No global symmetries

example of global symm.:

global $U(1)$ of complex scalar in D dim.s

$$\mathcal{L} = \partial_\mu \phi \, \partial^\mu \bar{\phi}$$

$$\phi \to e^{i\alpha} \cdot \phi = \phi + i\alpha \cdot \phi$$

α infinitesimal

$$\Rightarrow \mathcal{L} \to \mathcal{L} + i\alpha \cdot \partial_\mu \phi \, \partial^\mu \bar{\phi} - i\alpha \cdot \partial_\mu \phi \, \partial^\mu \bar{\phi} + O(\alpha^2)$$

$$= \mathcal{L} + O(\alpha^2)$$

invariant

conserved current & charge:

$\to$ localize symm. transf. to small region

$$\phi \to \phi + \rho(x^\mu) \cdot i\alpha \phi$$

$$\Rightarrow S = \int d^4x \, \mathcal{L} \to$$

$$\underbrace{S - \int d^4x \cdot \alpha \partial_\mu \rho \cdot \overbrace{\left[i\phi \partial^\mu \bar{\phi} - i\bar{\phi} \partial^\mu \phi \right]}^{\equiv j^\mu}}_{= -\int d^4x \cdot \alpha \cdot \rho \cdot \partial_\mu j^\mu}$$

look at quantum theory ...

path integral:

$$\int \mathcal{D}\phi \, e^{-S[\phi]}$$

only insertions outside small region of support for $\rho(x^\mu)$:

$$\Rightarrow\ 0 = \delta_{\phi \to \phi + \rho \cdot i\alpha\phi} \int \mathcal{D}\phi\, e^{-S} \ldots$$

$$= -\alpha \cdot \int d^4x \cdot \rho(x^\mu) \langle \partial_\mu j^\mu \ldots \rangle$$

$\Rightarrow\ \partial_\mu j^\mu = 0$ Noether's theorem

$\Rightarrow\ Q = \int d^3x\, j^0$ conserved charge

Now look at global (space-time) symm.s acting on string worldsheet

$$S = \frac{1}{\alpha'} \int d^2\sigma\, g_{\mu\nu}\, \partial_a X^\mu \partial^a X^\nu$$

space-time coord.s of string

$a = \tau, \sigma$

$a \to \tau \pm \sigma$: light-cone coord.s

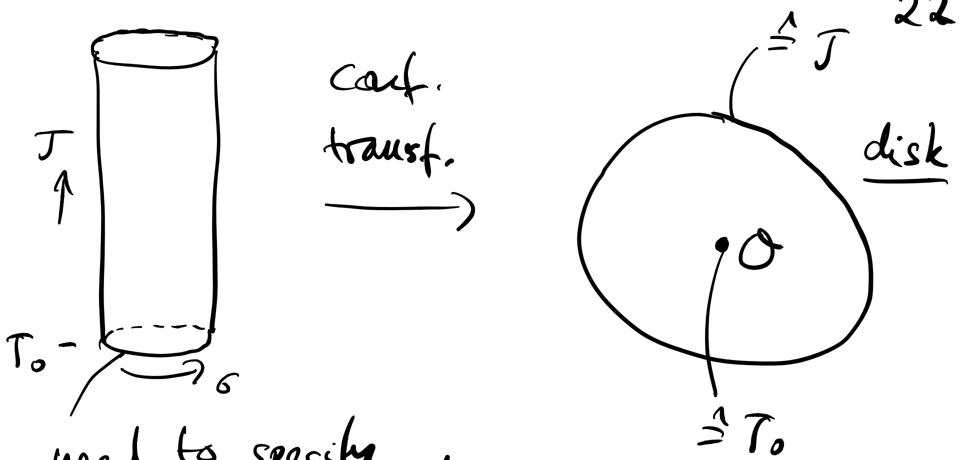
$$z, \bar{z} = e^{-(\tau \mp i\sigma)/\sqrt{\alpha'}}$$

$$S = \frac{1}{\alpha'} \int d^2z\, \partial X^\mu \bar{\partial} X_\mu$$

conformal invariance of S:

$z \to f(z)$ holomorphic is symm. of S

free propagator of string path integral:



need to specify initial state $\hat{=}$ insertion of vertex operator V at center point of disk

$$V = \int d^2z\, \mathcal{O}\, \tilde{\mathcal{O}}\, e^{ik\cdot X(z,\bar{z})}$$

closed bosonic string

$\Rightarrow$ $\mathcal{O}$ has conf. weight $(1,0)$

$\tilde{\mathcal{O}}$ " " " $(0,1)$

d^2z " " " $(-1,-1)$

for a global symm. on the worldsheet:

$\exists$ conserved currents j_z, $\tilde{j}_{\bar{z}}$

and charge: $Q = \frac{1}{2\pi i}\oint \left(dz\, j_z - d\bar{z}\, \tilde{j}_{\bar{z}}\right)$

$\Rightarrow$ j_z : weight $(1,0)$

$\tilde{j}_{\bar{z}}$: " " $(0,1)$

moreover: ∂X^μ: weight $(1,0)$

$\bar{\partial} X^\mu$: weight $(0,1)$

(from string action)

$\Rightarrow$ can build vertex op. from j_z, $\tilde{j}_{\bar{z}}$:

$$\underbrace{j_z \bar\partial X^\mu e^{ik\cdot X}, \; \tilde{j}_{\bar z} \partial X^\mu e^{ik\cdot X}}$$

$\searrow \swarrow$ space-time gauge bosons !

$\downarrow$

the global symm. becomes always gauged !

example : space-time translations

$$\delta X^\mu = \rho(z,\bar z)\cdot \epsilon \cdot a^\mu$$

$$\Rightarrow \delta S \sim \epsilon \cdot a^\mu \int d^2 z \cdot \left(\partial\rho\, \bar\partial X_\mu + \bar\partial\rho\, \partial X_\mu\right)$$

$$\Rightarrow j_z^\mu \sim \partial X^\mu, \; \tilde{j}^\mu_{\bar z} \sim \bar\partial X^\mu$$

$$\Rightarrow j_z^\mu \bar\partial X^\nu e^{ik\cdot X} \sim \underbrace{\partial X^\mu \bar\partial X^\nu e^{ik\cdot X}}$$

$$\underbrace{\subset g^{\mu\nu}, B^{\mu\nu}, \phi}$$

diffeomorph.s gauge global. translat. symm. $\longleftarrow$ gravitons as gauge bosons exceptions: full string action also contains $B_{\mu\nu}$-field

$$\Delta S_{B_2} \sim \int d^2\sigma \cdot \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$$

$$= \int B_2$$

invariance under

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu} \xi_{\nu]}$$

$\rightsquigarrow$ worldsheet current:

$$j_z \sim \partial_{[\mu} \xi_{\nu]} X^\mu \partial X^\nu$$

contains extra k_μ in momentum space

$\Rightarrow$ $j_z = 0$ at zero momentum

$\to$ no charge, no vertex op.

however: non-perturbative contribution to path from instantons

$$e^{-S_{\text{string}}} \supset e^{-i\int_{\Sigma_2} B_2}$$

Euclidean string worldsheet wrapped on 2-cycle Σ_2 of extra-dim.s

$$e^{-i\int_{\Sigma_2} B_2} \to e^{-i\int_{\Sigma_2} B_2} \quad \text{for } b = \int_{\Sigma_2} B_2 \to b + 2\pi$$

continuous shift symm. broken to discrete subgroup

→ <u>generic:</u> no global symm. s in ST

<u>comments:</u>

(i) bottom-up BH 'arguments':

- throw global charge into BH → no-hair theorem wipes any exterior trace
  → BH decays → charge is lost: non-conserved
  ⇒ global symm. broken

(ii) if AdS/CFT conjecture is

taken as exact property of quantum gravity: proof by Harlow & Ooguri 2018 that all global symm.s broken

<u>but:</u> broken at what scale?

Weak gravity conjecture (WGC)

<u>Motivation:</u>

take a 4D theory with GR and U(1) gauge field & states (m, q) with:

$$q < \frac{m}{M_p}$$

consider charged BH (Q, M) with:

$$Q < \frac{M}{M_p} \quad \text{(subextremal)}$$

Hawking decay will shrink Q faster than M

$\Rightarrow$ BH becomes extremal & stable

$\rightsquigarrow$ infinitely many stable BH remnants!

$\rightsquigarrow$ if(!) this is problematic (e.g. holographic entropy bounds)

$\rightarrow$ demand:

(electric WGC) $\exists$ at least 1 state with: $q > \frac{m}{M_p}$

if all charges quantized (as true in string solutions):

$$\Rightarrow \quad q_n = n \cdot g_{U(1)}, \; n \in \mathbb{Z}$$

Hence:

(WGC) $\Rightarrow$ $g_{U(1)} \rightarrow 0$ forbidden

WGC consequence of "no global symm.s" & specifies mass scale!

'magnetic' WGC:

$$(\text{WGC}) \Rightarrow m < g_{el.} \cdot M_P$$

$\Downarrow$

for magn. monopoles:

$$g_{mag.} \sim \frac{1}{g_{el.}}$$

and:

UV cutoff

$$m_{mag.} \sim \frac{\Lambda}{g_{el.}^2}$$ "field mass"

demand: monopole $\neq$ BH

$$\Rightarrow \Lambda < \frac{1}{R_S} = \frac{M_P^2}{m_{mag}}$$

$$\Rightarrow \boxed{\Lambda < g_{el.} \cdot M_P}$$

cutoff on EFT!

further evidence:

- SM electron satisfies WGC
- if WGC-state is <u>some</u> state in spectrum, <u>all</u> string theory sol.s known satisfy WGC

generalizations:

$D(p-1)$-branes with tension T_p and charge g_p under gauge potential C_p:

$$T_{p-1} < \sqrt{M_D^{D-2} g_p^2}$$

$$[g_p] = \text{mass}^{p+1-\frac{D}{2}}$$

$$[T_{p-1}] = \text{mass}^p$$

formally, can go to $p=0$:

D(−1)-brane charged under C_0: an instanton charged under the axion C_0

particle charged under $U(1)_{em}$		instanton charged under C_0
$m\int ds$	$\longrightarrow$	$S_{inst.}$
$q\int A_1$	$\longrightarrow$	$\frac{M_P}{f}$;

$$\mathcal{L}_{kin} = f^2 \partial_\mu C_0 \partial^\mu C_0$$

$$\Rightarrow \quad \frac{M_P}{f} > S_{inst.}$$

$$\Rightarrow \quad f < \frac{M_P}{S_{inst.}}$$

for a controlled instanton effect expansion of the path integral:

$$\sim \quad S_{inst.} > 1 \quad \Rightarrow \boxed{f < M_P}$$

Models of 'natural' inflation or quintessence use an axion:

$$V = V_0 \cdot \left(1 - \cos\left(\frac{\phi}{f}\right)\right)$$

both inflation & quintessence require:

$$\epsilon = \frac{M_p^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2 < 1$$

$$\eta = M_p^2 \, \frac{\partial_\phi^2 V}{V} < 1$$

$$\Rightarrow f > M_p \quad \lightning$$

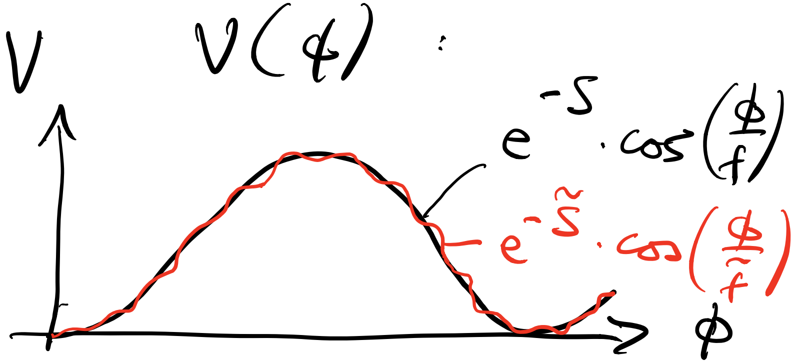
WGC would forbid axion inflation with instanton potentials ...

unless:

$\rightsquigarrow$ maybe there are 2 instantons (S, f) $(\tilde{S}, \tilde{f})$ and only:

$$\frac{M_p}{\tilde{f}} < \tilde{S}$$

while S dominates $V(\phi)$:



$\hat{=}$ is for $U(1)_{em}$ the WGC-state:

- the <u>lightest</u>:
  - strong WGC, lattice/ tower/ sublattice-WGC ...

OR

- just <u>some</u> state:
  - mild WGC (satisfied by all ST examples)

... many discussions in the literature !

<u>also recently:</u> argument from locality & unitarity of scatt. amplit. prove form of WGC arXiv:1902.03250