TMD splitting kernels project

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January 16, 2020

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Recap: Motivation

We want to implement TMD splitting functions in the PB method.
Why?

- Correct treatment of kinematics at each branching
- Connection with small-x
- Reproduces Kernels of different evolution equations in different limits

Evolution equations

$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu_0^2) + \sum_b \int \frac{d^2\boldsymbol{\mu}'_\perp}{\pi\mu'^2}\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)}\Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\times$$
$$\times\int_x^{z_M} dz\, P_{ab}^R(z)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp+(1-z)\boldsymbol{\mu}'_\perp,\mu'^2)$$

Insert TMD splitting functions

$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2) = \Delta_a(\mu^2,\mathbf{k}_\perp)\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu_0^2) + \sum_b \int \frac{d^2\boldsymbol{\mu}'_\perp}{\pi\mu'^2}\frac{\Delta_a(\mu^2,\mathbf{k}_\perp)}{\Delta_a(\mu'^2,\mathbf{k}_\perp)}\Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\times$$
$$\times\int_x^{z_M} dz\, \tilde{P}_{ab}^R(z,\mathbf{k}_\perp+a(z)\boldsymbol{\mu}'_\perp,a(z)\boldsymbol{\mu}'_\perp)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp+a(z)\boldsymbol{\mu}'_\perp,\mu'^2)$$

P: probabilty to split Δ: probability not to split

Evolution equations

$$\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu_0^2) + \sum_b \int \frac{d^2\boldsymbol{\mu}'_\perp}{\pi\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2) \times$$
$$\times \int_x^{z_M} dz P_{ab}^R(z) \tilde{\mathcal{A}}_b(\frac{x}{z}, \mathbf{k}_\perp + (1-z)\boldsymbol{\mu}'_\perp, \mu'^2)$$

$$\downarrow$$

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$$\times \int_x^{z_M} dz \tilde{P}_{ab}^R(z, \mathbf{k}_\perp + a(z)\boldsymbol{\mu}'_\perp, a(z)\boldsymbol{\mu}'_\perp) \tilde{\mathcal{A}}_b(\frac{x}{z}, \mathbf{k}_\perp + a(z)\boldsymbol{\mu}'_\perp, \mu'^2)$$

P: probabilty to split Δ: probability not to split

Intermediate step: collinear Sudakov (earlier work)

Generation of branching scale

$$\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\ z\ P_{ba}^R(z)\right) = \exp\left(-\int_{t_0}^{t} dt'\ f(t')\right)$$

with $f(t) = \sum_b \int_0^{z_M} dz\ z\ P_{ba}^R(z) \rightarrow$ probability that a branching wil happen and $t = \ln(\mu)$

Differential probability that a branching happens:
$\mathcal{P}(t) = -\frac{d\Delta_a}{dt} = f(t)\Delta_a(t) = f(t)\exp\left(-\int_{t_0}^{t} dt\ f(t)\right)$

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Select t through:

$R = \int_{t_0}^{t} \mathcal{P}(t')dt' = \Delta_a(t_0) - \Delta_a(t) = 1 - \Delta_a(t)$

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Select t through:

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In updfevolv code table of $\Delta_a(t)$ is calculated, but $\Delta_a(\mathbf{k}_\perp, t)$ additional parameter$\rightarrow$no table

Veto algorithm

One can use a simplified function to generate the scale
Condition $g(t) \geq f(t)$ for all t
Since $g(t) \geq f(t)$, there will be more branchings. Some branchings will be refused

Algorithm:

1. Start with $i = 0$, $t_i = t_0$
2. i=i+1. Select $t_i > t_{i-1}$ according to g(t)
3. if $f(t_i)/g(t_i) \leq R$ go to 2
4. else: t_i is generated scale

Veto algorithm in the code

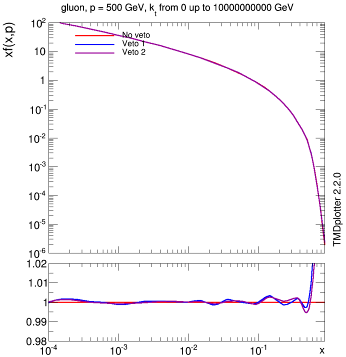
First step to k_T-dep. Sudakov:
Write intermediate step (TMD P, coll. Sudakov) with veto algorithm

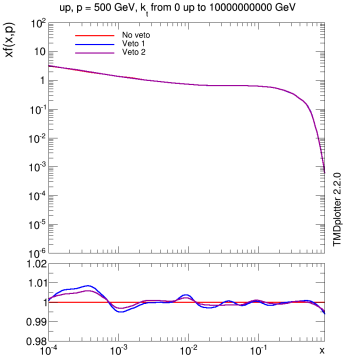
LO Splitting functions$\rightarrow$

$$f_g(t) = \int_0^{z_M} dz\ z\ (P^R_{qg}(z) + P^R_{gg}(z))$$
$$f_q(t) = \int_0^{z_M} dz\ z\ (P^R_{qq}(z) + P^R_{gq}(z))$$

2 different functions in veto algorithms

- Veto 1: $P_{ab} = 2P_{ab}$
- Veto 2: $P_{ab} = P_{ab} + 0.1/z$

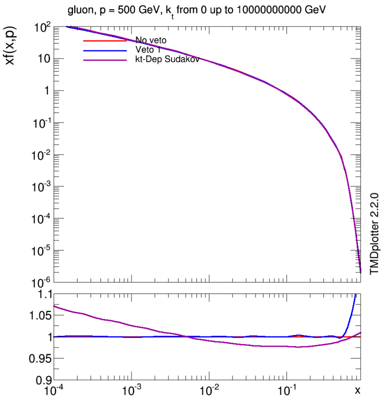


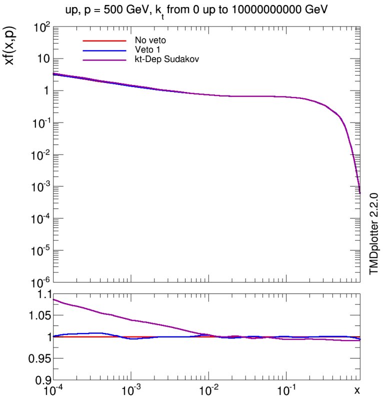


Events 'No veto':1000×10^7 'Veto1/2':200×10^6

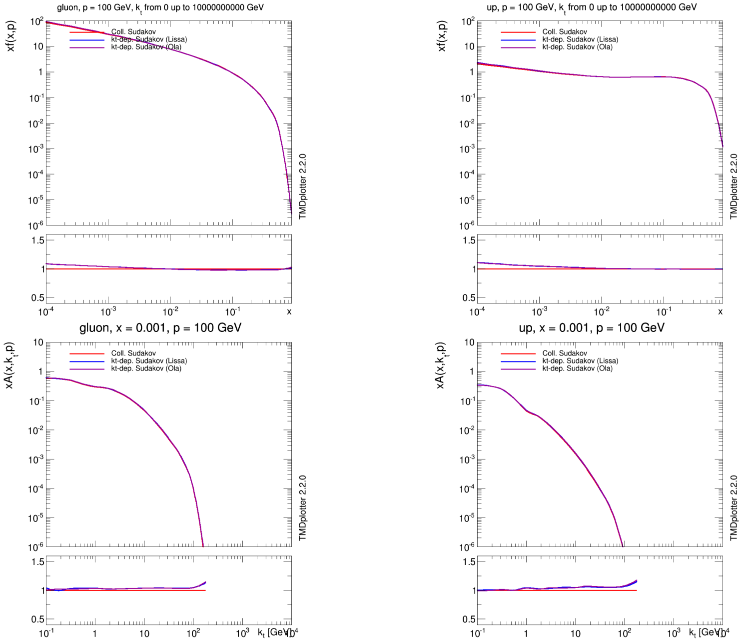
k_T-dep. Sudakov

Implementation of k_T-dep. Sudakov: much larger than effect of veto vs no veto



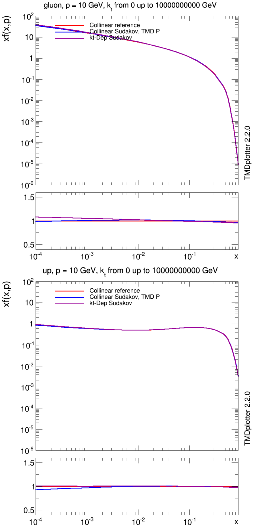


Cross-checked!!

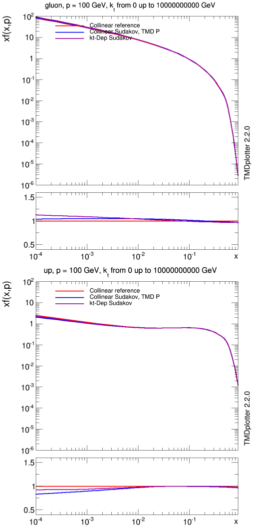


iTMDs vs Scale

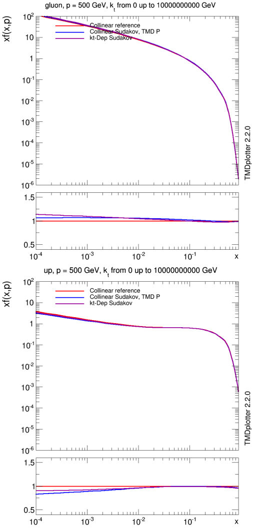
$\mu = 10$



$\mu = 100$



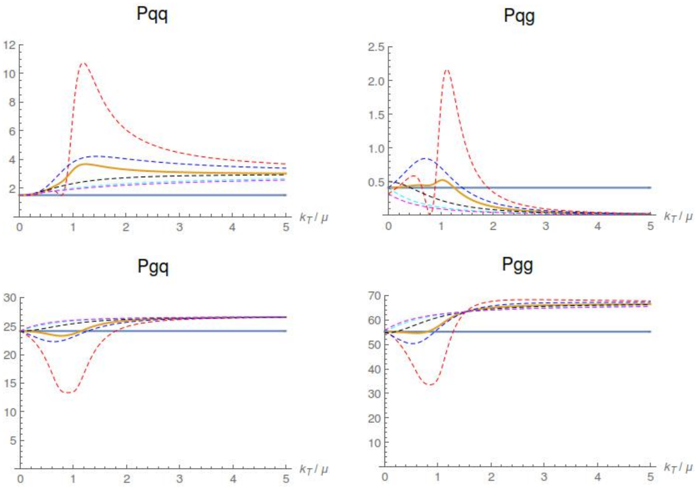
$\mu = 500$

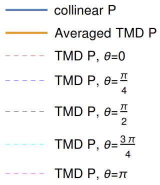


Backup

$k_\perp$-dependence of TMD splitting functions

Splitting functions (except P_{qg}) rise for large $k_\perp$





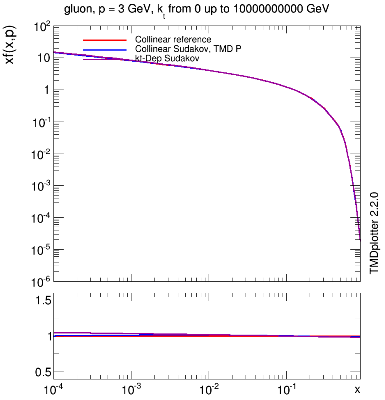
Functions in Veto-algorithm:

Pqq/Pgq/Pqq: Anti-coll+0.1z

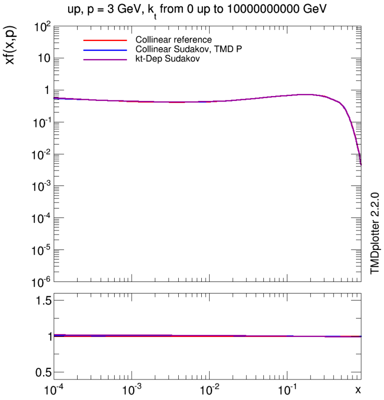
Pqg: Coll+0.1z

$z = 0.1,\ \mu = \frac{p_\perp}{1-z}$ with $p_\perp$ transverse momentum of the emitted parton

iTMDs $\mu = 3$



gluon, p = 3 GeV, k_t from 0 up to 10000000000 GeV
xf(x,p)
Collinear reference
Collinear Sudakov, TMD P
kt-Dep Sudakov
TMDplotter 2.2.0
x



up, p = 3 GeV, k_t from 0 up to 10000000000 GeV
xf(x,p)
Collinear reference
Collinear Sudakov, TMD P
kt-Dep Sudakov
TMDplotter 2.2.0
x

No veto vs Veto: TMDs vs k_T