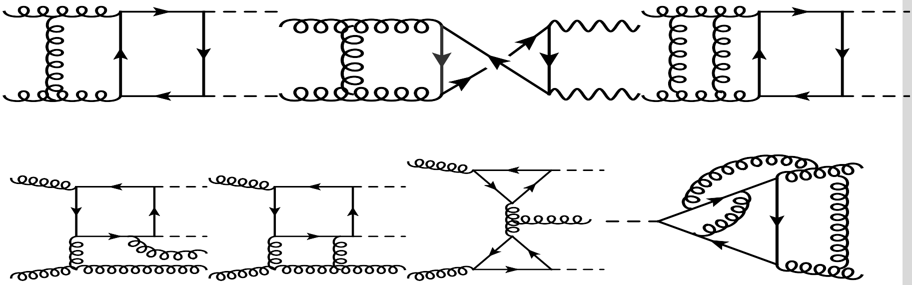
Higher order corrections to gluon fusion processes

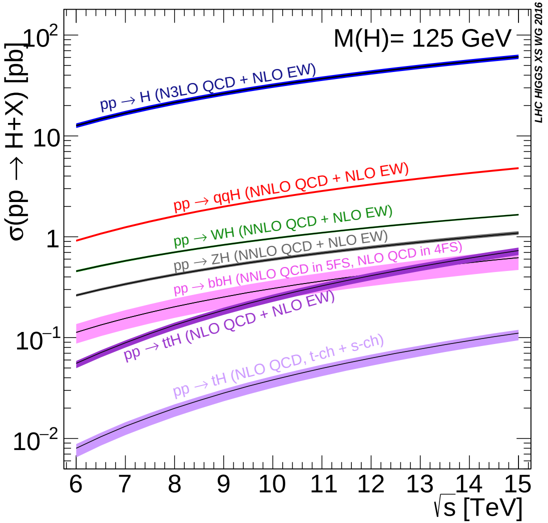
DESY, February 24, 2020

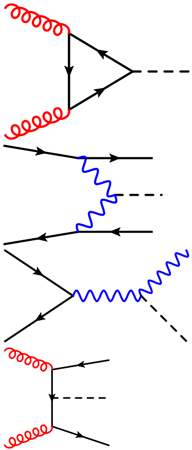
Matthias Steinhauser | in collaboration with J. Davies, F. Herren, G. Mishima, D. Wellmann



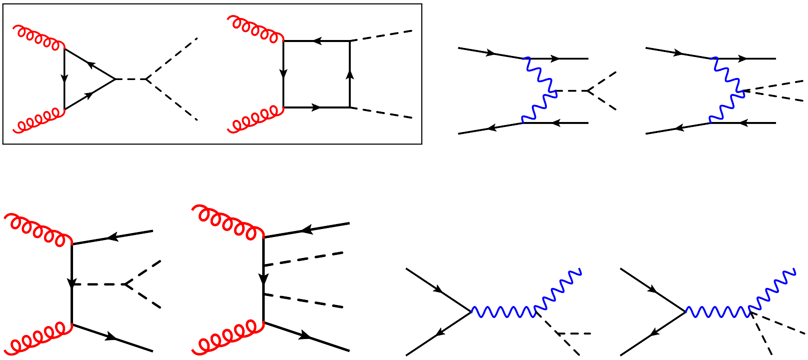
Higgs production in SM

[LHC HIGGS XS WG 2016]





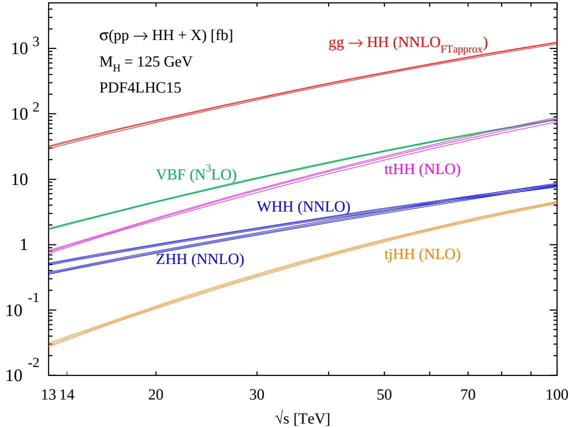
Double Higgs production in SM

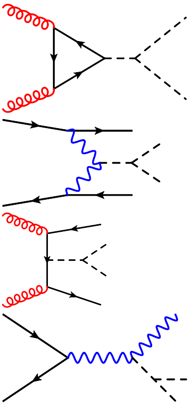


$$\text{SM: } \lambda = m_H^2/(2v^2) \approx 0.13\ldots$$

Double Higgs production in SM (2)

[Micco et al., 1910.00012]



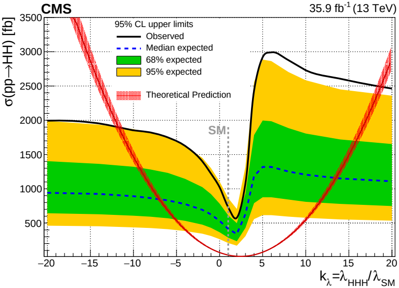


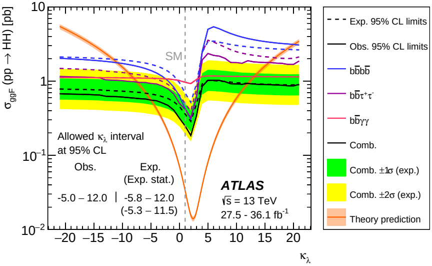
$K^{\rm NLO} \approx 1.9$

$K^{\rm NNLO} \approx 1.2$

λ from $gg \to HH$

combine several channels: $b\bar{b}b\bar{b}$, $b\bar{b}\tau^+\tau^-$, $b\bar{b}\gamma\gamma$, $b\bar{b}VV$





$-11.8 < \lambda/\lambda_{\rm SM} < 18.8$ $\qquad$ $-5.0 < \lambda/\lambda_{\rm SM} < 12.0$

HL-LHC: $\mathcal{O}(50-100\%)$

FCC-hh: $\mathcal{O}(5\%)$

$gg \to HH$ results

LO [Glover, van der Bij'88; Plehn,Spira,Zerwas'96]

NLO exact (numerical): [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]

[Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher'18]

$m_t \to \infty$ [Dawson,Dittmaier,Spira'98]

incl. $1/m_t$ terms [Grigo,Hoff,Melnikov,Steinhauser'13; Degrassi,Giardino,Gröber'16]

exact real rad.: [Maltoni,Vryonidou,Zaro'14]

Padé: [Gröber,Maier,Rauh'17]

small-p_T: [Bonciani,Degrassi,Giardino,Gröber'18]

high energy: [Davies,Mishima,Steinhauser,Wellmann'18'19; Mishima'18]

combination exact $\oplus$ high energy [Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]

NNLO $m_t \to \infty$ [de Florian,Mazzitelli'13; Grigo,Melnikov,Steinhauser'14]

incl. $1/m_t$ terms (virtual) [Grigo,Hoff,Steinhauser'15; Davies,Steinhauser'19]

$1/m_t$ terms (real) [Davies,Herren,Mishima,Steinhauser'19]

finite-m_t approx. ...,[Grazzini,Heinrich,Jones,Kallweit,Kerner,Lindert,Mazzitelli'18]

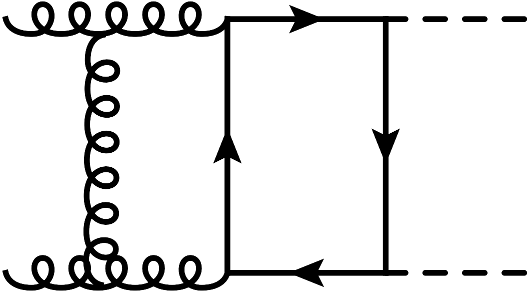
resummations [Shao,Li,Li,Wang'13],. . . ,[de Florian,Mazzitelli'18]

N^{3}LO C_{HH} [Spira'16;Gerlach,Herren,Steinhauser'18]

massless 2-loop box diagrams: [Banerjee,Borowka,Dhani,Gehrmann,Ravindran'18]

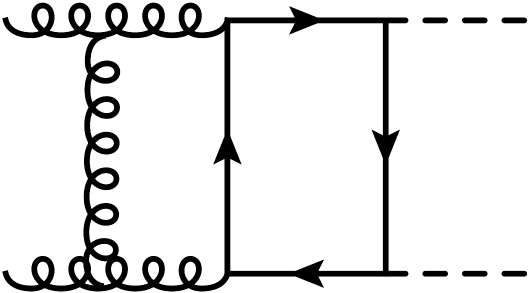
σ: [Chen,Li,Shao,Wang'19]

$gg \to HH$: NLO



$$s, t, m_t^2, m_H^2$$

$gg \to HH$: NLO



$$s, t, m_t^2, m_H^2$$

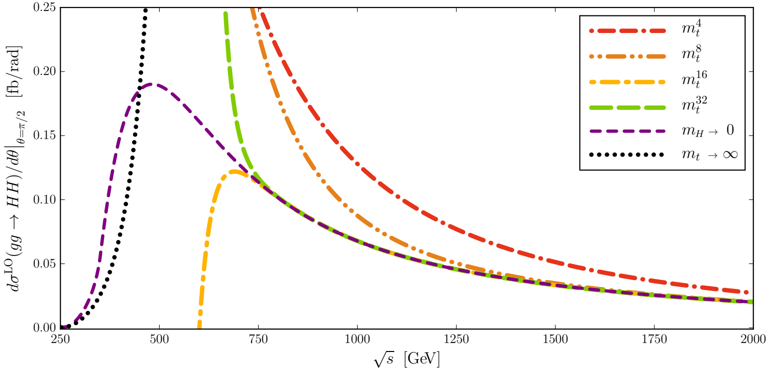
$$s, t \quad \gg \quad m_t^2 \quad \gg \quad m_H^2$$

reduce to MIs
expand MIs

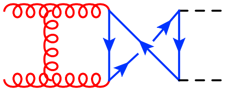
Taylor expansion

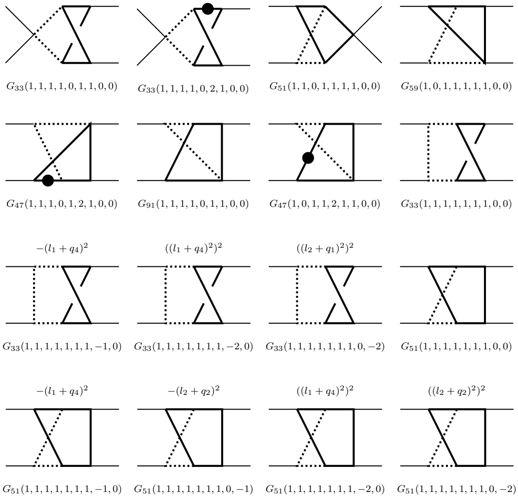
LO: exact vs $s, t \gg m_t^2 > m_H^2$ expansion

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}(s)$$



Non-planar MIs





total: 161 MIs$(s, t \gg m_t^2)$

Compute MIs

- differentiate MIs ($X = s, t, m_t^2$)

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{J} = M(s,t,m_t^2,\epsilon)\cdot\vec{J}$$

- expand in m_t^2 ➪ ansatz

see, e.g., [Melnikov,Tancredi,Wever'16]

$$J = \sum_i \sum_j \sum_k C_{ijk}(s,t)\,\epsilon^i\,(m_t^2)^j\,\log(m_t^2)^k$$

➪ system of linear equations for $C_{ijk}(s,t)$

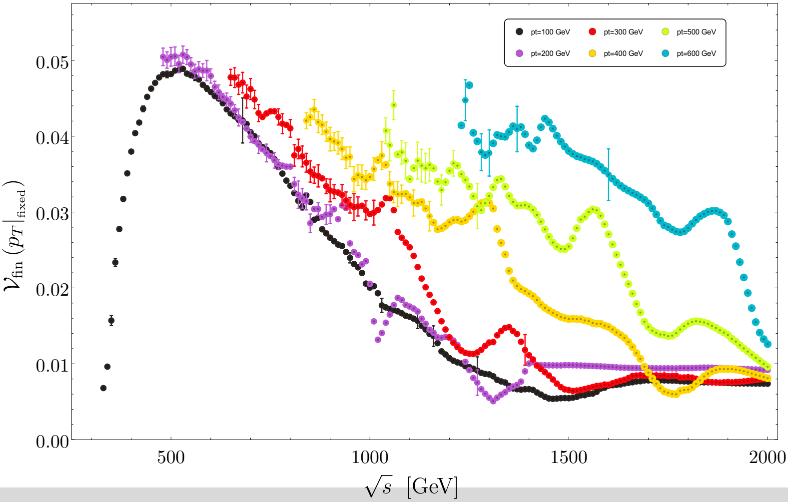
- solution requires BCs for $m_t \to 0$
- compute MIs such that $F_{\rm tri}$, $F_{\rm box1}$, $F_{\rm box2}$ are available up to m_t^{32}

[Davies,Mishima,Steinhauser,Wellmann'18'19; Mishima'18]

$$\mathcal{M} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\left(\mathcal{M}_1 A_1^{\mu\nu} + \mathcal{M}_2 A_2^{\mu\nu}\right) \qquad \mathcal{M}_1 \sim \frac{3m_H^2}{s-m_H^2}F_{\rm tri} + F_{\rm box1} \qquad \mathcal{M}_2 \sim F_{\rm box2}$$

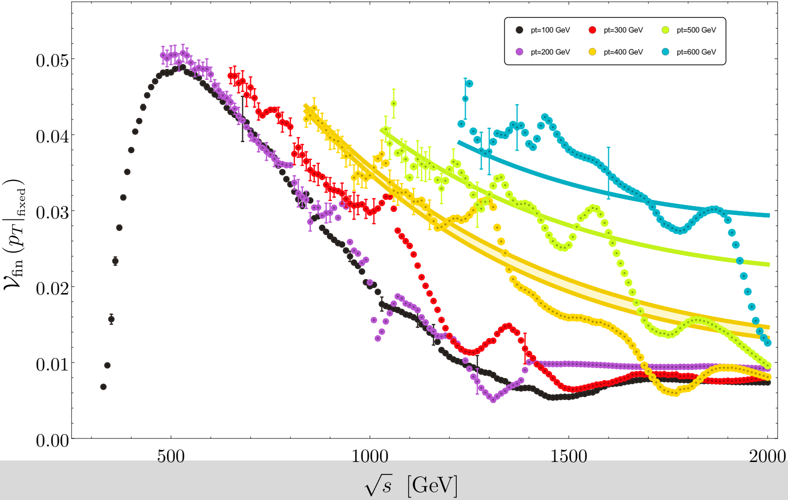
NLO $\mathcal{V}_{\text{fin}}$: grid vs. expansions

- hhgrid [Heinrich,Jones,Kerner,Luisoni,Vryonidou'17] 3398 points



NLO $\mathcal{V}_{\rm fin}$: grid vs. expansions

- hhgrid [Heinrich,Jones,Kerner,Luisoni,Vryonidou'17] 3398 points
- m_t^{30} and m_t^{32} terms



Padé improvement

Padé approximant:

$$[n/m](m_t^2) \quad = \quad \frac{a_0 + a_1 m_t^2 + \ldots + a_n (m_t^2)^n}{1 + b_1 m_t^2 + \ldots + b_m (m_t^2)^m}$$

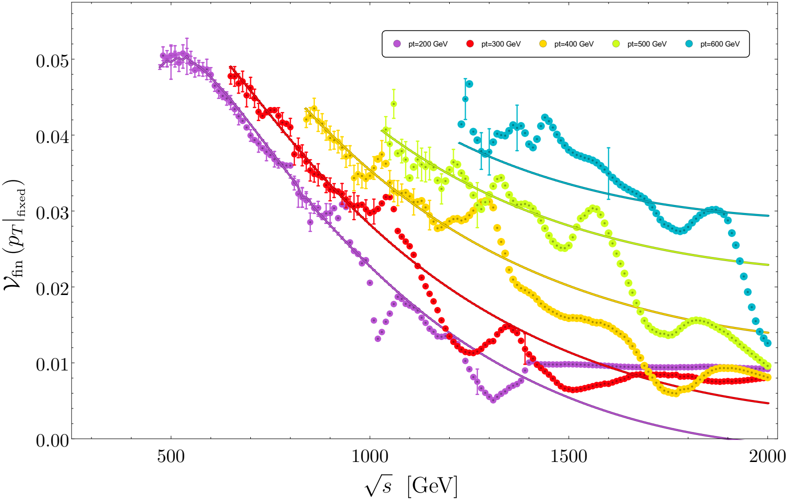
- determine a_i and b_i from expansion of $\mathcal{V}_{\rm fin}$

$$\mathcal{V}_{\rm fin} = \mathcal{V}_0 + \sum \mathcal{V}_i \, (m_t^2)^i$$

- $[8/8], [7/8], [8/7], [7/9], [9/7]$
- weighted mean value $\pm$ weighted stdev

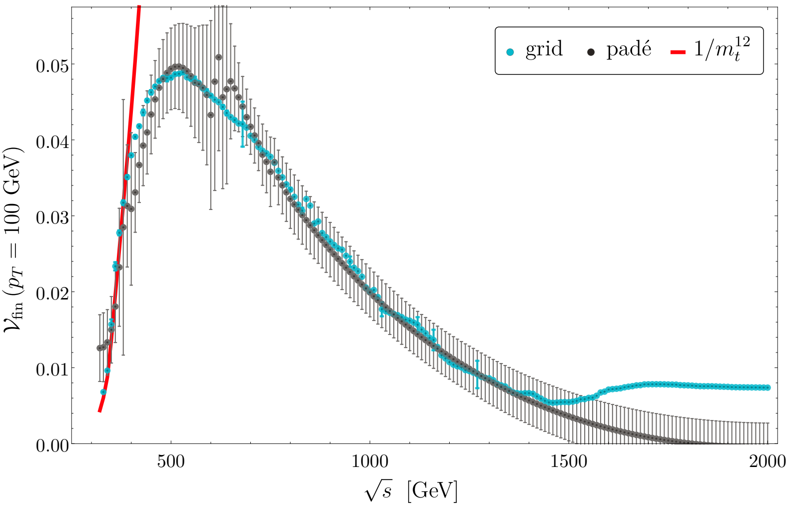
NLO $\mathcal{V}_{\text{fin}}$: grid vs. Padé

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]



NLO $\mathcal{V}_{\rm fin}$ for $p_T = 100$ GeV

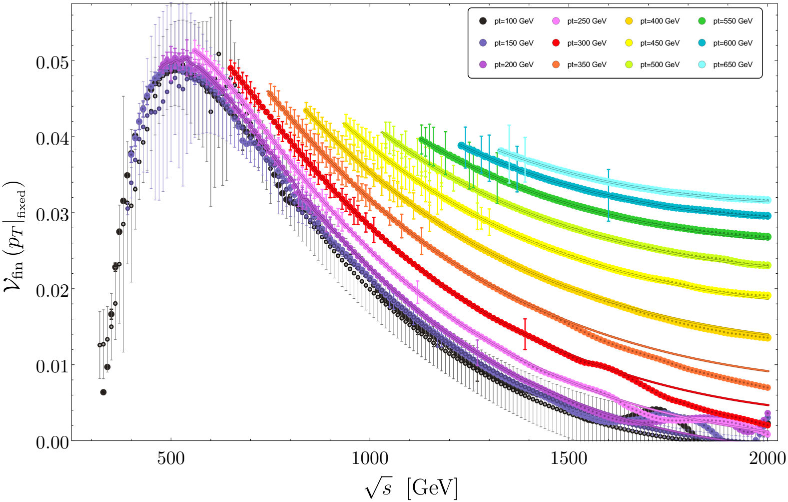
[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]



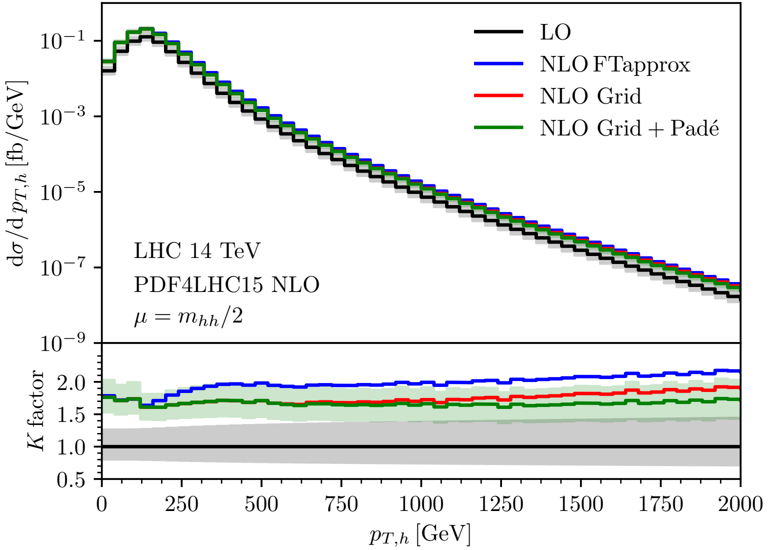
NLO $\mathcal{V}_{\rm fin}$ — improved grid

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]

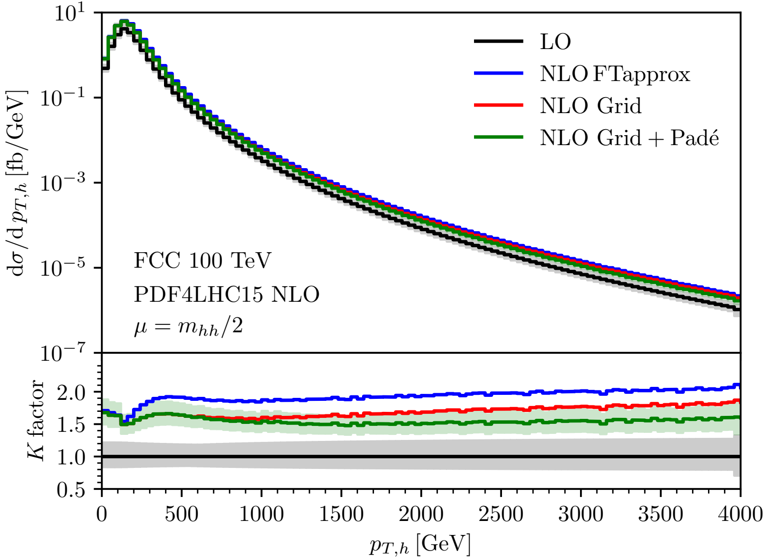
https://github.com/mppmu/hhgrid



$p_{T,h}$ distributions



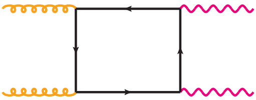
$p_{T,h}$ distributions

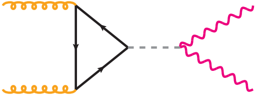


$gg \to ZZ$

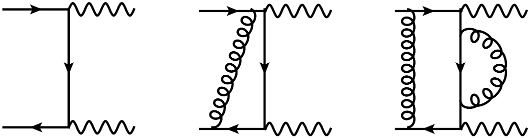
- (already) precise results from ATLAS and CMS
- important for indirect determination of Higgs width

[Kauer,Passerino'12; Caola,Melnikov'13; Campbell,Ellis,Williams'13]

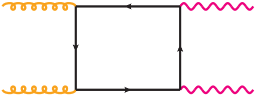




$gg \to ZZ$



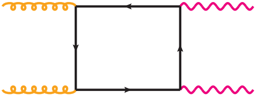
NNLO: [Cascioli,. . . '14; Gehrmann,. . . '14; Caolo,. . . '14; Grazzini,. . . '15; Heinrich,. . . '18; Kallweit,. . . '18]



u, d, s, c, b

$\approx$ 50% of NNLO

large K factor

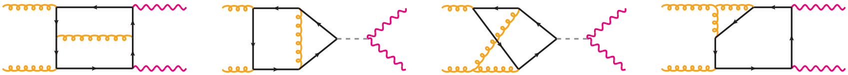


top

few % of u, d, s, c, b

[Glover,van der Bij'88]

$gg \to ZZ$: top at 2 loops

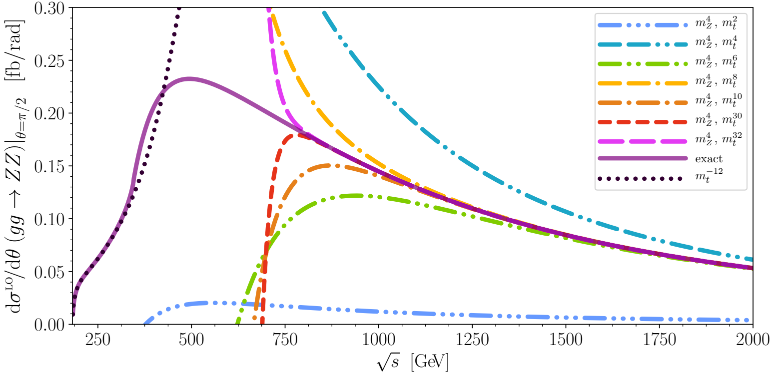


- 2 loops (massless): [von Manteuffel,Tancredi'15; Caola,Henn,Melnikov,Smirnov,Smirnov'15]
  2 loops (top): expansions for large m_t: [Melnikov,Dowling'15; Campbell et al.'16; Caola et al.'16]
  2 loops (top): large m_t + threshold + conf. mapping + Padé [Gröber,Maier,Rauh'19]

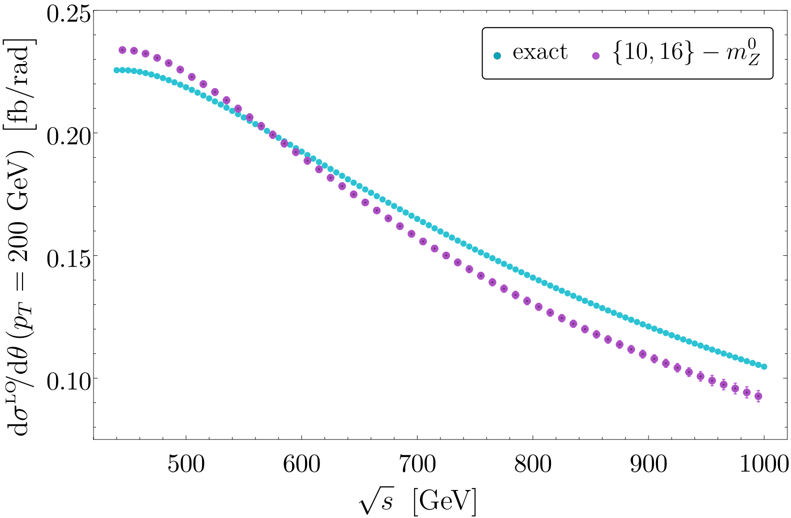
- $gg \to HH$: 3 form factors ➪ $gg \to ZZ$: 20 form factors
- reduction to MIs more involved
- MIs from $gg \to HH$
- expand up to m_t^{32} and m_Z^4
- large intermediate expressions
  inserting MIs and expansion in m_t and ϵ takes $\mathcal{O}$(weeks)

LO: exact vs $s, t \gg m_t^2 > m_Z^2$ expansion

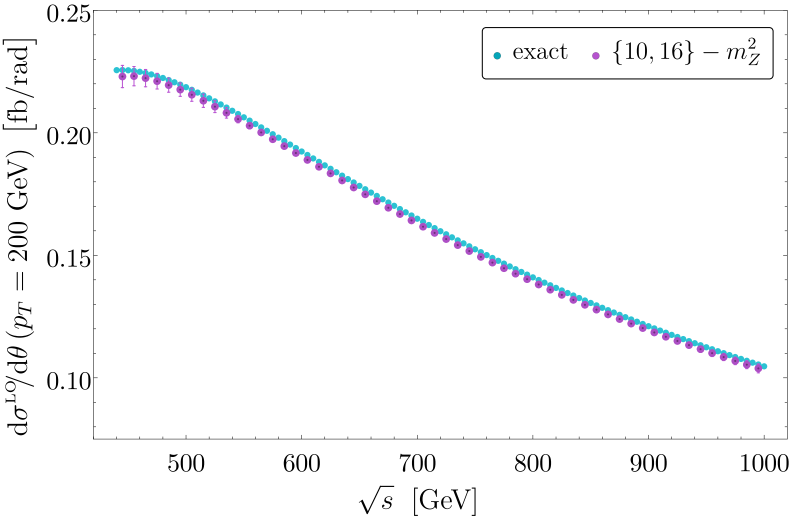
$$\frac{d\sigma}{d\theta}(s)$$



LO: $p_T = 200$ GeV

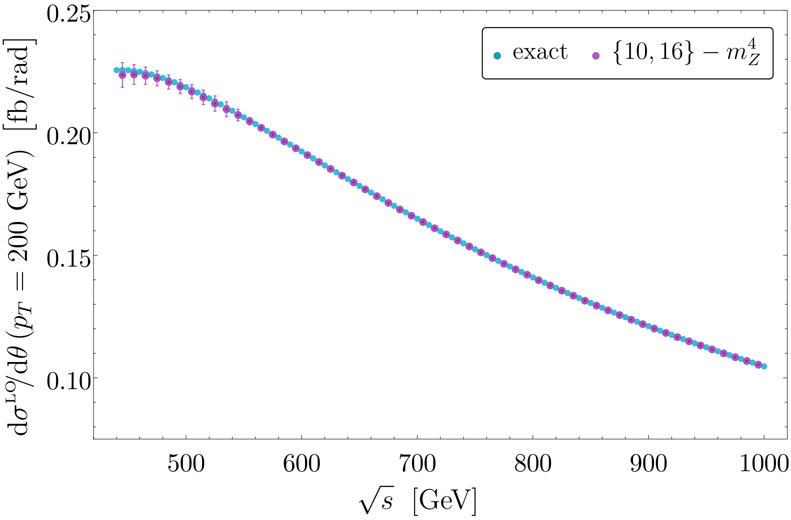

0.25
0.20
0.15
0.10
500
600
700
800
900
1000
$\sqrt{s}$ [GeV]
$\mathrm{d}\sigma^{\mathrm{LO}}/\mathrm{d}\theta\,(p_T = 200\ \mathrm{GeV})$ [fb/rad]
exact
$\{10, 16\} - m_Z^0$

LO: $p_T = 200$ GeV



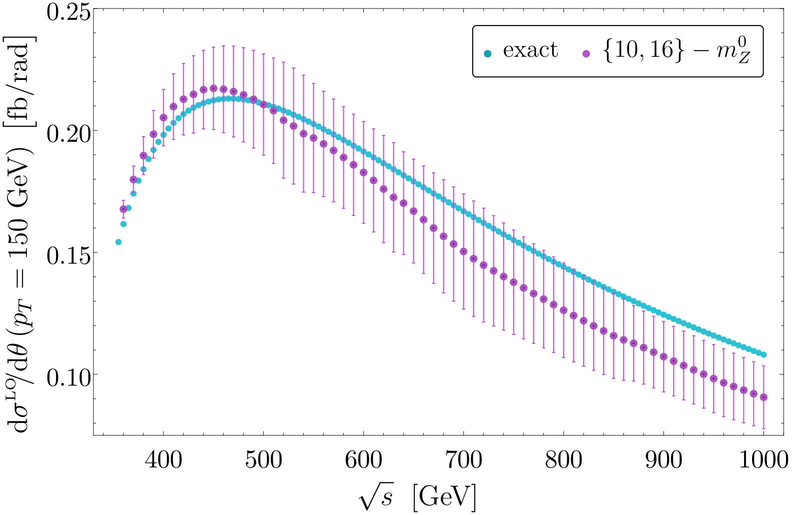
0.25
0.20
0.15
0.10
$d\sigma^{LO}/d\theta \, (p_T = 200 \text{ GeV})$ [fb/rad]
exact
$\{10, 16\} - m_Z^2$
500
600
700
800
900
1000
$\sqrt{s}$ [GeV]

LO: $p_T = 200$ GeV



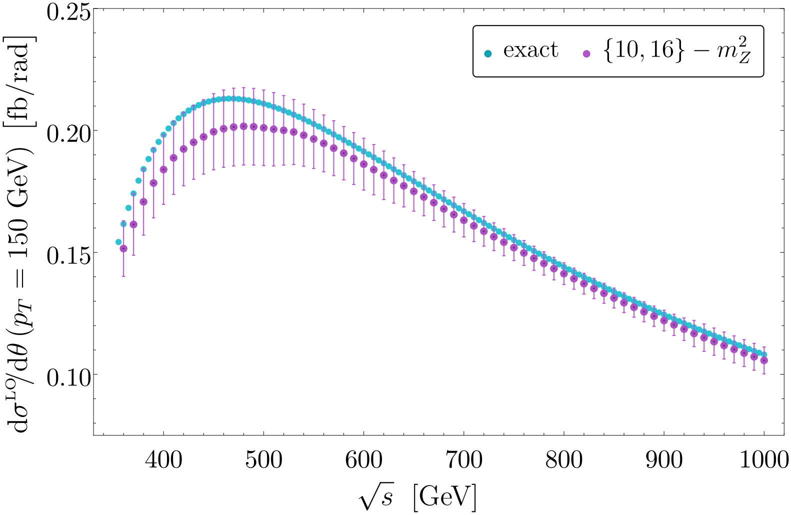
exact
$\{10, 16\} - m_Z^4$
0.25
0.20
0.15
0.10
$d\sigma^{LO}/d\theta\,(p_T = 200\text{ GeV})$ [fb/rad]
500
600
700
800
900
1000
$\sqrt{s}$ [GeV]

LO: $p_T = 150$ GeV



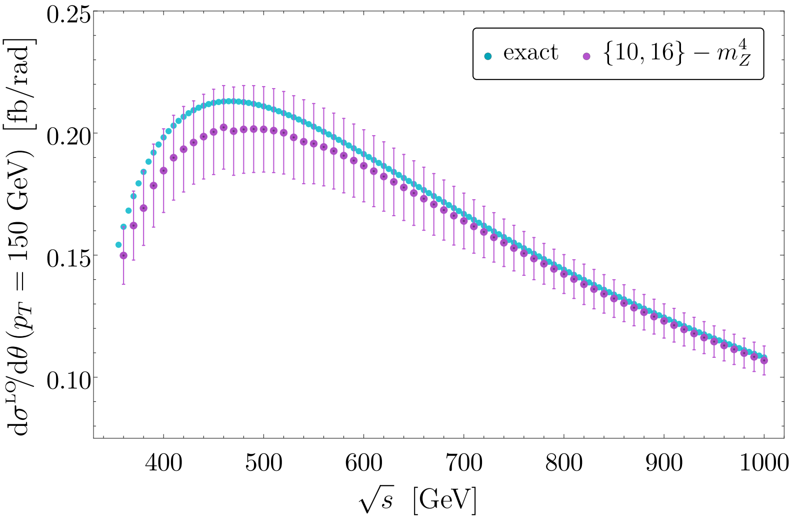
exact
$\{10, 16\} - m_Z^0$
0.25
0.20
0.15
0.10
$d\sigma^{LO}/d\theta\,(p_T = 150\text{ GeV})$ [fb/rad]
400
500
600
700
800
900
1000
$\sqrt{s}$ [GeV]

LO: $p_T = 150$ GeV



exact
$\{10, 16\} - m_Z^2$
0.25
0.20
0.15
0.10
400
500
600
700
800
900
1000
$\sqrt{s}$ [GeV]
$d\sigma^{LO}/d\theta\,(p_T = 150\text{ GeV})$ [fb/rad]

LO: $p_T = 150$ GeV



exact
$\{10, 16\} - m_Z^4$
0.25
0.20
0.15
0.10
$\mathrm{d}\sigma^{\mathrm{LO}}/\mathrm{d}\theta\,(p_T = 150\ \mathrm{GeV})\ [\mathrm{fb/rad}]$
400
500
600
700
800
900
1000
$\sqrt{s}\ [\mathrm{GeV}]$

NLO $\mathcal{V}_{\text{fin}}$

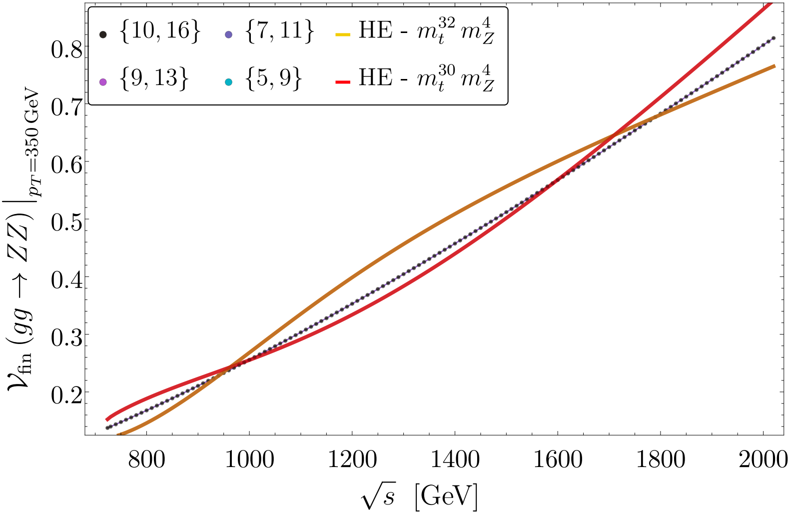
Karlsruhe Institute of Technology

$$\mathcal{V}_{\text{fin}} = \frac{\alpha_s^2(\mu)}{\pi^2}\frac{G_F^2 m_Z^4}{32}\sum_i \left[C_i + 2\left(F_i^{(0)*}F_i^{(1)} + F_i^{(0)}F_i^{(1)*}\right)\right]$$

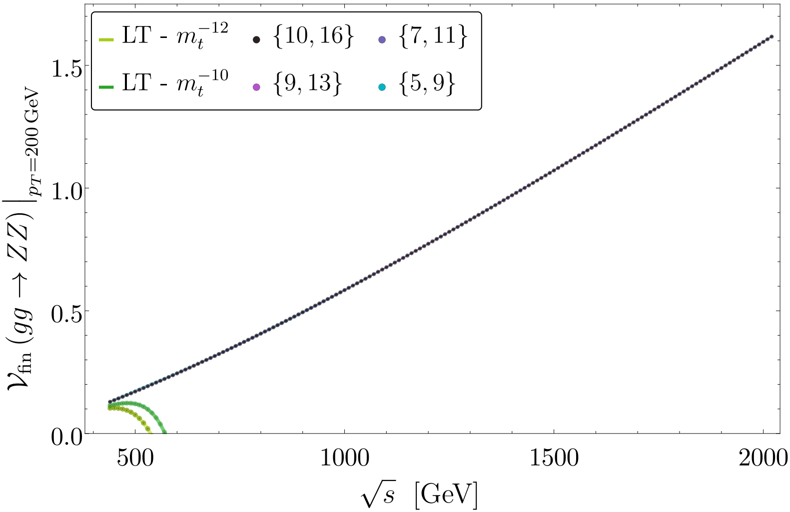
$$C_i = \left|F_i^{(0)}\right|^2 C_A \left(\pi^2 - \log^2\frac{\mu^2}{s}\right)$$

$F_i^{(0)}$: top + u, d, s, c, b
$F_i^{(1)}$: only top

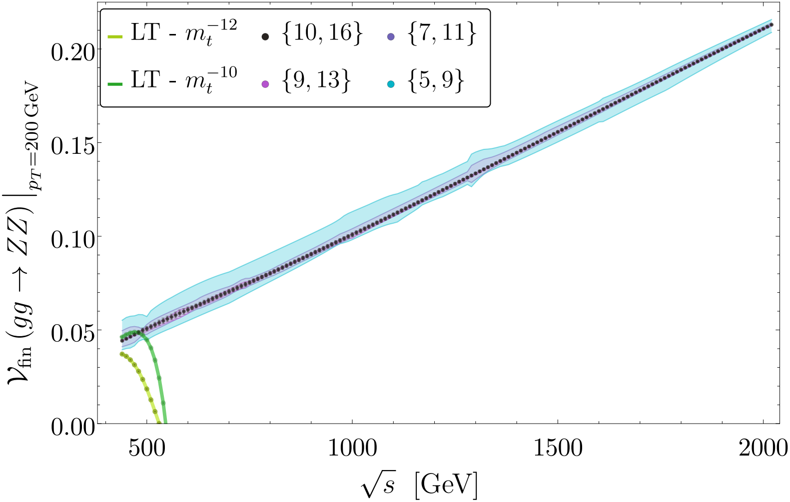
NLO $\mathcal{V}_{\rm fin}$: $p_T = 350$ GeV



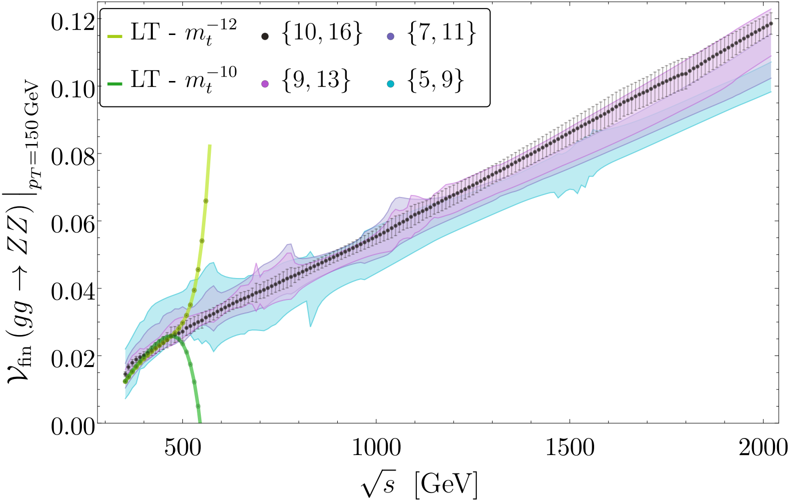
NLO $\mathcal{V}_{\text{fin}}$: $p_T = 200$ GeV



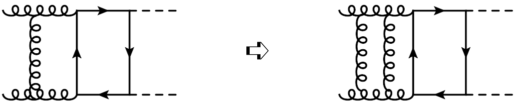
NLO $\mathcal{V}_{\text{fin}}$: $p_T = 200$ GeV only top



NLO $\mathcal{V}_{\rm fin}$: $p_T = 150$ GeV only top



From NLO to NNLO



- exact: NO
- high-energy: box integrals with s, t and m_t
  ➪ 3 scales: not yet at 3 loops (NNLO)
- large-m_t: tadpoles + massless triangles (+ 1-loop massless box)
  ➪ 1-scale integrals: doable at 3 loops
- threshold: non-analytic terms [Gröber,Maier,Rauh'17]
  ($s \approx 4m_t^2$)

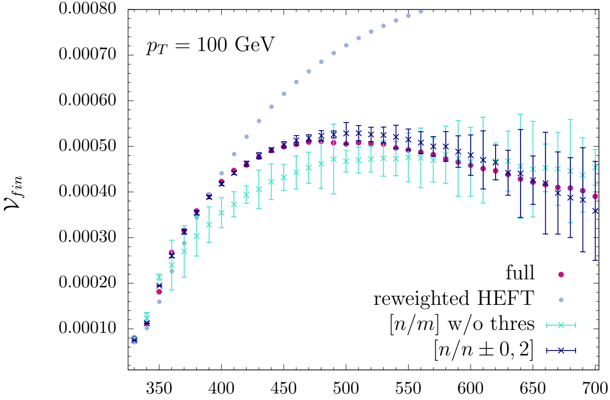
large m_t

- $gg \to H$: $m_H^2 \ll m_t^2$
- $gg \to HH$: $m_H^2, s \ll m_t^2$

  ➪ order of magnitude of finite-m_t terms
  ➪ cross-check of future (numerical) exact results
  ➪ input for approximations

NLO approximation for $gg \to HH$: large-m_t $\oplus$ threshold $\oplus$ conformal mapping $\oplus$ Padé

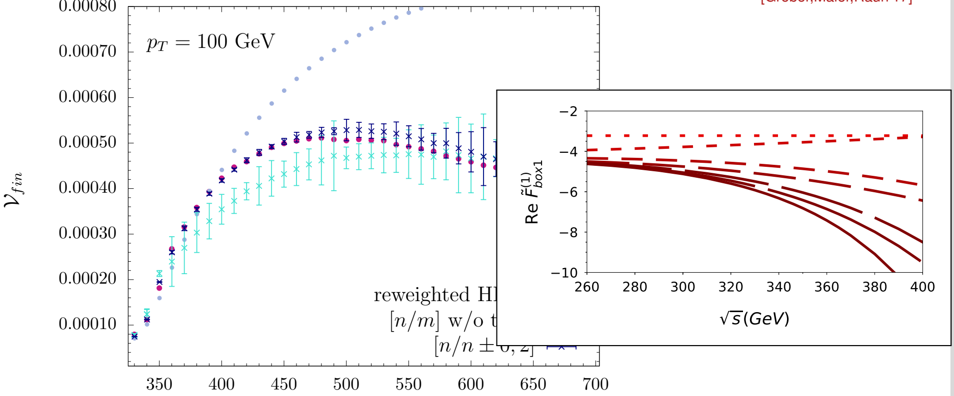
[Gröber,Maier,Rauh'17]



input for Padé:	large-m_t: up to $1/m_t^8$ threshold: non-analytic terms ("logs")

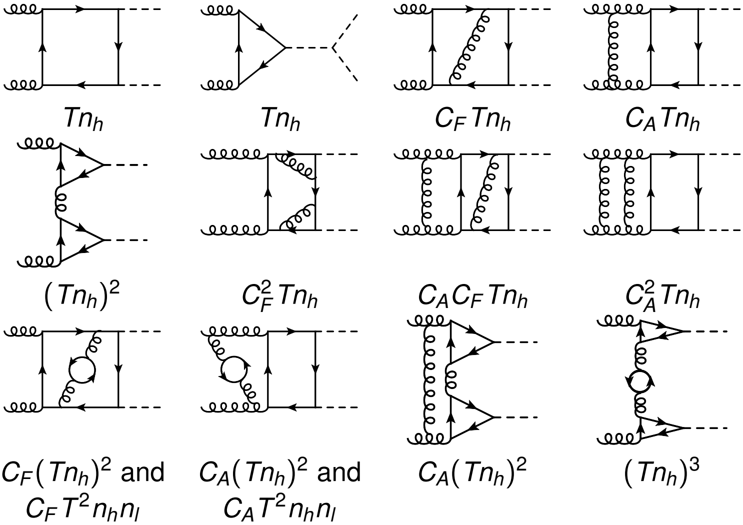
NLO approximation for $gg \to HH$: large-m_t $\oplus$ threshold $\oplus$ conformal mapping $\oplus$ Padé

[Gröber,Maier,Rauh'17]

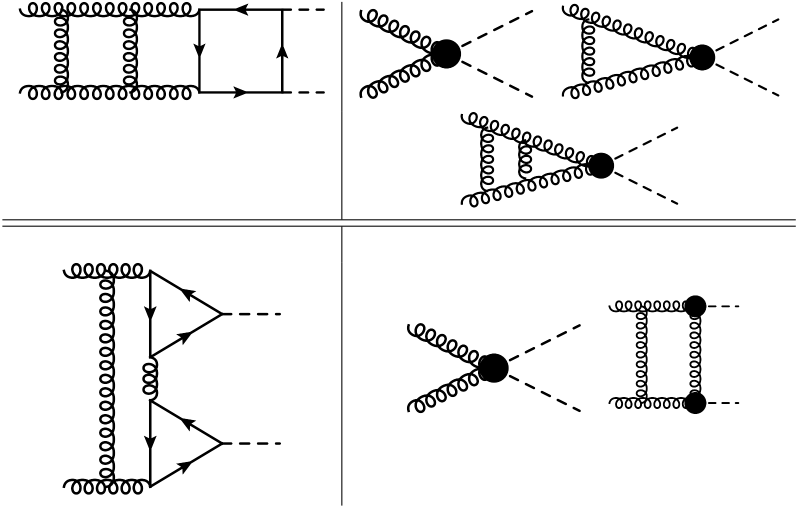


input for Padé: large-m_t: up to $1/m_t^8$
threshold: non-analytic terms ("logs")

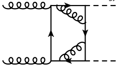
NNLO large-m_t: Feynman diagrams



Asymptotic expansion: $m_t \gg q_1, q_2, q_3$



NNLO $gg \to hh$ for large-m_t



- Avoid (vacuum) tensor integrals by projecting each diagram

$$A = \sum_{L=0}^{L_{max}} \sum_{k+l+m+n}^{L} C_{k,l,m,n} \, (q_3 \cdot q_3)^k \, (q_1 \cdot q_2)^l \, (q_1 \cdot q_3)^m \, (q_2 \cdot q_3)^n$$

  using projectors $P_{k,l,m,n} A = C_{k,l,m,n}$

  Example: $\Box_{a,b} = \frac{\partial}{\partial q_{a\,\mu}} \frac{\partial}{\partial q_b^{\mu}}$

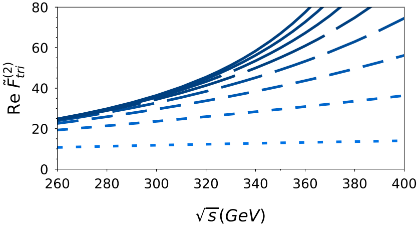
$$P_{0,0,0,2} = \frac{1}{2d^2 + 2d - 4} \Box_{2,3} \Box_{2,3} - \frac{1}{2d^3 + 2d^2 - 4d} \Box_{2,2} \Box_{3,3}$$

- Do $1/m_t$ expansion only once: 324 GB (gzipped) stored result (a few $\times \geq$ 96 GB RAM, 12 cores; $\approx$ 10 days wall time)
- Compute derivative. Load only necessary terms.
  ➪ $\approx$ 29.000 easy tasks   total time $\sim$ 4.5 yr ($\sim$ 1 month)
- heavy use of modern `(T)FORM` commands: [Ruijl,Ueda,Vermaseren'17]
  e.g. `ArgToExtraSymbols`
- $F_{\rm box1}, F_{\rm box2} \to 1/m_t^8$; $F_{\rm tri} \to 1/m_t^{14}$ [Davies,Steinhauser'19]

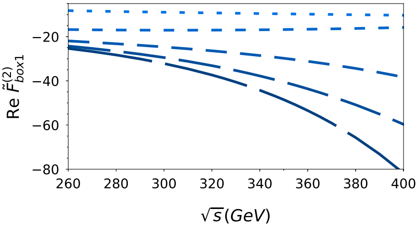
$gg \to hh$ for large-m_t: form factors

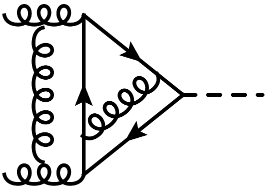
[Davies,Steinhauser'19]

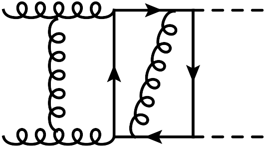
$F_{\rm tri}(1/mt^{14})$



$F_{\rm box1}(1/mt^{8})$

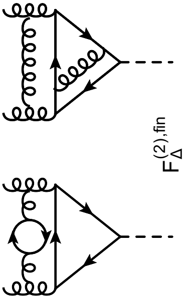


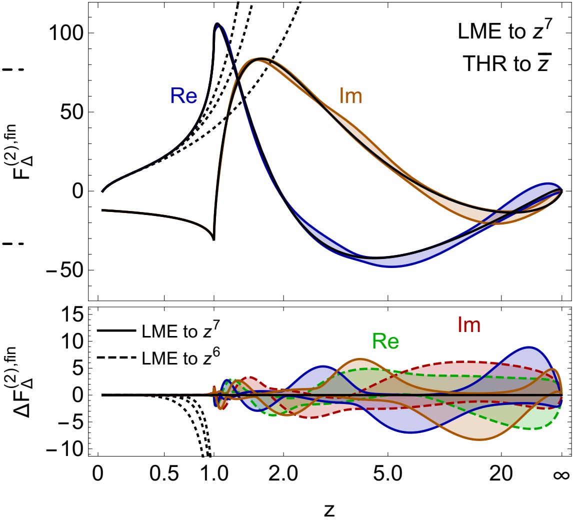




NNLO approximation for $gg \to h$: large-m_t $\oplus$ threshold $\oplus$ Padé

[Davies,Gröber,Maier,Rauh,Steinhauser'19]



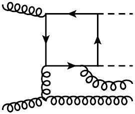


$z = \frac{s}{4m_t^2}$

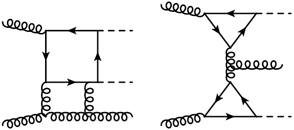
$\bar{z} = 1 - z$

(analytic n_l terms: [Harlander,Prausa,Usovitsch'19]; num. int. of diff. eqs.: [Czakon,Niggetiedt'20])

NNLO real corrections for large m_t

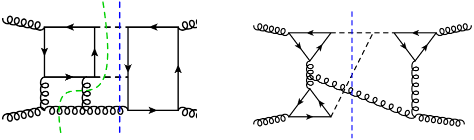
- 

  1-loop $2 \rightarrow 4$

  2-loop $2 \rightarrow 3$

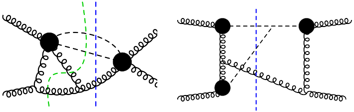
- use optical theorem

- asymptotic expansion for $m_t^2 \gg m_H^2, s$ ➪

tadpoles $\otimes$ “phase-space” integrals

1 and 2 loops



m_t^2

m_H^2, s

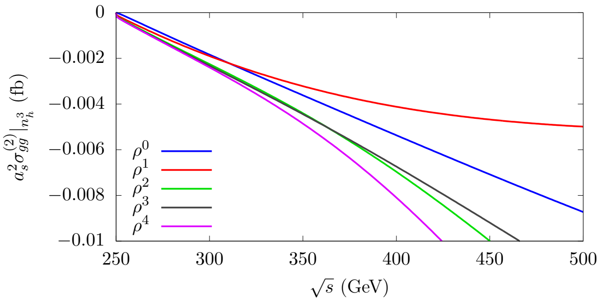
3-loop 3- and 4-particle cuts [DHMS WIP]

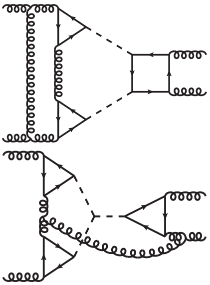
and 2-loop 3-particle cut [DHMS’19]

[DHMS = Davies,Herren,Mishima,Steinhauser]

n_h^3 results

- expansion up to $1/m_t^8$
- combine with virtual corrections [Grigo,Hoff,Steinhauser'15; Davies,Steinhauser'19]



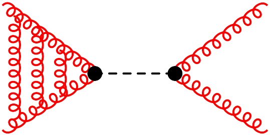


$\rho = m_H^2 / m_t^2$

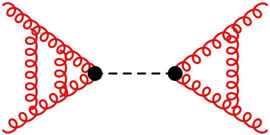
$gg \to H$

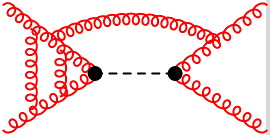
virtual N^3LO corrections for finite m_t

Higgs production at the LHC: $m_t \to \infty$

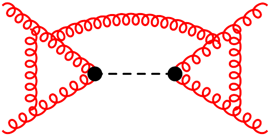


[Baikov,Chetyrkin,Smirnov,Smirnov, Steinhauser'09], [Gehrmann,Glover,Huber,Ikizlerli, Studerus'10]; [Lee,Smirnov'10]

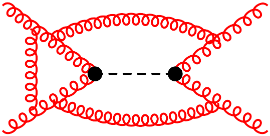




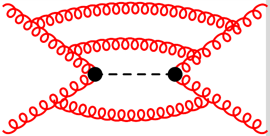
[Duhr,Gehrmann'13], [Li,Zhu'13], [Dulat,Mistlberger'14], [Duhr,Gehrmann,Jaquier'14]



[Anastasiou,Duhr,Dulat,Herzog, Mistlberger'13], [Kilgore'13]



[Anastasiou,Duhr,Dulat,Furlan,Gehrmann, Herzog,Mistlberger'14], [Li,von Manteuffel,Schabinger,Zhu'14]

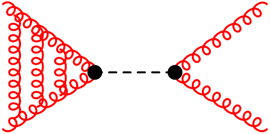


[Anastasiou,Duhr,Dulat,Mistlberger'13]

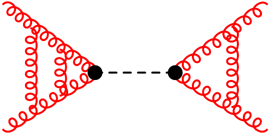
N^3LO: [Anastasiou,Duhr,Dulat,Herzog,Mistlberger'15]

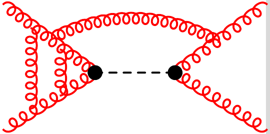
[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16; Mistlberger'18]

Higgs production at the LHC: $m_t \to \infty$

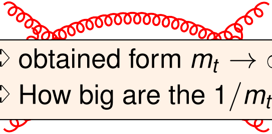


[Baikov,Chetyrkin,Smirnov,Smirnov, Steinhauser'09], [Gehrmann,Glover,Huber,Ikizlerli, Studerus'10]; [Lee,Smirnov'10]

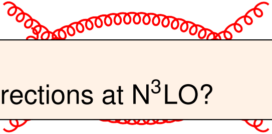




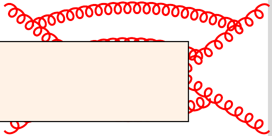
[Duhr,Gehrmann'13], [Li,Zhu'13], [Dulat,Mistlberger'14], [Duhr,Gehrmann,Jaquier'14]



[Anastasiou,Duhr,Dulat,Herzog, Mistlberger'13], [Kilgore'13]



[Anastasiou,Duhr,Dulat,Furlan,Gehrmann, Herzog,Mistlberger'14], [Li,von Manteuffel,Schabinger,Zhu'14]



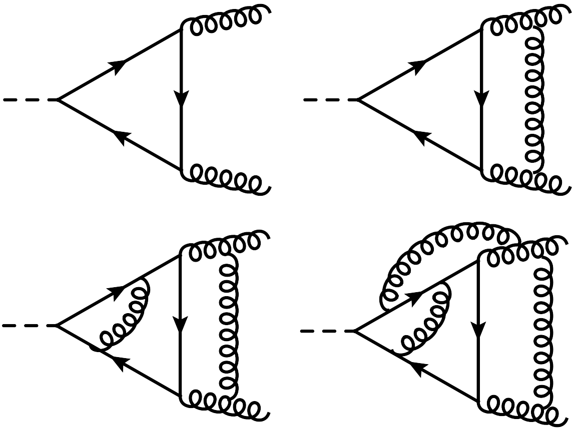
[Anastasiou,Duhr,Dulat,Mistlberger'13]

- ➪ obtained form $m_t \to \infty$
- ➪ How big are the $1/m_t$ corrections at N^{3}LO?

N^{3}LO: [Anastasiou,Duhr,Dulat,Herzog,Mistlberger'15]

[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16; Mistlberger'18]

Feynman diagrams

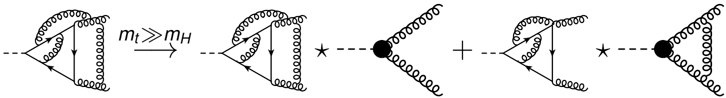


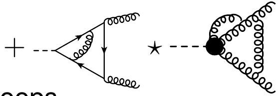
2 loops: [Harlander,Kant'05; Anastasiou,Beerli,Bucherer,Daleo,Kunszt'06; Aglietti,Bonciani,Degrassi,Vicini'07]

3 loops: expansion: [Harlander,Ozeren'09; Pak,Rogal,Steinhauser'09]

3 loops: [Davies,Gröber,Maier,Rauh,Steinhauser'19; Harlander,Prausa,Usovitsch'19; Czakon,Niggetiedt'20]

Asymptotic expansion





- $\rho = m_H^2/m_t^2 \approx 0.5$
- vacuum integrals, scale: m_t; 1,2,3,4 loops

  [Laporta'02; Schroder,Vuorinen'05; Chetyrkin,Sturm,...'06; Lee,Terekhov'10; ...Marquard, ...]

  tensor integrals up to rank 8 needed
- massless vertices, scale: $s = m_H^2$; 1,2,3 loops

  [Baikov,Chetyrkin,Smirnov,Smirnov,Steinhauser'09; Heinrich,Huber,Kosower,Smirnov'09;

  Gehrmann,Glover,Huber,Ikizlerli,Studerus'10]
- challenges:

  number of 4-loop diagrams: 23251

  organization of asymptotic expansion: `exp` [Harlander,Seidensticker,Steinhauser'97]

  reduction: `FIRE 6` [Smirnov,Chuharev'19] `LiteRed` [Lee'13], size of tables: $\approx$ 25 GB

$1/\epsilon$ poles

UV poles: renormalize m_t (OS or $\overline{\mathrm{MS}}$), α_s ($\overline{\mathrm{MS}}$) and gluon field (OS)

$1/\epsilon$ poles

IR poles: $\log(F) = \log(F)_{\rm poles} + \log(F)_{\rm finite}$ $\quad\quad (F = 1 + \ldots)$

$\log(F)_{\rm poles}$ see, e.g, [Becher and Neubert'09; Gehrmann,Glover,Huber,Ikizlerli,Studerus'10]

– has no expansion in m_H/m_t

– predicted from massless form factor

$$\log(F)_{\rm poles} =$$
$$\frac{\alpha_s}{4\pi}\left\{\frac{1}{\epsilon^2}\left[-\frac{1}{2}C_F\gamma^0_{\rm cusp}\right]+\frac{1}{\epsilon}\left[\gamma^0_q\right]\right\}$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{1}{\epsilon^3}\left[\frac{3}{8}\beta_0 C_F\gamma^0_{\rm cusp}\right]+\frac{1}{\epsilon^2}\left[-\frac{1}{2}\beta_0\gamma^0_q-\frac{1}{8}C_F\gamma^1_{\rm cusp}\right]+\frac{1}{\epsilon}\left[\frac{\gamma^1_q}{2}\right]\right\}$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left\{\frac{1}{\epsilon^4}\left[-\frac{11}{36}\beta_0^2 C_F\gamma^0_{\rm cusp}\right]+\frac{1}{\epsilon^3}\left[C_F\left(\frac{2}{9}\beta_1\gamma^0_{\rm cusp}+\frac{5}{36}\beta_0\gamma^1_{\rm cusp}\right)+\frac{1}{3}\beta_0^2\gamma^0_q\right]\right.$$
$$\left.+\frac{1}{\epsilon^2}\left[-\frac{1}{3}\beta_1\gamma^0_q-\frac{1}{3}\beta_0\gamma^1_q-\frac{1}{18}C_F\gamma^2_{\rm cusp}\right]+\frac{1}{\epsilon}\left[\frac{\gamma^2_q}{3}\right]\right\}$$

$\gamma_{\rm cusp}$: cusp anomalous dimension $\quad\quad$ γ_q: collinear anomalous dimension

$\log(F)_{\mathrm{finite}}$

Karlsruhe Institute of Technology

[Davies,Herren,Steinhauser'19]

$$\log(F)_{\mathrm{finite}} = \qquad\qquad \rho = m_H^2/m_t^2$$

$$+\frac{\alpha_s}{\pi}\left(\frac{11}{4}+\frac{1}{8}\pi^2-\frac{3}{4}l_{tH}^2+\frac{17}{135}\rho+\frac{3553}{226800}\rho^2\right)$$

$$+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{523}{108}+\frac{151}{192}\pi^2-\frac{499}{48}\zeta_3+l_{tH}\left(-\frac{155}{36}+\frac{23}{48}\pi^2+\frac{9}{8}\zeta_3\right)+l_{tH}^2\left(-\frac{151}{48}+\frac{3}{16}\pi^2\right)-\frac{23}{48}l_{tH}^3\right.$$

$$+\rho\left(-\frac{15765509}{829440}+\frac{7}{1080}\pi^2+\frac{7}{540}\log(2)\pi^2+\frac{1909181}{110592}\zeta_3+\frac{793}{10368}l_{tH}\right)$$

$$\left.+\rho^2\left(-\frac{1013177390077}{234101145600}+\frac{857}{907200}\pi^2+\frac{857}{453600}\log(2)\pi^2+\frac{267179777}{70778880}\zeta_3+\frac{580759}{43545600}l_{tH}\right)\right]$$

$$+\left(\frac{\alpha_s}{\pi}\right)^3\left[-\frac{18539405}{1119744}+\frac{441517}{62208}\pi^2-\frac{11549467}{82944}\zeta_3-\frac{50839}{311040}\pi^4-\frac{1949}{576}\pi^2\zeta_3+\frac{39307}{288}\zeta_5\right.$$

$$-\frac{39}{8}\zeta_3^2-\frac{193}{7560}\pi^6+l_{tH}\left(-\frac{322955}{31104}+\frac{665}{96}\pi^2-\frac{3043}{144}\zeta_3-\frac{1801}{5760}\pi^4-\frac{15}{16}\pi^2\zeta_3-\frac{27}{4}\zeta_5\right)$$

$$+l_{tH}^2\left(-\frac{58745}{3456}+\frac{1435}{576}\pi^2+\frac{25}{8}\zeta_3-\frac{33}{320}\pi^4\right)+l_{tH}^3\left(-\frac{3995}{864}+\frac{23}{96}\pi^2\right)-\frac{529}{1152}l_{tH}^4$$

$$+\rho\left(-\frac{542872693595}{3218890752}+\frac{65743583}{55987200}\pi^2-\frac{4691}{9720}\log(2)\pi^2-\frac{6788585826089}{107296358400}\zeta_3\right.$$

$$-\frac{11421210133}{1149603840}\log^4(2)+\frac{11364084757}{1149603840}\log^2(2)\pi^2+\frac{244657561171}{55180984320}\pi^4-\frac{11421210133}{47900160}\mathrm{Li}_4(1/2)$$

$$+\frac{718337}{9979200}\log^5(2)-\frac{718337}{5987520}\log^3(2)\pi^2+\frac{46111267}{239500800}\log(2)\pi^4-\frac{10073}{25920}\pi^2\zeta_3-\frac{3254515597}{31933440}\zeta_5$$

718337 … 5327119 … 25639 …

$+ \rho\Big(\frac{\cdots}{234101145600} + \frac{\cdots}{907200}\pi^{\cdots} + \frac{\cdots}{453600}\log(2)\pi^{\cdots} + \frac{\cdots}{70778880}\zeta_3 + \frac{\cdots}{43545600} l_{tH} \Big)\Big]$

$$
\begin{aligned}
\log(F)_{\rm finite}\left(\frac{\alpha_s}{\pi}\right)^3 \Big[& -\frac{18539405}{1119744} + \frac{441517}{62208}\pi^2 - \frac{11549467}{82944}\zeta_3 - \frac{50839}{311040}\pi^4 - \frac{1949}{576}\pi^2\zeta_3 + \frac{39307}{288}\zeta_5 \\
& - \frac{39}{8}\zeta_3^2 - \frac{193}{7560}\pi^6 + l_{tH}\Big(-\frac{322955}{31104} + \frac{665}{96}\pi^2 - \frac{3043}{144}\zeta_3 - \frac{1801}{5760}\pi^4 - \frac{15}{16}\pi^2\zeta_3 - \frac{27}{4}\zeta_5 \Big) \\
& + l_{tH}^2\Big(-\frac{58745}{3456} + \frac{1435}{576}\pi^2 + \frac{25}{8}\zeta_3 - \frac{33}{320}\pi^4 \Big) + l_{tH}^3\Big(-\frac{3995}{864} + \frac{23}{96}\pi^2 \Big) - \frac{529}{1152} l_{tH}^4 \\
& + \rho\Big(-\frac{542872693595}{3218890752} + \frac{65743583}{55987200}\pi^2 - \frac{4691}{9720}\log(2)\pi^2 - \frac{6788585826089}{107296358400}\zeta_3 \\
& - \frac{11421210133}{1149603840}\log^4(2) + \frac{11364084757}{1149603840}\log^2(2)\pi^2 + \frac{244657561171}{55180984320}\pi^4 - \frac{11421210133}{47900160}\mathrm{Li}_4(1/2) \\
& + \frac{718337}{9979200}\log^5(2) - \frac{718337}{5987520}\log^3(2)\pi^2 + \frac{46111267}{239500800}\log(2)\pi^4 - \frac{10073}{25920}\pi^2\zeta_3 - \frac{3254515597}{31933440}\zeta_5 \\
& - \frac{718337}{83160}\mathrm{Li}_5(1/2) + \frac{5327119}{11197440} l_{tH} + \frac{25639}{746496} l_{tH}^2 \Big) \\
& + \rho^2\Big(-\frac{10557943614178824870061}{668136655521054720 0} + \frac{4077367559}{23514624000}\pi^2 + \frac{231579175005397170 53}{11783715264921600 0}\zeta_3 \\
& - \frac{110153}{1632960}\log(2)\pi^2 + \frac{2712037738087}{2391175987200}\log^4(2) - \frac{2729355664999}{2391175987200}\log^2(2)\pi^2 - \frac{150868470717581}{229552894771200}\pi^4 \\
& + \frac{2712037738087}{99632332800}\mathrm{Li}_4(1/2) + \frac{46902913}{202176000}\log^5(2) - \frac{46902913}{121305600}\log^3(2)\pi^2 - \frac{8632107859}{33965568000}\log(2)\pi^4 \\
& - \frac{1233223}{21772800}\pi^2\zeta_3 + \frac{49563452909}{4528742400}\zeta_5 - \frac{46902913}{1684800}\mathrm{Li}_5(1/2) + \frac{103150403081}{658409472000} l_{tH} + \frac{18740929}{3135283200} l_{tH}^2 \Big) \Big]
\end{aligned}
$$

$\log(F)_{\rm finite}$

[Davies,Herren,Steinhauser'19]

$\log(F)_{\rm finite} =$ $\qquad\qquad\qquad\qquad \rho = m_H^2/m_t^2$

$+ \alpha_s \Big(11 \; + \; 1 \; \pi^2 - 3 \; l_{tH}^2 + 17 \; \rho + 3553 \; \rho^2 \Big)$

$\mu = m_t^{\rm OS}$:

$$\begin{aligned}\log(F)_{\rm finite} \approx\; & a_t\,[(11.07 - i3.06) + 0.07 + 0.004] \\ & + a_t^2\,[(22.59 + i13.24) + (1.02 + i0.13) + (0.07 + i0.01)] \\ & + a_t^3\,[(-73.18 + i51.55) + (7.61 + i0.85) + (0.70 + i0.14)]\end{aligned}$$

$$a_t = \alpha_s(m_t)/\pi$$

- mass corrections more important at 4 loops (10%)
  ρ^2 term $< 1\%$ $\qquad m_H^2/(4m_t^2) \approx 13\%$
- $m_t^{\overline{\rm MS}}$: ρ^1 terms contribute 0.1%, 2.5% and 1.4%
  ➪ $1/m_t$ corrections smaller

$$- \frac{11421210133}{1149603840}\log^4(2) + \frac{11364084757}{1149603840}\log^2(2)\pi^2 + \frac{244657561171}{55180984320}\pi^4 - \frac{11421210133}{47900160}{\rm Li}_4(1/2)$$

$$+ \frac{718337}{9979200}\log^5(2) - \frac{718337}{5987520}\log^3(2)\pi^2 + \frac{46111267}{239500800}\log(2)\pi^4 - \frac{10073}{25920}\pi^2\zeta_3 - \frac{3254515597}{31933440}\zeta_5$$

718337 … 5327119 … 25639 …

Conclusions

$gg \to H$

finite m_t corrections at N^{3}LO (virtual)

fast convergence in m_H^2/m_t^2

$gg \to HH$

NLO: many approximations

exact result is expensive! ➪ combine with approximations

test bed for NNLO

NNLO: $1/m_t$ expansion (virtual + real)

convergence below top threshold

input for approximation procedures (Padé, ...)

$gg \to ZZ$

NLO: high-energy expansion $\oplus$ Padé

= benchmark for future numeric calculation