Stochastic fluctuations

of bosonic dark matter and statistical inference

G. Centers[1]   The CASPEr Collaboration

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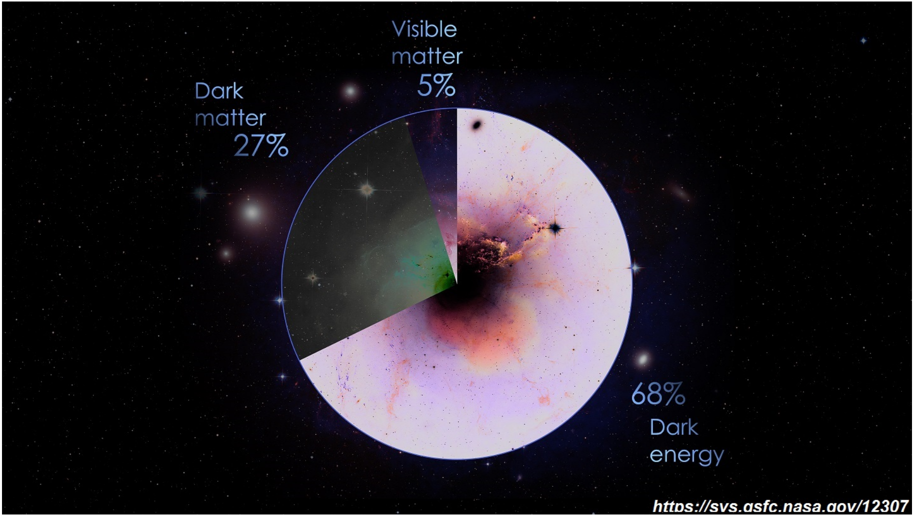
Fuzzy Dark Matter Workshop, 2020

The Dark Matter

It's there, but what is it?



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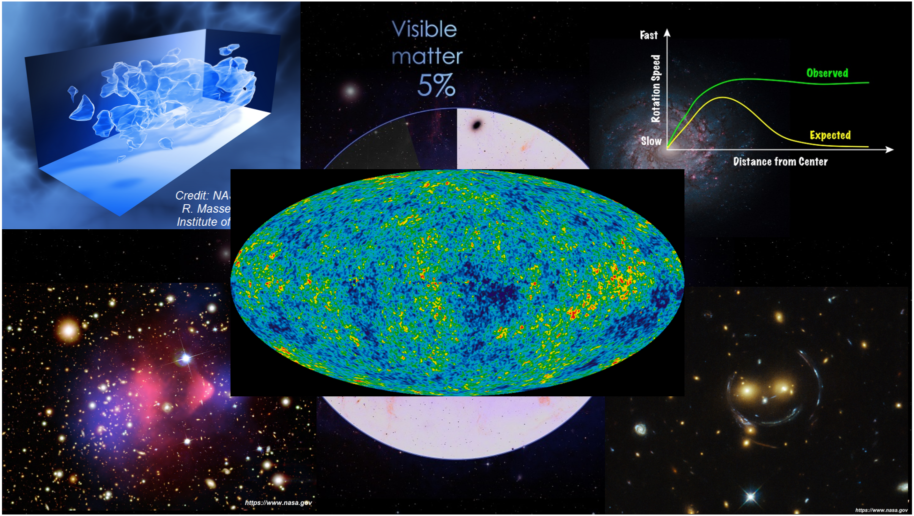

The Budker Group

DM Constraints (see all publications at budker.uni-mainz.de)

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Global Network of Optical Magnetometers for Exotic physics searches (GNOME):

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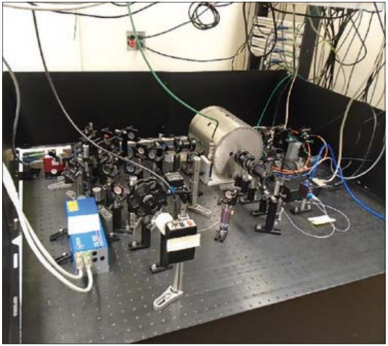

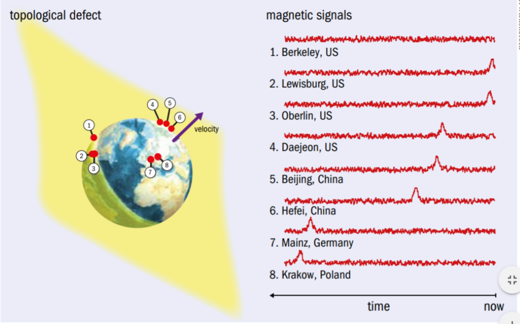

A Wickenbrock

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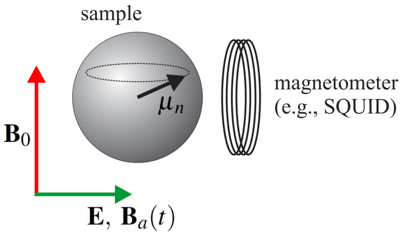

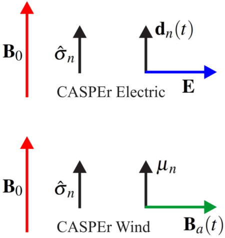

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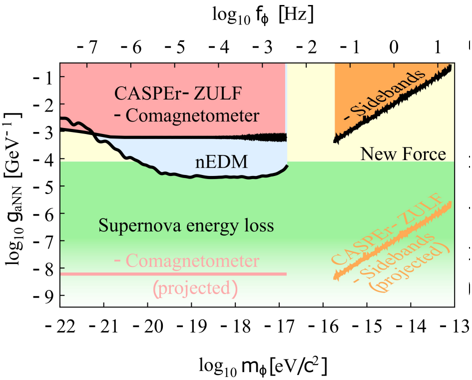

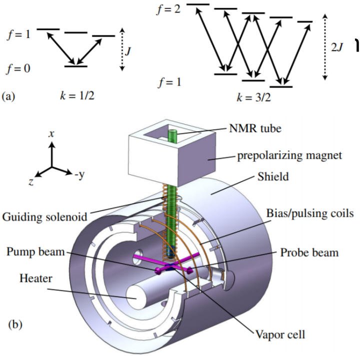

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Bosonic Dark Matter

Stochastic fluctuations summary

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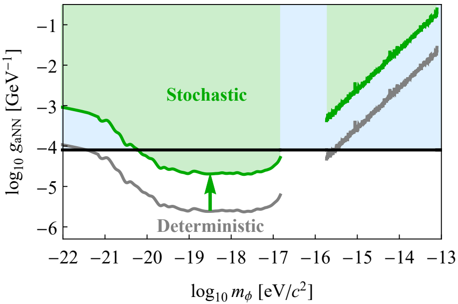

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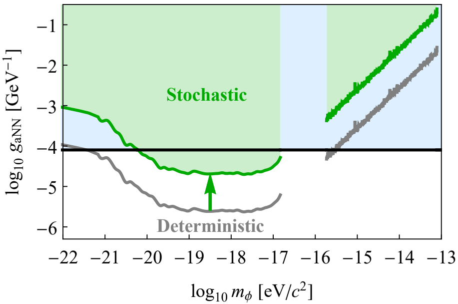

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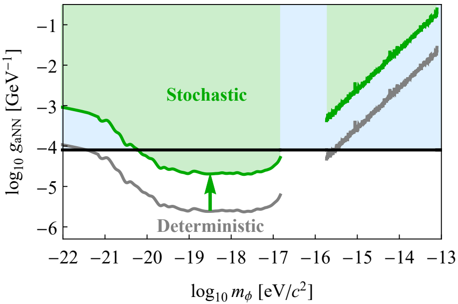

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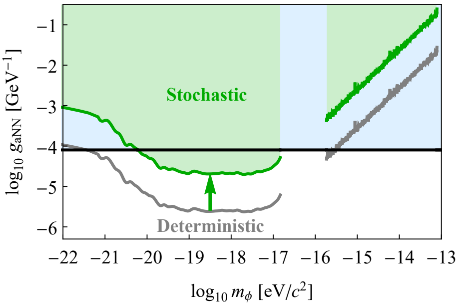

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Virialized UltraLight Fields (VULFs)

For $T \ll \tau_c$ where $\tau_c \equiv \left(f_c v_{\mathrm{vir}}^2/c^2\right)^{-1}$ with $f_c = m_\phi c^2 h^{-1}$:

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$$\text{scalar:} \qquad s(t) = \gamma\xi\phi(t) = \gamma\,\xi\,\Phi_0\,\cos(2\pi f_\phi t + \theta)$$

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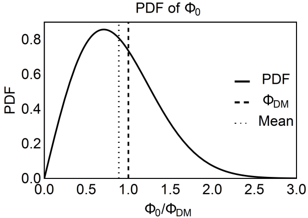
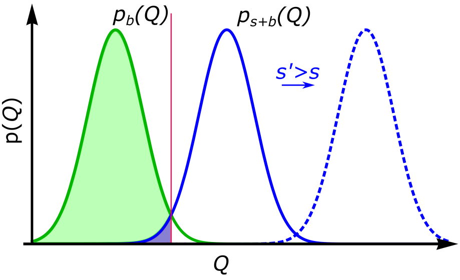
$$\boxed{\Phi_0 \rightarrow \Phi_{\mathrm{DM}} = \hbar(m_\phi c)^{-1}\sqrt{2\rho_{\mathrm{DM}}}}$$

$$\rightarrow p(\Phi_0|\Phi_{\mathrm{DM}}) = \begin{cases} \delta(\Phi_0 - \Phi_{\mathrm{DM}})\,, & \text{deterministic} \\ \frac{2\Phi_0}{\Phi_{\mathrm{DM}}^2}\exp\left(-\frac{\Phi_0^2}{\Phi_{\mathrm{DM}}^2}\right)\,, & \text{stochastic} \end{cases}$$



Comparing one sides limits

General/frequentist hypothesis testing



- Red line is "detection threshold," defined by choice of α the Type-I error (false-positive rate under null hypothesis)
- Excluded parameter space for $s' > s$ when Type-II error $\beta < \alpha$ (false-negative rate given alternative hypothesis)

Comparing one sides limits

Lindley's paradox

$$\text{scalar:} \quad s(t) = \gamma\xi\phi(t) = \gamma\,\xi\,\Phi_0\,\cos(2\pi f_\phi t + \theta)$$

$$p(\Phi_0|\Phi_{\rm DM}) = \begin{cases} \delta(\Phi_0 - \Phi_{\rm DM})\,, & \text{deterministic} \\ \frac{2\Phi_0}{\Phi_{\rm DM}^2}\exp\left(-\frac{\Phi_0^2}{\Phi_{\rm DM}^2}\right)\,, & \text{stochastic} \end{cases}$$

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$$\implies \boxed{\text{Determine } \gamma_{95\%}^{\rm stoch}/\gamma_{95\%}^{\rm det}}$$

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$\Longrightarrow$ Determine $\gamma_{95\%}^{\rm stoch}/\gamma_{95\%}^{\rm det}$

Approach	$\gamma_{95\%}^{\rm stoch}/\gamma_{95\%}^{\rm det}$
Frequentist	≈ 3
Bayesian	≈ 10

- Expected? Bayesian and frequentist inference try to answer different questions - Lindley's paradox

Bayesian inference

Basic probability

Conditional probability, Bayes Theorem:

$$p(B|A) = p(A|B)p(B)/p(A)$$

Bayesian inference

Basic probability

Conditional probability, Bayes Theorem:

$$p(\gamma|D) = \mathcal{L}(D|\gamma)p(\gamma)/p(D)$$

- $\mathcal{L}(D|\gamma)$ is the likelihood
- $p(\gamma)$ is the prior probability distribution
- $p(\gamma|D)$ is the posterior distribution for γ
- $p(D)$ can be treated as a normalization constant

Bayesian inference

Basic probability

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- $\mathcal{L}(D|\gamma)$ is the likelihood
- $p(\gamma)$ is the prior probability distribution
- $p(\gamma|D)$ is the posterior distribution for γ
- $p(D)$ can be treated as a normalization constant
- Bayesian vs. frequentist inference

Bayesian inference

Nuisance parameters

How to deal with them?

$$p(\gamma|D) = \mathcal{L}(D|\gamma)p(\gamma)/p(D)$$

Bayesian inference

Nuisance parameters

How to deal with them?

$$p(\gamma, \Phi_0|D) = \mathcal{L}(D|\gamma, \Phi_0)p(\gamma, \Phi_0)/p(D)$$

$$p(\gamma|D) = \int d\Phi_0 p(\gamma, \Phi_0|D)$$

- Straightforward approach within Bayesian framework
- There are several frequentist approaches: profile likelihood, marginalized likelihood, MC based approaches, etc.

The uniform prior

Something curious

- Results depend on choice of variable (Data in power vs. amplitude e.g.)
- Posterior is improper (divergent normalization) when working in power + nuisance variable marginalization
- Objective priors are one potential solution! See:
  - Kass and Wasserman, Journal of the American Statistical Association 91, 1343 (1996)
  - Berger and Bernardo (1992)

Choice of prior is critical!

Resolution of Lindley's paradox

Approach	$\gamma_{95\%}^{\mathrm{stoch}}/\gamma_{95\%}^{\mathrm{det}}$
Frequentist	≈ 3
Bayesian ($p(\gamma) = 1$)	≈ 10
Bayesian ($p(\gamma)$ =objective)	≈ 3

- Objective priors yield favorable properties of posterior distributions
- Using the Berger-Bernardo reference prior (similar to Jeffreys' prior but more general) Bayes/freq. agree
- For pseudoscalar coupling correction factor is up to 8.4

Acknowledgments

HELMHOLTZ GEMEINSCHAFT
Helmholtz-Institut Mainz

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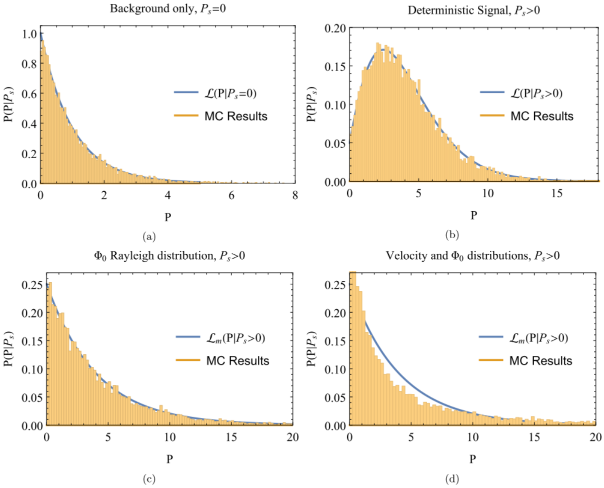
Pseudoscalar case

Brute-force Monte Carlo

pseudoscalar: $s(t) = \gamma\xi\Phi_0 m_\phi \cos(2\pi f_\phi t + \theta)\vec{v}\cdot\vec{e}$

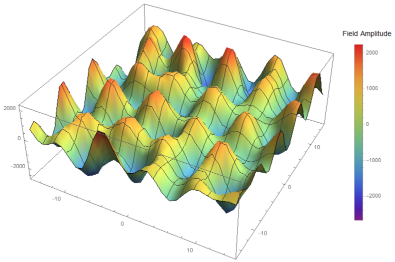


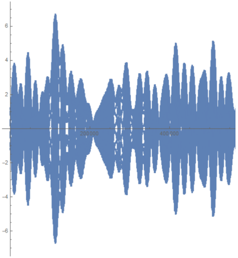
Correction factor up to 8.4

Chaotic light analogy

Rayleigh distribution and random phase

- Add N plane waves in a box: $\sum_{j=0}^{N} \exp -i(\omega_a t + \vec{k_j} \cdot \vec{x_j} + \phi)$
- Resulting field amplitude is just a random walk in the complex plane
- Quick simulation: $\omega_a = 2\pi$, $N = 1000$, $|v|$ is MB dist. $v_{avg} = 10^{-3}$





See Foster/Safdi 2018