BROWNIAN MOTION OF FDM SOLITONS

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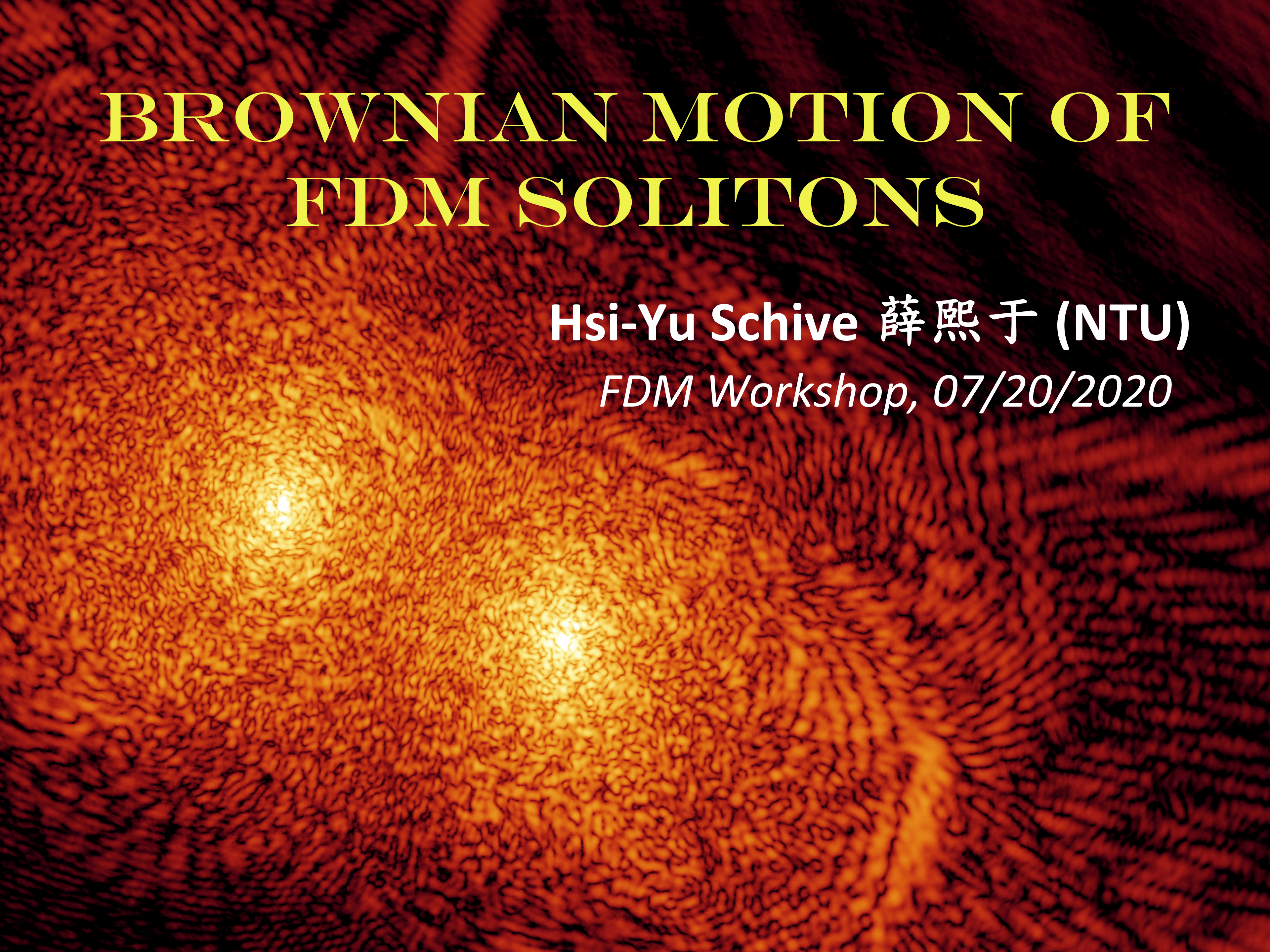
FDM Workshop, 07/20/2020



Outline

- Fuzzy/Wave Dark Matter (FDM/ψDM)
- Simulations
- Soliton Random Walk

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Fuzzy Dark Matter (FDM)

- Extremely light particles
  - $m_{22} \equiv m_\psi$ / 10^{-22} eV ~ 1.0 → 10^{31} lighter than cold dark matter (CDM)
  - de Broglie wavelength becomes astronomical (kpc) scale
  - Wavelike properties (e.g., interference)
  - References:
    - D. Marsh. Physics Reports 643, 1 (2016)
    - L. Hui, J. Ostriker, S. Tremaine, & E. Witten. PRD 95, 043541 (2017)
    - E. Ferreira. arXiv:2005.03254 (2020)

- Governing eq.: Schrödinger-Poisson eq.

$$\boxed{\begin{aligned} & i\frac{\partial \psi(x)}{\partial t} = -\frac{1}{2m_\psi}\nabla^2\psi(x) + m_\psi \varphi(x)\psi(x) \\ & \nabla^2\varphi(x) = 4\pi G a(t)\left(|\psi(x)|^2 - 1\right) \end{aligned}}$$

ψ: wave function

ϕ: Newton potential

a: scale factor

ħ: 1

Particle mass (m_ψ) → the ONLY free parameter in FDM

Quantum Fluid

- **Rewrite Schrödinger eq. into conservation laws**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0,$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \nabla \left(\frac{1}{2 m_\psi^{\ 2}} \frac{\nabla^2 f}{f} \right) - \nabla \varphi$$

$$\psi = f e^{iS}$$

$$\rho = m_\psi f^2$$

$$v = m_\psi^{-1} \nabla S$$

$$\text{Hydro}: \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{1}{\rho} \nabla P - \nabla \varphi$$

$$\tilde{P}_{ij} = \frac{1}{m_\psi} \left(\partial_i f \partial_j f - \frac{1}{4} \delta_{ij} \nabla^2 f^2 \right)$$

quantum stress

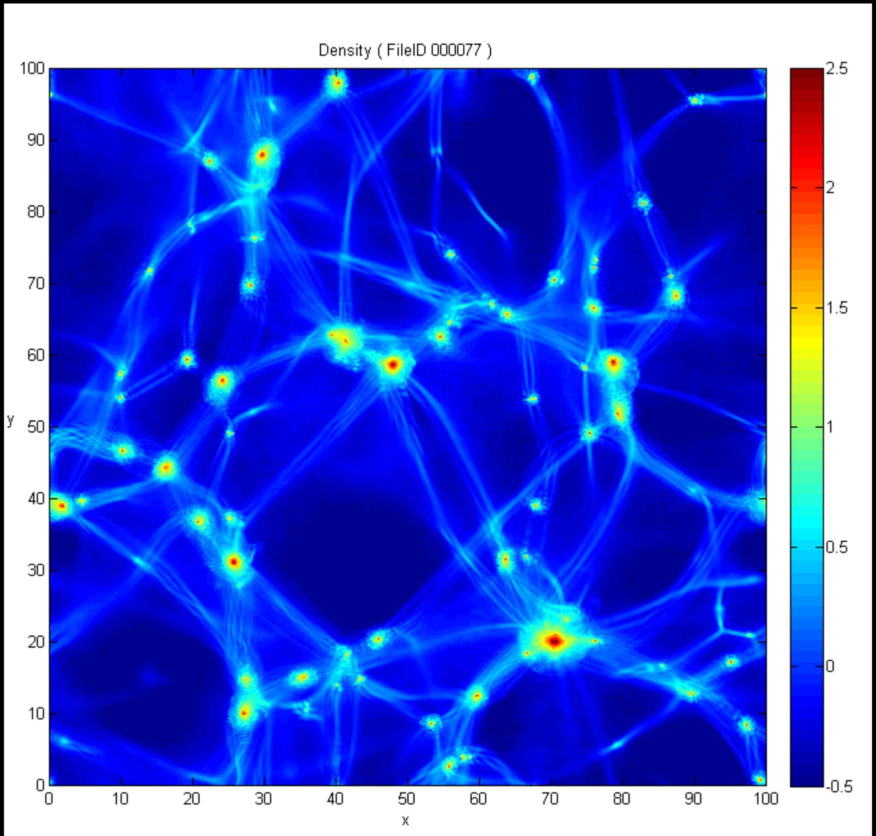
$$k_J = (6a)^{1/4} (H_0 m_\psi)^{1/2}$$

Jeans wave number in FDM

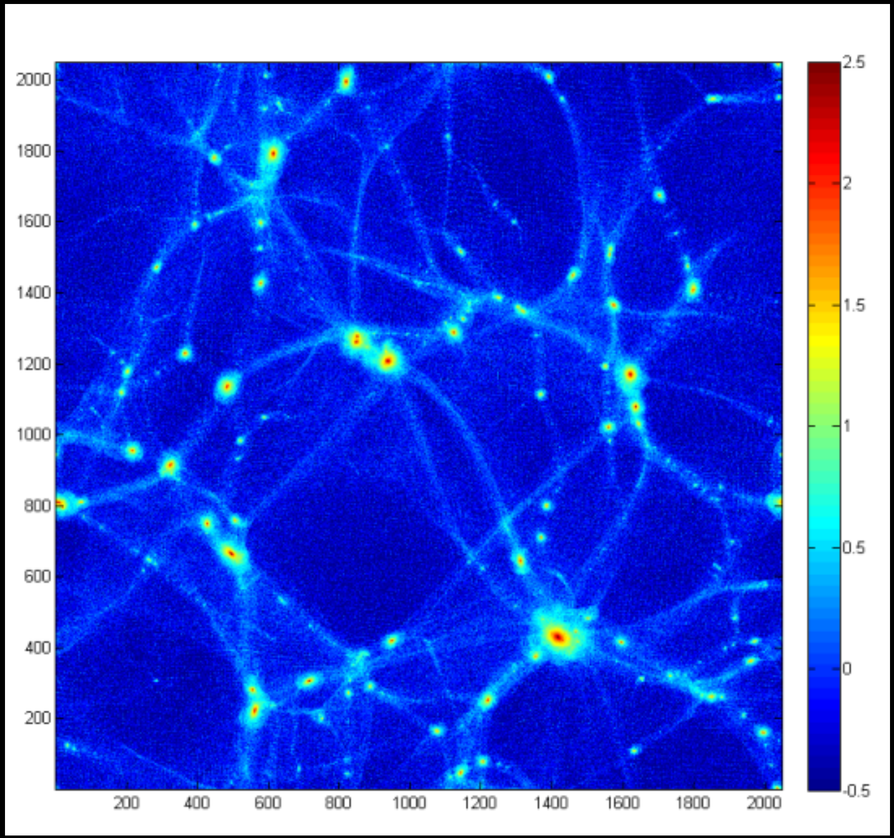
→ Suppressing small-scale structures

FDM vs. CDM (Large Scales)

FDM



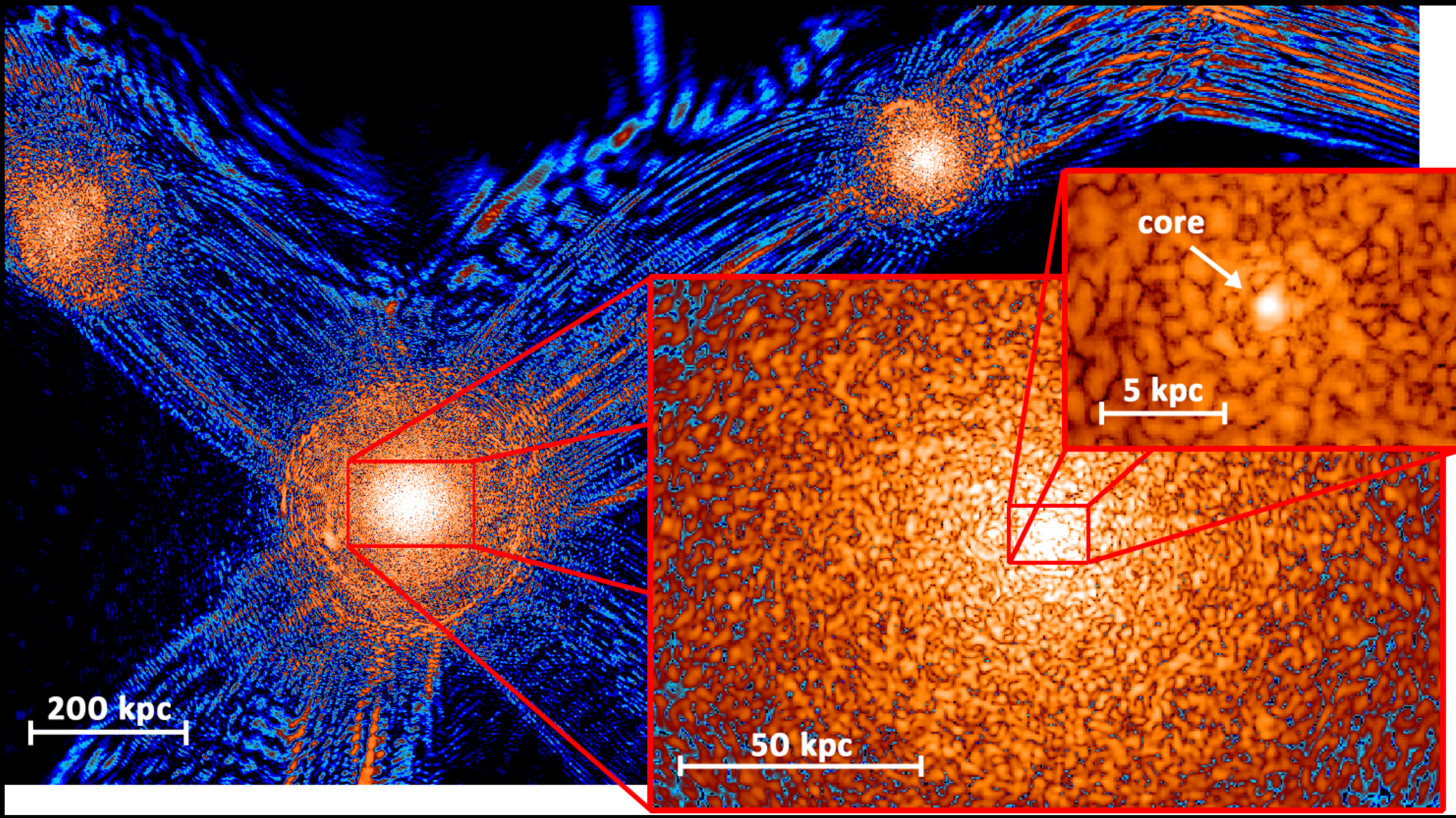
CDM



50 Mpc/h box

- Large-scale structures are indistinguishable

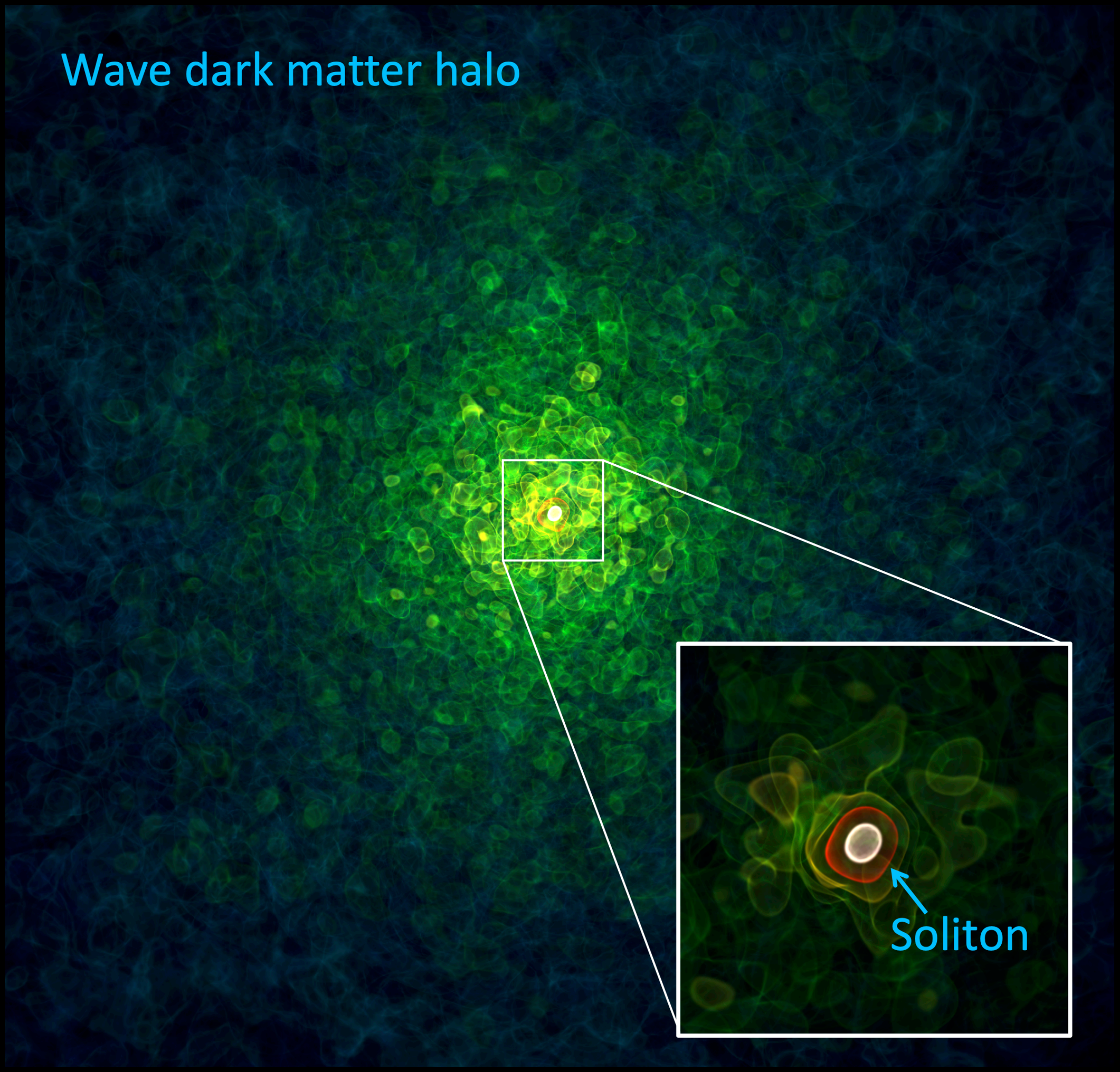
Interference Patterns (Small Scales)



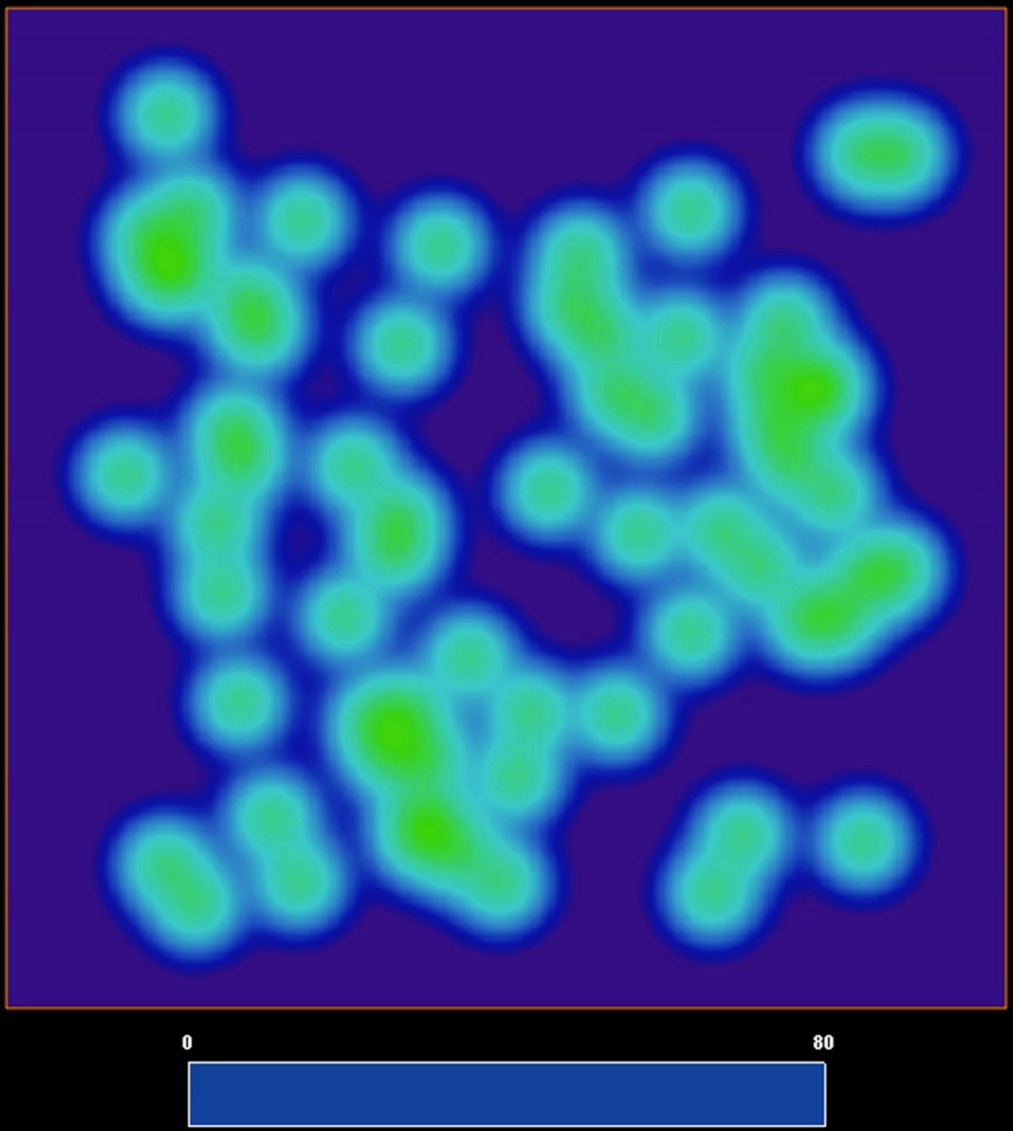
- Interference is everywhere: filaments, density granules, and central cores ↔ CDM predicts cuspy profiles

Schive, Chiueh, & Broadhurst 2014, *Nature Physics*, 10, 496

Soliton-Halo Pair



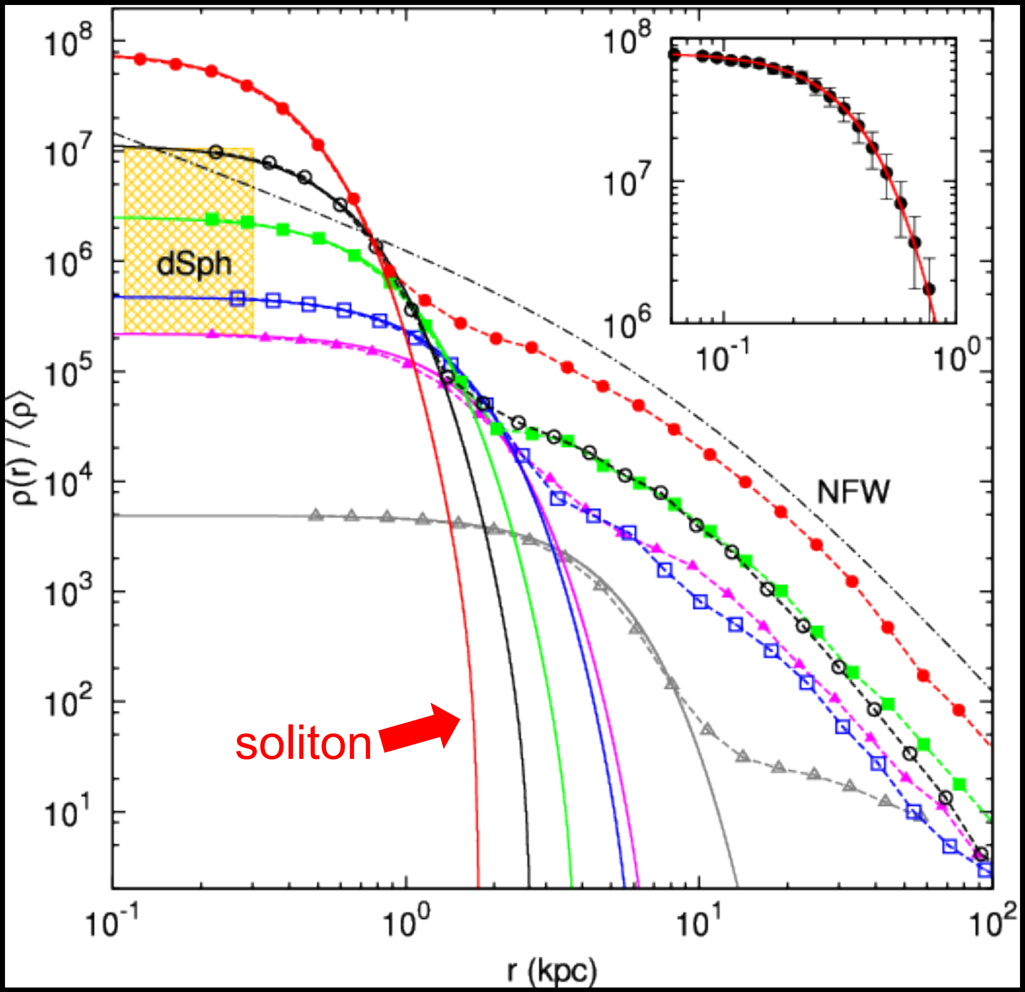
Core-Halo Relation



Q1: is there a prominent core in every halo?

Schive et al. 2014, *PRL*, 113, 261302

Core-Halo Relation



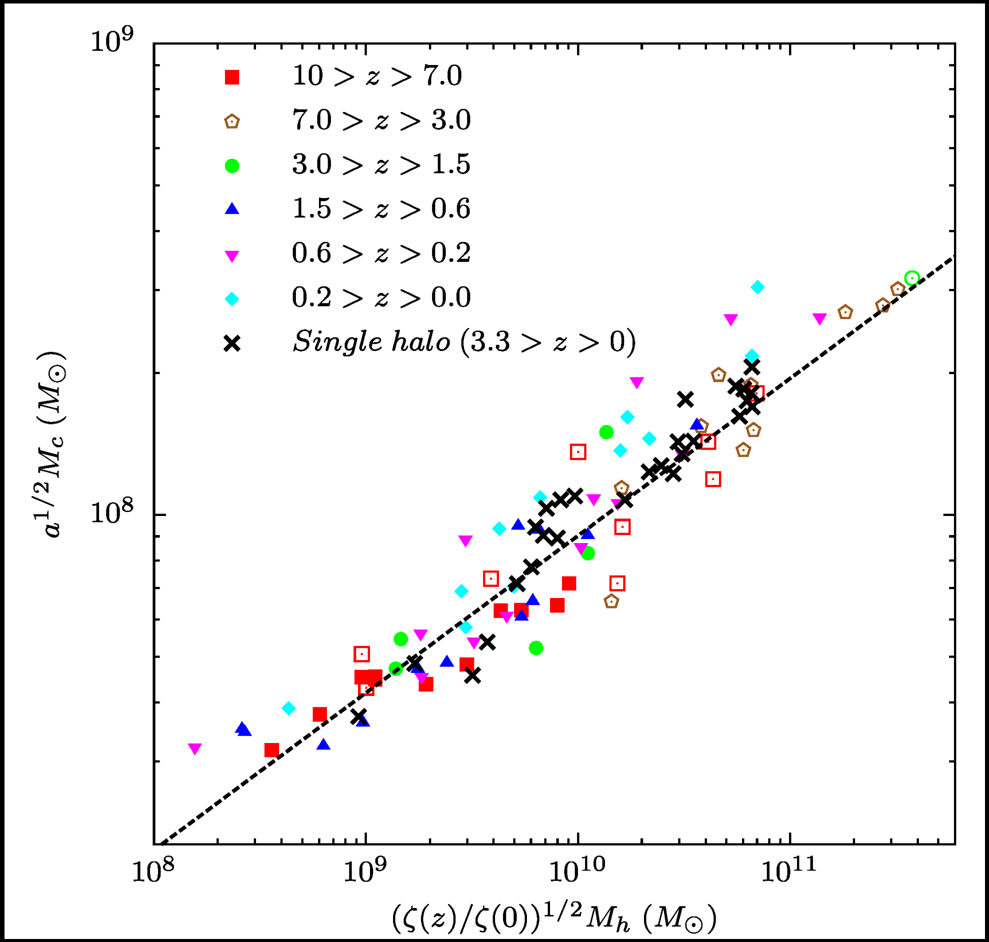
Q1: is there a prominent core in every halo?

YES; core ≈ soliton !!

Q2: for a given halo, can we predict its core properties?

Schive et al. 2014, *PRL*, 113, 261302

Core-Halo Relation



- **Core mass (M_c) vs. halo mass (M_h) at different z**
- **Solid line: theoretical prediction**

Schive et al. 2014, *PRL*, 113, 261302

Q1: is there a prominent core in every halo?

YES; core ≈ soliton !!

Q2: for a given halo, can we predict its core properties?

$$\boldsymbol{M_c \propto r_c^{-1} \propto (1+z)^{1/2} M_h^{1/3}}$$

$\boldsymbol{M_h \uparrow,\ z \uparrow \Rightarrow M_c \uparrow,\ r_c \downarrow}$

- **Dwarfs: kpc-scale cores**
- **Minimum halos: $M_h \approx 10^8\ M_\odot$**
- **MW: ~300 pc core with $M \approx 10^9\ M_\odot$**
- **More compact cores at higher z**

What May Break the Core-Halo Relation?

- Unrelaxed systems
  - For example, just after major mergers
- Non-isolated systems
  - For example, subhalos
- Massive halos
  - The isothermal assumption may become invalid
- Non-negligible baryon mass around the center
- Strong baryonic feedback around the center
- Simulations with insufficient resolution
  - An example will be given shortly

More references on the core-halo relation:

Schwabe et al. 2016, PRD, 94, 043513; Du et al. 2017, PRD, 95, 043519; Mocz et al. 2017, MNRAS, 471, 4559; Mina et al. 2020, arXiv: 2007.04119; Nori & Baldi 2020, arXiv: 2007.01316; … (incomplete)

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- Simulations
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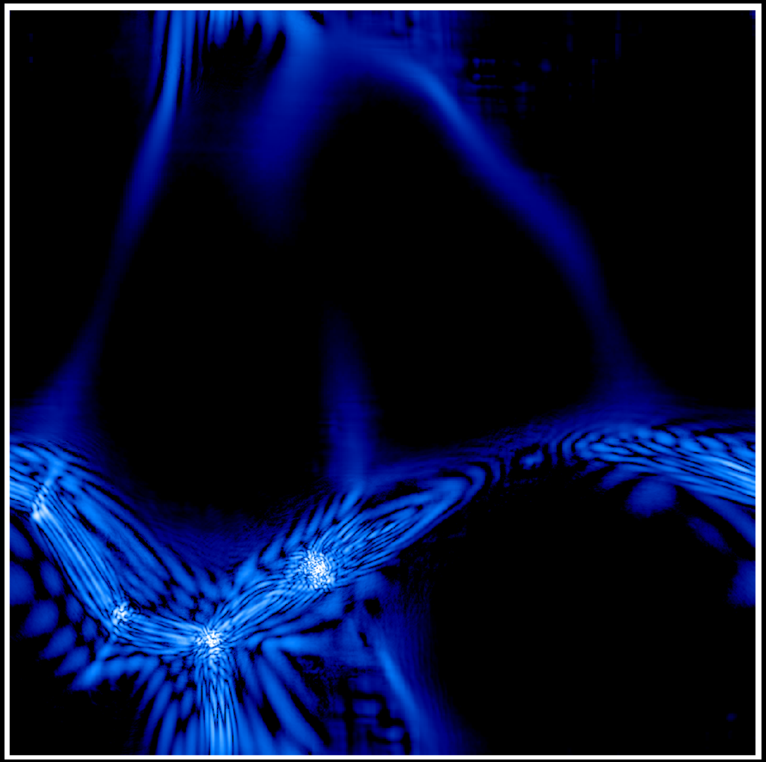
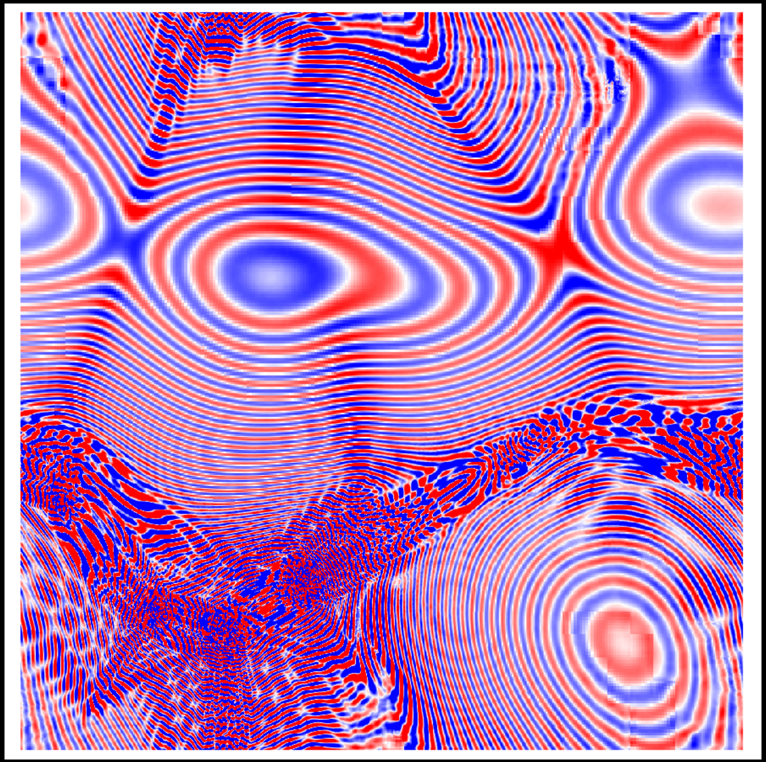
Simulation Challenges

Density



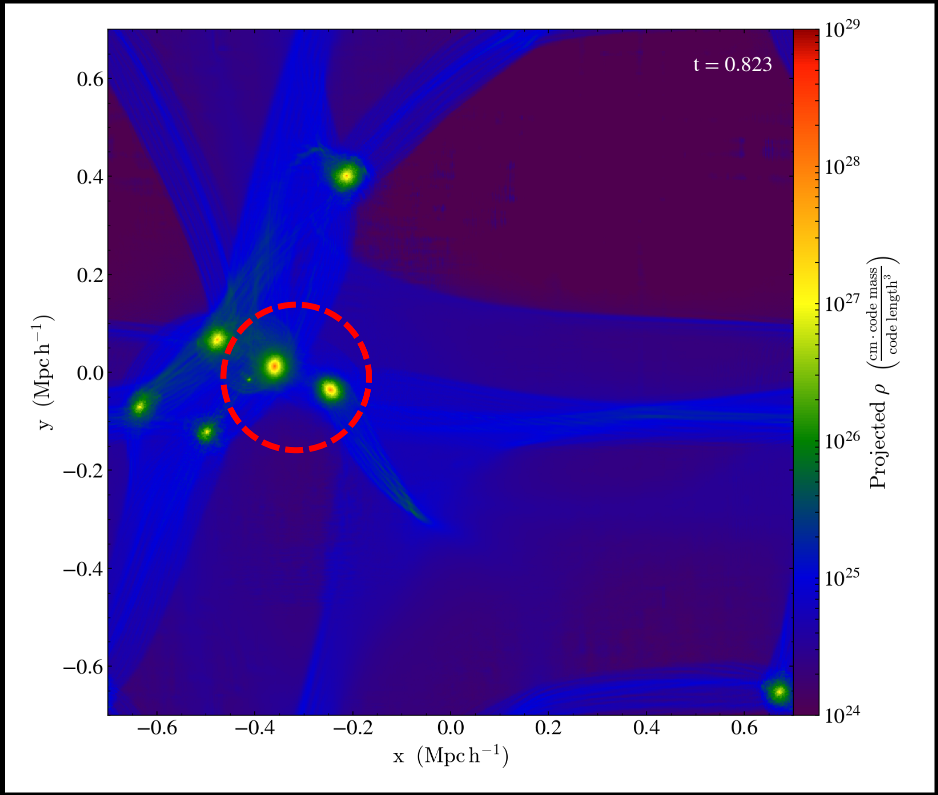
Wave function



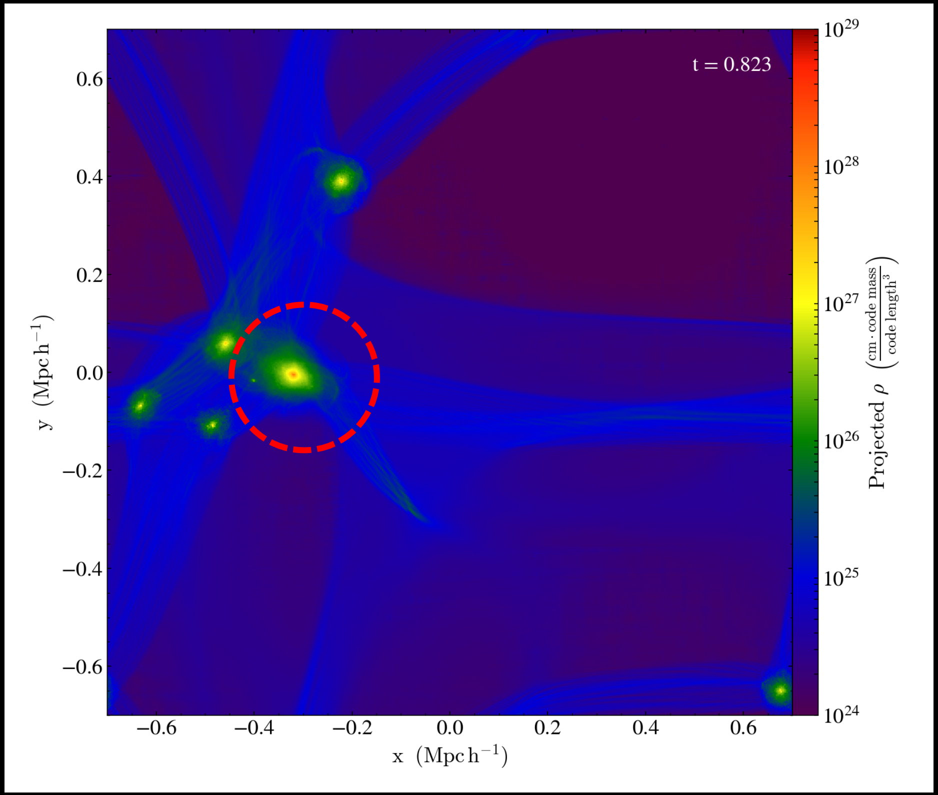
- Ultra-high resolution is required
- GAMER : *GPU-accelerated Adaptive MEsh Refinement Code*

Simulation Resolution Is Critical

LOWER resolution

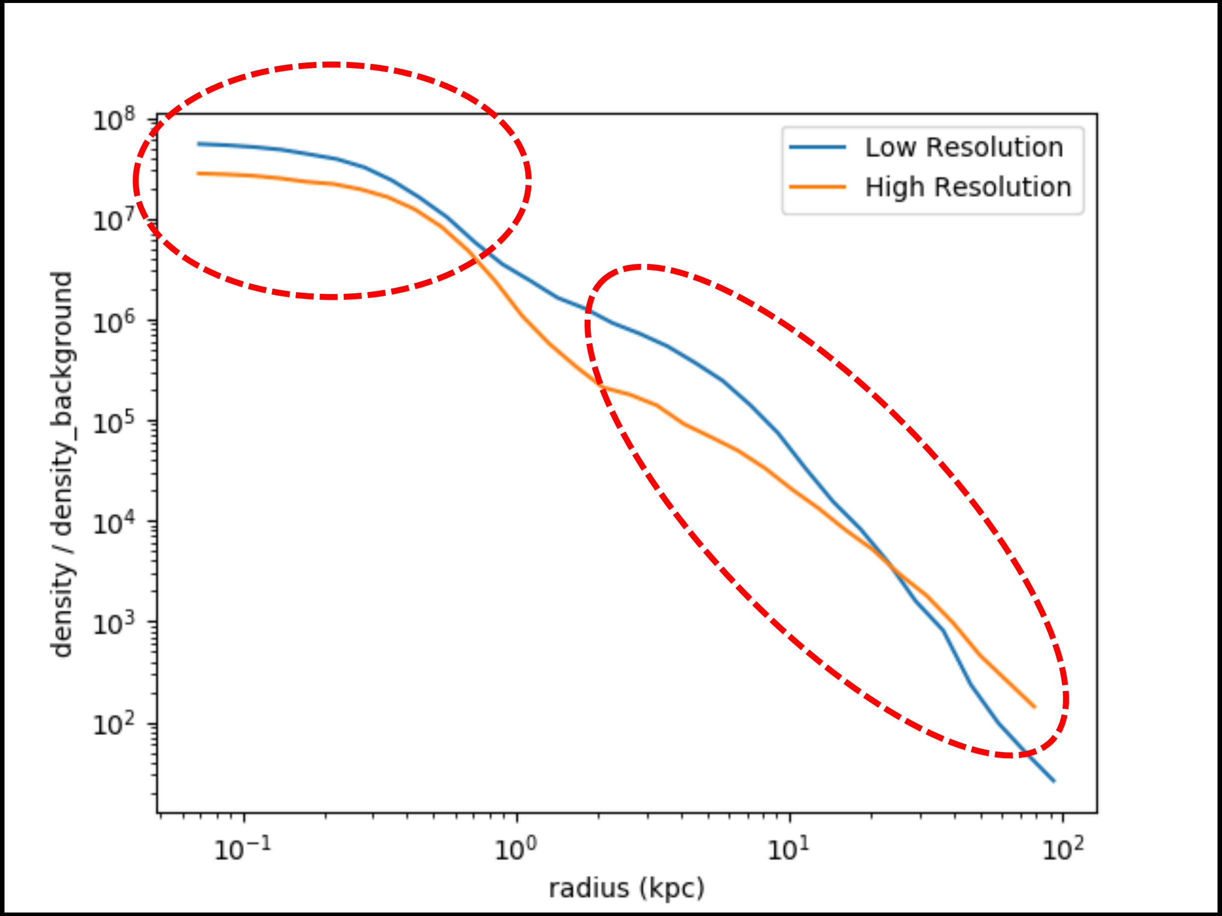


HIGHER resolution



- Structure evolution is delayed for simulations with insufficient resolution

Simulation Resolution Is Critical



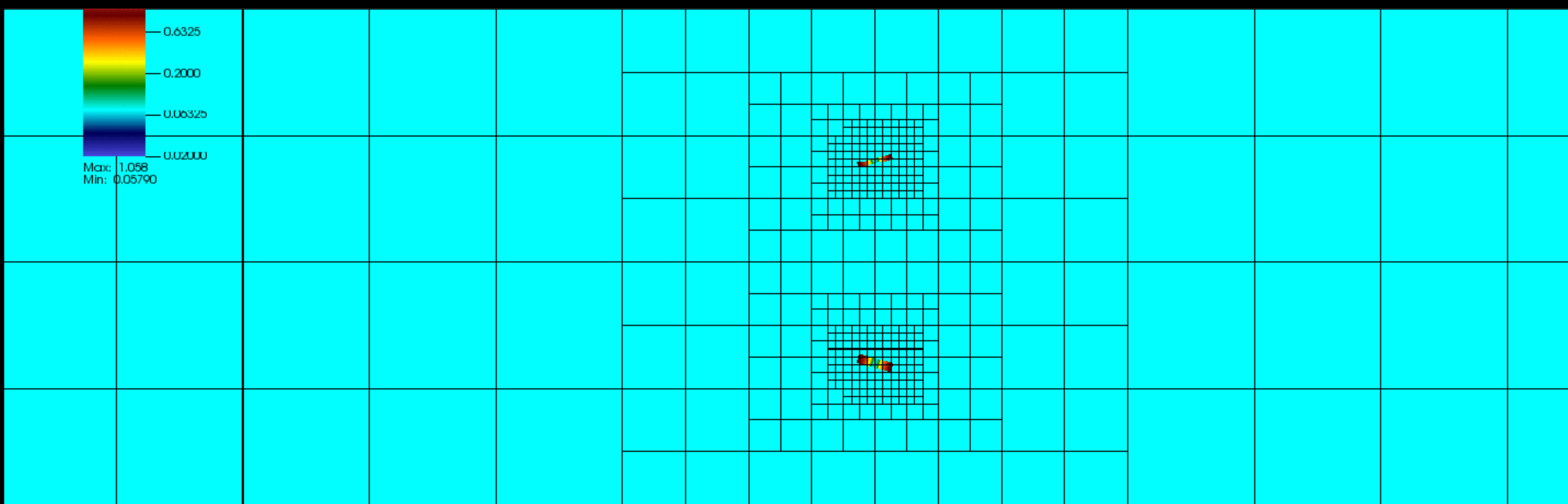
- **Stepper halo profile and more massive soliton for simulations with insufficient resolution**

Simulation Resolution Is Critical

- **Insufficient resolution**
  - → Not properly resolve the phase and density oscillation of wave function
  - → Underestimate flow velocity and quantum pressure
  - → Incorrect merger history and halo/soliton profiles
- **Making solitons is easy, but obtaining a correct core-halo relation can be challenging**
  - ◆ More challenging for massive halos and larger FDM particle mass that require higher resolution
  - ◆ Insufficient resolution → halo becomes too compact → gravitational potential becomes too deep → soliton becomes too massive and compact → $M_c \propto M_h^{\beta}$ with $\beta > 1/3$

Adaptive Mesh Refinement (AMR)

- Astrophysical simulations require a large dynamic range
  - $10^4 - 10^9$ spatial scales
  - Uniform-resolution simulations become impractical
- AMR: allow resolution to adjust locally and automatically
  - Problem-specific refinement criteria



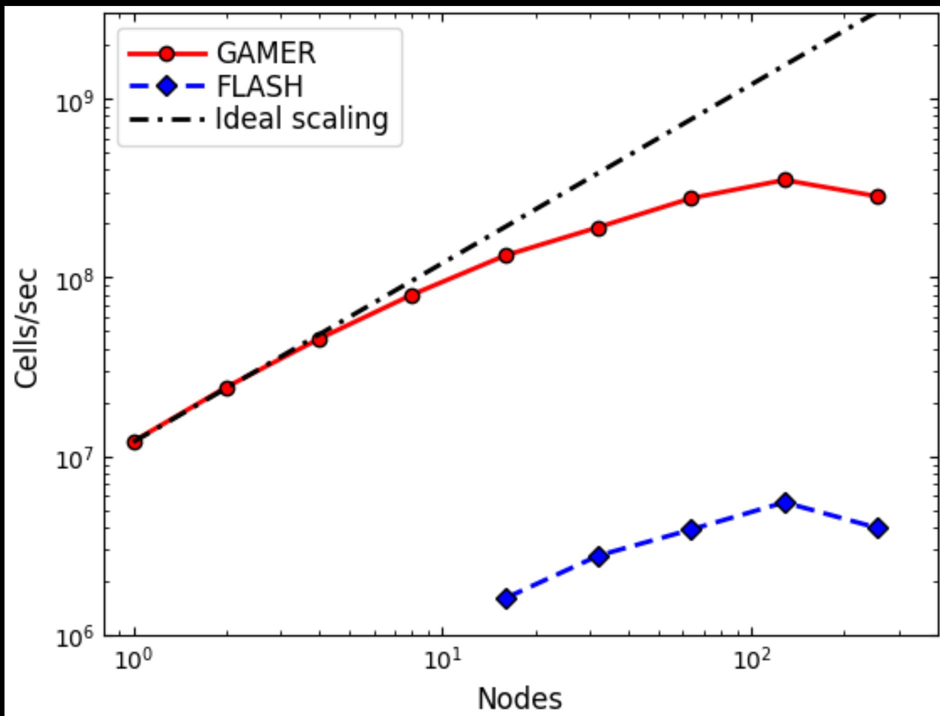
Colliding active galactic nucleus jets using the GAMER code (Molnar, Schive, et al. 2017, ApJ)

GAMER → GPU-accelerated AMR

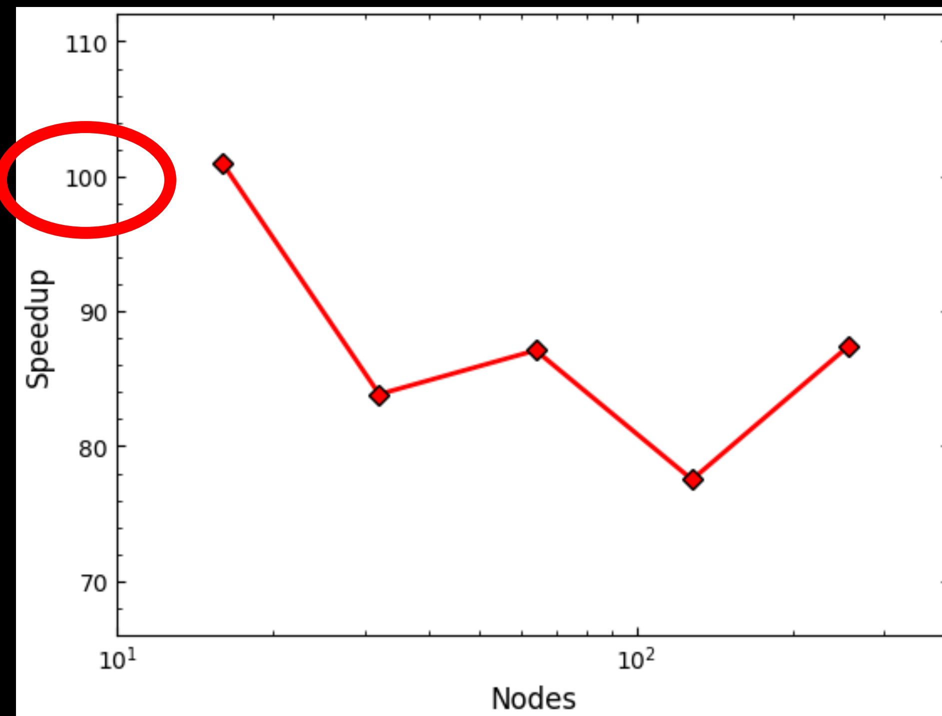
- **Physics modules**
  - FDM
  - Hydrodynamics
  - Magnetohydrodynamics
  - Self-gravity
  - Particles
  - Star formation
  - Chemistry
  - Radiative cooling/heating
- **Features**
  - AMR (no external library)
  - Adaptive time-step
  - Hybrid MPI/OpenMP/GPU
  - Load balance by Hilbert curve
  - Data analysis with *yt*
  - Bitwise reproducibility
  - Open source
    - github.com/gamer-project/gamer
    - FDM module will be released soon
  - Designed from scratch
    - 177,488 code lines as of Mar. 2018

Performance vs. FLASH

Overall performance



GAMER speed-up



Schive et al. 2018, *MNRAS*

- **Two orders of magnitude speed-up**
  - ◆ **Using 16 – 256 nodes (→ 256 GPUs and 8,192 CPU cores)**
- **Similar parallel scalability**
  - ◆ **Speed-up only drops slightly when using more nodes**
  - → **Optimum data communication and load balancing in GAMER**

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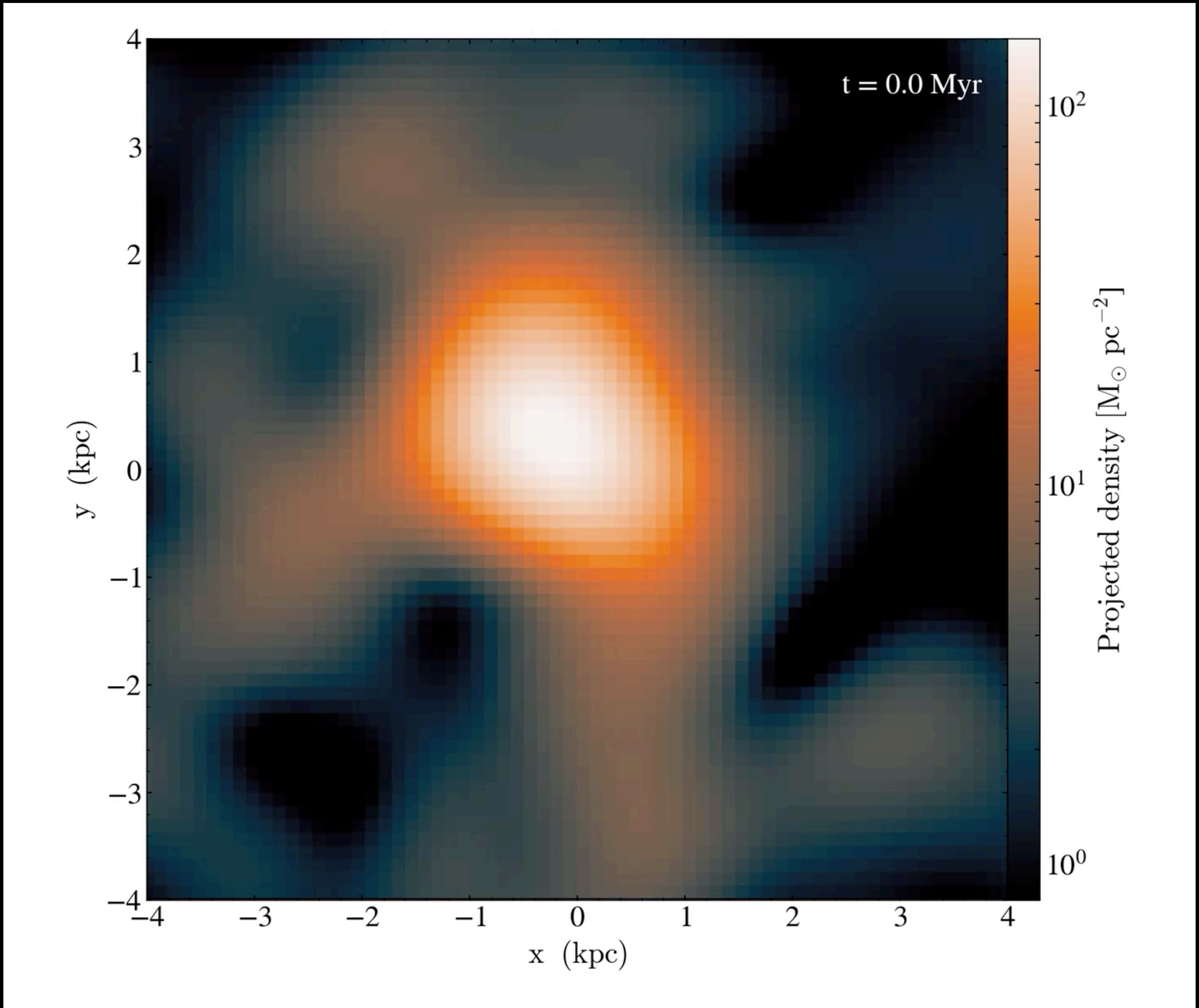
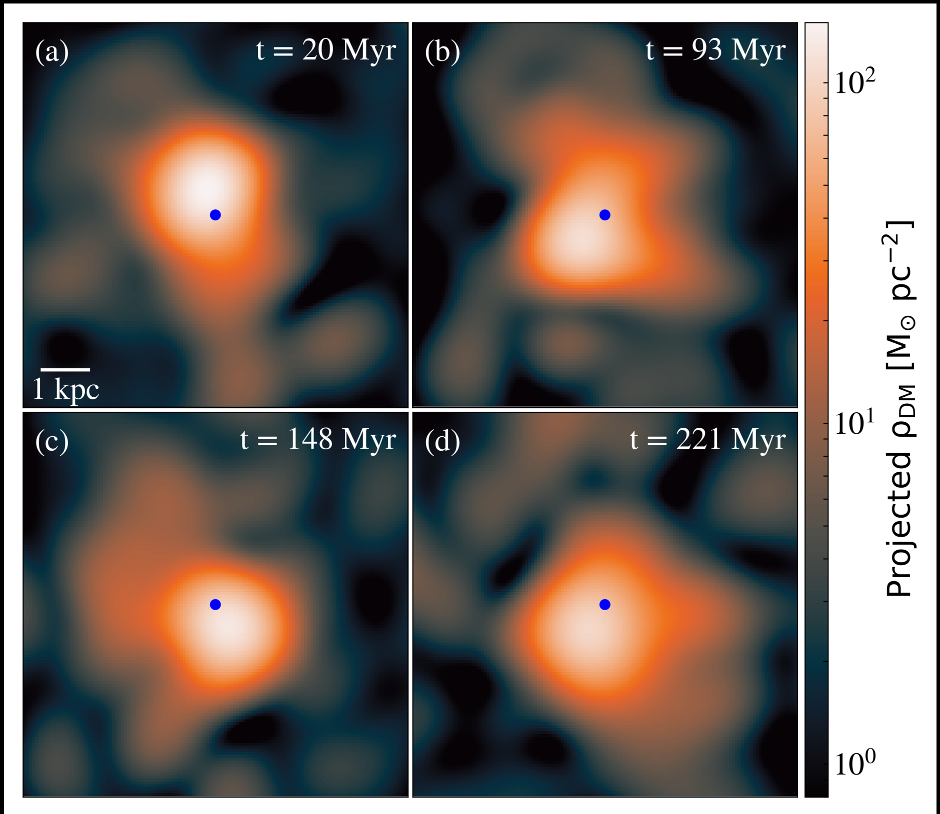
Soliton Random Walk

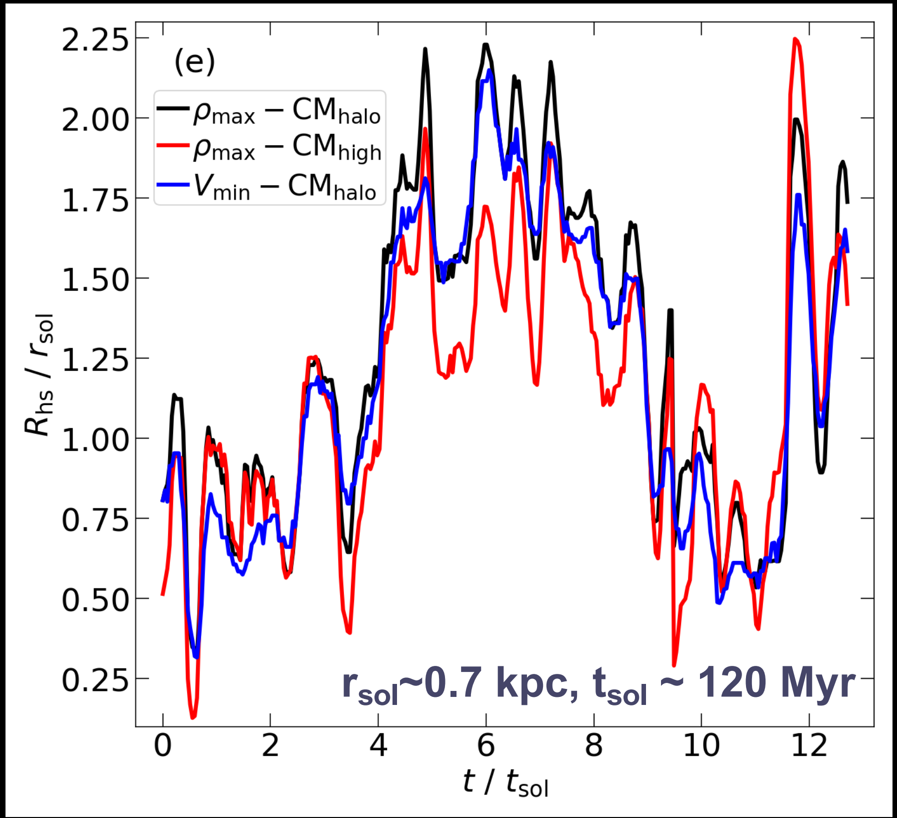


Soliton Random Walk

Projected dark matter density

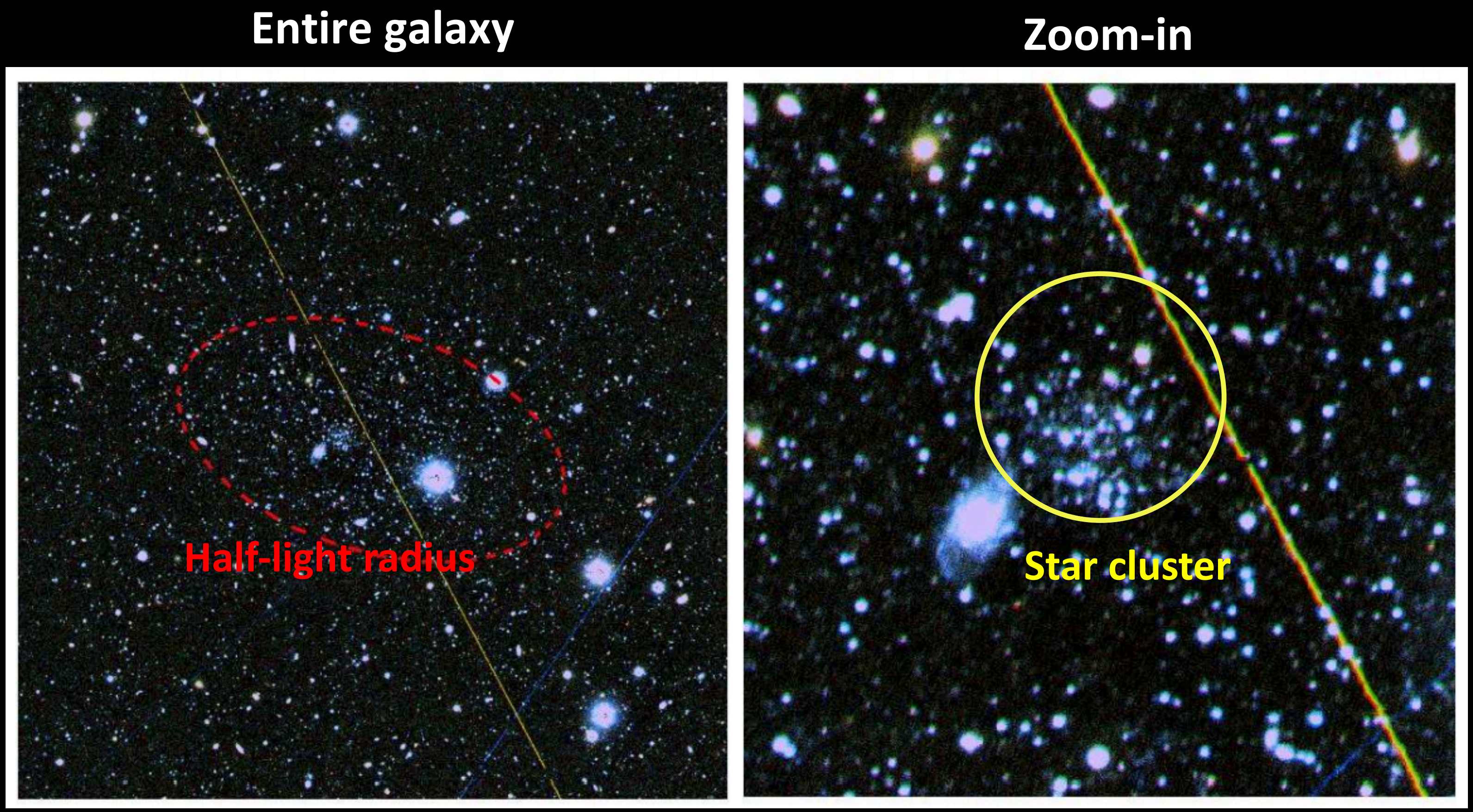


Offset between soliton and halo



- **Feature**: confined random walk at the base of the halo potential
- **Characteristic scales**: close to the length and time scales of soliton itself
- **Numerical robustness**: results are insensitive to the definitions of centers

Central Star Cluster in Eridanus II



Crnojevic et al. (2016)

- **Key questions:**
  - **Can this star cluster survive in an FDM halo?**
  - **How does soliton random walk affect its survival?**

Eridanus II Parameters

Baryons (Observation)

- **Ultrafaint dwarf galaxy**
  - Half-light radius: 0.3 kpc
  - Enclosed mass: $1.2\times10^{7}\ M_{\odot}$
  - Mass-to-light ratio: $420\ M_{\odot}/L_{\odot}$
  - Velocity dispersion: 6.9 km/s
  - Galactocentric distance: 370 kpc
- **Central star cluster**
  - Half-light radius (r_{EII}): 13 pc
  - Luminosity: $2\times10^{3}\ L_{\odot}$
  - Age: $\gtrsim$ 3 Gyr

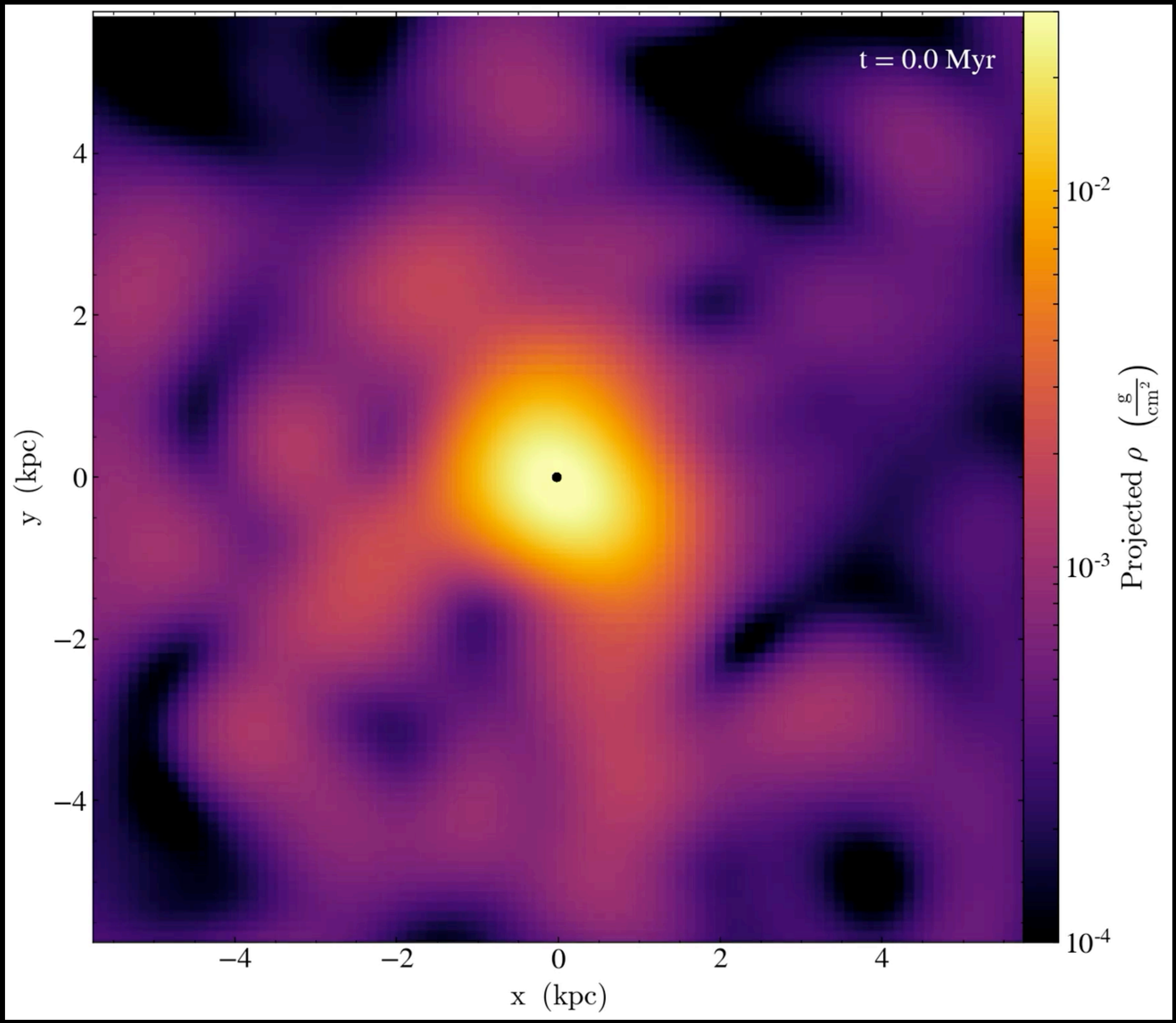
Dark Matter (Simulation)

- **FDM particle mass:** 8x10-23 eV
- **Halo**
  - Virial radius: 50 kpc
  - Virial mass: $6\times10^{9}\ M_{\odot}$
- **Soliton**
  - Half-density radius (r_{sol}): 0.7 kpc
  - Enclosed mass: $1\times10^{8}\ M_{\odot}$
    - Consistent with Eridanus II
  - Peak density: $0.1\ M_{\odot}/pc^{3}$
  - Wave function oscillation period (t_{sol}): 120 Myr

Simulation Setup

- Code: GAMER-2 with adaptive mesh refinement
- Box size: 250 kpc → cover the entire halo
- Model the star cluster by a Plummer sphere
- Resolution
  - Spatial resolution: 3.8 pc
  - Temporal resolution: 150 yr
  - Stellar mass resolution: $3.6\text{x}10^{-2}\ M_\odot$
  - → Well resolve the star cluster
- Numerical robustness
  - Numerical convergence with different resolutions
  - Drift of halo center of mass (caused by numerical errors) $< 10^{-2}\ r_{sol}$

Tidal Disruption of Star Cluster



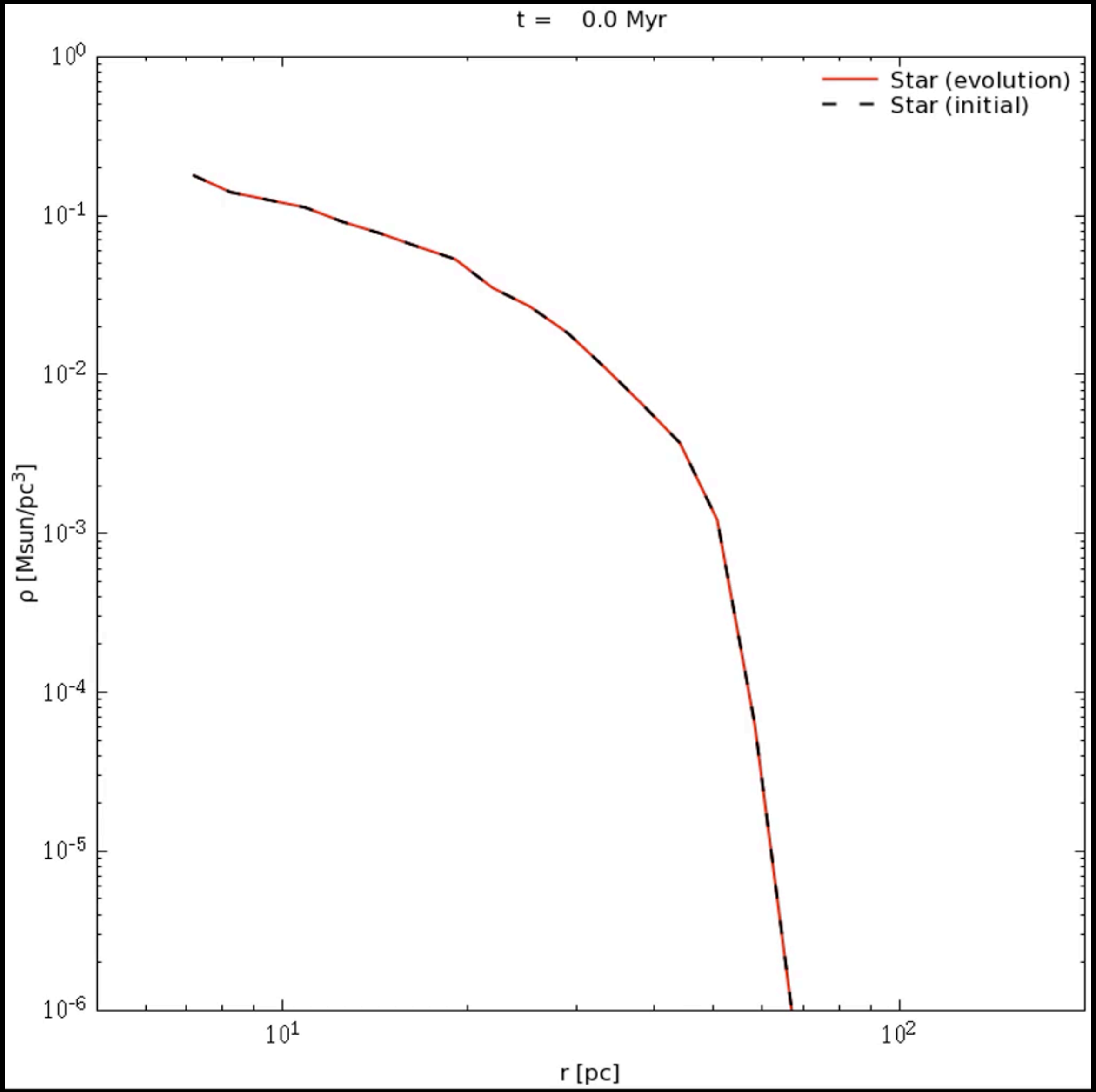
Star cluster traces soliton

Clear separation between star cluster and soliton caused by soliton random walk

Maximum separation ~ r_{sol}

Star cluster is disrupted by the tidal field of soliton within 1 Gyr

Tidal Disruption of Star Cluster



Star cluster traces soliton

Clear separation between star cluster and soliton caused by soliton random walk

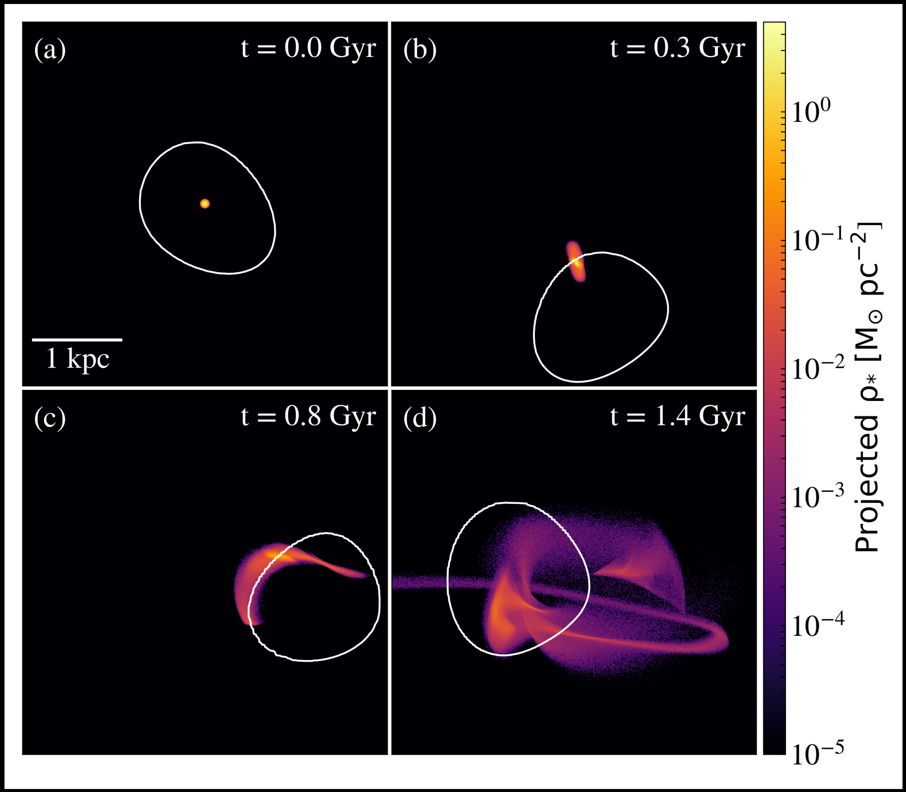
Maximum separation ~ r_{sol}

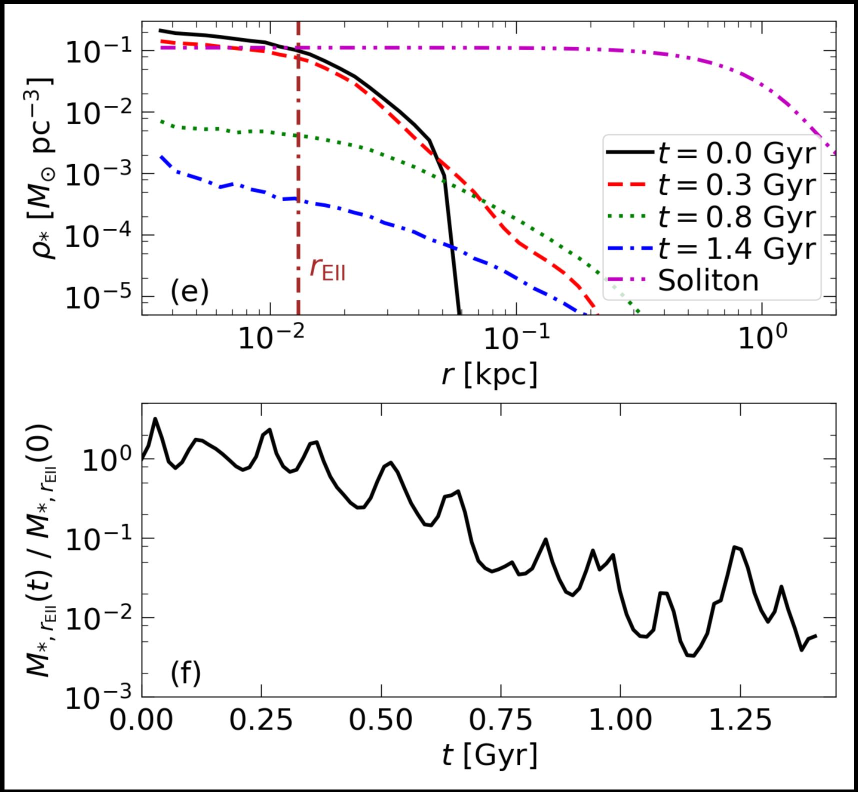
Star cluster is disrupted by the tidal field of soliton within 1 Gyr

Tidal Disruption of Star Cluster

COLOR: Projected stellar mass density
CONTOUR: Soliton

UPPER: Stellar density profiles
LOWER: Enclosed stellar mass fraction

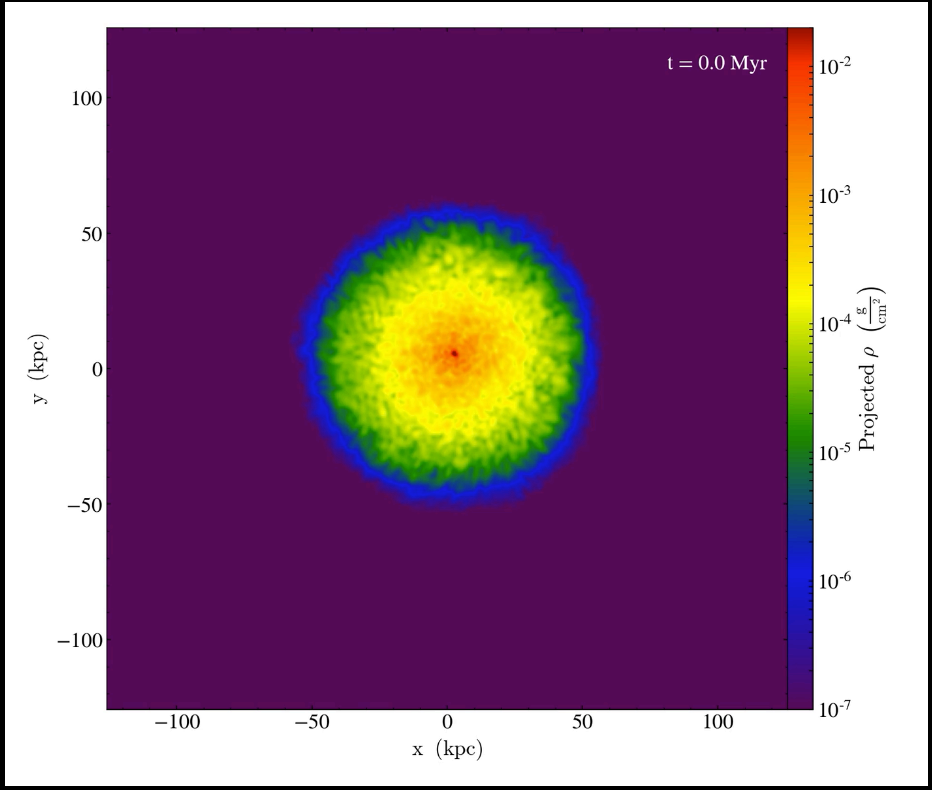




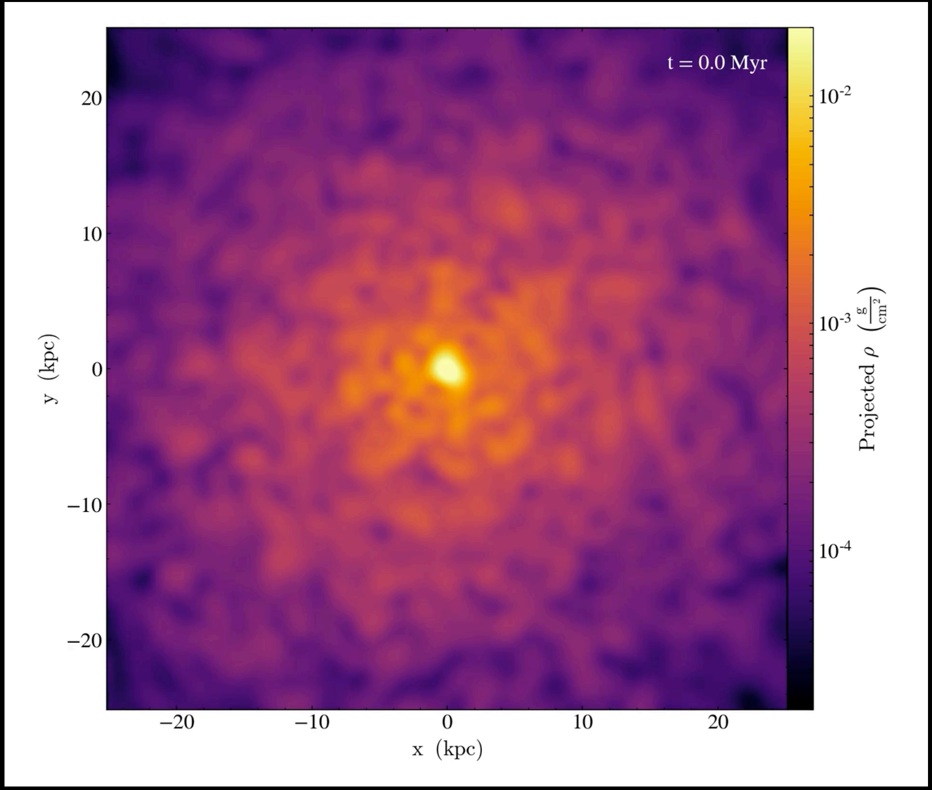
- Flat core → Tidal stripping is most effective at the soliton edge (F_{tidal} ~ F_*)
- Star cluster loses ~99% of mass after ~1 Gyr → Shorter than the estimated minimum age of the star cluster (3 Gyr) → Crisis for FDM !?

Tidal Stripping of Eri II's Halo

Entire halo



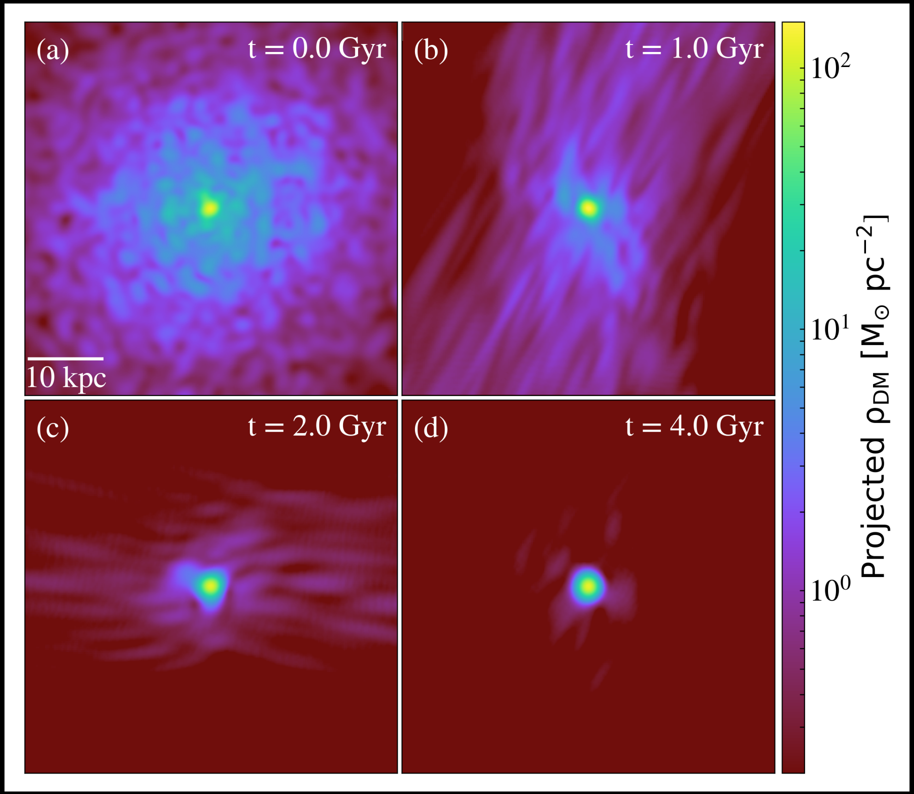
Zoom-in



- **Question**: whether the Milky Way tides can affect the soliton random motion?
- **Simulation setup**:
  - Adding a Milky Way tidal field
  - Circular orbit of radius 100 kpc for Eri II
  - Simulation coordinates move together with Eri II (so Milky Way is orbiting)

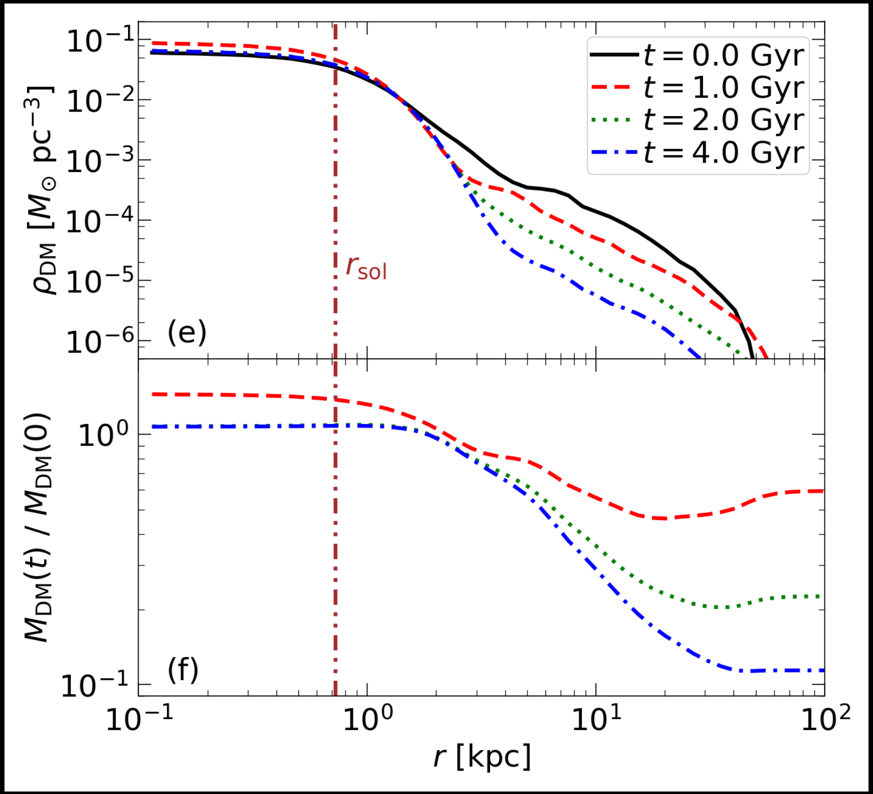
Tidal Stripping of Eri II's Halo

Projected dark matter (DM) density



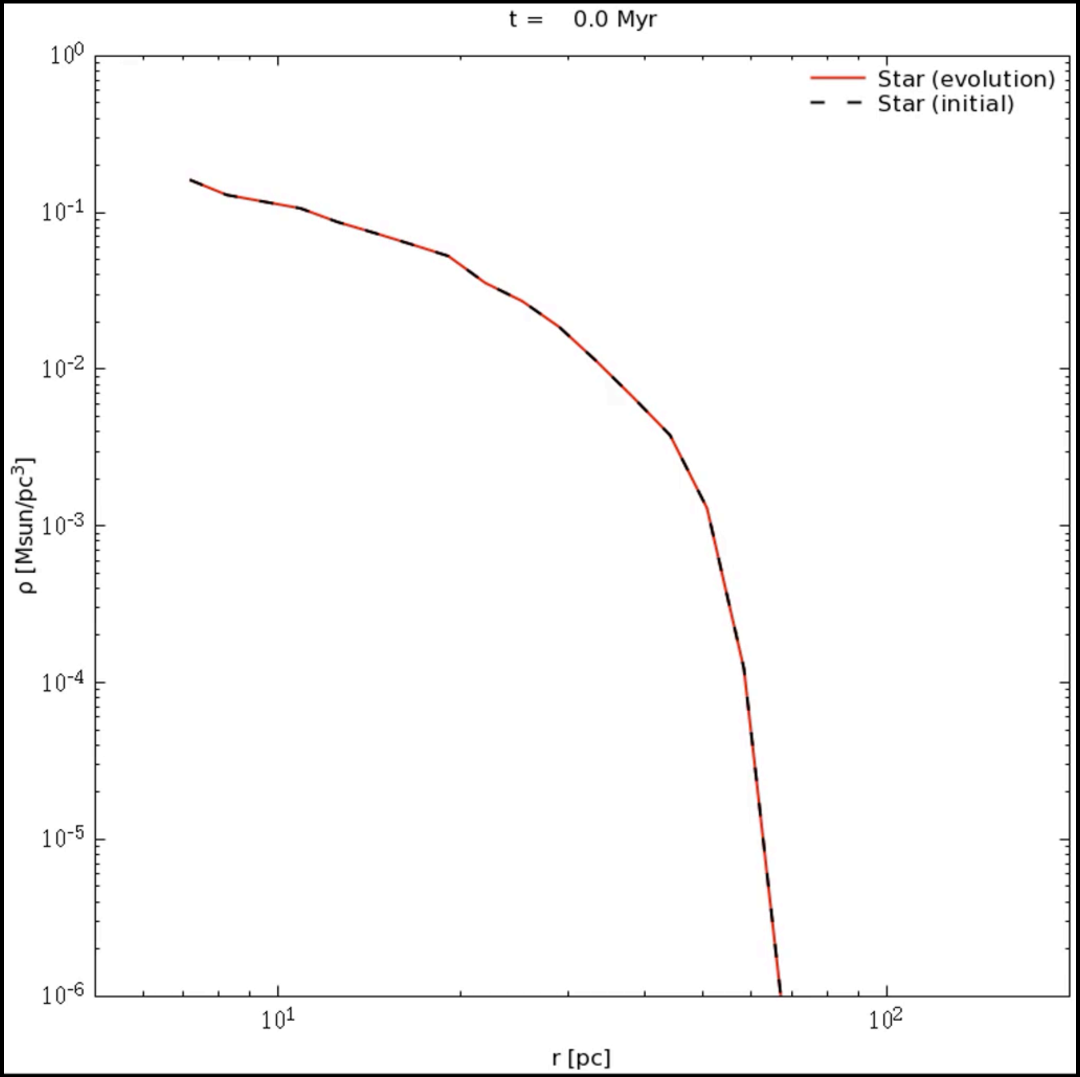
UPPER: DM density profiles

LOWER: Enclosed DM mass fraction



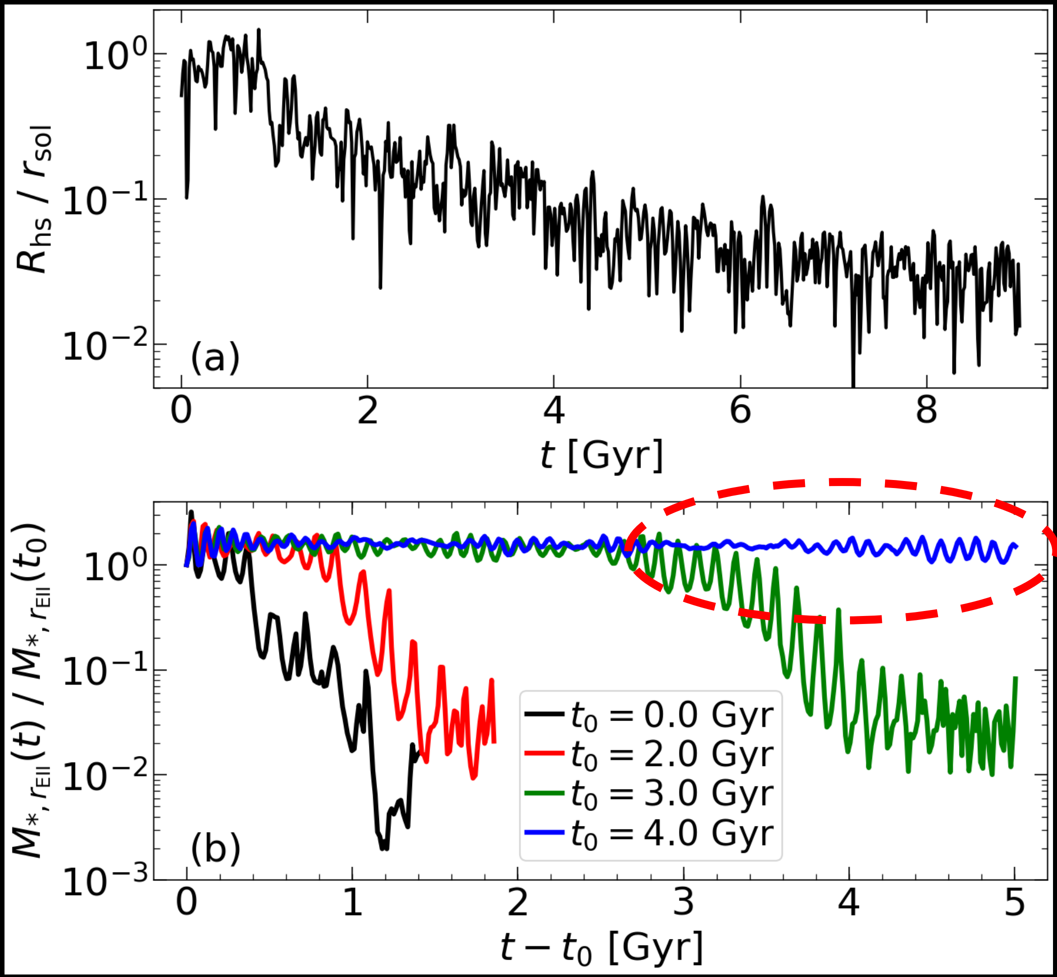
- Soliton remains intact ($\gtrsim$ 9 Gyr)
- FDM halo ($r \gtrsim 3\ r_{sol}$) is tidally stripped after a few Gyr
- Without the interplay between halo and soliton:
  - Soliton random motion is significantly diminished
  - Star cluster and soliton can coexist!

Star Cluster in a Tidally Stripped Halo



- Star cluster **can survive** longer than 5 Gyr in a tidally stripped FDM halo

Star Cluster in a Tidally Stripped Halo



Soliton random motion becomes smaller over time as the halo being tidally stripped

The later the star cluster is added, the more stable it is

For t_0 = 4 Gyr, the star cluster can remain intact for more than 5 Gyr

Avoid the aforementioned FDM crisis!

- **UPPER: Offset between soliton and halo**
- **LOWER: Enclosed stellar mass fraction (adding a star cluster at four different time t_0)**

Open Questions

- More accurate orbital parameters of Eridanus II
  - Is it on its second or third orbit around the Milky Way?
- How does soliton random walk depend on other parameters?
  - FDM particle mass
  - Halo mass
  - Baryonic components
- How does it affect the observational predictions of FDM?
  - Dynamical friction and core stalling in dwarf galaxies
  - Galactic rotation curve
  - Excess mass in the Milky Way center (e.g., central molecular zone)
  - Black hole formation and evolution

Summary

- Simulation resolution is critical in FDM
- Soliton random walk
  - ◆ Caused by the soliton-halo interaction
  - ◆ Characteristic length and time scales close to the soliton itself
- FDM crisis?
  - ◆ Soliton random walk can displace the central star cluster in Eri II slightly outside the soliton

  → Lead to efficient tidal disruption of the star cluster
- Possible solution
  - ◆ Milky Way tides can disrupt the halo of Eri II (but soliton remains intact)
  - ◆ Without the interplay between soliton and halo, the soliton random motion is significantly diminished

  → Star cluster and soliton can coexist