Session X: Clockwork approach to the Hierarchy problem

Felix Giese

January 14, 2020

Abstract

In this talk we present how the clockwork mechanism arouse from work on the relaxion approach to solve the hierarchy problem. We then wonder whether Clockwork is a new solution to the hierarchy problem for its own.

Contents

1 Prélude

The clockwork starts from the observation that masses and interaction scales are not the same. In a unit system where one differentiates length scales from energies, one gets

$$\begin{gathered}[\hbar] = EL, \quad [\mathcal{L}] = EL^-3, \quad [\phi] = [A_\mu] = E^{\frac{1}{2}}L^{-\frac{1}{2}},\ [\psi] = E^{\frac{1}{2}}L^{-\frac{1}{2}}, \\ [\partial] = [\tilde{m}] = L^{-1} \quad [g] = [y] = E^{-\frac{1}{2}}L^{\frac{1}{2}}\end{gathered} \tag{1}$$

Not only masses but also couplings are dimensionful quantities. Canonical dimensions $\hbar = 1$ are recovered by identifying $E = L^{-1}$. We set units of mass and coupling by

$$\tilde{M} = L^{-1} \quad \text{and} \quad C \equiv E^{-\frac{1}{2}}L^{-\frac{1}{2}} \tag{2}$$

Now look at a general operator of canonical dimension d

$$\frac{1}{\Lambda^{d-4}}\partial^{n_D}\phi^{n_B}\psi^{n_F} \tag{3}$$

where $d = n_D + n_B + \frac{3}{2}n_F$. Then what Λ does is to define the effective interaction strength called *scale*

$$[\Lambda] = \frac{\tilde{M}}{C^{\frac{n-2}{d-4}}} \tag{4}$$

where $n = n_B + n_F$. Each field carries an inverse power of C, the Lagrangian has units of $[\mathcal{L}] = \tilde{M}C^{-2}$.

Since $\frac{n-2}{d-4} > 0$: *scales and masses are incommensurable quantities.*

- A mass is asscociated with $E_m = \tilde{m\hbar}$ at which new degrees of freedom appear.
- A scale is the energy at which the theory becomes strongly coupled if no new degrees of freedom intervene. It tells you about the interaction strength and carries no information on new dynamics. This information would be given by Scale $\times$ Coupling.

We will now introduce the Clockwork mechanism which is a tool to explicitly construct concrete examples where extremely small couplings can be generated [without introducing any exponentially large parameters].

2 The original clockwork scalar

> "Although these approaches have been around for a long time, a move of marketing genius has recently renamed them as clockworkmodels." – A. Hook: TASI lectures on the Strong CP Problem and Axions 2018 , p.18

The clockwork mechanism was popularized by G. Giudice & M. McCullough in 2016 [1, 2] where the validity of the mechanism was shown for all kind of fields. However the original idea comes from axion model building where the same idea has been developed by two groups independent from each other [3, 4]. See also proceedings from Moriond EW 2017(18) [5, 6]. Also you can find a good blackboard talk by Sutherland

The set-up: Consider a theory with a spontaneously broken $U(1)^{\mathrm{N+1}}$ symmetry at high scale f. As usual we parametrize the expected $N+1$ Goldstone fields π_j by

$$U_j(x) = e^{i\pi_j/f} \qquad j = 0, \ldots, N \tag{5}$$

Further we break $U(1)^{\mathrm{N}}$ symmetries explicitly by introducing linking mass terms m_j between neighboring sites. This explicit breaking pattern can be understood by thinking of m_j as spurion field charged under the corresponding $\mathrm{U}(1)$ with charge

$$Q_i[m_j] = \delta_{ij} - q\delta_{ij+1} \tag{6}$$

under $\mathrm{U}(1)_i$. We assume $q > 1$ and the explicit breaking to be small. Then the hierarchy $\frac{m_j^2}{f^2}$ is technically natural because the $\frac{m_j^2}{f^2} \to 0$ restores the orginial symmetry of the model.

Writing the theory down explicitly

$$\mathcal{L} = -\frac{f^2}{2}\sum_{j=0}^{N} \partial_\mu U_j^\dagger\, \partial^\mu U_j + \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left(U_j^\dagger\, U_{j+1}^q + \text{h.c.}\right)\,. \tag{7}$$

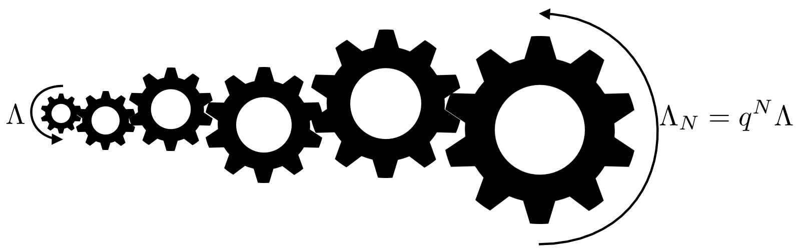

Figure 1: Picture taken from [1] to convince you the model is appropriately named.

In terms of the fields π_j, eq. 7 becomes

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} \partial_\mu \pi_j \partial^\mu \pi_j - V(\pi) \tag{8}$$

$$V(\pi) = \frac{m^2}{2}\sum_{j=0}^{N-1} (\pi_j - q\,\pi_{j+1})^2 + \mathcal{O}(\pi^4) = \frac{1}{2}\sum_{i,j=0}^{N} \pi_i\, M^2_{\pi\, ij}\, \pi_j + \mathcal{O}(\pi^4)\,. \tag{9}$$

The mass matrix M_π^2 is given by

$$M_\pi^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix}\,. \tag{10}$$

To see the clockwork inside (10) we construct the corresponding mass spectrum. The eigenvalues can be determined analytically to be

$$m_{a_0} = 0 \qquad m_{a_k}^2 = m^2\left[q^2 + 1 - 2q\cos\frac{k\pi}{N+1}\right] \tag{11}$$

where we have

- a_0 is the massless Goldstone that we still expect to be there from the nonbroken U (1) factor
- a_k are called "Clockwork gears".

The mass eigenvalues are determined by solving following recursion relation

$$-\,qa_{k-1}+1+q^2a_k-qa_{k+1}=\lambda_k a_k \tag{12}$$

by using the Ansatz

$$a_k=\sin k\theta+A\cos k\theta \tag{13}$$

from which $\lambda=1+q^2-2q\cos\theta$. The boundary conditions set by the 0^{th} and n^{th} site can be written as

$$A=-\frac{\sin\theta}{q-cos\theta} \tag{14}$$

and

$$0=(1+q^2)\sin(N+1)\theta-q\sin N\theta-q\sin\left(N+2\right)\theta \tag{15}$$

The second constraint gives N solutions $\theta_i=\frac{i\pi}{N+1}$ From this the Eigenvalues of the clockwork gears follow.
For the eigenvectors a_j one obtains at nth site

$$\pi_n=\sum O_{nj}\ a_j \tag{16}$$

with

$$O_{j0}=\frac{\mathcal{N}_0}{q^j}\,,\quad O_{jk}=\mathcal{N}_k\left[q\sin\frac{jk\pi}{N{+}1}-\sin\frac{(j+1)k\pi}{N{+}1}\right]\,,\quad j=0,..,N;\quad k=1,..,N$$

$$\mathcal{N}_0\equiv\sqrt{\frac{q^2-1}{q^2-q^{-2N}}}\,,\qquad \mathcal{N}_k\equiv\sqrt{\frac{2}{(N{+}1)\lambda_k}}\,. \tag{17}$$

This exponential suppression at n^{th} site supressed side localization of the zero mode gives clockwork its name (and possibly also the localization of the "gears"). Note that the mass gap Δm and the state density δm in the band are given by

$$\frac{\Delta m}{m_{a_1}}=2\left(q-1\right),\qquad\qquad \frac{\delta m_k}{m_{a_k}}\sim\frac{q\pi}{N\lambda}\sin\frac{k\pi}{N+1}\sim\mathcal{O}\left(1/N\right). \tag{18}$$

One might guess that the continuum limit $N\to\infty$ is described by the following "clockwork metric" (or Linear dilaton metric):

$$\mathrm{d}s^2=e^{\frac{4k|y|}{3}}\left(\mathrm{d}x^2+\mathrm{d}y^2\right) \tag{19}$$

A massless scalar in this background can be deconstructed to 4D to give the same spectrum as the clockwork which leads to the conjecture that a 5D clockwork can be realized in form of a linear dilaton model. The 5D model for KK modes in the dilaton model gives

$$m_n^2=\sqrt{k^2+n^2/R^2} \tag{20}$$

We will come back to 5D Clockwork and show this later.

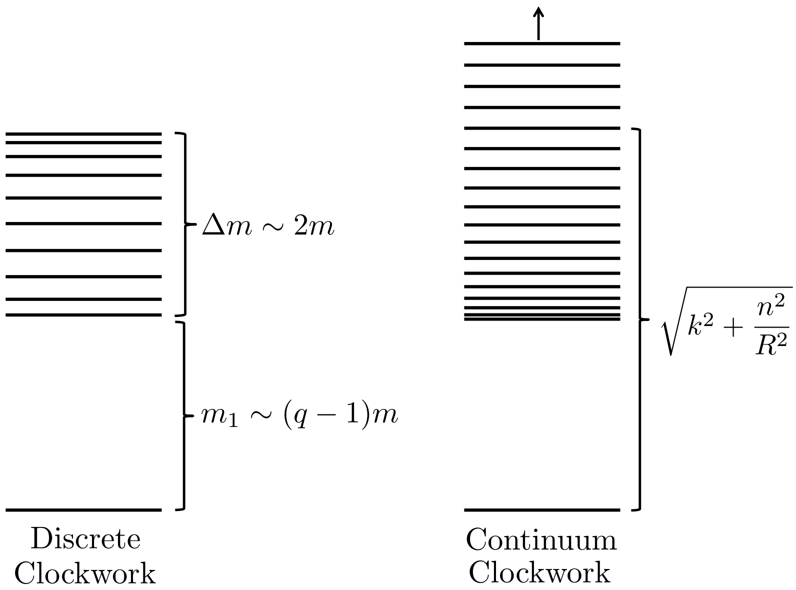

Figure 2: Mass spectrum of the clockwork theory. k is called clockwork spring. Picture taken from [1].

Example of a possible application (relaxion): Originally the Clockwork was invented to explain large axion constants.

Take for instance the relaxion solution to the hierarchy problem [7] which we have seen in Philip a couple of sessions ago. The ingredients you need in order to make the mechanism work are

$$V(\phi, H) = \left(-\Lambda^2 + g\phi\right)|H|^2 + \left(g\Lambda^2\phi + g^2\phi^2 + \dots\right) + \frac{1}{32\pi}\frac{\phi}{f}G\tilde{G} \tag{21}$$

where the last term after confinement becomes

$$\Lambda_b^4(H)\cos\frac{\phi}{f}. \tag{22}$$

For details describing the relaxion mechanism, see Philips talk. To remind you of one observation, we had found:

- The relaxion needs to stop, slopes need to be comparable: $gM^2f \sim \Lambda_b^4 \sim \frac{vf_\pi^3}{f}$. Typical values for the relaxion mechanism are $M \sim 10^7$ GeV and $f \sim 10^9$ GeV. Also the scan needs to include all field values from M to $-M$. Thus we expect
$$\Delta\phi \sim \frac{M^2}{g} \sim 10^{37}\text{ GeV} \tag{23}$$
which should be carefully generated.

To implement a clockwork (see [8] for details) we rewrite the relaxion potential as

$$V(\phi, H) = \Lambda_N^4\cos\frac{\phi}{F} + \Lambda_b^4\cos\frac{\phi}{f} \tag{24}$$

where we interpreted the slope as linear piece expanded from another stronly coupled sector. Imagine that the relaxion is the zero mode in a clockwork theory and we have the two strong sectors coupling two the axion located at different end sites of a clockwork, e.g. $\sim \frac{1}{32\pi^2}\phi_N H\tilde{H}$. With this

$$\frac{1}{32\pi^2} H_{\mu\nu} H^{\mu\nu} \left(\frac{a_0}{F} - \sum (-1)^k \frac{a_k}{f_k} \right) \tag{25}$$

where $F = q^N f/\mathcal{N}_0 = \Delta\phi$. With this the hierarchy of scales is

$$\Lambda_b \ll \Lambda \sim \Lambda_N \ll f \ll F \tag{26}$$

Note that so far we did not really do something fundamentally new nor something fundamentally wrong. People generate hierachies by mass matrix diagonalization since many years. What is new, we have used a very particular theory with $N+1$ axions and a weird pattern of global symmetry breaking. So the question of interest is whether there is some kind of UV completion of the clockwork. The main interest in the literature so far has been whether there is a 5D completion which can be dimensionally deconstructed to 4D. This question has lead to some delicate tensions in the literature which we also will have a look at.

3 Clockworks exist in all colors

Starting from the ideas in axion context, clockwork did get its own dynamics. The idea has been generalized to many other branches of BSM physics. Maybe most interestingly, a new solution to the hierarchy problem would be to see whether gravity can be clockworked itself and the Planck scale might look large because localization of a zero mode in theory space. Generalizations of the clockwork mechanism are subject to the next section.

Before we turn to the question on how to clockwork the graviton, we want to point out that "the clockwork theory" aims to give a very complete picture. The clockwork cannot only be implemented for scalar fields, but [1] also gives examples on how to construct a clockworked fermion and clockworked photons.

Fermions Make use of N left-chiral symmetries and $N+1$ rightchiral symmetries with breaking $U(1)_L \times U(1)_R \to U(1)_R$. One ends up with a slightly different mass matrix $\bar{\psi}_L M_\psi \psi_R + \text{h.c.}$

$$m \sum_{j=0}^{N-1} \left(\bar{\psi}_{Lj} \psi_{Rj} - q\, \bar{\psi}_{Lj} \psi_{Rj+1} + \text{h.c.} \right) \equiv \left(\bar{\psi}_L M_\psi \psi_R + \text{h.c.} \right) \tag{27}$$

$$M_\psi = m \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ 0 & 1 & -q & \cdots & & 0 \\ 0 & 0 & 1 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & -q & 0 \\ 0 & 0 & 0 & \cdots & 1 & -q \end{pmatrix} , \tag{28}$$

but there will still be an underlying clockwork mechanism because of $M_\psi^2 = M_\phi^2$. This then relates to observed hierarchies related to small fermion masses (for instance there is clockwork neutrinos [9], clockworked flavor physics [10], WIMP Dark matter [11] ... list goes on)

Photons For the photon field $\mathrm{U}(1)^{N+1}$ gauge symmetries are broken to U(1)

$$\mathcal{L} = -\sum_{j=0}^{N} \frac{1}{4} F^j_{\mu\nu} F^{j\,\mu\nu} - \sum_{j=0}^{N-1} \left[|D_\mu \phi_j|^2 + \lambda(|\phi_j|^2 - f^2/2)^2 \right] \tag{29}$$

$$D_\mu \phi_j \equiv \left[\partial_\mu + ig \left(A^j_\mu - q A^{j+1}_\mu \right) \right] \phi_j . \tag{30}$$

Below the scale f, working in unitary gauge, we find the effective Lagrangian for the gauge fields

$$\mathcal{L} = -\sum_{j=0}^{N} \frac{1}{4} F^j_{\mu\nu} F^{j\,\mu\nu} + \sum_{j=0}^{N-1} \frac{g^2 f^2}{2} (A^j_\mu - q A^{j+1}_\mu)^2 \quad . \tag{31}$$

This theory gives rise to gauge millicharges.

Gravitons (EW hierarchy problem): Finally, we apply the clockwork mechanism to gravity itself by considering $N+1$ copies of GR. On would of course start with $N+1$ gravitons and $N+1$ diffeomorphism invariances $(i, 0 \ldots N)$.

$$g^i_{\mu\nu} \to g^i_{\mu\nu} + \nabla_{(\mu} A^i_{\nu)} \tag{32}$$

Again introducing nearest neighbor interactions breaks these symmetries down to

$$g^i_{\mu\nu} \to g^i_{\mu\nu} + \frac{1}{q^i} \nabla_{(\mu} \tilde{A}_{\nu)} \tag{33}$$

where $\tilde{A}$ acts on all sides. Working in linearized gravity $g^i_{\mu\nu} = \eta^i_{\mu\nu} + \frac{2h^i_{\mu\nu}}{M_i}$ the interaction lagrangian is given by Pauli-Fierz terms which may arise in a deconstruction from 5D gravity:

$$\mathcal{L} = -\frac{m^2}{2} \sum_{j=0}^{N-1} \left(\left[h^{\mu\nu}_j - q h^{\mu\nu}_{j+1} \right]^2 - \left[\eta_{\mu\nu} (h^{\mu\nu}_j - q h^{\mu\nu}_{j+1}) \right]^2 \right) \quad . \tag{34}$$

which is indeed invariant under the clockwork gauge symmetry and respected by the clockwork structure of mass terms. The combination of Eigenstates is the same as in the scalar case. Now couple the SM sector $T^{\mu\nu}_{\mathrm{SM}}$ to the last site of the clockwork. Then

$$-\frac{1}{M_N} h^{\mu\nu}_N T_{\mu\nu} \to -\frac{1}{M_{\mathrm{Pl}}} \tilde{h}^{\mu\nu}_0 T_{\mu\nu} \tag{35}$$

This suggests that there is a clockwork solution to the hierarchy problem. In [1] one can find the statement that multigravity theories are plagued from theoretical subtleties which they claim can be solved by constructing a $N \to \infty$ Clockwork theory in 5D which is well behaved. This solution the authors identify with linear dilaton theories which originates as a holographic dual from Little string theory [12].

Concerning the question whether Clockwork can be thought of as solution to the EW hierarchy problem there exists a critical paper by N. Craig, I. Garcia & D. Sutherland [13]. They disagree strongly to two points, namely

- On the question whether there are 5D counterparts to clockwork.
- On the kind of theories that can be clockworked in 4D. To them Clockwork is a strictly abelian phenomenon. A 4D clockworked gravity cannot not work.

The criticism has been addressed again [14, 15]. It seems that by now all people technically agree with each others arguments but would like to stick to their definition of what is meant by a successful "clockwork theory".
Let us try to wonder about these statements a bit in the next section while we discuss the linear dilaton.

4 Linear dilaton and phenomenology

In addition to maybe solving the hierarchy problem, the ultimate aftermath of Clockwork might be the rich pheno, which was discovered as site effect of trying to solve the hierarchy problem. The clockwork mechanism has been acknowledged for raising more attention to previously less thought of signatures for collider experiments like displaced vertices and in particular compressed KK spectra[1]. We will shorten the derivation of the clockwork metric from the Linear dilaton by a lot. For details, see [1]. After changing to Einstein frame $g_{MN} \to e^{-\frac{2S}{3}} g_{MN}$, the dilaton action reads

$$\begin{aligned} S &= \int d^4x\, dy \sqrt{-g} \left\{ \frac{M_5^3}{2} \left(\mathcal{R} - \frac{1}{3} g^{MN} \partial_M S\, \partial_N S + e^{-\frac{2S}{3}} 4k^2 \right) \right. \\ &\left. - \frac{e^{-\frac{S}{3}}}{\sqrt{g_{55}}} \left[\delta(y - y_0)\Lambda_0 + \delta(y - y_\pi)\Lambda_\pi \right] \right\} \end{aligned} \tag{36}$$

where S is the dilaton field and k some (negative) Bulk vacuum energy. Λ_0 and Λ_π denote 4D vacuum energies. Parametrizing the metric as $\mathrm{d}s^2 = e^{2\sigma(y)} (\mathrm{d}x^2 + \mathrm{d}y^2)$

[1]Although there are some that did this in the past [12]

one finds by solving Einstein equations, the dilaton equation of motion, and the four junction conditions that

$$\sigma = \frac{2k|y|}{3} e^{\left(\sigma_0 - \frac{S_0}{3}\right)+\sigma_0} \,, \qquad S = 2k|y| e^{\left(\sigma_0 - \frac{S_0}{3}\right)} + S_0 \tag{37}$$

under the condition $-\Lambda_0 = \Lambda_\pi = 4kM_5^3$. S_0 and σ_0 are integration constants that can be neglected without any consequence. Then one finds

$$3\sigma = S = 2k|y| \qquad \rightarrow \qquad \mathrm{d}s^2 = e^{\frac{4k|y|}{3}} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right) \tag{38}$$

Often one generally writes

$$ds^2 = X(|y|)dx^2 + Y(|y|)dy^2, \qquad y = \in (0, \pi R) \tag{39}$$

so we in our special case have $X = Y = e^{\frac{4k|y|}{3}}$. Randall-Sundrum has $X = Y \sim e^{2k|y|}$ and $Y = 1$. We will now deconstruct the linear dilaton to recover 4D clockwork. By putting a massless scalar in the theory one finds

$$\begin{aligned}
\mathcal{S} &= 2\int_0^{\pi R} dy \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{MN} \partial_M \phi \, \partial_N \phi\right) \\
&= -\int_0^{\pi R} dy \int d^4x \, X^2 Y^{1/2} \left[\frac{(\partial_\mu \phi)^2}{X} + \frac{(\partial_y \phi)^2}{Y}\right] \\
&= -\int_0^{\pi R} dy \int d^4x \left[(\partial_\mu \phi)^2 + \frac{X^2}{Y^{1/2}} \left(\partial_y \frac{\phi}{X^{1/2} Y^{1/4}}\right)^2\right] \quad \rightarrow
\end{aligned}$$

$$\boxed{y \to ja, \quad Na = \pi R, \quad \int \mathrm{d}y \to \sum, \quad \partial_y \phi \to \frac{1}{a}\left(\phi_{j+1} - \phi_j\right), \qquad X(ja) \to F_j}$$

$$\rightarrow \mathcal{S} = -\frac{1}{2} \int d^4x \left[\sum_{j=0}^{N} (\partial_\mu \phi_j)^2 + \sum_{j=0}^{N-1} m_j^2 \left(\phi_j - q_j \phi_{j+1}\right)^2\right] \tag{40}$$

with

$$m_j^2 \equiv \frac{N^2 X_j}{\pi^2 R^2 Y_j} = \frac{N^2}{\pi^2 R^2} \,, \quad q_j \equiv \frac{X_{j+1}^{1/2} Y_{j+1}^{1/4}}{X_j^{1/2} Y_j^{1/4}} = e^{\frac{k\pi R}{N}} \tag{41}$$

and using

$$X(ja) = Y(ja) = e^{+\frac{4kjN}{R}} \qquad \text{(sign flip } k \to -k). \tag{42}$$

So we got the clockwork in case of a massless scalar in a dilaton background (putting graviton instead of the scalar works the same).[2]

[2] According to N. Craig et al. the reason that this works has nothing to do with geometry. They provide an example of flat extradimension, where they obtain the same matrix in the end also using a dilaton.

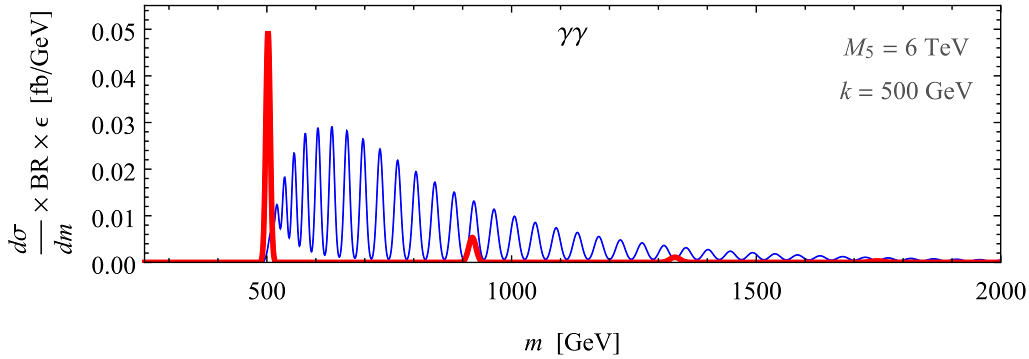

Figure 3: Comparison between RS (red, scaled down by a factor of 30) and CW/LD (blue). Picture taken from [2].

From the metric one also gets the following relation between fundamental scale M_5 (cutoff of the theory at scale somewhat larger than v) and $M_{\rm pl}$

$$M_P^2 = \frac{M_5^3}{k}\left(e^{2\pi kR} - 1\right) . \tag{43}$$

which resembles the relation you get in the traditional Randall-Sundrum set-up. However the interpretation of k is different in the Clockwork as well as the question what the size of the of the extradimension is. See details also in [1]. For comparison the values of the KK spectrum, the KK mode interaction scale, the Planck mass, and length of extradimension.

	m_n^2	Λ_n^2	M_P^2	L_5
RS	$\approx [(n+\frac{1}{4})\pi\hat{k}]^2$	$\approx \frac{M_5^3}{\hat{k}}$	$\frac{M_5^3}{\hat{k}}(e^{2\hat{k}\pi R} - 1)$	$2\pi R$
CK	$k^2 + \frac{n^2}{R^2}$	$M_5^3\pi R\left(1 + \frac{k^2R^2}{n^2}\right)$	$\frac{M_5^3}{k}(e^{2k\pi R} - 1)$	$\frac{3}{k}\left(e^{\frac{2}{3}k\pi R} - 1\right)$

The required electroweak/gravity hierarchy is generated by

$$kR \simeq \frac{1}{\pi}\ln\left(\frac{M_P}{M_5}\sqrt{\frac{k}{M_5}}\right) \approx 10 + \frac{1}{2\pi}\ln\left(\frac{k}{\text{TeV}}\right) - \frac{3}{2\pi}\ln\left(\frac{M_5}{10\ \text{TeV}}\right) . \tag{44}$$

In principle of course the general LHC search strategy is the same for CW and RS. The higher mode gravitons couple to the SM, so bump hunt in dimass spectra is possible (in particular $m_{\gamma\gamma}$, m_{ee}). However the compressed spectrum of CW/LD makes alternative search strategies possible. In particular the periodic structure allows to search for a peak in the power spectrum of LHC data. This gives

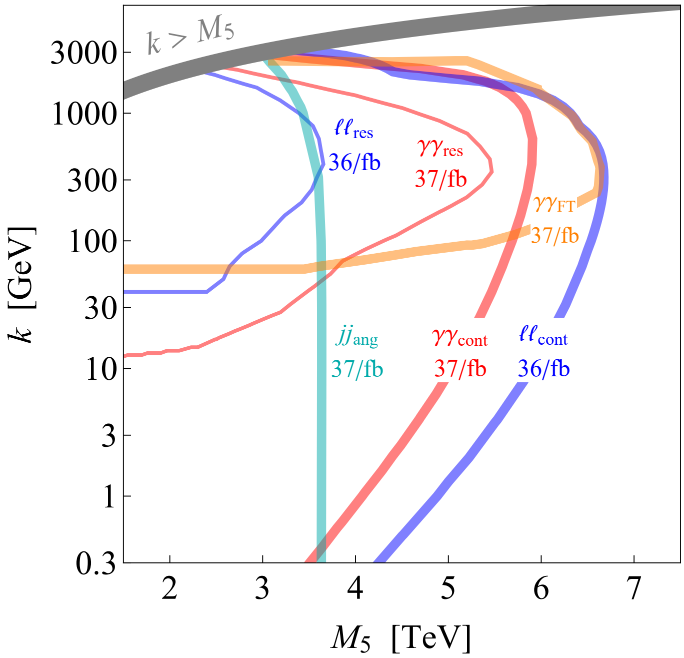

Figure 4: CW/LD estimated sensitivity. Picture taken from [2]. See also [12].

References

[1] G. F. Giudice and M. McCullough, *A Clockwork Theory*, *JHEP* **02** (2017) 036, [1610.07962].

[2] G. F. Giudice, Y. Kats, M. McCullough, R. Torre and A. Urbano, *Clockwork/linear dilaton: structure and phenomenology*, *JHEP* **06** (2018) 009, [1711.08437].

[3] K. Choi and S. H. Im, *Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry*, *JHEP* **01** (2016) 149, [1511.00132].

[4] D. E. Kaplan and R. Rattazzi, *Large field excursions and approximate discrete symmetries from a clockwork axion*, *Phys. Rev.* **D93** (2016) 085007, [1511.01827].

[5] M. Mccullough, *A Clockwork Theory*, .

[6] R. Torre, *Clockwork/Linear Dilaton: Structure and phenomenology*, .

[7] P. W. Graham, D. E. Kaplan and S. Rajendran, *Cosmological Relaxation of the Electroweak Scale*, *Phys. Rev. Lett.* **115** (2015) 221801, [1504.07551].

[8] W. Tangarife, K. Tobioka, L. Ubaldi and T. Volansky, *Dynamics of Relaxed Inflation*, *JHEP* **02** (2018) 084, [1706.03072].

[9] S. Hong, G. Kurup and M. Perelstein, *Clockwork Neutrinos*, *JHEP* **10** (2019) 073, [1903.06191].

[10] R. Alonso, A. Carmona, B. M. Dillon, J. F. Kamenik, J. Martin Camalich and J. Zupan, *A clockwork solution to the flavor puzzle*, *JHEP* **10** (2018) 099, [1807.09792].

[11] T. Hambye, D. Teresi and M. H. G. Tytgat, *A Clockwork WIMP*, *JHEP* **07** (2017) 047, [1612.06411].

[12] I. Antoniadis, A. Arvanitaki, S. Dimopoulos and A. Giveon, *Phenomenology of TeV Little String Theory from Holography*, *Phys. Rev. Lett.* **108** (2012) 081602, [1102.4043].

[13] N. Craig, I. Garcia Garcia and D. Sutherland, *Disassembling the Clockwork Mechanism*, *JHEP* **10** (2017) 018, [1704.07831].

[14] G. F. Giudice and M. McCullough, *Comment on "Disassembling the Clockwork Mechanism"*, 1705.10162.

[15] K. Choi, S. H. Im and C. S. Shin, *General Continuum Clockwork*, *JHEP* **07** (2018) 113, [1711.06228].