



Model-independent energy budget for cosmological PTs

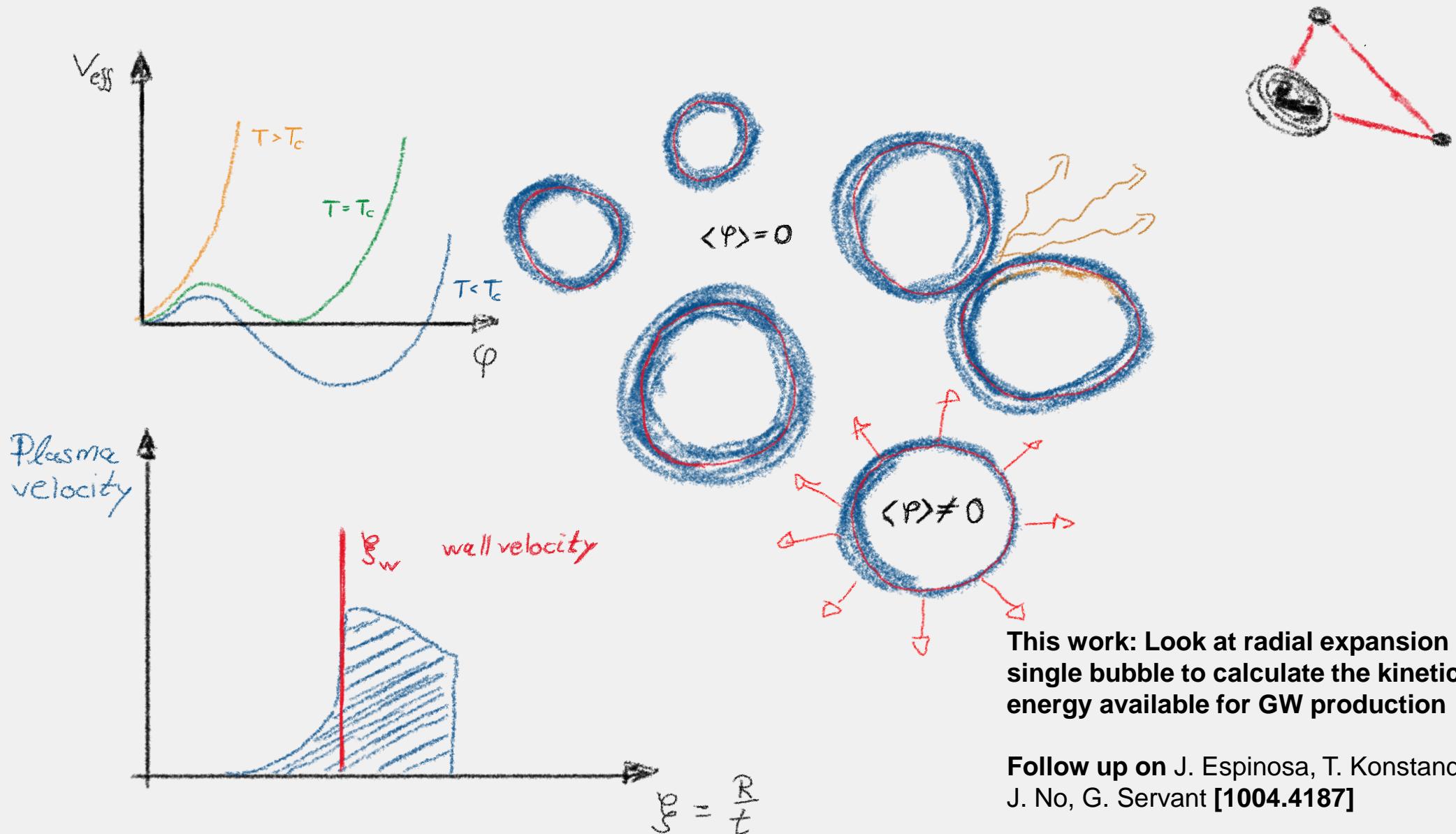
Felix Giese

Hamburg, 25.09.20

in collaboration with:

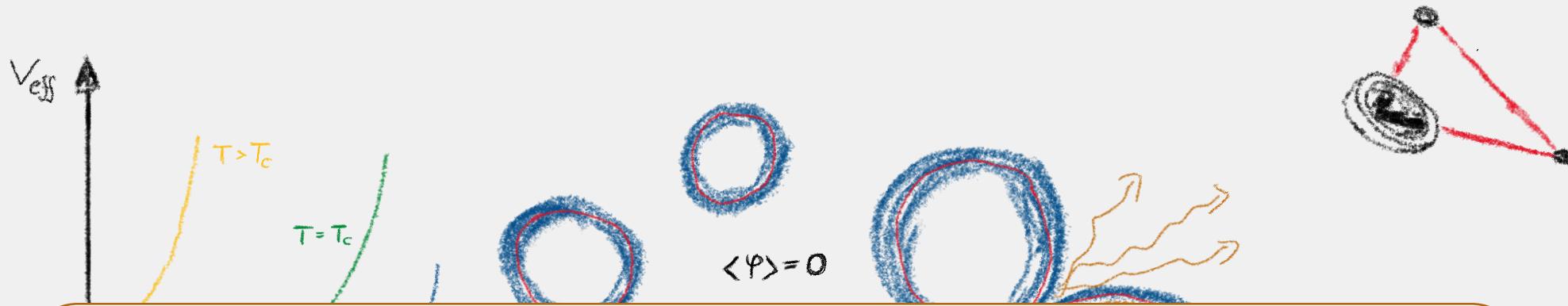
T. Konstandin, J. van de Vis, **2004.06995, JCAP07(2020)05**

T. Konstandin, K. Schmitz, J. van de Vis, **In preparation**



This work: Look at radial expansion of a single bubble to calculate the kinetic energy available for GW production

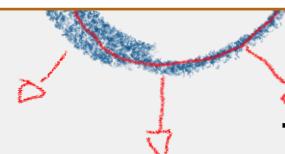
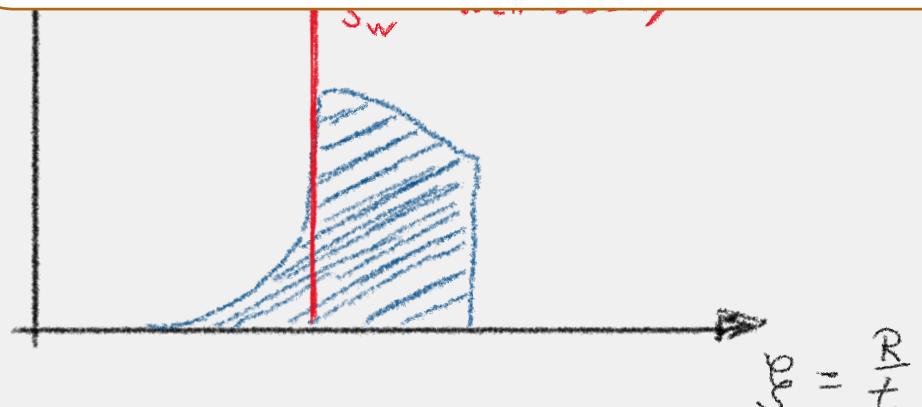
Follow up on J. Espinosa, T. Konstandin,
J. No, G. Servant [1004.4187]



$$\Omega_{tot} = 7.53 \cdot 10^{-5} \min\{1, H_* \tau_{sh}\} \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \tilde{\Omega}_{GW} \frac{\max\{c, v_w\}}{\beta/H_*} * K^2$$

Plasma
velocity

C. Caprini et al [1512.06239, 1910.13125]
K. Schmitz [2005.10789]



This work: Look at radial expansion of a single bubble to calculate the kinetic energy available for GW production

Follow up on J. Espinosa, T. Konstandin,
J. No, G. Servant [1004.4187]

Kinetic energy fraction $K = \rho_{kin} / e_n$

- estimated by hydrodynamics of single expanding bubble in a plasma

$$K = \frac{\rho_{kin}}{e_n} = \frac{3}{e_n \xi_w^3} * \int d\xi \xi^2 v^2 \gamma^2 w$$

- reshuffle terms to:

$$K = \frac{w_n}{e_n} * \frac{DX}{4 w_n} * \kappa(\alpha_x, \xi_w)$$

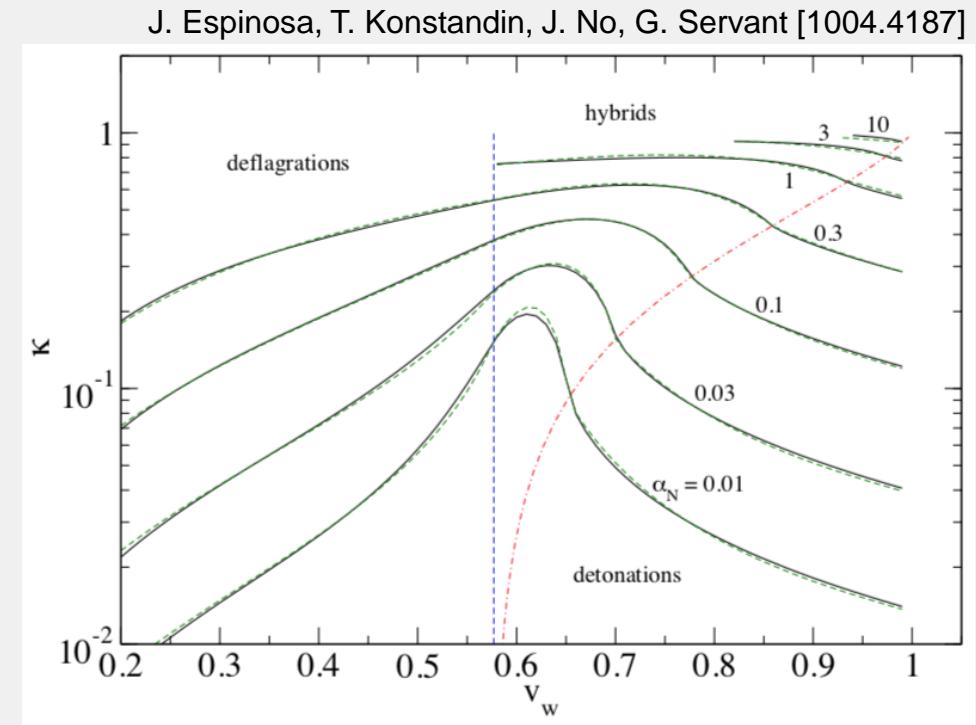


Adiabatic index Γ

\propto Phase transition strength α_x
(different definitions exist)



Model-independent efficiency factor



literature approach: Calculate in bag equation of state (assume relativistic gas in both phases separated by vacuum contribution ϵ)

Model dependence in hydrodynamics

Sound arguments to go beyond the bag model

- start from perfect fluid:

$$T_{\mu\nu} = u^\mu u^\nu w - g^{\mu\nu} p$$

- hydrodynamic equations:

$$\partial_\mu T^{\mu\nu} = 0$$

- for constant speed of sound these decouple into*:

$$\frac{dv}{d\xi} = \frac{2v(1-v^2)}{\xi(1-v\xi)} \left(\frac{\mu^2}{c^2} - 1 \right)^{-1},$$

$$\frac{dw}{dv} = w \left(1 + \frac{1}{c^2} \right) \gamma^2 \mu$$

*with: $\mu = \frac{\xi - v}{1 - \xi v}$, $\gamma^2 = (1 - v^2)^{-1}$

- energy momentum conservation at the wall front leads to:

$$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)},$$

$$v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$

- different classes of solutions depending on wall velocity ξ_w (detonations, deflagrations, etc.)
- also dependent on speed of sound via p, e

$$c_{s,b}^2(T) = \frac{dp_{s,b}/dT}{de_{s,b}/dT}$$

Expansion of matching equations and speed of sound

rewrite differences between phases as:

$$\Delta p = \underbrace{[p_s(T_+) - p_b(T_+)]}_{\equiv Dp} + \underbrace{[p_b(T_+) - p_b(T_-)]}_{\equiv \delta p_b}$$

Under assumption $T_n \sim T_+ \sim T_-$:

$$c_b^2 = \frac{\delta p_b}{\delta e_b} = \left. \frac{dp_b/dT}{de_b/dT} \right|_{T=T_n}$$

with this the first matching equation becomes:

$$\delta p \left(1 - \frac{v_+ v_-}{c_b^2} \right) \simeq v_+ v_- D e - D p$$

plugging into the second:

$$\frac{v_+}{v_-} \simeq \frac{(v_+ v_- / c_b^2 - 1) + 3\alpha_{\bar{\theta}}}{(v_+ v_- / c_b^2 - 1) + 3v_+ v_- \alpha_{\bar{\theta}}}$$

→ matching only depends on $\alpha_{\bar{\theta}}, c_b^2$

Key definitions:

„pseudotrace“

PT strength parameter

$$\bar{\theta} \equiv e - \frac{p}{c_b^2}, \quad \alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3w_N}$$

agrees with definition of alpha from literature
for $c_b^2 = \frac{1}{3}$

Generalization of the bag model

- consistent treatment of energy budget needs to allow for deviations from $c^2 = \frac{1}{3}$
- construct „bag-like“ equation of state that has soundspeed in symmetric and broken phase as free parameters

$$p_s = \frac{1}{3}a_+T^\mu - \epsilon$$

$$p_b = \frac{1}{3}a_-T^\nu$$

$$\nu = 1 + \frac{1}{c_b^2}$$

$$e_s = \frac{1}{3}a_+(\mu - 1)T^\mu + \epsilon$$

$$e_b = \frac{1}{3}a_-(\nu - 1)T^\nu$$

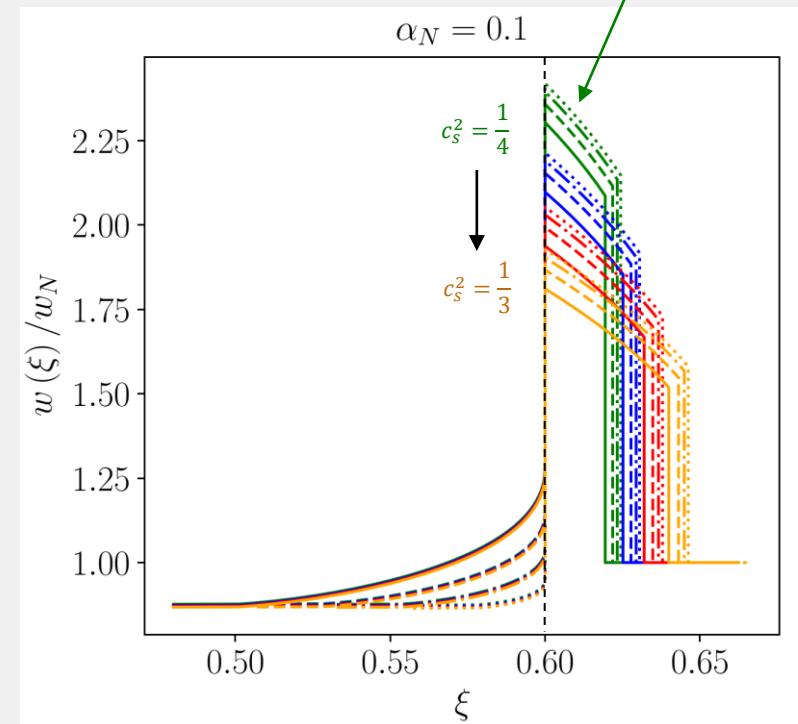
$$\mu = 1 + \frac{1}{c_s^2}$$

vμ-model
(L. Leitao, A. Megevand
2014)

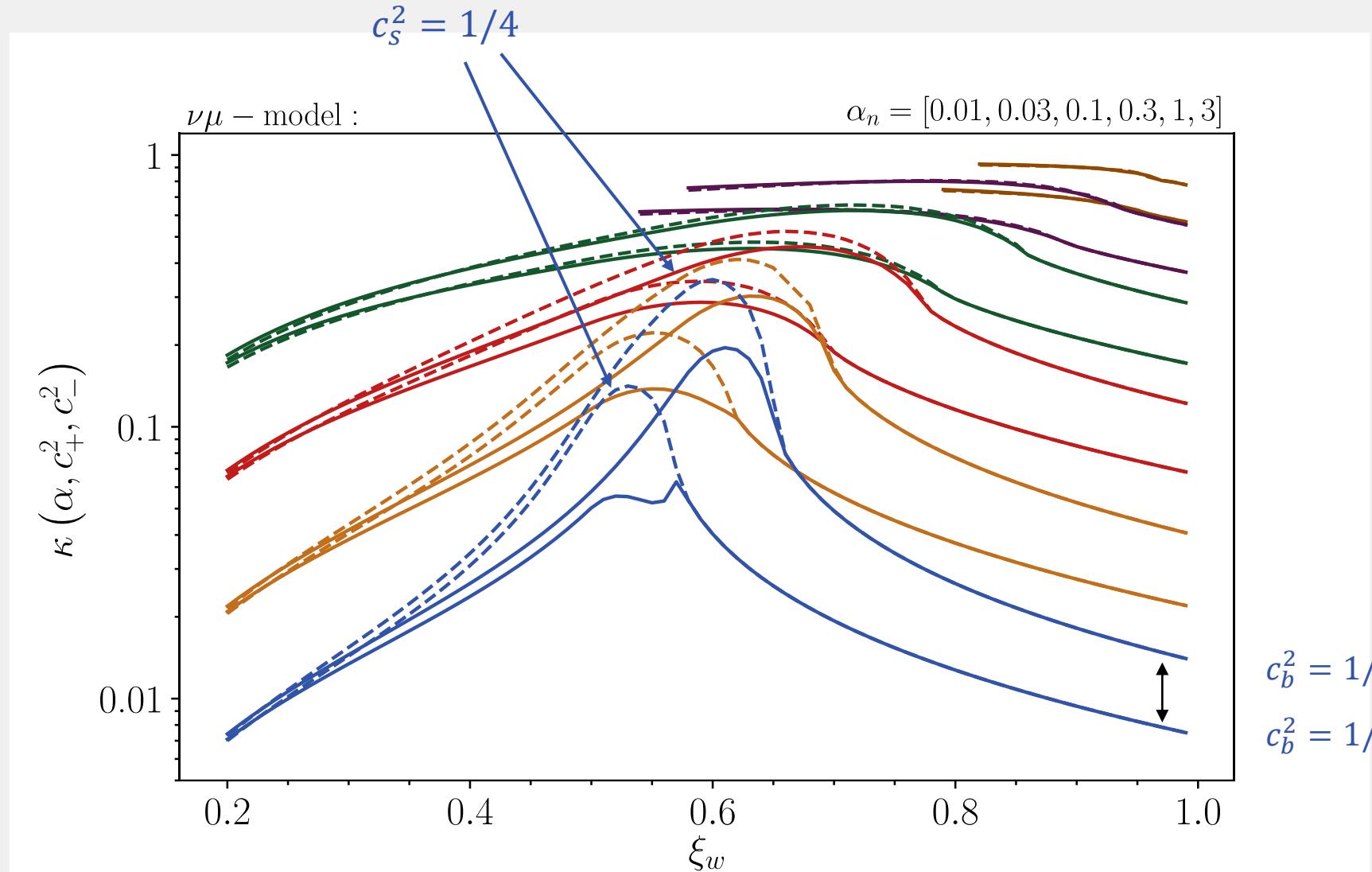
$$c_b^2: \frac{1}{3} \rightarrow \frac{1}{4}$$

- for $\mu = \nu = 4$, the bag equation of state is recovered.
- the expansion from last page is exact for this model

$$\frac{\nu_+}{\nu_-} = \frac{(\nu_+\nu_-/c_b^2 - 1) + 3\alpha_{\bar{\theta}}}{(\nu_+\nu_-/c_b^2 - 1) + 3\nu_+\nu_-\alpha_{\bar{\theta}}}$$



Efficiency factor in the $\nu\mu$ model



How to use our method?

- map your favorite model onto the $\nu\mu$ -model by calculating $\alpha_{\bar{\theta}}$, c_s^2 and c_b^2 in your model
- use python code to compute $\kappa_{\bar{\theta}}(\alpha_{\bar{\theta}}, c_s^2, c_b^2)$
- rescale result consistently by using

$$K = \frac{D\bar{\theta}}{4e_n} \kappa_{\bar{\theta}}$$

- result model-independent up to T dependence in $c_{s,b}^2(T)$.



```
01 import numpy as np
02 from scipy.integrate import odeint
03 from scipy.integrate import simps
04
05 def kappaNuModel(cs2,al,vp):
06     nu = 1./cs2+1.
07     tmp = 1.-3.*al+vp**2*(1./cs2+3.*al)
08     disc = 4*vp**2*(1.-nu)+tmp**2
09     if disc<0:
10         print("vp too small for detonation")
11         return 0
12     vm = (tmp+np.sqrt(disc))/2/(nu-1.)/vp
13     wm = (-1.+3.*al+(vp/vm)*(-1.+nu+3.*al))
14     wm /= (-1.+nu-vp/vm)
15
16     def dfdv(xiw, v, nu):
17         xi, w = xiw
18         dxidv = (((xi-v)/(1.-xi*v))**2*(nu-1.)-1.)
19         dxidv *= (1.-v*xi)*xi/2./v/(1.-v**2)
20         dwdv = nu*(xi-v)/(1.-xi*v)*w/(1.-v**2)
21         return [dxidv,dwdv]
22
23     n = 501 # change accuracy here
24     vs = np.linspace((vp-vm)/(1.-vp*vm), 0, n)
25     sol = odeint(dfdv, [vp,1.], vs, args=(nu,))
26     xis, ws = (sol[:,0], -sol[:,1]*wm/al*4./vp**3)
27
28     return simps(ws*(xis*vs)**2/(1.-vs**2), xis)
```

So far available for
detonations.
deflagrations and
hybrids in next
publication.

Different methods to determine K

different estimates for K exist in the literature. Approximations that can be found in the literature are:

$$K = \frac{w_n}{e_n} * \frac{DX}{4 w_n} * \kappa(c_b^2, c_s^2, \alpha)$$

$$\sim \alpha_X$$

$$\Gamma \sim \frac{4}{3} \frac{1}{1 + \alpha_X}$$

(True in bag model)

X ~ θ, p, e
trace difference
pressure difference
energy difference

all methods from the literature use the efficiency factor from the bag model $\kappa_{bag}(\alpha) = \kappa\left(\frac{1}{3}, \frac{1}{3}, \alpha\right)$

Codes for methods we use:

M1	K
M2	$\left(\frac{D\bar{\theta}}{4e_+}\right) \kappa(\alpha_{\bar{\theta}}, c_{s,s}, c_{s,b}) _{\nu\mu}$
M3	$\left(\frac{D\theta}{4e_+}\right) \kappa(\alpha_\theta) _{bag}$
M4	$\left(\frac{\alpha_\theta}{\alpha_\theta+1}\right) \kappa(\alpha_\theta) _{bag}$
M5	$\left(\frac{\alpha_p}{\alpha_p+1}\right) \kappa(\alpha_p) _{bag}$
M6	$\left(\frac{\alpha_e}{\alpha_e+1}\right) \kappa(\alpha_e) _{bag}$

Exact hydrodynamics

This work

Example for a realistic model: SM+singlet

(Z2 symmetry)

- two step EW phase transition via addition of a real scalar s with Z2 symmetry

$$\Delta V_0 = -\frac{\mu_s^2}{2} s^2 + \frac{\lambda_s}{4} s^4 + \frac{\lambda_m}{2} H^\dagger H s^2$$

Set Barrier in h-s plane

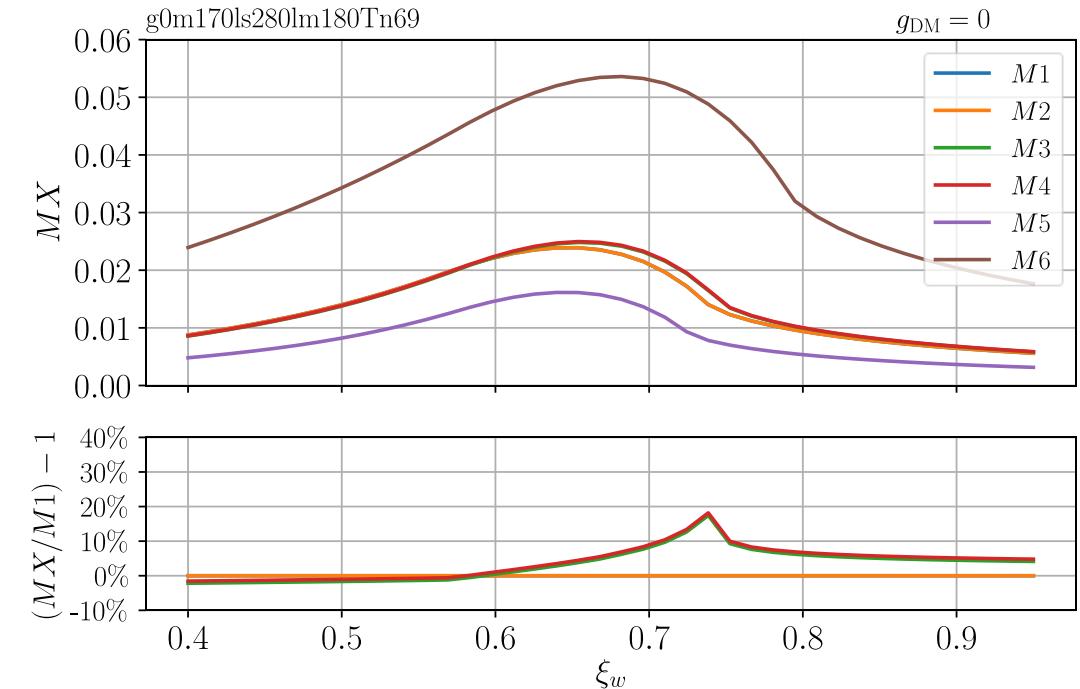
- thermodynamic potentials from

$$p_b(T_n) = -V_{eff}(h(T_n), s = 0, T_n) \quad \text{EW br. minimum}$$

$$p_s(T_n) = -V_{eff}(h = 0, s(T_n), T_n) \quad \text{Z2 br. mininum}$$

- we use the FindBounce [2002.00881] to calculate the nucleation temperature $T_n \rightarrow \alpha_{\bar{\theta}}, c_b^2, c_s^2$
- daisy diagrams wash out the contribution to soundspeed reduction

$\alpha_{\bar{\theta}} = 0.067$	$m_s = 170 \text{ GeV}$	$\lambda_s = 2.8$	$c_s^2(T_n) = 0.331$
	$T_n = 69.4 \text{ GeV}$	$\lambda_m = 1.8$	$c_b^2(T_n) = 0.321$



Example for a realistic model: SM+singlet

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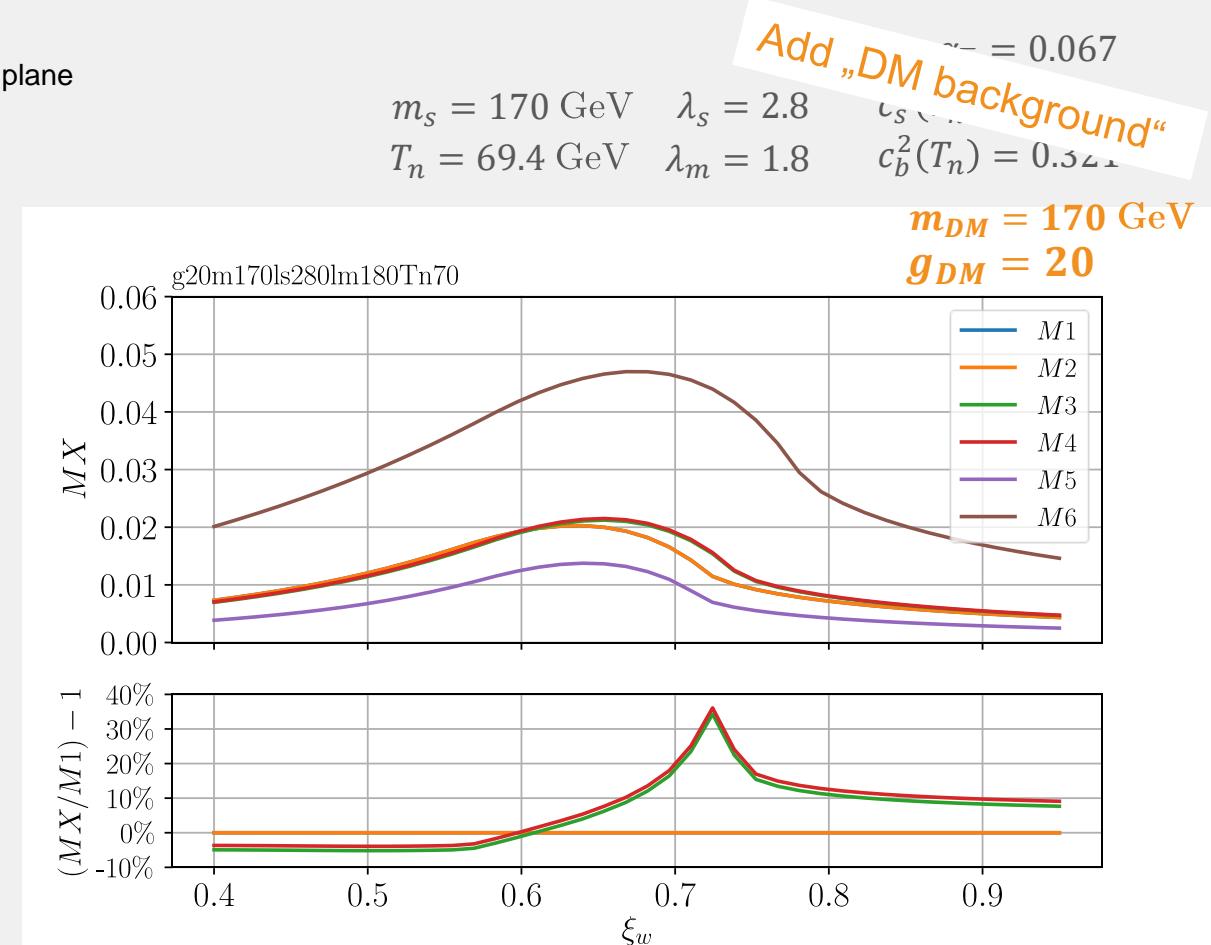
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Conclusions

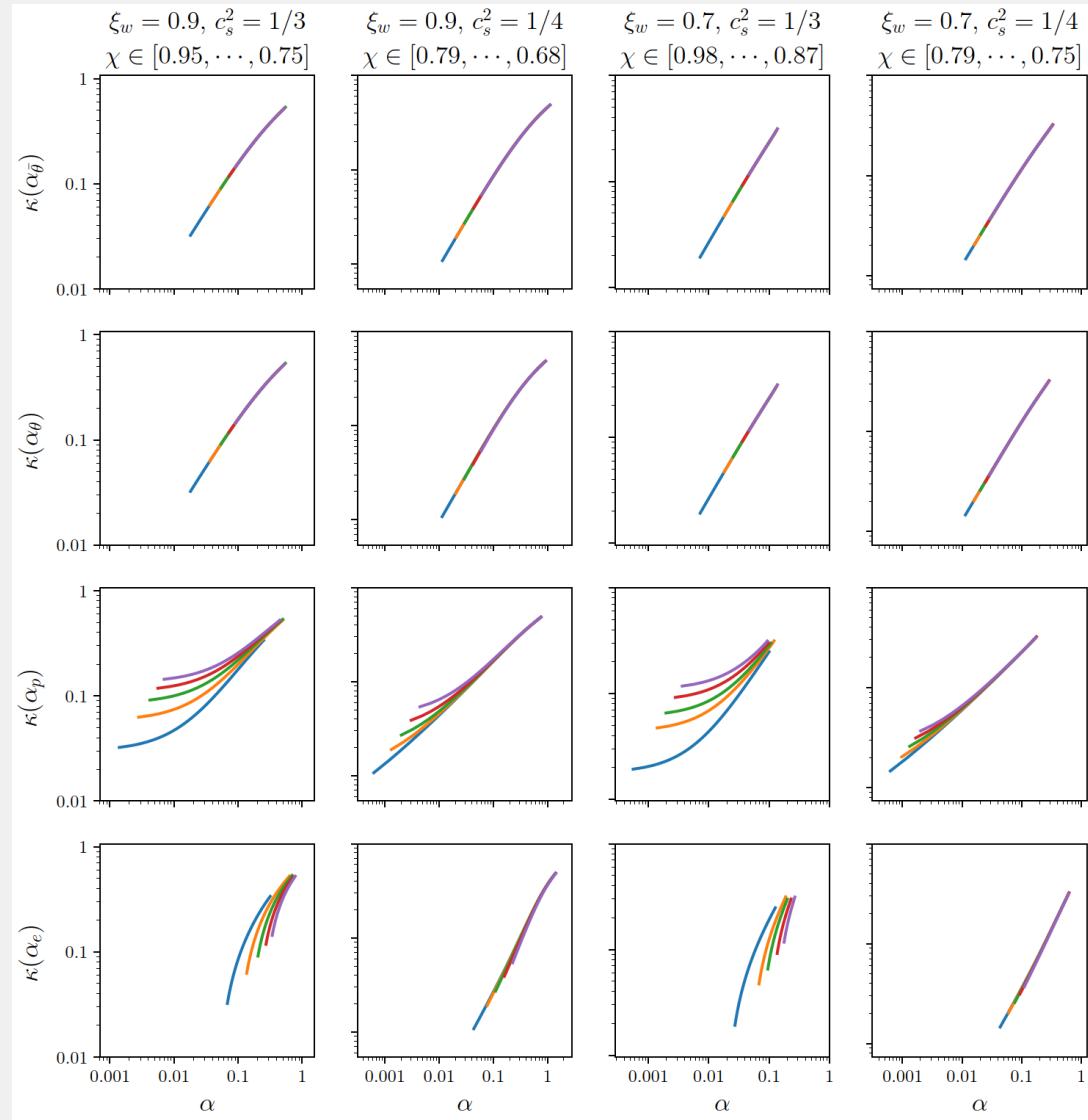
- gravitational waves are an interesting probe to new physics.
- parametrization of K in terms of $\alpha_{\bar{\theta}}, \xi_w, c_s^2, c_b^2$:

$$\kappa_{\bar{\theta}} = \frac{4\rho_{fl}}{D\bar{\theta}}, \quad \bar{\theta} \equiv e - \frac{p}{c_b^2}, \quad \alpha_{\bar{\theta}} \equiv \frac{\bar{\theta}_s(T_n) - \bar{\theta}_b(T_n)}{3w_N}$$

- taking soundspeed corrections into account by mapping to the $\nu\mu$ model (M2) gives the best approximation to the exact hydrodynamics (M1)
- advise against general use of α_p, α_e even for $c_{s,b}^2 \simeq \frac{1}{3}$

Thank you!

Model independence $\nu\mu$ model (table of different α s)



χ is the free parameter in the new model

$$\chi = \frac{a_-}{a_+} T_{cr}^{\nu-4}$$

For our method the result is independent from χ .