Growth and Evaporation of Axion Soliton

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Motivation

- How a soliton lives in a halo?
- core-halo mass relation $M_c \propto M_h^{1/3}$?
- grow? evaporate? how fast?
- BEC in the kinetic regime (Levkov et al. 2018)

To long to wait

put a **soliton** in a **gas** field

Equation

- Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi$$

- Poisson equation

$$\nabla^2\Phi = 4\pi G\rho$$

$$\rho = m|\psi|^2$$

ψ = wave function
m = particle mass
Φ = self-gravitational potential
ρ = the mass density

- spectral method with a uniform mesh
- drift-kick-drift scheme
- GPU

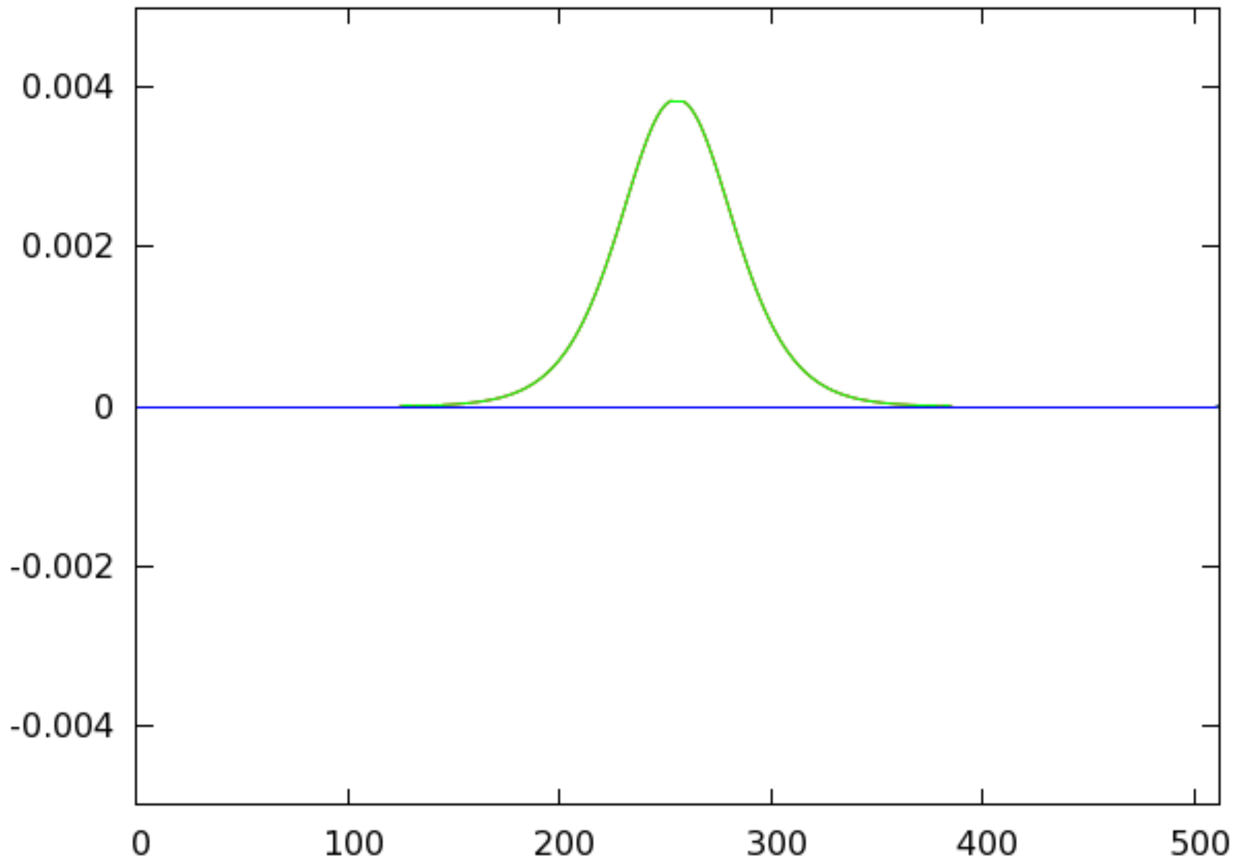
Chan, Schive, Woo, and Chiueh (2018)

Soliton

$\sqrt{\text{density}} = \sqrt{\text{Re}^2 + \text{Im}^2}$

Re

Im



$$\rho_{\rm sol}(r) = \frac{\rho_{\rm c}}{\left[1 + (\frac{r}{r_{\rm c}})^2\right]^8}$$

$$\rho_{\rm c} = 1.9\frac{4\pi G}{\hbar^2}(10^{-23}{\rm eV}/c^2)^2{\rm kpc}^4 \cdot r_{1/2}{}^{-4} \cdot (M_\odot/{\rm pc}^3)$$
$$= 337.387 \cdot r_{\rm c}{}^{-4}$$

Schive et al. 2014

half-peak radius:
$r_{1/2} \sim 0.3 r_{\rm c}$

Background Gas:

halo size ≫ de Broglie wavelength

$(mvR \gg 1)$

$$\psi(\mathbf{r}) = \sqrt{\frac{1}{V}} \sum_{\mathbf{k}} \psi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

Gaussian distribution:

$$|\psi_{\mathbf{k}}|^2 = A e^{-\frac{k^2}{k_0{}^2}} \qquad V = N^3 \qquad \bar{\rho} = \frac{A k_0{}^3}{8\pi^{3/2}}$$

N: resolution, box size

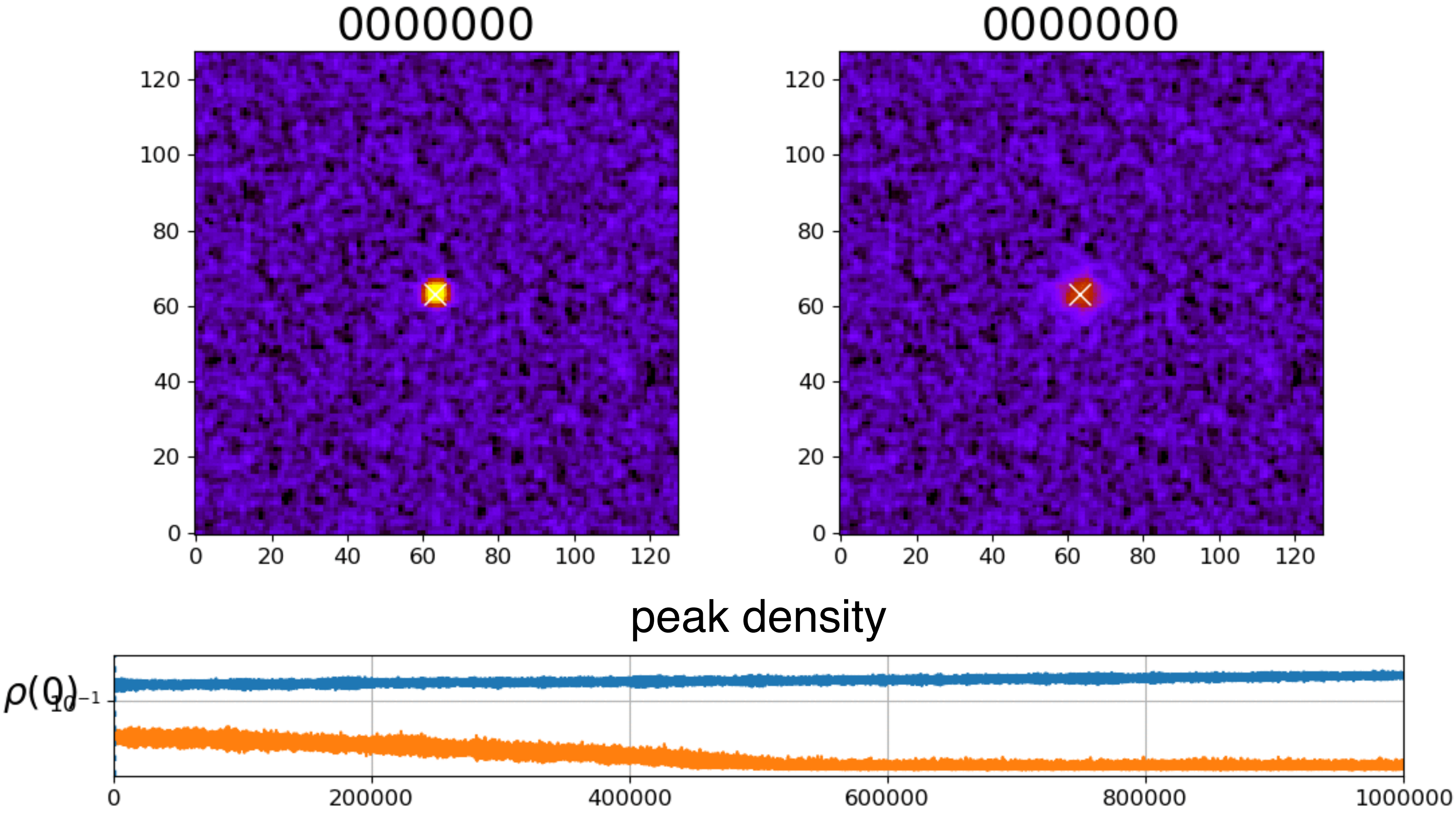
k_0: temperature, velocity of gas

A: number of particles

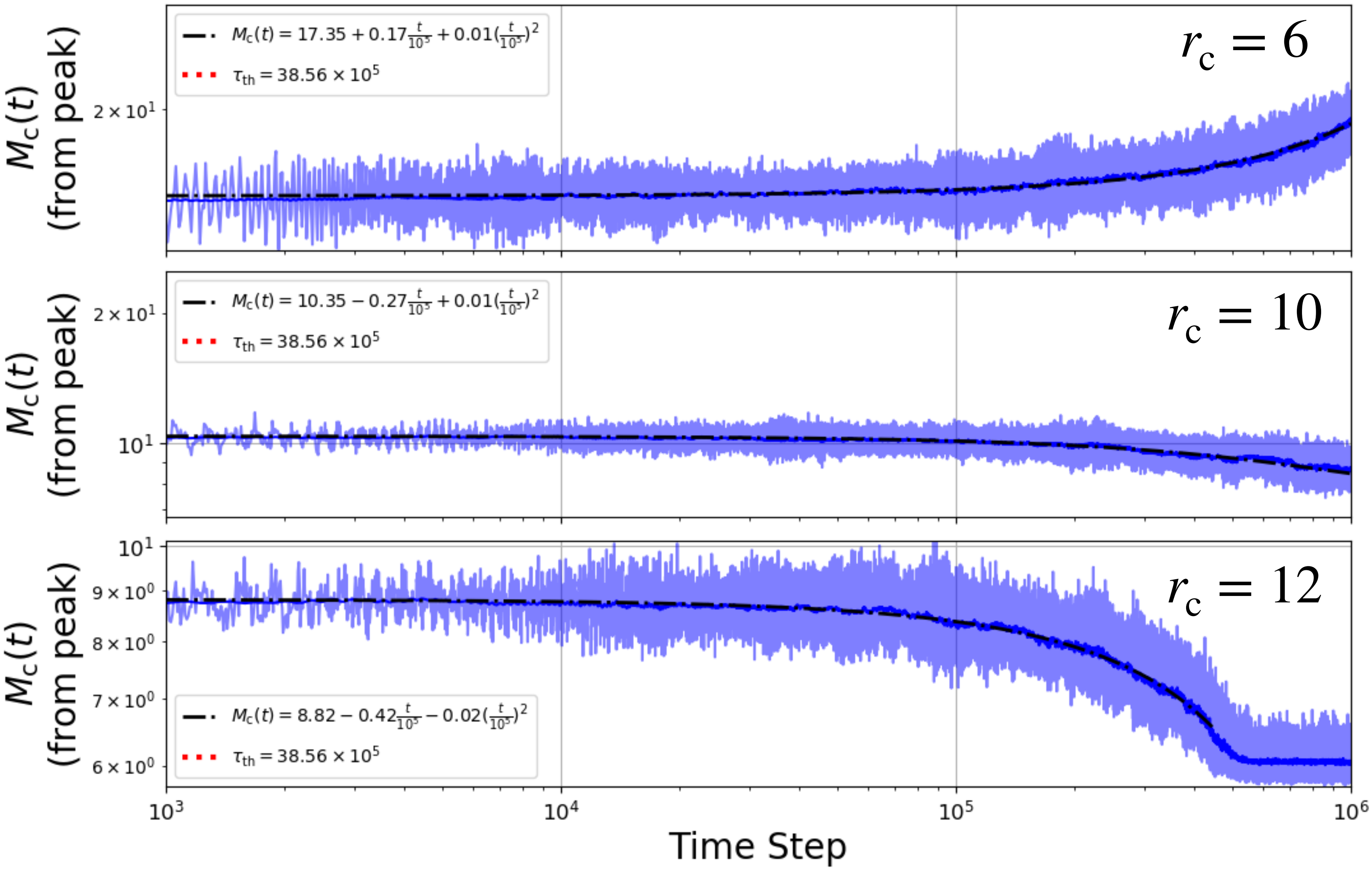
Khlebnikov 1999,
Levkov et al. 2018

All solitons grow?

$k_0 = 1.0,\ A = 0.01,\ r_c = 6$ $\quad$ $k_0 = 1.0,\ A = 0.01,\ r_c = 12$



$k_0 = 1.0,\ A = 0.01,\ N = 128$



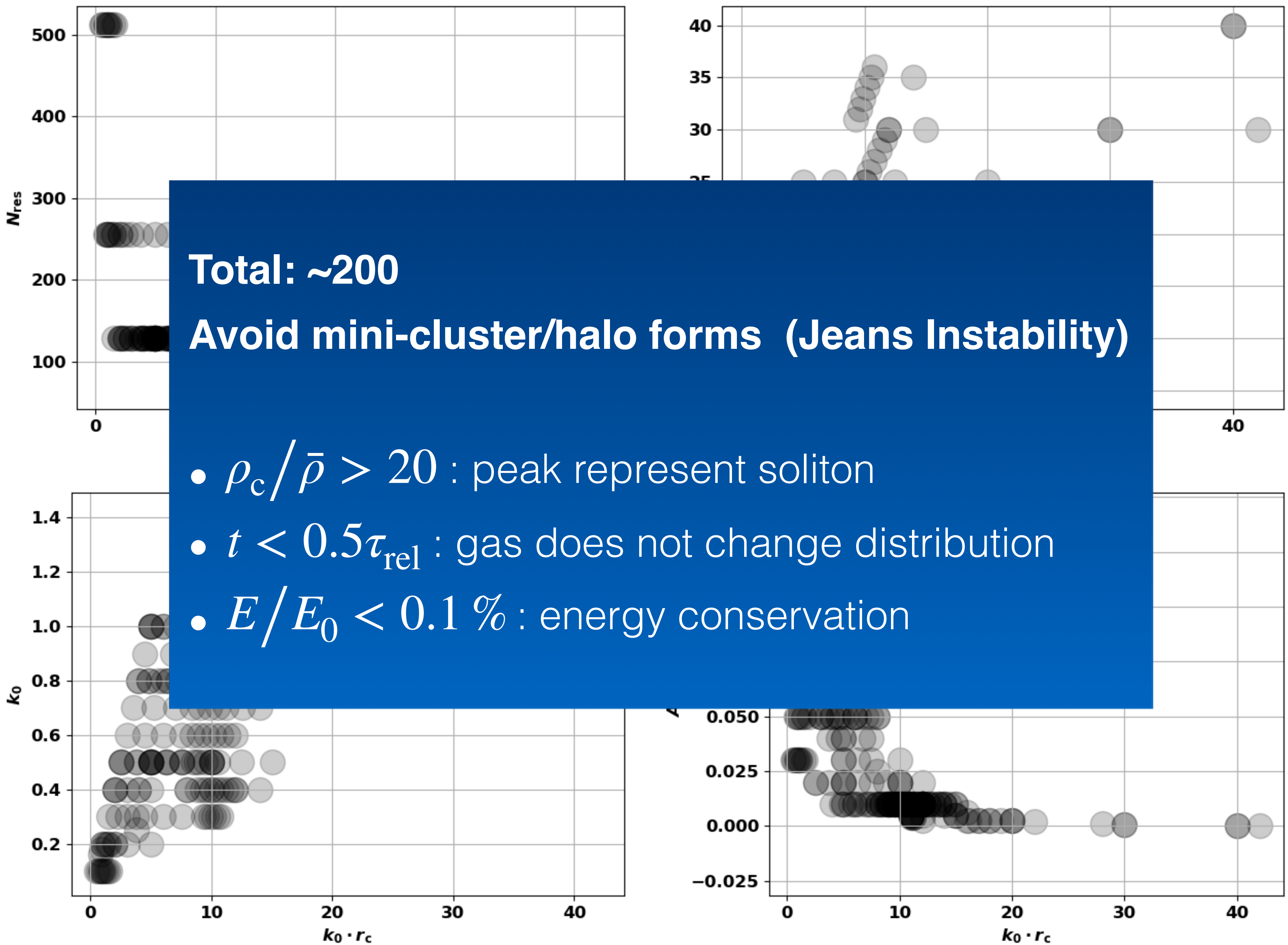
Fitting function:

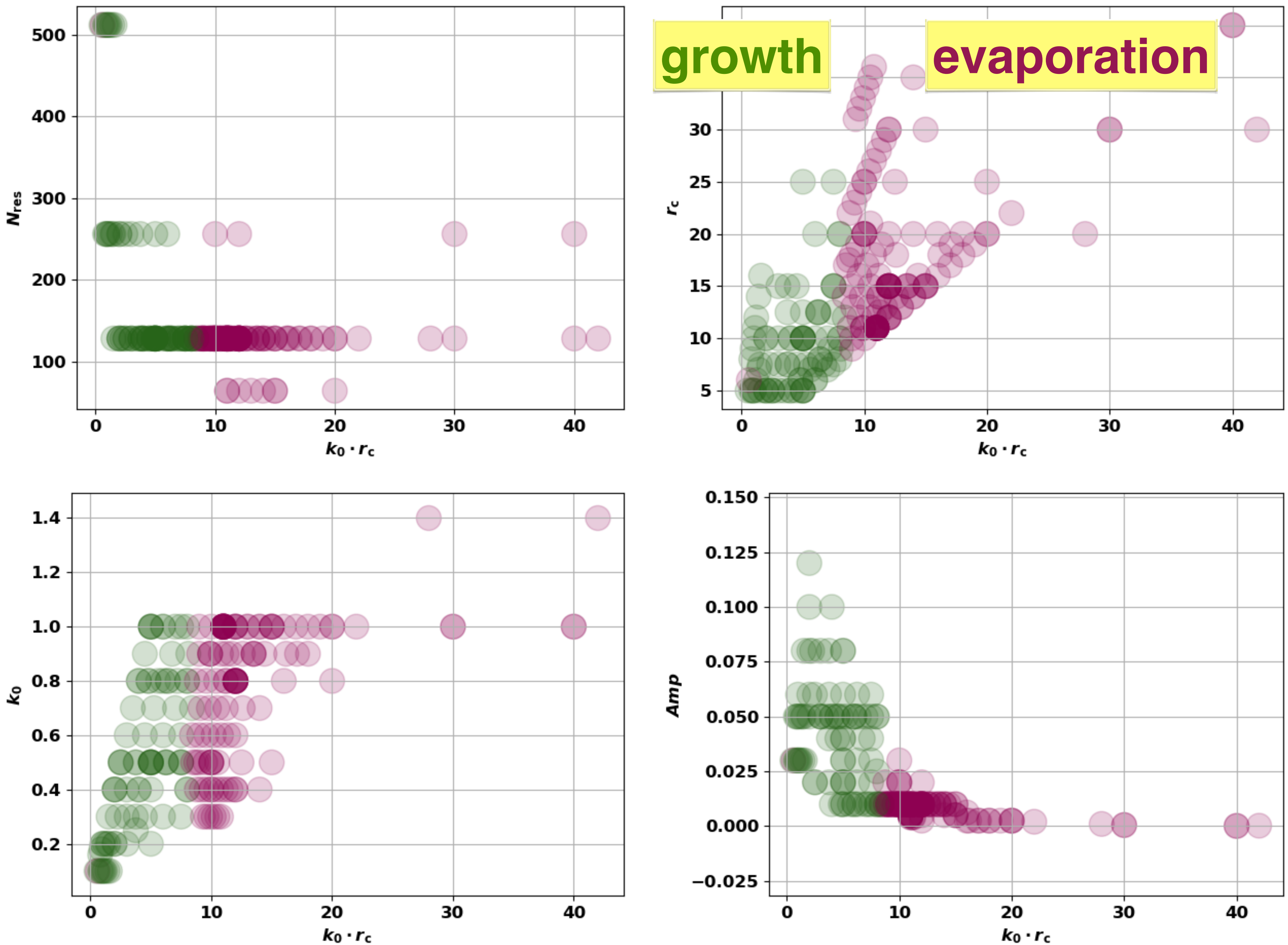
$$M_c(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2 \qquad M_c \propto \rho_c^{1/4}$$

Total: ~200

Avoid mini-cluster/halo forms (Jeans Instability)

- $\rho_c / \bar{\rho} > 20$: peak represent soliton
- $t < 0.5\tau_{rel}$: gas does not change distribution
- $E / E_0 < 0.1\ \%$: energy conservation

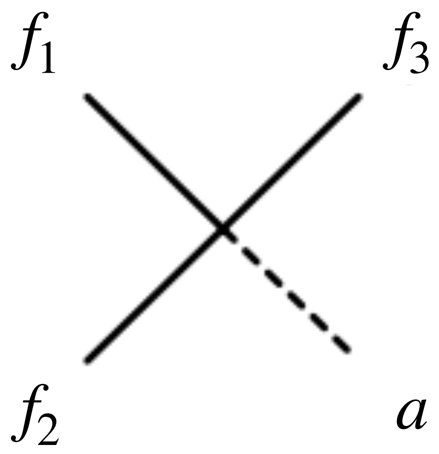




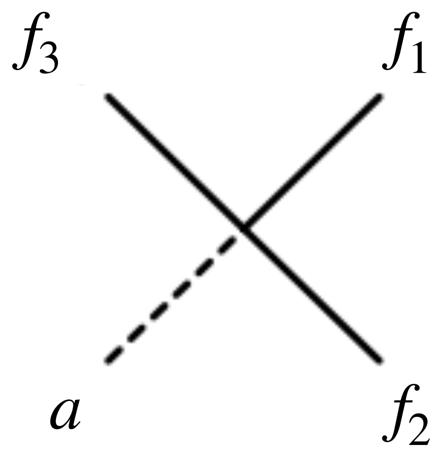
growth
evaporation
N_{res}
r_c
k_0
Amp
$k_0 \cdot r_c$

Average Growth/Evaporation Rate

absorption



emission



f_1 : gas in bound state

f_2, f_3 : gas

a : axion soliton

$$\langle \delta N_s \rangle = \frac{1}{2} \sum_{1,2,3} \big(f_1 f_2 (1 + f_3) - (1 + f_1)(1 + f_2) f_3 \big) \, | M_{1s,23} + M_{2s,13} |^2$$

$$\simeq \frac{1}{2} \sum_{1,2,3} (f_1 f_2 - f_1 f_3 - f_2 f_3) \, | M_{1s,23} + M_{2s,13} |^2$$

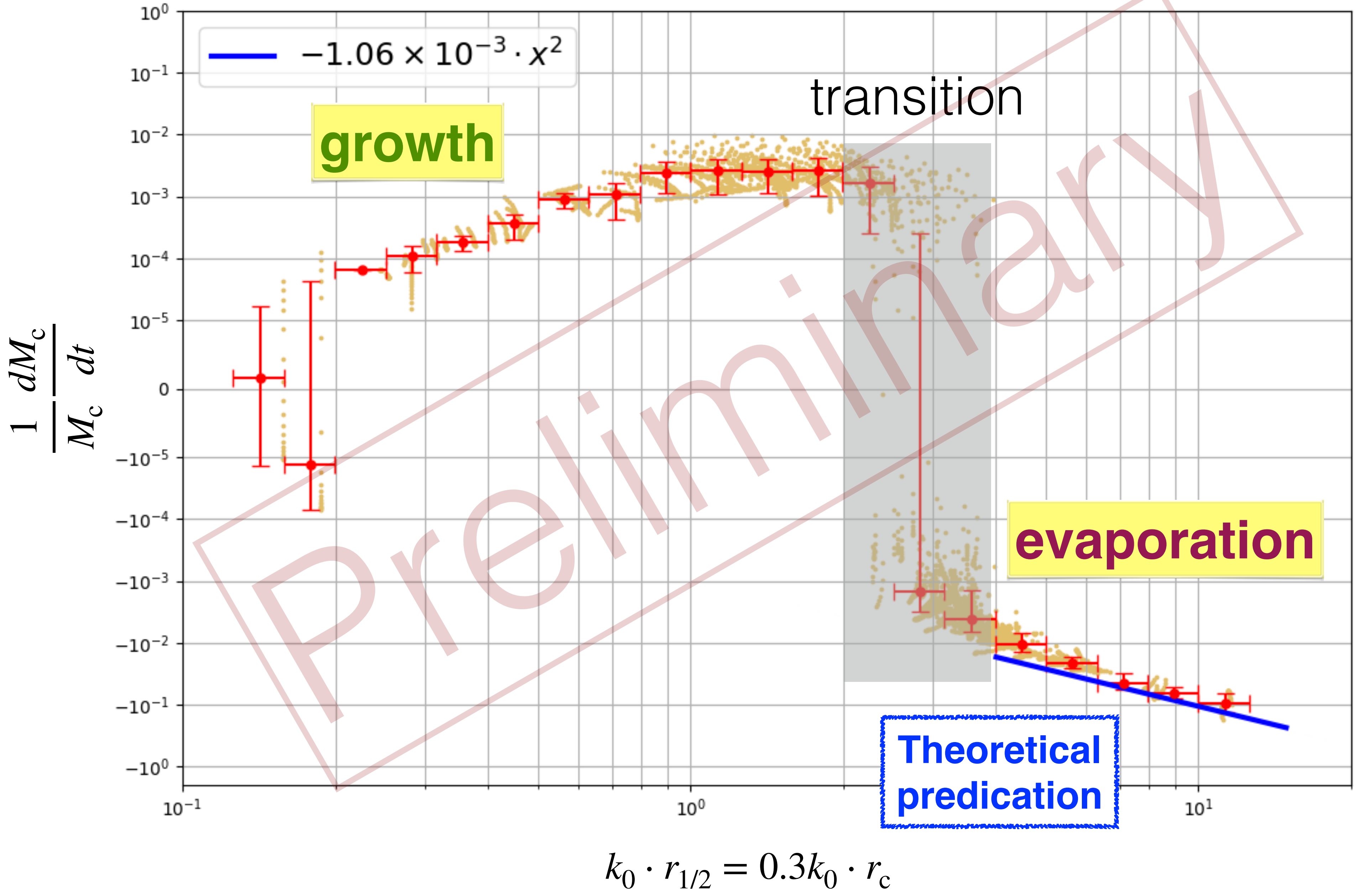
evaporation

hot gas: $k_0 \cdot r_{1/2} \gg 1$

[Chan, Sibiryakov, and Xue in prep.]

Fitting function:

$$M_{\mathrm{c}}(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2$$

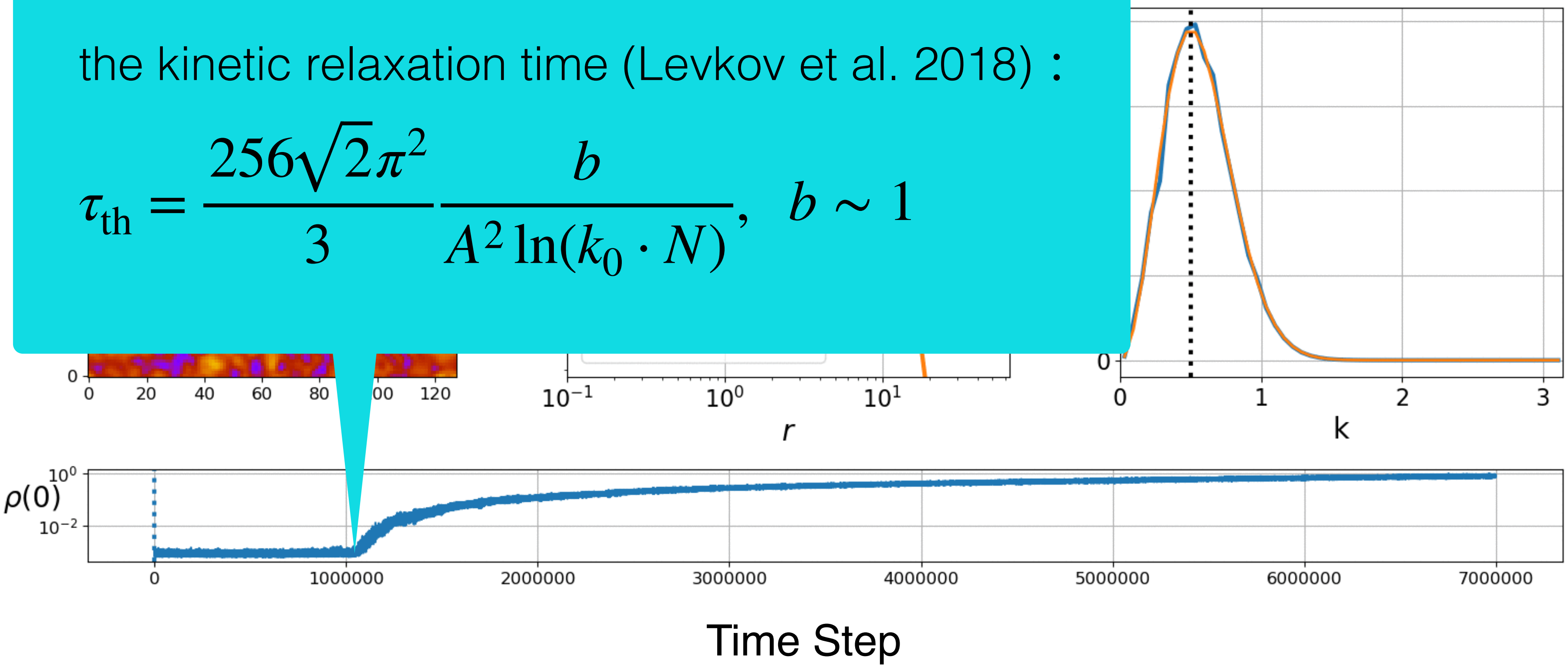


Formation of soliton in the kinetic regime

$$N = 128,\ k_0 = 0.5,\ A = 0.02$$

the kinetic relaxation time (Levkov et al. 2018) :

$$\tau_{\rm th} = \frac{256\sqrt{2}\pi^2}{3}\frac{b}{A^2\ln(k_0 \cdot N)},\quad b \sim 1$$



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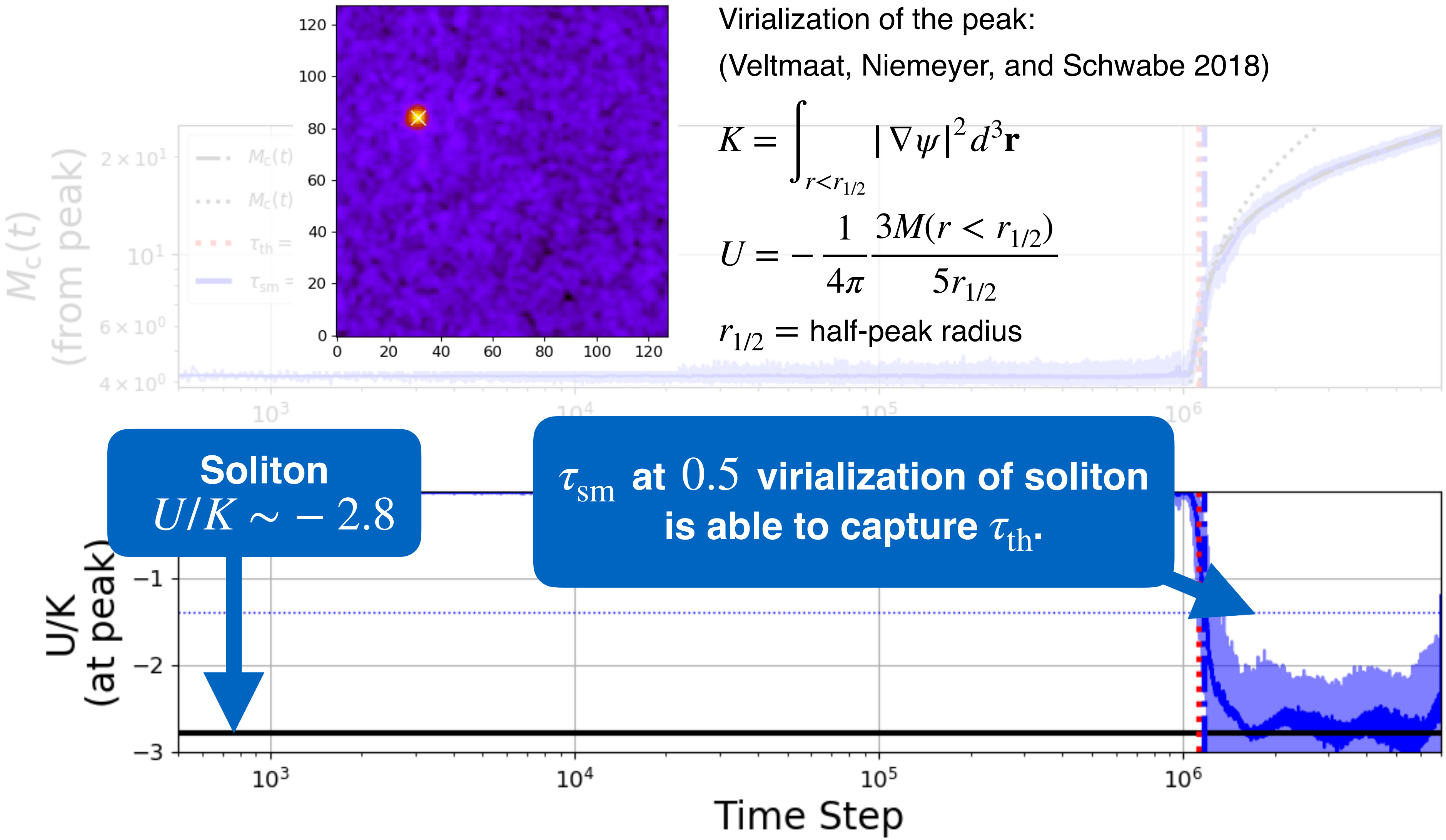


Virialization of the peak:
(Veltmaat, Niemeyer, and Schwabe 2018)

$$K = \int_{r<r_{1/2}} |\nabla\psi|^2 d^3\mathbf{r}$$

$$U = -\frac{1}{4\pi}\frac{3M(r<r_{1/2})}{5r_{1/2}}$$

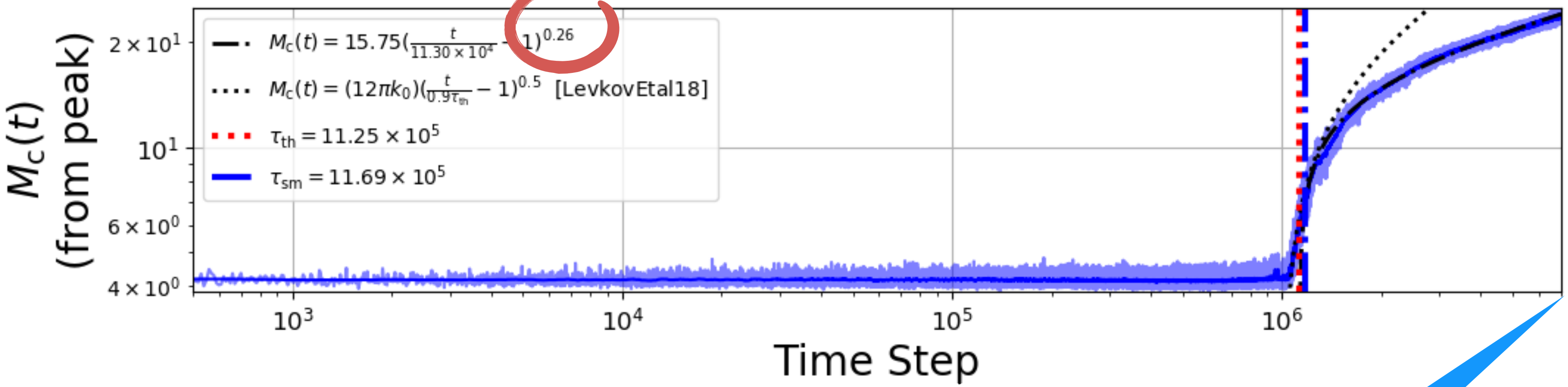
$r_{1/2}$ = half-peak radius

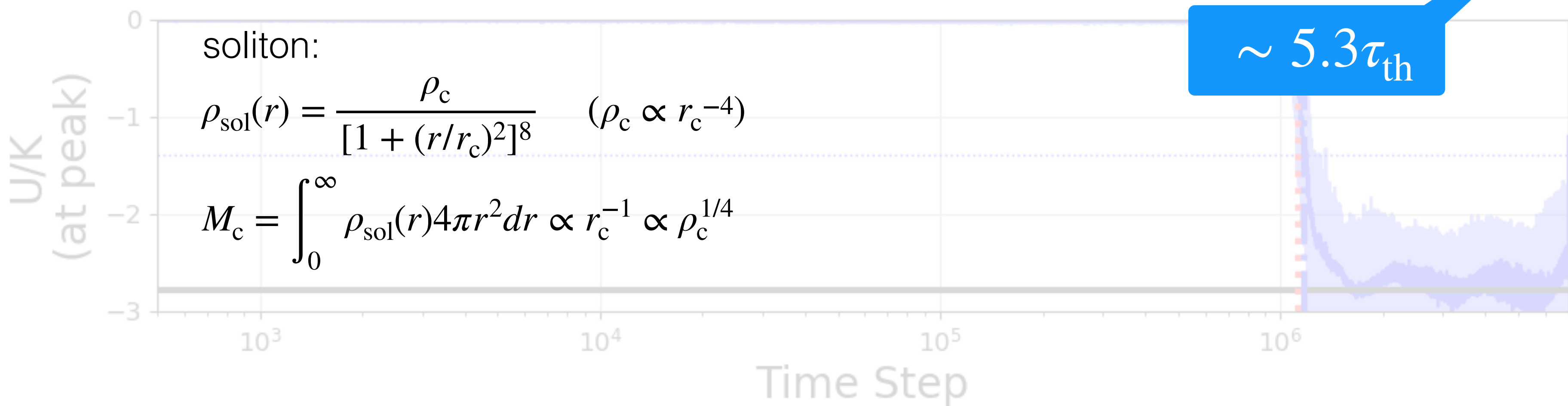
Soliton
$U/K \sim -2.8$

τ_{sm} at 0.5 virialization of soliton is able to capture τ_{th}.

$$N = 128,\ k_0 = 0.5,\ A = 0.02$$

$$M_c(t) = M_0\left(\frac{t}{\tau_0} - 1\right)^{\alpha} \quad [\alpha,\ \tau_0,\ M_0]$$





soliton:

$$\rho_{sol}(r) = \frac{\rho_c}{[1 + (r/r_c)^2]^8} \qquad (\rho_c \propto r_c{}^{-4})$$

$$M_c = \int_0^\infty \rho_{sol}(r) 4\pi r^2 dr \propto r_c^{-1} \propto \rho_c^{1/4}$$

$$M_c(t) = M_0\left(\frac{t}{\tau_0} - 1\right)^{\alpha} \quad [\alpha,\ \tau_0,\ M_0]$$

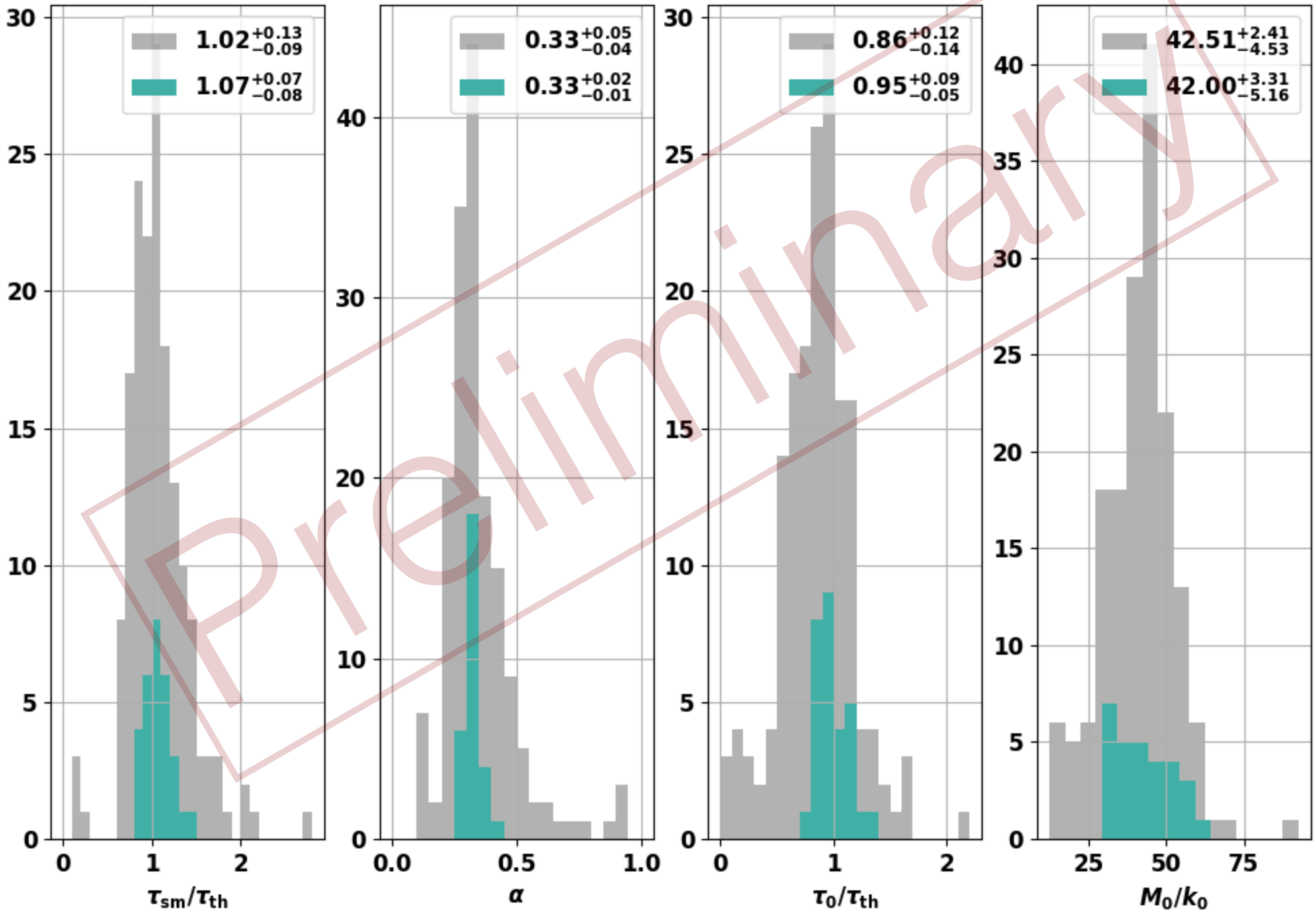
$$M_c(t) = 12\pi k_0\left(\frac{t}{\tau_{th}} - 1\right)^{0.5} \quad \text{(Levkov et al.2018)}$$

$k_0 N > 40$

$Ak_0 < 0.5$JeansCond.

$t_{fit} > 3\tau_{th}$

30 data points



Result

- **transition at** $k_0 \cdot r_{1/2} \sim 3$
  - $k_0 \cdot r_{1/2} < 3$ **: soliton grows**
  - $k_0 \cdot r_{1/2} > 3$ **: soliton evaporates**
- **growth rate decreases when gas/soliton becomes cooler/heavier (**$k_0 \cdot r_{1/2} \searrow$ **)**
- **evaporation rate increases when gas/soliton besoms hotter/lighter (**$k_0 \cdot r_{1/2} \nearrow$ **)** **[theoretical calculation matches]**
- **soliton formation in the kinetic regime:**
  - We confirm the expression for the condensation time (Levkov et al. 2018).
  - We find a different growth rate (slower).
  - gas reappears as **Maxwellian-like distribution** when a soliton becomes massive.