

# Precision Calculations in QCD

Bahman Dehnadi

Deutsches Elektronen-Synchrotron



# Precision Calculations in QCD

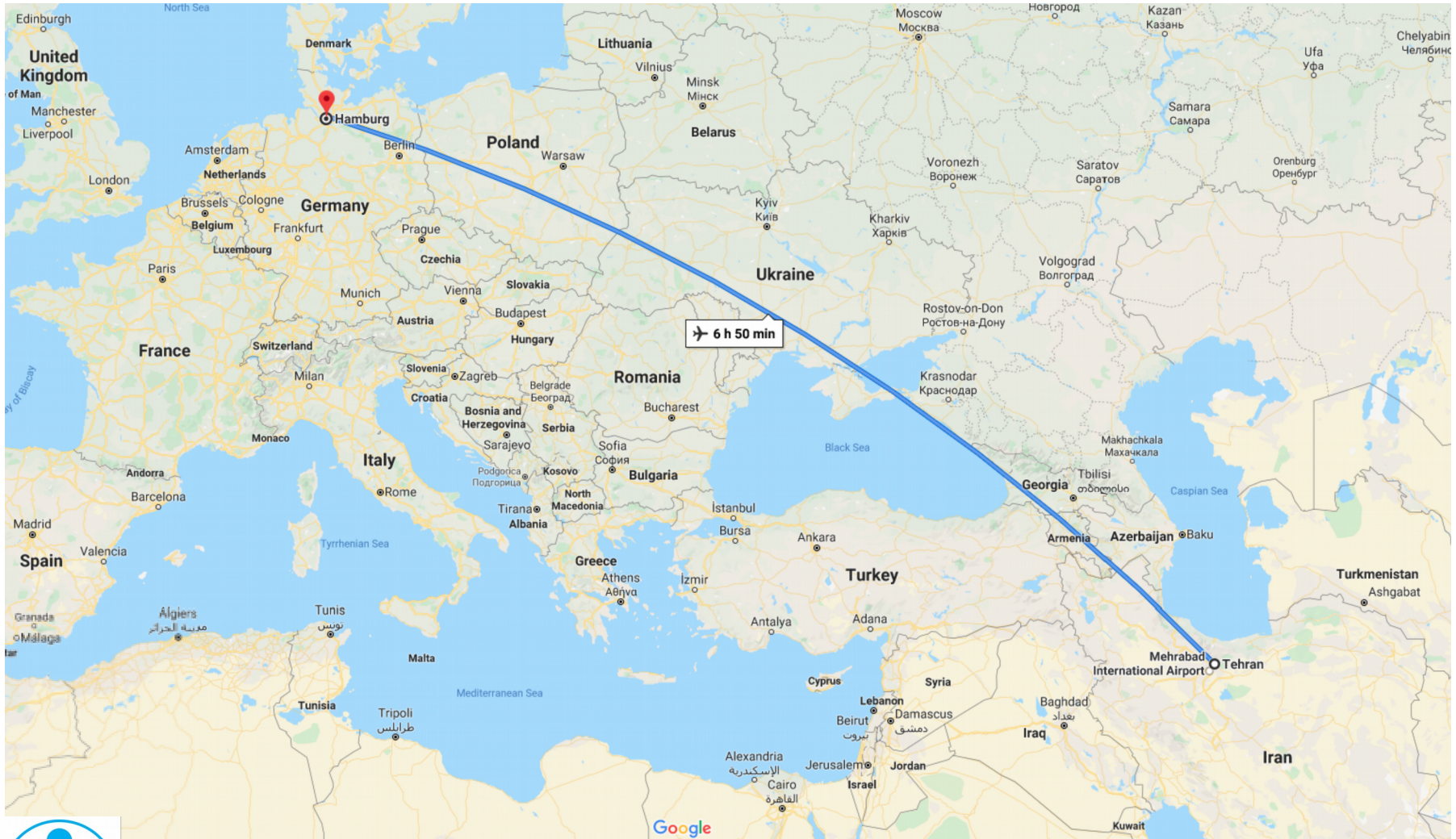
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# About me...

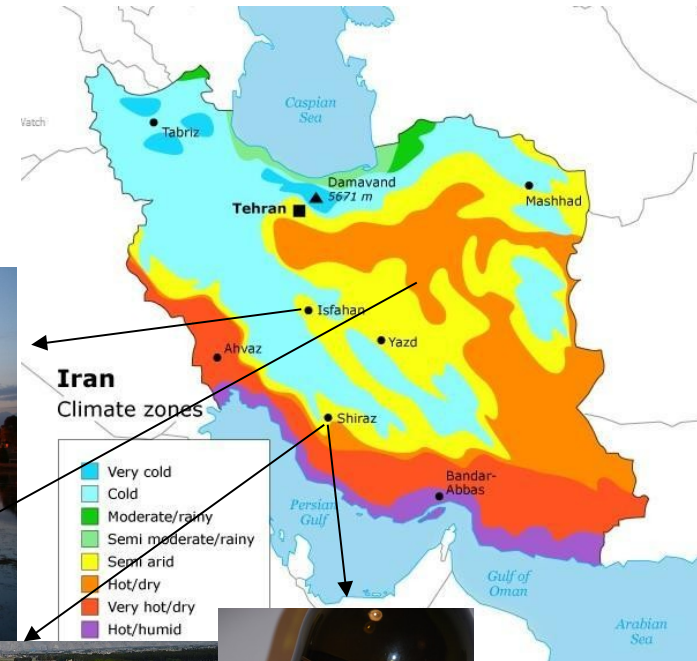
- I am from Tehran, Iran (4,404 Km away)



# About Iran

## Some Facts

- Muslims, Zoroastrians, Christians and Jews are living “peacefully”
- Two salt deserts covers 25% of Iran’s land area
- Iran has the third highest number of UNESCO world sites in Asia



# About me...

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## Academic Carrier

- 2012 – 2016      PhD from University of Vienna under supervision of Andre Hoang
- 2016 – 2019      First postdoc at University of Siegen
- Since Oct. 2019    Second postdoc at DESY, Hamburg

## Hobbies

- Playing persian music (Kamanche)
- Dancing *LiNdY HoP*



# Research Overview

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- **Charm and Bottom Quark Mass Determinations from QCD Sum Rules**  
[BD, A.H. Hoang, V. Mateu, I. W. Stewart, S. M. Zebarjad (2013) ]  
[BD, A.H. Hoang, V. Mateu (2015) ]
- **Top Quark Mass Calibration for Monte-Carlo Event Generators**  
[M. Beutenschoen, BD, A.H. Hoang, V. Mateu, I. W. Stewart (2016) ]
- **Automated Calculation of N-jet Soft Functions at NNLO**  
[G. Bell, BD, T. Mohrmann, R. Rahn (2018)]
- **Inclusive Top Quark Pair Production at Future Linear Colliders**  
[BD, A.H. Hoang, V. Mateu, M. Stahlhofen, A. Widl (to appear soon) ]
- **Evolution Kernel of B-meson Light Cone Distribution Amplitude at Two Loops**  
[G. Bell, BD, J. Piclum, B. Lange (w.i.p.)]



# Research Overview

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# Charm and Bottom Quark Masses



# Charm Quark Mass

$$\bar{m}_c(\bar{m}_c) = 1.27 \pm 0.02 \text{ GeV}$$

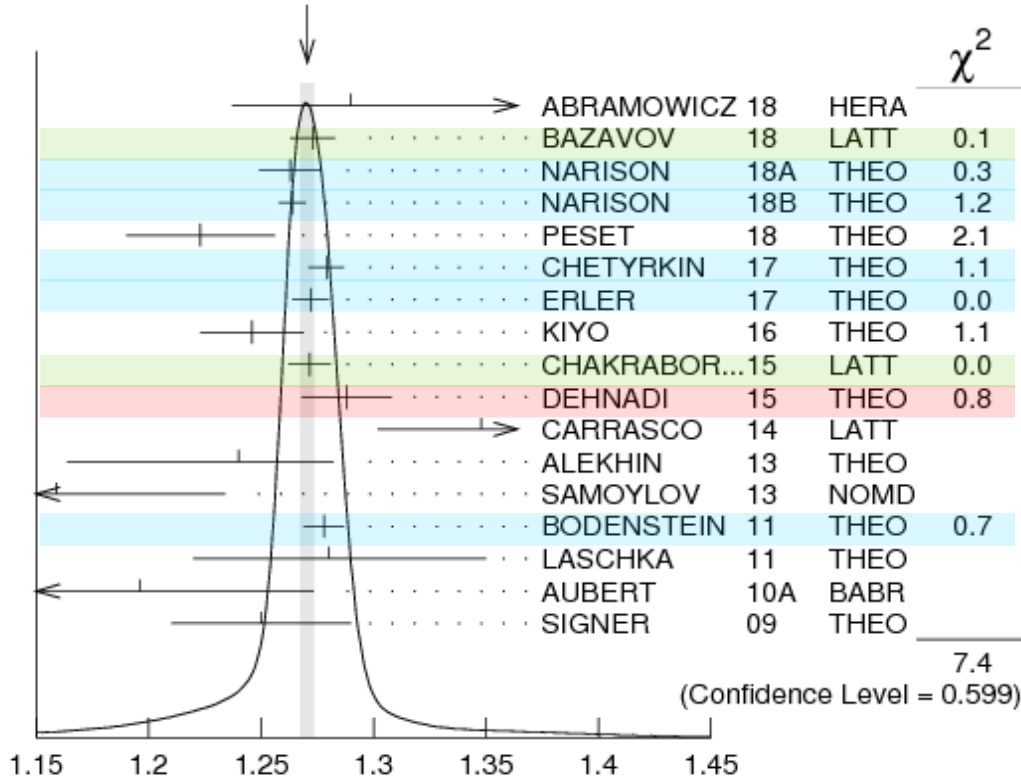
[PDG 2019]

➤ **Most precise determinations:**

QCD sum rules  $\rightarrow M_n^{\text{exp}} = \int_{4m^2}^{\infty} \frac{ds}{s^{n+1}} R_{e^+e^- \rightarrow c\bar{c}+X}(s)$

Lattice

WEIGHTED AVERAGE  
1.2705±0.0031 (Error scaled by 1.0)



[Taken from PDG 2019]

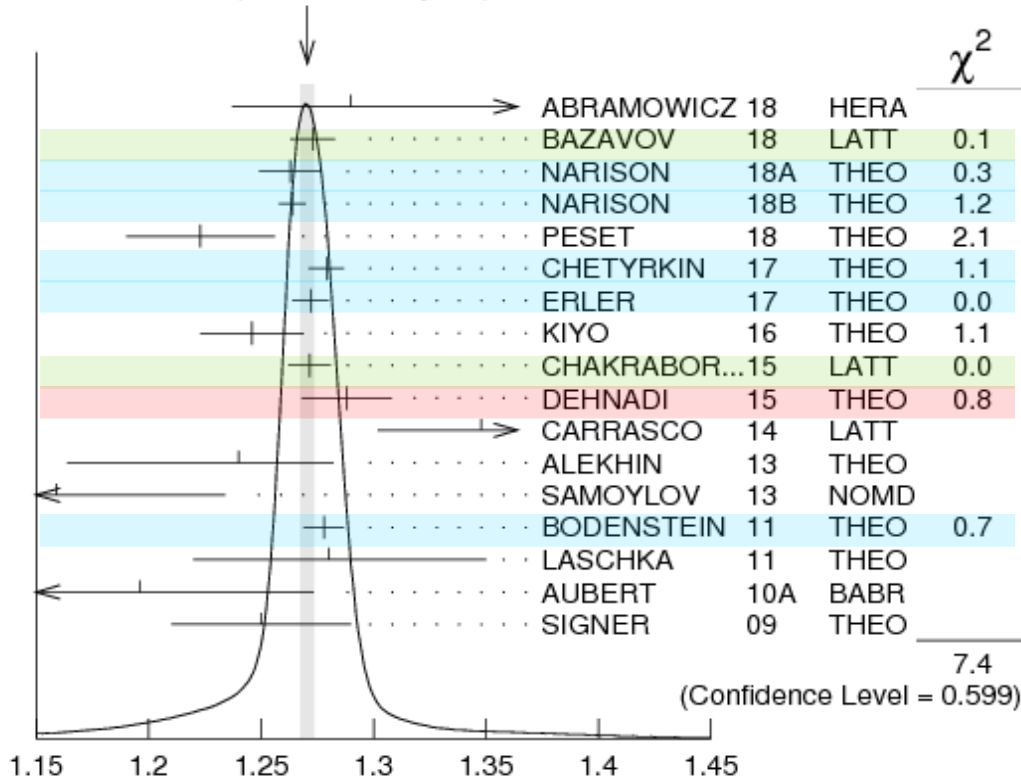


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Lattice

➤ How much **reliable**?

- ✗ Relied heavily on theoretical input (significant modeling uncertainties)
- ✗ Same algorithms & perturbative series
- ✗ Common method for estimating **Unc.**
- ✗ Consistency to each other for justification

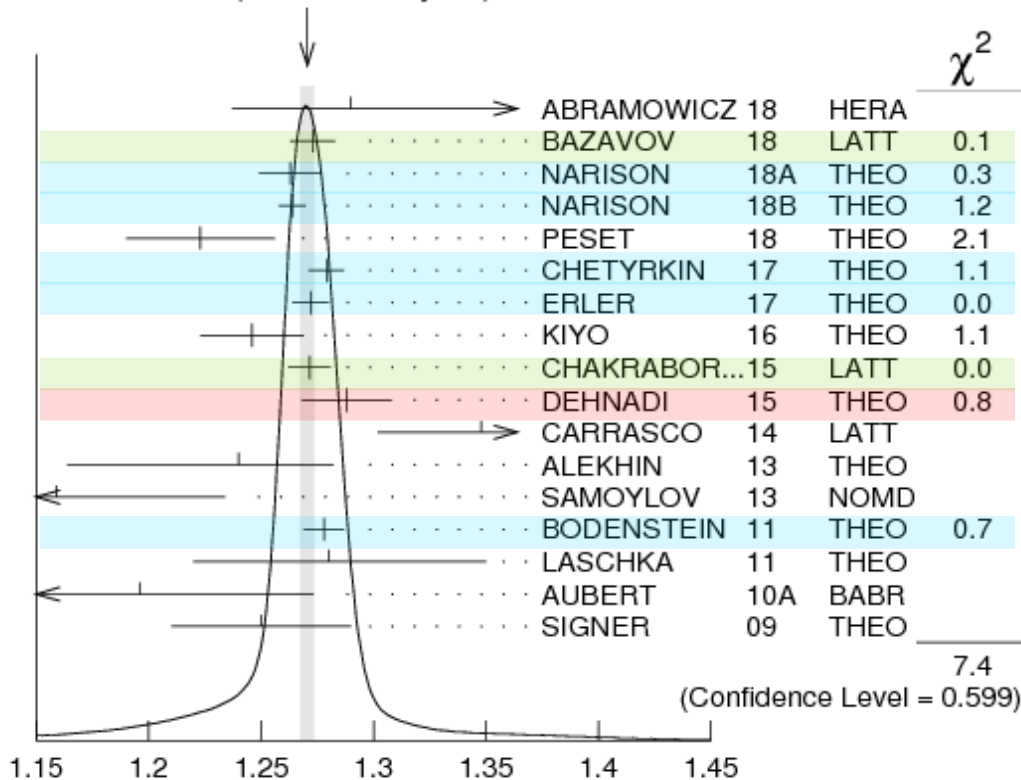


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- ✗ **Same algorithms** & perturbative series
- ✗ **Common** method for estimating **Unc.**
- ✗ **Consistency to each other** for justification
- **Our results (QCD sum rules)**
- ✓ Combine **all experimental** data
- ✓ **Different algorithms** for determining the mass
- ✓ **New method** for estimating **Unc.**  
Double scale variation

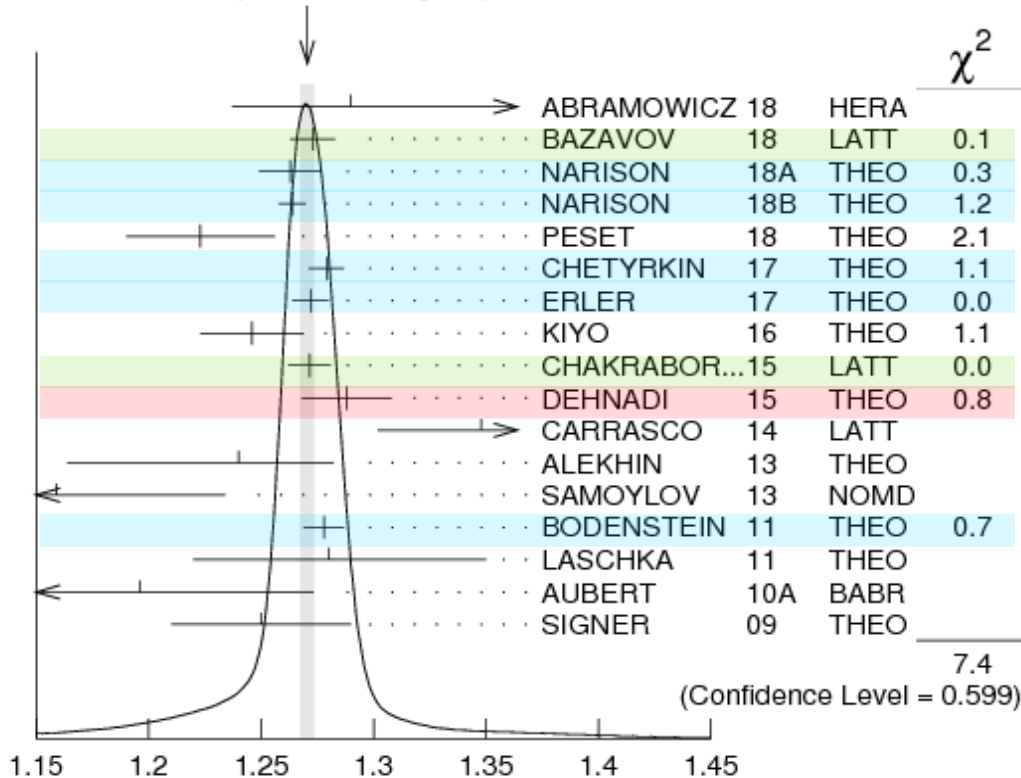


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- ✗ **Same algorithms** & perturbative series
- ✗ **Common** method for estimating **Unc.**
- ✗ **Consistency to each other** for justification
- **Our results (QCD sum rules)**

$$\bar{m}_c(\bar{m}_c) = 1.288 \pm (0.006)_{\text{stat}} \pm (0.009)_{\text{syst}} \pm (0.014)_{\text{pert}} \pm (0.010)_{\alpha_s} \pm (0.002)_{(GG)} \text{ GeV}$$

$$\alpha_s(m_Z) = 0.1184 \pm 0.0021$$

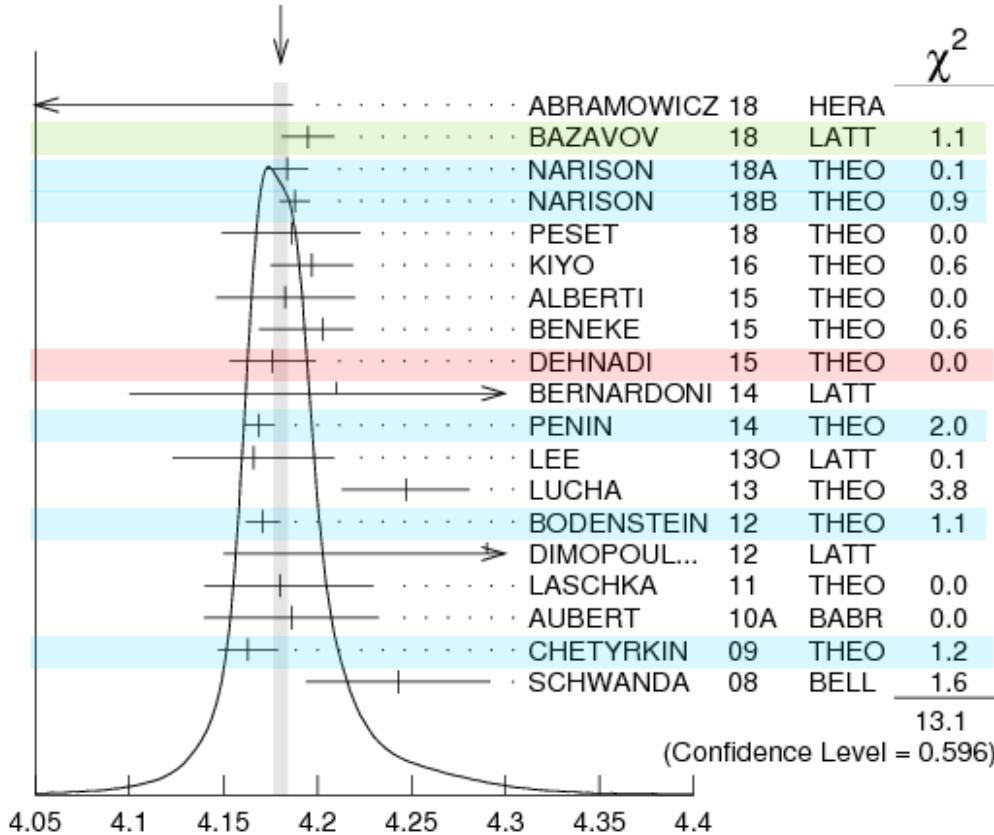


# Bottom Quark Mass

$$\bar{m}_b(\bar{m}_b) = (4.18 \pm 0.03) \text{ GeV}$$

[PDG 2019]

WEIGHTED AVERAGE  
4.181±0.004 (Error scaled by 1.0)



➤ **Most precise determinations:**

QCD sum rules

Lattice

✗ **Similar caveats**

➤ **Our results (QCD sum rules)**

$$\bar{m}_b(\bar{m}_b) = 4.176 \pm (0.004)_{\text{stat}} \pm (0.019)_{\text{syst}} \pm (0.010)_{\text{pert}} \pm (0.007)_{\alpha_s} \pm (0.0001)_{\langle GG \rangle} \text{ GeV}$$

$$\alpha_s(m_Z) = 0.1184 \pm 0.0021$$

[Taken from PDG 2019]



# Top Quark Mass

# Top Quark Mass

## 2019 Review of Particle Physics.

M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update.

### QUARKS

[INSPIRE search](#)

$$t \quad I(J^P) = 0(1/2^+)$$

$$\text{Charge} = \frac{2}{3} e \quad \text{Top} = +1$$

See related review:

[Top Quark](#)

### *t*-QUARK MASS

*t*-Quark Mass (Direct Measurements)  $172.9 \pm 0.4 \text{ GeV} (S = 1.3)$

*t*-Quark Mass from Cross-Section Measurements  $160^{+5}_{-4} \text{ GeV}$

*t*-Quark Pole Mass from Cross-Section Measurements  $173.1 \pm 0.9 \text{ GeV}$

[Taken from PDG 2019]



# Top Quark Mass

## 2019 Review of Particle Physics.

M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update.

## *t*-Quark Mass (Direct Measurements)

[INSPIRE search](#)

The following measurements extract a *t*-quark mass from the kinematics of  $t\bar{t}$  events. They are sensitive to the top quark mass used in the MC generator that is usually interpreted as the pole mass, but the theoretical uncertainty in this interpretation is hard to quantify. See the review "The Top Quark" and references therein for more information.

OUR AVERAGE of  $172.9 \pm 0.4$  (GeV) is an average of top mass measurements from LHC and Tevatron Runs. The latest Tevatron average,  $174.30 \pm 0.35 \pm 0.54$  GeV, was provided by the Tevatron Electroweak Working Group (TEVEWWG).

VALUE (GeV)	DOCUMENT ID	TECN	COMMENT
<b><math>172.9 \pm 0.4</math></b>	<b>OUR AVERAGE</b> Error includes scale factor of 1.3.		
$172.25 \pm 0.08 \pm 0.62$	<a href="#">1 SIRUNYAN</a> <a href="#">2018DE</a>	CMS	$\ell + \geq 4j$ (2b)
$172.95 \pm 0.77^{+0.97}_{-0.93}$	<a href="#">2 SIRUNYAN</a> <a href="#">2017L</a>	CMS	<i>t</i> -channel single top production
$172.84 \pm 0.34 \pm 0.61$	<a href="#">3 AABOUD</a> <a href="#">2016T</a>	ATLAS	combination of ATLAS
$172.44 \pm 0.13 \pm 0.47$	<a href="#">4 KHACHATRYAN</a> <a href="#">2016AK</a>	CMS	combination of CMS
$174.30 \pm 0.35 \pm 0.54$	<a href="#">5 TEVEWWG</a> <a href="#">2016</a>	TEVA	Tevatron combination

Conceptual problem: **What is  $m_t^{MC}$  ?**

→ Additional conceptual uncertainty in  $m_t^{MC} \sim O(1 \text{ GeV})$

[Hoang, Stewart (2008), Hoang (2014)]



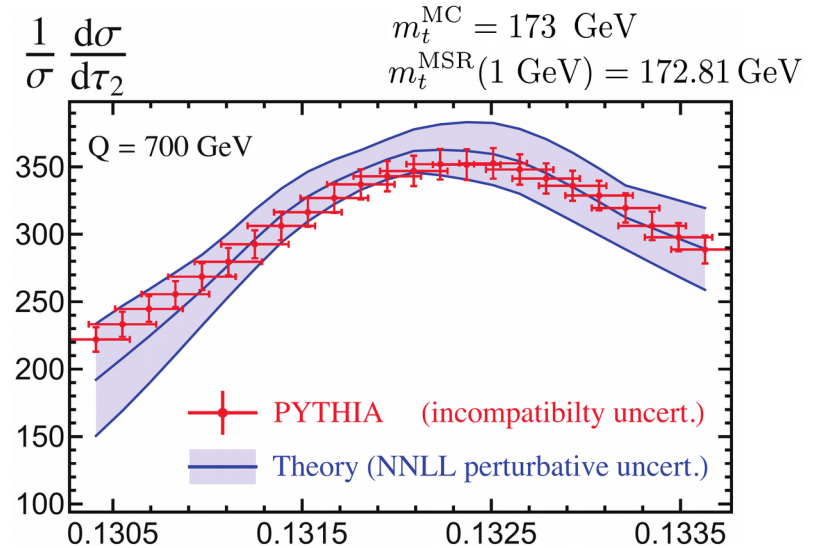
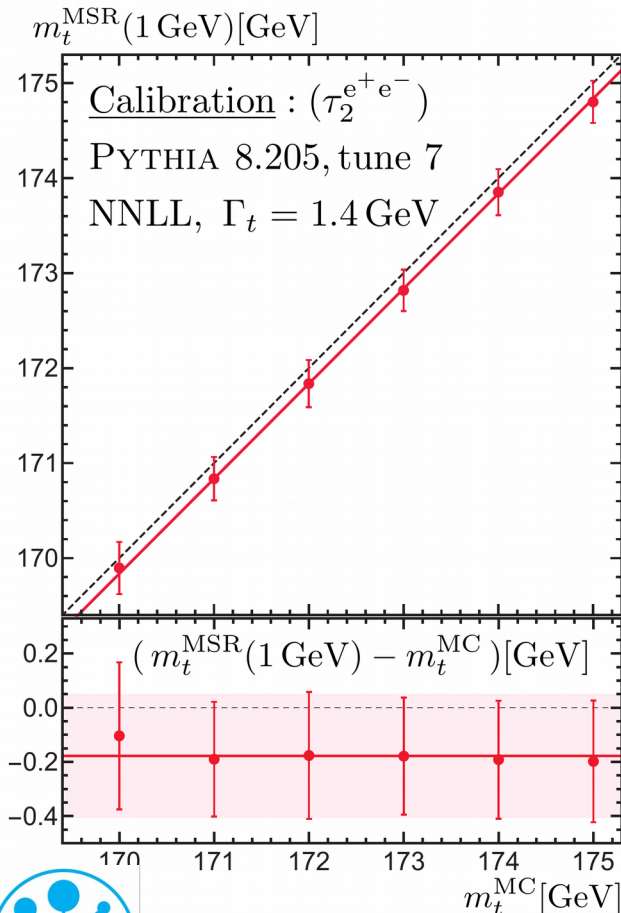


# Top Quark Mass Calibration

Basic idea: calibrate the top mass

$$m_t^{MC} = m_t^{short-distance} + O(1 \text{ GeV})$$

[M. Beutenschoen, BD, V. Mateu, A.H. Hoang, I. W. Stewart (2016) ]



## Strategy for calibration:

- Strong mass-sensitive observable (**2-jettiness**)
- Accurate **analytic hadron-level QCD predictions** with full control over the quark mass-scheme dependence
- Compare to sample data from MC generators
- Find a numerical relation:

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(1 \text{ GeV}) + (0.18 \pm 0.22) \text{ GeV}$$

$$m_t^{\text{MC}} = m_t^{\text{pole}} + (0.57 \pm 0.28) \text{ GeV}$$



# ***N-jet soft function***

# N-jettiness soft function

## N-jettiness cross section:

[I. W. Stewart, F. J. Tackmann, W. J. Waalewijn (2010)]

factorization theorems in **soft/collinear** limits

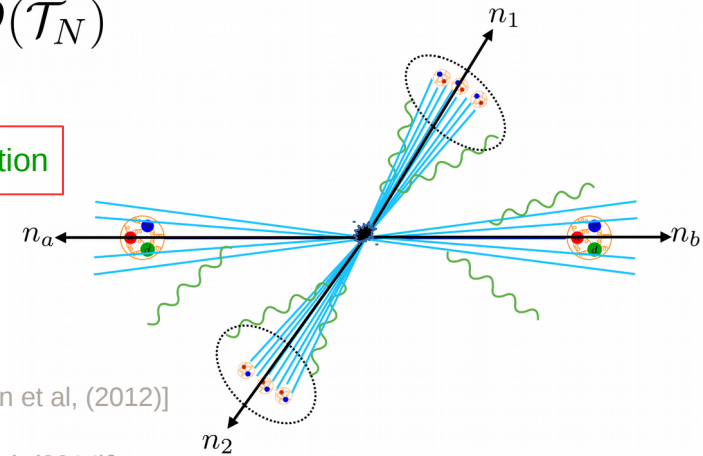
$$\frac{d\sigma(X)}{d\mathcal{T}_N} = \mathbf{H} \otimes \underbrace{B_a \otimes B_b}_{\text{Beam functions}} \otimes \underbrace{J_1 \otimes \dots \otimes J_N}_{\text{Jet functions}} \otimes \underbrace{S_N}_{\text{N-jet soft function}} + \mathcal{O}(\mathcal{T}_N)$$

Hard function

Beam functions

Jet functions

N-jet soft function



**Hard function:** for many processes known to NNLO

[Gehrmann et al, (2012)]

**Beam function:** known to NNLO

[Gaunt et al, (2014)]

**Jet function:** known to NNNLO

[Brüser et al. (2018)]

**Soft function:** for **N=0,1** known to NNLO

[Boughezal et al (2015), Compbell et al (2017)]

Aim: calculating N-jettiness soft function for N>1 to NNLO



# *N-jettiness soft function*

## Idea: Automation

[G. Bell, BD, T. Mohrmann, R. Rahn (2018)]

- Find generic strategy to evaluate soft functions (to NNLO)
- Set up a numerical method based on universal structure of divergences
  - ✓ Isolate singularities with universal phase-space parametrization, e.g. at NLO:

$$\begin{aligned}
 & \text{Soft divergence} \\
 S_{ab}(\tau, \mu) & \sim \frac{\Gamma(-2\epsilon)}{\Gamma(-\epsilon)} \left( \sqrt{n_{ab}/2} \tau e^{\gamma_E} \mu \right)^{2\epsilon} \\
 & \times \int_0^1 dy \int_{-1}^1 d\cos\theta_1 d\cos\theta_2 \sin^{-1-2\epsilon}\theta_1 \sin^{-2-2\epsilon}\theta_2 \underbrace{y^{-1+n\epsilon} \left[ f(y, \theta_1, \theta_2) \right]^{2\epsilon}}_{\text{Measurement function}} \\
 & \qquad \qquad \qquad \text{Collinear divergences (} y \rightarrow \text{rapidity measure)}
 \end{aligned}$$

- ✓ Compute observable dependent integrations numerically

$$\int_0^1 dx x^{-1+n\epsilon} f(x) = \int_0^1 dx x^{-1+n\epsilon} \left[ \underbrace{f(x) - f(0)}_{\text{finite}} + \underbrace{f(0)}_{1/\epsilon} \right]$$



# *Current Research Focus*

# Current Research Focus

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- **N-jettiness subtraction technique** for studying various **multi-jet processes**
- **Quark mass effects** in exclusive **Drell-Yan**
- Flavor Physics: **SIMBA**

The goal is to provide a **global**  $B \rightarrow X_s \gamma + B \rightarrow X_u \ell \bar{\nu}$  fit using **Belle II** measurements (determine the shape function,  $|V_{ub}|$ ,  $|C_7^{\text{incl}} V_{tb} V_{ts}^*|$ ,  $m_b$ )

*Thank you for your attention!*

