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Asymptotic dynamics on the worldline for spinning particles

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Outline

Introduction

First quantized approach

Schwinger's method (1951): inverse scalar propagator interpreted as a Hamiltonian governing evolution in **proper time** T

$$\frac{i}{p^2 - m^2 + i\epsilon} = \int_0^\infty dT \, e^{i\,(\overbrace{p^2 - m^2}^{H(x,p)} + i\epsilon)T}$$

Path integral via classical action $S = \int dt\,(p \cdot \dot{x} - H)$

Relativistic scalar particle described in terms of fields $x^\mu(t)$ and $p^\mu(t)$ leaving in a one-dimensional space of length T: the **wordline**

Wordline representation

Spin typically implemented with additional variables $\psi_i^\mu(t)$
[Barducci,Casalbuoni,Lusanna(1976), Brink,DiVecchia,Howe(1977), Gershun,Tkach(1979), Howe,Penati,Pernici,Townsend(1988-89), ...]

$$S = \int dt \left(p_\mu \dot{x}^\mu + \frac{i}{2}\psi_i^\mu \dot{\psi}_\mu^i - \frac{1}{2} e p_\mu p^\mu - i\chi_i \psi_\mu^i p^\mu - \frac{i}{2} a_{ij} \psi_i^\mu \psi_\mu^j \right)$$

with $i = 1, ..., N$ for spin $N/2$

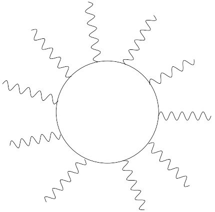
Many symmetries!

- reparametrization invariance $\to e(t)$
- local susy $\to \chi_i(t)$
- $O(N)$ invariance $\to a_{ij}(t)$

Wordline representation

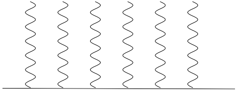
Many studies for **closed** topology (1-loop effective action): solving path integral at order g^N, one gets amplitude with N external gauge bosons

[Strassler(1992), Bern,Kosower(1992), Schmidt,Schubert(1993), Bastianelli et al, ...]

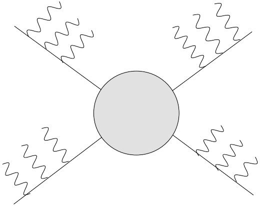


Open topology (dressed propagator) less explored

[DiVecchia,Ravndal(1979), Fradkin,Gitman(1991), Pierri,Rivelles(1990), Marnelius(1994), vanHolten(1995), Reuter,Schmidt,Schubert(1996), Ahmadiniaz,Bastianelli,Corradini(2015), ...]



An intermediate situation is given by **asymptotic** propagators: solving path integral to all-orders in the coupling g, but order-by-order in the soft expansion, one gets exponentiation



Generalized Wilson Line

[Laenen,Stavenga,White(2008), White(2011), DB,Laenen,Magnea,Vernazza,White(2016)]

$$
\begin{aligned}
&\widetilde{W}_n(0,\infty) \\
&= \mathcal{P}\exp\left[\int_k \tilde{A}_\mu(k)\left(-\frac{n^\mu}{n\cdot k} + \frac{k^\mu}{2n\cdot k} - k^2\frac{n^\mu}{2(n\cdot k)^2} - \frac{\mathrm{i}k_\nu J^{\nu\mu}}{n\cdot k}\right)\right. \\
&\quad + \int_k\int_l \tilde{A}_\mu(k)\tilde{A}_\nu(l)\left(\frac{\eta^{\mu\nu}}{2n\cdot(k+l)} - \frac{n^\nu l^\mu n\cdot k + n^\mu k^\nu n\cdot l}{2(n\cdot l)(n\cdot k)\left[n\cdot(k+l)\right]}\right. \\
&\quad \left.\left. + \frac{(k\cdot l)n^\mu n^\nu}{2(n\cdot l)(n\cdot k)\left[n\cdot(k+l)\right]} - \frac{\mathrm{i}J^{\mu\nu}}{n\cdot(k+l)}\right)\right],
\end{aligned}
$$

$J^{\mu\nu} = S^{\mu\nu} + L^{\mu\nu}$ total angular momentum

- LP (eikonal approximation, usual Wilson line)
- NLP (next-to-eikonal, next-to-soft): pairwise correlations, spin and recoil taken into account
- $\frac{\mathrm{i}k_\nu J^{\nu\mu}}{n\cdot k}$ next-to-soft theorems [Cachazo,Strominger(2014), Casali(2014), Sen (2017), ...]

Why bother?

Derivation for generic spinning particle is desirable

- QCD phenomenology: NLP factorization with quarks and gluons as external states (Drell-Yan, Higgs, ...)
  [DB,Laenen,Magnea,Vernazza,White(2016), Bahjat-Abbas,DB,Damstè,Laenen,Magnea,Vernazza,White(2019)]
- Faddeev-Kulish asymptotic states
  [Kapec,Perry,Raclariu,Strominger(2017), Gonzo,McLaughlin,Medrano,Spiering(2019), Choi,Akhoury(2019), Hannesdottir,Schwartz(2019), ...]
- Semiclassical interpretation for (next-to-)soft theorems
  [Sen, Laddha, Sahoo (2017-20)]
- High energy limit of scattering amplitudes
  [Luna,Melville,Naculich,White(2017), Damour(2018), Ciafaloni,Colferai,Veneziano(2018), Collado,DiVecchia,Russo(2019), Kosower,Maybee,O'Connell(2019), ...]
- Spin in gravity [Levi(2013-2018), Steinhoff,Vines(2016), Guevara,Ochirov,Vines(2019), Porto et al(2017), Bern et al(2020), ...]

Spin 0

Free scalar particle

Action is well-known:

$$S = \int dt \left(p \cdot \dot{x} - e \frac{1}{2} \overbrace{(p^2 - m^2)}^{\text{constraint}} \right) .$$

Gauge invariance (reparametrization invariance) generated by $Q_0 \equiv \frac{1}{2} (p^2 - m^2)$

$$\delta x_\mu = \xi p^\mu , \qquad \delta p_\mu = 0 , \qquad \delta e = \dot{\xi} ,$$

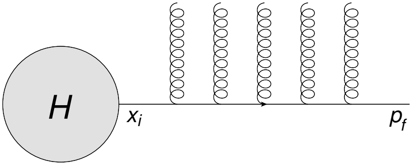
Quantization à la Dirac: Hilbert space generated by eigenstates of $\hat{x}$ and $\hat{p}$, where physical states satisfy $Q_0|\psi\rangle = 0$

Free scalar propagator

Usual QFT propagator in position space

$$\langle \phi(x_f)\phi(x_i)\rangle = \frac{1}{2}\int_{x(0)=x_i}^{x(1)=x_f} \mathcal{D}e\mathcal{D}x\mathcal{D}p\, e^{-i\int_0^1 dt\,(p\cdot\dot{x}-e(Q_0+i\epsilon))}\,,$$

Fixing the gauge $e(t) = T \to$ Schwinger's representation.



Asymptotic propagators need **mixed position-momentum** representation:

$$\frac{1}{2}\int_0^\infty dT \int_{x(0)=x_i}^{p(T)=p_f} \mathcal{D}x\mathcal{D}p\, e^{ip(T)\cdot x(T)-i\int_0^T dt\,(p\cdot\dot{x}-Q_0-i\epsilon)}\,,$$

Background field

Introduced via $p^\mu \to p^\mu - A^\mu$ in the Noether's charge

$$Q_0^A(\hat{x},\hat{p}) \equiv \frac{1}{2}\left((\hat{p}_\mu - A_\mu(\hat{x}))^2 - m^2\right)$$

Dressed propagator becomes (integration over p is Gaussian)

$$\frac{\langle p_f|(2Q_0^A + i\epsilon)^{-1}|x_i\rangle}{\langle p_f|x_i\rangle} = \frac{1}{2}\int_0^\infty dT\, e^{i\frac{1}{2}\overbrace{(p_f^2 - m^2 + i\epsilon)}^{\textbf{free}\text{ propagator}}T}\ \overbrace{f(x_i,p_f,T)}^{\text{radiative factor}}$$

$$f(x_i,p_f,T) = \int_{\tilde{x}(0)=0} \mathcal{D}\tilde{x}\ e^{i\int_0^T dt\left[\frac{1}{2}\dot{\tilde{x}}^2 + (p_f+\dot{\tilde{x}})\cdot A(x(t)) + \frac{i}{2}\partial\cdot A(x(t))\right]}$$

ASYMPTOTIC PROPAGATOR

Truncating external **free** propagator à la LSZ

$$i(p_f^2 - m^2)\frac{\langle p_f|(2Q_0^A + i\epsilon)^{-1}|x_i\rangle}{\langle p_f|x_i\rangle} = \lim_{T\to\infty} f(x_i, p_f, T)\ .$$

The asymptotic propagator is given by the radiative factor f for $T \to \infty$

Now we can evaluate f order by order in the soft expansion, but to all-orders in the coupling constant!

Soft expansion

Rescale $p_f \to \lambda n$ and $t \to t/\lambda$ and solve path integral (1-dim QFT for $x^\mu(t)$) order-by-order in $1/\lambda$

$$f(x_i, p_f, \infty) = \int_{\tilde{x}(0)=0} \mathcal{D}\tilde{x}\, \mathcal{P}\, e^{i\int_0^\infty dt\, \left[\frac{\lambda}{2}\dot{\tilde{x}}^2 + (p_f + \dot{\tilde{x}})\cdot A(x(t)) + \frac{i}{2\lambda}\partial\cdot A(x(t))\right]}$$

- Crucially, propagator for $\tilde{x}$ is of order $1/\lambda$, so finite number of diagrams at given order in $1/\lambda$.
- Expanding $A^\mu(nt + \tilde{x}) = A^\mu(nt) + x^\nu \partial_\nu A^\mu(nt) + \ldots$ generates powers of $\tilde{x}(t)$.

Soft expansion

- LP: path integral on its stationary phase: Wilson line on the straight classical path
- NLP: include fluctuations along classical path. Only 2 diagrams!

Sum of connected diagrams exponentiate! (Non-abelian equivalent requires "webs")

$$
\begin{aligned}
= \mathcal{P} \exp \Bigg[g \int_k \tilde{A}_\mu(k) \left(-\frac{n^\mu}{n \cdot k} + \frac{k^\mu}{2n \cdot k} - k^2 \frac{n^\mu}{2(n \cdot k)^2} n \cdot k \right) \\
+ \int_k \int_l \tilde{A}_\mu(k) \tilde{A}_\nu(l) \left(\frac{\eta^{\mu\nu}}{2n \cdot (k+l)} - \frac{n^\nu l^\mu n \cdot k + n^\mu k^\nu n \cdot l}{2(n \cdot l)(n \cdot k)\left[n \cdot (k+l)\right]} \right. \\
\left. + \frac{(k \cdot l) n^\mu n^\nu}{2(n \cdot l)(n \cdot k)\left[n \cdot (k+l)\right]} \right) \Bigg] ,
\end{aligned}
$$

Spin 1/2

FREE DIRAC FERMION

Include spin variable $\psi^\mu(t)$:

$$S = \int dt \left(p_\mu \dot{x}^\mu + \frac{i}{2}\psi^\mu \dot{\psi}_\mu - \frac{1}{2} e \overbrace{p_\mu p^\mu}^{Q_0} - i\chi \overbrace{\psi_\mu p^\mu}^{Q_1} \right) \, ,$$

Q_0 and Q_1 generate larger gauge invariance:

- reparametrization invariance $e(t)$
- local susy $\chi(t)$

$$\delta x^\mu = \xi p^\mu + i\zeta\psi^\mu \qquad \delta p^\mu = 0 \qquad \delta\psi^\mu = -\zeta p^\mu$$
$$\delta e = \dot{\xi} \qquad \delta\chi = \dot{\zeta}$$

Background field introduced again via $p^\mu \to p^\mu - A^\mu$ and susy generated via $Q_i \to Q_i^A$

Dressed propagator

Fixing gauge multiplet $(e, \chi) \to (T, \theta)$

$$\langle p_f | \frac{Q_1^A}{2Q_0^A + i\epsilon} | x_i \rangle = \frac{1}{2} \int_0^\infty dT \int d\theta$$
$$\int_{\psi(T)=\Gamma} \mathcal{D}\psi \int_{x(0)=x_i}^{p(T)=p_f} \mathcal{D}x\mathcal{D}p \, e^{ip(T)\cdot x(T) - i\int_0^T dt \left(p\cdot\dot{x} + \frac{i}{2}\psi\cdot\dot{\psi} - Q_0^A - \frac{\theta}{T} Q_1^A - i\epsilon \right)} .$$

- proper time T exponentiates numerator Q_0^A
- Grassmann “supertime” θ exponentiates denominator Q_1^A

Asymptotic Propagator

$$\frac{1}{2}\int_0^\infty dT\, e^{\frac{i}{2}\overbrace{(p_f^2+i\epsilon)}^{\text{free denom}}T}\int d\theta\, e^{i\overbrace{\Gamma\cdot p_f}^{\text{free num}}\theta}\overbrace{f(x_i,p_f,\Gamma,T,\theta)}^{\text{radiative factor}}\ ,$$

$$f(x_i,p_f,\Gamma,T,\theta)=\int_{\psi(T)=\Gamma}\mathcal{D}\psi\int_{\tilde{x}(0)=0}\mathcal{D}\tilde{x}\ \overbrace{e^{-i\theta\psi\cdot A(x(t))}}^{\text{prevents }\theta\text{ int}}$$
$$e^{i\int_0^T dt\,\left(-\frac{i}{2}\psi\cdot\dot{\psi}+\frac{1}{2}\dot{\tilde{x}}^2+(p_f+\dot{\tilde{x}})\cdot A(x(t))+\frac{i}{2}\partial\cdot A(x(t))+\frac{1}{2}\psi_\mu\psi_\nu F^{\mu\nu}\right)}\ .$$

Truncate external free propagator: multiply by $\bar{u}(p_f)\frac{i}{\not p_f}p_f^2$ to get

$$\lim_{T\to\infty}\bar{u}(p_f)\frac{1}{\not p_f}\int d\theta\ e^{i\Gamma\cdot p_f\theta}\,f(x_i,p_f,\Gamma,T,\theta)$$

Asymptotic Propagator

Problem: numerators do not cancel due to factor $\theta \cdot A(x(t))$ in f

But $Q_1^A = \Gamma \cdot (p_f - A(x(t)))$ is a conserved charge!
We can evaluate it at $t = T$, where $A \to 0$.
Effectively, we replace $Q_1^A \to Q_1$

Then, the asymptotic propagator is given by

$$\lim_{T\to\infty} \bar{u}(p_f) f(x_i, p_f, \Gamma, T) \ .$$

Background gauge field in the numerator does not contribute to the asymptotic dynamics! This is reminiscent of 1-loop effective action. Supersymmetry is key to make this manifest.

DENOMINATOR

Contribution of the background soft field is contained only in the denominator. Difference w.r.t. scalar case is $\psi_\mu \psi_\nu F^{\mu\nu}$:

$$\int_{\psi(\infty)=\Gamma} \mathcal{D}\psi \, e^{i\int_0^\infty dt\,\left(-\frac{i}{2}\psi\cdot\dot{\psi}+\frac{1}{2\lambda}\psi_\mu\psi_\nu F^{\mu\nu}\right)}$$

This integral factorizes from x-path-integral and gives rise to two additional vertices, in agreement with the GWL:

$$\frac{i}{\lambda} S^{\mu\nu} \int \frac{d^d k}{(2\pi)^d} \frac{k_\nu}{n\cdot k} \tilde{A}_\mu(k)$$

$$\frac{i}{\lambda} S^{\mu\nu} \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{n\cdot(k+l)} \tilde{A}_\mu(k)\tilde{A}_\nu(l)$$

Both terms correspond to a chromagnetic interaction $S^{\mu\nu}F_{\mu\nu}$ as in the one-loop effective action

Spin 1 and higher

GLUONS

Susy model needed to deal with gauge field in numerators: not necessary if it is unity! In background-field-gauge:

$$\mathcal{L}_{A^2} = \frac{1}{2} A^a_\mu \Bigg[\eta^{\mu\nu} (\widetilde{D}^{ab})^2 + i g f^{abc} \widetilde{F}^c_{\rho\sigma} \underbrace{(S^{\mu\nu})^{\rho\sigma}}_{\text{spin-1 gen.}} \Bigg] A^b_\nu = \frac{1}{2} A^a_\mu H^{\mu\nu}_{ab} A^b_\nu$$

Analogous to scalar case:

$$\frac{\langle p_f | (H_{\mu\nu} + i\epsilon)^{-1} | x_i \rangle}{\langle p_f | x_i \rangle} = \frac{1}{2} \int_0^\infty dT\, e^{i\frac{1}{2} \overbrace{(p_f^2 + i\epsilon)}^{\text{free propagator}} T} \overbrace{f_{\mu\nu}(x_i, p_f, T)}^{\text{radiative factor}} \;,$$

$$f_{\mu\nu}(x_i, p_f, T) =$$

$$= \int_{\tilde{x}(0)=0} \mathcal{D}\tilde{x}\, \mathcal{P} e^{i \int_0^T dt \left[\frac{1}{2} \dot{\tilde{x}}^2 + (p_f + \dot{\tilde{x}}) \cdot A(x(t)) + \frac{i}{2} \partial \cdot A(x(t)) \right] \eta_{\mu\nu} + g (S_{\mu\nu})^{\rho\sigma} F_{\rho\sigma}}$$

Asymptotic propagator given by $\lim_{T\to\infty} \epsilon^{*\mu}(p_f)\, f_{\mu\nu}(x_i, p_f, T)$

So far we have derived the GWL for spin 0, 1/2 and 1 so we proved the exponentiation of next-to-soft emissions in QCD. So the primary goal is achieved.

Still, there are open questions:

- Can we extend the previous picture to to other gauges?
- What about massive vector bosons?
- Higher spin?

We need a representation with worldline fermions in analogy with spin 0 and spin 1/2

Worldline representation

Spin 1 free particle quantization well-known

$$S = \int dt \left(p_\mu \dot{x}^\mu + i\bar{\psi}^\mu \dot{\psi}_\mu - \frac{1}{2} e \overbrace{p_\mu p^\mu}^{Q_0} - i\bar{\chi} \overbrace{\psi_\mu p^\mu}^{Q_1} - i\chi \overbrace{\bar{\psi}_\mu p^\mu}^{Q_2} + a \overbrace{\bar{\psi}_\mu \psi^\mu}^{J} \right) ,$$

Propagator corresponds to $F_{\mu\nu}$ rather than A_μ

$$\int_0^\infty dT \int d\theta d\bar{\theta} \int_{\psi(T)=\Gamma}^{\bar{\psi}(T)=\bar{\Gamma}} \mathcal{D}\psi \mathcal{D}\bar{\psi}$$
$$\int_{x(0)=x_i}^{p(T)=p_f} \mathcal{D}x \mathcal{D}p \, e^{ip(T)\cdot x(T) - i\int_0^T dt \left(p\cdot\dot{x} + i\bar{\psi}\cdot\dot{\psi} - Q_0 - \frac{\bar{\theta}}{T} Q_1 - \frac{\theta}{T} Q_2 - i\epsilon \right)} ,$$

Worldline representation

Bigger problems with background field: equations of motion inconsistent unless background field is constant. Moreover, no susy unless $F^{\mu\nu} = 0$.

However, in the soft limit $F^{\mu\nu} \sim k^{\mu} A^{\nu} \to 0$. Susy is only softly broken. Hence, the corresponding charges are approximately conserved and the derivation can be repeated in analogy with the Dirac case, i.e.

- contribution of the background field to the numerator is negligible up to NLP
- contribution of the background field to the denominator is still $(S_{\mu\nu})^{\rho\sigma} F_{\rho\sigma}$, as in the 1-loop effective action

Similar structure for higher spin

Conclusions

The problem

- The GWL is a powerful tool to describe asymptotic states dressed by soft radiation at subleading power
- Issues to be clarified for spin 1/2 while no proof for spin 1 and higher

This work

- Revisited spin 1/2: wordline susy crucial to understand contribution of background field
- Proof for spin 1: next-to-soft exponentiation extended to Yang-Mills
- Argument for higher spin and analogy with 1-loop effective action

Outlook

- massive gauge bosons/different gauges
- gravity, spin and high energy scattering
- color with Grassmann variables
- double copy on the worldline