DESY Theory Workshop Seminar
New approaches to the Hierarchy Problem IX

Self-organized Higgs Criticality

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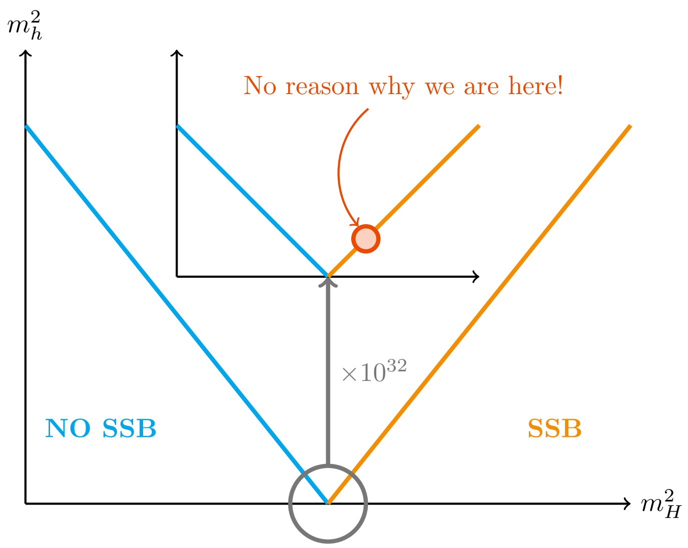
1 Introduction and Motivation

Yet another formulation of the Hierarchy Problem: The Standard Model seems to be *very very* close to the critical point, without any symmetry reasons.



It looks like the nature has adjusted the bare mass parameter m_H^2, as if an experimentalist has adjusted the temperature T to an accuracy of one part in 10^{32}.

Most solutions proposed to solve this puzzle, predicts new physics and particles not too far from the electroweak scale, around $\mathcal{O}(\text{TeV})$, but none of them have been observed. $\Rightarrow$ We need novel approaches!

Goal of this work: Explore the possibility that the Higgs is close to its critical point not due to symmetry, but rather due to a *self-organization* principle, as suggested in [1].

1.1 Self-organized Criticality

The systems who exhibit *self-critical* behavior

1. are driven *naturally* to their critical points, without any fine-tuning,
2. and remain critical under slow temporal loading of the system.

1.2 Complex Scaling Laws

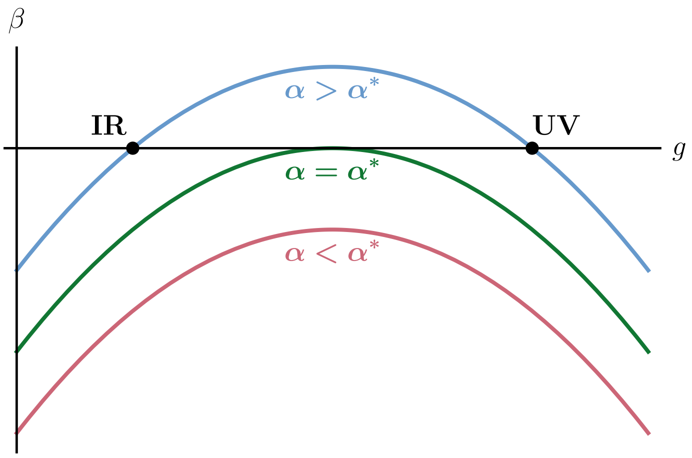
It has been hypothesized that at least some self-organized systems exhibit *log-periodic* scaling, i.e. complex scaling dimensions, at the threshold of criticality. The log-periodic power law implies discrete scale invariance:

$$\Delta = \delta \pm i\gamma \Rightarrow \langle \mathcal{O}(0)\mathcal{O}(x)\rangle \propto \frac{1}{|x|^{2\Delta}} \sim \frac{1}{|x|^{2\delta}} \cos(\gamma \log |x|). \tag{1.1}$$

This can be interesting, but discrete scale invariance/limit cycles are not an allowed endpoint for a 4D RG flow [2, 3]. This means such a behavior should somehow cause a phase transition, leads to confinement, and spontaneous breaking of conformal symmetry.

Lost of Conformality: Such a breakdown of conformality as a result of scaling dimensions becoming complex has been studied before by Kaplan, Lee, Son, and Stephanov [4]. They studied a model, where an operator $\mathcal{O}$ has different scaling dimensions $\Delta_{\mathrm{UV}}/\Delta_{\mathrm{IR}}$ at the UV/IR fixed points. The location of these fixed points are determined by an external parameter α.

The RG equations for this model can be such that under variation of α, fixed points and scaling dimensions move to each other, merge at some critical α^*, and become complex after that.



The AdS Dual: At each fixed point we have an exact CFT, and provided it is strongly coupled, it is dual to a classical gravity theory on AdS_5.

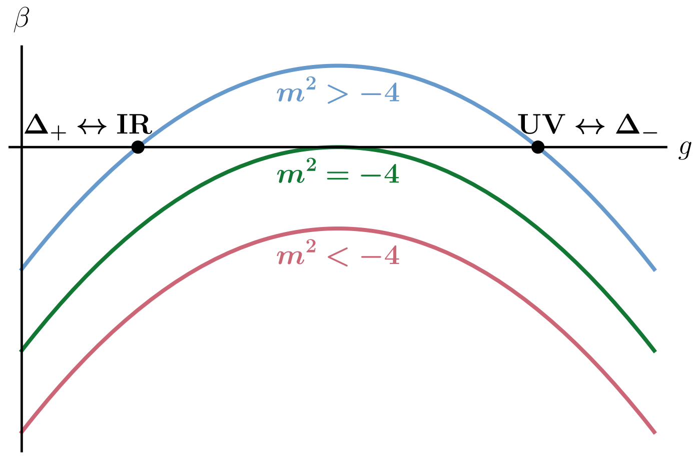
A scalar field ϕ with mass $-4 < m^2 < -3$ on AdS_5 has two scaling solutions near the AdS_5 boundary at $z \sim 0$:

$$\phi(z \sim 0) = c_+(x) z^{\Delta_+} + c_-(x) z^{\Delta_-}. \tag{1.2}$$

These two scaling dimensions correspond to *different* CFT's where $\mathcal{O}$ has the scaling dimensions

$$[\mathcal{O}_{\mathrm{CFT}}] = \Delta_\pm = 2 \pm \sqrt{4 + m^2}. \tag{1.3}$$

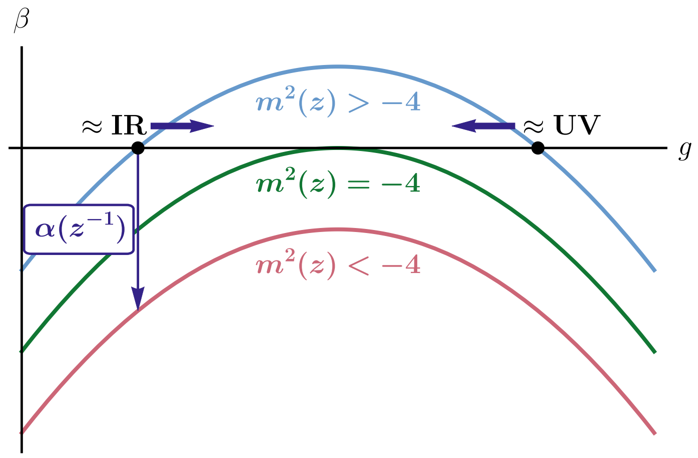
At the special point $m^2 = -4$, the Breitenlohner-Freedman (BF) bound [5], these two solutions merge with each other, below which the theory obtains an AdS tachyon, and it becomes unstable. The theory mush gap to fix the instability.



Our proposal: In the model of [4], moving out of the conformal window was a result of an external parameter, hence it is not *dynamical*. We want to promote this to a dynamical one.

Consider a model where the bulk mass m^2 is a *slowly decreasing* function of the bulk coordinate z. The dual theories are *approximate* CFT's with different *quasi* fixed points. These points walk with the RG flow (moving in the AdS_5 bulk) and merge at the BF bound resulting loss of conformality and condensation of ϕ.

$$m^2(z) \leftrightarrow \alpha(\mu \sim z^{-1}) \leftrightarrow \text{temporal loading in SOC} \tag{1.4}$$



We will construct and study two models which exhibit above mentioned features.

1. Toy model: Vary Higgs mass explicitly
2. Dynamical model: Use a Goldberger-Wise field to drive the Higgs mass

2 Toy Model

We consider this model in asymptotically AdS_5 space without a UV brane, but with a IR brane located at $z = z_1$. Working in units where the AdS curvature k is set to 1, the metric is given by

$$\mathrm{d}s^2 = \frac{1}{z^2}\left[\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \frac{\mathrm{d}z^2}{G(z)}\right], \quad z \in [0, z_1] \tag{2.1}$$

where $G(z)$ encodes the backreaction due to non-trivial bulk profile of scalar field(s). By writing this metric ansatz, we have restricted ourselves to background solutions obeying 4D Lorentz invariance.

The action is

$$\begin{aligned} S = &\int \mathrm{d}^4x\,\mathrm{d}z\,\sqrt{g}\left[g^{MN}\partial_M H^\dagger \partial_N H + \frac{6}{\kappa^2} - m^2(z)|H|^2 - \frac{1}{2\kappa^2}\mathcal{R}\right], \\ &- \int \mathrm{d}^4x\,\sqrt{-g_0}V_0(|H|)\Big|_{z\to 0} - \int \mathrm{d}^4x\,\sqrt{-g_1}V_1(|H|)\Big|_{z\to z_1}, \end{aligned} \tag{2.2}$$

where $\kappa^2 = 1/(2M_{\rm pl}^3)$. In this model, the z-dependence of the Higgs mass is set explicitly, and given by

$$m^2(z) = -4 + \delta m^2 - \lambda z^\epsilon, \quad \delta m^2 > 0. \tag{2.3}$$

V_0 and V_1 are UV and IR brane potentials respectively, and are given as

$$V_0(|H|) = T_0 + m_0^2|H|^2, \tag{2.4}$$
$$V_1(|H|) = T_1 + \lambda_H|H|^2(|H|^2 - v_H^2), \tag{2.5}$$

where T_0 and T_1 are brane tensions. The Higgs may, for some regions of parameter space, pick a nonvanishing vacuum expectation value (VEV), $\langle H \rangle = \phi_h(z)/\sqrt{2}$. In that case, the HiggsVEV has a nontrivial profile alond the z-coordinate.

By using the metric ansatz (2.1) and the action (2.2), one can get the Einstein equations, and an expression for the scalar curvature $\mathcal{R}$. Then one plugs these back into (2.2), and finds that the total action is given by a pure boundary term[1]:

$$S = \int \mathrm{d}^4 x \left(-V_{\text{eff}}\right), \tag{2.6}$$

where V_{eff} is the effective 4D potential of the system[2]:

$$V_{\text{eff}} = \underbrace{\frac{1}{z_0^4}\left[V_0(\phi_h) - \frac{6}{\kappa^2}\sqrt{G}\right]\Bigg|_{z=z_0}}_{\text{UV contribution}} + \underbrace{\frac{1}{z_1^4}\left[V_1(\phi_h) + \frac{6}{\kappa^2}\sqrt{G}\right]\Bigg|_{z=z_1}}_{\text{IR contribution}}. \tag{2.7}$$

In this model, the UV contribution vanished in the $z_0 \to 0$ limit, with the expection of a constant term which is tuned to zero to give a vanishing effective cosmological constant. Using Einstein equations, one can relate the metric function G to the behavior of the Higgs VEV in the bulk:

$$G = \frac{-\frac{\kappa^2}{6}V(\phi_h)}{1 - \frac{\kappa^2}{12}\left(z\phi_h'\right)^2} = \frac{1 - \frac{\kappa^2}{12}m^2(z)\phi_h^2}{1 - \frac{\kappa^2}{12}\left(z\phi_h'\right)^2}. \tag{2.8}$$

Therefore, the effective potential can be obtained exactly once the bulk profile of the Higgs is solved.

The Higgs bulk profile is obtained by solving the scalar field equation of motion, which in the limit of small κ^2 is

$$\phi_h'' - \frac{3}{z}\phi_h' - \frac{1}{z^2}\frac{\partial V}{\partial \phi} = 0. \tag{2.9}$$

This equation is supplemented by the boundary conditions:

$$z\phi_h'\Big|_{z=z_{0,1}} = \pm\frac{1}{2}\frac{\partial V_{0,1}}{\partial \phi_h}. \tag{2.10}$$

The full solution of (2.9), without imposing any boundary condition, is the combination of two scaling solutions:

$$\phi_h \sim \phi_{h,\pm} z^2 J_{\pm\nu}\left(\frac{2\sqrt{\lambda}}{\epsilon}z^{\epsilon/2}\right), \quad \nu \equiv \frac{2\sqrt{\delta m^2}}{\epsilon}. \tag{2.11}$$

[1]For more details, and generalization of this result to multiple scalar fields, see [6] and the references therein.

[2]This procedure can also be interpreted as "integrating out" the extra dimension.

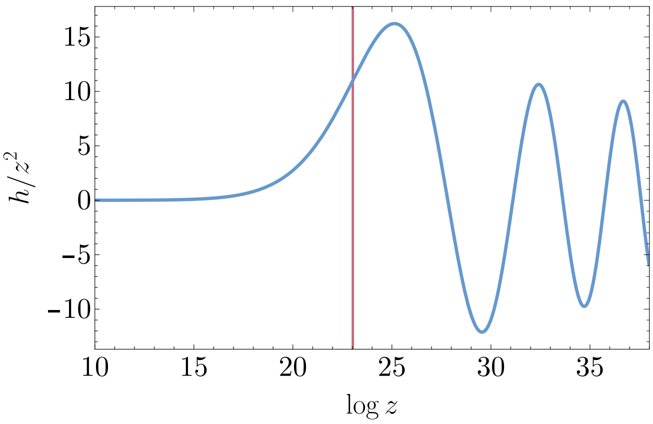
Note that near the AdS boundary $z \to 0$, these solutions are power law in z, with the expected behavior given in (1.2).

For a generic choice of the UV boundary condition, $m_0^2 \neq 2 - \sqrt{\delta m^2}$, only positive scaling solution is relevant; while the negative scaling solution is only relevant for the special case $m_0^2 = 2 - \sqrt{\delta m^2}$. We will only consider the former by setting $m_0^2 = 0$.

For large z, and small ϵ, the Bessel functions exhibit log-periodic scaling, so the Higgs profile behaves as

$$\phi_h \propto z^{2-\epsilon/4} \cos\left(\sqrt{\lambda} \log z + \gamma\right), \tag{2.12}$$

which is a sign of complex scaling dimensions, which might be a common feature in self-organized critical systems.



The condition for formation of a condensate is met when the IR brane boundary condition favors a nontrivial value for the coefficients of the bulk solution:

$$\frac{1}{\epsilon}\left(\lambda_H v_H^2 - 4\right) \geq \frac{x J_\nu'(x)}{J_\nu(x)}, \quad x \equiv \frac{2\sqrt{\lambda} z_1^{\epsilon/2}}{\epsilon}, \tag{2.13}$$

where equality indicates critical points. Note that there are many of them as a result of the quasi-periodicity of the RHS at large z_1. We shall label the i-th critical point as z_c^i.

Since the Higgs VEV turns on at these critical points, the character of the 4D effective potential also changes dramatically. In the vicinity of the first critical point, it can be approximated as

$$V_{\text{eff}} \approx \begin{cases} \frac{1}{z_1^4}\delta T_1, & z_1 < z_c^1 \\ \frac{1}{z_1^4}\left[\delta T_1 + \frac{\lambda_H \sigma^4}{8}\left(\frac{z_1}{z_c^1} - 1\right)\right], & z_1 > z_c^1, \end{cases} \tag{2.14}$$

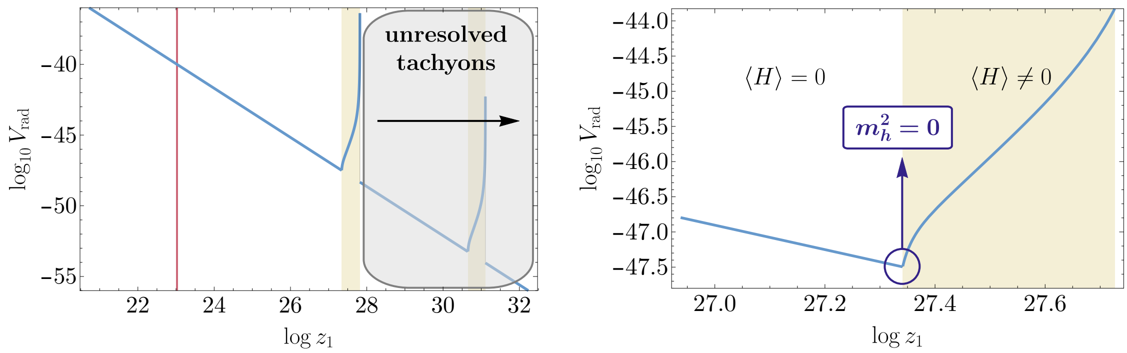
where

$$\sigma^2 = \frac{-4m^2(z_c^1) + \lambda_H v_H^2\left(\lambda_H v_H^2 - 8\right)}{2\lambda_H}, \tag{2.15}$$

and δT_1 is the mistune between the IR brane tension and the bulk cosmological constant. If this brane mistune satisfies another condition

$$0 < \delta T_1 < \frac{\lambda_H \sigma^4}{32}, \tag{2.16}$$

then the derivate of the effective potential changes sign, and creates *kink minimums* at critical points.



Higgs Fluctuations To investigate the tachyon instabilities and confirm that the Higgs is massless at the critical point, we need to study the spectrum of Higgs fluctuations. Presuming a vanishing Higgs VEV, the equation of motion for the fluctuations are given by

$$h_n''(z) - \frac{3}{z} h_n'(z) - \frac{1}{z^2} m^2(z) h_n(z) = -m_n^2 h_n(z), \tag{2.17}$$

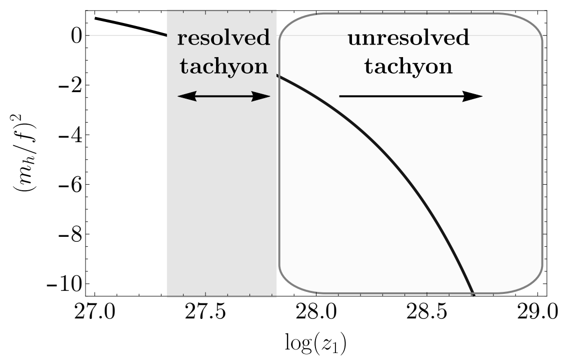
with the IR boundary condition given by

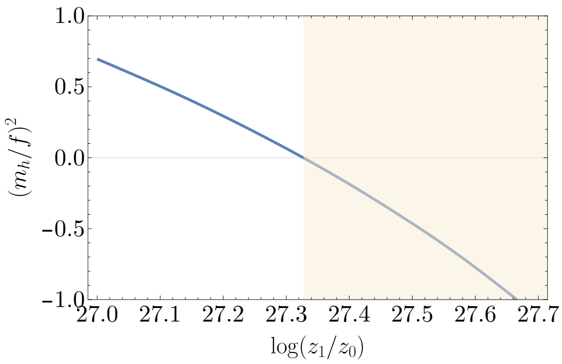
$$h_n'(z_1) = \frac{\lambda_H v_H^2}{2 z_1} h_n(z_1). \tag{2.18}$$

The UV boundary condition is

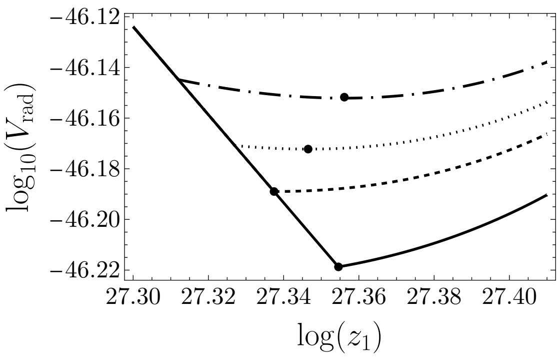
$$z_0 h_n'(z_0) = \frac{m_0^2}{2} h_n(z_0), \tag{2.19}$$

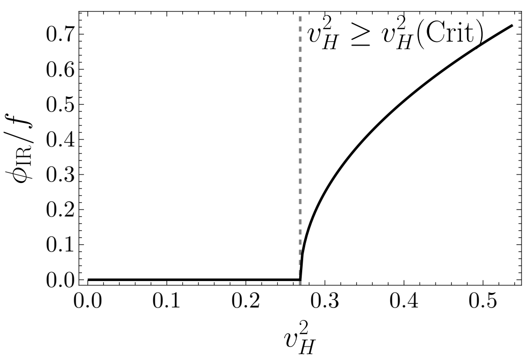
in the $z_0 \to 0$ limit. Solving this boundary value problem gives the spectrum of states. Below, we show the lowest eigenvalue, expressed as the ratio $(m_h/f)^2$, where $f^{-1} = z_1$ represents the conformal breaking scale, i.e. the KK scale.





Broad Critical Region: Crucial to the success of the model as one of self-organized Higgs criticality is the existence of broad critical region, over which the Higgs remains light. This is the case in this model, as can be seen in the plots below.





Metric Boundary Conditions: This model in its current form has a major bug (or feature). The brane localized potentials do also impose junctions conditions on the metric function $G(y)$, in addition to boundary conditions imposed on the scalar fields. These conditions enforce

$$\sqrt{G(z_0)} = \frac{\kappa^2}{6} V_0 \quad \text{and} \quad \sqrt{G(z_1)} = -\frac{\kappa^2}{6} V_1. \tag{2.20}$$

By comparing these conditions with the expression for the effective potential (2.7), one can immediately see that the metric junction conditions requires UV and IR contributions separetely vanish.

In the usual Goldberger-Wise scenario, one performs the tuning of the UV brane tension which sets the UV contribution to zero, which is equivalent to tuning of the bare cosmological constant. However, the IR contribution automatically vanishes at the minimum of the potential, thus tuning of the IR brane tension is not necessary.

The situation is quite different for this model. At the kink minimum generated by the Higgs contribution, these junction conditions cannot be met unless two tunings are performed — both the bare cosmological constant and the mistune in the IR brane tension. This problem does not necessarily mean that this model is useless or ruled out, but rather tells us that our metric ansatz is too simple to study this model. It *must be relaxed* to include a nontrivial 4D cosmology.

3 Dynamical Model

In this section, how much of the physics of the toy model studied in the previous section can be derived in a dynamical and more realistic model.

This model has two bulk scalar fields. In addition to a Higgs scalar, there is also a Goldberger-Wise-like scalar field ϕ_d. Both fields are coupled in such a way that a varying VEV of the scalar ϕ_d drives the effective bulk mass of the Higgs, making it a function of the extra-dimensional coordinate. The 5D action is

$$\begin{aligned} S = & \int \mathrm{d}^4x\,\mathrm{d}z\,\sqrt{g}\left[g^{MN}\partial_M H^\dagger \partial_N H + \frac{1}{2}g^{MN}\partial_M\phi_d\partial_N\phi_d + \frac{6}{\kappa^2} - \frac{1}{2\kappa^2}\mathcal{R}\right] \\ & + \int \mathrm{d}^4x\,\mathrm{d}z\,\sqrt{g}\left[-\left(m_H^2 - \lambda\phi_d\right)|H|^2 - \frac{1}{2}\epsilon(\epsilon-4)\phi_d^2\right] \\ & - \int \mathrm{d}^4x\,\sqrt{-g_0}V_0(\phi_d,|H|)\bigg|_{z\to z_0} - \int \mathrm{d}^4x\,\sqrt{-g_1}V_1(\phi_d,|H|)\bigg|_{z\to z_1}, \end{aligned} \tag{3.1}$$

where $\epsilon \sim \mathcal{O}(0.1)$. Unlike the toy model, there is a UV brane located at $z = z_0$. The brane potentials are assumed to take the form

$$V_{0,1} = T_{0,1} + V_{0,1}^{\phi_d} + V_{0,1}^H, \tag{3.2}$$

where for the brane Higgs potentials, we take

$$V_0^H = m_0^2|H|^2 \quad \text{and} \quad V_1^H = \lambda_H|H|^2\left(|H|^2 - v_H^2\right). \tag{3.3}$$

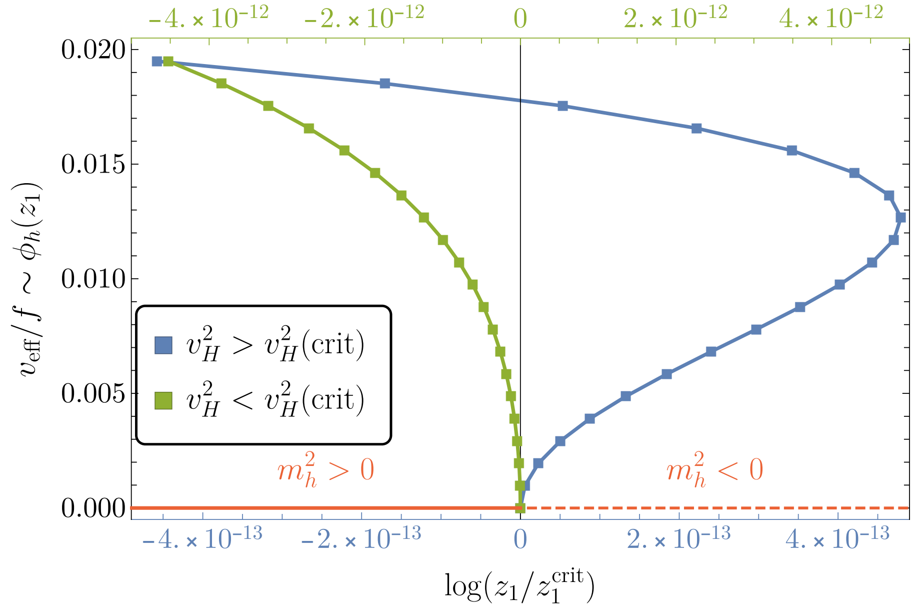
We choose the brane GW potentials $V_{0,1}^{\phi_d}$ such that they impose the stiff wall boundary conditions for ϕ_d, which are $\phi_d(z_{0,1}) = v_{0,1}$. Thus, the value of the bulk Higgs mass varies from $m_H^2 - \lambda v_0$ on the UV brane to $m_H^2 - \lambda v_1$ in the IR. It is possible, and not fine-tuned, to arrange for the effective Higgs mass to vary such that it crosses the BF bound somewhere in the bulk, and evolves from power law behavior in the UV to a log-periodic power law in the IR.

Non-linear BVP: Due to the explicit coupling between the two fields, the bulk equations governing the behavior of the field VEVs are nonlinear. Existence and uniqueness are not guaranteed in the case of general nonlinear boundary value problem. Thus, in order to solve the system, we do not search for solutions with fixed z_1, but rather search for solutions with varying values of the Higgs VEV. For a given Higgs VEV, we use a shooting method to obtain a value for z_1 and the associated bulk profiles ϕ_d and ϕ_h.

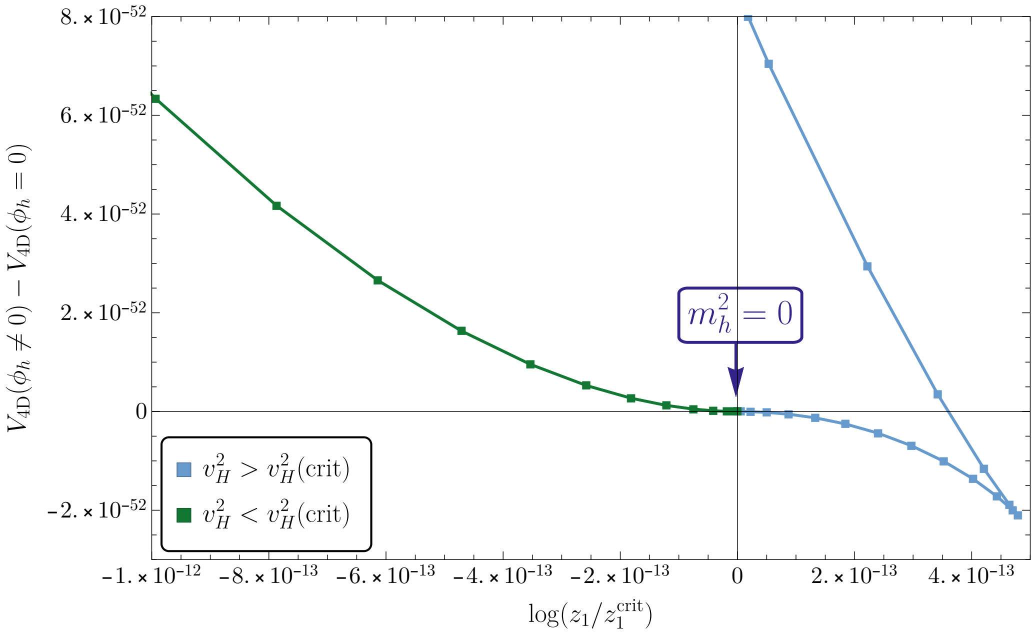
Effective Higgs VEV: A convenient measure of the size of the Higgs VEV, with physical meaning to a low energy observer, is the mass that would be given to a gauge field by the Higgs mechanism. For a Higgs field in the bulk, it is given by

$$\left(\frac{v_{\rm eff}}{f}\right)^2 = z_1^2 \int_{z_0}^{z_1} \frac{\mathrm{d}z}{z^3}\phi_h(z)^2. \tag{3.4}$$

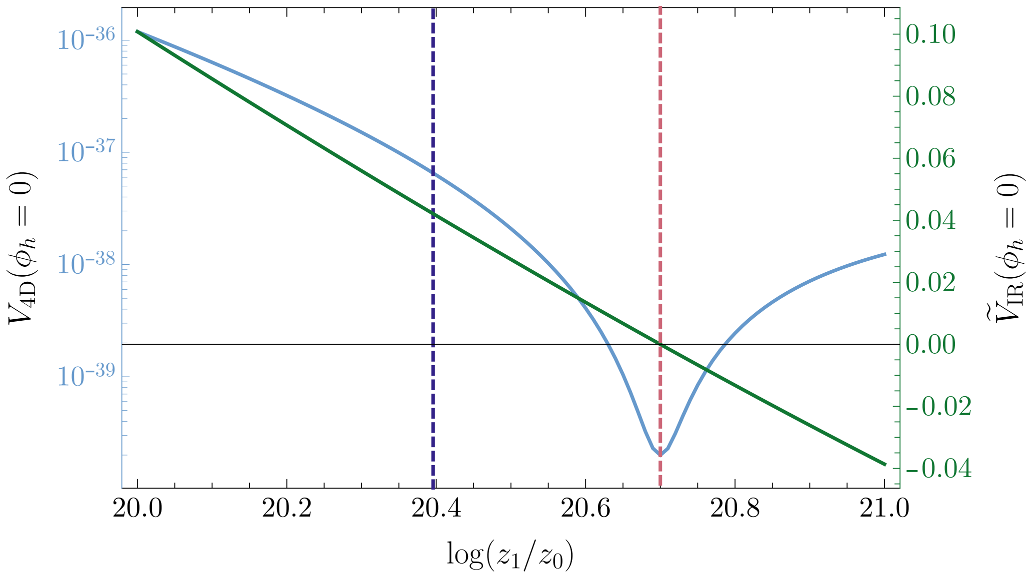
Two Class of Solutions: The types of solutions one obtains can be divided into two classes based on the value of the IR-brane localized Higgs mass.



The effective potential can be obtained in a similar way as in the previous section.



Tension Between Two Minima: Also in this model, the metric junction conditions are not satisfied at the minimum of the effective potential. However, they are satisfied at the minimum of the effective potential in the absence of Higgs VEV. This point might be larger or smaller than the critical point. If it is larger, then the theory has a tension between two competing minima. The minimum of the gravity sector and the minimum of the Higgs sector.



4 The Frustrated Dilaton

In this section we discuss how we can interpret the features we have observed in the dynamical model.

In an approximately scale-invariant theory that undergoes spontaneous symmetry breaking, the effective potential for the dilaton has the form

$$V_{\rm dil} = \alpha(f) f^4, \tag{4.1}$$

where $\alpha(f)$ has some mild dependence on f. This potential can have non-trivial minimum at some finite value of f.

If many operators are condensing simultaneously, they all contribute to the scale of conformal breaking f, but there might be relations between individual VEVs which are not encoded in the dilaton potential. The relations can be such that there is no solution between VEVs below some critical $f_{\rm crit}$. In this case, the theory gets "trapped" at a larger f, although the potential (4.1) is minimized at a lower (or zero) f.

5 Speculation: Cosmology

In this section, we shall make some speculations how the tension between the two minima might be solved by incorporating the cosmology into the picture.

Imagine that we have knob to tune the trilinear coupling λ between the Higgs and GW fields. We can arrange it such that the GW potential gets minimized at $z_1 < z_c$, where the electroweak symmetry is unbroken. There is no tension in this case, the potential gets minimized at a region without instabilities.

Then as one increase λ, the minimum of the GW potential gets closer to z_c. There is a fine tuned value of λ, where the GW minimum coincides with z_c. At this point, there is a massive radion, and a fine tuned massless Higgs.

What happens when we further increase λ remains mysterious. Since the static metric ansatz does not work in this region, we suspect that there might be a transition to a spontaneously Lorentz-violating dynamical background.

6 Conclusions

- We have discussed a new possible approach to the hierarchy problem which is inspired by aspects of SOC.
- We have explored 5D constructions that resemble SOC features and found that there is a large region of parameter space where a novel type of phase transition appears to be taking place.
- We have found that cosmological dynamics might play a big role to correctly describe this transition, providing a novel link between particle physics and cosmology.

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