Cold birth and bright death of Bose stars

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Fuzzy Dark Matter Workshop 2020
Your computer, 20/07/2020

DL, A.G. Panin, I.I. Tkachev, arXiv:1804.05857

arXiv:2004.05179

Light bosonic (ALP) dark matter

- **Fits into the galaxy:**

  $(mv)^{-1} \lesssim R \qquad \Rightarrow \boxed{m \gtrsim 10^{-22}\ \text{eV}}$

  $\ll$ in kinetic regime

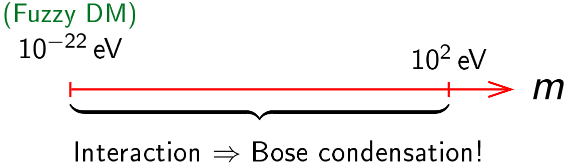
- **Can be only bosonic:**

  $f \sim \dfrac{\rho/m}{(mv)^3} \gtrsim 1 \qquad \Rightarrow \boxed{m \lesssim 10^{2}\ \text{eV}}$

  $\gg$ for classical field

- **Forms BE condensate:**

  $(mv)^{-1} > n^{-1/3} \qquad \Rightarrow \boxed{m \lesssim 10^{2}\ \text{eV}}$



Fornax dwarf galaxy

parameters are known!

$\rho \sim 0.1\, M_\odot/\text{pc}^3$
$R \sim \text{kpc}$
$v \sim 10\ \text{km/s}$

Do they interact strongly enough?

Answer: YES, gravitationally!

Describing light DM

1. $f \gg 1 \Rightarrow$ **classical field $\mathbf{a}(t, \mathbf{x})$**

$$\Box \mathbf{a} + m^2 \mathbf{a} - m^2\, \mathbf{a}^3/6f_a^2 + \ldots = 0$$

self-interaction

2. $v \ll 1 \Rightarrow$ **nonrelativistic axions $E_a \approx m$**

$$\mathbf{a} = f_a\, \psi(t, \mathbf{x})\, \mathrm{e}^{-imt}/\sqrt{2} + \text{h.c.}$$

$$\partial_{t,\mathbf{x}}\psi \ll m\psi$$

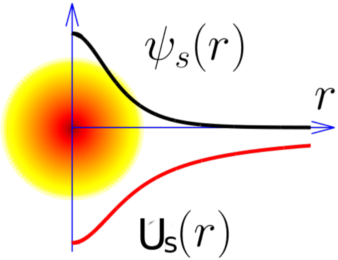
$\Downarrow$

$$i\partial_t \psi = -\Delta\psi/2m + mU\psi - \cancel{mg_4^2|\psi|^2\psi/8}$$

3. **Newtonian gravity:** $U(t, \mathbf{x}) \ll 1$

$$\Delta U = 4\pi G m^2 f_a^2 |\psi|^2$$

Scrödinger-Poisson system: $\psi(t, \mathbf{x})$, $U(t, x)$



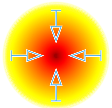
Stationary solutions of SP system

- $\psi = \psi_s(\mathbf{r})\, e^{-i\omega_s t}$
  ↑
  Ground state of U
- $U = U_s(r)$ — potential of ψ_s
- $\omega_s < 0$ — binding energy
- $M = M(\omega_s)$ — only parameter

Ruffini, Bonazolla '69; Tkachev '86

Properties:

1. Bose star = BE condensate in state $\psi_s(\mathbf{r})$
2. Attractive self-interaction: $-g_4^2 < 0$
   Large mass: $M > M_{cr} = 10\, f_a M_{pl} / m g_4$
   } ⇒ collapse



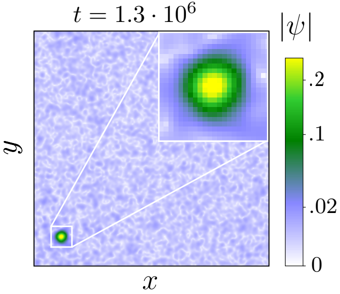
- $\boxed{t = 0}$: **Virialized initial state:**

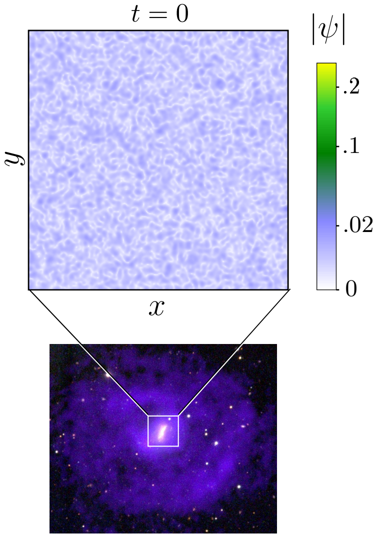
$$\psi_{\mathbf{p}} \propto \underbrace{e^{-\mathbf{p}^2/2(mv_0)^2}}_{\text{momentum distribution}} \times \underbrace{e^{iA_{\mathbf{p}}}}_{\text{random phases}}$$

Kinetic regime:

$$\boxed{l_{coh} \sim (mv_0)^{-1} \ll R}$$

- **Numerical evolution** $\Rightarrow$





Virialized halo

DL, Panin, Tkachev '18

Unsatisfactory description?

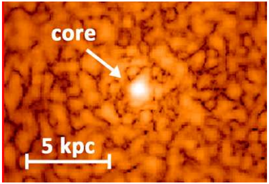
Many simulations:

- Fuzzy DM in dwarf galaxies, $m \sim 10^{-22}$ eV

  Schive et al '14; Veltmaat et al '18

- QCD axions in miniclusters, $m \sim 10^{-5}$ eV

  Eggemeier, Niemeyer '19



What is wrong?

1. Hard to simulate: $mvR \gg 1$
2. No analytic estimates!
3. No prediction for $f_p(x)$

Solution: Kinetic description at $mvR \gg 1$

Kinetic equation: Derivation

Levkov, Panin, Tkachev '18

1. Occupation numbers $\leftrightarrow$ Wigner distribution

$$f_{\mathbf{p}}(t,x) = \int d^3\mathbf{y}\, \mathrm{e}^{-i\mathbf{p}\mathbf{y}} \langle \psi(\mathbf{x}+\mathbf{y}/2)\psi^*(\mathbf{x}-\mathbf{y}/2)\rangle$$

2. Bogolyubov chain of equations

SP system: $\mathbf{i}\partial_{\mathbf{t}}\psi = -\Delta\psi/2m \quad + \quad \underbrace{U\psi}_{G\psi\,\Delta^{-1}\,|\psi|^2}$ $\leftarrow$ short notations

(with Δ, $2m$ and Δ^{-1} struck through)

3. Perturbative approximation
4. Expand in $(px)^{-1} \ll 1 \Rightarrow$ Local equation for $f \propto \langle\psi^*\psi\rangle$
5. Interaction at $p^{-1} \ll |x - x'| \ll R$ — Landau approximation

Kinetic equation: Derivation

Levkov, Panin, Tkachev '18

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2. Bogolyubov chain of equations

   SP system: $\mathbf{i}\partial_{\mathbf{t}}\psi = \psi + \mathbf{G}\psi\psi^*\psi$ $\leftarrow$ short notations

   Infinite chain:

$$\partial_t\langle\psi^*\psi\rangle = \langle\psi^*\psi\rangle + G\langle\psi^*\psi\psi^*\psi\rangle$$
$$\partial_t\langle\psi^*\psi\psi^*\psi\rangle = \langle\psi^*\psi\psi^*\psi\rangle + G\langle\psi^*\psi\psi^*\psi\psi^*\psi\rangle$$
$$\dots$$

3. Perturbative approximation
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2. Bogolyubov chain of equations + perturbative expansion

SP system: $\mathbf{i}\partial_{\mathbf{t}}\psi = \psi + \mathbf{G}\psi\psi^*\psi$ $\leftarrow$ short notations

~~Infinite~~ chain:

$$\partial_t\langle\psi^*\psi\rangle = \langle\psi^*\psi\rangle + G\langle\psi^*\psi\rangle\langle\psi^*\psi\rangle + G\langle\psi^*\psi\psi^*\psi\rangle_{conn}$$
$$\partial_t\langle\psi^*\psi\psi^*\psi\rangle_{conn} = \langle\psi^*\psi\psi^*\psi\rangle_{conn} + G\langle\psi^*\psi\rangle\langle\psi^*\psi\rangle\langle\psi^*\psi\rangle + O(G^2)$$

~~$\vdots$~~

3. Perturbative approximation

$$\langle\psi^*\psi\psi^*\psi\rangle = \underbrace{\langle\psi^*\psi\rangle\langle\psi^*\psi\rangle}_{\text{Free field}} + \underbrace{\langle\psi^*\psi\psi^*\psi\rangle_{conn}}_{\text{Interaction: } O(G)}$$

4. Expand in $(px)^{-1} \ll 1 \Rightarrow$ Local equation for $f \propto \langle\psi^*\psi\rangle$

5. Interaction at $p^{-1} \ll |x - x'| \ll R$ — Landau approximation

Landau kinetic equation

$$\partial_t f_{\mathbf{p}} + \frac{\mathbf{p}}{m}\nabla_x f_{\mathbf{p}} - m\nabla_x \bar{U}\nabla_p f_{\mathbf{p}} + \underset{(t\text{-reversal})}{\text{terms}} = \underbrace{\mathrm{St}\, f_{\mathbf{p}}}_{f_{\mathbf{p}}^3} \sim \frac{f_{\mathbf{p}}}{\tau_{gr}}$$

relaxation time

Bose amplification

cf. Landau, Lifshitz, X

Time to Bose star formation:

$$\tau_{gr} = \frac{b\sqrt{2}}{12\pi^3}\,\frac{m^3 v^6}{G^2 \Lambda\, \rho^2} = \underbrace{\frac{4\sqrt{2}}{\sigma_{gr} f_{\mathbf{p}} v n}}_{\text{kinetic}} \simeq \frac{R}{v}\,(Rmv)^3$$

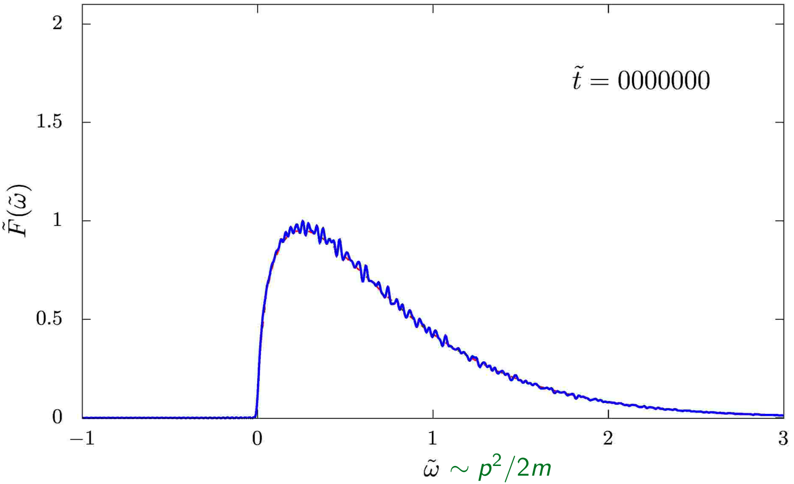
$$\Lambda = \log(mvR) \text{ — Coulomb logarithm}$$

In minicluster/galaxy:

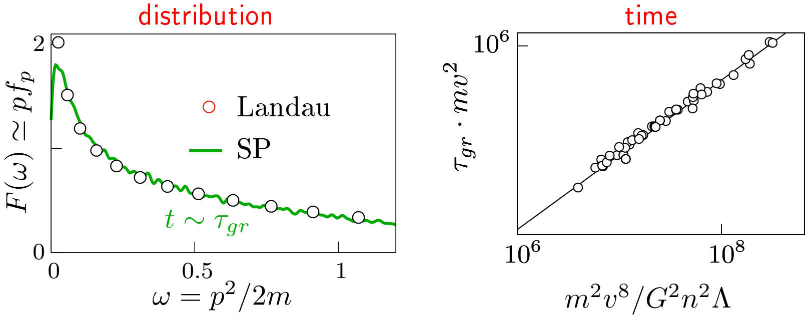
- $\tau_{gr} \gg R/v \leftarrow$ free-fall time
- $Rmv \sim 1$ — condense immediately!
  correct for Fuzzy DM & QCD axions

Distribution function in simulation

$$\underbrace{F(\omega,\ t)}_{pf_p} \equiv \frac{dN}{d\omega} = \int d^3\mathbf{x}\, dt_1\, \psi^*(t,\, \mathbf{x})\psi(t + t_1,\, \mathbf{x})\, \mathrm{e}^{i\omega t_1 - t_1^2/\tau_1^2}$$



- **Solve Landau Eq. numerically!**



One parameter to fit: $\Lambda = \log(mvL) + a \leftarrow$ correction

$a \approx 5$

Conclusions-1

- **We understand kinetic mechanism well**
- Fuzzy DM in Dwarf galaxies, QCD axions in miniclusters $\Big\}$ $\tau_{gr} \sim R/v \leftarrow$ fast formation
- **Still, prediction:** $m \gtrsim 10^{-20}$ eV $\Rightarrow \tau_{gr} > 10^{10}$ yr in dwarf galaxies
- **Q: How do Bose stars grow?**

Eggemeier, Niemeyer '20

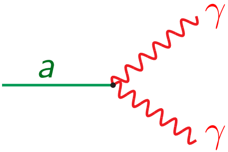
Veltmaat, Schwabe, Niemeyer '20

- **Q: Other kinetic processes?**

ALP: Interaction with photons

$$\mathcal{L}_{\rm int} = \frac{g_{a\gamma\gamma}}{4}\, \mathbf{a}\, F_{\mu\nu}\tilde{F}_{\mu\nu} \qquad \text{or} \qquad \frac{g_{a\gamma\gamma}}{4}\, \mathbf{a}\, F_{\mu\nu}F_{\mu\nu}$$

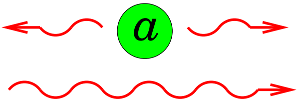
pseudoscalar — scalar



For QCD axions

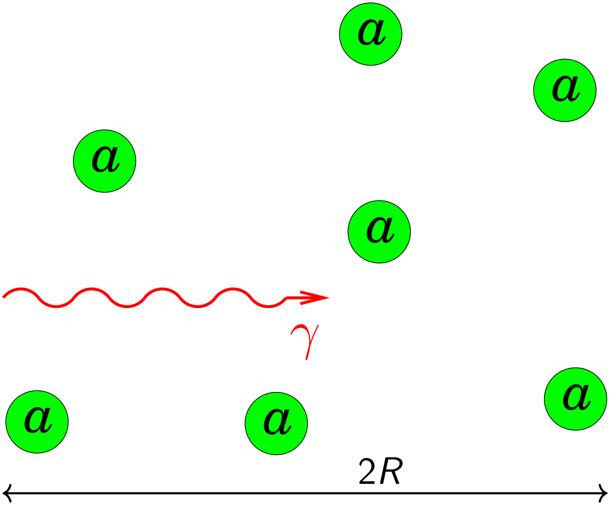
- "Simple" models: $g_{a\gamma\gamma} \sim \alpha/2\pi f_a \lesssim f_a^{-1}$
  KSVZ, DFSZ — unitarity
- "Clockwork": $g_{a\gamma\gamma} \gg f_a^{-1}$

Spontaneous decay



Stimulated decay

Parametric resonance: $a \to \gamma\gamma$ avalanche



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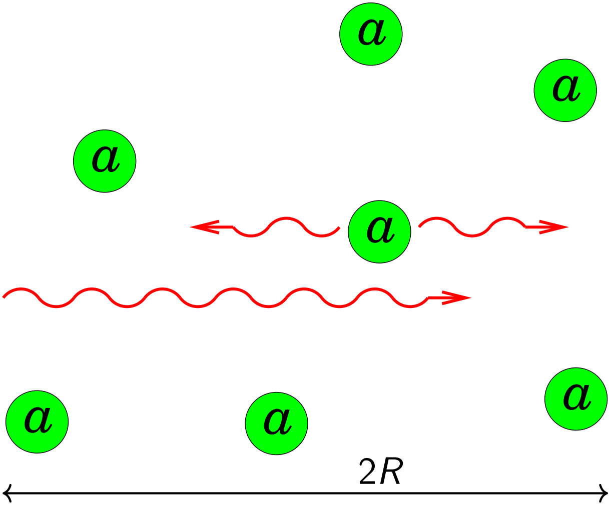

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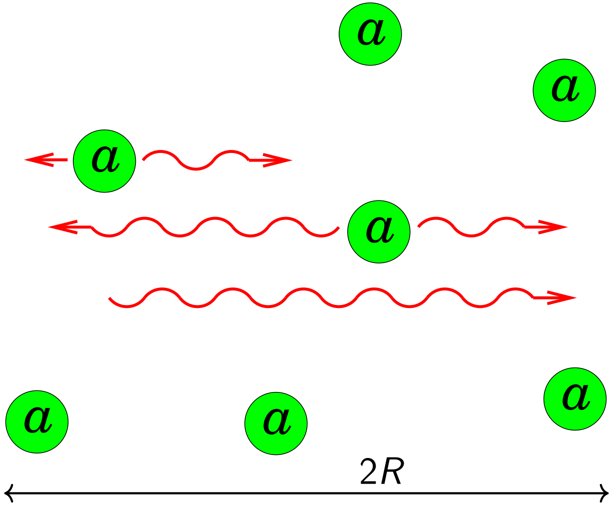

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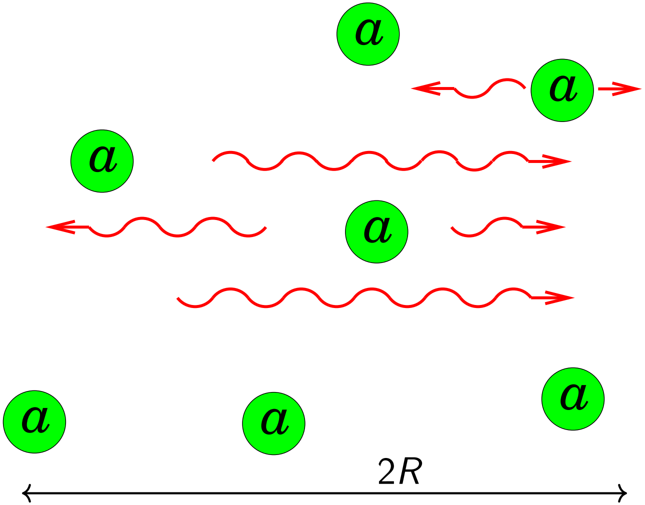
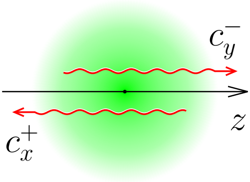


Condition: at least one axion decay due to γ

Tkachev '87; Riotto, Tkachev '00; Hertzberg, Schiappacasse '18

$$\partial_\mu (F_{\mu\nu} + g_{a\gamma\gamma}\, a\, \tilde{F}_{\mu\nu}) = 0$$



1. **Nonrelativistic axions:**

$\gamma : E_\gamma \sim p_\gamma \sim m/2$

$E_a \approx m$

$$A_i = \underbrace{c_i^+(t,x)\, \mathrm{e}^{im(z+t)/2}}_{\text{left-moving}} + \underbrace{c_i^-(t,x)\, \mathrm{e}^{im(z-t)/2}}_{\text{right-moving}} + \text{h.c.}$$

$$\partial_t c_x^+ = \partial_z c_x^+ + i g_{a\gamma\gamma} f_a m\, \psi^* c_y^- / 2^{3/2}$$

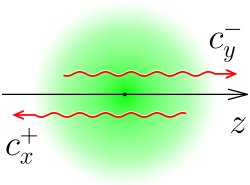
$$\partial_t c_y^- = -\partial_z c_y^- - i g_{a\gamma\gamma} f_a m\, \psi c_x^+ / 2^{3/2}$$

$\leftarrow$ $\partial_{t,x} c_i^\pm \ll m c_i^\pm$

left $\leftrightarrow$ right

2. **Quasi-stationary approximation:**
3. **Boundary conditions:**
4. **Restoring the solution:**

$$\boxed{\partial_\mu (F_{\mu\nu} + g_{a\gamma\gamma}\, a\, \tilde{F}_{\mu\nu}) = 0}$$



1. **Nonrelativistic axions:**

$$\mu \not{\partial}_t c_x^+ = \partial_z c_x^+ + i g_{a\gamma\gamma} f_a m\, \psi^* c_y^- / 2^{3/2}$$

$$\mu \not{\partial}_t c_y^- = -\partial_z c_y^- - i g_{a\gamma\gamma} f_a m\, \psi\, c_x^+ / 2^{3/2}$$

2. **Quasi-stationary approximation:**

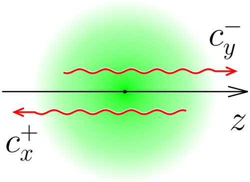
$$\underset{R}{\underset{\wr}{t_\gamma}} \ll \underset{R/v}{\underset{\wr}{t_a}} \quad \Rightarrow \left\{ \begin{array}{l} c_i^\pm \propto \mathrm{e}^{\int^t \mu(t')\, dt'} \\ vR^{-1} \ll \mu \ll R^{-1} \end{array} \right.$$

(start of resonance)

3. **Boundary conditions:**
4. **Restoring the solution:**

Simple equations for photons

$$\partial_\mu(F_{\mu\nu} + g_{a\gamma\gamma}\, a\, \tilde{F}_{\mu\nu}) = 0$$



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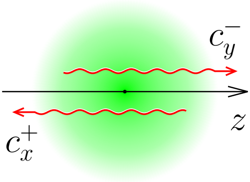
2. **Quasi-stationary approximation:** $vR^{-1} \ll \mu \ll R^{-1}$
3. **Boundary conditions:** $c_i^\pm \to 0$ as $z \to \pm\infty$
   (see the figure)
4. **Restoring the solution:**

Simple equations for photons

$$\partial_\mu(F_{\mu\nu} + g_{a\gamma\gamma}\, a\, \tilde{F}_{\mu\nu}) = 0$$



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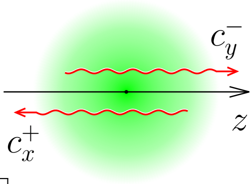
$$A_i = \int d\mathbf{n}_z\; c_i^{(\mathbf{n}_z)}(\mathbf{x})\; \mathrm{e}^{\int^t \mu\, dt' + im(\mathbf{n}_z \mathbf{x} + t)/2} + \text{h.c.}$$

Welcome to Quantum Mechanics!

$$\mathbf{v} = \mathbf{0}, \quad \mu \ll R^{-1}$$



1. **Analytic solution for any ψ!**

$$\left.\begin{aligned} c_x^+ &= A\,\mathrm{e}^{\mu z}\cos D(z) \\ c_y^- &= -iA\,\mathrm{e}^{-\mu z}\sin D(z) \end{aligned}\right\} \times \mathrm{e}^{\int^t \mu\, dt}$$

$$D(z) = g_{a\gamma\gamma} f_a m\, 2^{-3/2} \int_{-\infty}^{z} \psi\, dz'$$

2. **Growth exponent:** $\mu = \dfrac{D(+\infty) - \pi/2}{\int dz\, \sin[2D(z)]}$

3. **Condition for resonance:** $D(+\infty) \geq \pi/2$

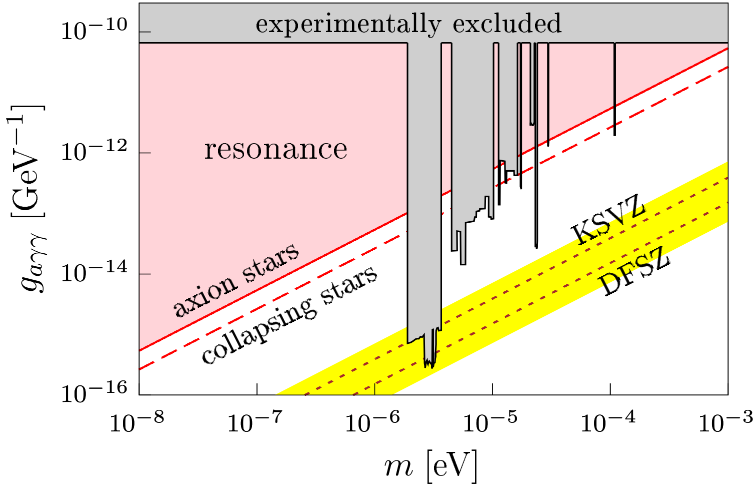
   Need massive Bose stars! $M \geq M_0 = 7.66\, M_{pl}/(m g_{a\gamma\gamma})$

4. **QCD axions:** $M \leq M_{cr} \Rightarrow g_{a\gamma\gamma} \geq 0.31\,/f_a$

   or collapse

Exclusion plot for QCD axions

$g_{a\gamma\gamma} \geq 0.31\,/f_a$

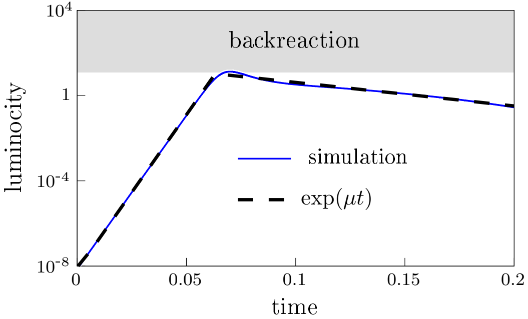


Hertzberg, Schiappacasse '18; DL, Panin, Tkachev '18

- **Solve eqs. for a(t, r)**, $A_\mu(t, \mathbf{x})$

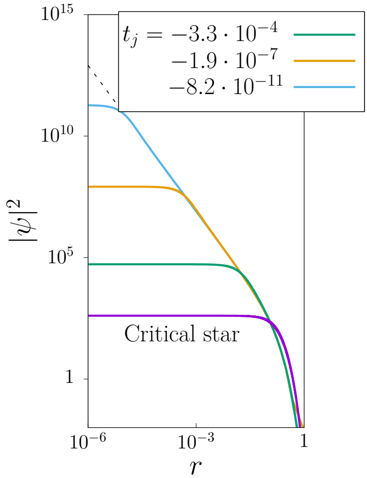
  ↑ sph-symmetric (r) ↑ general (x)

- **Compare with analytics**



$$\mu = \frac{D(+\infty) - \pi/2}{\int dz \, \sin[2D(z)]} = 0.197 \, \frac{m^2}{M_{pl}^2} (M - M_0)$$

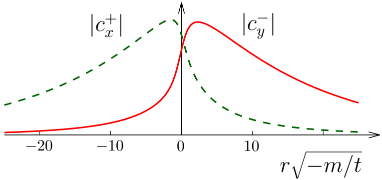
Attractive ψ^4 self-interaction $\Rightarrow$ self-similar collapse (Bosenova)



$$|\psi(\mathbf{r}, \mathbf{t})| = \chi_*(\underbrace{r\sqrt{-m/t}}_{\zeta})/(mrg_4)$$

$\uparrow$ universal

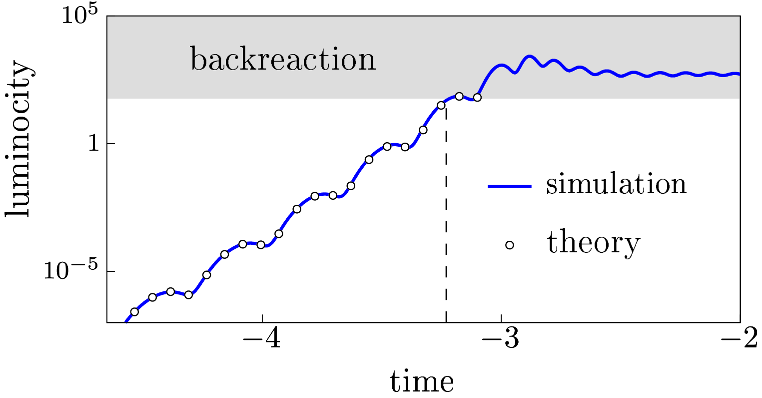
1. **Compute $c^{\pm}(\zeta)$ for $\psi \propto \chi_*$**



Two modes!

2. $\mathrm{Re}\,\mu > 0 \Rightarrow g_{a\gamma\gamma} \geq 0.15\,/\,f_a$
   QCD axions ($\times 2$)

$$\mu = \tilde{\mu}\sqrt{-m/t}$$



Simulation of collapsing star

DL, Panin, Tkachev '20

Conclusions-2

We described:

- **Simple equations for photons & axions @** $\boxed{v \ll 1}$

$$\left.\begin{array}{c}\text{eikonal}\\ +\\ \text{quasi-stationary}\end{array}\right\} \text{approximations} \Rightarrow \left\{\begin{array}{c}\text{Spectral problem}\\ \left[\hat{H} - \mu\right]\begin{pmatrix} c_x^+ \\ c_y^- \end{pmatrix} = 0 \\ \text{Like in QM!}\end{array}\right.$$

- **Analytic solutions** $\Rightarrow$ **exact results for Bose stars**

Applications: (see *DL, Panin, Tkachev '20*)

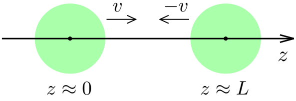
Conclusions-2

We described:

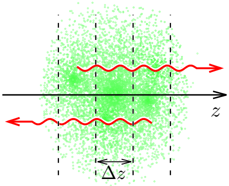
- Simple equations for photons & axions @ $\boxed{v \ll 1}$
- Analytic solutions $\Rightarrow$ exact results for Bose stars

Applications: (see *DL, Panin, Tkachev '20*)

1. **Two stars**
   Resonance is easier: $2D(\infty) \geq \pi/2$
   But: $v \lesssim (mR)^{-1}$, $L > R$

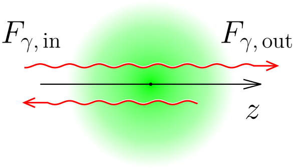


2. **Diffuse axions**
   Coarse-graining $\Rightarrow$ kinetic eq. for n_γ
   $D_{\rm diff} \equiv \frac{g^2_{a\gamma\gamma}}{8} \int \rho(z) l_{coh}(z) dz \geq 1$



3. **Radio amplification, $\omega_\gamma \approx m/2$:**
   $F_{\gamma,\, out} = F_{\gamma,\, in} / \cos^2 D$ — Bose star
   (dominant if $\rho_{\rm stars}/\rho_{\rm diffuse} > 10^{-4}$)

   $F_{\gamma,\, out} = F_{\gamma,\, in}/(1 - D_{\rm diff})$ – diffuse axions



Thank you!