Classicalization

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based on. 1010.1415, 1011.0114, 1103.5963, 1607.07422, ...

▨ Introduction : BH formation from 2-particle collision.

- Scattering w/ Center of mass energy $\sqrt{s}$, impact parameter b

  $\longrightarrow$ BH formation if $b < r_s = \frac{\sqrt{s}}{M_p^2}$. ($\leftarrow$ increase with energy).

- BH decays by Hawking rad. $\Rightarrow$ (BH) $\to$ 2 particles : exponentially suppressed.

  $\longrightarrow$ 2to2 amp. actually dominated by $b \gg r_s$,

  or low momentum transfer process.

$\therefore$ { produce "classical object" if going to deep-UV regime.
Unitarize theory even w/. higher dim operator.

[**Q.** Is this phenomenon unique to gravity ?

A. No !! ($\leftarrow$ See this in the following)]

※ **caveat** Connection to hierarchy prob. : hard to formalize.

In general, (classical obj.) $\to$ 2-particles, exponentially suppressed.

$\longrightarrow$ 1010.1415 says,

Quantum correction to Higgs also suppressed once you are in a regime to produce classical object.

But, hard to incorporate production of classical object into computation of corrections to the Higgs mass...

(More focus on the property of the classical obj. itself. at least so far...)

Definition and requirement of "classicalization".

<u>def</u>. $\sigma = r_*^2(\sqrt{s})$, w/ r_* : increasing function of $\sqrt{s}$.

(※ if you produce classical obj. r_* will be the radius of that object.)

<u>necessary ingredients.</u>

(1) Source term. $\left(\Leftarrow \frac{1}{M_p} h_{\mu\nu} T^{\mu\nu} \text{ for gravity} \right)$

(2) Self interaction $\left(\Leftarrow R \sim (\partial h)^2 + h(\partial h)^2 + \cdots \text{ for gravity} \right)$.

※ (2) is also important to redistribute the source "energy" within r_*.

$\left(\nabla^2 \phi = Q \rightarrow \phi = \frac{Q}{4\pi r} \text{ : no well-defined } r_* \right)$

$\Rightarrow$ Let's think about something like

(Sometimes I use $M_*^{-1} \equiv L_*$.)

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{M_*^{n+4k-4}} \phi^n (\partial\phi)^{2k} + \frac{\phi}{M_*} J,$$

w/ J : Source term. (can depend on ϕ → self-sourcing)

Example I : Galileon.

$$\mathcal{L} = (\partial\phi)^2 \left[\frac{1}{2} + \frac{L_*^3}{4} \Box\phi \right] + \frac{\phi}{M_*} J.$$

$$\longrightarrow \Box\phi + \frac{L_*^3}{2} \left[(\Box\phi)^2 - (\partial_\mu \partial_\nu \phi)^2 \right] = \frac{J}{M_*}.$$

$L^{-1} = M$: energy.

· Try to find spherical-sym. static configuration.

$\Phi = \Phi(r)$ for localized source $J = \dfrac{4\pi\,\delta^{(3)}(\vec{x})}{L}$

$$\to \partial_i\left[\frac{x^i}{r}\left(\Phi' - \frac{L_*^3}{r}(\Phi')^2\right)\right] = -4\pi\left(\frac{L_*}{L}\right)\delta^{(3)}(\vec{x})$$

Here. $\delta^{(3)}(\vec{x}) = \dfrac{1}{4\pi}\partial_i\left(\dfrac{x^i}{r^3}\right)$

$$\therefore \Phi' - \frac{L_*^3}{r}(\Phi')^2 = -\frac{L_*}{L r^2}$$

$$\to \Phi' = \frac{r}{2L_*^3}\left[1 \pm \sqrt{1 + \frac{4r_*^3}{r}}\right].$$

$r_* \equiv L_*\left(\dfrac{L_*}{L}\right)^{1/3} = L_*(L_* M)^{1/3}$: increasing function of M (or $\sqrt{s}$)

→ plays the role of the radius!!

Example 2 : more generic int.

$$\mathcal{L} = \frac{1}{2}(\partial\Phi)^2 + \frac{1}{M_*^{n+4k-4}}\Phi^n(\partial\Phi)^{2k} + \frac{\Phi}{M_*}J.$$

Estimate r_* for $J = \dfrac{4\pi\,\delta^{(3)}(\vec{x})}{L}$.

· first ignore non-linear self-int.

$$\to \Box\Phi = 4\pi\left(\frac{L_*}{L}\right)\delta^{(3)}(\vec{r})$$

$$\to \Phi(r) = \left(\frac{L_*}{L}\right)\frac{1}{r} \quad (\text{w/ b.c. } \Phi(\infty) = 0)$$

Gravity
$\sim (k,n) = (1,1)$

Non-linear term becomes important when

$(\partial\phi)^2 \sim \frac{1}{M_*^{n+4k-4}} \cdot \phi^n \cdot (\partial\phi)^{2k}$. or $r_* \sim L_* \cdot \left(\frac{L_*}{L}\right)^{\frac{2k+n-2}{4k+n-4}}$

$\rightarrow$ r_* depends strongly on (k, n)

e.g. · no classicalization for $k=0$, $n \leq 4$

since r_* does not grow with energy

($n=4 \rightarrow r_*$: indep. of energy)

· pure derivative case $n=0$, all are of the same order.

· pure potential case $k=0$, Smaller n (≥ 5) is more important

▨ Example 3 : Goldstone bosons, or self-sourcing (← talk only if I have Time)

· "classicalizer" can be sourced by itself.

$\longrightarrow$ source and self-int. can also be of the same origin.

e.g. $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{L_*^4}{4}(\partial\phi)^4$

Expand $\phi = \phi_0 + \phi_1$, $\begin{cases} \phi_0 : \text{initial particles, source energy} \\ \phi_1 : \text{"classicalizer" sourced by } \phi_0 \end{cases}$

$\phi_0 = A \cdot \frac{e^{-(t+r)^2/a^2}}{r}$ (spherical, ingoing wave packet)

$\Box\phi_0 = 0$.

1st order in perturbation

$\Box\phi_1 = -L_*^4\, \partial_\mu\left(\partial^\mu\phi_0 \cdot (\partial\phi_0)^2\right)$

- Take an spatial ave. (exact sol. is also given in [011.0114])

$\longrightarrow \quad \Box\phi_1 = \delta^{(3)}(\vec{x}) \cdot Q(t),$

$$Q(t) = -L_*^4 \int d^3x \cdot \partial_\mu \left(\partial^\mu \phi_0 \cdot (\partial\phi_0)^2\right)$$

$$= -16\pi L_*^4 A^3 \cdot \int dr \left[\frac{2}{3} \cdot \frac{1}{a^2 r^3} + \frac{1}{9r^3} \partial_r^2 - \frac{a}{r^5} \right] \cdot e^{-3(r+t)^2/a^2}.$$

$$\xrightarrow[a \ll |t|, r]{} \frac{32\pi}{3} \sqrt{\frac{\pi}{3}} L_*^4 A^3 \cdot \frac{1}{at^3}.$$

$$\longrightarrow \quad \phi_1 = \frac{Q(t-r)}{4\pi r} = \frac{8}{3} \sqrt{\frac{\pi}{3}} \cdot \frac{L_*^4 A^3}{a(t-r)^3 \cdot r.} \quad \sim \quad \frac{L_*^4 A^3}{ar^4.}$$

- two ways to estimate r_* $\qquad\qquad$ $A^2/a \sim M.$

(a) $\phi_0 \sim \phi_1$ @ $|t+r| \lesssim a.$

$$\longrightarrow \quad \frac{L_*^4 A^2}{a r_*^3} \sim 1 . \quad \text{or} \quad r_* \sim L_* \cdot \left(L_* \frac{A^2}{a}\right)^{1/3} \sim L_* \cdot (L_* M)^{1/3}.$$

(b) $(\partial\phi_1)^2 \sim L_*^4 \cdot (\partial\phi_1)^4.$

$$\longrightarrow \quad L_*^4 \cdot \left(\frac{L_*^4 A^3}{a r_*^5}\right)^2 \sim 1 \quad \text{or} \quad r_* \sim L_* \cdot (L_*^2 M^3 \cdot a)^{1/5}.$$

if we take $a \sim r_*$: softest wave packet (why?)

$r_* \sim L_* \cdot (L_* M)^{1/3}$: coincides w/ previous one.

※: Longitudinal gauge bosons in Higgsless theory are essentially the same.

$\longrightarrow$ Maybe unitarized even w/o Higgs.

since 2-to-2 scattering does not probe deep-UV regime.

Examples 4 : Higgs

$\kappa = \pm 1$

$$\mathcal{L}_{int} = \frac{\kappa}{M_*^2} \cdot H^\dagger H \cdot T^\mu_\mu$$

※ contains also non-linearity since $T^\mu_\mu \ni |\partial H|^2$

- Ignore λ and non-linearity of Higgs for simplicity.

$$H^\dagger H = \frac{\rho^2(r)}{2}, \quad T^\mu_\mu = \frac{3\theta(L-r)}{L^4}$$

$$\rightarrow \frac{1}{r}\partial_r^2(r\rho) = -\frac{3\kappa}{M_*^2 L^4}\theta(L-r)\cdot\rho$$

Sol. for $\rho(\infty) = v$, $\rho(r) = v$ for $M_*^2 \to \infty$:

$$\rho(r) \simeq v + \frac{\kappa v L_*^2}{L r}$$

(a) $\kappa = -1$.

$$\rho \sim 0 \ @ \ r \sim r_*^{(-)} \equiv \frac{L_*^2}{L}.$$

(b) $\kappa = +1$

$$|\partial H|^2 \sim \frac{H^\dagger H}{M_*^2} \cdot |\partial H|^2 \longrightarrow r \sim r_*^{(+)} \equiv \frac{v L_*^3}{L}.$$

A finite λ gives $e^{-\lambda v r}$

$$\longrightarrow r_*^{(\pm)} < \frac{1}{\lambda v}$$ is required for classicalization.

- 1010.1415 (and subsequent papers) says it can be used to solve hierarchy prob. since (classical obj) $\to$ 2. is exponentially suppressed.
BUT YE feels it is far from obvious how to incorporate this classical obj. into the loop computation. and so on.

▨ Collider signatures

- Expected to be similar to Hawking rad. in BH case

  → dominance of $2 \to n$ process w/ large n soft quanta.

▨ What is done / yet to be done.

- Done
  - perturbative and spherically symmetric analysis for
    Goldstone bosons, Higgs, gauge bosons, graviton
    (1010.1415, 1011.0114, 1103.5963, ...)
  - Connection to BH physics (entropy. etc.) (1103.5963, 1409.7405, ...)

- To be done (in YE's opinion)
  - Static solution beyond perturbation for generic cases.
  - Formation of classical obj. w/o spherical symm. etc.
  - Confirm the decay into a bunch of soft quanta.
  - Incooperate it to loop (quantum) computations.

    → connection to hierarchy problem.
  - Relations to Vainshtein mechanism, Skyrme model, etc.... (?)
  - Axion classicalization by shooting photons (??)

    ⋮

In the end, YE feels the theory is not quite matured yet.