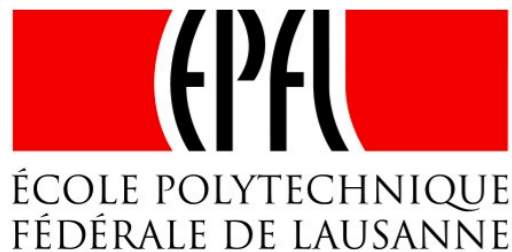


Using an amplitude analysis to measure the photon polarisation in $B \rightarrow K\pi\pi\gamma$ decays

V. Bellee, F. Blanc, P. Pais, A. Puig, O. Schneider, K. Trabelsi, G. Veneziano



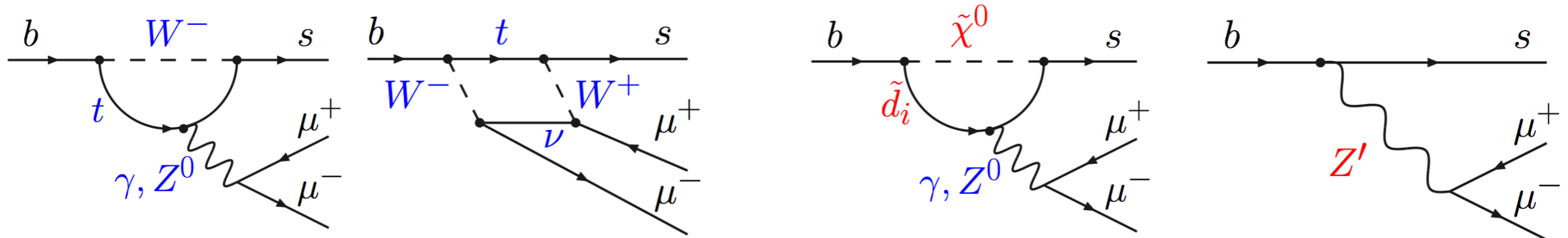
DESY Pizza (+HEP) Seminar



November 27, 2019

RARE DECAYS

- ‘Flavour-changing neutral current’ (FCNC) transitions
- Proceed via electroweak loops; suppressed in the SM
- **New particles** could also contribute at loop level



Potential effects observed via:

- Anomalies in decay rates, differential branching fraction measurements
- Analyses of angular distributions
- Tests of lepton flavour universality and lepton flavour violation

PARAMETRISING RADIATIVE DECAYS

Effective Hamiltonian described by an operator product expansion

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\underbrace{\sum_{i=1}^8 C_i(\mu) Q_i(\mu)}_{\text{left handed}} + \underbrace{\sum_{i=7}^8 C'_i(\mu) Q'_i(\mu)}_{\text{right handed (suppressed in the SM)}} \right)$$

- The operators Q_i encode long-distance effects

Q_{1-6} SM 4-quark operators

$$Q_7^{(\prime)} = \frac{e}{16\pi^2} m_b (\bar{s}_{L/R} \sigma_{\mu\nu} b_{R/L}) F^{\mu\nu}$$

Electromagnetic dipole operator

$$Q_8^{(\prime)} = \frac{g_s}{16\pi^2} m_b (\bar{s}_{L/R} \sigma_{\mu\nu} T^a b_{R/L}) G^{a\mu\nu}$$

Chromomagnetic dipole operator

PARAMETRISING RADIATIVE DECAYS

Effective Hamiltonian described by an operator product expansion

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\underbrace{\sum_{i=1}^8 C_i(\mu) Q_i(\mu)}_{\text{left handed}} + \underbrace{\sum_{i=7}^8 C'_i(\mu) Q'_i(\mu)}_{\text{right handed (suppressed in the SM)}} \right)$$

- The Wilson coefficients encode perturbative, short-distance effects
- Define an effective Wilson coefficient C_7^{eff} :

$$C_7^{\text{eff}}(\mu) = C_7(\mu) + \sum_{i=1}^6 y_i C_i(\mu)$$

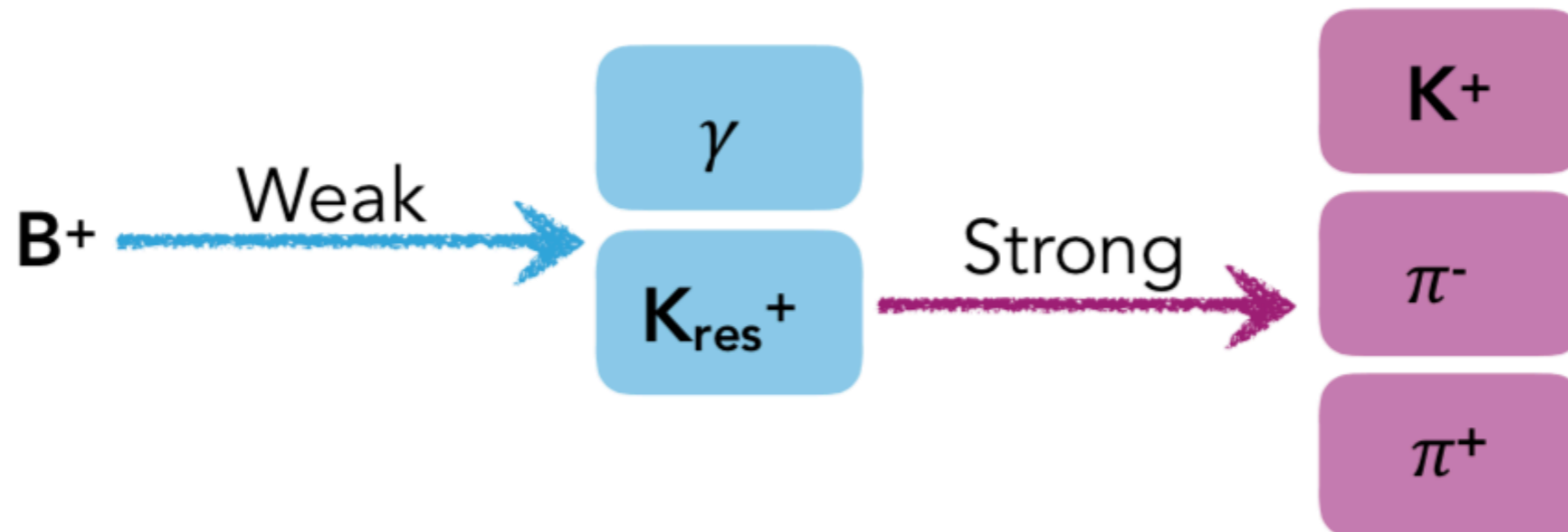
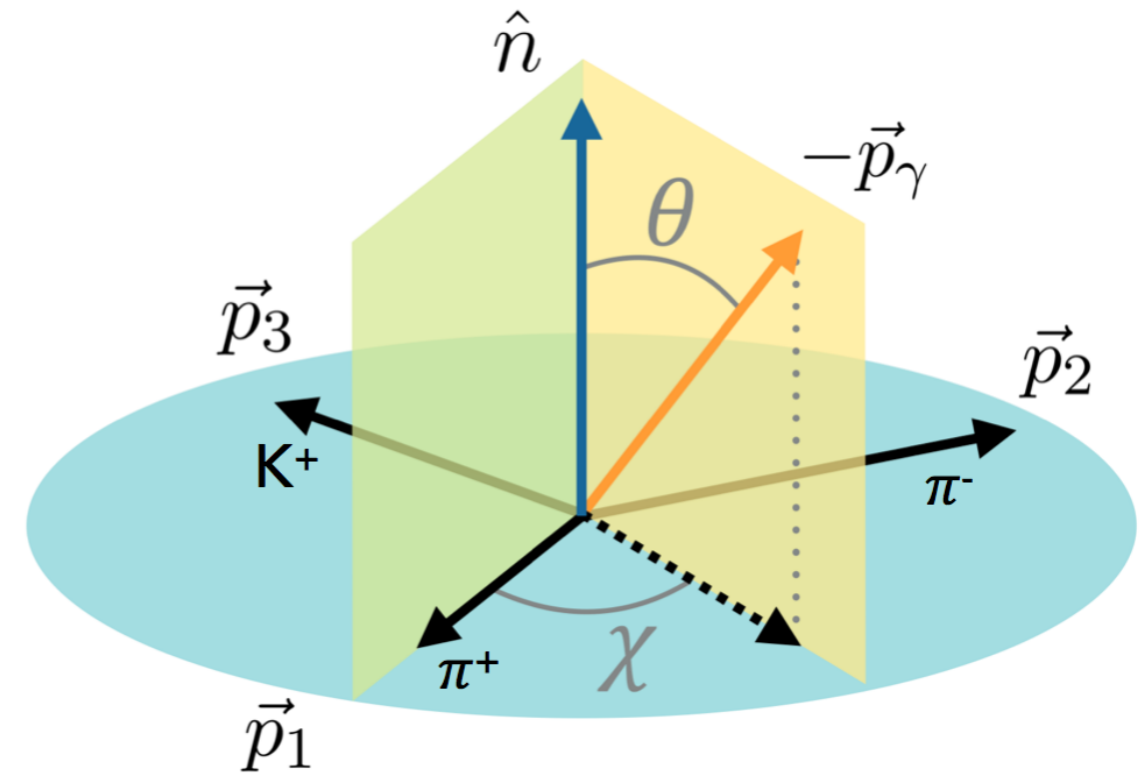
- In the SM, $C_7'/C_7^{\text{eff}} = m_s/m_b \sim 0.02$
 - Left-handed photons are dominant
- NP contributions could enhance fraction of right-handed photons

PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS

The $K\pi\pi\gamma$ final state can be described in terms of 5 independent variables:

- Three invariant masses ($m^2(K\pi\pi)$, $m^2(K\pi)$ and $m^2(\pi\pi)$)
- Two angular variables (χ and θ) that describe the orientation of the photon with respect to the hadronic plane

This is a $\bar{b} \rightarrow \bar{s}$ transition; in the SM, photons are expected to be predominantly right-handed



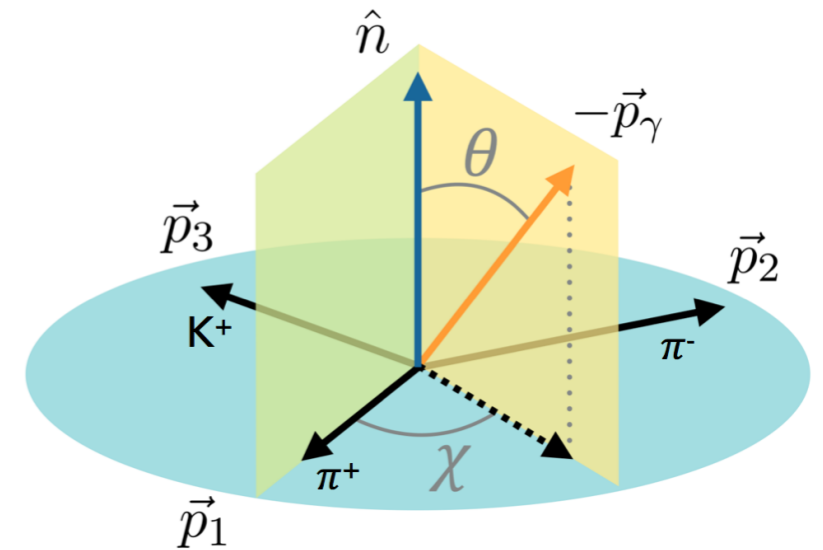
PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS

Can measure the photon polarisation using the recoil hadron distribution:

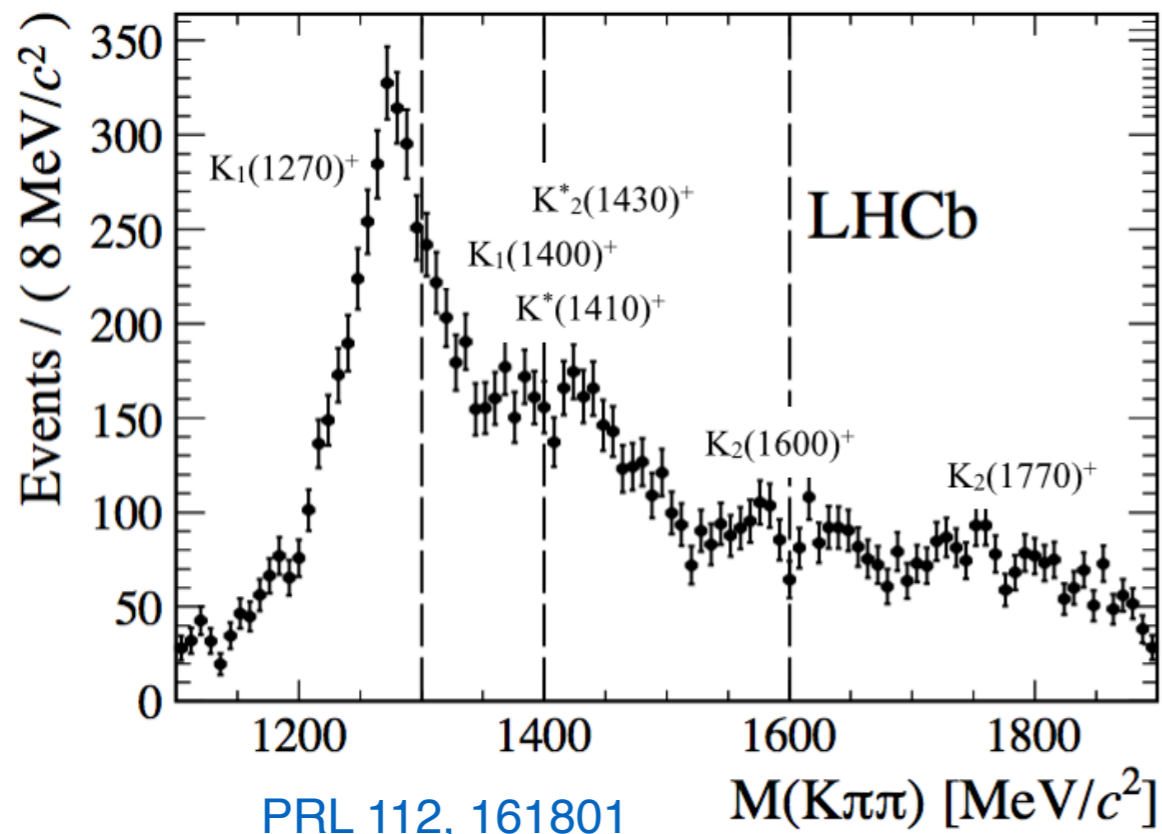
- Photon helicity is odd under parity
- Need three tracks in the final state; can form a parity-odd triple product from final-state particle momenta

$$\vec{p}_\gamma \cdot (\vec{p}_1 \times \vec{p}_2)$$

- Requires interference between various decay amplitudes that contribute to the final state



Gronau et. al:
[PRL 88, 051802](#)
[PRD 66, 054008](#)

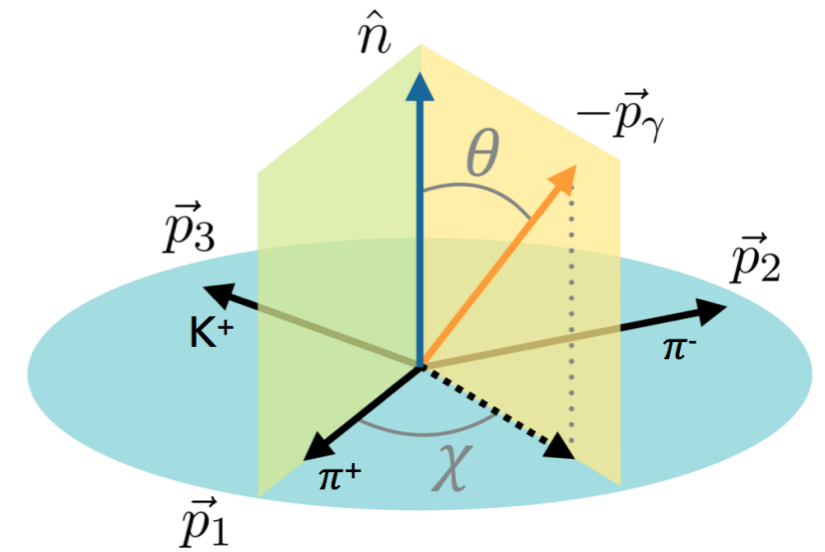


- System populated by a multitude of resonances
 - Interferences give sensitivity to photon polarisation parameter
 - Very complex!

PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS

Simplify the problem:

- Compute an up-down asymmetry (between the number of photons emitted on one side and on the other of the $K\pi\pi$ decay plane)
- Proportional to photon polarisation parameter

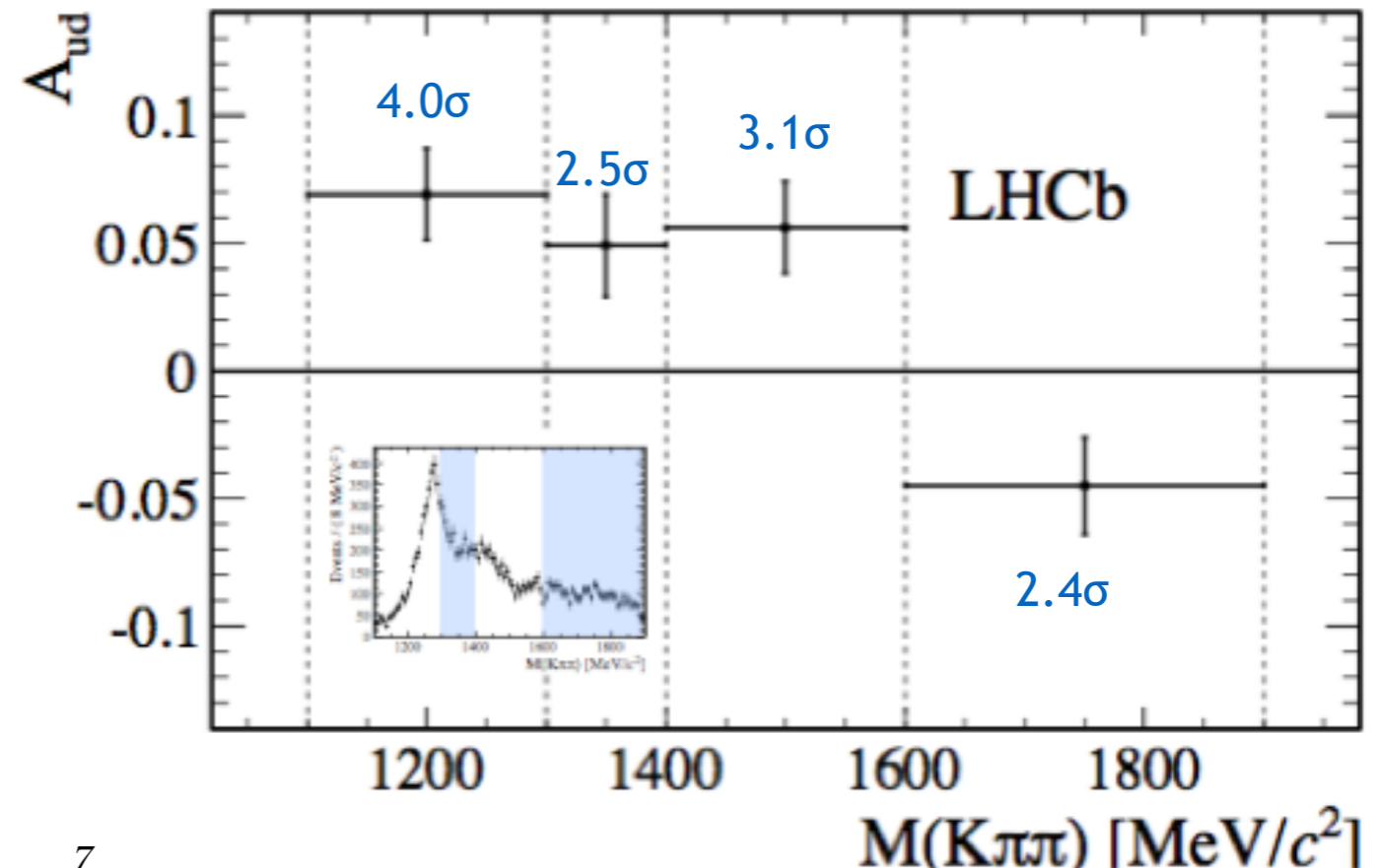


[PRL 112, 161801](#)

$$A_{ud} \equiv \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{d\cos\theta}}{\int_{-1}^1 d\cos\theta \frac{d\Gamma}{d\cos\theta}} = C\lambda_\gamma$$

- 14,000 signal events selected from full LHCb Run 1 dataset
- $\cos\theta$ fit performed in four $m(K\pi\pi)$ bins

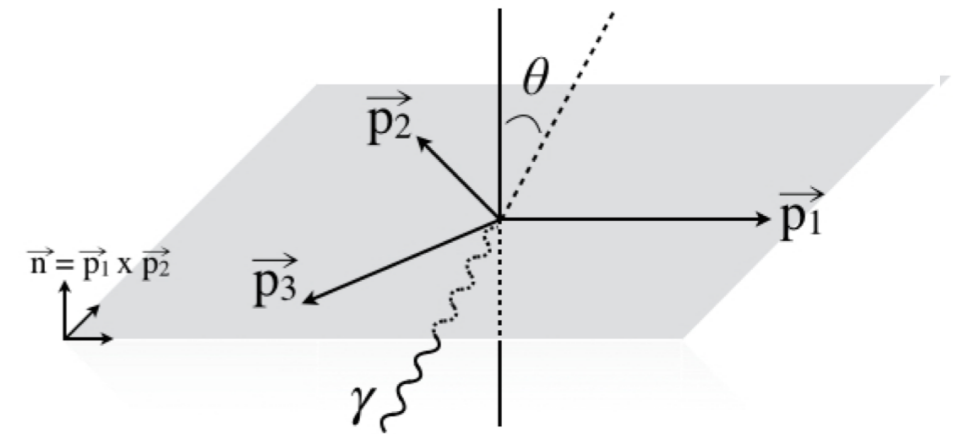
First observation of a non-zero photon polarisation in $b \rightarrow s\gamma$ transitions (**5.2 σ** significance)



PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS

Simplify the problem:

- Compute an up-down asymmetry (between the number of photons emitted on one side and on the other of the $K\pi\pi$ decay plane)
- Proportional to photon polarisation parameter

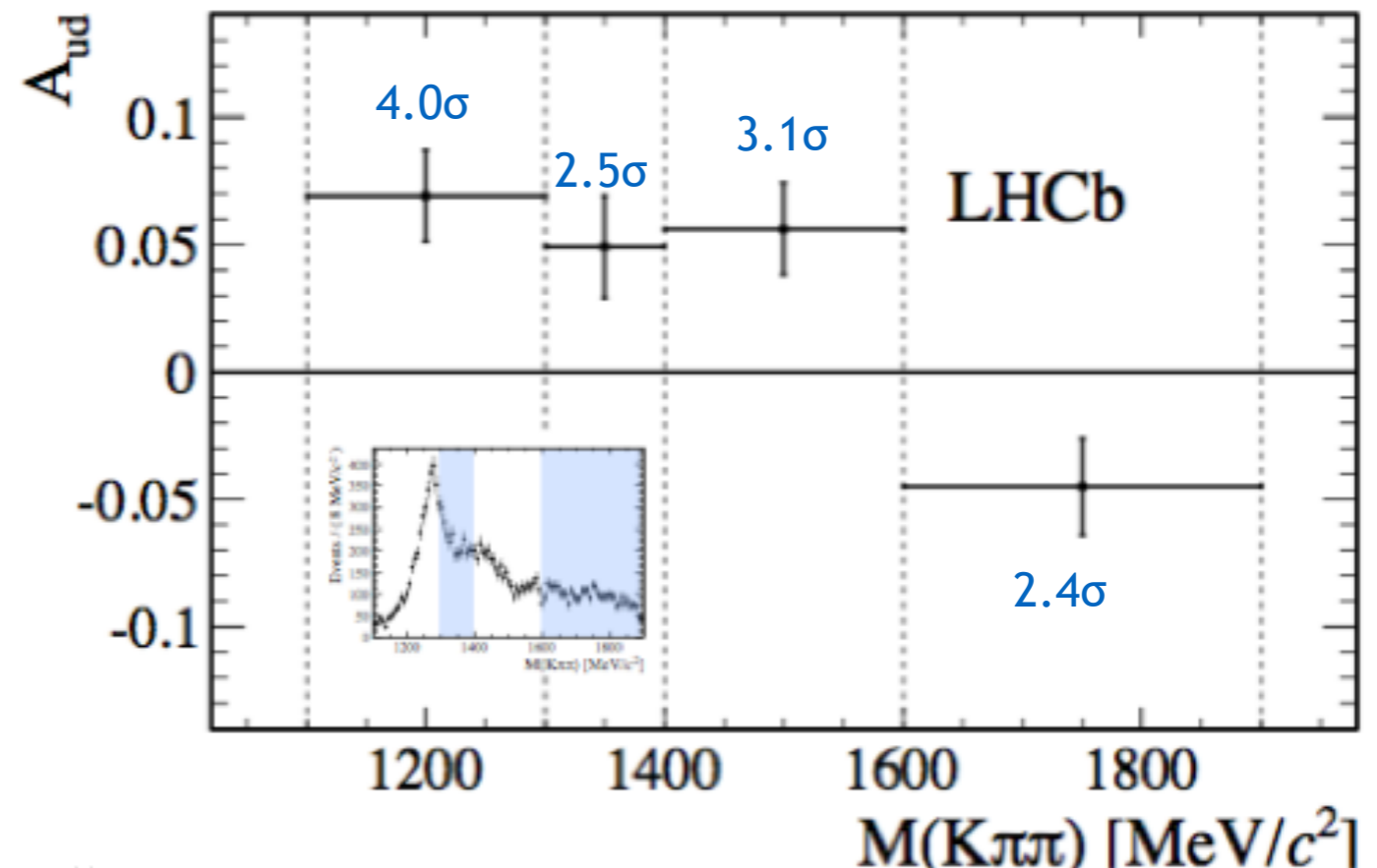


$$A_{ud} \equiv \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{d\cos\theta}}{\int_{-1}^1 d\cos\theta \frac{d\Gamma}{d\cos\theta}} = C\lambda_\gamma$$

[PRL 112, 161801](#)

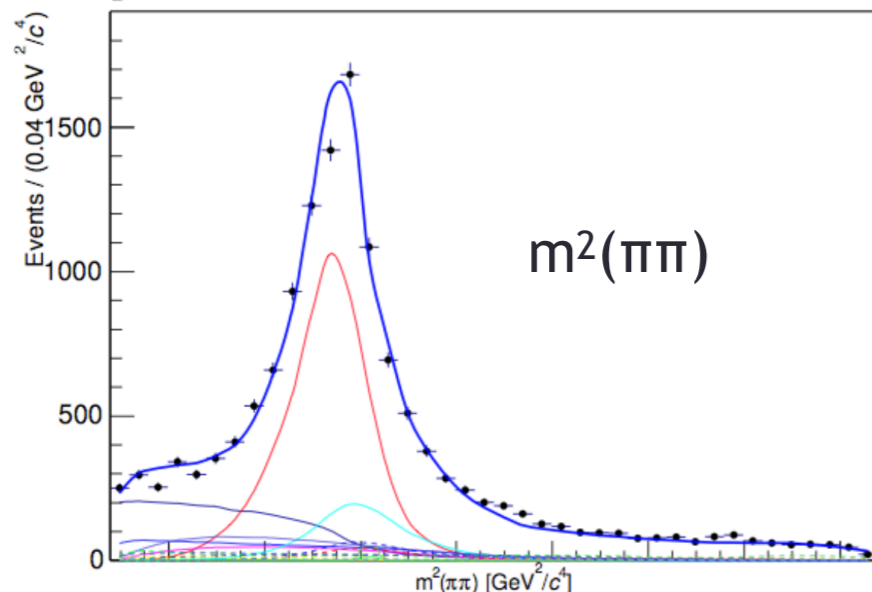
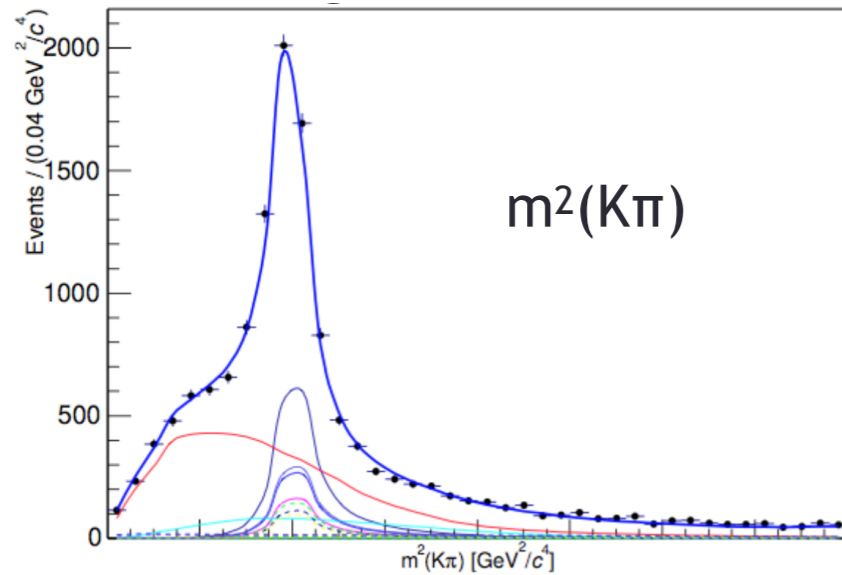
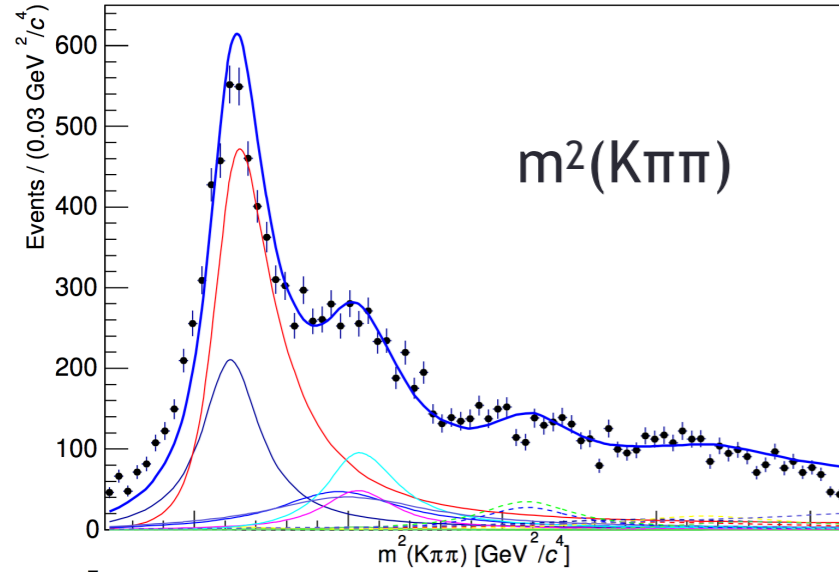
Cannot translate this to a value of the photon polarisation parameter without exact knowledge of the resonances that populate the system

Need a full amplitude analysis



3-D AMPLITUDE ANALYSIS OF THE $K\pi\pi$ SYSTEM

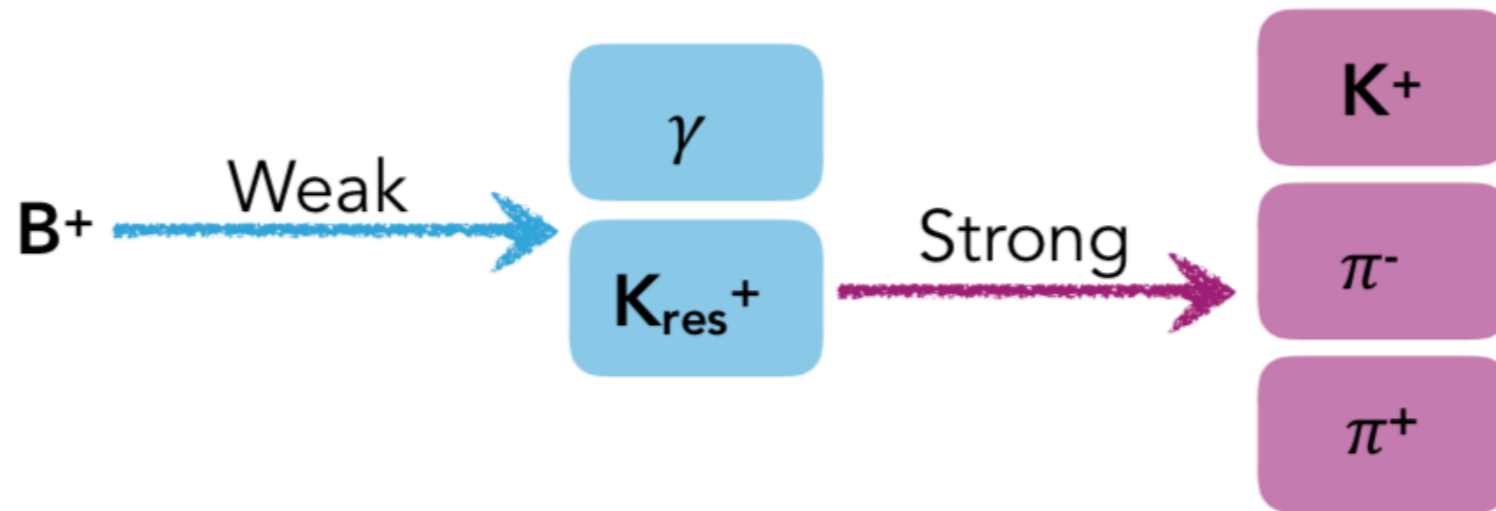
[CERN-THESIS-2015-287](#)



Integrate over photon angular variables; fit in terms of invariant masses $m^2(K\pi\pi)$, $m^2(K\pi)$ and $m^2(\pi\pi)$

Decay channel k	$\mathcal{R}e[f_k]$	$\mathcal{I}m[f_k]$	$FF_k (10^{-2})$
$K_1(1270)^+ \rightarrow K^*(892)^0\pi^+$	1 (fixed)	0 (fixed)	16.8 ± 0.9
$K_1(1270)^+ \rightarrow K^+\rho(770)^0$	1.072 ± 0.050	-1.379 ± 0.047	$39.9^{+0.6}_{-0.7}$
$K_1(1270)^+ \rightarrow K^+\omega(782)^0$	0.288 ± 0.086	0.090 ± 0.081	$0.068^{+0.028}_{-0.180}$
$K_1(1270)^+ \rightarrow K^*(1430)^0\pi^+$	-0.025 ± 0.062	-0.381 ± 0.055	$0.69^{+0.22}_{-0.20}$
$K_1(1400)^+ \rightarrow K^*(892)^0\pi^+$	0.306 ± 0.024	-0.288 ± 0.020	7.8 ± 0.8
$K^*(1410)^+ \rightarrow K^*(892)^0\pi^+$	-0.479 ± 0.042	0 (fixed)	$8.4^{+2.8}_{-3.3}$
$K^*(1680)^+ \rightarrow K^*(892)^0\pi^+$	0.198 ± 0.020	0.094 ± 0.028	$3.5^{+1.7}_{-2.1}$
$K^*(1680)^+ \rightarrow K^+\rho(770)^0$	0.019 ± 0.025	0.1104 ± 0.0097	2.4 ± 0.4
$K_2^*(1430)^+ \rightarrow K^*(892)^0\pi^+$	-0.509 ± 0.034	0 (fixed)	4.8 ± 1.0
$K_2^*(1430)^+ \rightarrow K^+\rho(770)^0$	-0.115 ± 0.047	0.497 ± 0.024	9.0 ± 0.8
$K_2^*(1430)^+ \rightarrow K^+\omega(782)^0$	-0.234 ± 0.072	-0.236 ± 0.084	$0.30^{+0.13}_{-0.26}$
$K_2(1600)^+ \rightarrow K^*(892)^0\pi^+$	-0.1666 ± 0.0088	0.044 ± 0.021	$4.4^{+0.9}_{-1.0}$
$K_2(1600)^+ \rightarrow K^+\rho(770)^0$	-0.073 ± 0.011	0.061 ± 0.013	$3.33^{+0.34}_{-0.50}$
$K_2(1770)^+ \rightarrow K^*(892)^0\pi^+$	0.1072 ± 0.0078	0 (fixed)	$3.0^{+0.6}_{-0.8}$
$K_2(1770)^+ \rightarrow K^+\rho(770)^0$	-0.0147 ± 0.0044	0.0103 ± 0.0050	$0.23^{+0.08}_{-0.32}$
$K_2(1770)^+ \rightarrow K_2^*(1430)^0\pi^+$	-0.041 ± 0.012	-0.0772 ± 0.0077	$0.67^{+0.10}_{-0.09}$
$K_2(1770)^+ \rightarrow K^+f_2(1270)^0$	0.1673 ± 0.0071	-0.029 ± 0.015	$1.30^{+0.15}_{-0.16}$
Non resonant	-0.0511 ± 0.0021	0 (fixed)	4.1 ± 0.5

PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS



Differential decay rate for a particular decay mode $B^+ \rightarrow K_{res}^{+(i)} \gamma \rightarrow K^+ \pi^- \pi^+ \gamma$:

$$\frac{d\Gamma(B^+ \rightarrow K^+ \pi^- \pi^+ \gamma)}{ds} = \left| \sum_i c_R^i B^i(s) A_R^i \right|^2 + \left| \sum_i c_L^i B^i(s) A_L^i \right|^2$$

Decay amplitude
for $B^+ \rightarrow K_{res}^{+(i)} \gamma$

Propagator
for $K_{res}^{+(i)}$

Decay amplitude
for $K_{res}^{+(i)} \rightarrow K^+ \pi^- \pi^+$

The photon polarisation parameter λ_γ for a decay mode i is then defined as:

$$\lambda_\gamma^i \equiv \frac{|c_R^i|^2 - |c_L^i|^2}{|c_R^i|^2 + |c_L^i|^2}$$

Gronau et. al:
[PRL 88, 051802](#)
[PRD 66, 054008](#)

PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS

Use parity invariance of the strong interaction to relate the amplitudes for emitting left- and right- handed photons:

$$\langle K_{\text{res}}^{+(i)R} \gamma_R | \mathcal{O}_{7R} | B^+ \rangle = \mathcal{P}_i (-1)^{(J_i-1)} \langle K_{\text{res}}^{+(i)L} \gamma_L | \mathcal{O}_{7L} | B^+ \rangle$$

[PRL 88, 051802](#)
[PRD 66, 054008](#)

The weak amplitudes are then proportional to the Wilson coefficients:

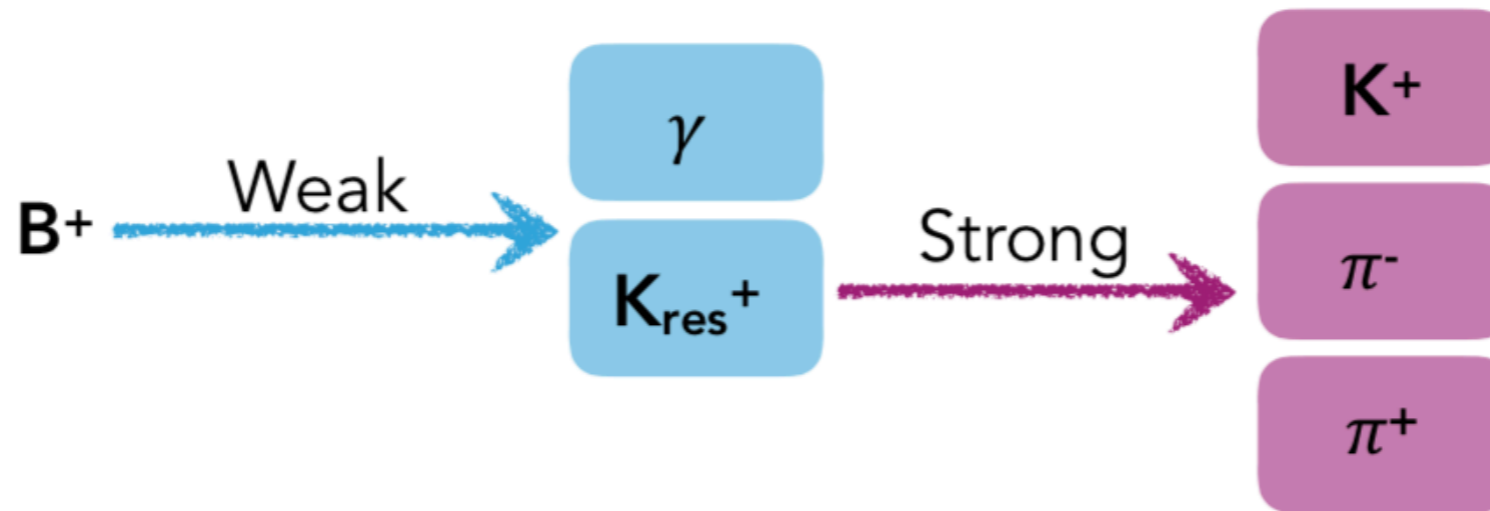
$$\begin{pmatrix} c_R^i \\ c_L^i \end{pmatrix} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \begin{pmatrix} C_7^{\text{eff}} g^i(0) \\ C_7' P_i (-1)^{J_i-1} g^i(0) \end{pmatrix} \rightarrow \begin{array}{l} \text{hadronic} \\ \text{matrix} \\ \text{element of } \mathcal{O}_7 \end{array}$$

We can therefore derive λ_γ in terms of the Wilson coefficients:

$$\frac{|c_R^i|}{|c_L^i|} = \frac{|C_{7R}|}{|C_{7L}|} \implies \lambda_\gamma^i = \frac{|C_{7R}|^2 - |C_{7L}|^2}{|C_{7R}|^2 + |C_{7L}|^2} \equiv \lambda_\gamma$$

**λ_γ is the same for
all decay modes**

PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS



Can rewrite the total decay rate for all decay modes $B^+ \rightarrow K_{\text{res}}^+ \gamma \rightarrow K^+ \pi^- \pi^+ \gamma$:

$$d\Gamma(B^+ \rightarrow K_{\text{res}}^{+(i)} \gamma \rightarrow K^+ \pi^- \pi^+ \gamma) \propto (|\mathcal{M}_R|^2 + |\mathcal{M}_L|^2) + \lambda_\gamma (|\mathcal{M}_R|^2 - |\mathcal{M}_L|^2)$$

Right handed
amplitudes

Photon polarisation parameter

$$\frac{|C_7^{\text{eff}}|^2 - |C_7'|^2}{|C_7^{\text{eff}}|^2 + |C_7'|^2} \equiv \lambda_\gamma$$

Left handed
amplitudes

B → Kππγ FORMALISM

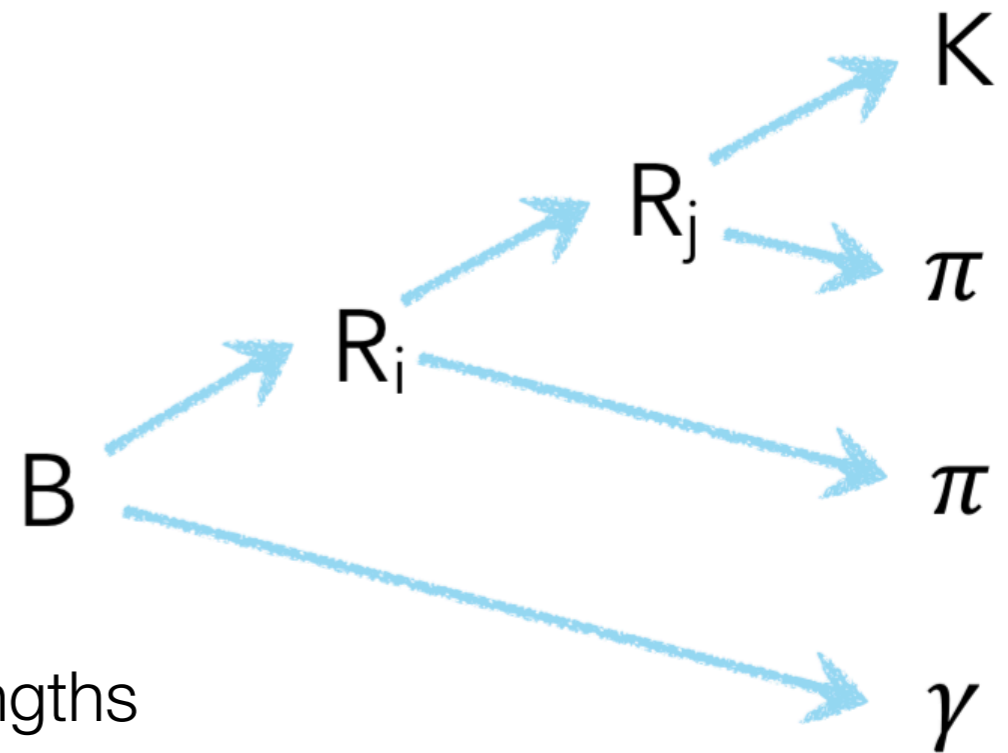
[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

$$d\Gamma(B^+ \rightarrow K_{\text{res}}^{+(i)} \gamma \rightarrow K^+ \pi^- \pi^+ \gamma) \propto (|\mathcal{M}_R|^2 + |\mathcal{M}_L|^2) + \lambda_\gamma (|\mathcal{M}_R|^2 - |\mathcal{M}_L|^2)$$

Use the isobar model to construct decay amplitudes

$$\mathcal{M}_{R/L} = \sum_l f_k \mathcal{A}_{k,R/L}(\mathbf{x})$$

$$f_k = a_k e^{i\phi_k}$$



The complex coefficients f_k encode relative strengths and phases for each amplitude

B → Kππγ FORMALISM

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

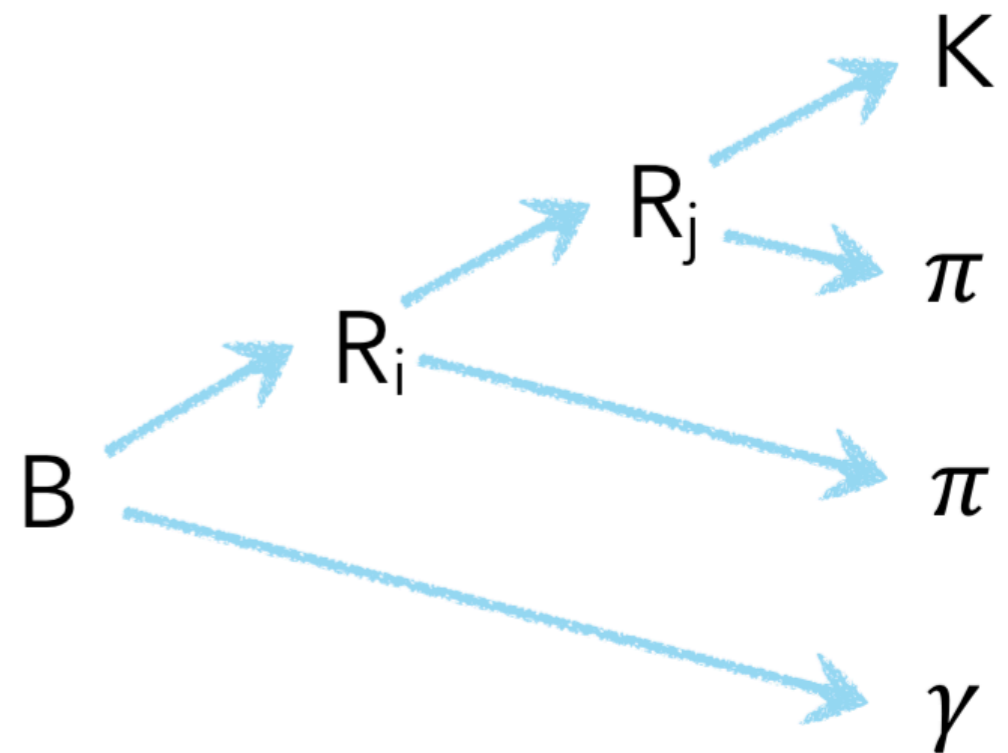
$$\mathcal{A}_R^k(\mathbf{x}) = B_{LB}(q_B(\mathbf{x}), 0) \mathcal{T}_i^k(\mathbf{x}) \mathcal{T}_j^k(\mathbf{x}) \mathcal{S}_{ij,R}^k(\mathbf{x})$$

$$\mathcal{A}_L^k(\mathbf{x}) = P_i(-1)^{J_i-1} B_{LB}(q_B(\mathbf{x}), 0) \mathcal{T}_i^k(\mathbf{x}) \mathcal{T}_j^k(\mathbf{x}) \mathcal{S}_{ij,L}^k(\mathbf{x})$$

Resonance
propagators

Barrier factor for a B
meson of angular
momentum L

Spin factors
Encode the phenomenology
of the decay, computed with
the covariant formalism



B → Kππγ FORMALISM (RECAP)

The (normalised) probability density function for B → Kππγ decays:

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

[PRL 88, 051802](https://arxiv.org/abs/1902.09201)

[PRD 66, 054008](https://arxiv.org/abs/1902.09201)

$$\mathcal{F}(\mathbf{x}|\Omega) = \frac{\xi(\mathbf{x})\mathcal{P}_s(\mathbf{x}|\Omega)\Phi_4(\mathbf{x})}{\int \xi(\mathbf{x})\mathcal{P}_s(\mathbf{x}|\Omega)\Phi_4(\mathbf{x}) d\mathbf{x}}$$

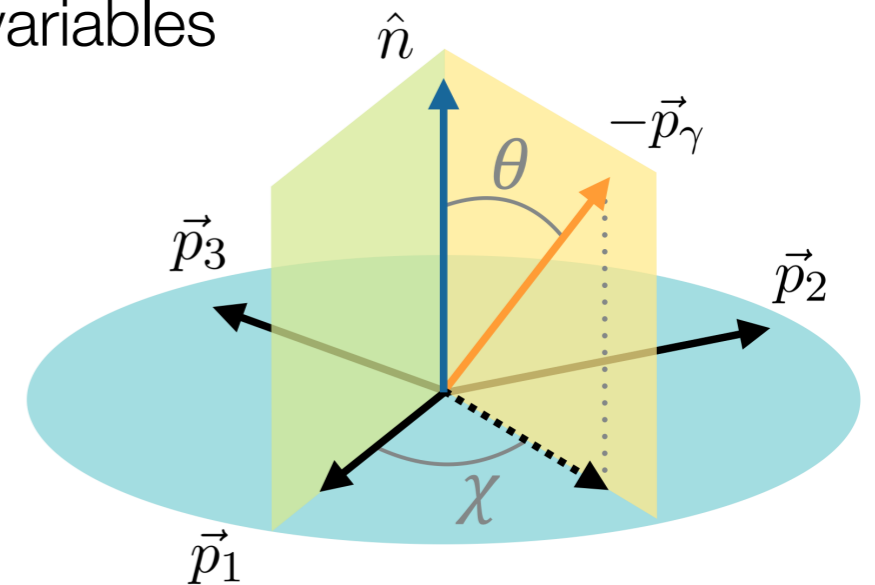
where

$\mathbf{x} = (m_{K^+\pi^-\pi^+}^2, m_{K^+\pi^-}^2, m_{\pi^-\pi^+}^2, \cos\theta, \chi)$: phase space variables

$\Omega = \lambda_\gamma, \{a_k\}, \{\phi_k\}$: fit parameters

$\xi(\mathbf{x})$: efficiency

$\Phi_4(\mathbf{x})$: phase space volume



The signal function \mathcal{P}_s encodes the dependence on λ_γ

$$\mathcal{P}_s = \frac{(1 + \lambda_\gamma)}{2} |\mathcal{M}_R|^2 + \frac{(1 - \lambda_\gamma)}{2} |\mathcal{M}_L|^2$$

B → Kππγ AMPLITUDE ANALYSIS: METHOD

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

The (normalised) probability density function for B → Kππγ decays:

$$\mathcal{F}(\mathbf{x}|\Omega) = \frac{\xi(\mathbf{x})\mathcal{P}_s(\mathbf{x}|\Omega)\Phi_4(\mathbf{x})}{\int \xi(\mathbf{x})\mathcal{P}_s(\mathbf{x}|\Omega)\Phi_4(\mathbf{x}) d\mathbf{x}}$$

This PDF is implemented in a modified version of the MINT2 generator-fitter framework

- Enables generation of multi-amplitude models with interferences, performs unbinned maximum likelihood minimisation

The normalisation integral is computed numerically using MC generated events according to an approximate PDF \mathcal{P}_{gen} :

$$\int \xi(\mathbf{x})\mathcal{P}_s(\mathbf{x})\phi_4(\mathbf{x}) d\mathbf{x} = \frac{I_{\text{gen}}}{N_{\text{sel}}} \sum_j^{N_{\text{sel}}} \frac{\mathcal{P}_s(\mathbf{x}_j)}{\mathcal{P}_{\text{gen}}(\mathbf{x}_j)}$$

The term $I_{\text{gen}} = \int \xi(\mathbf{x})\mathcal{P}_{\text{gen}}(\mathbf{x})\phi_4(\mathbf{x}) d\mathbf{x}$ is independent of all fit parameters, so can be neglected in the minimisation

$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ WITH TWO DECAY AMPLITUDES

- Test the method with a simplified, two-amplitude model:



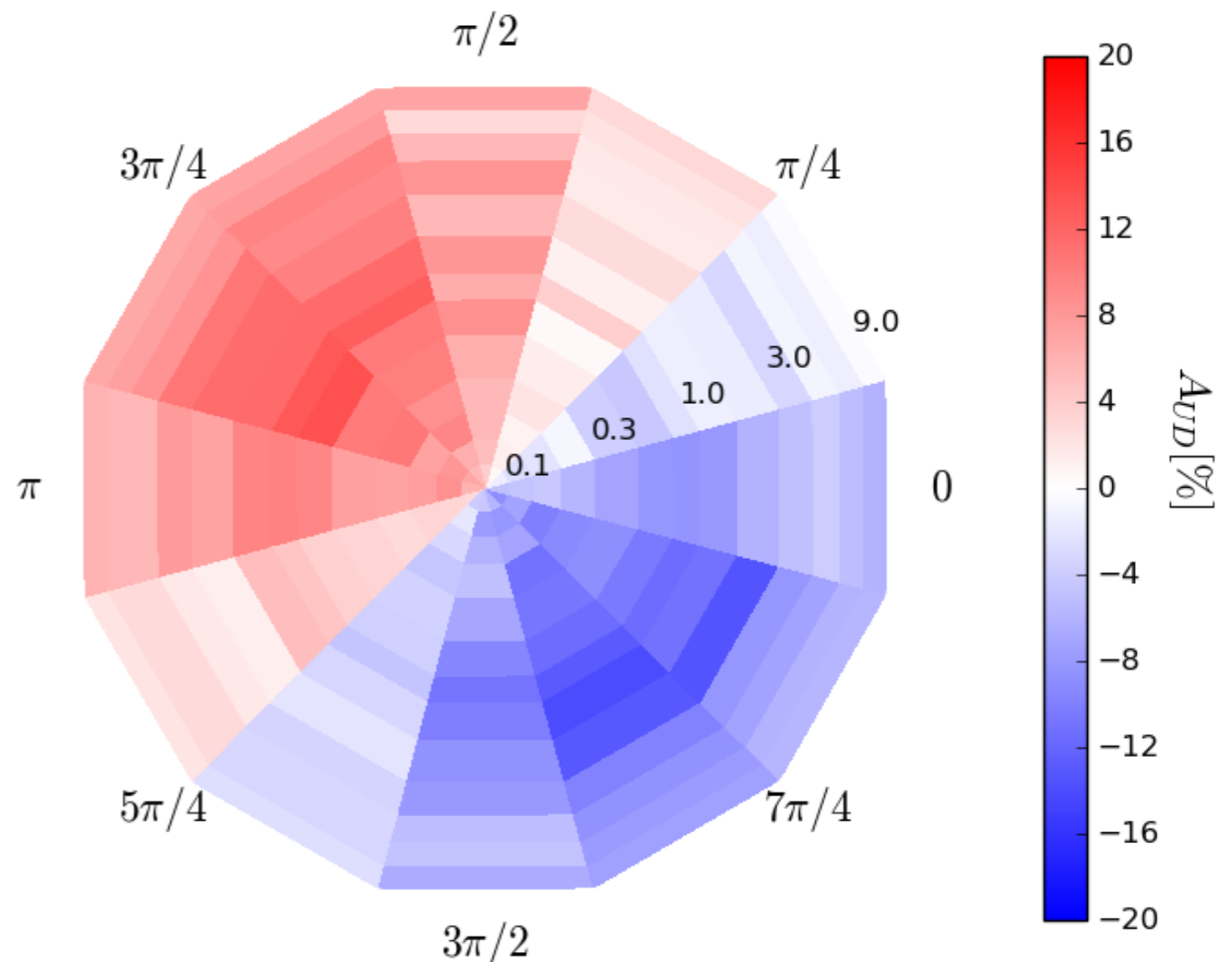
- System described by three free parameters:
 - The relative fraction r between the two amplitudes
 - Their phase difference $\Delta\phi$
 - The photon polarisation parameter λ_γ
- Study performance of the fit for a range of model parameters
 - At each point, 10 data sets of 8000 events each are generated and fit

REMINDER: LIMITATIONS OF A_{ud}

The up-down asymmetry A_{ud} is proportional to λ_γ

- The proportionality constant C depends on the resonance content of the system
- Cannot calculate λ_γ from up-down asymmetry without full characterisation of the resonances

- As an illustration, use the two amplitude model
- Generate samples for a range of relative fractions and phases between the amplitudes; $\lambda_\gamma=1$ (always)
- For each generated sample, calculate A_{ud}
- Up-down asymmetry varies as a function of the relative fraction and phase
 - Regions where $A_{ud}=0$, so no sensitivity to λ_γ

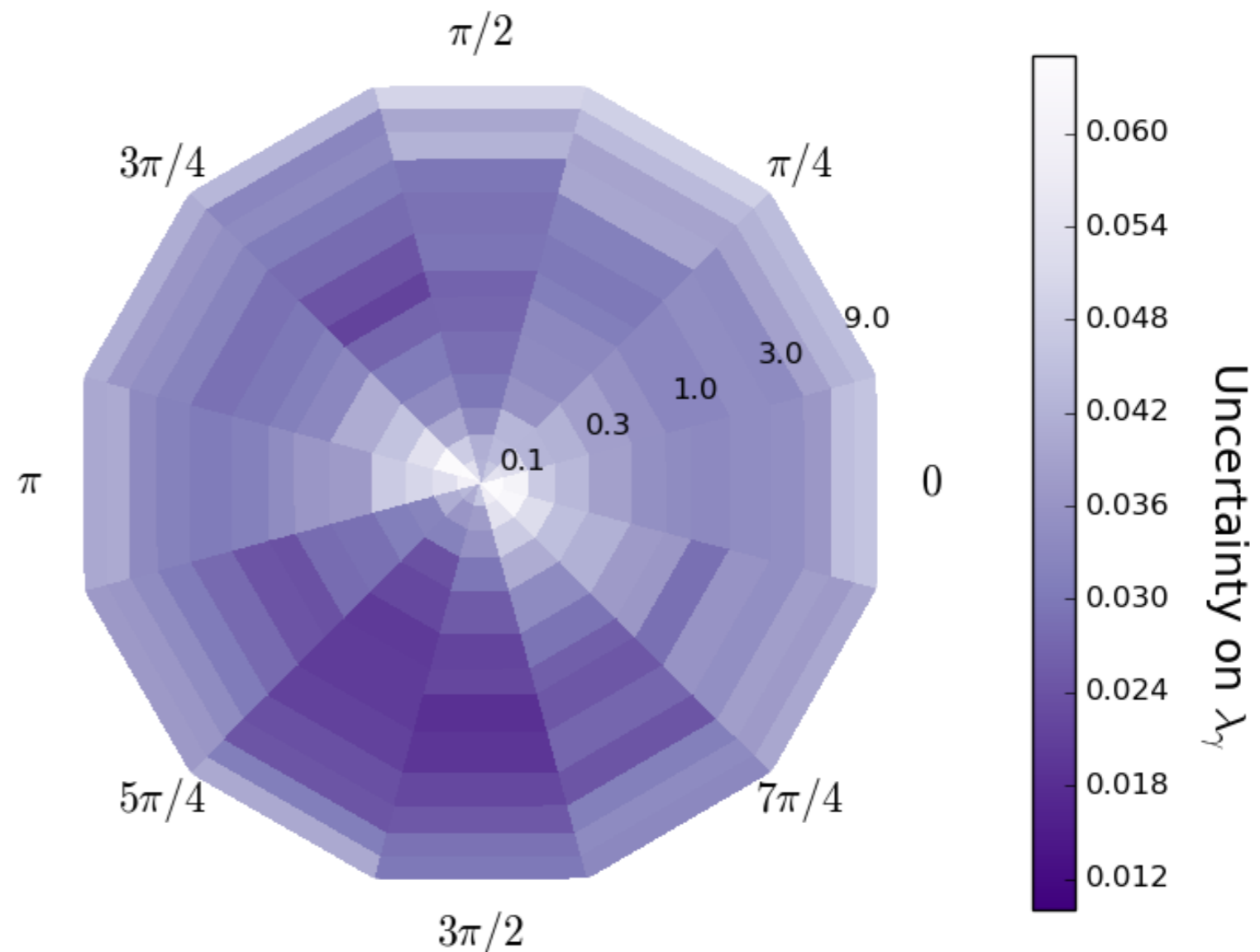


$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$: TWO-AMPLITUDE MODEL

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

- Use MINT to perform amplitude fits for the same generated data sets
- Fits mostly converge with no errors
 - Resulting fit parameters are centred around the ‘true’ (generated) values

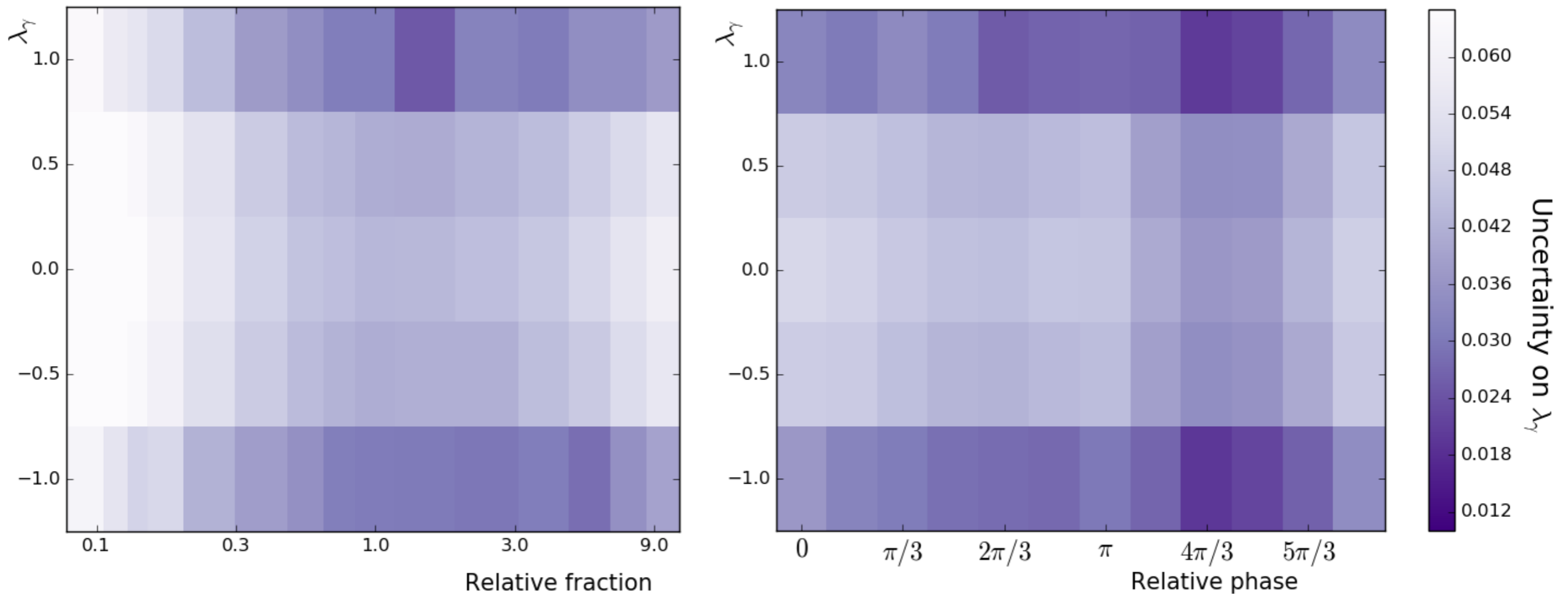
- The method is sensitive to all data points studied
- Variations in uncertainty on seen as a function of both r and $\Delta\phi$
 - Uncertainty is highest when one amplitude is dominant, tending towards a single amplitude model



$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$: TWO-AMPLITUDE MODEL

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

- Repeat the previous test for ranges of (r, λ_γ) and $(\Delta\phi, \lambda_\gamma)$
- Fit is sensitive to all values studied
 - The uncertainty on λ_γ increases as values tend towards 0



$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$: TWO-AMPLITUDE MODEL

- More detailed check of behaviour of the fit:
 - Perform 100 fits for a subset of points for each of the previous plots
 - Check distributions of fit parameters, associated errors, and pull parameter g :

$$\text{if (fit result) } \leq \text{(true value): } g = \frac{\text{(true value)} - \text{(fit result)}}{\text{(positive error)}}$$

$$\text{otherwise: } g = \frac{\text{(fit result)} - \text{(true value)}}{\text{(negative error)}}$$

- Pull distributions are unbiased with widths mostly consistent with unity in all cases

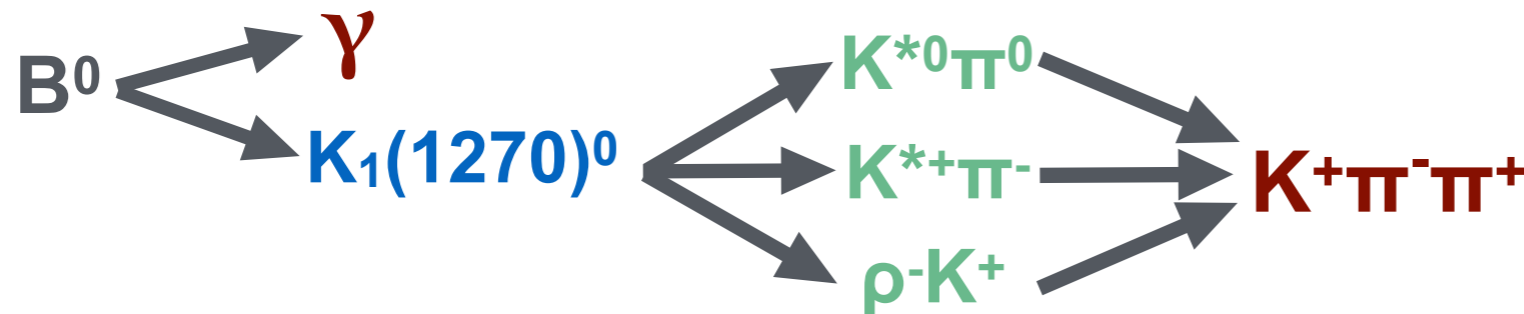
Parameter	True value	Mean value	Std deviation	μ_{pull}	σ_{pull}
a	2.02	2.020	0.04	0.01 ± 0.11	1.14 ± 0.07
ϕ	0.82	0.823	0.02	-0.09 ± 0.09	0.94 ± 0.07
λ_γ	1	1.001	0.04	-0.09 ± 0.12	1.17 ± 0.08
a	2.02	2.023	0.04	-0.06 ± 0.11	1.17 ± 0.07
ϕ	0.82	0.823	0.03	-0.13 ± 0.09	0.97 ± 0.06
λ_γ	0.875	0.870	0.04	0.11 ± 0.11	1.17 ± 0.08
a	2.02	2.022	0.04	-0.03 ± 0.09	1.03 ± 0.07
ϕ	0.82	0.822	0.03	-0.07 ± 0.09	0.92 ± 0.06
λ_γ	0.75	0.741	0.04	0.20 ± 0.09	1.03 ± 0.07

$B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$: THREE-AMPLITUDE MODEL

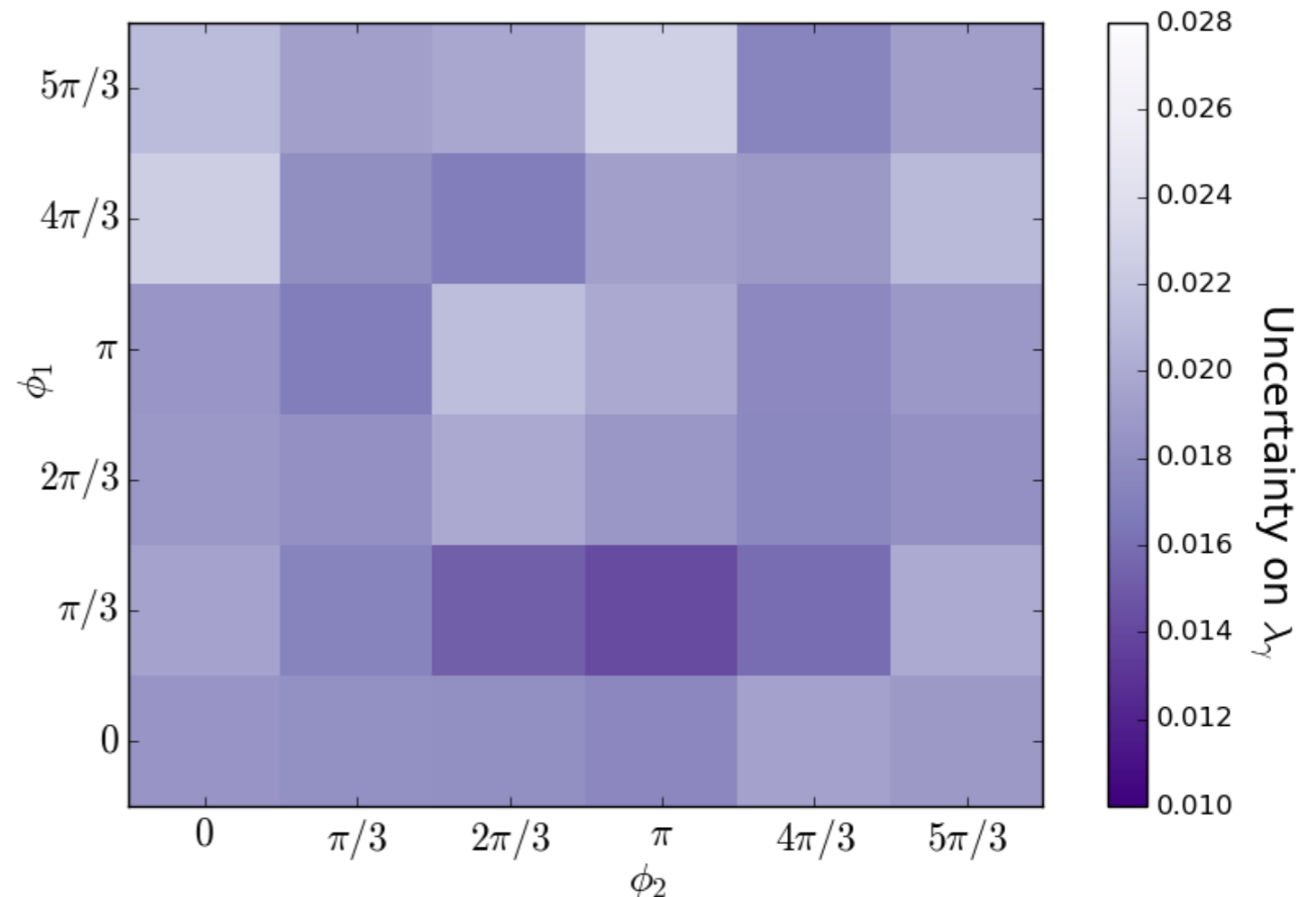
- Perform a similar test with $B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$ decays

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

- Gronau et. al. noted an increased sensitivity in up-down symmetry with additional interference terms, would the same hold true for a full amplitude analysis?



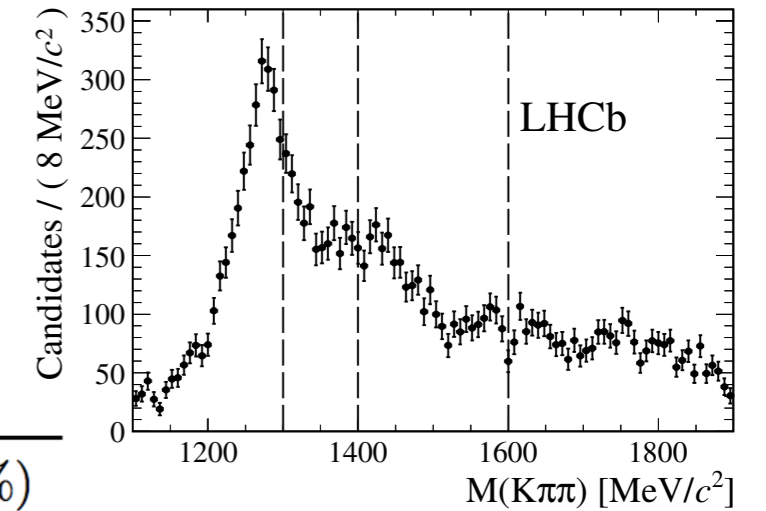
- Generated ratios of fit fractions ~ 1 , $\lambda_\gamma = 1$
- Scan over a range of relative phases
- Uncertainties within the range seen for the charged mode



$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$: REALISTIC MODEL

[CERN-THESIS-2015-287](#)

- Build a model with 15 decay amplitudes
- Generate 100 datasets with 14000 events each, $(r, \Delta\phi)$ generated values chosen correspond to 3-dimensional amplitude fit results from LHCb Run1 data analysis

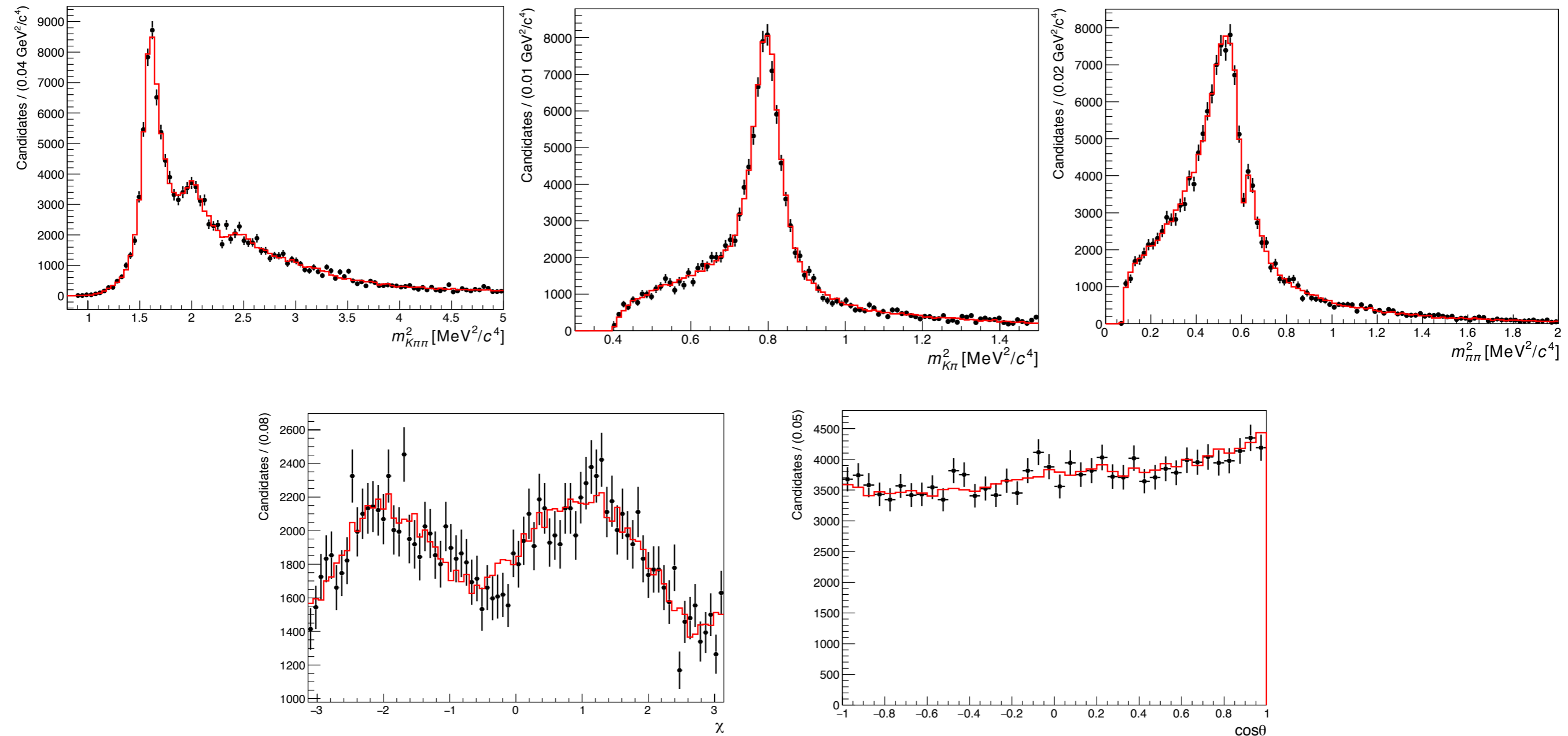


J^P	Amplitude k	a_k	ϕ_k	Fraction (%)
1^+	$K_1(1270)^+ \rightarrow K^*(892)^0 \pi^+$ [S-wave]	1 (fixed)	0 (fixed)	15.3
	$K_1(1270)^+ \rightarrow K^*(892)^0 \pi^+$ [D-wave]	1.00	-1.74	0.6
	$K_1(1270)^+ \rightarrow K^+ \rho(770)^0$	2.02	-0.91	37.9
	$K_1(1400)^+ \rightarrow K^*(892)^0 \pi^+$	0.59	-0.76	7.4
1^-	$K^*(1410)^+ \rightarrow K^*(892)^0 \pi^+$	0.11	0.00	7.9
	$K^*(1680)^+ \rightarrow K^*(892)^0 \pi^+$	0.05	0.44	3.4
	$K^*(1680)^+ \rightarrow K^+ \rho(770)^0$	0.04	1.40	2.3
2^+	$K_2^*(1430)^+ \rightarrow K^*(892)^0 \pi^+$	0.28	0.00	4.5
	$K_2^*(1430)^+ \rightarrow K^+ \rho(770)^0$	0.47	1.80	8.9
2^-	$K_2(1580)^+ \rightarrow K^*(892)^0 \pi^+$	0.49	2.88	4.2
	$K_2(1580)^+ \rightarrow K^+ \rho(770)^0$	0.38	2.44	3.2
	$K_2(1770)^+ \rightarrow K^*(892)^0 \pi^+$	0.35	0.00	2.8
	$K_2(1770)^+ \rightarrow K^+ \rho(770)^0$	0.08	2.53	0.2
	$K_2(1770)^+ \rightarrow K_2^*(1430)^0 \pi^+$	0.07	-2.06	0.6

$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$: REALISTIC MODEL

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

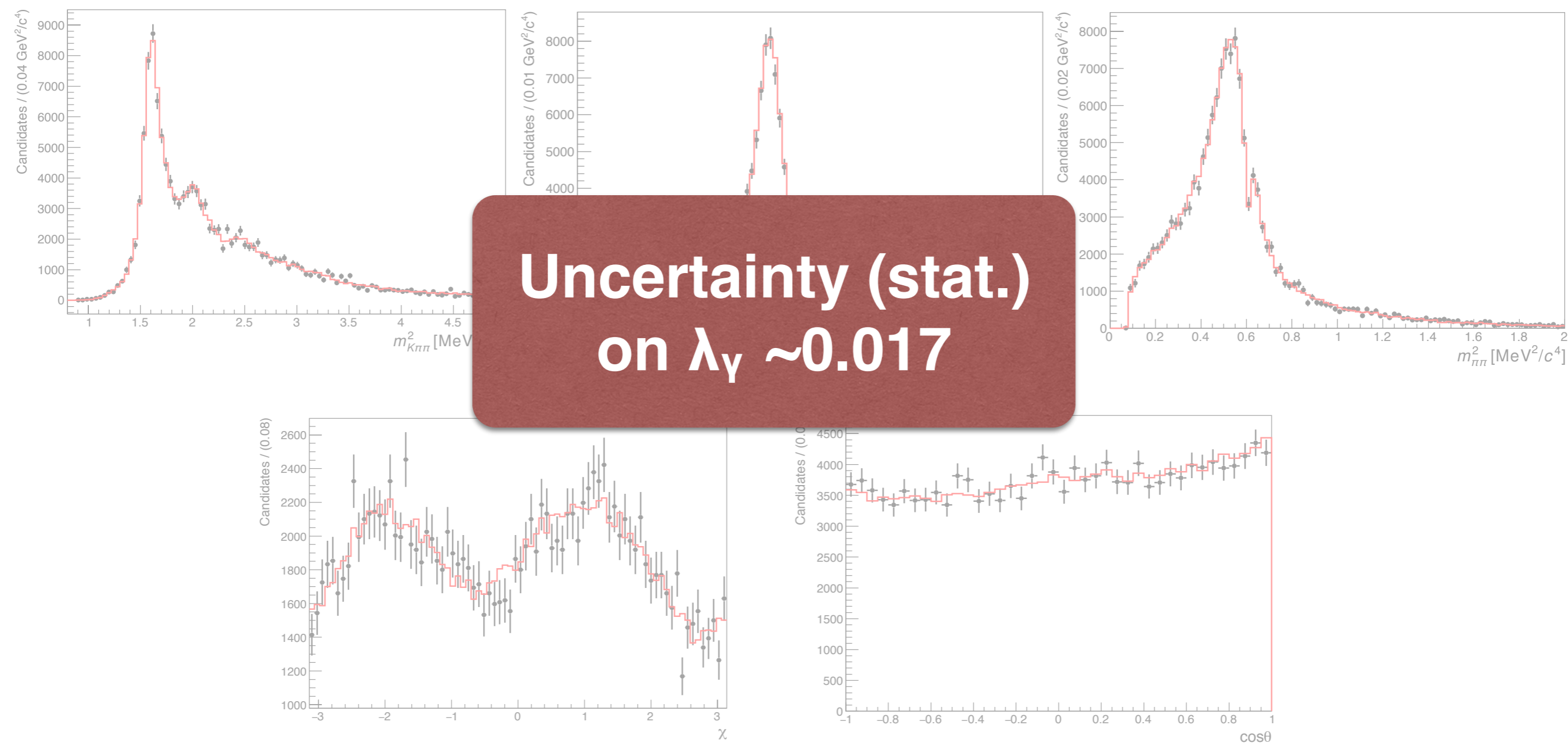
- Fits converge, pulls mostly unbiased, with unit width
- Sample fit projection shown below:



$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$: REALISTIC MODEL

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

- Fits converge, pulls mostly unbiased, with unit width
 - Pull width for λ_γ slightly greater than unity, correct average uncertainty from fit to compensate



$B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$: REALISTIC MODEL

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

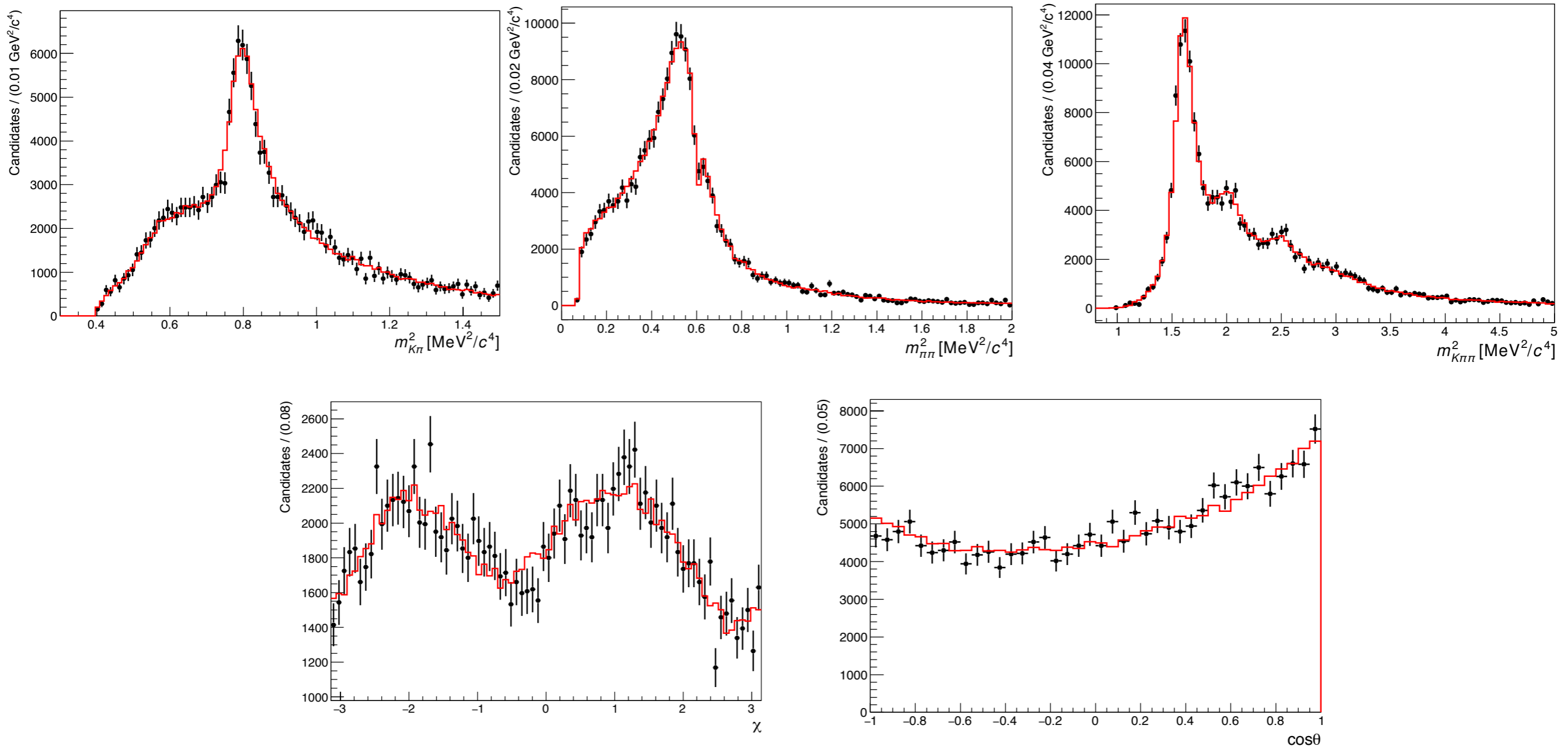
- Build analogous model for $B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$ decays
- Generate and fit 100 data sets with 10000 events each

J^P	Amplitude k	a_k	ϕ_k	Fraction (%)
1 ⁺	$K_1(1270)^0 \rightarrow K^*(892)^0 \pi^0$ [S-wave]	1(fixed)	0 (fixed)	8.0
	$K_1(1270)^0 \rightarrow K^*(892)^+ \pi^-$ [S-wave]	1.01	0.00	8.0
	$K_1(1270)^0 \rightarrow K^*(892)^+ \pi^-$ [D-wave]	0.98	-1.74	0.3
	$K_1(1270)^0 \rightarrow K^*(892)^0 \pi^0$ [D-wave]	0.99	-1.74	0.3
	$K_1(1270)^0 \rightarrow K^+ \rho(770)^-$	2.86	-0.91	39.7
	$K_1(1400)^0 \rightarrow K^*(892)^+ \pi^-$	0.60	-0.76	3.8
	$K_1(1400)^0 \rightarrow K^*(892)^0 \pi^0$	0.59	-0.76	3.8
1 ⁻	$K^*(1410)^0 \rightarrow K^*(892)^+ \pi^-$	0.11	0.00	3.9
	$K^*(1410)^0 \rightarrow K^*(892)^0 \pi^0$	0.11	0.00	3.9
	$K^*(1680)^0 \rightarrow K^*(892)^+ \pi^-$	0.05	0.44	1.7
	$K^*(1680)^0 \rightarrow K^*(892)^0 \pi^0$	0.05	0.44	1.7
	$K^*(1680)^0 \rightarrow K^+ \rho(770)^-$	0.06	1.40	2.4
2 ⁺	$K_2^*(1430)^0 \rightarrow K^*(892)^+ \pi^-$	0.27	0.00	2.3
	$K_2^*(1430)^0 \rightarrow K^*(892)^0 \pi^0$	0.27	0.00	2.3
	$K_2^*(1430)^0 \rightarrow K^+ \rho(770)^-$	0.63	1.80	8.9
2 ⁻	$K_2(1580)^0 \rightarrow K^*(892)^+ \pi^-$	0.49	2.88	2.2
	$K_2(1580)^0 \rightarrow K^*(892)^0 \pi^0$	0.49	2.88	2.2
	$K_2(1580)^0 \rightarrow K^+ \rho(770)^-$	0.54	2.44	3.2
	$K_2(1770)^0 \rightarrow K^*(892)^+ \pi^-$	0.35	0.00	1.5
	$K_2(1770)^0 \rightarrow K^*(892)^0 \pi^0$	0.35	0.00	1.5
	$K_2(1770)^0 \rightarrow K^+ \rho(770)^-$	0.11	2.53	0.2
	$K_2(1770)^0 \rightarrow K_2^*(1430)^+ \pi^-$	0.07	-2.06	0.3
	$K_2(1770)^0 \rightarrow K_2^*(1430)^0 \pi^0$	0.07	-2.06	0.3

$B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$: REALISTIC MODEL

[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

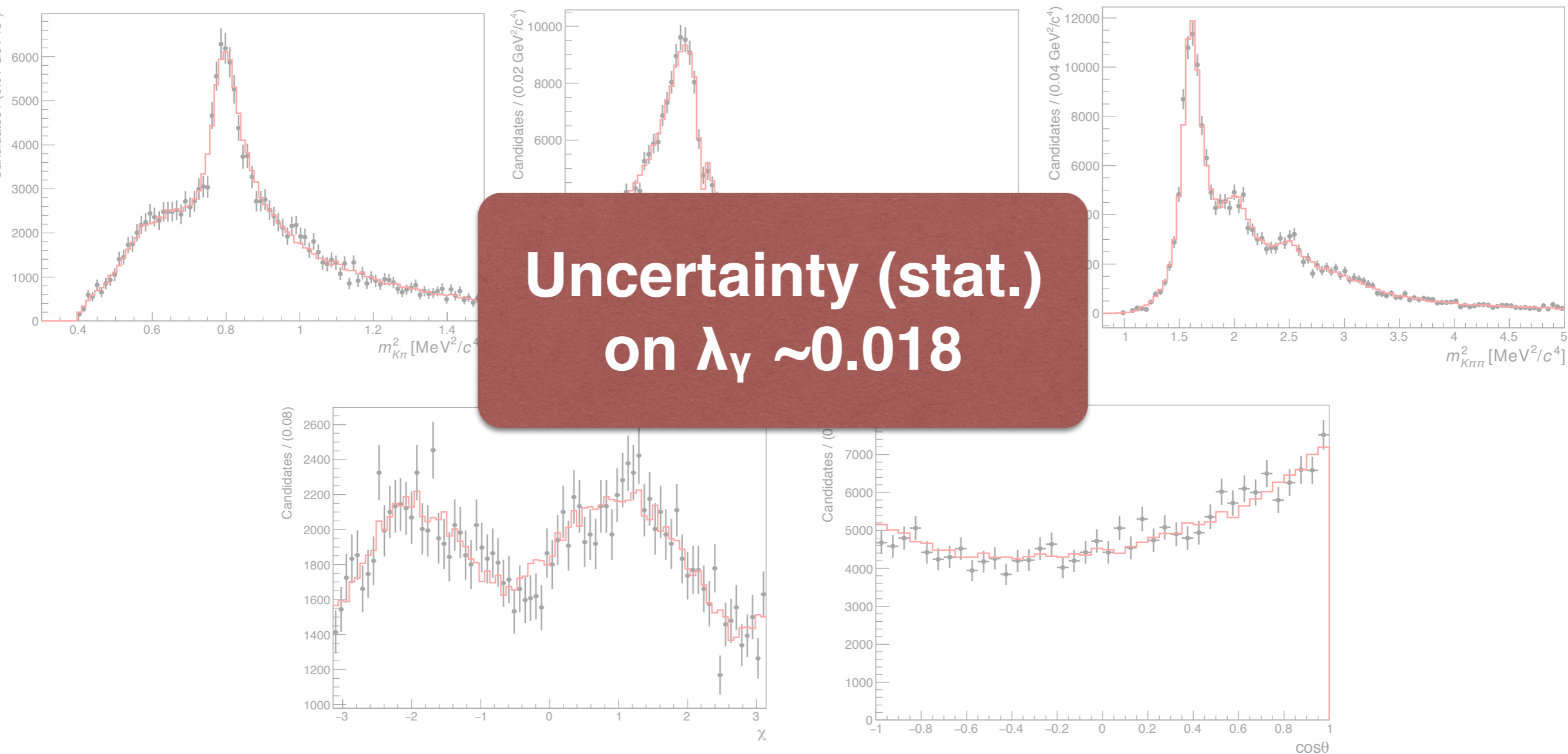
- Fits converge, pulls mostly unbiased, with unit width
- Sample fit projection shown below:



$B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$: REALISTIC MODEL

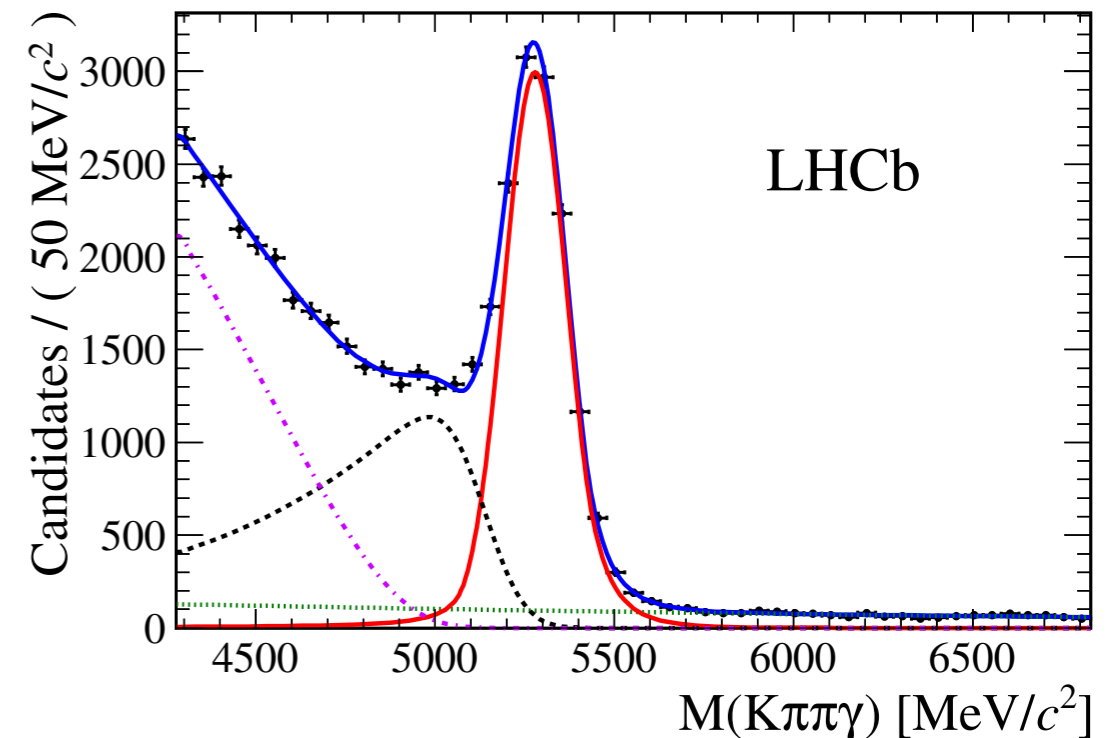
[arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

- Fits converge, pulls mostly unbiased, with unit width
- Pull width for λ_γ slightly greater than unity, correct average uncertainty from fit to compensate



PROSPECTS FOR PHOTON POLARISATION MEASUREMENTS

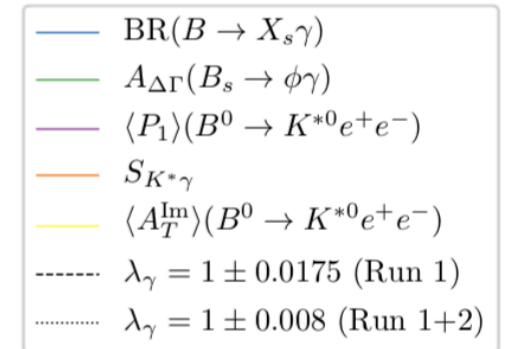
- ◆ Sensitivity to photon polarisation parameter studied for both simplified and realistic signal-only models
 - ◆ The method is sensitive to both simplified and realistic models with reasonable values of $(r, \Delta\phi, \lambda_\gamma)$ in the ideal, signal-only case
- ◆ LHCb has collected $\sim 6.4 \text{ fb}^{-1}$ of data during 2011, 2012, 2016 and 2017
 - ◆ Expect around 50000 $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ signal events
 - ◆ Signal-only toy studies show an expected statistical sensitivity of ~ 0.01
- ◆ Full analysis ongoing with LHCb Run 1 + partial Run 2 data set
 - ◆ Event selection + mass fits finalised
 - ◆ Pieces in place to proceed with amplitude analysis with data
- ◆ Belle-II could expect around 10000 $B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$ signal events with $\sim 5 \text{ ab}^{-1}$
 - ◆ Signal-only toy studies show an expected statistical sensitivity of ~ 0.018



INTERPRETATION IN TERMS OF WILSON COEFFICIENTS

- ◆ Sensitivity to photon polarisation parameter from these decays provides complementary information to that from complementary analyses

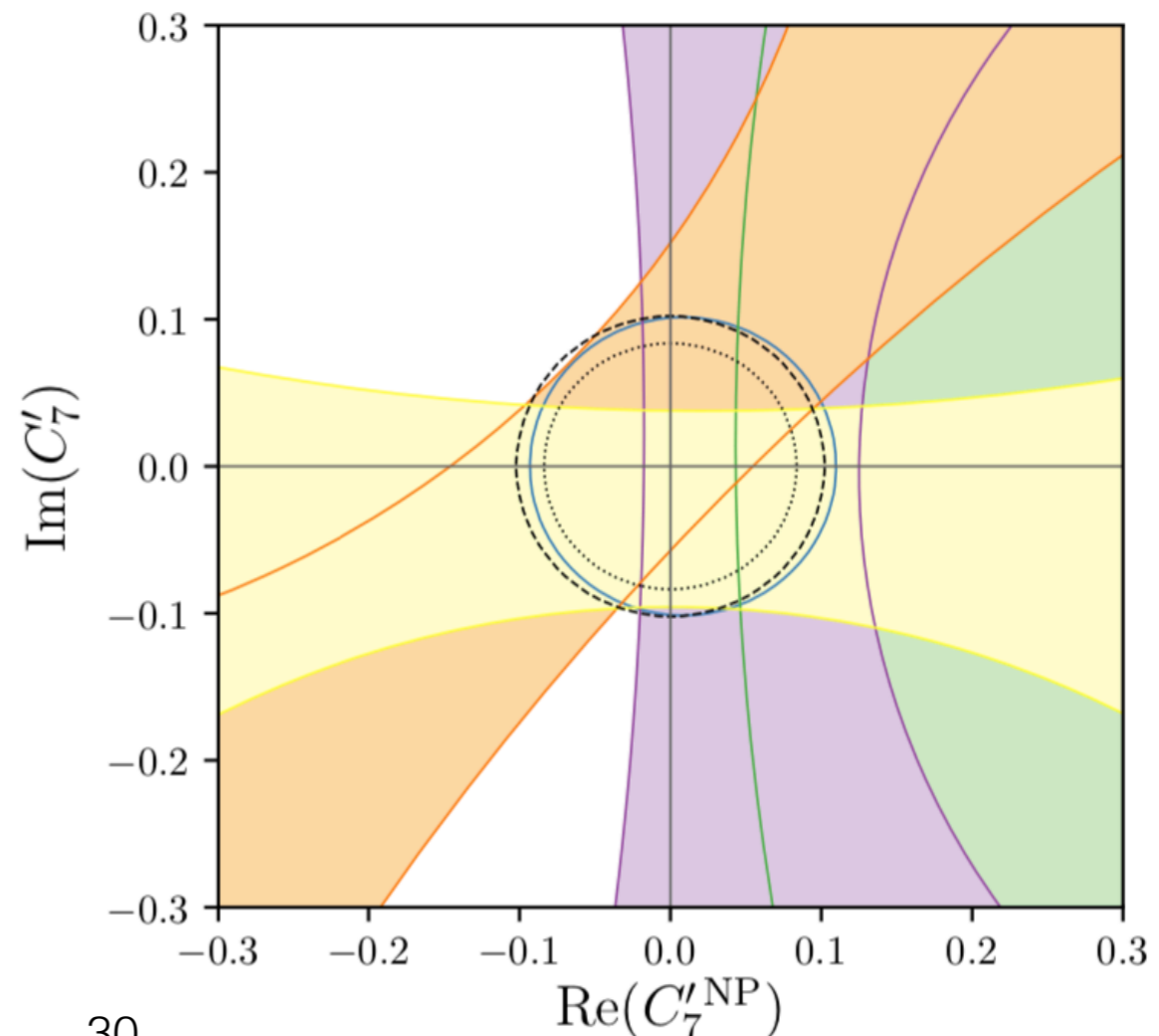
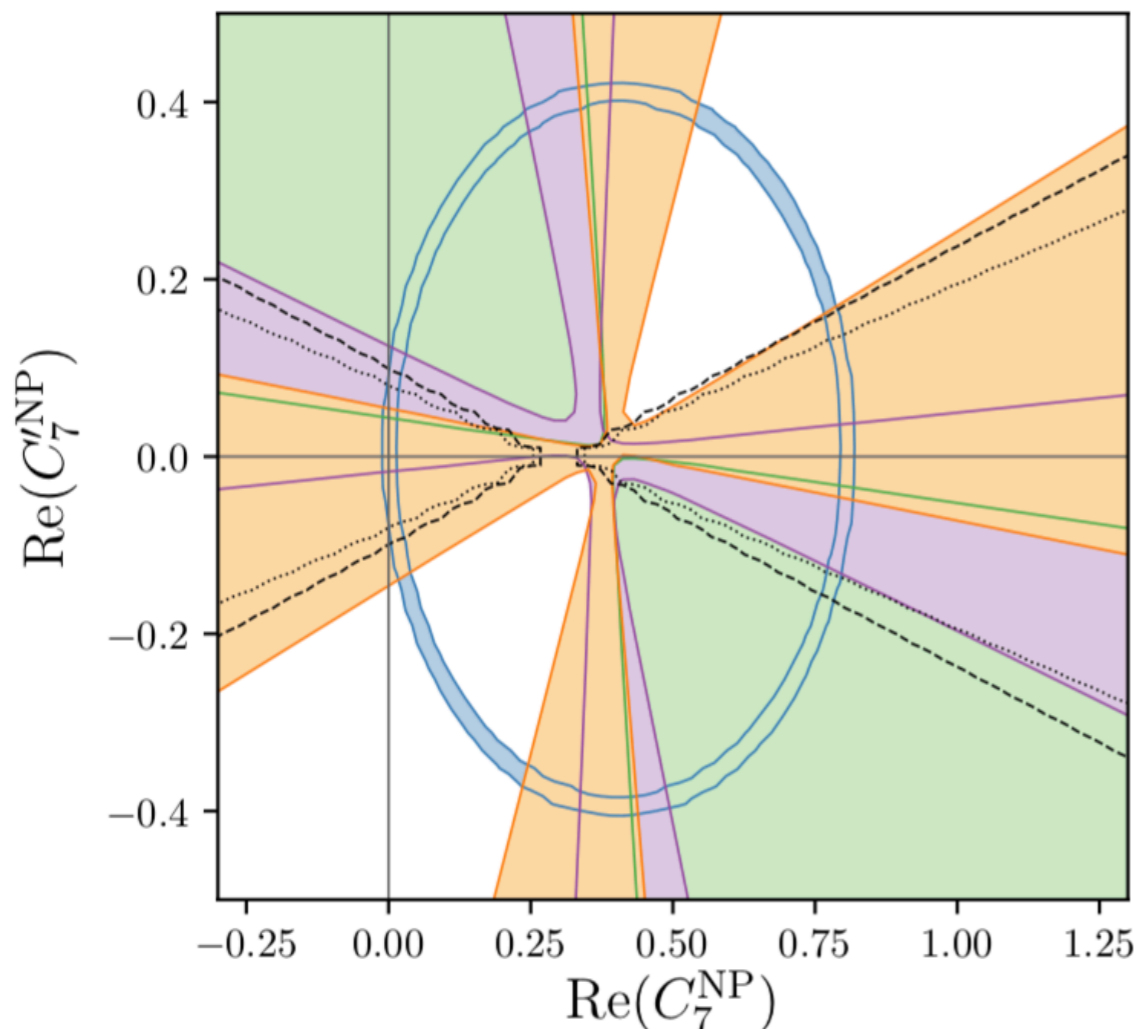
- ◆ In particular, could help resolve ambiguities in $\text{Re}(C_7')$



A. Puig

flavio: D. Straub et al.

[JHEP 04 \(2017\) 027](#)



INTERPRETATION IN TERMS OF WILSON COEFFICIENTS

- ◆ However, interpretation may not be quite as straightforward
 - ◆ Reminder - weak decay amplitudes written in terms of the Wilson coefficients

$$\begin{pmatrix} c_{\text{R}}^i \\ c_{\text{L}}^i \end{pmatrix} = -\frac{4G_{\text{F}}}{\sqrt{2}} V_{tb} V_{ts}^* \begin{pmatrix} C_7^{\text{eff}} g^i(0) \\ C_7' P_i (-1)^{J_i-1} g^i(0) \end{pmatrix}$$

- ◆ Could define a single photon polarisation parameter for all decay modes

$$\lambda_{\gamma}^i = \frac{|C_{7\text{R}}|^2 - |C_{7\text{L}}|^2}{|C_{7\text{R}}|^2 + |C_{7\text{L}}|^2} \equiv \lambda_{\gamma}$$

INTERPRETATION IN TERMS OF WILSON COEFFICIENTS

- ♦ Could have non-negligible contributions from other operators

$$\begin{pmatrix} c_R^i \\ c_L^i \end{pmatrix} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \begin{pmatrix} C_7^{\text{eff}} g^i(0) + h_R^i \\ C_7' P_i (-1)^{J_i-1} g^i(0) + h_L^i \end{pmatrix}$$

- ♦ How could we deal with this?

- ♦ Theoretical estimation of $h_{R/L}$?

- ♦ Treat them as nuisance parameters in the analysis

- ♦ Compute the photon polarisation parameter in different invariant mass bins

- ♦ Check compatibility between channels with different contributions, like $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ and $B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$ decays

INTERPRETATION IN TERMS OF WILSON COEFFICIENTS

- ♦ Could have non-negligible contributions from other operators

$$\begin{pmatrix} c_R^i \\ c_L^i \end{pmatrix} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \begin{pmatrix} C_7^{\text{eff}} g^i(0) + h_R^i \\ C_7' P_i (-1)^{J_i-1} g^i(0) + h_L^i \end{pmatrix}$$

- ♦ How could we deal with this?

- ♦ Theoretical estimation of $h_{R/L}$?

- ♦ Treat them as nuisance parameters in the analysis

→ **technically challenging**

- ♦ Compute the photon polarisation parameter in different invariant mass bins

→ **difficult due to long tails of (broad) high mass resonances**

- ♦ Check compatibility between channels with different contributions, like $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ and $B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$ decays

✓ **For now, the way to go**

SUMMARY

- ◆ PDF that describes $B \rightarrow K\pi\pi\gamma$ decays implemented in a generator-fitter framework
 - ◆ Generated single amplitude MC samples validated (where possible) against EvtGen
- ◆ Sensitivity to photon polarisation parameter studied for both simplified and realistic signal-only models
 - ◆ Ideal case: signal-only samples with perfect efficiency
 - ◆ The method is sensitive to both simplified and realistic models with reasonable values of $(r, \Delta\phi, \lambda_\gamma)$
- ◆ Amplitude analysis method can be used to study both charged and neutral decay modes
- ◆ Analysis of charged decay mode with LHCb data sample ongoing

BACKUP

PARAMETRISING FCNC TRANSITIONS

Effective Hamiltonian for radiative $b \rightarrow s$ gamma transitions:

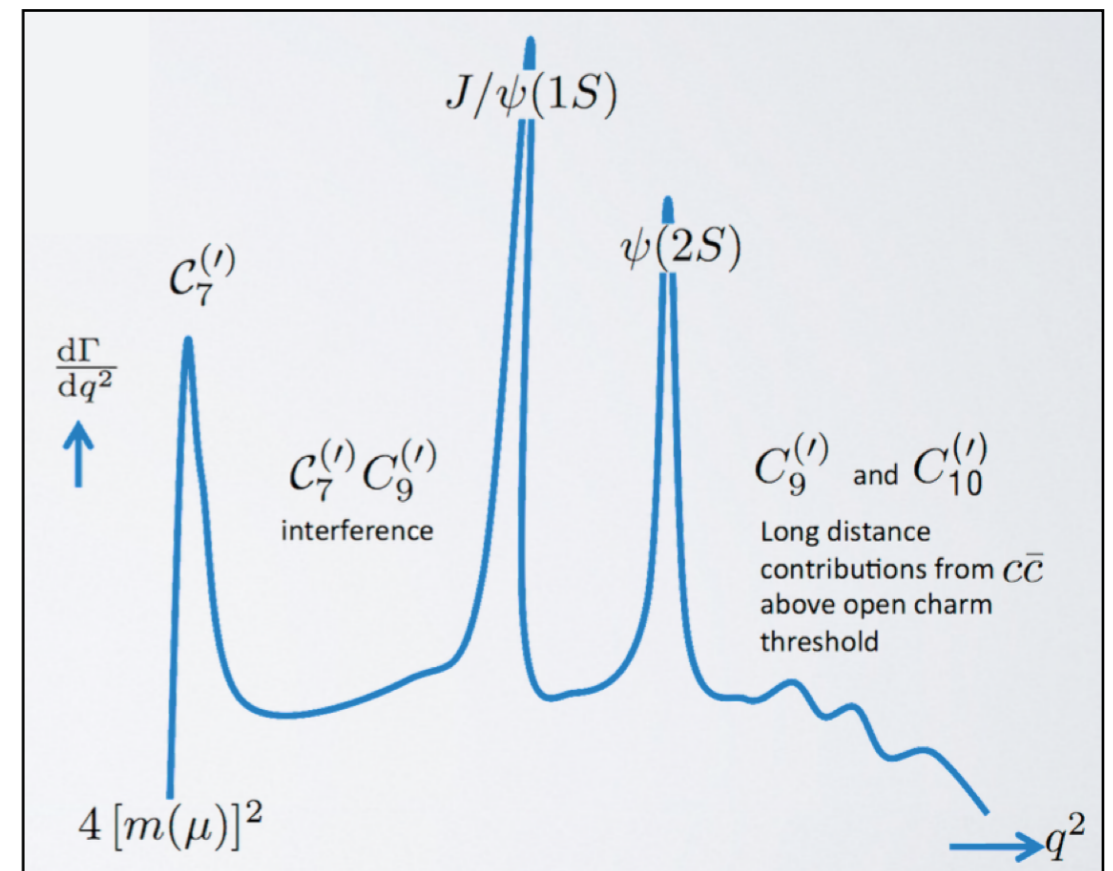
$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left[\underbrace{C_i(\mu) \mathcal{O}_i(\mu)}_{\text{left handed}} + \underbrace{C'_i(\mu) \mathcal{O}'_i(\mu)}_{\text{right handed (suppressed in the SM)}} \right]$$

Operators (\mathcal{O}_i) - long-distance effects (non-perturbative)

Wilson coefficients (C_i) - perturbative, short-distance physics

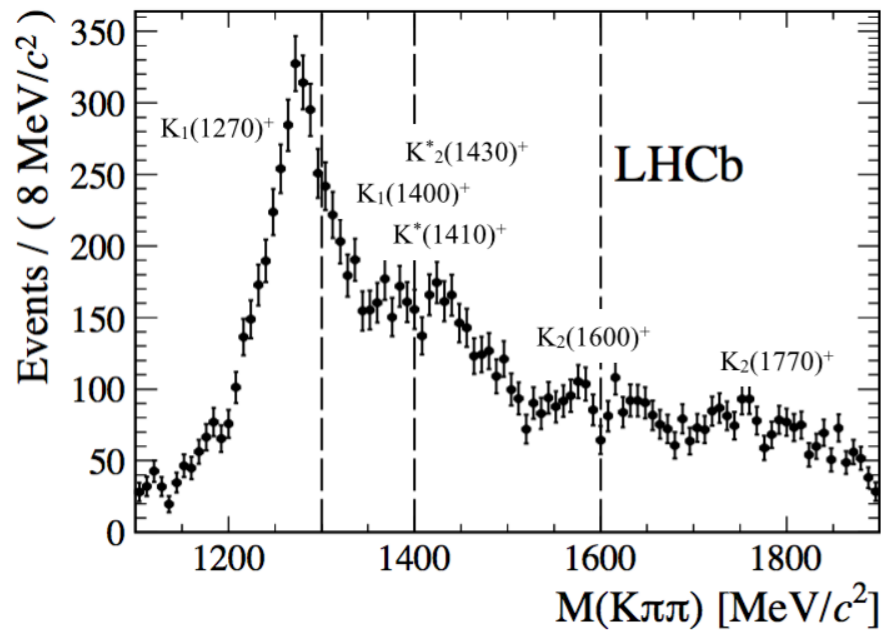
$i=1, 2$	Tree
$i=3-6, 8$	Gluon penguin
$i=7$	Photon penguin
$i=9, 10$	Electroweak penguin
$i=S$	Higgs (scalar) penguin
$i=P$	Pseudoscalar penguin

different regions of q^2 probe
different processes

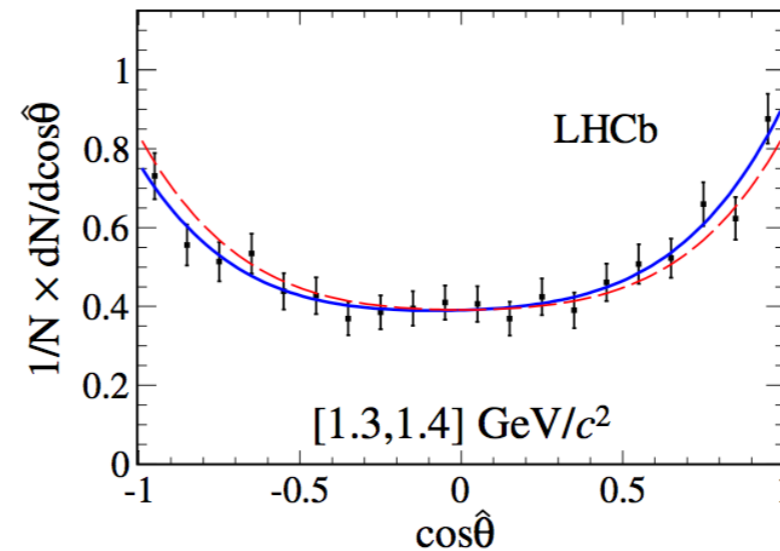
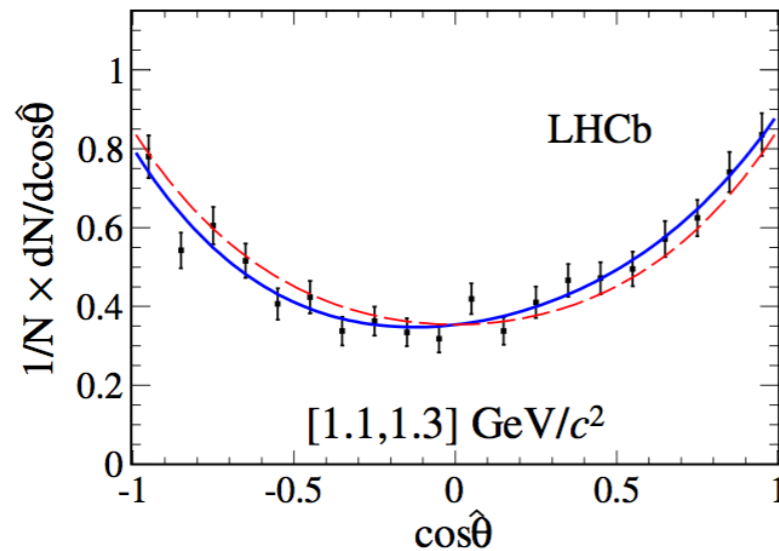


MEASURING THE UP-DOWN ASYMMETRY

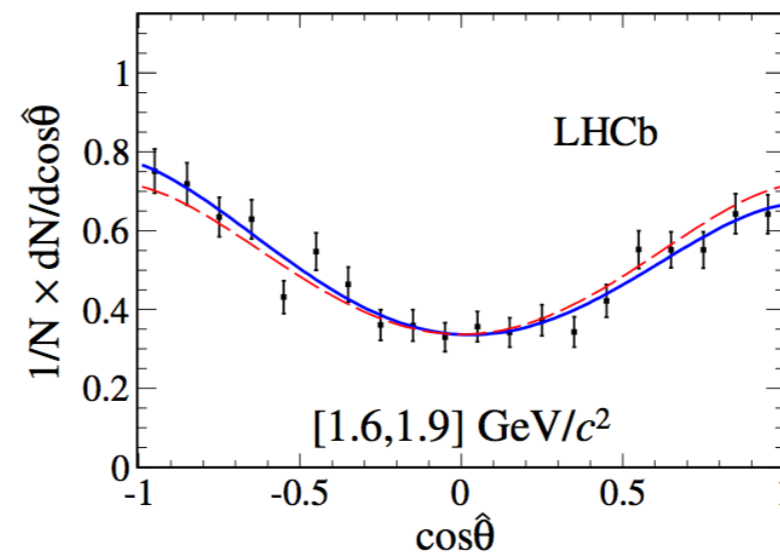
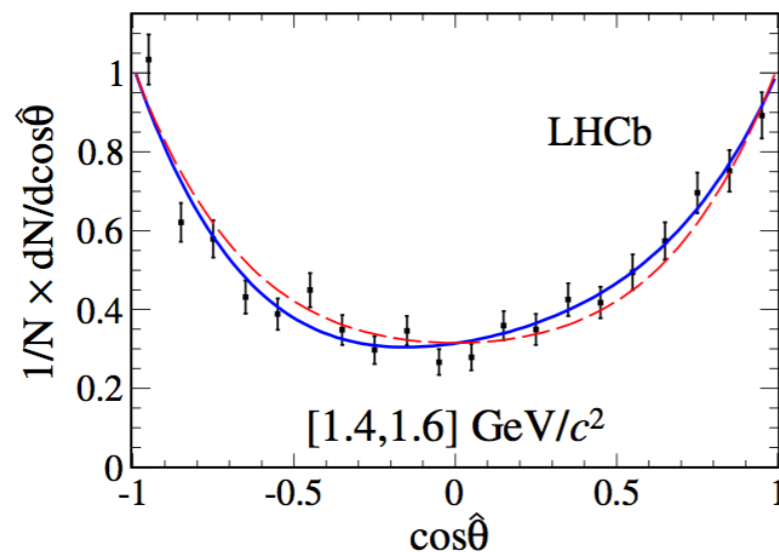
[PRL 112, 161801](#)



- 14,000 events selected from full LHCb Run 1 dataset
- Obtained background-subtracted Kππ mass spectrum
- Cosθ fit performed in four m(Kππ) bins



— Fit
- - - Fit without asymmetry
+ Data



TWO-AMPLITUDE MODEL

Decay rate for a system with a single 1+ resonance:

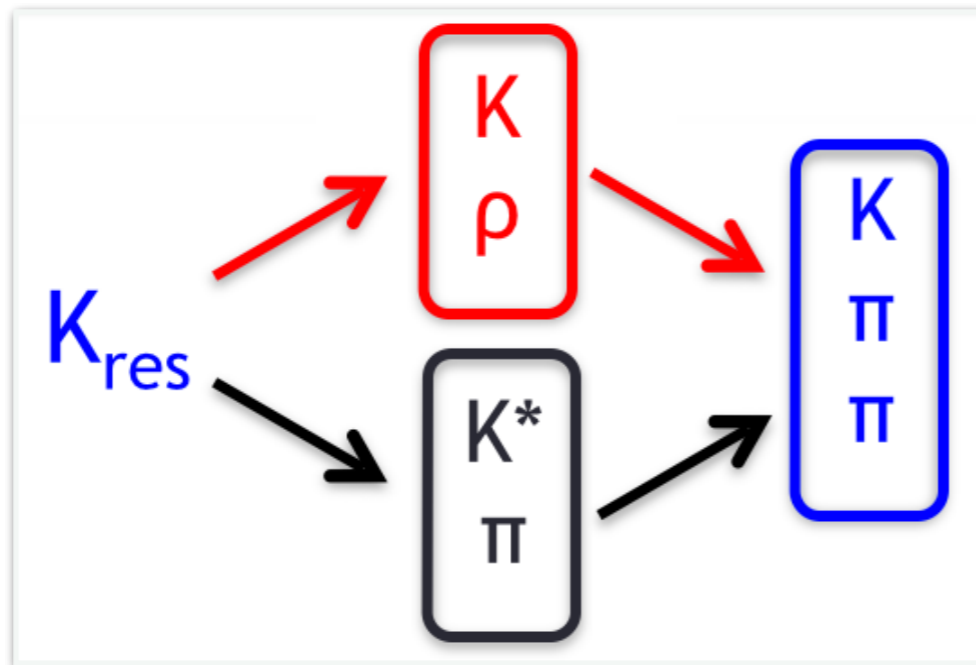
[Gronau et al, PRD66 (2002) 054008]

$$\frac{d\Gamma(B \rightarrow K\pi\pi\gamma)}{ds ds_{13} ds_{23} d\cos\theta} \propto \frac{1}{2} |\vec{\mathcal{J}}|^2 (1 + \cos^2\theta) + \lambda_\gamma \cos\theta \operatorname{Im}[\vec{n} \cdot (\vec{\mathcal{J}} \times \vec{\mathcal{J}}^*)]$$

invariant mass dependencies

where $s \rightarrow m^2(K\pi\pi)$, $s_{13} \rightarrow m^2(K\pi)$ and $s_{23} \rightarrow m^2(\pi\pi)$

Interferences
between decay
modes



$B \rightarrow K\pi\pi\gamma$ AMPLITUDE ANALYSIS: METHOD

Notes on the normalisation integral and its implementation in MINT [arxiv:1902.09201](https://arxiv.org/abs/1902.09201)

- Makes use of ‘importance sampling’, i.e. sample the function more frequently in regions where its value is large (to minimise uncertainties)
 - Use a mix of approximate signal events with a small amount of phase space events
- Explicit functional form of the efficiency not needed (non-trivial to parametrise in five dimensions)
- Often need a large number of official (full-chain) MC simulated events in order to obtain good precision on the fit
- Generation of normalisation MC toy events + computation of integrals for more complicated models takes up the majority of CPU time in MINT

$$\int \xi(\mathbf{x}) \mathcal{P}_s(\mathbf{x}) \phi_4(\mathbf{x}) \, d\mathbf{x} = \frac{I_{\text{gen}}}{N_{\text{sel}}} \sum_j^{N_{\text{sel}}} \frac{\mathcal{P}_s(\mathbf{x}_j)}{\mathcal{P}_{\text{gen}}(\mathbf{x}_j)}$$

More details on the (practical) challenges of amplitude analyses in Albert’s talk

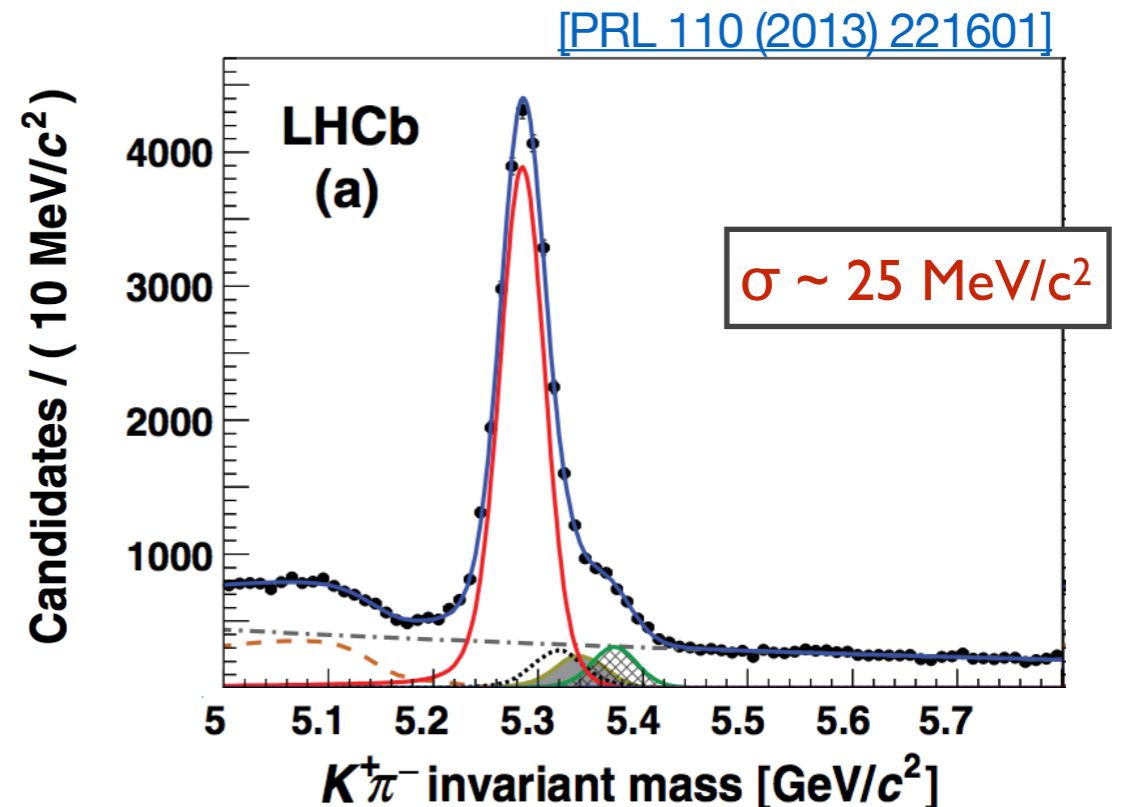
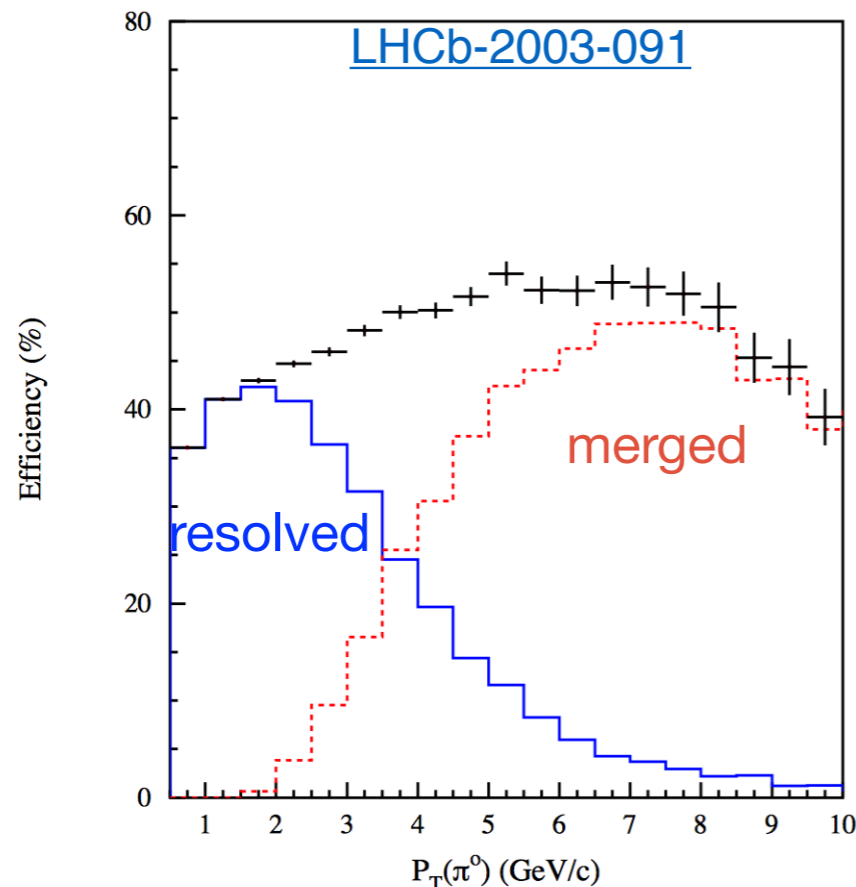
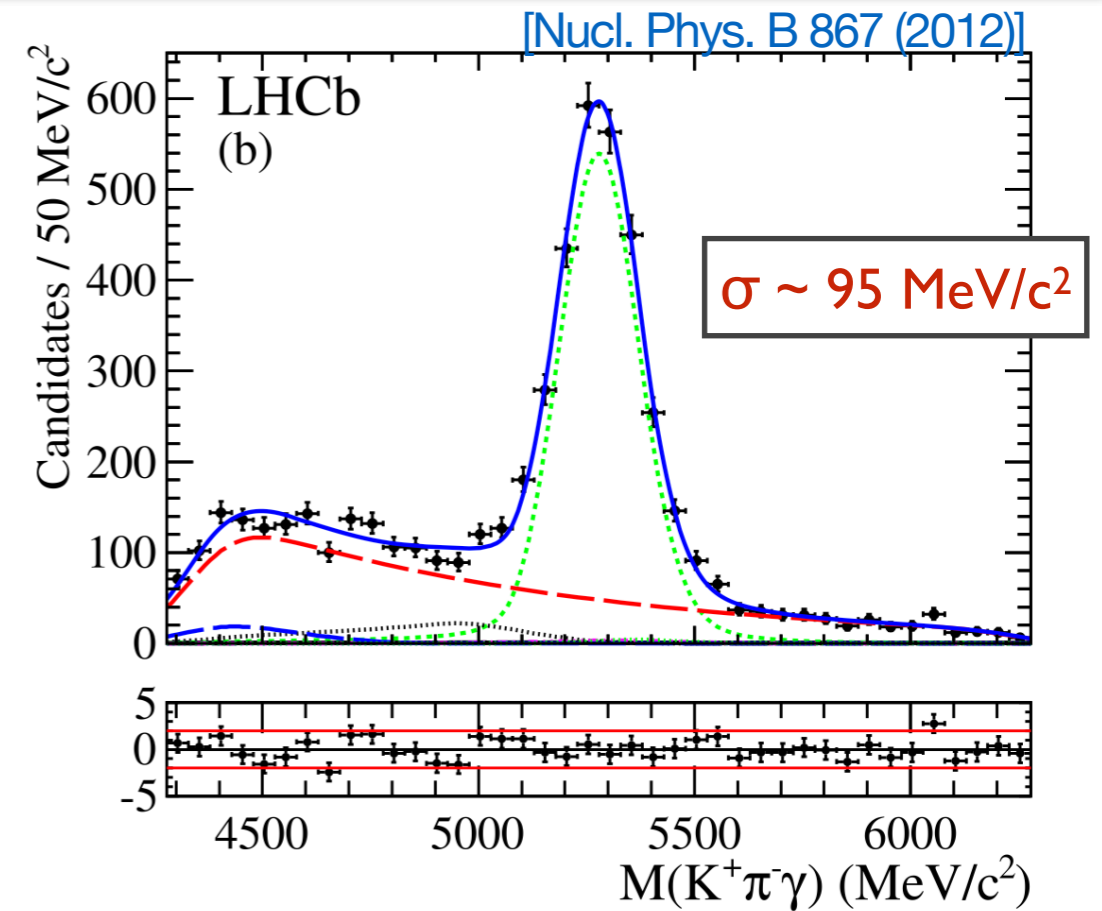
PHOTON POLARISATION IN $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ DECAYS

Fit pull values for realistic model:

Amplitude k	Magnitude a_k		Phase ϕ_k	
	μ_{pull}	σ_{pull}	μ_{pull}	σ_{pull}
$K_1(1270)^+ \rightarrow K^*(892)^0 \pi^+$ [D-wave]	0.12 ± 0.10	0.97 ± 0.07	-0.01 ± 0.10	1.02 ± 0.07
$K_1(1270)^+ \rightarrow K^+ \rho(770)^0$	0.08 ± 0.09	0.91 ± 0.06	0.02 ± 0.11	1.08 ± 0.07
$K_1(1400)^+ \rightarrow K^*(892)^0 \pi^+$	-0.44 ± 0.09	0.95 ± 0.06	0.87 ± 0.10	1.06 ± 0.07
$K^*(1410)^+ \rightarrow K^*(892)^0 \pi^+$	-0.45 ± 0.09	0.94 ± 0.06	0.06 ± 0.10	1.04 ± 0.07
$K^*(1680)^+ \rightarrow K^*(892)^0 \pi^+$	0.04 ± 0.09	0.94 ± 0.06	0.02 ± 0.10	1.08 ± 0.07
$K^*(1680)^+ \rightarrow K^+ \rho(770)^0$	-0.02 ± 0.11	1.11 ± 0.07	0.02 ± 0.10	1.05 ± 0.07
$K_2^*(1430)^+ \rightarrow K^*(892)^0 \pi^+$	0.51 ± 0.10	1.07 ± 0.07	0.45 ± 0.09	0.86 ± 0.06
$K_2^*(1430)^+ \rightarrow K^+ \rho(770)^0$	0.36 ± 0.09	0.98 ± 0.07	-0.01 ± 0.09	0.94 ± 0.06
$K_2(1580)^+ \rightarrow K^*(892)^0 \pi^+$	-0.39 ± 0.10	1.03 ± 0.07	-0.06 ± 0.11	1.10 ± 0.07
$K_2(1580)^+ \rightarrow K^+ \rho(770)^0$	0.04 ± 0.09	0.90 ± 0.06	0.14 ± 0.10	0.97 ± 0.07
$K_2(1770)^+ \rightarrow K^*(892)^0 \pi^+$	0.08 ± 0.11	1.11 ± 0.07	-0.10 ± 0.12	1.21 ± 0.08
$K_2(1770)^+ \rightarrow K^+ \rho(770)^0$	-0.13 ± 0.10	0.97 ± 0.06	-0.04 ± 0.09	0.97 ± 0.06
$K_2(1770)^+ \rightarrow K_2^*(1430)^0 \pi^+$	0.17 ± 0.10	1.05 ± 0.07	0.05 ± 0.10	1.01 ± 0.07

EXPERIMENTAL CHALLENGES

- ◆ Mass resolution dominated by photon reconstruction
 - $\sigma \sim 95 \text{ MeV}/c^2$ for $B \rightarrow K^* \gamma$ decays, compared to $\sim 25 \text{ MeV}/c^2$ for $B \rightarrow K \pi$ decays.
- ◆ Backgrounds:
 - Above transverse energies of 4 GeV, $\pi^0 \rightarrow \gamma \gamma$ reconstructed as a single cluster in the calorimeter
 - Combinatorial: $O(10)$ reconstructed photons per event



EXPERIMENTAL CHALLENGES

Without analysis improvements, many analyses would be systematics-limited by Run 5

Primary known/expected sources of systematic uncertainty:

- ◆ Partially reconstructed background, due to large invariant mass resolution
 - Correlation between decay time and reconstructed mass in $B_s \rightarrow \phi \gamma$ decays
 - Uncertainty in background modeling in A_{CP} and branching fraction measurements
 - Effects on angular distributions in $K\pi\pi\gamma$ decays
- ◆ Detector effects
 - Decay time resolution for C,S measurements in tagged $B_s \rightarrow \Phi \gamma$ analysis
 - Detection asymmetry in A_{CP} measurement
- ◆ Modeling of acceptances
 - Main source of uncertainty for $\Lambda_b \rightarrow \Lambda^0 \gamma$ angular analysis