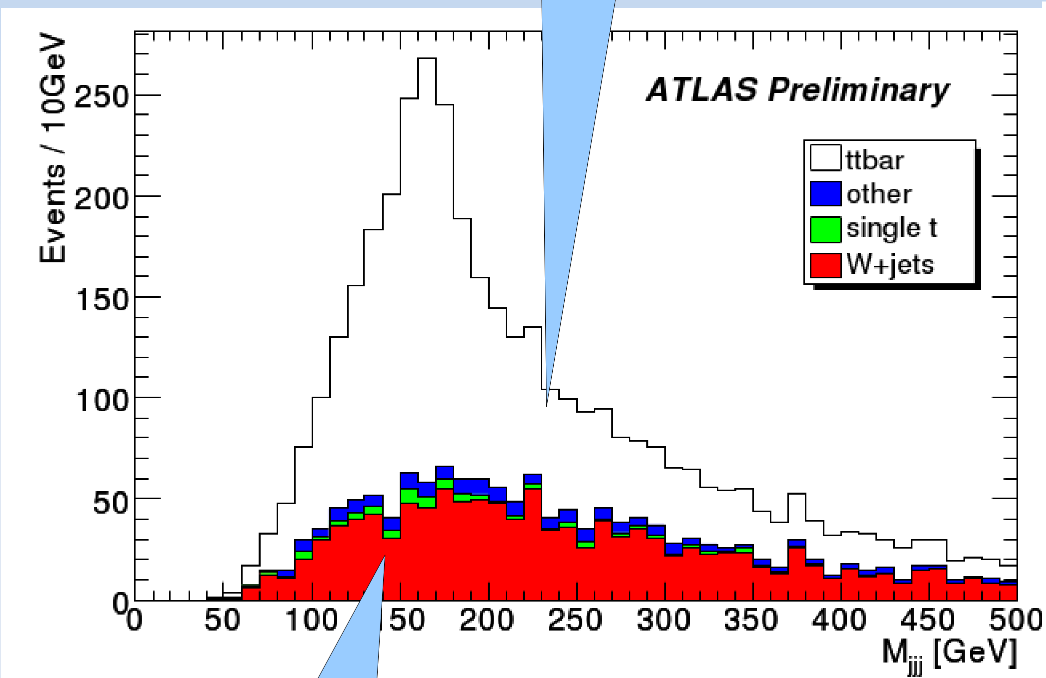
Top quark mass

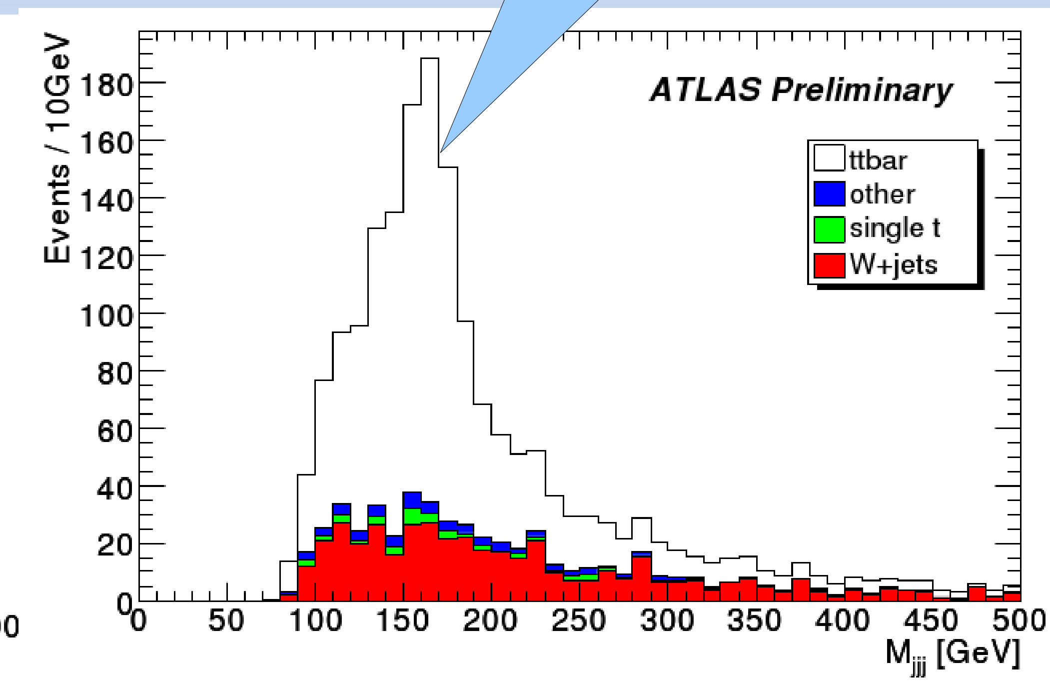


Background and combinatorics

Combinatoric background

Narrower peak
Shoulder structure





"Real" background

Require 2 jets to have invariant mass consistent with W mass

Signal and background

- Cut and Count
  - Define a signal range
  - Total number of events
  - Estimate background
  - Subtract background from total
- ATLAS top
  - $141 < m_t < 189$ GeV
  - N_{tot} = 4771 (2101)
  - N_{bkg} = 1497 (495)
  - N_{sig} = 3274 (1606)
  - S/B = 2.2 (3.2)

With W mass cut

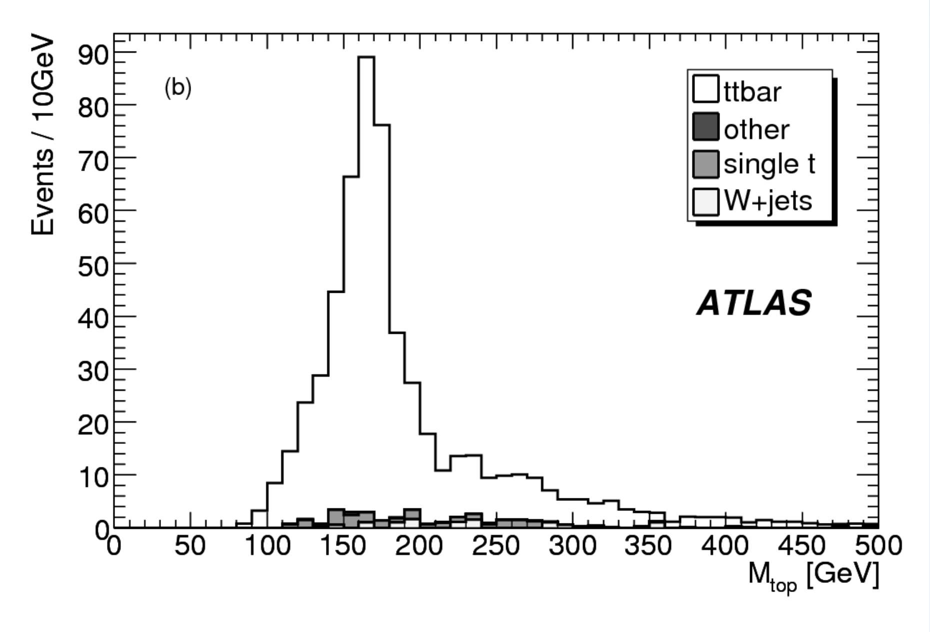
Signal and background

- Error on background?
- W+jet cross-section x Lumi
  - 20% lumi error → 30% error on cross-section (at 10 TeV centre-of-mass energy, S/B = 1.4)
- Or determine background from the data itself
  - e.g. use Z+jets events to estimate W+jets in signal dominated region ($N_{jet} \geq 4$)

Further Improvements?

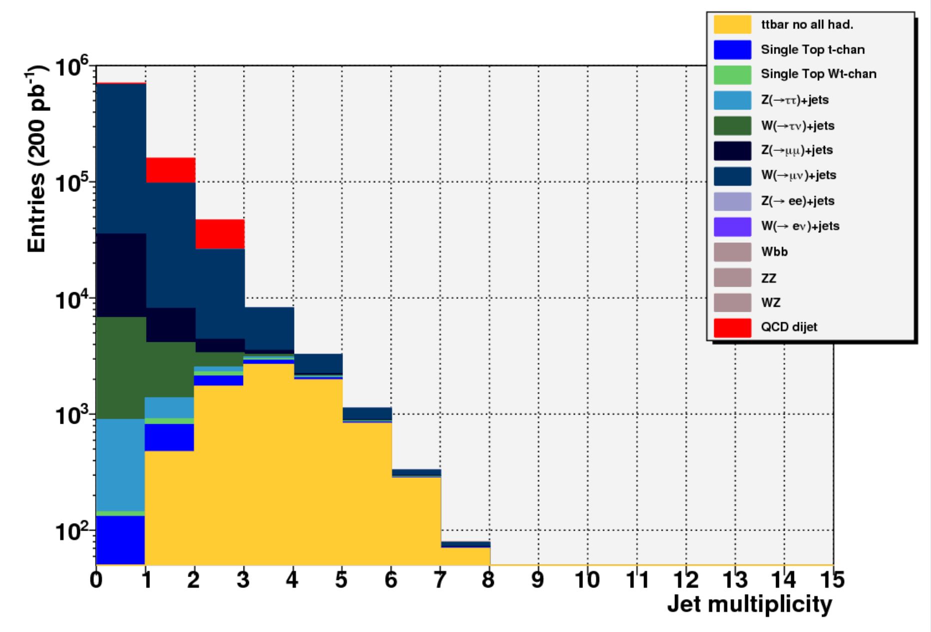
- Reduce the background
- Usually reduces efficiency, so increases statistical error
- But be careful of increased efficiency error

$$\left(\frac{\Delta\sigma}{\sigma}\right)^2 = \left(\frac{1}{N_{tot}+N_{bkg}}\right)^2 + \left(\frac{\Delta\epsilon}{\epsilon}\right)^2 + \left(\frac{\Delta L}{L}\right)^2$$



Further Improvements?

- Look at extra/different variables to separate signal from background
- Be very careful with number of jets!

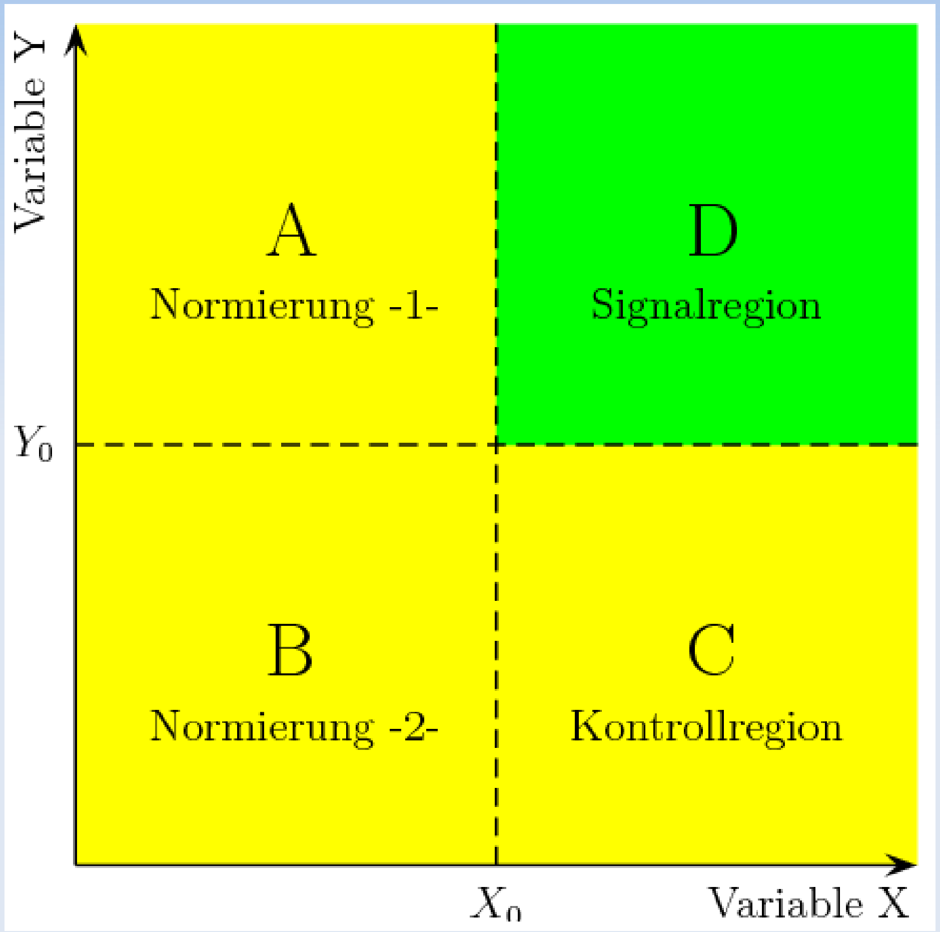


ABCD (Matrix) method

- Two variables to separate signal from background
- A, B, C background dominated

$$N_{bkg}^{D} = \frac{N_{bkg}^{A}}{N_{bkg}^{B}} N_{bkg}^{C}$$

$$\Rightarrow \quad N_{bkg}^{D} = \frac{N_{tot}^{A}}{N_{tot}^{B}} N_{tot}^{C}$$



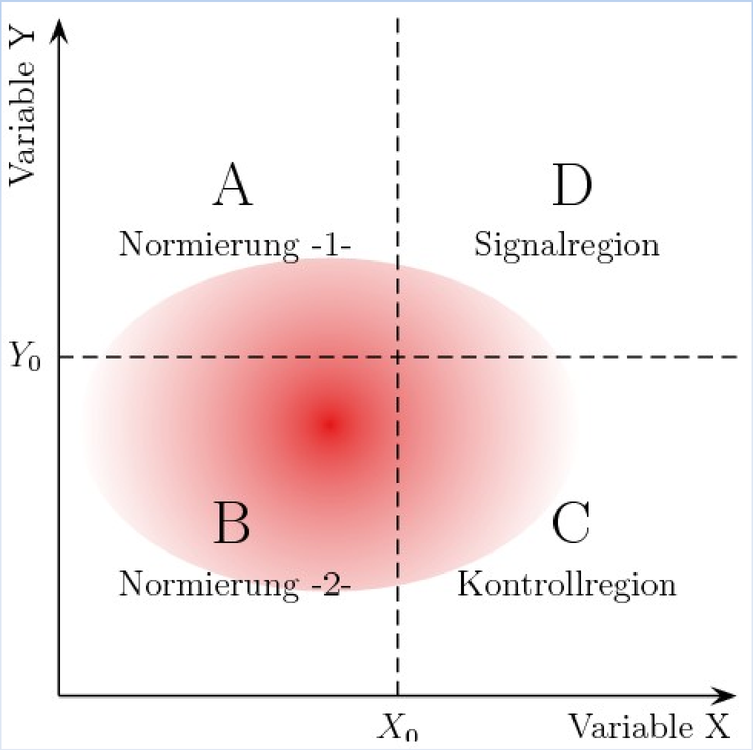
$$N_{sig}^{D} = N_{tot}^{D} - N_{bkg}^{D}$$

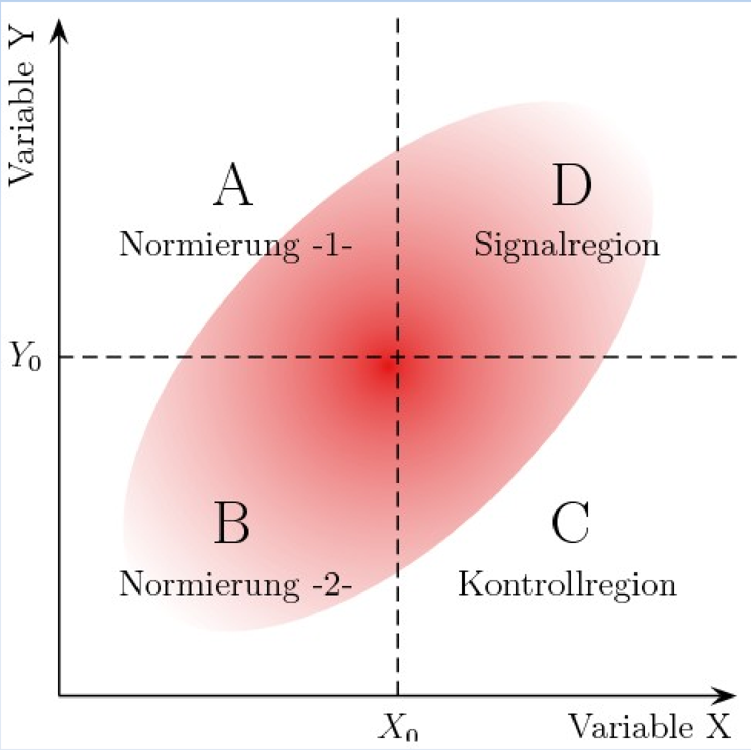
ABCD (Matrix) method

- Restrictions:
  - Signal contamination in A,B,C
  - Signal in C leads to overestimate of background
  - Signal in A, B leads to wrong ratio A/B
  - Cut values (X_0, Y_0) and correlations need MC

ABCD (Matrix) method

- Correlations are dangerous!



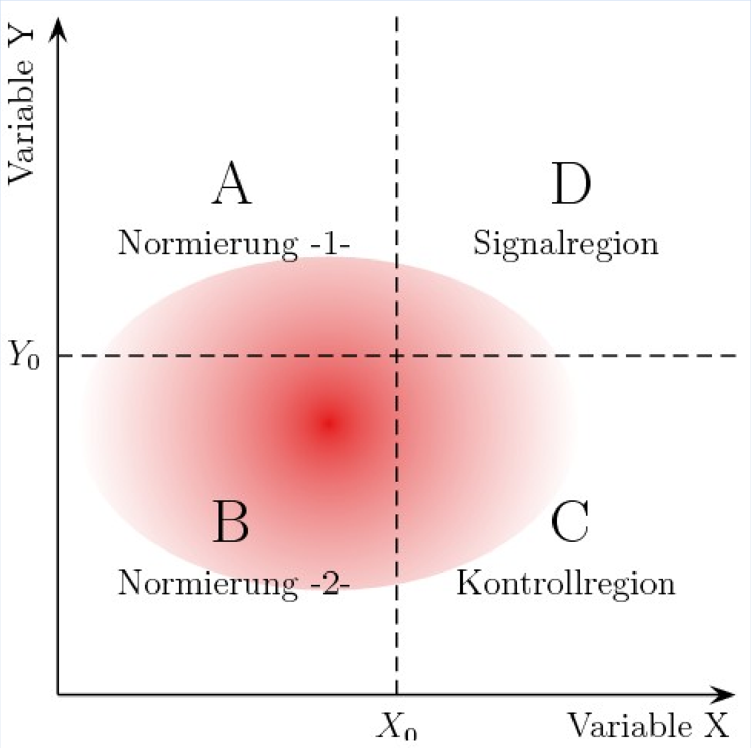


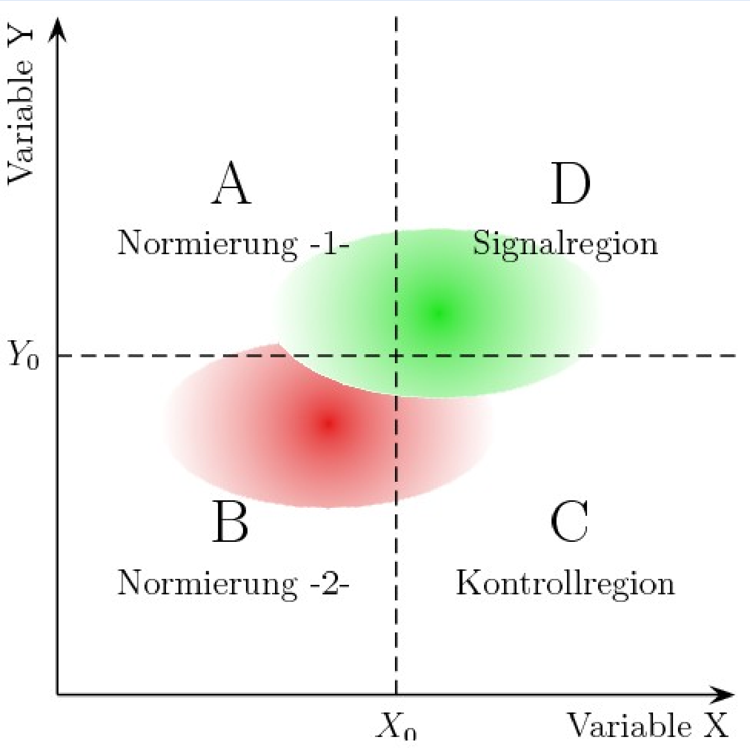
$$\frac{N_{bkg}^{D}}{N_{bkg}^{C}} = \frac{N_{bkg}^{A}}{N_{bkg}^{B}}$$

$$\frac{N_{bkg}^{D}}{N_{bkg}^{C}} > \frac{N_{bkg}^{A}}{N_{bkg}^{B}}$$

ABCD (Matrix) method

- Sum of 2 uncorrelated backgrounds can still give a correlation!





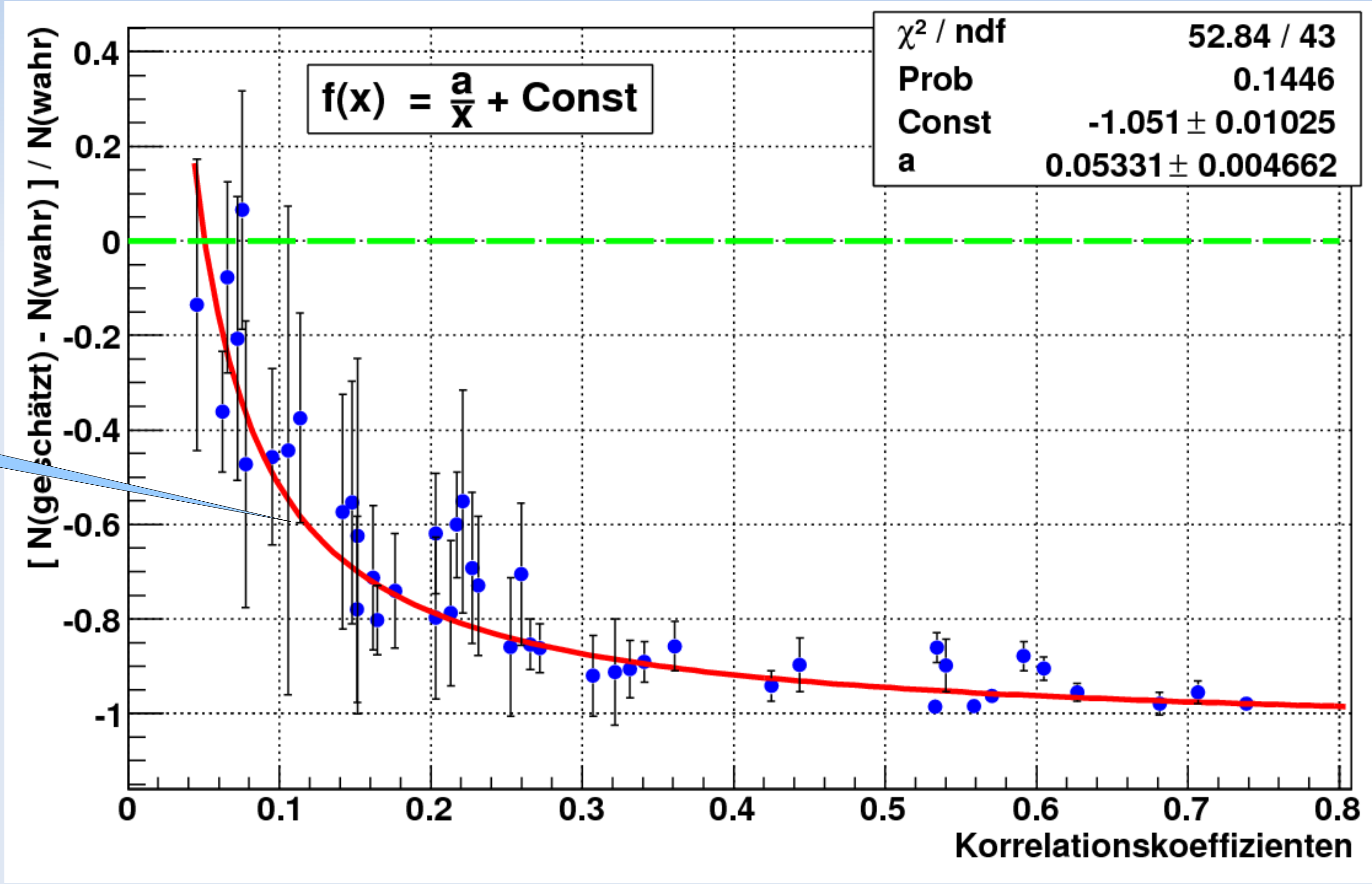
$$\frac{N^D_{bkg}}{N^C_{bkg}} = \frac{N^A_{bkg}}{N^B_{bkg}}$$

$$\frac{N^D_{bkg}}{N^C_{bkg}} > \frac{N^A_{bkg}}{N^B_{bkg}}$$

ABCD (Matrix) method

- What size correlation causes what effect?

Small correlations can have big effects!



Powerful method,
But be careful!

Acceptance, Efficiency, Purity

- Efficiency:

$$\frac{\text{Number of signal events passing cuts}}{\text{Number of signal events}}$$

- Purity:

$$\frac{\text{Number of signal events passing cuts}}{\text{Number of events passing cuts}}$$

- Acceptance (not universally accepted defn.)

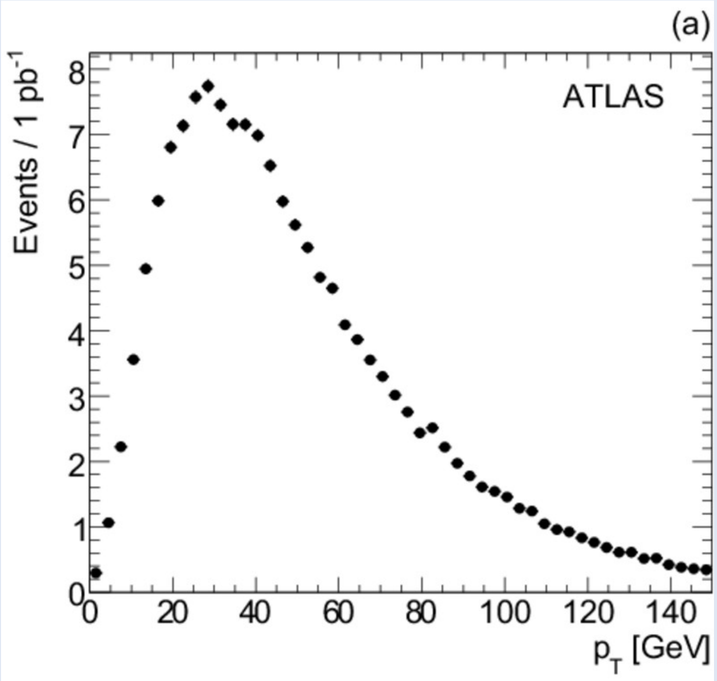
$$\frac{\text{Number of events passing cuts}}{\text{Number of signal events}}$$

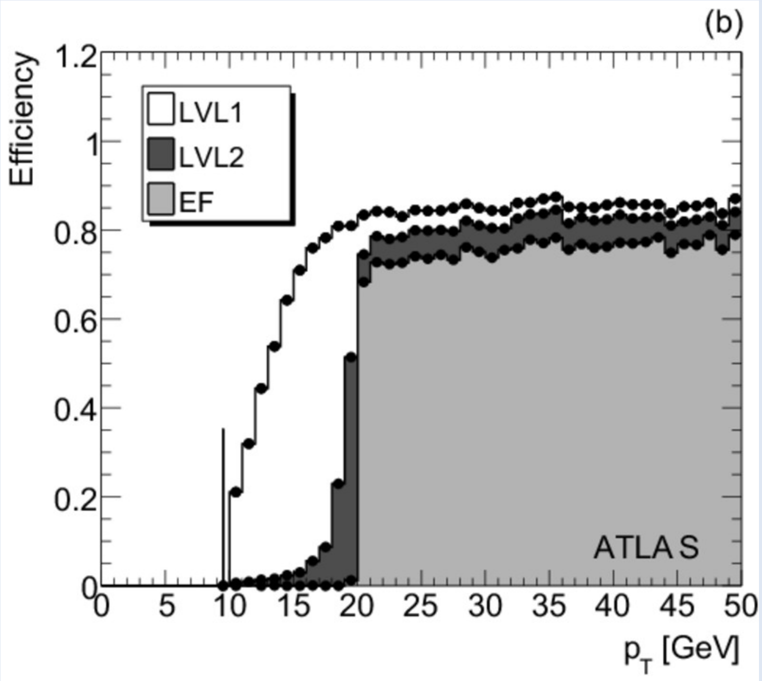
Efficiency Determination

- Often use Monte Carlo simulation
  - Simulate 4-vectors of interaction
  - Simulate decays of unstable particles
  - Simulate response of detector to particles passing through it
- Apply same reconstruction to real data and Monte Carlo events
- To reduce statistical fluctuations, need more MC events than data (signal)

Trigger Efficiency

- Specify cuts used for final selection
- Determine trigger efficiency for such events
  - e.g. Muons from leptonic $t\bar{t}$ events





Tag and Probe

- Use data to measure efficiencies
- Identify events of a certain type without using information to be investigated
- Best are events with “doubled” signatures, e.g. pair production of top quarks
  - Tag one half of event for signature
  - Probe the other half to measure efficiency

ATLAS example

- Use $Z \to \ell\ell$ events to measure lepton trigger efficiency
- Select clean $Z \to \ell\ell$ sample
- Take one triggered lepton as tag
- Probe the other lepton to measure efficiency

Matrix method (loose & tight cuts)

- Can also set up a set of equations using loose and tight cuts to estimate background
- Measure efficiency using tag&probe
- Measure fake rate with background dominated sample
- Look at number of events satisfying loose-loose, loose-tight and tight-tight cuts
- Use truth information to find number of events with true-true, true-fake, fake-true
- Set up set of equations and solve for number of fakes

Differential cross-section

- Often want cross-section as a function of a variable, e.g. p_T, η

$\eta = -\ln\tan(\theta/2)$

- Divide data into bins in the variable (bin widths do not all have to be the same)
  - Ensure enough entries in each bin
- For bin i

$$\frac{d\sigma}{dx} = \frac{N_i}{\Delta x_i \epsilon \int L\,dt}$$

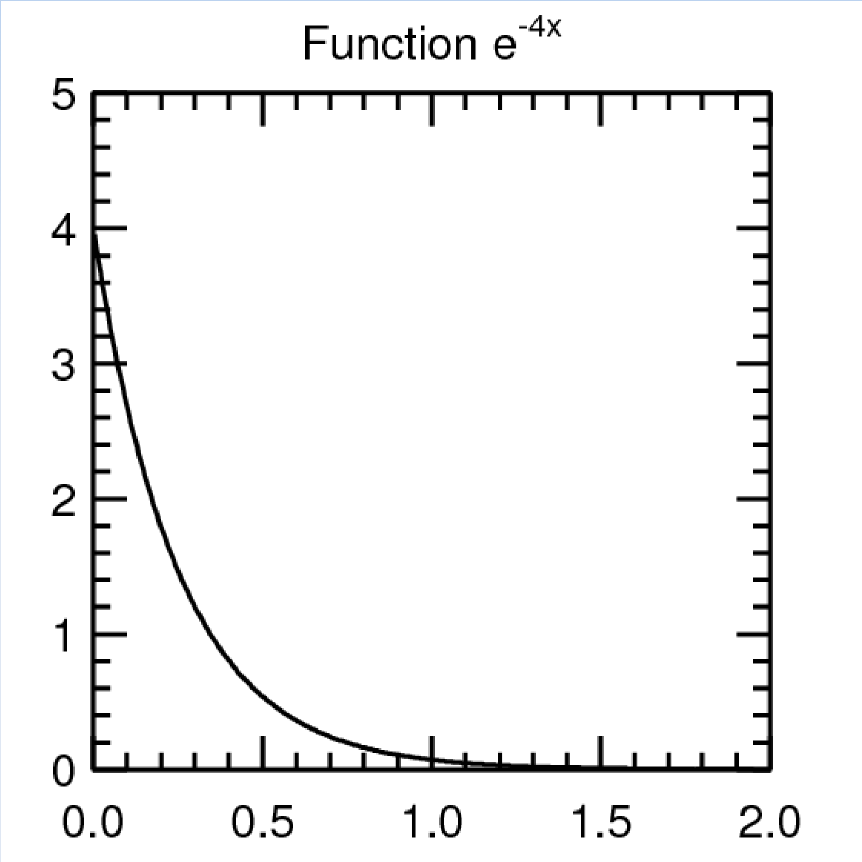
Δx_i is width of bin i

Migration

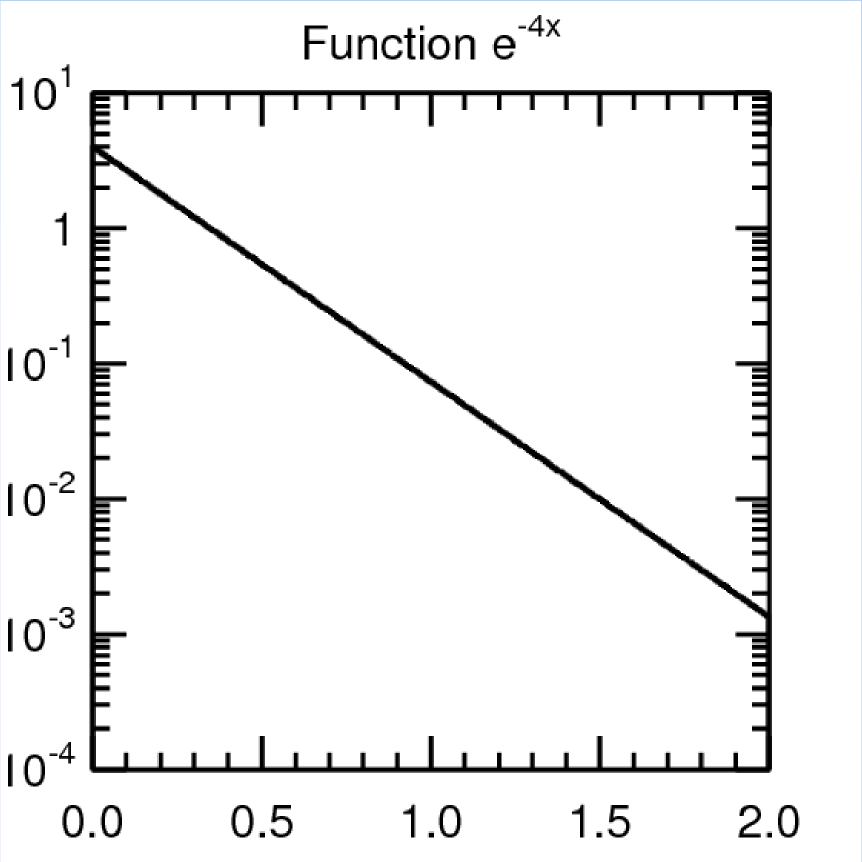
- You've determined the background
- What about the resolution?
  - Detector response to energy deposit fluctuates:
    - EM calorimeter $\sigma_E/E = 10\text{-}20\%/\sqrt{E}$ (GeV)
    - Hadron calorimeter $\sigma_E/E = 50\text{-}100\%/\sqrt{E}$ (GeV)
  - Tracking resolution $\sigma_{pT}/p_T \propto p_T^2$
  - Jet energy resolution a combination of both
- pp cross-sections fall fast

Toy Example

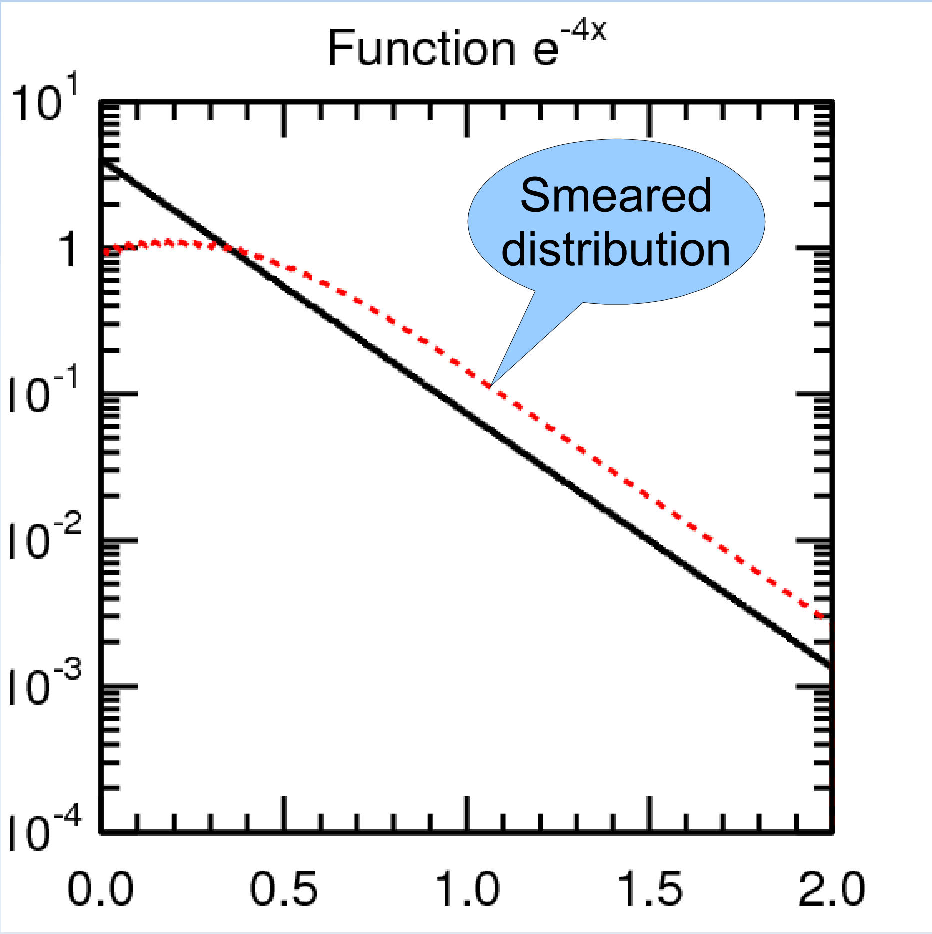
Linear scale



Log scale



Resolution effects



- Smeared with a Gaussian, σ = 0.3
- Bin contents change by large amount
- Purity ~50%!
- Use acceptance to unfold true distribution

Unfolding migration effects

$$\left(\frac{d\sigma}{dx}\right)_i^{data} = \frac{N_i^{data}}{N_i^{MC}} \cdot \left(\frac{d\sigma}{dx}\right)_i^{MC}$$

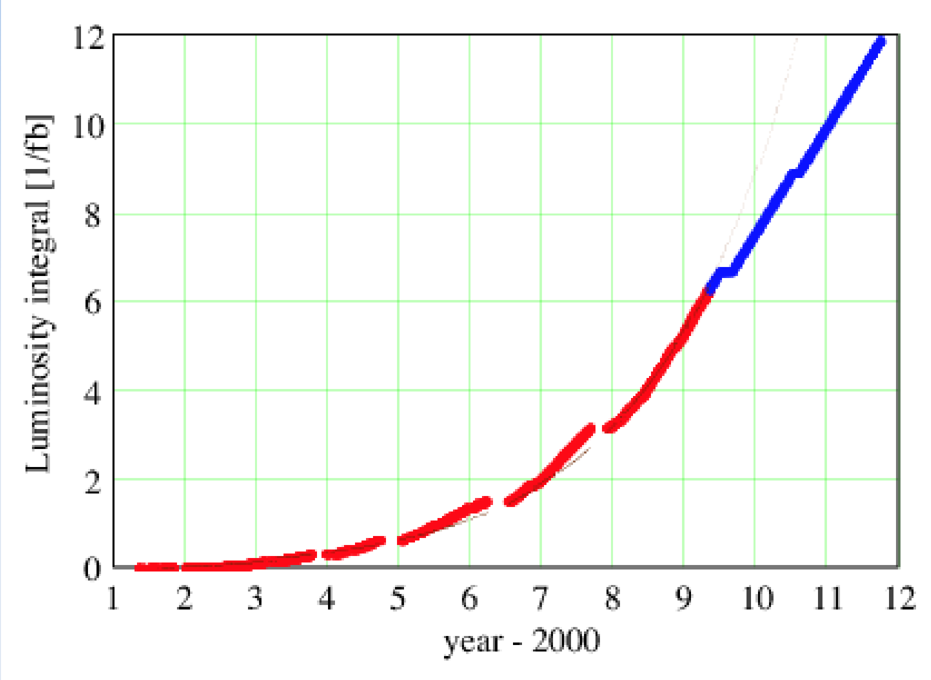
- i.e. simply scale MC by ratio of data to MC
- Method is very nice, but assume MC provides good description of (shape of) data
- May need iterations
- Reduce migration effects by choosing bin width $\gg$ resolution
- Rule of thumb: purity in each bin $\geq$ 50%

Luminosity

- Number of particles that can interact per unit area per second
- From machine parameters
- e^+e^-, ep and pp
- Absolute and relative
- Yesterday's discovery is today's tool and tomorrow's background
  - W production for lumi measurement

Typical luminosities

- Usual units are $cm^{-2}s^{-1}$
- LEP $10^{31} - 10^{34}$
- HERA $10^{31} - 10^{32}$
- Tevatron $10^{30} - 10^{33}$
- LHC $10^{32} - 10^{34}$
- It usually takes a while to reach design / maximum luminosity



1 nb = 10^{-33} cm^2
L = 10^{33} $cm^{-2}s^{-1}$
Event rate 1 Hz

From machine parameters

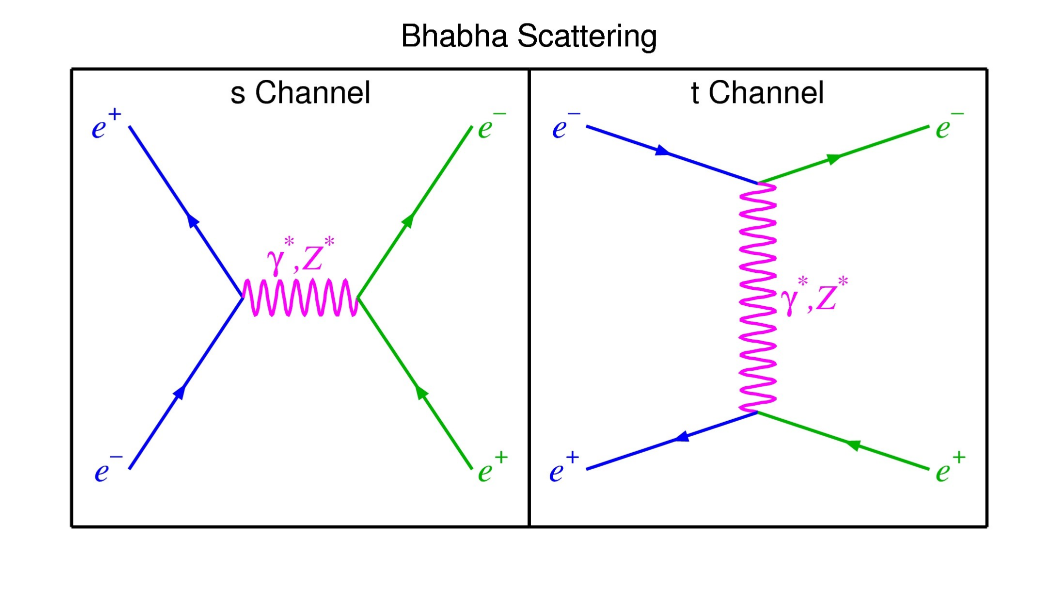
- n bunches, N_1, N_2 particles per bunch
- Particles passing crossing point per second: $N_1 N_2 n f$

$$L = f\, n \frac{N_1 N_2}{A} = f\, n \frac{N^2}{4\pi\sigma_x \sigma_y}$$

- Beam-beam effects can cause weaker dependence than N^2
- What are beam sizes?
- Do bunches overlap fully?

e^+e^- machines

- Use Bhabha process
  - Dominated by QED at small angles
  - High rate
  - Can be calculated to high precision
- 1‰ experimental and theoretical precision achieved at LEP



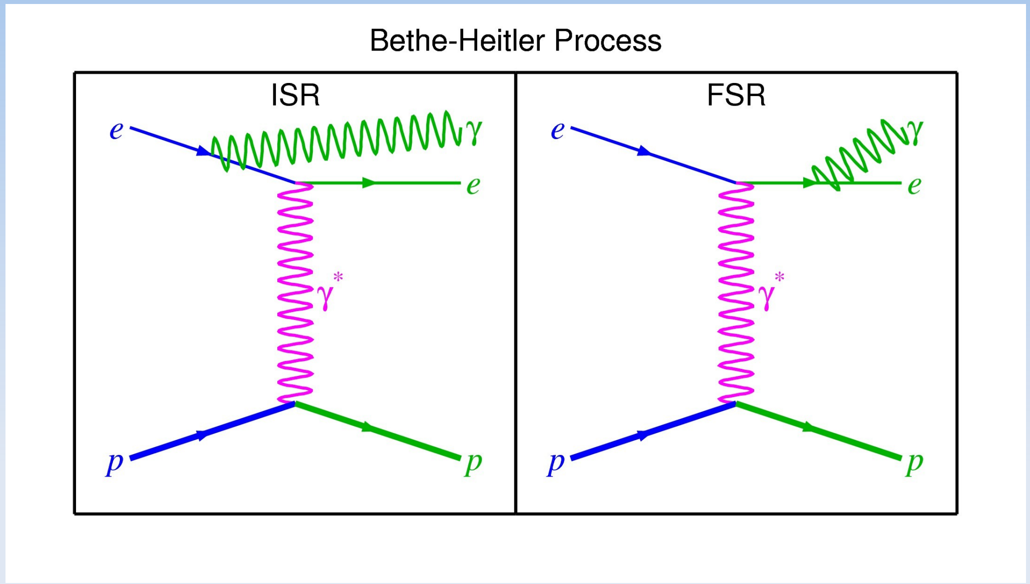
$$\sigma = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta_{min}^2} - \frac{1}{\theta_{max}^2}\right)$$

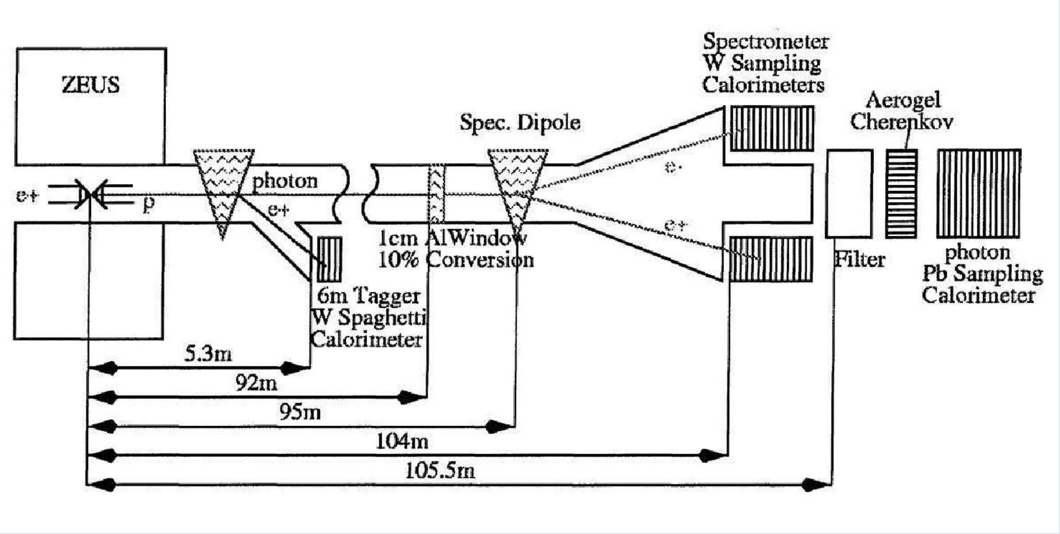
ep machine (HERA)

- Use Bethe-Heitler process

$$e\,p \rightarrow e\,p\,\gamma$$

  - QED process
  - High rate
  - Good theory precision
- 1-3% precision achieved at HERA





pp machine

- No obvious QED process
- Have to cope with multiple interactions per bunch crossing
- pp elastic scattering at very small angles:
  - $t = (p_{in} - p_{out})^2 \approx (p\theta)^2$
- Measure relative rate using small angle detector or even hadron calorimeter endcap
- Special detector to measure absolute lumi (only works for low lumi, so need to extrapolate)

pp machine

- ATLAS has >3 different devices that contribute to lumi measurement!
- Count number of interactions in short time period 1-2 mins (luminosity block)
- Have to keep track of which lumi blocks used in analysis!
- Expect initial accuracy of 10-20%
- May reach 5-10% after detailed studies
- NNLO predictions of W production cross-section now exist, with accuracy of <5%
- Rate is high enough to use as a lumi measurement!

Connecting theory with experiment

- Factorisation of cross-section (ep)

$$d\sigma(ep \to e'X) = \sum_{\text{partons}} \int_0^1 dx\, f_{i/p}(x, \mu_f^2) \cdot d\hat{\sigma}(\hat{s}, \alpha_S(\mu_R), \mu_R, \mu_F)$$

$$\hat{s} = x\, s$$

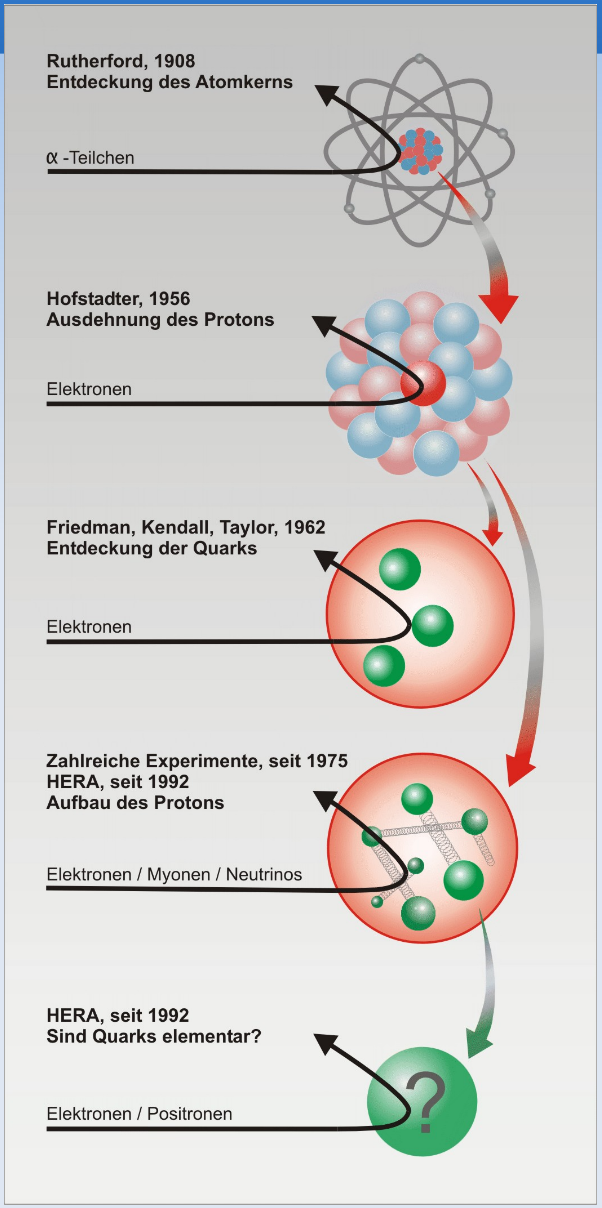
- Factorisation of cross-section (pp)

$$d\sigma(pp \to X) = \sum_{p_j} \sum_{p_i} \int_0^1 \int_0^1 dx_1\, dx_2\, f_{i/p}(x_1, \mu_f^2)\, f_{j/p}(x_2, \mu_f^2) \cdot$$
$$d\hat{\sigma}(\hat{s}, \alpha_S(\mu_R), \mu_R, \mu_F)$$

$$\hat{s} = x_1 x_2 s$$

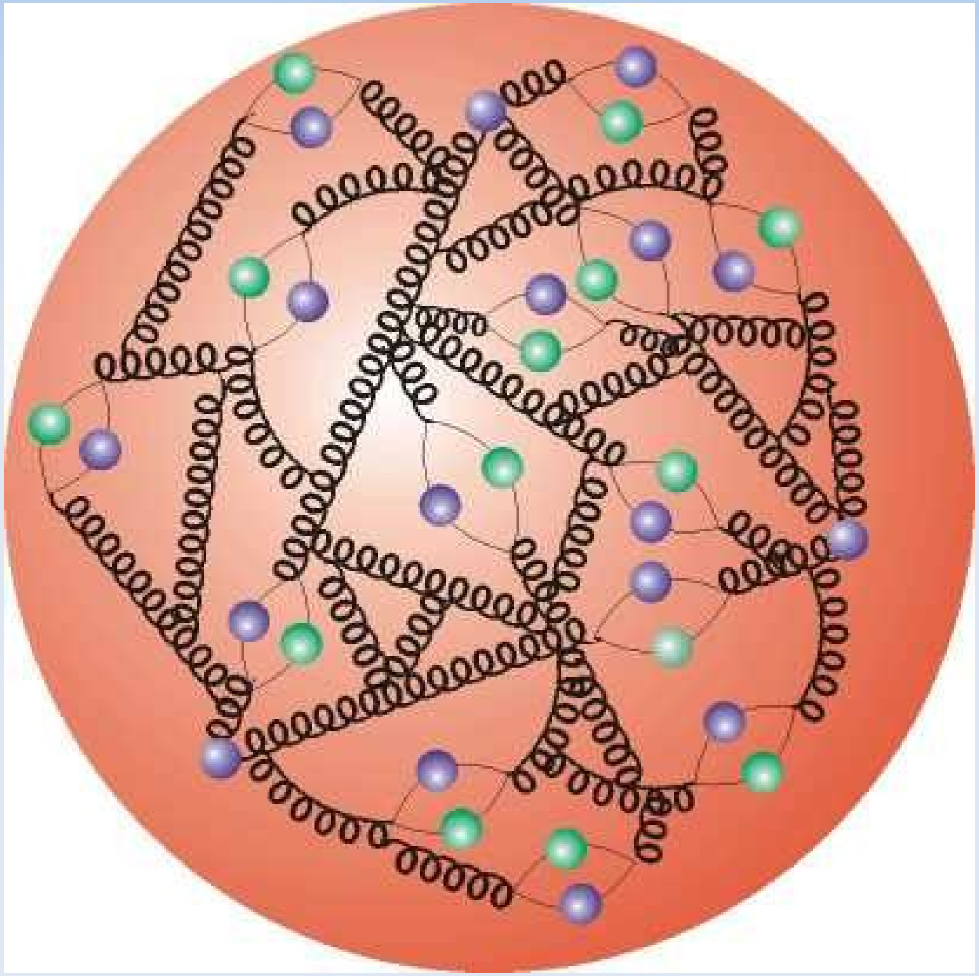
- $f_i(x, \mu_F)$ is probability to find parton of type i with momentum fraction x in proton

Inside a Proton



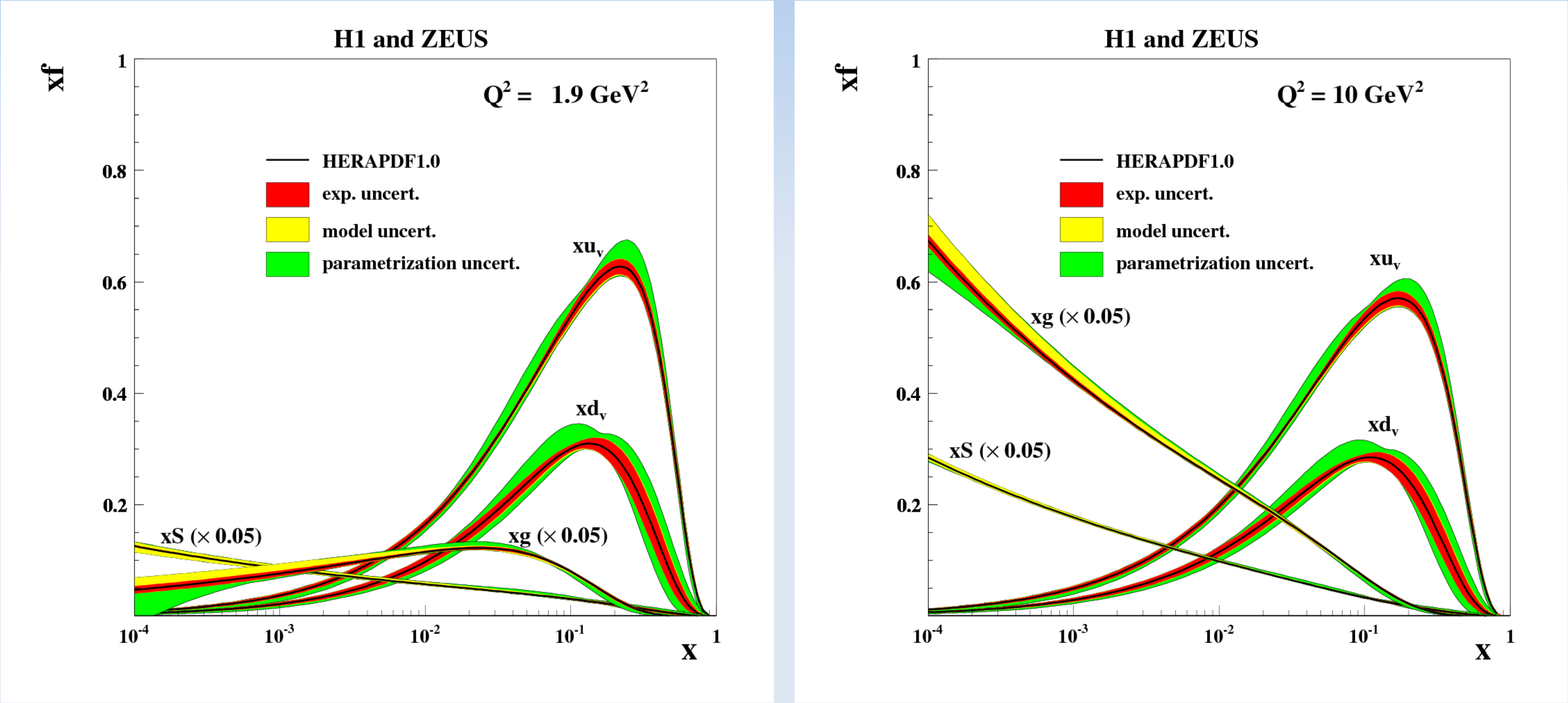
- Scattering experiments can resolve substructure
- The higher the energy the better the resolution
- First glance: a proton consists of 3 quarks (uud)
  - The quarks are pointlike
  - ~50% of the momentum carried by gluons

Inside a proton with HERA



- The proton is much more complicated!
- Several hundred quarks und gluons
- The more accurately you look the more you see

Inside a proton with HERA



Systematic Uncertainties

- Jet energy scale (few %)
  - Use constraints
    - Meson masses
    - W,Z mass
    - Photon opposite jet
- Trigger (few %)
  - Try to measure with data as much as possible
- Monte Carlo simulation
  - Tricky!
  - Vary renormalisation and factorisation scales by a factor of 2 !?

How big is a 1σ systematic uncertainty?
Is there a 68% chance that true value lies in given range?

Cooking up a cross-section

- Counting number of events is the easy part!
  - Background, migration, ...
- Efficiency
  - Can use MC, but better to use data as much as possible
- Luminosity
  - Someone else probably provides the numbers, but bookkeeping is not simple
- Never say, only have to determine systematics