Geometries for scattering amplitudes and beyond

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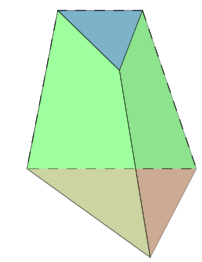
DESY Virtual Theory Forum, 24.09.2020

Introduction

N=4 sYM

(Momentum) Amplituhedron

Wavefunction



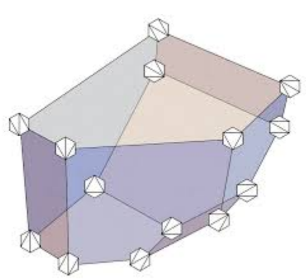
Cosmological polytope

...

positive geometries

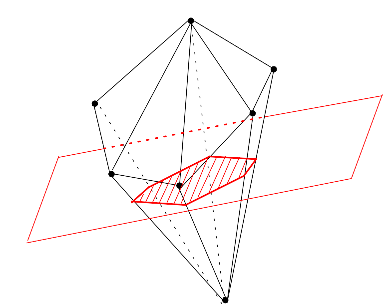
...

Biadjoint φ^3



Kinematic Associahedron

CFTs



Cyclic polytope

Introduction

positive geometries

- Real, oriented, closed geometry with boundaries of all codimension
- Each boundary is again a positive geometry
- Every positive geometry has a unique differential form with logarithmic singularities along all boundaries: the canonical form
- The residue along a boundary is given by the canonical form on the boundary

for physically relevant positive geometries the canonical form is a physical quantity

Introduction

positive geometries

for physically relevant positive geometries
the canonical form is a physical quantity

* Locality and unitarity = when approaching one of the boundaries, the quantity appropriately factorises into smaller pieces ~ recursion relations constructing more complicated objects from simpler ones

* given a set of external kinematic data, there exists a geometrical object defined by imposing particular positivity and/or topological constraints

Introduction

positive geometries

for physically relevant positive geometries the canonical form is a physical quantity

this talk: focus on scattering amplitudes

Introduction

Particle Physics described by QFT

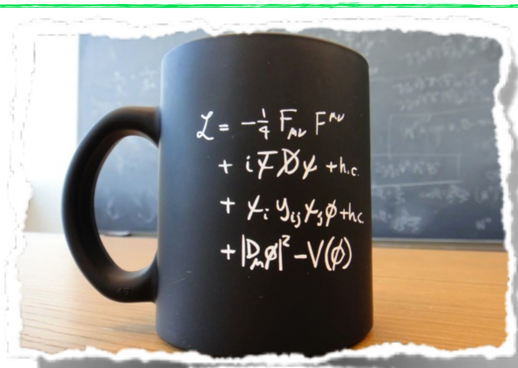
scattering amplitudes

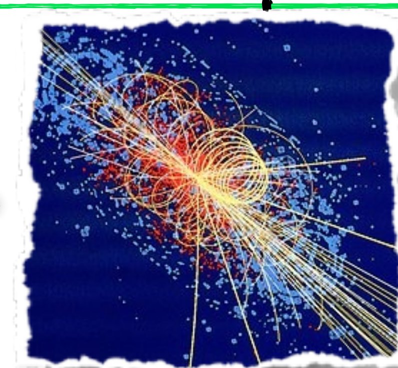
central observables in perturbative QFT

describe interactions between particles

driving force for theoretical developments in QFT

link between theory and experiment





Introduction

Particle Physics described by QFT

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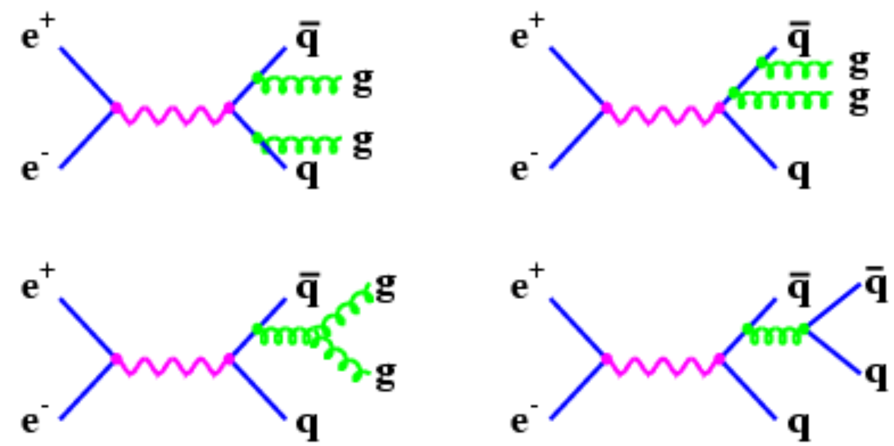
+

powerful help in understanding a theory

Standard approaches to amplitudes

- Write down a Lagrangian consistent with particles and symmetries
- Derive Feynman rules and compute Feynman diagrams



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Final answer usually simpler than intermediate steps
gauge redundancies, off-shell processes...

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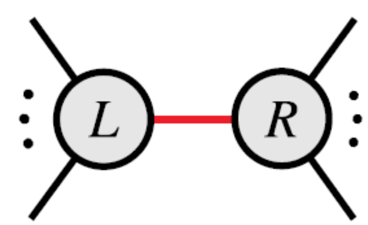
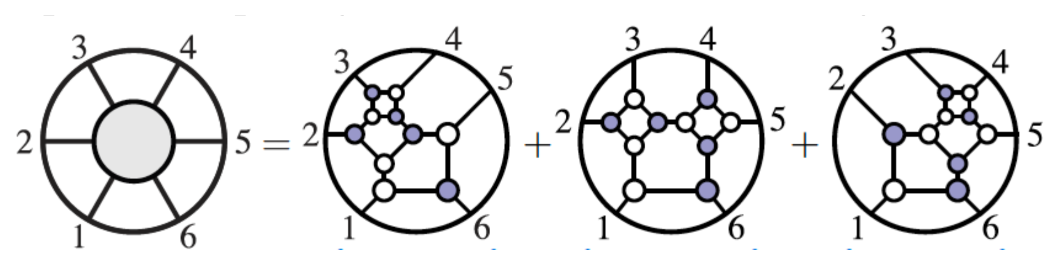
use properties of amplitudes

* Locality and unitarity: ampls factorize in smaller pieces on physical poles
* On-shell recursion relations, on-shell diagrams

$$P_{i,j}^2 = (p_i + p_{i+1} + \ldots + p_j)^2 \qquad A_n^{\text{tree}}(1,\ldots,n) \xrightarrow{P_{i,j}^2 \to 0} \sum_{L,R} A_L^{\text{tree}} \frac{1}{P_{i,j}^2} A_R^{\text{tree}}$$





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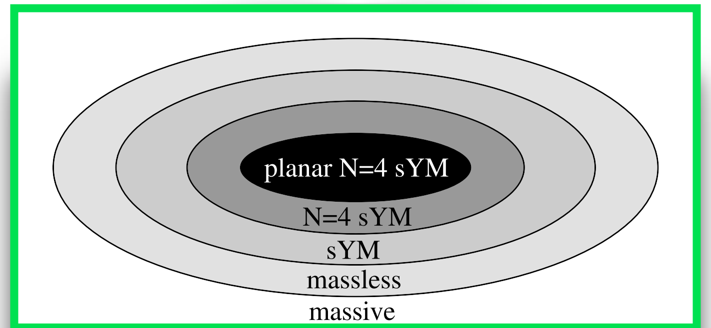
Spurious unphysical poles

encode sings in an efficient way?

From rational functions to differential forms

Scattering amplitudes in N=4 sYM



(picture by L. Dixon)

N=4 super Yang-Mills

Interacting 4d QFT with highest degree of symmetry

prototype of QCD

Symmetries

Novel formulations for QFTs

hint: simplicity of final results

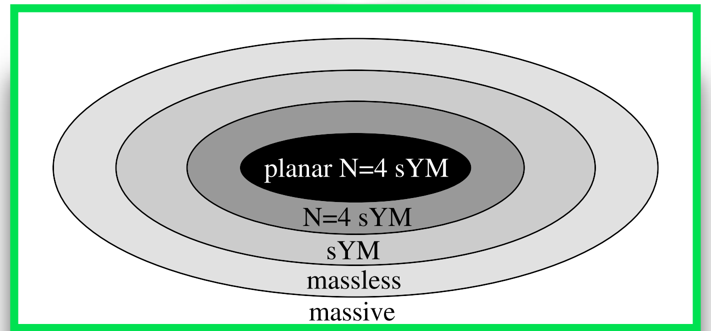
despite complexity of calculations

Geometry

developments not possible from
Lagrangian point of view

locality & unitarity
emergent concepts

Scattering amplitudes in N=4 sYM



(picture by L. Dixon)

N=4 super Yang-Mills

Geometrization of ampls in planar N=4 sYM: Amplituhedron
(N. Arkani-Hamed, J. Trnka)

Geometrization of tree-level ampls in N=4 sYM:
Momentum Amplituhedron (D. Damgaard, L. Ferro, T. Lukowski, M. Parisi)

Scattering amplitudes in N=4 sYM

On-shell supermultiplet described by a superfield:

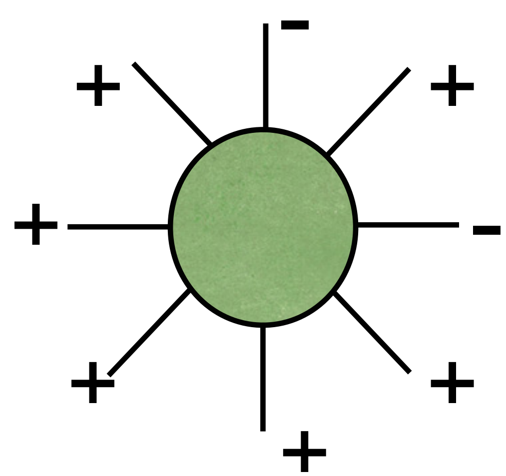
$$\Phi = G^+ + \eta^A \Gamma_A + \frac{1}{2!}\eta^A\eta^B S_{AB} + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\overline{\Gamma}^D + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-$$

* $p^2 = 0 \iff p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha}, \quad q^{\alpha A} = \lambda^\alpha \eta^A$ on-shell superspace $\begin{cases} \alpha, \dot\alpha = 1,2 \\ A = 1,\ldots,4 \end{cases}$

$$\mathcal{A}_n = \langle \Phi(\lambda_1, \tilde\lambda_1, \eta_1)\Phi(\lambda_2, \tilde\lambda_2, \eta_2)\ldots\Phi(\lambda_n, \tilde\lambda_n, \eta_n)\rangle$$

Component amplitudes labeled by two numbers:

* number of particles - n
* helicity - k



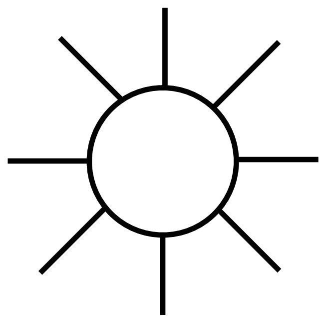
MHV tree level

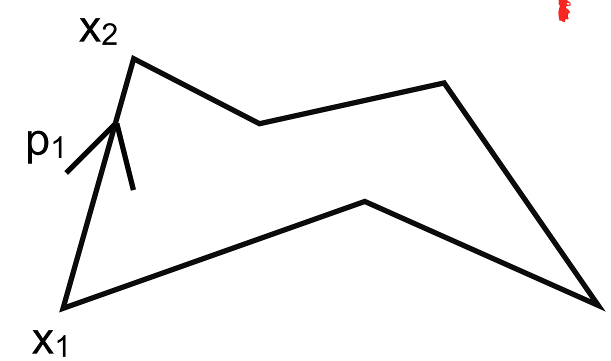
[Parke-Taylor]

$$A^{\text{tree}}_{n,2} = \frac{\delta^4(p)\delta^8(q)}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}, \quad \langle ij\rangle = \lambda_i^\alpha \lambda_{j\alpha}$$

Scattering amplitudes in N=4 sYM

Amplitude duality $\longleftrightarrow$ **Wilson loop**





$$\boxed{\begin{aligned} p_i^{\alpha\dot\alpha} &= x_i^{\alpha\dot\alpha} - x_{i+1}^{\alpha\dot\alpha} \\ q_i^{\alpha A} &= \theta_i^{\alpha A} - \theta_{i+1}^{\alpha A} \end{aligned}}$$

on-shell superspace $\longleftrightarrow$ dual superspace

$$(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A) \qquad (\lambda_i^\alpha, x_i^{\alpha\dot\alpha}, \theta_i^{\alpha A})$$

Fourier transform on λ_i^α

Incidence relations

$$\boxed{\begin{aligned} \tilde\mu_i^{\dot\alpha} &:= x_i^{\alpha\dot\alpha}\lambda_{i\alpha} \\ \chi_i^A &:= \theta_i^{\alpha A}\lambda_{i\alpha} \end{aligned}}$$

twistor superspace

$$\mathscr{W}_i^{\mathscr{A}} = (\mu_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$$

momentum-twistor superspace

$$\mathscr{Z}_i^{\mathscr{A}} = (\lambda_i^\alpha, \tilde\mu_i^{\dot\alpha}, \chi_i^A)$$

+ bosonization

Scattering amplitudes in N=4 sYM

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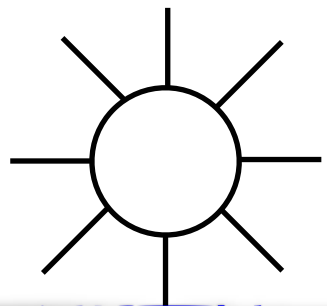

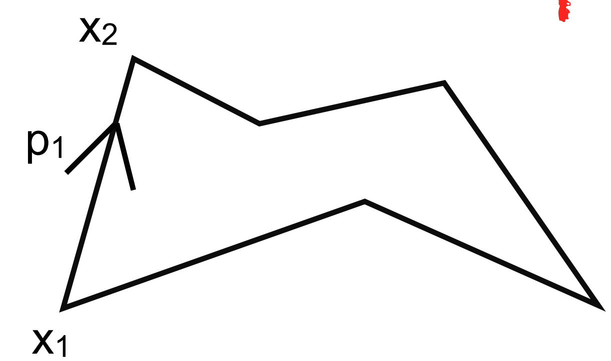

$$p_i^{\alpha\dot\alpha} = x_i^{\alpha\dot\alpha} - x_{i+1}^{\alpha\dot\alpha}$$
$$q_i^{\alpha A} = \theta_i^{\alpha A} - \theta_{i+1}^{\alpha A}$$

on-shell superspace $(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$ $\longleftrightarrow$ **dual superspace** $(\lambda_i^\alpha, x_i^{\alpha\dot\alpha}, \theta_i^{\alpha A})$

Momentum Amplituhedron

Fourier transform on λ_i^α

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Amplituhedron

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Amplituhedron

- Positive geometry for planar N=4 sYM: $\mathscr{A}^{(m)}_{n,k'}$
- Defined on the momentum twistor space $z_i^A, A = 1,2,\ldots,m$
- Region of momentum twistor space satisfying certain positivity conditions
- We can associate a logarithmic differential form $\Omega(z_i^A)$ with logarithmic singularities on all boundaries of $\mathscr{A}^{(m)}_{n,k'}$
- This differential form encodes tree-level amplitudes in planar N=4 sYM (for m=4)

$$A_{n,k'} = \Omega\left(z_i^A\right)|_{dz_i^A \to \chi_i^A}$$

m

- Easier to define using the bosonized momentum twistor space

$$Z_i = \begin{pmatrix} z_i \\ \varphi_1^A \chi_{iA} \\ \vdots \\ \varphi_{k'}^A \chi_{iA} \end{pmatrix}$$

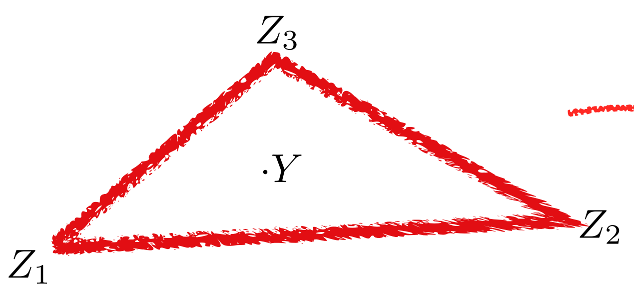
k'

φ_α^A = Grassmann-odd auxiliary variables

Amplituhedron is a generalization of polytope into Grassmannian space

Amplituhedron: the space

Amplituhedron is a generalization of polytope into Grassmannian space



$$Y^I = c_1 Z_1^I + c_2 Z_2^I + c_3 Z_3^I$$

$$k' = 1, \quad m = 2, \quad c_i > 0$$

$$Y_\alpha^I = \sum_{a=1}^{n} c_{\alpha a} Z_a^I$$

$$I = 1,2,\ldots,k' + m$$
$$\alpha = 1,2,\ldots,k'$$

$$\Phi_Z : G_+(k', n) \to G(k', k' + m)$$

- physics: m=4
- tree: k'=1 polytope, k'>1 more complicated object
- loops: similar, more complicated formulae

* $Z \in M_+(k' + m, n)$ - fixed positive external data
* $c \in G_+(k', n)$ - vary over all positive matrices
* $Y \in G(k', k' + m)$ - image of the positive Grassmannian through the map Φ_Z

Amplituhedron: the dlog form

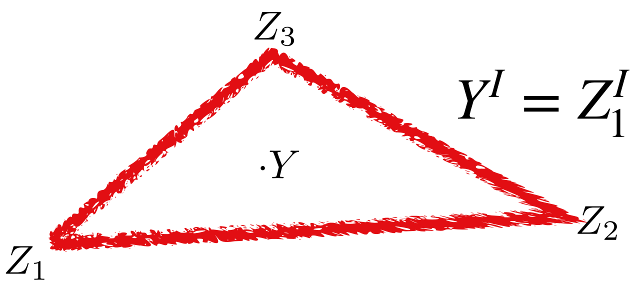
For each amplituhedron $\mathscr{A}^{(m)}_{n,k'}$ one defines a logarithmic form

$$\tilde{\Omega}^{(m)}_{n,k'} = \prod_{\alpha} \langle Y_1 \dots Y_{k'} d^m Y_\alpha \rangle \Omega^{(m)}_{n,k'}$$

$\tilde{\Omega}^{(m)}_{n,k}$ has logarithmic singularities at all boundaries of $\mathscr{A}^{(m)}_{n,k'}$

Example:

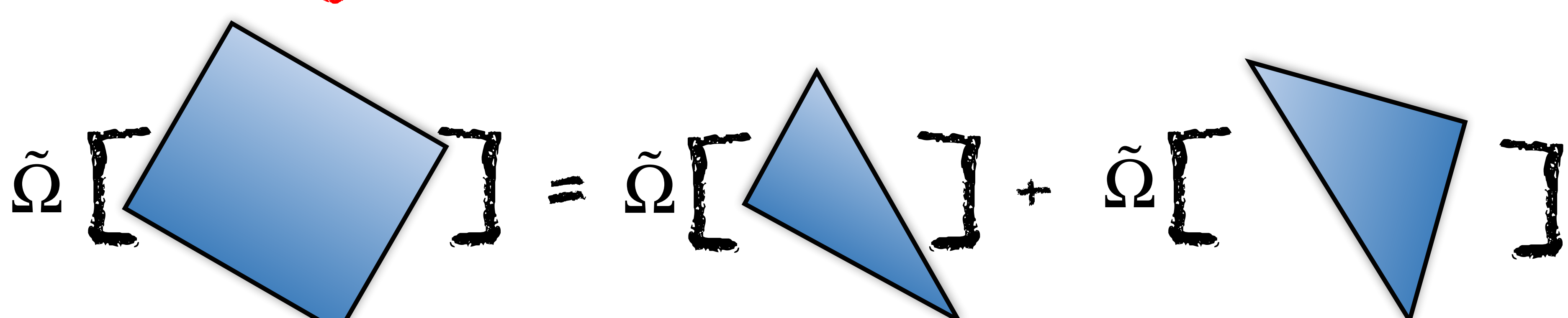
$$\langle 123 \rangle = \langle Z_1 Z_2 Z_3 \rangle = \epsilon_{ABC} Z_1^A Z_2^B Z_3^C$$



$$Y^I = Z_1^I + c_2 Z_2^I + c_3 Z_3^I \longrightarrow \tilde{\Omega}^{(2)}_{3,1} = \frac{dc_2}{c_2}\frac{dc_3}{c_3} = d\log\frac{\langle Y13 \rangle}{\langle 123 \rangle} d\log\frac{\langle Y12 \rangle}{\langle 123 \rangle} = \frac{\langle Y dY dY \rangle \langle 123 \rangle^2}{\langle Y12 \rangle \langle Y23 \rangle \langle Y31 \rangle} =: [123]$$

pushforward through Φ_Z

Compatible with triangulations:



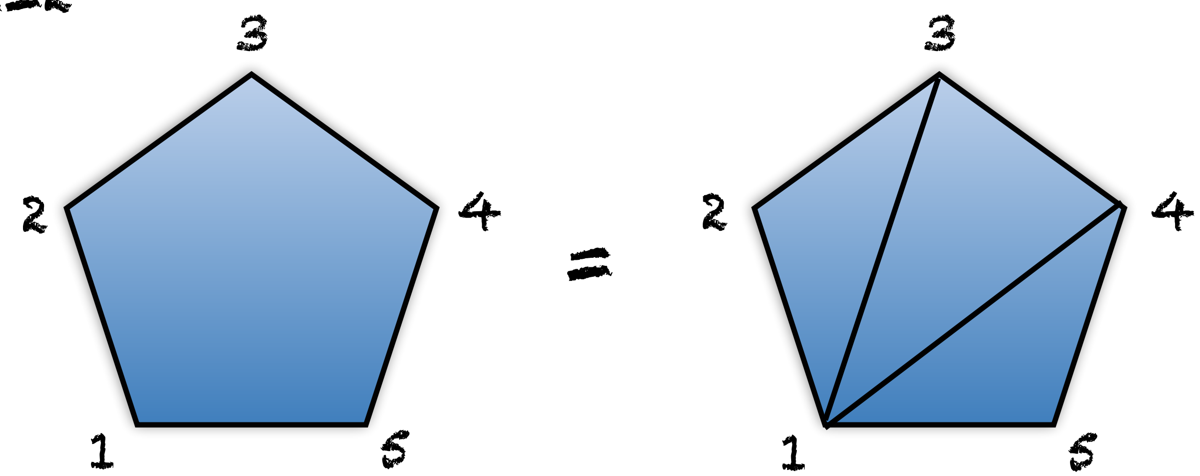
Tree amplitudes in planar N=4 sYM extracted from $\tilde{\Omega}^{(4)}_{n,k'}$

Amplituhedron: the log form

How to find the logarithmic form?

Geometrically: triangulate $\mathscr{A}^{(m)}_{n,k'}$ and sum over the known volume fcs of triangles

Example: n=5, k'=1, m=2



$$\Omega^{(2)}_{5,1} = [123] + [134] + [145]$$

Analytically: evaluate the contour integral

$$\Omega^{(m)}_{n,k'} = \int_\gamma \frac{d^{k'n}c}{(12\cdots k')\ldots(n1k'-1)} \prod_{\alpha,A} \delta(Y^A_\alpha - \sum_i c_{\alpha i} Z^A_i)$$

different contours <-> different triangulations

Back to spinors

* Amplituhedron relies on planarity -> suitable for planar N=4 sYM
* What about non-planar theories?

On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$

Non-chiral superspace

$$(\lambda_i^a, \eta_i^r \,|\, \tilde{\lambda}_i^{\dot{a}}, \tilde{\eta}_i^{\dot{r}}),\ a, \dot{a}, r, \dot{r} = 1{,}2$$

$$\begin{cases} \tilde{q}^{\dot{a}r} = \sum_{i=1}^{n} \tilde{\lambda}_i^{\dot{a}} \eta_i^r \\ q^{ar} = \sum_{i=1}^{n} \lambda_i^a \tilde{\eta}_i^{\dot{r}} \end{cases}$$

Associate a,ȧ with SU(2)xSU(2) R-symmetry indices:

$$\eta^a \to d\lambda^a\,, \quad \tilde{\eta}^{\dot{a}} \to d\tilde{\lambda}^{\dot{a}}$$

n-point super-amplitude in non-chiral space <-> 2n form

(S. He, C. Zhang)

Amplitudes as forms

$$(\lambda_i^a, \eta_i^r \,|\, \tilde{\lambda}_i^{\dot{a}}, \tilde{\eta}_i^{\dot{r}}),\ a, \dot{a}, r, \dot{r} = 1,2$$

n-point super-amplitude in non-chiral space <-> 2n form

$$A_{n,k} := (dq)^4 \wedge \mathbf{\Omega_{n,k}}$$

2n-4 form

$$\begin{cases} d\tilde{q}^{\dot{a}r} = \sum_{i=1}^{n} \tilde{\lambda}_i^{\dot{a}} d\lambda^r \\ dq^{a\dot{r}} = \sum_{i=1}^{n} \lambda_i^a d\tilde{\lambda}_i^{\dot{r}} \end{cases}$$

Example: 4-point MHV

$$\mathbf{\Omega_{4,2}} = \frac{(d\tilde{q})^4}{st} = \mathrm{dlog}\frac{\langle 12 \rangle}{\langle 13 \rangle} \wedge \mathrm{dlog}\frac{\langle 23 \rangle}{\langle 13 \rangle} \wedge \mathrm{dlog}\frac{\langle 34 \rangle}{\langle 13 \rangle} \wedge \mathrm{dlog}\frac{\langle 41 \rangle}{\langle 13 \rangle}$$

Geometry whose canonical form gives the amplitude form?

Momentum amplituhedron

=

Positive geometry whose log diff form is the amplitude in spinor-helicity space

Momentum Amplituhedron

Bosonized spinor helicity variables:

$$\tilde{\Lambda}_i^{\dot{A}} = \begin{pmatrix} \tilde{\lambda}_i^{\dot{a}} \\ \tilde{\phi}_{\dot{a}}^{\dot{\alpha}} \cdot \tilde{\eta}_i^{\dot{a}} \end{pmatrix}, \qquad \dot{A} = (\dot{a}, \dot{\alpha}) = 1,\ldots,k+2 \qquad \Lambda_i^A = \begin{pmatrix} \lambda_i^a \\ \phi_a^\alpha \cdot \eta_i^a \end{pmatrix}, \qquad A = (a,\alpha) = 1,\ldots,n-k+2$$

$$\left\{ \text{matrix } \tilde{\Lambda} \text{ positive} \quad \text{and} \quad \text{matrix } \Lambda^\perp \text{ positive} \right\}$$

Positive region

Momentum amplituhedron: Image of the positive Grassmannian $G_+(k,n)$ through the map

$$\Phi_{(\Lambda,\tilde{\Lambda})} : G_+(k,n) \to G(k,k+2) \times G(n-k,n-k+2)$$

defined as:

$$\tilde{Y}_{\dot{\alpha}}^{\dot{A}} = \sum_{i=1}^n c_{\dot{\alpha}i} \tilde{\Lambda}_i^{\dot{A}} \qquad Y_\alpha^A = \sum_{i=1}^n c_{\alpha i}^\perp \Lambda_i^A$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_{i} \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^{A}_{i}$$

Facets: codimension-1 boundaries

$\langle Y\, i\, i+1\rangle = 0\,, \qquad [\tilde{Y}\, i\, i+1] = 0$ Collinear limits

$S_{i,i+1\ldots,i+p} = 0\,, \qquad p = 2,\ldots,n-4$ Factorizations

Uplift of planar Mandelstam variables

$$S_{i,i+1\ldots,i+p} = \sum_{i \le j_1 < j_2 \le i+p} \langle Y j_1 j_2\rangle[\tilde{Y} j_1 j_2]$$

Momentum Amplituhedron

$$\tilde{Y}_{\dot\alpha}^{\dot A} = \sum_{i=1}^{n} c_{\dot\alpha i}\,\tilde\Lambda_i^{\dot A} \qquad\qquad Y_\alpha^A = \sum_{i=1}^{n} c_{\alpha i}^{\perp}\,\Lambda_i^A$$

Facets: codimension-1 boundaries

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$S_{i,i+1\ldots,i+p} = 0\,,$	$p = 2,\ldots,n-4$	Factorizations

Volume form
=
Differential form with log singularities on all boundaries
=
Sum over cells of push-forwards of canonical diff-form

Momentum Amplituhedron

$$\tilde{Y}_{\dot{\alpha}}^{\dot{A}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}_i^{\dot{A}} \qquad\qquad Y_{\alpha}^{A} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda_i^{A}$$

➡ Volume form

$$\mathbf{\Omega}_{n,k} = \sum_{\sigma} \mathrm{dlog}\, \alpha_1^{\sigma} \wedge \mathrm{dlog}\, \alpha_2^{\sigma} \wedge \ldots \wedge \mathrm{dlog}\, \alpha_{2n-4}^{\sigma}$$

➡ Volume function

$$\mathbf{\Omega}_{n,k} \wedge d^4P\, \delta^4(P) = \prod_{\alpha=1}^{n-k} \langle Y_1 \ldots Y_{n-k} d^2 Y_{\alpha} \rangle \prod_{\dot{\alpha}=1}^{k} [\tilde{Y}_1 \ldots \tilde{Y}_k d^2 \tilde{Y}_{\dot{\alpha}}]\, \delta^4(P)\, \Omega_{n,k}$$

➡ Amplitude

Reference subspaces

$$A_{n,k}^{\mathtt{tree}} = \delta^4(p) \int d\phi_a^1 \ldots d\phi_a^{n-k} \int d\tilde{\phi}_{\dot{a}}^1 \ldots d\tilde{\phi}_{\dot{a}}^k\, \Omega_{n,k}(Y^*, \tilde{Y}^*, \Lambda, \tilde{\Lambda})$$

$$Y^* = \begin{pmatrix} \mathbf{0}_{2\times(n-k)} \\ \mathbf{I}_{(n-k)\times(n-k)} \end{pmatrix} \qquad \tilde{Y}^* = \begin{pmatrix} \mathbf{0}_{2\times k} \\ \mathbf{I}_{k\times k} \end{pmatrix}$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Example

* MHV$_4$ amplitude:

$$\alpha_1 = \frac{\langle Y12\rangle}{\langle Y13\rangle}\,, \alpha_2 = \frac{\langle Y23\rangle}{\langle Y13\rangle}\,, \alpha_3 = \frac{\langle Y34\rangle}{\langle Y13\rangle}\,, \alpha_4 = \frac{\langle Y14\rangle}{\langle Y13\rangle}$$

$$\boldsymbol{\Omega}_{4,2} = \bigwedge_{j=1}^{4} \mathrm{dlog}\alpha_j = \mathrm{dlog}\frac{\langle Y12\rangle}{\langle Y13\rangle} \wedge \mathrm{dlog}\frac{\langle Y23\rangle}{\langle Y13\rangle} \wedge \mathrm{dlog}\frac{\langle Y34\rangle}{\langle Y13\rangle} \wedge \mathrm{dlog}\frac{\langle Y14\rangle}{\langle Y13\rangle}$$

$$= \frac{\langle 1234\rangle^2}{\langle Y12\rangle\langle Y23\rangle\langle Y34\rangle\langle Y41\rangle}\langle Yd^2Y_1\rangle\langle Yd^2Y_2\rangle \to A^{\texttt{tree}}_{4,2} = \delta^4(p)\frac{\delta^4(q)\delta^4(\tilde{q})}{\langle 12\rangle_\lambda\langle 23\rangle_\lambda[12]_{\tilde{\lambda}}[23]_{\tilde{\lambda}}}$$

Divergences on the 4 facets of the momentum amplituhedron:

$$\langle Yii+1\rangle = 0\,, i = 1,\ldots,4$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Examples

* NMHV$_6$ amplitudes:

$$\Omega_{6,3} = \Omega^{(612)}_{6,3} + \Omega^{(234)}_{6,3} + \Omega^{(456)}_{6,3} = \Omega^{(123)}_{6,3} + \Omega^{(345)}_{6,3} + \Omega^{(561)}_{6,3}$$

$$\Omega^{(123)}_{6,3} = \frac{\left(\langle Y12\rangle[12456] + \langle Y13\rangle[13456] + \langle Y23\rangle[23456]\right)^2 \left([\tilde{Y}45]\langle 12345\rangle + [\tilde{Y}46]\langle 12346\rangle + [\tilde{Y}56]\langle 12356\rangle\right)^2}{S_{123}\langle Y12\rangle\langle Y23\rangle[\tilde{Y}45][\tilde{Y}56]\boxed{\langle Y1|5+6|4\tilde{Y}]\langle Y3|4+5|6\tilde{Y}]}}$$

Spurious singularities

Divergences on the 15 facets of the momentum amplituhedron:

$$\langle Yii+1\rangle = 0\ , i = 1,\ldots,6\,, \quad [\tilde{Y}ii+1] = 0\,, i = 1,\ldots,6\,, \quad S_{i,i+1,i+2} = 0\,, i = 1,2,3$$

Geometry gives unique insight into singularity structure of ampls

Singularities from Boundaries

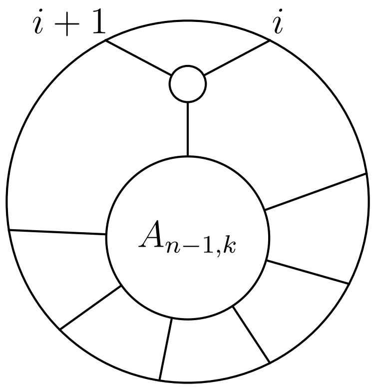
Geometry gives unique insight into singularity structure of ampls

(LF, T. Lukowski, R. Moerman)

Boundary stratification

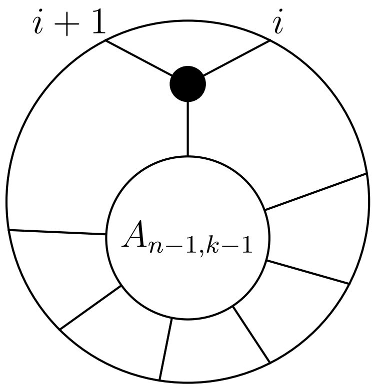
Codim 1 boundaries

collinear



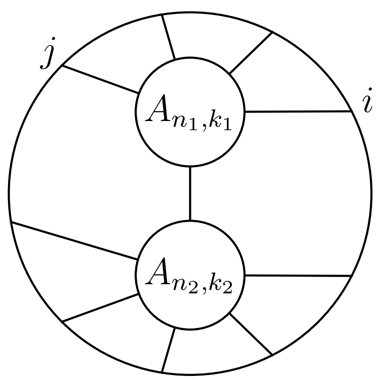
$\langle Y\, i\, i+1\rangle = 0$

collinear



$[\tilde{Y}\, i\, i+1] = 0$

factorization



$S_{i,i+1\dots,j} = 0$

Singularities from Boundaries

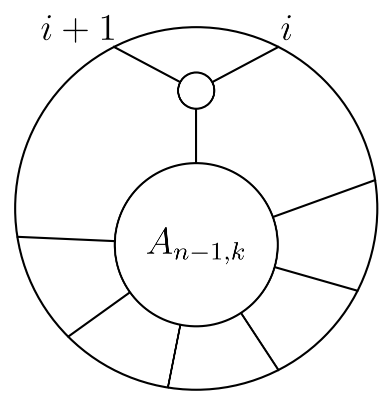
Geometry gives unique insight into singularity structure of ampls

(LF, T. Lukowski, R. Moerman)

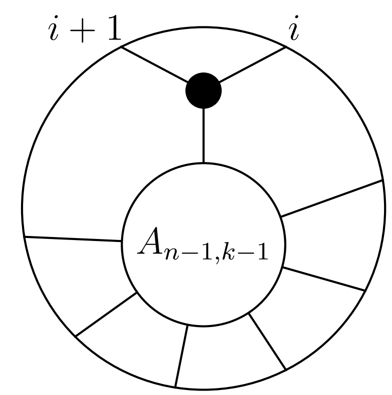
Boundary stratification

collinear | collinear | factorization

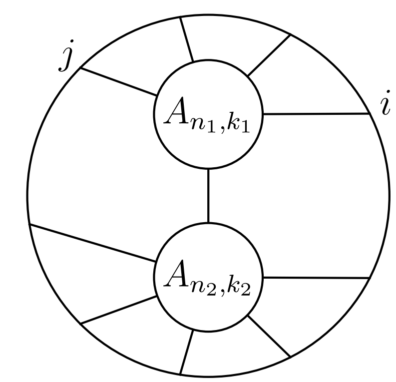
Codim 1 boundaries



$\langle Y\, i\, i+1\rangle = 0$



$[\tilde{Y}\, i\, i+1] = 0$

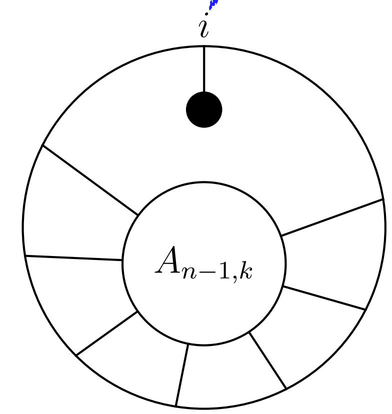


$S_{i,i+1\ldots,j} = 0$

Codim 2 boundaries

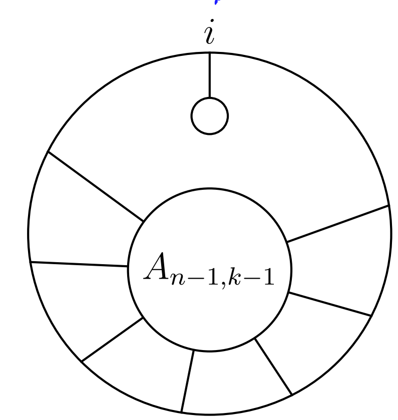
further factorizations
and collinear limits

soft



$\langle Y\, i\, i+1\rangle = 0 = \langle Y\, i-1\, i\rangle$

soft



$[\tilde{Y}\, i\, i+1] = 0 = [\tilde{Y}\, i-1\, i]$

Intersection of two consecutive
codim 1 collinear boundaries

Singularities from Boundaries

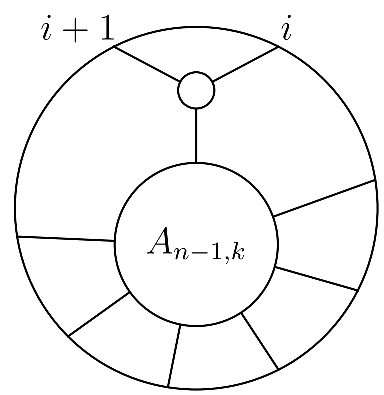
Geometry gives unique insight into singularity structure of ampls

(LF, T. Lukowski, R. Moerman)

Boundary stratification

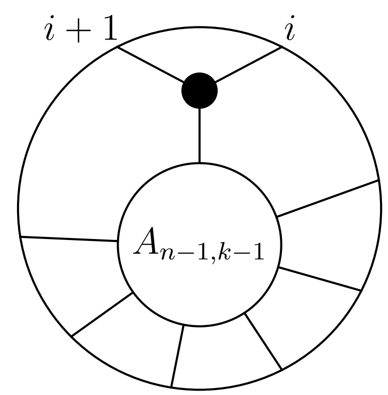
Codim 1 boundaries

collinear



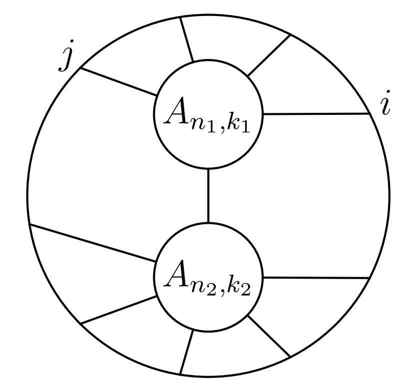
$\langle Y\, i\, i+1\rangle = 0$

collinear



$[\tilde{Y}\, i\, i+1] = 0$

factorization

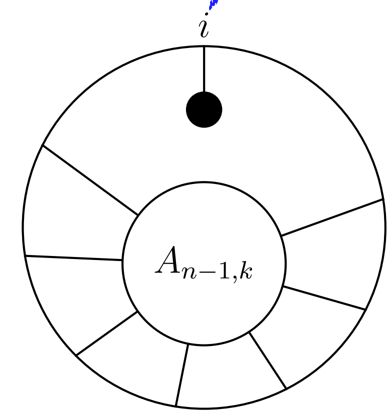


$S_{i,i+1\ldots,j} = 0$

Codim 2 boundaries

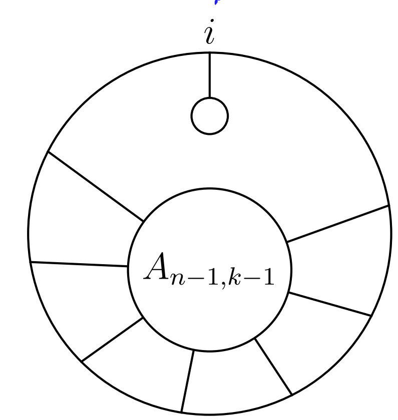
further factorizations
and collinear limits

soft



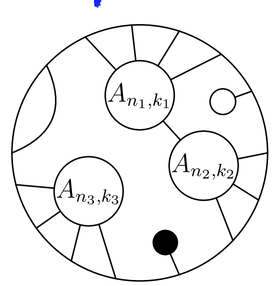
$\langle Y\, i\, i+1\rangle = 0 = \langle Y\, i-1\, i\rangle$

soft



$[\tilde{Y}\, i\, i+1] = 0 = [\tilde{Y}\, i-1\, i]$

+ generalization deeper in the geometry



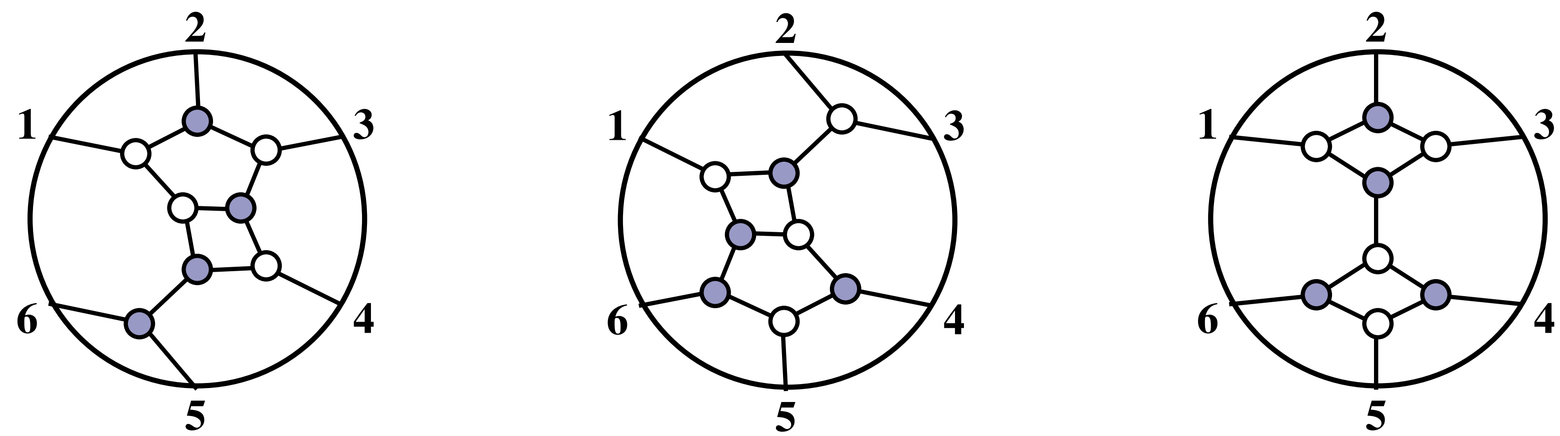
Singularities from Boundaries

Examples

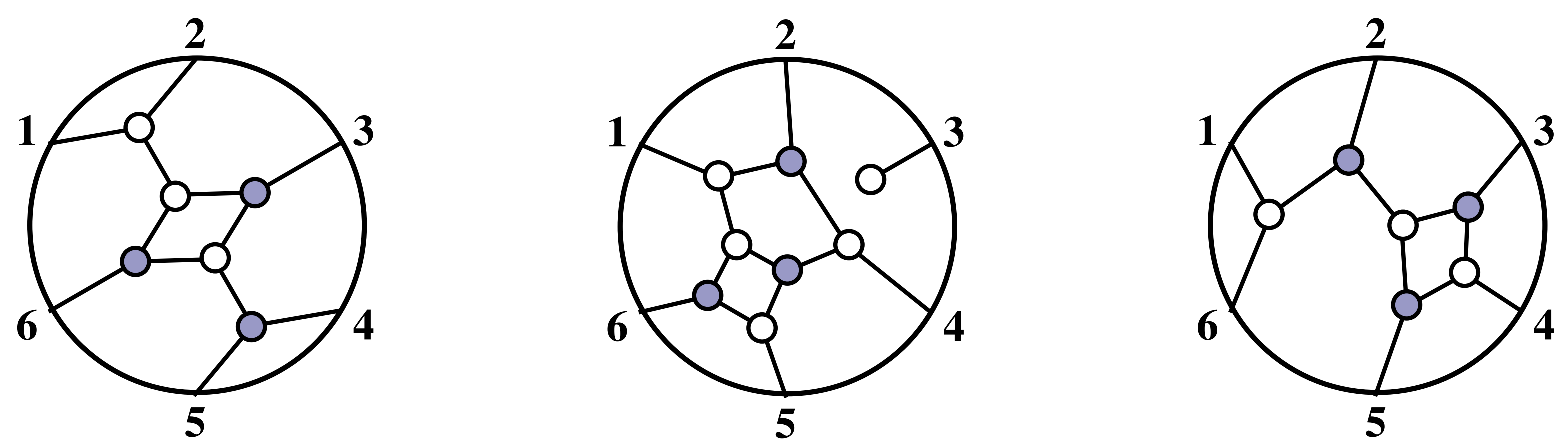
☆ $NMHV_6$

Representatives:

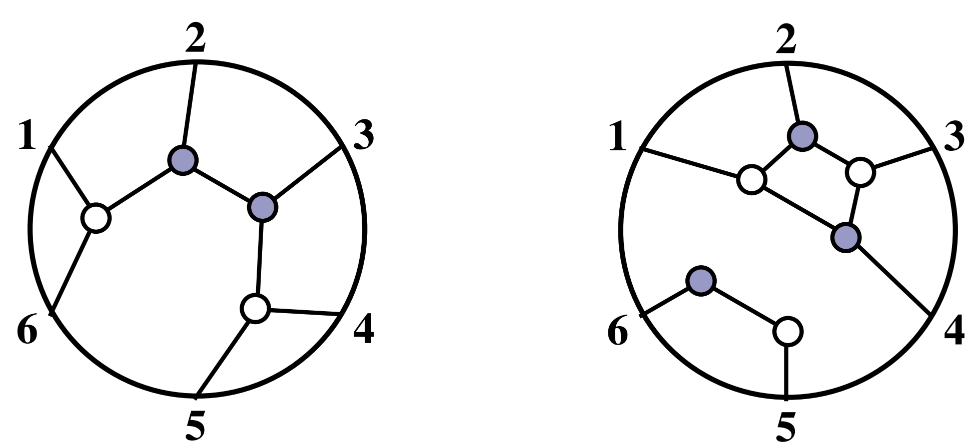
Codim 1 boundaries



Codim 2 boundaries



Codim 3 boundaries



...

Conclusions

Summary

- [x] Space-time independent approach to scattering amplitudes
- [x] Positivity plays a crucial role
- [x] Locality and unitarity emerge from the geometry

Open problems

- [ ] Extension to other theories: e.g. non-planar, less-, non-susy?
- [ ] Geometry for integrated loop ampls, not just integrands?
- [ ] Classify all triangulations of a positive geometry?
- [ ] ...


Thank you!