Alternative tuning concepts & scale-invariant gravity (agravity)

literature:

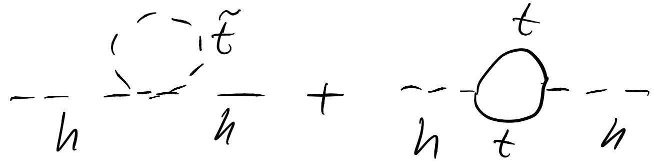
- Natural tuning: 1305.6939
- Modified naturalness: 1303.7244

  used to justify:

- Agravity: 1403.4226

(i) Natural tuning

2 standard options:

- $\Delta m_h^2 \sim$ 

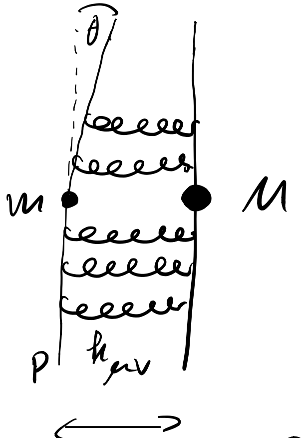
  $$\overset{!}{\sim} \ln\left(\frac{\Delta m_{t\tilde{t}}}{\bar{\mu}}\right) + \text{finite}$$

  (technicolor, TeV-scale SUSY)

- $e^{\#}$-many vacua with varying Higgs vev + environmental selection (e.g. Gamow-resonance needed for ${}^{12}_{6}C$-cycle fusion in stars) explains/accommodates $v \simeq v_{EW}$.

3rd option:

gravitational 2-2-scattering



$b >> R_S = \frac{2M}{M_P^2}$

semi-classical limit:

above ladder diagram with many gravitons $h_{\mu\nu}$, $m << |\vec{p}|$

$\hat{=}$ eikonal approx in shockwave background of M

S-matrix: $S_{GGRT} = e^{i2\cdot\delta_{eik}(s)} \cdot \mathbb{1}$

phase: $e^{i2\cdot\delta_{eik}(s)} = e^{i\ell^2 \cdot s/4}$

$\leftrightarrow$ 2D integrable quantum gravity

$\sim$ critical string theory:

$$\ell^2 \sim G_N\, b^{4-d} \xrightarrow[d\to4]{} G_N \cdot \ln \frac{R_{IR}}{b}$$

properties:

- on-shell: UV-soft
- classical limit:
  $S_{NG} = -\ell^2 \cdot \int d^2\sigma \sqrt{-\det(\eta_{\alpha\beta} + \partial_\alpha X^\mu \partial^\alpha X_\mu)}$
- IR-limit: EFT with local higher-dim. op.s
- UV-limit: <u>not</u> a local EFT <u>off-shell</u>

- $e^{i2\cdot\delta_{eik}}$ exp. damped on-shell in IR where $\mathrm{Im}\, s > 0$, but has <u>essential</u> singularity <u>off-shell</u> for $s = \infty$, $\mathrm{Im}\, s < 0$.

↝ asymptotic fragility, no RGE flow of local EFT from UV QFT to IR EFT, looks like non-renorm. EFT in IR — but on-shell well behaved in UV!

⇒ can import this for arbitrary matter sectors:

$$\hat{S}_n(p_i) = e^{i\ell^2/4 \cdot \sum_{i<j} p_i * p_j} \cdot S_n(p_i)$$

$$p_i * p_j \equiv \epsilon_{\alpha\beta}\, p_i^\alpha p_j^\beta$$

(rapidity-ordered antisymm. product ⟷ reminiscent of non-commutative space-time theories, i.e. Moyal product, but better behaved: rapidity-ordered cyclicity works off-shell at loop level without acausalities...)

example: massive free scalar

$$e^{i2\delta_{eik}(s)} = e^{i\frac{\ell^2}{4}\sqrt{s(s-4m^2)}}$$

reconstruct $\mathcal{L}$ order by order:

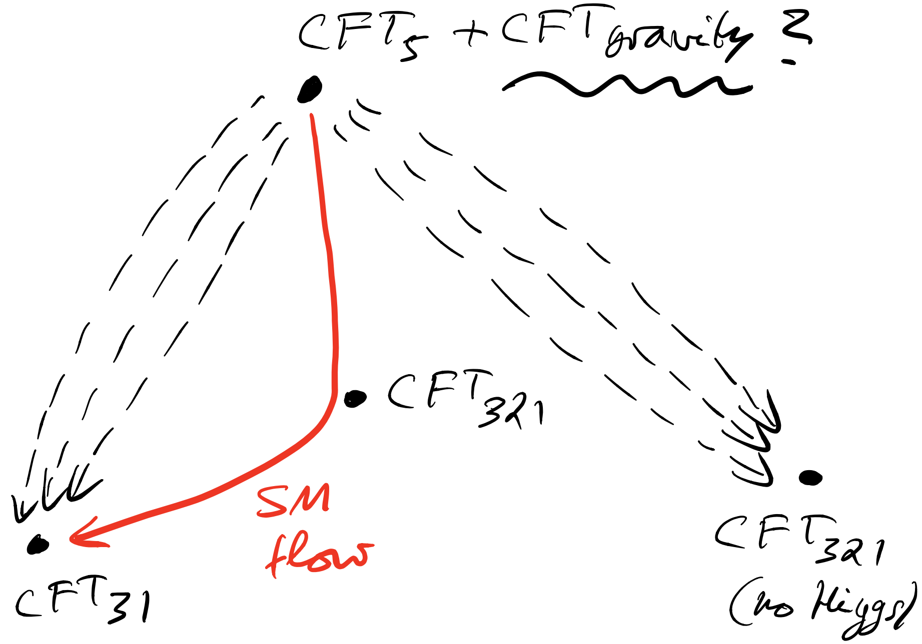
$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 + \frac{\ell^2}{8}\left[(\partial\varphi)^4 - m^4\varphi^4\right] + O(\ell^4)$$

on-shell amplitudes match at given order in ℓ up to finite polynomials in $\partial\varphi$ and $m\varphi$ at next-higher order in ℓ.

$\rightsquigarrow$ Corrections remove on-shell the quadrat. divergences, but <u>no</u> <u>off-shell</u> <u>local</u> UV <u>QFT</u> for this.

result:

maybe our wish



does not work, because e.g. Weinberg-Witten ('80) makes $CFT_{gravity}$ very hard ...

so instead only:

CFT_{321} + "TeV-scale copies"

$\bullet$

$\downarrow$ SM flow

$\bullet$

CFT_{31}

⊕ gravity: <u>no</u> space-time CFT, but gravitationally dresses

⇒ • EW scale is fundamental non-gravitational scale!

• No heavy non-grav. scales above EW, only gravity itself ...

⇝ nature is "naturally tuned"

(ii) "finite naturalness":

∼ arrives at the same eff. proposal as (i)

observation: in dim-reg.

$$\delta m_h^2 \Big|_{1\text{-loop}}^{\text{gauge}+\text{top}} \sim \ln \frac{M}{\bar{\mu}} + \text{finite}$$

M : $W^\pm$, Z, top mass-scales

$\bar{\mu}$: renormalization scale

⇒ <u>rule</u>: no new non-grav. scales at $\gg$ TeV, and then drop all

quadratic div.
(keep only logs +
finite)

$\leadsto$ leads for neutrino see-saw to RH ν-masses $\lesssim 10^7$ GeV; and for e.g. minimal fermion DM, scalar singlet DM, KSVZ axions to new states at a few TeV ...

$\leadsto$ essentially same rule as in (i)

(iii) applies outcome of (i) & (ii) to SM, but then ignores arguments about gravity in (i) (and much other literature!) and tries to work out implications of assuming a 4D CFT gravity in the UV:

don't start with GR + SM, but

$$S = \int d^4x \sqrt{-g} \left[\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \mathcal{L}_{SM} + \mathcal{L}_{BSM} \right]$$

$$\mathcal{L}_{SM} = -\frac{F_{\mu\nu}^2}{4g^2} + i\bar{\Psi}\not{D}\Psi + |D_\mu H|^2 - (y H \Psi \Psi + h.c.) - \lambda_H |H|^4 - \xi_H |H|^2 R$$

$$\mathcal{L}_{BSM} = |D_\mu S|^2 - \lambda_S |S|^4 + \lambda_{HS} |H|^2 |S|^2 - \xi_S |S|^2 R$$

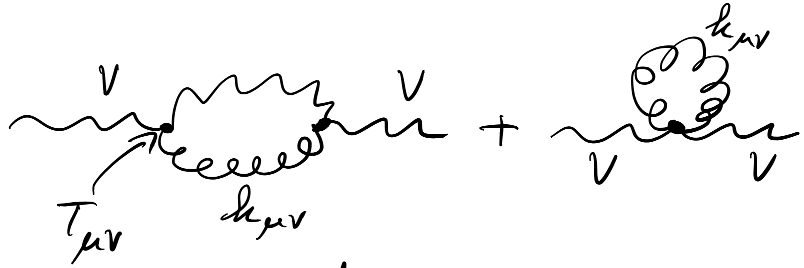
note: on simply connected space-time all combinations of R^2, $R_{\mu\nu}^2$, $R_{\mu\nu\rho\sigma}^2$ are equivalent to:

$$W^2_{\mu\nu\rho\sigma} = \frac{1}{2} R^2_{\mu\nu\rho\sigma} - R^2_{\mu\nu} + \frac{1}{6} R^2 \simeq R^2_{\mu\nu} - \frac{1}{3} R^2$$

BRST quantize & use dim-reg.

$\longleftrightarrow$ neglect all quadratic & quartic divergences, keep logs + finite

$\Rightarrow$ for gauge fields, only diagrams from gravity:



$$+ \ln \frac{\Lambda}{\bar{\mu}} \qquad\qquad - \ln \frac{\Lambda}{\bar{\mu}}$$

$\Rightarrow$ finite, $\beta_V^{full} = \beta_V^{SM}$

gravitational RGE:

$$(4\pi)^2 \frac{df_2^2}{d\ln\bar{\mu}} = -f_2^4 \cdot \left(\frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60}\right)$$

always asymptotically free

$$(4\pi)^2 \cdot \frac{df_0^2}{d\ln\bar{\mu}} = \frac{5}{3} f_2^4 + 5 f_2^2 f_0^2 + \frac{5}{6} f_0^4 + \frac{1}{12} f_0^4 \cdot (\delta_{ab} + 6\xi_{ab})^2$$

$\Rightarrow$ f_0^2 asymptotically free only, if $f_0^2 < 0$

$\Rightarrow$ Starobinsky scalar χ has mass $M_0^2 < 0$ (could use χ as inflaton ...)

dynamical generation of M_P:

$\sim$ vacuum for S

e.o.m.: $$\frac{\partial V}{\partial S} + \frac{1}{2}\frac{\partial f}{\partial S}\cdot R = 0$$

$-\frac{1}{2}f(S)\cdot R$ in $\mathcal{L}$

trace of gravitational field eq.s:

$$f\cdot R + 4\cdot V = \mathcal{O}(R^2/f_{0,2}^2)$$

(Einstein gravity: $R = -4T = -4V$)

$$\Rightarrow \quad \frac{\partial V}{\partial S} - \frac{2}{f}\frac{\partial f}{\partial S} V = 0$$

(near flat space: $R^2 \ll R$)

same as: $$\frac{\partial V_E}{\partial S} = 0, \quad V_E = \frac{V}{f^2}$$

Einstein frame scalar potential

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}|\partial S|^2 - \frac{1}{2} f R - V\right]$$

$$\rightarrow S = \int d^4x \sqrt{-g_E}\left[-\frac{1}{2}R - \frac{K(S)}{2}|\partial S|^2 - V_E\right]$$

here we have approx.:

$$V(S) = \lambda(\bar{\mu} \approx S)\,\frac{S^4}{4}$$

$$f(S) = \xi_S(\bar{\mu} \approx S)\cdot S^2$$

$$\Rightarrow \quad V_E = \frac{1}{4}\cdot\frac{\lambda_S(S)}{\xi_S^2(S)}$$

$$\Rightarrow 4\frac{\partial V_E}{\partial S} = \frac{\partial \lambda_S}{\partial S}\cdot\frac{1}{\xi_S^2} - \frac{2\lambda_S}{\xi_S^3}\cdot\frac{\partial \xi_S}{\partial S}$$

$$\overset{!}{=} \beta_{\lambda_S} \cdot \frac{1}{s \cdot \xi_S^2} - \beta_{\xi_S} \cdot \frac{2\lambda_S}{s \cdot \xi_S^3} \overset{!}{=} 0$$

$$\Rightarrow \frac{\beta_{\lambda_S}}{\lambda_S} - 2 \cdot \frac{\beta_{\xi_S}}{\xi_S} = 0$$

different vacuum eq. than that from non-gravitational Coleman-Weinberg potential!

so, agravity can generate M_P and keep c.c. small if:

$$\begin{cases} \lambda_S(s) = 0 & (1) \\ \beta_{\lambda_S}(s) = 0 & (2) \\ \xi_S(s)\, s^2 = M_P^2 & (3) \end{cases}$$

(1), (2) approximately fulfillable (model-dependent), see e.g. SM Higgs: $\lambda_h(h)$, $\beta_{\lambda_h}(h)$ can vanish at scales close to each other ...

$\rightarrow$ but minimum c.c. at $M_0^2 M_P^2$, $M_0 \gtrsim$ eV

$\rightsquigarrow$ 10^{60} too large, still need landscape ...

to get (1), (2) approx., S needs to be charged under other sector:

e.g. 2nd SM copy with Z_2-symm. with our SM

$\Rightarrow$ λ_{HS} loop-generated

$\Rightarrow$ m_S^2 heavy, few orders of magnitude below M_P

$\Rightarrow$ m_h^2 gets log-contribution from agravity sector (β-functions)

final note:

can use e.g. S as inflaton

$\rightarrow$ $V(S) = \lambda_S S^4$

$\Rightarrow$ $V_E(S) \sim \lambda_S = \text{const.}$

since at 1-loop:

$$\lambda_S(S) \sim \ln S$$

$\Rightarrow$ $V(S) \sim S^4 \ln S$

$\Rightarrow$ $V_E(S) = \frac{S^4}{f^2} \ln S \sim \ln S$

$S \sim e^{c \cdot S_E} \Rightarrow V_E \sim S_E^p$

monomial, not

Starobinsky $\Rightarrow r \simeq 0.1$

instead of

0.001

all of this neglects

<u>main problems</u>:

- no quantum 4D CFT of gravity known!

- all quadratic ~~gravity~~ theories except $f(R)$ contain a spin-2 <u>ghost</u>! How to make sense of that?

Stelle (1977)