Fellow's Meeting 2019

From Scattering Equations to S-Matrices

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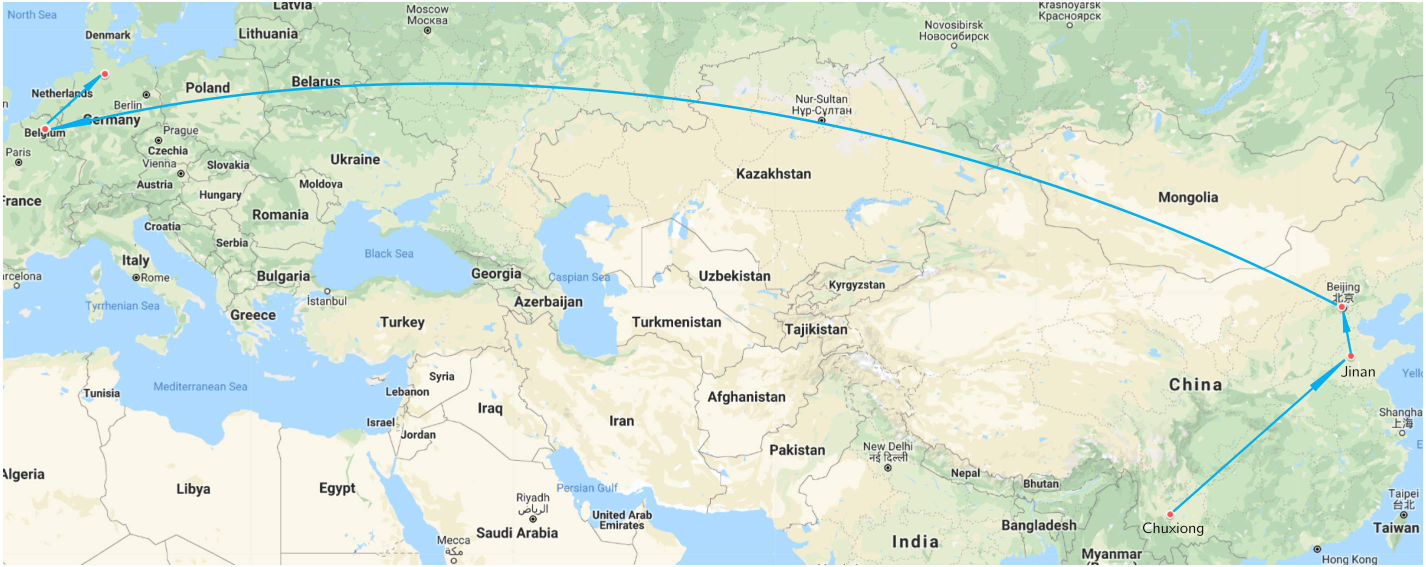
DESY Theory Group (Mathematical Physics & Strings Subgroup)

HELMHOLTZ
RESEARCH FOR GRAND CHALLENGES

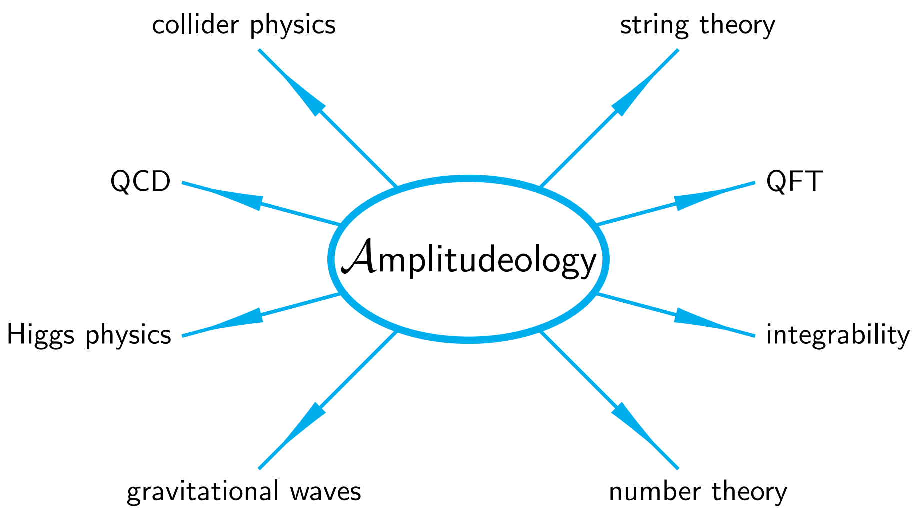
Hamburg, December 3, 2019

My worldline



- Born and raised in Southwest China (Chuxiong, Yunnan)
- Bachelor & Master's degrees in North China (Jinan & Beijing respectively)
- PhD at Université catholique de Louvain in Louvain-la-Neuve, Belgium (supervisor: Claude Duhr)
- Postdoctoral fellow at DESY (supervisor: Volker Schomerus)
- Hobbies: reading, traveling, art...

Research Interests



As fundamental objects in QFT, amplitudes play an important role in many interrelated subjects.

- Allow us to make predictions for physical observables in collider experiments, e.g. LHC

- Amplitudes have a remarkably rich mathematical structure. A good understanding of amplitudes may lead to a deeper understanding of QFT and new approachs to perform computations.

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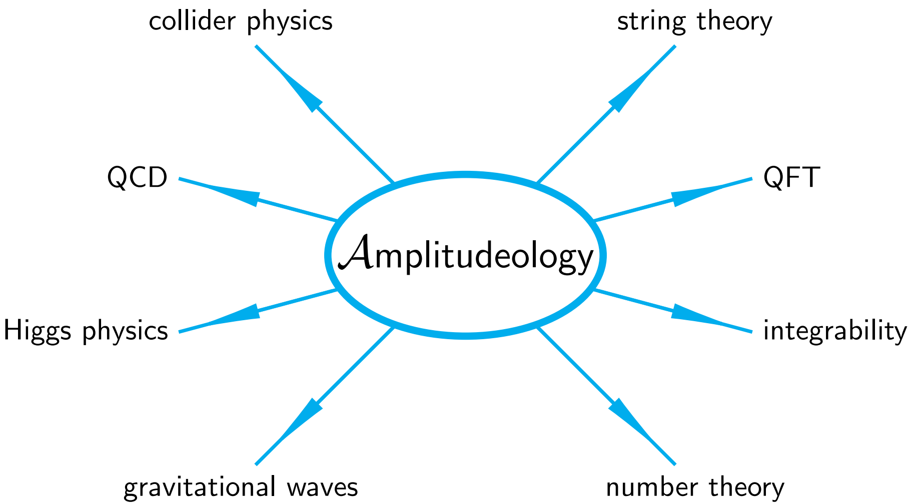


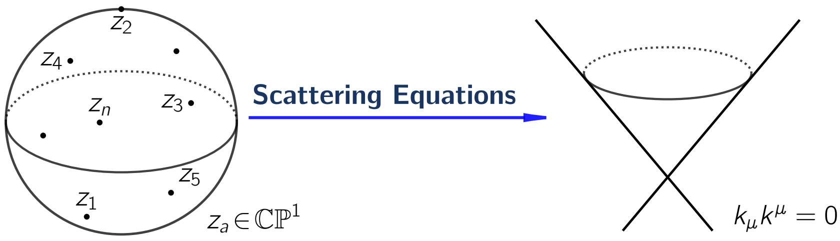
I am interested in scattering amplitudes and related physics and mathematics

- New formulations of computing scattering amplitudes
- Novel (hidden) mathematical structures behind amplitudes and even QFT
- Applications to particle phenomenology and gravitational physics

Scattering equations

- The **scattering equations** link the kinematic data with the moduli space of n-punctured spheres

[Cachazo-He-Yuan 2013]



$$f_a(z,k) = \sum_{b \neq a} \frac{k_a \cdot k_b}{z_a - z_b} = 0, \quad a = 1, \ldots, n$$

- This system has a SL$(2, \mathbb{C})$ redundancy, only $(n-3)$ out of n equations are independent.

- $(n-3)$ independent equations and $(n-3)$ independent unknowns z_a

- The total number of independent solutions is $(n-3)!$.

Reformulate the S-matrix

Any tree-level S-matrix in massless theories may be reformulated as a multiple localized integral

[Cachazo-He-Yuan 2013]

$$\mathcal{A}_n \sim \underbrace{\oint_{\mathcal{C}} \frac{d^n z_a}{\prod'_a f_a(z,k)}}_{\textbf{universal!}} \mathcal{I}_n(z,k)$$

$$f_a = \sum_{b \neq a} \frac{k_a \cdot k_b}{z_a - z_b}$$

- The contour $\mathcal{C}$ is entirely determined by the zeros of the scattering equations
- The scattering equations, $f_a(z,k)$, are universal for all theories.
- The rational function $\mathcal{I}_n(z,k)$ encodes dynamics of the specific theory.

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- The scattering equations, $f_a(z,k)$, are universal for all theories.
- The rational function $\mathcal{I}_n(z,k)$ encodes dynamics of the specific theory.
- The multiple integral is completely fixed by the zeros of the scattering equations:

$$\mathcal{A}_n = \sum_{\text{all solutions}} \frac{\mathcal{I}_n(z,k)}{\det'(\partial f_a/\partial z_b)}$$

- Therefore, it is very interesting and important to find
  - ▶ new contour-integral representations for various theories (i.e. construct $\mathcal{I}_n$)
  - ▶ an efficient way to solve the scattering equations

New representations

On-shell amplitudes in EFTs

- We proposed the new representations for on-shell scattering amplitudes in effective field theories, including maximally supersymmetric ($\mathcal{N}=4$) Dirac-Born-Infeld-Volkov-Akulov, NLSM and a special Galileon theory.
- Our new formulas provide new ways to study amplitudes in these theories. [He, ZL & Wu, JHEP 1607 060]
  - ▶ Mysterious relation between different theories!

$$\mathcal{I}^{(\text{Born-Infeld})} \sim \mathcal{I}^{(\text{YM})} \times \mathcal{I}^{(\text{NLSM})}, \qquad \mathcal{I}_n^{(\text{Galileon})} \sim \mathcal{I}_n^{(\text{NLSM})} \times \mathcal{I}_n^{(\text{NLSM})}$$

  - ▶ Vanishing single soft limit (Adler's zero); universal double-soft limits

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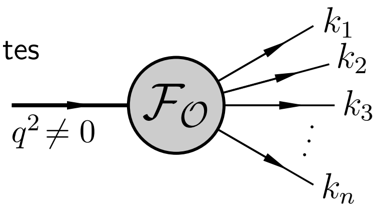
Form factors with off-shell momenta

• Form factor: the overlap of an off-shell composite operator and n on-shell states

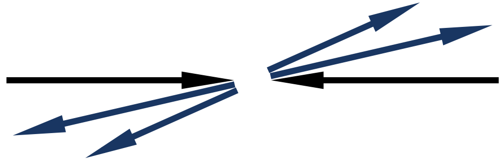
$$\langle 1, \ldots, n|\mathcal{O}|0\rangle$$



• Based on a set of modified scattering equations, we proposed new compact formulas for form factors with some operators in $\mathcal{N}=4$ SYM, e.g. scalar operators $\mathcal{O}_2 = \mathrm{Tr}[(\phi_{12})^2]$. [He & ZL, JHEP 1612 006]

• Our results strongly support the universality of the scattering equations for off-shell/massive quantities.

Multi-Regge kinematics



- Multi-Regge kinematics/limit: $2 \to n{-}2$ scattering
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$$y_3 \gg y_4 \gg \cdots \gg y_n$$

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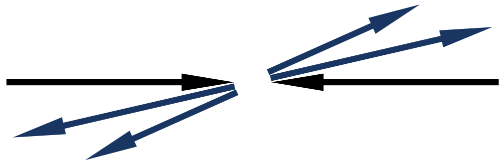


- In MRK, we observed for any solution of scattering eqs [Duhr & ZL, JHEP 1901 146]

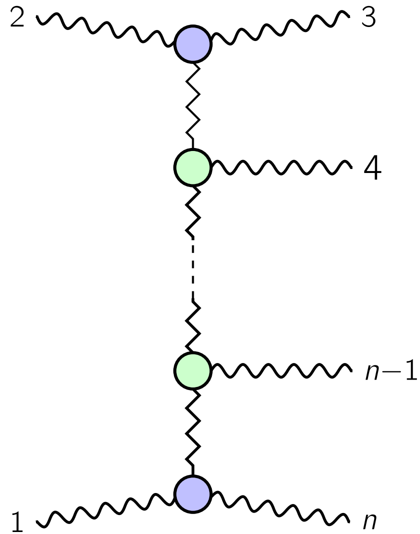
$$\text{“}z_3 \gg z_4 \gg \cdots \gg z_n\text{”}$$

- We obtained the exact solution for any multiplicity and for any helicity sector!

$$z_a = \frac{k_a^+}{k_a^\perp} \times \begin{cases} \left(\prod_{l\in\mathfrak{N}_{<a}} \frac{q_l^\perp}{q_{l+1}^\perp}\right)^* \left(\prod_{l\in\overline{\mathfrak{N}}_{>a}} \frac{q_l^\perp}{q_{l+1}^\perp}\right), & a\in\mathfrak{P} \\ \frac{k_a^\perp}{q_{a+1}^\perp} \left(\frac{q_a^\perp}{k_a^\perp}\right)^* \left(\prod_{l\in\overline{\mathfrak{N}}_{<a}} \frac{q_l^\perp}{q_{l+1}^\perp}\right)^* \left(\prod_{l\in\overline{\mathfrak{N}}_{>a}} \frac{q_l^\perp}{q_{l+1}^\perp}\right), & a\in\overline{\mathfrak{N}} \end{cases}$$

- We derived the amazingly simple factorized form of amplitudes in gauge theory and gravity: [Duhr & ZL, JHEP 1901 146; ZL, JHEP 1902 112]

$$\mathcal{A}_n \simeq -s\, \mathcal{C}_{2;3} \frac{1}{t_4} \mathcal{V}_4 \cdots \frac{1}{t_{n-1}} \mathcal{V}_{n-1} \frac{1}{t_n} \mathcal{C}_{1;n}$$



Homotopy continuation

- For generic kinematics, it is very difficult to obtain all solutions valid for a high multiplicity.
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- It naturally induces a homotopy continuation of the scattering equations [ZL & Zhao, JHEP 1902 071]

$$f_a(t) = \sum_{b \neq a} \frac{s_{ab}(t)}{z_a(t) - z_b(t)} = 0$$

- The scattering equations (and their solutions) with different kinematics become related each other!

Homotopy continuation

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- The scattering equations (and their solutions) with different kinematics become related each other!
- Differentiating $\mathrm{d}f_a(t) = 0$ gives a system of ODEs:

$$\frac{\mathrm{d}z_i}{\mathrm{d}t} + \Phi_{ij}^{-1} f_j' = 0, \qquad \Phi_{ij} \equiv \frac{\partial f_i(z,t)}{\partial z_j}, \qquad f_i' \equiv \frac{\partial f_i(z,t)}{\partial t}$$

- Solving the scattering equations becomes integrating ODEs.
- Because of using the physical homotopy continuation $s_{ij}(t)$, our algorithm is stable and efficient! We can easily generate all solutions of the scattering equations with a high accuracy. [https://github.com/zxrlha/sehomo]
- Pave a way towards a more efficient method of generating amplitudes via scattering equations.

Work at DESY

Currently, I focus on the formal aspect of scattering amplitudes

- Scattering equations and related topics
- 4D scattering amplitudes/2D conformal correlators duality
- Soft theorems and asymptotic symmetry

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During my postdoc at DESY, I hope to expand/shift my research

- Feynman integrals and special (elliptic) functions
- Apply new techniques of amplitudes to do precision computations in phenomenology and gravitational physics.

Thanks for your attention!