N naturalness. 1607.06821 Nima Arkani-Hamed et al.

§ Setup

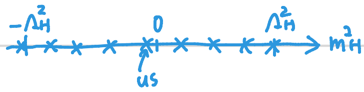
(each theory can be regarded as a theory localized @ each brane.)

$$\mathcal{L}_{SM,i} = |DH|^2 - \left(m_{H,i}^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2\right) + \mathcal{L}_{\rm others}\left(3\times(Q,U,D,L,E)_i,\ U(1)_i\times SU(2)_i\times SU(3)_i\right)$$

$-\Lambda_H^2 \lesssim m_{H,i}^2 \lesssim \Lambda_H^2$ w/ $-\frac{N}{2} \leq i \leq \frac{N}{2}$ //

For simplicity; uniform distribution $m_{H,i}^2 = -\frac{\Lambda_H^2}{N}(2i+r)$ // $i=0 \Rightarrow$ our vac. w/ $m_{H,0}^2 = -\frac{\Lambda_H^2}{N} r$

fine tuning.

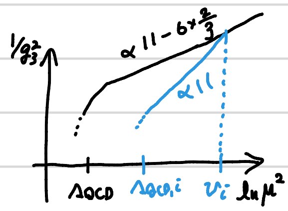


Z_N-sym. softly broken by $m_{H,i}$; reheaton (discussed later)

Setup is $\mathcal{L} = \sum_i \mathcal{L}_{SM,i} + \mathcal{L}_R$ (+ mixing btw sectors)

- Cutoff : $\Lambda_H,\ \Lambda_G \sim \frac{M_{Pl}}{\sqrt{N}}$ // ※ examples. GUT : $M_{GUT} \lesssim \Lambda_G \Rightarrow N \lesssim 10^4 \leftrightarrow \Lambda_H \lesssim 10\,{\rm TeV}$ //
  No hierarchy : $\Lambda_G \sim \Lambda_H \sim 10^{10}\,{\rm GeV}$ @ $N \sim 10^{16}$ //
  (c.f., 't Hooft coupling $N\frac{\Lambda_G^2}{M_{Pl}^2} \lesssim 1$, 0710.4344)

- Spectrum of each i :
  $m_{H,i}^2 < 0 \Rightarrow v_i \sim v\sqrt{i}$, For $i \gtrsim 10^8$, $m_{u,i} \sim m_u\sqrt{i} > \Lambda_{QCD,i}$ ($\propto i^{2/11}$ at most)
  leptons & meson, glueballs. (cf. $\Lambda_{QCD,i} \sim 10\,{\rm GeV}$ @ $i \sim 10^8$)



$m_{H,i}^2 > 0 \Rightarrow$ QCD induced EWPT (cf. 1704.04955, 1711.11554)

$y_t h\langle \bar{t}t\rangle \sim y_t \Lambda_{QCD}^3 h \Rightarrow v \sim y_t \frac{\Lambda_{QCD}^3}{m_{H,i}^2}$, $m_f \sim y_f y_t \frac{\Lambda_{QCD}^3}{m_{H,i}^2} \lesssim 100\,{\rm eV}$

Fermionic & gauge dofs are extremely light!

↳ extra rel. dof $\Delta N_{\rm eff} \sim N$, Need to make sure that they are never produced.

⇓

※ Reheaton ($\mathcal{L}_R$) : Gauge singlet. → couples to all the sectors.

Reheaton domination ⟹ Decay ※ make sure that others sectors than us are not "reheated".

§ Reheaton

Not to produce heavy particles for $m_{H,i}^2 < 0$

Suppress production of light particles for $m_{H,i}^2 > 0$.

Assumptions : ① Gauge singlet, ② $m_R \lesssim \Lambda_H/\sqrt{N}$, ③ Couples to Higgs dominantly

- On condition ②

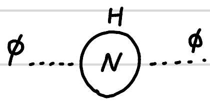
  It just replaces the hierarchy problem of Higgs w/ that of reheaton … Need justification

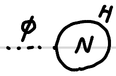
"scalar (ϕ-model)"

Consider a scalar w/ shift sym. w/ $\phi \mapsto \phi + \text{const.}$

It is broken by $\frac{1}{\sqrt{N}} \Lambda_H \phi \sum_i a_i |H_i|^2$, w/ $a_i = +a$ or $-a$ (random sign)

spurion

 $\sim \frac{\Lambda_H^2}{N} a^2 \times N \sim a^2 \Lambda_H \Rightarrow a \lesssim \frac{1}{\sqrt{N}}$ to have $m_\phi^2 \lesssim \frac{\Lambda_H^2}{N}$

Also  $\sim \frac{\Lambda_H^3}{\sqrt{N}} a \sum_i (\pm) \sim a \Lambda_H^3 \Rightarrow \langle \phi \rangle \sim \frac{a \Lambda_H^3}{m_\phi^2} \sim a N \Lambda_H \lesssim \sqrt{N} \Lambda_H$

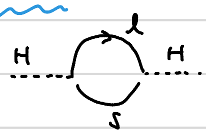
This leads to $\delta m_{H,i}^2 \sim a_i \frac{\Lambda_H}{\sqrt{N}} \langle \phi \rangle \lesssim \mathcal{O}\left(\frac{\Lambda_H^2}{\sqrt{N}}\right) \ll \Lambda_H^2$,

possibilities ? ① They claim it can be renormalized so that it has unif. dist.

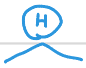
② Random sign. $\Rightarrow$ random dist. but still one could find $m_{H,i}^2 \sim 0$ by a slight tuning of b.

"fermion (ℓ-model)"

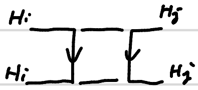
$$\mathcal{L} \supset -\frac{\lambda}{\sqrt{N}} S^\dagger \sum_i \ell_i H_i - m_S S^\dagger S + \text{H.c.} \quad \text{w/ } m_S \lesssim \frac{\Lambda_H}{\sqrt{N}},$$

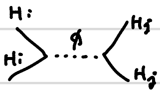
 $\sim \frac{\lambda^2}{N} \Lambda_H^2 \sim \lambda^2 m_{H,0}^2$ natural,

relevant for Higgs mass. (Those relevant for ΔN_{eff} will be discussed later)

- Mixing $\kappa |H_i|^2 |H_j|^2 \Rightarrow$  $\sim \kappa \Lambda_H^2 \times N \ll \Lambda_H^2$ for $\kappa \ll \frac{1}{N}$

Radiative corrections from reheaton.

"ℓ-model"  $\sim \frac{\lambda^4}{N^2} \ll \frac{1}{N}$ controlled.

"ϕ-model"  $\sim \pm \frac{a^2}{N} \frac{\Lambda_H^2}{m_\phi^2} \sim \pm a^2$ w/ $a \lesssim \frac{1}{\sqrt{N}}$ marginal ?

 $\sim a^2 \Lambda_H^2 \gtrless (-1) \sim \frac{\Lambda_H^2}{\sqrt{N}} \ll \Lambda_H^2$, safe.

§ Reheating (※ How to suppress the production of light particles)

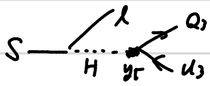
Basic assump. ϕ-domination $\Rightarrow$ reheating via perturbative decay (No preheating)

※ BBN < T_R < EW scale.

- "ℓ-model"

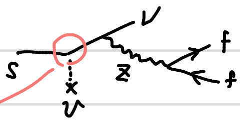
For $m_{H,i}^2 > 0$, reheating occurs $T \gg T_{EW}$ i.e. $\langle H \rangle = 0$,

Integrate out Higgs for $m_{H,i}^2 \gg m_s^2$.

$$\mathcal{L} \supset C_2^\ell \frac{\lambda}{\sqrt{N}} \frac{y_t}{m_H^2} S \ell Q_3^\dagger u_3^\dagger$$  $\Rightarrow \Gamma_{m_H^2 > 0} \propto \frac{1}{m_H^4} \propto \frac{1}{i^2}$

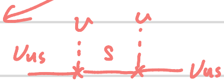
For $m_{H,i}^2 < 0$, vev can be involved.

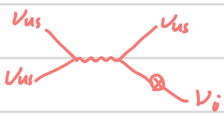
Integrate out Higgs for $|m_{H,i}^2| \gg m_s^2$.

$$\mathcal{L} \supset C_i^\ell \frac{\lambda}{\sqrt{N}} \frac{v}{m_Z^2 m_s} (\nu^\dagger \bar{\sigma} S) \cdot (f^\dagger \bar{\sigma} f)$$  $\Rightarrow \underbrace{\Gamma_{m_H^2 < 0} \propto \frac{1}{m_H^2} \propto \frac{1}{i}}_{\sum \frac{1}{i} \propto \log N}$

$\Rightarrow$ Reheaton dominantly decays into $m_H^2 \sim 0$ i.e. Our sector.

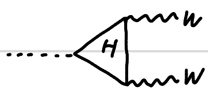
BR sensitive to N...

Mixes w/ ν_{us}  $\sim \frac{\lambda^2}{N} \frac{v^2}{m_s}$

 Freeze-in production occurs... $N \lesssim 10^3$

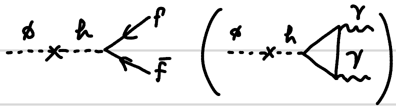
- "ϕ-model"

For $m_{H,i}^2 > 0$, $\mathcal{L} \supset C_3^\phi a \frac{\Lambda_H}{\sqrt{N}} \frac{g^2}{16\pi^2} \frac{\phi}{m_{H,i}^2} WW$ or $\gamma\gamma$



$\Rightarrow \Gamma \propto \frac{1}{m_{H,i}^4}$, $\propto \frac{1}{i^2}$

For $m_{H,i}^2 < 0$, $\mathcal{L} \supset C_i^\phi a \frac{\Lambda_H}{\sqrt{N}} y_f \frac{v}{m_{H,i}^2} \phi \bar{q} q$

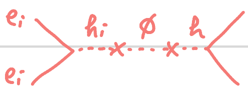


$\Rightarrow \Gamma \propto \frac{1}{m_{H,i}^2}$, $\propto \frac{1}{i}$

↳ For $m_\phi < 2 m_{c,i}$, Yukawa is too small & this process is negligible.

$i \gtrsim 10^8$, $\phi\gamma\gamma$ dominates. $\Gamma \propto \frac{1}{m_{H,i}^4} \propto \frac{1}{i^2}$,

⇓ Hence BR is not sensitive to N.



Freeze-out of e_i $\Rightarrow N \lesssim 10^4$ ($f_{e_i} < 1\%$ of DM ☺ Charged)

ⓡ 1610.04611

- "L_4-model" Not sensitive to $\log N$, No direct coupling to U_{us}.

Add 4th generation vector-like $(L_4, L_4^c), (E_4, E_4^c), (N_4, N_4^c)$

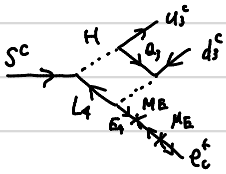
$$\mathcal{L} \supset -\lambda S^c \sum_i (L_4 H)_i - \mu_E \sum_i (e^c E_4)_i - \sum_i \left[Y_E (H^\dagger L_4 E_4^c)_i + Y_N (H L_4 N_4^c)_i + \text{H.c.} \right]$$

$$- \sum_i \left[M_E (E_4^c E_4)_i + M_L (L_4^c L_4)_i + M_N (N_4^c N_4)_i \right] - m_S S S^c.$$

Assume mixing w/ single flavor l_R.

For $m_{H_i}^2 > 0$ & $m_S^2 < m_{H_i}^2$,

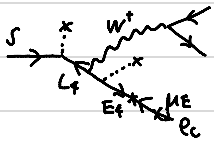
$$\mathcal{L} \supset C_2^{L_4} \lambda \frac{y_t y_b}{16\pi^2} \frac{Y_E M_E \mu_E}{m_{H,i}^4} (e_c^\dagger \bar{\sigma} S^c) \cdot (u_3^{c\dagger} \bar{\sigma} d_3^c)$$



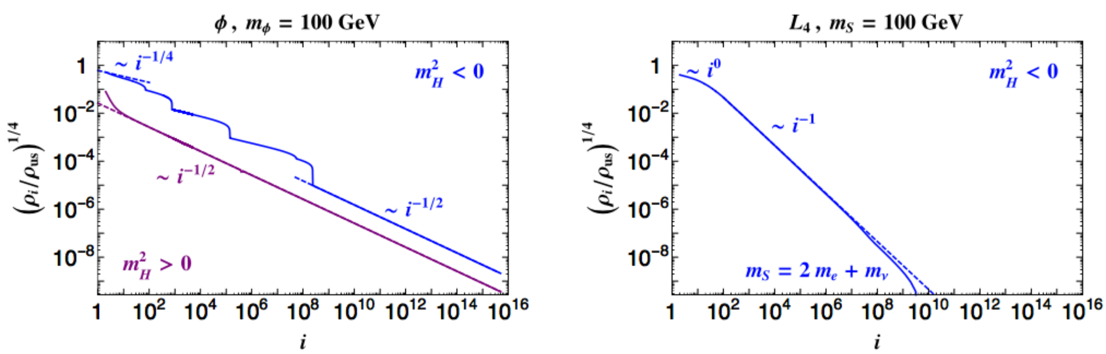
$\Rightarrow \Gamma \propto \frac{1}{m_{H,i}^8} \propto \frac{1}{i^4}$

For $m_{H_i}^2 < 0$ & $m_S^2 < |m_{H_i}^2|$,

$$\mathcal{L} \supset C_1^{L_4} \lambda \frac{v_i^2 \mu_E M_E}{M_{4_i}^4} \frac{g^2}{m_W^2} (e_c^\dagger \bar{\sigma} S_c) \cdot (f^\dagger \bar{\sigma} f)$$



$\Rightarrow \Gamma \propto \begin{cases} \text{const.} & i \sim O(1) \\ \frac{1}{v_i^8} \propto \frac{1}{i^4} & \end{cases}$



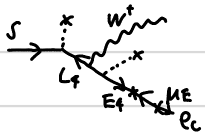
- Reheating Temperature.

$$T_R \sim 100\,\mathrm{GeV} \times \sqrt{\frac{\Gamma_R}{10^{-14}\,\mathrm{GeV}}} < \frac{\Lambda_H}{\sqrt{N}} \qquad \left(\text{otherwise } m^2_{H,i} \text{ suppression does not work}\right)$$

"ϕ-model" upper bounds on mixing. ⊛ $\Gamma \sim \theta_{\phi h}^2 y_f^2 m_\phi$

$$\theta_{\phi h} \sim \frac{a}{\sqrt{N}} \frac{\Lambda_H v}{m^2_{h,us}} \lesssim 10^{-6} \left(\frac{100\,\mathrm{GeV}}{m_\phi}\right)^{1/2} \Rightarrow \underbrace{10^{-11}\left(\frac{100\,\mathrm{GeV}}{m_\phi}\right)^{1/2}}_{\text{BBN}} \lesssim a \lesssim 10^{-6}\left(\frac{100\,\mathrm{GeV}}{m_\phi}\right)^{1/2}$$

"L_4 model"

Decay is dominated by 

$$\frac{\lambda v_{us}^2 \mu_E M_E}{M_{4us}} \lesssim 10^{-7}$$

(⊛ even w/ $\lambda/\sqrt{N}$ w/ $N \sim 10^{16}$, T_R can be as large as $\mathcal{O}(1)$ GeV)

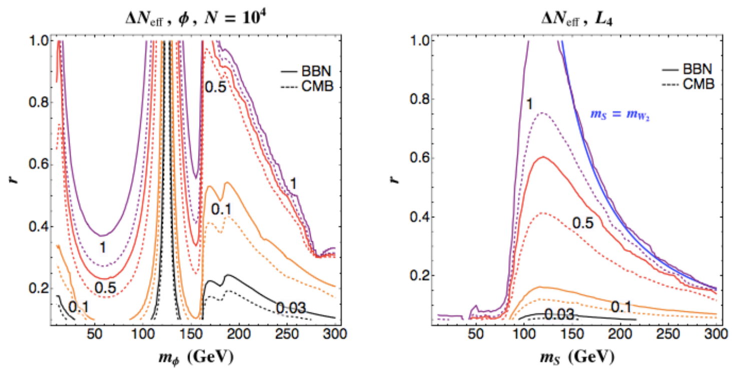


FIG. 5: $\Delta N_{\rm eff}$ contours as a function of reheaton mass and the r parameter defined in Eq. (1). $\Delta N_{\rm eff} \simeq 0.03$ corresponds to the sensitivity of CMB stage 4 experiments. The current upper bound at the CMB epoch is around 0.6. The left panel is for the ϕ model with $a = 1$ MeV. The right panel is for the L_4 model with $\lambda \times \mu_E = 1$ MeV, $M_L = 400$ GeV, $M_{E,N} = 500$ GeV, $Y_E = Y_N = 0.2$, and $Y_E^c = Y_N^c = -0.5$. As discussed in the text, the L_4 result is valid for a large range of N, namely $30 \lesssim N \lesssim 10^9$. Both figures were made using the zero temperature branching ratios of the reheaton; see the end of Sec. II for a discussion.

§ Constraints

- Relativistic dof in $m^2_{H,i} > 0 \Rightarrow \Delta N_{\rm eff}$

- Massive stable particles. ℓ-model $\Rightarrow$ freeze-in of ν_i; ϕ-model $\Rightarrow$ freeze-out of e_i
  ↳ $N \lesssim 10^3$ ↳ $N \lesssim 10^4$

- Mixing btw other sectors. e.g. kinetic mixing $\varepsilon_i F F_i \Rightarrow$ stellar cooling $\sqrt{\sum_i \varepsilon_i^2} \lesssim 10^{-14}$

  essentially no mixing.

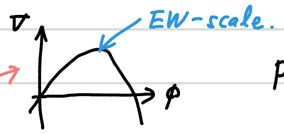
  ⊛ radiative corr. from reheaton?
  Can be controlled. (L_4 model)

- Heavy axion?? Assume S_N-sym. softly broken by $m^2_{H,i}$
  ↳ @ UV, $\theta_i = \theta \Rightarrow$ shared axion?

Inflating to the weak scale 1809.07338 Michael Geller, Yonit Hochberg, Eric Kuflik

§ Idea

scanning field ϕ : random walk during inflation.



1811.12390 discusses a class of models which has this feature.

patch w/ large V inflates most. (i.e., most probable)
identify it as our EW vacuum.

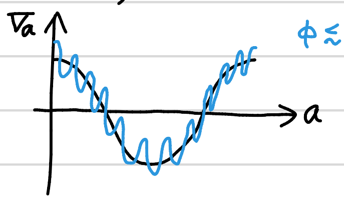
§ Setup

spurion of ~~shift~~ SU(3) confining scale of new group

$$V = (M^2 + yM\phi + \cdots)h^2 + \lambda h^4 + yM^3\phi + \cdots + \frac{a}{f} G\tilde{G} + \Lambda_H^4 \cos\frac{a}{F}, \quad \text{w/ } F \gg f,$$

Cutoff scale Fine tuning for strong CP...

For $\phi \lesssim -M/y$, $\sqrt{-\mu^2(\phi)/\lambda} \sim v_\phi$ w/ $\mu^2(\phi) = M^2 + yM\phi + \cdots \Rightarrow \langle h \rangle$ grows for $\phi \lesssim -M/y$,



$$V_a = \Lambda(\phi)^4 \cos a/f + \Lambda_H^4 \cos a/F, \quad \text{w/ } \Lambda(\phi)^4 \sim y_u v_\phi \Lambda_{QCD}^3 + \cdots$$

Choose the prm so that a starts to roll down when $\mu^2(\phi) = \mu_{obs}^2$, $\left(\frac{\Lambda_H^4}{F} \sim \frac{m_\pi^2 f_\pi^2}{f}\right)$

For $\mu^2(\phi) < \mu_{obs}^2$, a is trapped @ its maxima.

$$v(\phi) = \sqrt{-\mu^2(\phi)/2\lambda}, \quad V_h(v_\phi) = -\frac{\mu(\phi)^4}{4\lambda} + \frac{\mu_{obs}^4}{4\lambda} \simeq \frac{\mu_{obs}^2}{2\lambda}\left(\mu_{obs}^2 - \mu^2(\phi)\right) \propto + \frac{1}{2\lambda}(-\mu_{obs}^2)\, yM\phi,$$

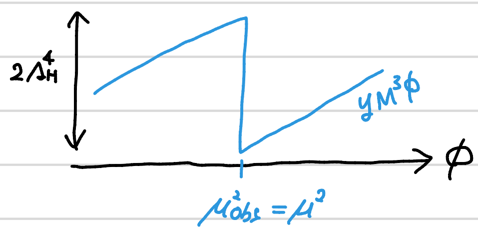
for $\phi < \phi_{obs}$

$$\hookrightarrow V_\phi = \left[\frac{1}{2\lambda}(-\mu_{obs}^2) + M^2\right] yM\phi + \cdots \simeq yM^3\phi + \cdots$$

For $\mu^2(\phi) > \mu_{obs}^2$, a goes to its minima. $V_\phi \simeq -2\Lambda_H^4 + yM^3\phi + \cdots$

$$\Rightarrow V_\phi \simeq -2\Lambda_H^4\, \Theta\left(\mu^2(\phi) - \mu_{obs}^2\right) + yM^3\phi$$

w/ $\Lambda_H > M$ so that vac. energy is dominated by Λ_H^4,



Assump. global extrema for $|\phi| \lesssim M/y$,

§ Requirements – conditions so that this mechanism works –

Inflation dominates the vac. energy. $V_{inf} \gg \Lambda_H^4$.

$$H^2 = H_{inf}^2 + \frac{V}{3M_{pl}^2} \Rightarrow H \simeq H_{inf} + \Delta H \quad \text{w/ } \Delta H = \frac{V}{6M_{pl}^2 H_{inf}} \quad \& \quad V \simeq V_h(v_\phi) + V_a + yM\phi + \cdots$$

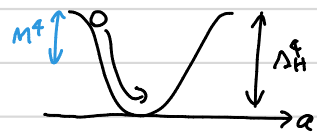
Requirements ① axion classically rolls down to its each minimum.

② axion rolls down sufficiently so that $\Delta V_a \ll M^4$.

③ ϕ climbs up its potential.

④ Probability of finding ϕ_{EW} is larger than others.

⑤ distribution of $\mu^2 - \delta\mu^2$ should be narrow, i.e., $\delta\mu^2 \ll \mu_{obs}^2$.



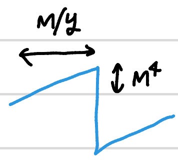
① Quantum $\Delta a_q \sim H_{inf}$, Classical $\Delta a_c \sim \frac{1}{H_{inf}^2}\frac{\partial V}{\partial a} \sim \frac{(m_\pi^2 f_\pi^2)}{H_{inf}^2 f}$ @ each $1/H_{inf}$

$$\Delta a_c > \Delta a_q \Leftrightarrow M < M_{inf} < \frac{(m_\pi f_\pi)^{1/3} M_{pl}^{1/2}}{f^{1/6}} \sim 10^7\,\text{GeV}\left(\frac{10^8\,\text{GeV}}{f}\right)^{1/6}$$

② $$N > M^4 \Big/ \left(\frac{m_\pi^2 f_\pi^2}{f H_{inf}}\right)^2 \sim 10^{44}\left(\frac{f}{10^8\,\text{GeV}}\right)^2\left(\frac{M M_{inf}}{(10^7\,\text{GeV})^2}\right)^4$$

③ Quantum $\Delta\phi_q \sim H_{inf}$, Classical $\Delta\phi_c \sim \frac{yM^3}{H_{inf}^2}$

$$\Delta\phi_q > \Delta\phi_c \Leftrightarrow y < \frac{H_{inf}^3}{M^3} \sim 10^{-33}\left(\frac{M_{inf}}{10^7\,\text{GeV}}\right)^6\left(\frac{10^7\,\text{GeV}}{M}\right)^3$$



$\frac{V}{2M_{pl}^2 H_{inf}^2}N$

④ $$\text{Prob.}(\phi_{EW}, t) \propto \exp\left(-\frac{\overbrace{(\phi_{EW} - \phi_{c})^2}^{M^2/y^2}}{H_{inf}^2}\frac{1}{N}\right) \times \exp(3\Delta H t)$$

$$\Rightarrow \frac{M^4}{M_{pl}^2 H_{inf}^2}N > \frac{M^2}{y^2 H_{inf}^2}\frac{1}{N} \Leftrightarrow N > \frac{M M_{pl}}{y M^2} \sim \frac{M_{pl}}{yM} \sim 10^{44}\left(\frac{10^7\,\text{GeV}}{M}\right)\left(\frac{10^{-33}}{y}\right)$$

⑤ After long enough inf., Prob. $\propto \exp(3\Delta H t)\,\Theta(\phi - \phi_{EW}) \Rightarrow \text{Prob.}(\mu^2 < \mu_{obs}^2 - \delta\mu^2) \sim \exp\left(-\frac{M^2\delta\mu^2}{H_{inf}^2 M_{pl}^2}N\right)$

$$\frac{M^2|\mu_{obs}^2|}{H_{inf}^2 M_{pl}^2}N > 1 \Rightarrow N > \frac{H_{inf}^2 M_{pl}^2}{M^2|\mu_{obs}^2|} \sim 10^{10}\left(\frac{M_{inf}}{10^7\,\text{GeV}}\right)\left(\frac{10^7\,\text{GeV}}{M}\right)^2$$

§ Evolution after inflation

(※) They never discuss reheating. Does this affect the cosmic history?

- $T > T_{EW}$.

QCD potential disappears.

typical mass. $m^2 \sim \frac{\Lambda_H^4}{F^2} \sim \frac{f}{F}\frac{m_\pi^2 f_\pi^2}{f^2} \lesssim 10^{-34}\,\mathrm{eV}^2$ (for $f \sim 10^8\,\mathrm{GeV}$)

↳ $T_{osc} \sim \sqrt{M_{pl} m} \lesssim 0.1\,\mathrm{MeV} \ll T_{QCD}$. Trapped again before it rolls down.

- $T < T_{EW}$. Trapped.

If ϕ rolls down after inflation, EW scale / Vac. energy changes.

$$\Delta\phi \sim \frac{gM^3}{H_0^2}$$

⑥ Change of EW scale is small.

$$\Delta\mu^2 \simeq gM\Delta\phi \ \& \ |\Delta\mu^2| < |\mu^2_{obs}| \Rightarrow g < \frac{|\mu_{obs}| H_0}{M^2} \sim 10^{-54}\left(\frac{10^7\,\mathrm{GeV}}{M}\right)^2$$

⑦ Change of Vac. energy is small.

$$gM^3\Delta\phi < \Lambda_{cc}M_{pl}^2 \Leftrightarrow (gM^3)^2 < \Lambda_{cc}^2 M_{pl}^2 \Rightarrow g < \frac{\Lambda_{cc}M_{pl}}{M^3} \sim 10^{-86}\left(\frac{10^7\,\mathrm{GeV}}{M}\right)^3$$

↳ $N > 10^{97}\left(\frac{10^7\,\mathrm{GeV}}{M}\right)\left(\frac{10^{-86}}{g}\right)$ extremely large e-fold is required.

To avoid this extremely large e-folds, they include another periodic potential.

↳ $\Lambda_\phi^4 \cos\frac{\phi}{f_\phi} \Rightarrow$ Ineffective for $H_{inf} > \Lambda_\phi$ during inflation. (?? $T_H \sim H_{inf}$)

After inf, ϕ relaxes @ local minima for $\frac{\Lambda_\phi^4}{f_\phi} \gtrsim gM^3$ (? Assume Λ_ϕ sector is never thermalized)

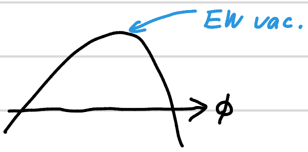
Also, the wiggle should be fine enough $gMf_\phi \lesssim |\mu^2_{obs}|$

⑥' Fluctuation of ϕ for last $N_{60} = 60$ is smaller than f_ϕ so that ϕ has the same value in our patch.

$$H_{inf}\sqrt{N_{60}} < f_\phi \ + \ g \lesssim \frac{\Lambda_\phi^4}{f_\phi M^3} < \frac{H_{inf}^4}{f_\phi M^3} \Rightarrow g \lesssim \frac{1}{\sqrt{N_{60}}}\frac{H_{inf}^3}{M^3} \sim 10^{-34}\left(\frac{10^7\,\mathrm{GeV}}{M}\right)^3$$

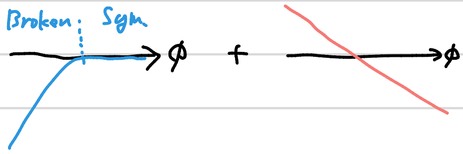
Mass Hierarchy & Vacuum Energy 1811.12390 Clifford Cheung & Prashant Saraswat

§ Class of models w/ criticality



Introduce a class of models which have EW vac. @ maxima.

$$V_{vac}(\phi) = \underbrace{V_{vac,0}(\phi)}_{\text{classical}} + \underbrace{\Delta V_{vac}(\phi)}_{\text{quantum}}$$



- Classical potential. $V(H,\phi) = V_H(H) + \phi\, \mathcal{O}(H)$ order prm. of SSB. w/ $\mathcal{O}(0) = 0$,

  e.g. $\mathcal{O}(H) \propto H^2$ (relaxion)

@ each ϕ, vev of H depends on ϕ. $0 = V_H'(v(\phi)) + \phi\, \mathcal{O}'(v(\phi))$

$\hookrightarrow$ $V_{vac,0}(\phi) = V_H(v(\phi)) + \phi\, \mathcal{O}(v(\phi))$ //

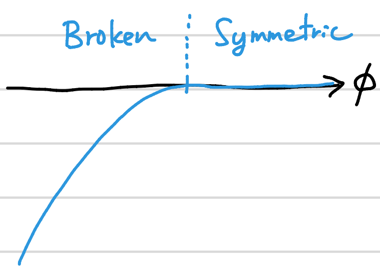
Property of V_{vac}.

$V_{vac,0}'(\phi) = \mathcal{O}(v(\phi))$ $\Rightarrow$ slope of ϕ is non-zero only if SSB occurs.

$V_{vac,0}''(\phi) = \mathcal{O}'(v(\phi))\, v'(\phi) = -m_h^2(\phi)\, v'(\phi)^2 \leq 0$

physical Higgs mass around $v(\phi)$,

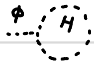
$\because$ $0 = \left[V_H''(v(\phi)) + \phi\, \mathcal{O}''(v(\phi))\right] v'(\phi) + \mathcal{O}'(v(\phi))$

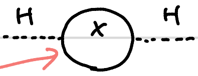
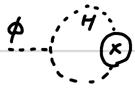


• Quantum potential. (Consider $\mathcal{O}(H) \propto |H|^2$)

"indirect coupling"

Suppose that H couples to new heavy particles w/ mass Λ (to avoid confusions about scheme dep.)

(? They drop  $\sim \frac{\mu^4(\phi)}{16\pi^2}$ @ dim reg.)

 $\sim -\frac{\Lambda^2}{16\pi^2} H^\dagger H$,  $\sim + \frac{\mu^2(\phi)}{16\pi^2}\left(-\frac{\Lambda^2}{16\pi^2}\right) = -\overbrace{\frac{\Lambda^2}{(16\pi^2)^2}}^{\Delta M^2} \mu^2(\phi)$

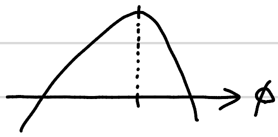
Fermion? They don't specify but its sign depends

$\hookrightarrow V + \Delta V = \underbrace{\left[\mu^2(\phi) - \frac{\Lambda^2}{16\pi^2}\right]}_{\text{redef. } =: \mu^2(\phi)} |H|^2 + \lambda |H|^4 - \Delta M^2 \mu^2(\phi) + \cdots$ → drop const. & $\mathcal{O}\left(\frac{\mu^4(\phi)}{16\pi^2}\right)$

$= \mu^2(\phi)|H|^2 + \lambda|H|^4 - \Delta M^2 \mu^2(\phi) + \cdots$

∴ This sign is an assump. of UV completion.

⊛ However, the shift gives $c\frac{\mu^4(\phi)}{16\pi^2} \mapsto c\frac{\mu^4(\phi)}{16\pi^2} + \underbrace{2c\Delta M^2 \mu^2(\phi)}_{\text{contribute to } \Delta M^2\mu^2(\phi)} + \cdots$

$\Rightarrow V_{vac} + \Delta V_{vac} = \underbrace{-\Delta M^2 \mu^2(\phi) + \cdots}_{\text{quantum}} + \underbrace{\begin{cases} 0 & \text{for } \mu^2(\phi) \geq 0 \\ -\frac{\mu^4(\phi)}{4\lambda} & \text{for } \mu^2(\phi) < 0 \end{cases}}_{\text{classical}}$ = 

Maxima is $0 = V'_{vac} + \Delta V'_{vac} = -\left[2\Delta M^2 \mu(\phi_{max}) + \frac{\mu^3(\phi_{max})}{\lambda}\right]\mu'(\phi_{max})$ ∴ $\mu^2(\phi_{max}) = -2\lambda\Delta M^2$

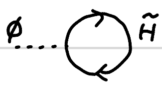
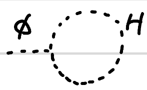
$\hookrightarrow \underbrace{v^2(\phi_{max})}_{v^2_{obs}} = \langle H^\dagger H\rangle = \Delta M^2 \sim \frac{\Lambda^2}{(16\pi^2)^2} = \left(\frac{\Lambda}{16\pi^2}\right)^2$ ← gain from 2 loop factor. $\sim \mathcal{O}(100)$

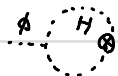
$\Lambda \sim \mathcal{O}(10)$ TeV

"direct coupling"

$\mathcal{L} \supset -(|\mu(\phi)|^2 - \underbrace{\Lambda^2}_{\text{SUSY}})|H|^2 + \mu(\phi)\underbrace{\tilde{H}\tilde{H}}_{\text{Higgsino}} + \text{c.c.}$

$\mu(\phi) = \mu_0 + y\phi$

$+\frac{\mu^4(\phi)}{16\pi^2}$  + $-\frac{\mu^4(\phi)}{16\pi^2}$  $= 0$ for $\Lambda = 0$

$\hookrightarrow$ L.O.  $-\Lambda^2$

$\hookrightarrow \Delta V_{vac} = -\Delta M^2 |\mu(\phi)|^2 + \cdots$ w/ $\Delta M^2 \sim \frac{\Lambda^2}{16\pi^2}$

$\hookrightarrow |v(\phi_{max})|^2 = \Delta M^2 \sim \frac{\Lambda^2}{16\pi^2}$ ← gain from 1 loop factor: $\Lambda \sim \mathcal{O}(1)$ TeV

⊛ This Λ should not be confused w/ ~~SUSY~~ scale. It is just a mass splitting btw Higgs & Higgsino.

i.e., Λ^2 could be $\frac{\Lambda^2_{SUSY}}{16\pi^2}$

§ Minima on the Maximum

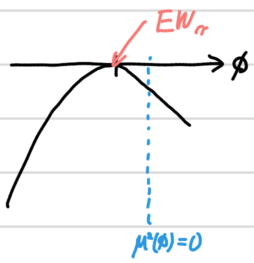
For the SM w/ $\mathcal{O}(H) \propto |H|^2$,

$$V_{vac}(\phi) + \Delta V_{vac}(\phi) = -v_{obs}^2 \mu^2(\phi) - \frac{\mu(\phi)^4}{4\lambda} - \lambda v_{obs}^4 = -\lambda v(\phi)^4 + 2\lambda v_{obs}^2 v(\phi)^2 - \lambda v_{obs}^4$$

$$= -\lambda \left[v^2(\phi) - v_{obs}^2\right]^2 = -\frac{g^2\phi^2}{4\lambda} \quad \text{for } \mu^2(\phi) \le 0,$$

(※) $0 = \mu^2(\phi) + 2\lambda v(\phi)^2$, & $\mu^2(\phi) = -2\lambda v_{obs}^2 + g\phi$, ((※) $g \sim \mu_0 y$)

$$V_{vac}(\phi) + \Delta V_{vac}(\phi) = 2\lambda v_{obs}^2 v(\phi)^2 - \lambda v_{obs}^4 = \lambda v_{obs}^4 - g v_{obs}^2 \phi \quad \text{for } \mu^2(\phi) > 0$$



- Make the extrema metastable. Add $V_{mod} = M^4 \cos\frac{\phi}{f}$,

↳ Around the EW vac. ($\phi \simeq 0$), $V(\phi) \simeq -\frac{g^2\phi^2}{4\lambda} + M^4 \cos\frac{\phi}{f}$,

Condition to have local min. $\frac{M^4}{f^2} \gtrsim \frac{g^2}{\lambda}$,

The wiggle should be fine enough $gf \lesssim \lambda v_{obs}^2$ ($\sim$ EW scale^2)

Two possibilities

① We have many local min other than ours.

② Our vac. is the only local min. around maxima

$$\frac{M^4}{f} \lesssim g v_{obs}^2 \implies m_\phi^2 \sim \frac{M^4}{f^2} \lesssim M^2 \sim \sqrt{gf}\, v_{obs} \lesssim \sqrt{\lambda}\, v_{obs}^2$$

$$\Downarrow \quad g \gtrsim \frac{M^4}{f v_{obs}^2} \sim \frac{m_\phi^2 f}{v_{obs}^2} \gtrsim \frac{m_\phi^3}{v_{obs}^3},$$

◎ Stability constraints.

Vacuum decay. $\Gamma(\phi) \sim f^4 e^{-S(\phi)}$ $\qquad S \propto (\text{large }\#) \times \dfrac{M^8 f}{g^2[v(\phi)^2 - v_{obs}^2]^3}$

S can be much larger than unity.

Finite T.

$T < T_{EW}$; safe & no destabilization.

$T > T_{EW}$; $\sim T^2\mu^2(\phi) - \cdots$ for $\dfrac{|\mu^2(\phi)|}{T^2} \ll 1$,

$\cup$

$gT^2\phi$,

↳ This potential could destabilize ϕ for $gT^2 \gtrsim \dfrac{M^4}{f}$, $\equiv gT_*^2$

Slow roll $\dot\phi \sim \dfrac{gT^2}{H} \sim gM_{Pl}$, $\Rightarrow \Delta\phi \sim \dot\phi \times \dfrac{M_{Pl}}{T_*^2} \sim \dfrac{g^2 M_{Pl}^2 f}{M^4} < \Delta\phi_{weak} \sim \dfrac{\lambda v_{obs}^2}{g}$

$\therefore\ g^3 \lesssim \dfrac{\lambda v_{obs}^2 M^4}{M_{Pl}^2 f}$.

◎ Observational constraints.

Mixing w/ Higgs. ← One can suppress g,

Coherent oscillation can be DM.

§ How to put ϕ on a top of the potential

① Same mechanism as "Inflating to the weak scale" 1809.07338

② Anthropic?