Detector and Magnet Geometry for Spectrometer

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Beam position after the magnet

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R[Ee_] = Sqrt[Ee^2 - me^2] / (clight * B)
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$$\frac{\sqrt{Ee^2 - me^2}}{B\,clight}$$

```
S[Ee_] = R[Ee] - Sqrt[R[Ee]^2 - zm^2]
```

$$\frac{\sqrt{Ee^2 - me^2}}{B\,clight} - \sqrt{\frac{Ee^2 - me^2}{B^2\,clight^2} - zm^2}$$

```
sint[Ee_] = zm / R[Ee]
```

$$\frac{B\,clight\,zm}{\sqrt{Ee^2 - me^2}}$$

```
tgt[Ee_] = sint[Ee] / Sqrt[1 - sint[Ee]^2]
```

$$\frac{B\,clight\,zm}{\sqrt{Ee^2 - me^2}\,\sqrt{1 - \frac{B^2\,clight^2\,zm^2}{Ee^2 - me^2}}}$$

```
xd [Ee_] = S[Ee] + zd * tgt[Ee]
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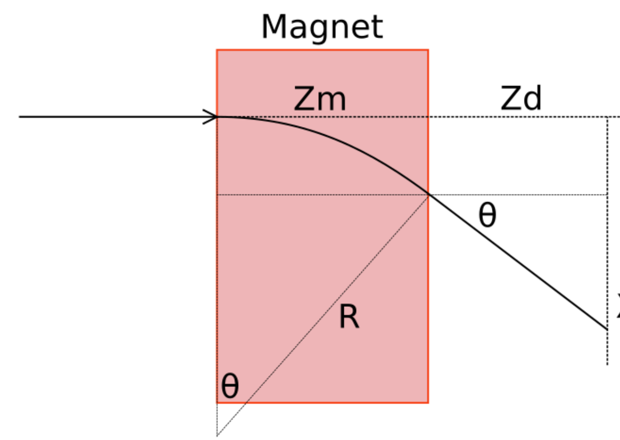
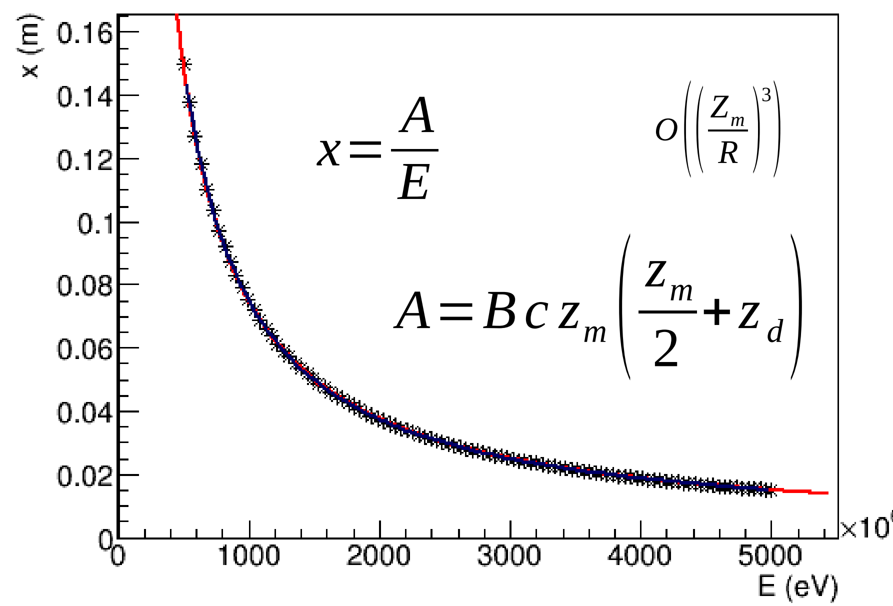
$$\frac{\sqrt{Ee^2 - me^2}}{B\,clight} - \sqrt{\frac{Ee^2 - me^2}{B^2\,clight^2} - zm^2} + \frac{B\,clight\,zd\,zm}{\sqrt{Ee^2 - me^2}\,\sqrt{1 - \frac{B^2\,clight^2\,zm^2}{Ee^2 - me^2}}}$$



$$x(E) = \frac{B c z_m}{E}\left(\frac{z_m}{2} + z_d\right)$$

$$\frac{dx(E)}{dE} = -D$$

1. $z_d = z_d(E)$
2. $z_m = z_m(E)$




```
Minimizer is Minuit / Migrad
Chi2                      =   6.30474e-08
NDf                       =           100
Edm                       =   3.34551e-21
NCalls                    =            15
p0                        =   7.4997e+07   +/-    3870.76
B*c*Zm*(Zm/2+Zd): 7.49481e+07
```

1. Zd = Zd(E)

$$x(E)=\frac{B c\, z_m}{E}\left(\frac{z_m}{2}+z_d\right)$$

Instead of requirement $\frac{dx(E)}{dE}=-D$

one can use also $x(E)=x_0+D(E_0-E)$

and also find $z_{d/m}=z_{d/m}(E)$ from the algebraic equation

```
xd[Ee_] = zm / R[Ee] * (zm / 2 + zd[Ee])
```

$$\frac{\text{B clight zm}\left(\frac{\text{zm}}{2}+\text{zd[Ee]}\right)}{\text{Ee}}$$

```
dxd[Ee_] = ∂Ee xd[Ee]
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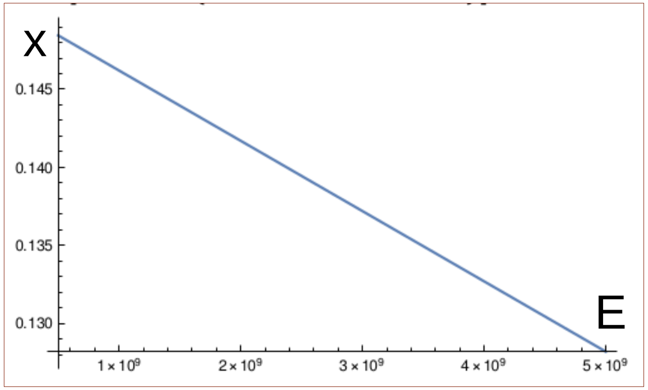
$$-\frac{\text{B clight zm}\left(\frac{\text{zm}}{2}+\text{zd[Ee]}\right)}{\text{Ee}^2}+\frac{\text{B clight zm zd}'\text{[Ee]}}{\text{Ee}}$$




```
zaeqsol = DSolve[{dxd[Ee] == -D, zd[E0] == z0}, zd, Ee]
```

$$\left\{\left\{\text{zd}\to\text{Function}\left[\{\text{Ee}\},\ \frac{2\,\text{D E0}^2\,\text{Ee}-2\,\text{D E0 Ee}^2+2\,\text{B clight Ee z0 zm}-\text{B clight E0 zm}^2+\text{B clight Ee zm}^2}{2\,\text{B clight E0 zm}}\right]\right\}\right\}$$

```
xx[Ee_] = Simplify[ xd[Ee] /. zaeqsol[[1]]]
```

$$\text{D (E0 - Ee)}+\frac{\text{B clight zm (2 z0 + zm)}}{\text{2 E0}} \rightarrow x_0$$

$$\Longrightarrow \quad x(E)=x_0+D(E_0-E)$$

Detector Shape

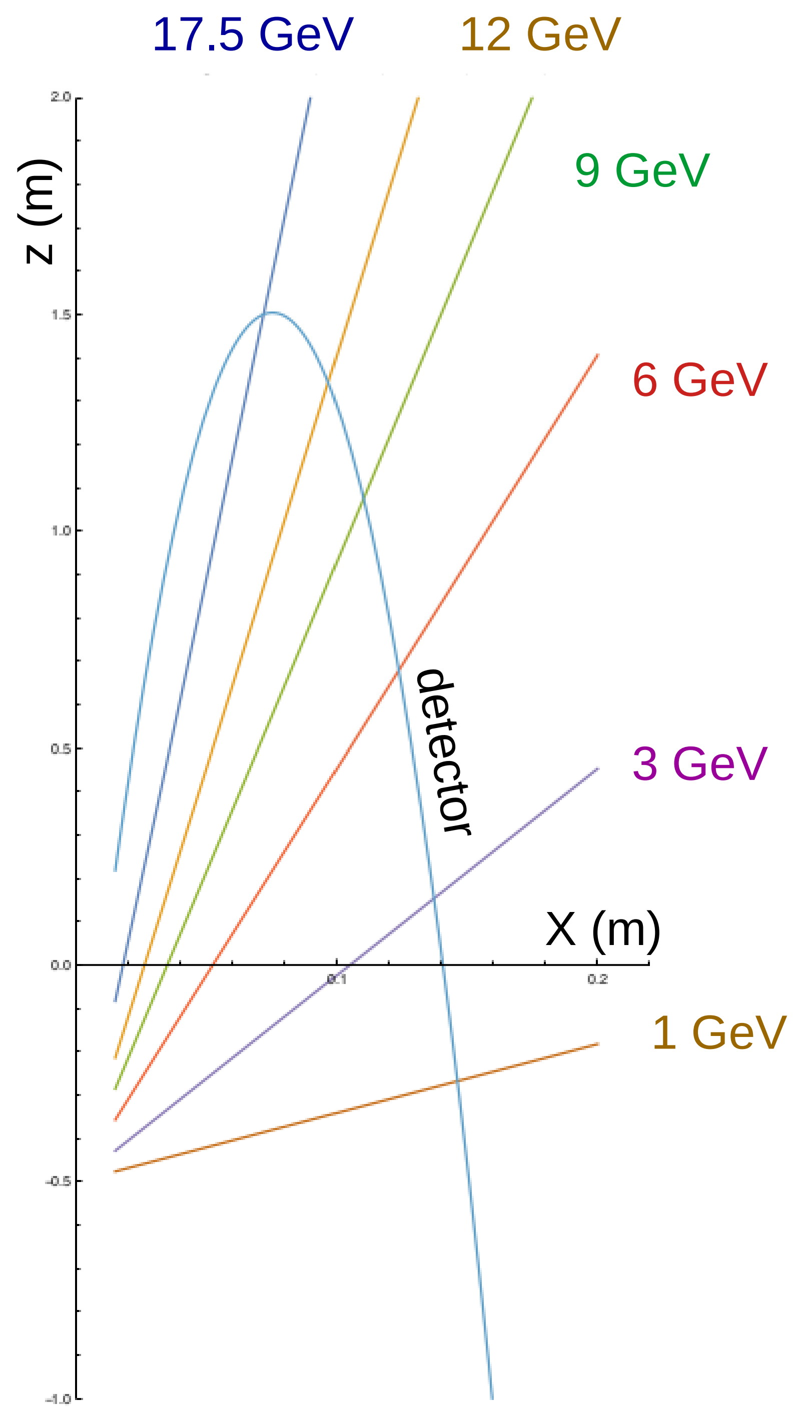
D, E_0 and z_0/x_0 are parameters which can be used to adjust the detector:

- D = -dX/dE – arbitrary constant essentially defines the distance from the magnet to the detector;
- E_0 is maximum expected energy in this case;
- $z_0=z(E_0)$ or $x_0=x(E_0)$;

Does not seem to be practical solution for the calorimeter:

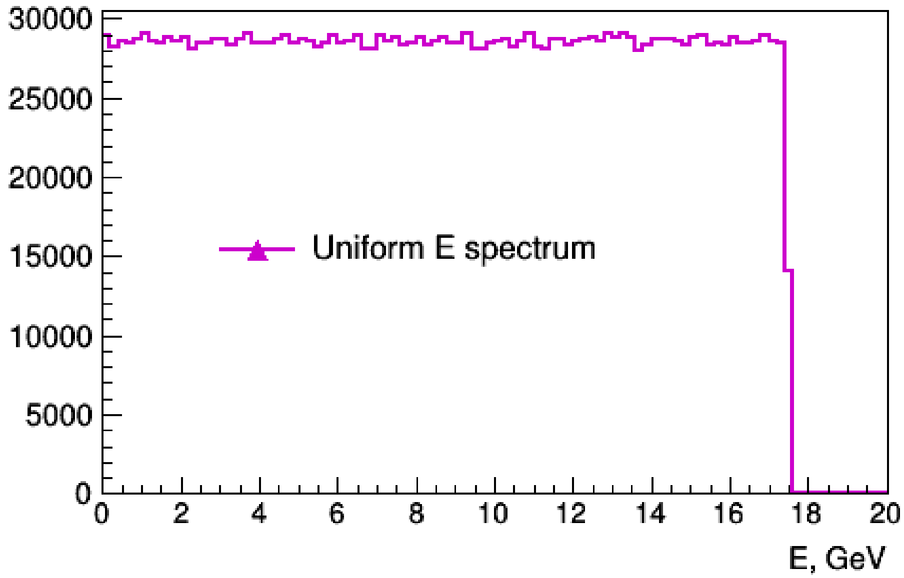
- Longer size compered to z=const;
- Track angle with respect to detector surface is rather acute.

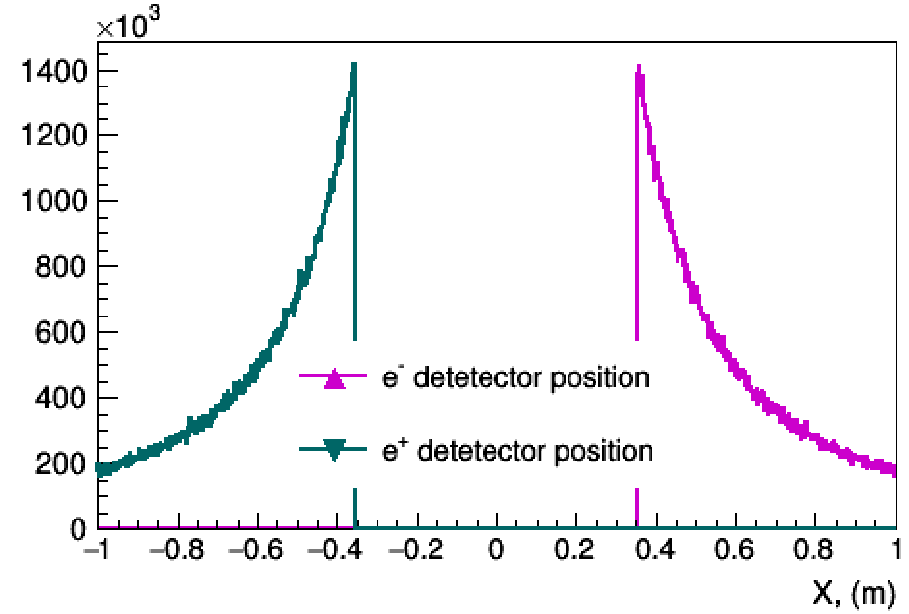
Might work for Cherenkov detectors.



2. $Zm = Zm(E)$

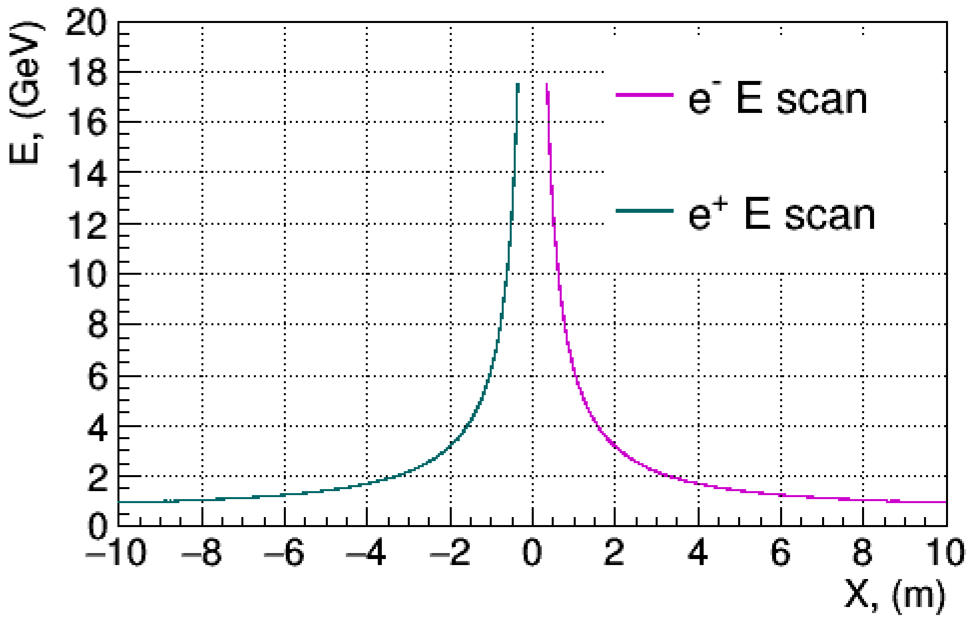
Position of e^-, e^+ in detector





Position of the particles can be determined using energy scan.

Drift_L	1.0	8.0
B_length	1.08	
B_field	2.24	



Position of electron with E = 1 GeV : 8.89636 m

Position of positron with E = 1 GeV : -8.89948 m

Position for E = 17.5 GeV : ±35.42 cm

Magnet Geometry

```
(%i3)   tg: d/sqrt(R^2-d^2);
        x: R-sqrt(R^2-d^2);
        L: (z-d)*tg+x;
```

(tg) $\frac{d}{\sqrt{R^2-d^2}}$

(x) $R-\sqrt{R^2-d^2}$

(L) $\frac{d\,(z-d)}{\sqrt{R^2-d^2}}-\sqrt{R^2-d^2}+R$

```
(%i4)   L1: (L-R)^2;
```

(L1) $\left(\frac{d\,(z-d)}{\sqrt{R^2-d^2}}-\sqrt{R^2-d^2}\right)^2$

```
(%i5)   R1: (a+b*R-R)^2;
```

(R1) $(R\,b+a-R)^2$

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(%i6)   L2: L1-R1;
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(L2) $\left(\frac{d\,(z-d)}{\sqrt{R^2-d^2}}-\sqrt{R^2-d^2}\right)^2-(R\,b+a-R)^2$

```
(%i7)   L3: ratsimp(L2*(d^2-R^2));
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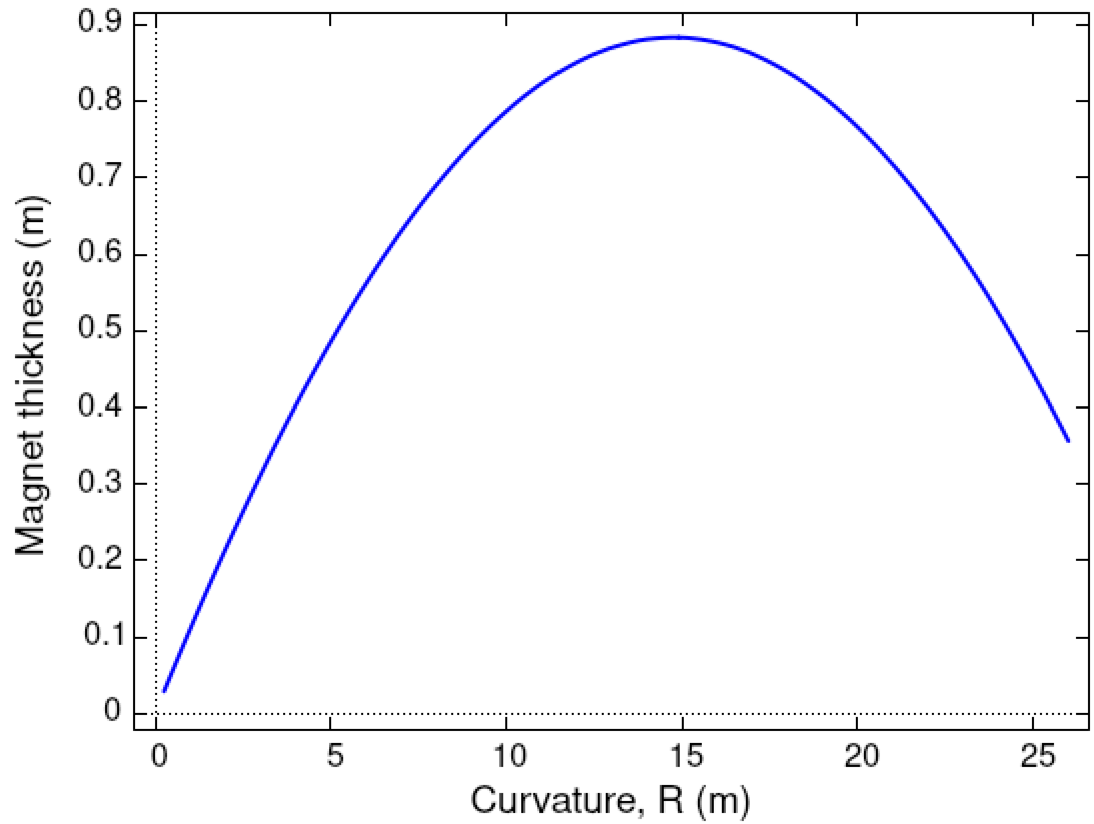
(L3) $-d^2\,z^2+2\,R^2\,d\,z+\left(-R^2\,b^2+\left(2\,R^2-2\,R\,a\right)b-a^2+2\,R\,a-R^2\right)d^2+R^4\,b^2+\left(2\,R^3\,a-2\,R^4\right)b+R^2\,a^2-2\,R^3\,a$

```
(%i8)   G: solve(L3, d);
```

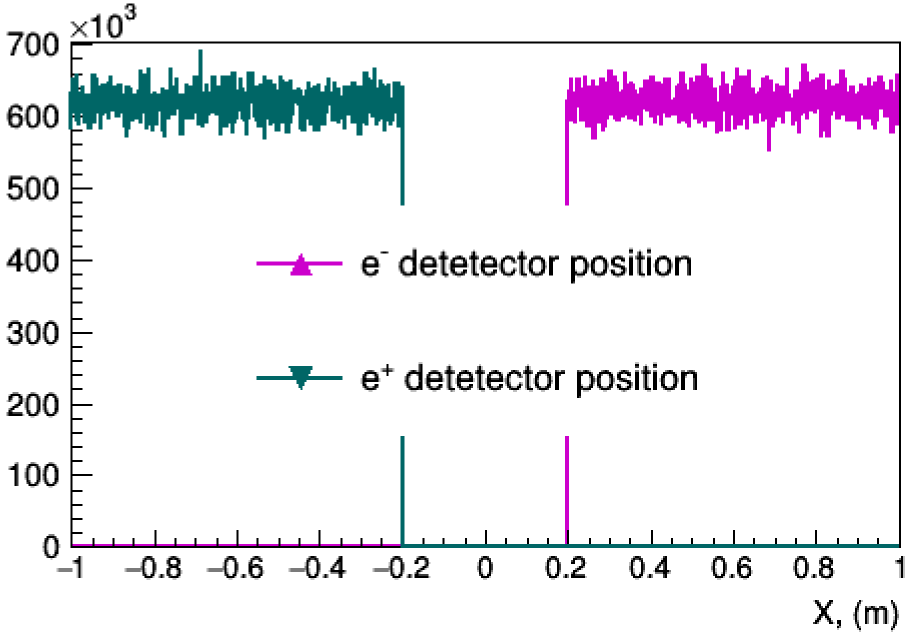
(G) $\left[\,d=-\frac{\left(R^2\,b+R\,a-R^2\right)\sqrt{z^2+R^2\,b^2+\left(2\,R\,a-2\,R^2\right)b+a^2-2\,R\,a}-R^2\,z}{z^2+R^2\,b^2+\left(2\,R\,a-2\,R^2\right)b+a^2-2\,R\,a+R^2},\; d=\frac{\left(R^2\,b+R\,a-R^2\right)\sqrt{z^2+R^2\,b^2+\left(2\,R\,a-2\,R^2\right)b+a^2-2\,R\,a}+R^2\,z}{z^2+R^2\,b^2+\left(2\,R\,a-2\,R^2\right)b+a^2-2\,R\,a+R^2}\,\right]$

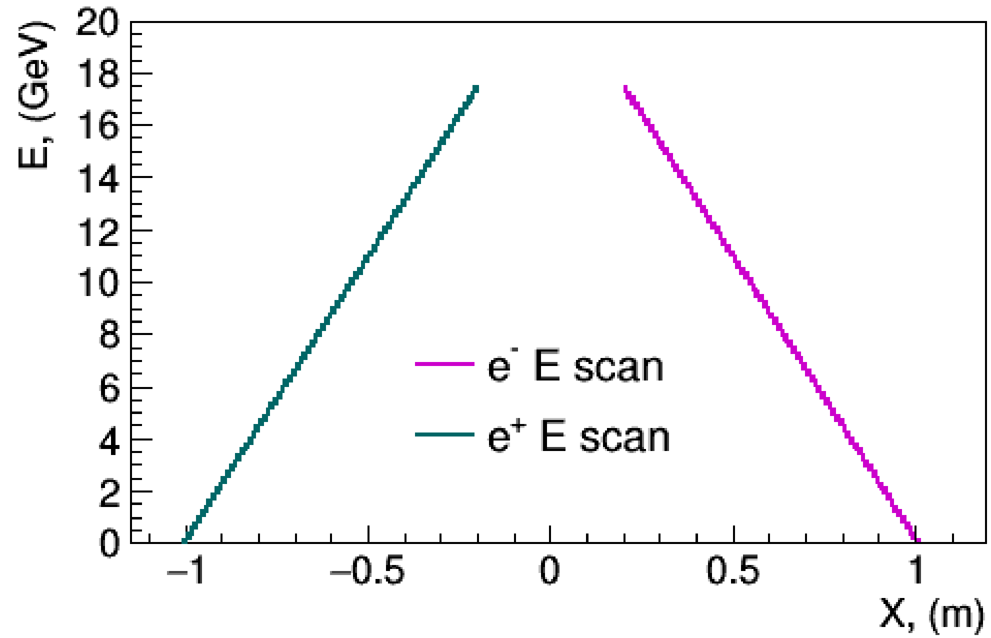



```
equals a+b*R. a and b are constants defined by R_min and R_max and X_min and X_max (not here, but in principle)
b = (X_min-X_max)/(R_max-R_min); a = X_min - b*R_max;
```

```
z: 9;
b: (0.1-1)/(26.06-0.26);
a: 0.1-b*26.6;
```

Magnet with cut






```
(%i10) D: subst(G1,d);
```

(D) $$\frac{\left(R^2 b+R a-R^2\right)\sqrt{z^2+R^2 b^2+\left(2 R a-2 R^2\right) b+a^2-2 R a}+R^2 z}{z^2+R^2 b^2+\left(2 R a-2 R^2\right) b+a^2-2 R a+R^2}$$

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(%i11) X: subst(G1,x);
```
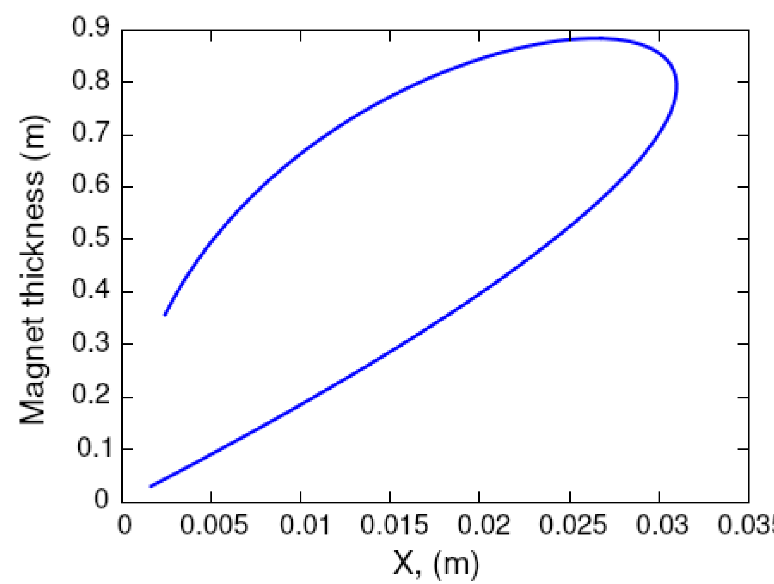
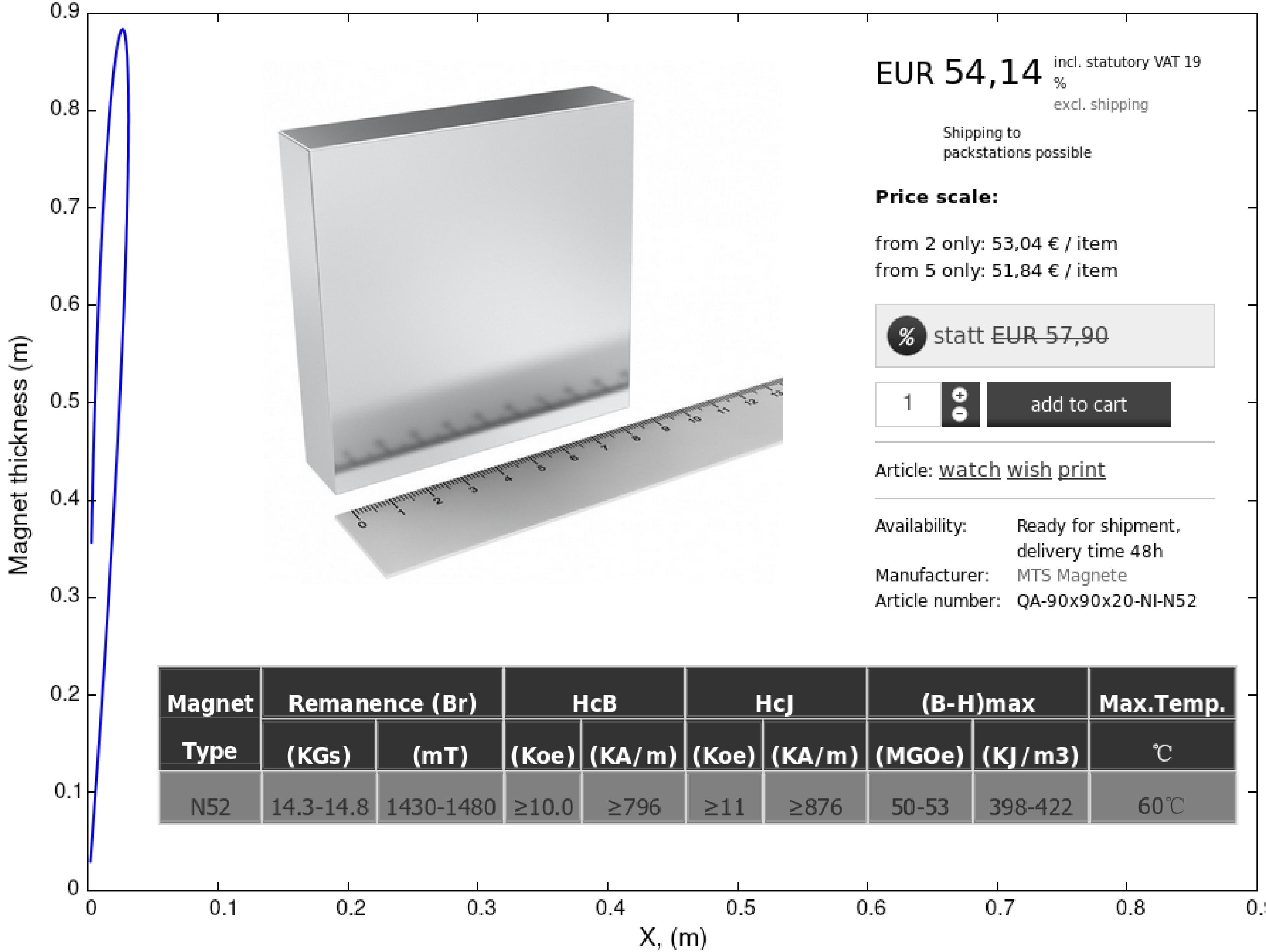

(X) $$R-\sqrt{R^2-\frac{\left(\left(R^2 b+R a-R^2\right)\sqrt{z^2+R^2 b^2+\left(2 R a-2 R^2\right) b+a^2-2 R a}+R^2 z\right)^2}{\left(z^2+R^2 b^2+\left(2 R a-2 R^2\right) b+a^2-2 R a+R^2\right)^2}}$$



Equal scale on x,y



Magnet	Remanence (Br)		HcB		HcJ		(B-H)max		Max.Temp.
Type	(KGs)	(mT)	(Koe)	(KA/m)	(Koe)	(KA/m)	(MGOe)	(KJ/m3)	℃
N52	14.3-14.8	1430-1480	≥10.0	≥796	≥11	≥876	50-53	398-422	60℃