UV Sensitivity of the Axion Mass from Instantons in Partially Broken Gauge Groups

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based on: arXiv:1912.02197 (JHEP)
with C. Csáki (Cornell) *and* Y. Shirman (UC Irvine)

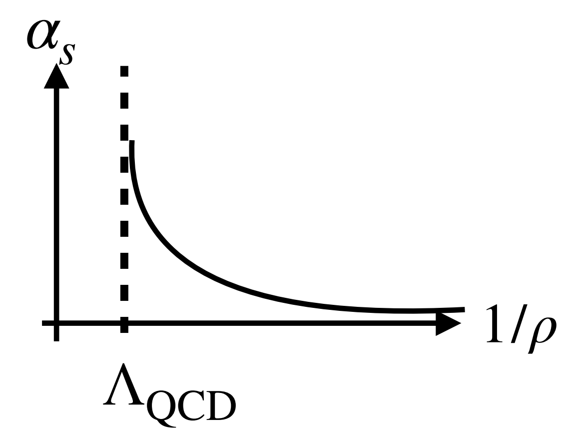
Motivation: Axion Mass

- **Standard folklore**: axion mass determined by non-perturbative **IR** physics / instantons

→ instanton density dominated by large (IR) instantons

$$d(\rho) \sim e^{-\frac{2\pi}{\alpha_s(\rho)}}$$

't Hooft '76



→ chiral symmetry relates axion mass to pion mass

$$m_a^2 f_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} m_\pi^2 f_\pi^2$$

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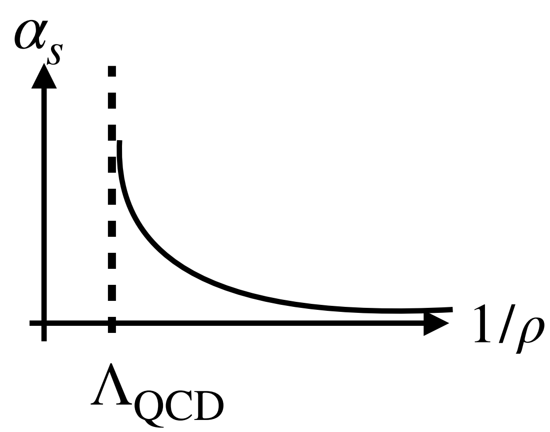

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- Non-trivial embedding of QCD in UV gauge group

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How can weakly-coupled small instantons dominate over large QCD instantons?

What is the underlying dynamics?

Axion Potential from Instantons

$$\mathcal{L} \supset \left(\bar{\theta} - \frac{a}{f_a}\right)\frac{g^2}{16\pi^2}\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- Axion shift symmetry broken in instanton background $\langle G_{\mu\nu}\tilde{G}^{\mu\nu}\rangle_{\mathrm{inst}} \neq 0$

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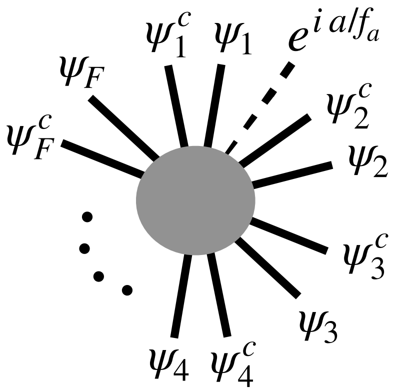
$$\frac{1}{\mu^{3F-4}}\left(\frac{\Lambda}{\mu}\right)^{b_0} \prod_{i=1}^{F} \psi_i \bar{\psi}_i \, e^{i(\bar{\theta}-a/f_a)} + h.c.$$



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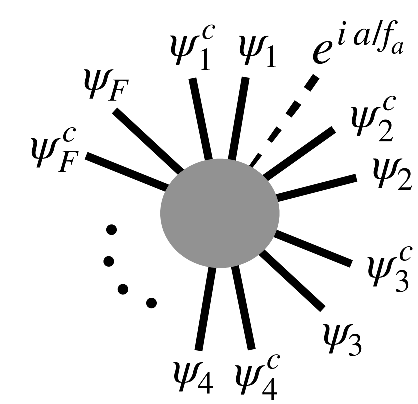

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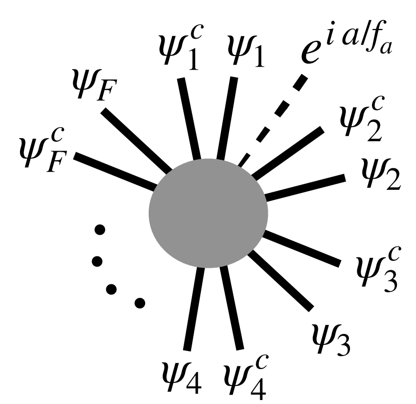

beta function coefficient

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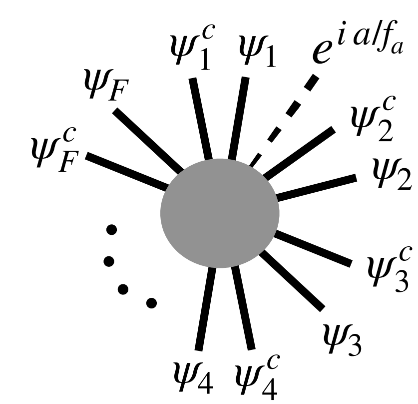

IR cutoff

beta function coefficient

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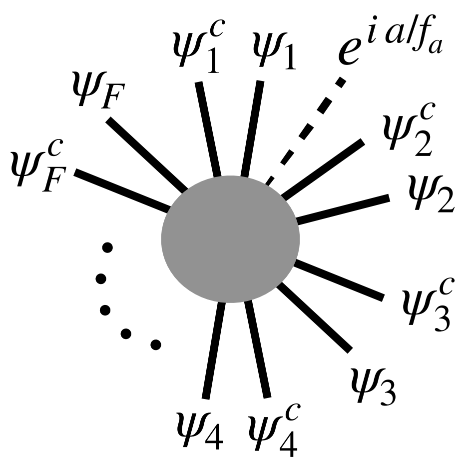
Axion Mass from QCD Instantons

- Estimate of axion potential in F-flavor QCD with $m_i \ll \Lambda_{\mathrm{QCD}}$



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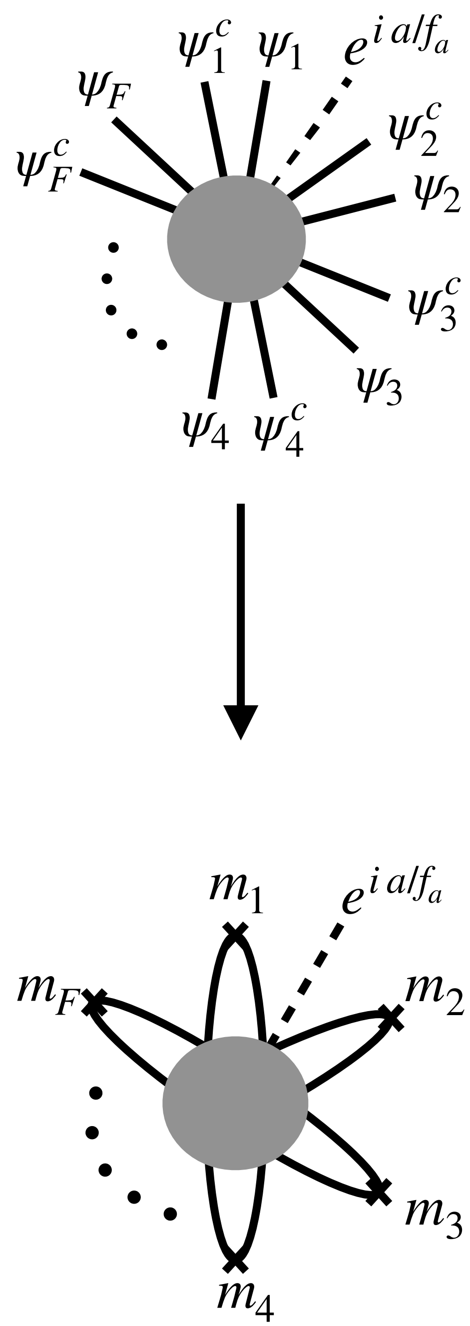


Close fermion legs with masses

Extrapolate to non-perturbative region $\mu \to \Lambda_{\rm QCD}$

$$m_{a,{\rm QCD}}^2\, f_a^2 \sim \frac{\prod_{i=1}^F m_i}{\Lambda_{\rm QCD}^{F-4}}$$

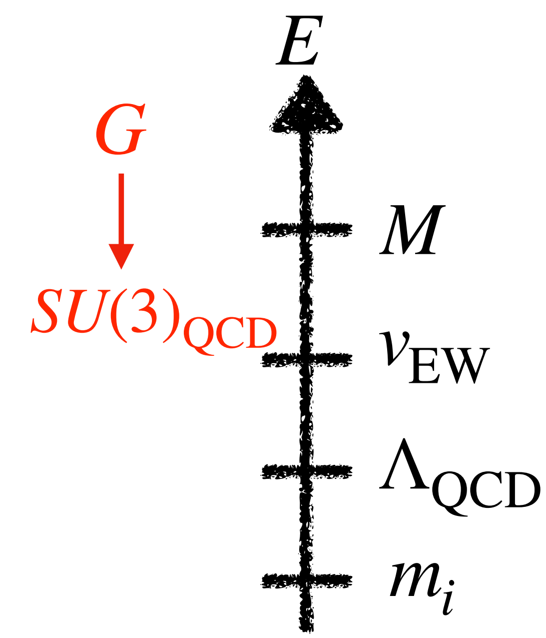
Axion Mass in Partially Broken Gauge Groups

Csáki, MR, Shirman arXiv:1912.02197

- $G \overset{M}{\rightarrow} SU(3)_{\rm QCD}$ weakly coupled at M

➡ assume hierarchy of scales $M \gg v_{\rm EW} \gg \Lambda_{\rm QCD}$

How do instantons in G contribute to axion mass?



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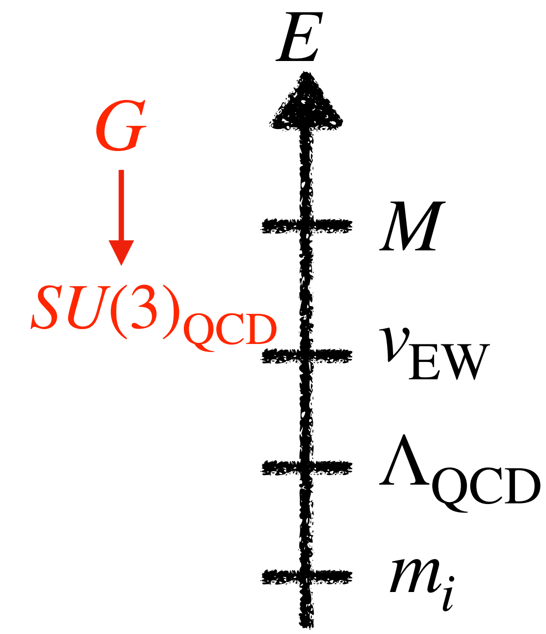

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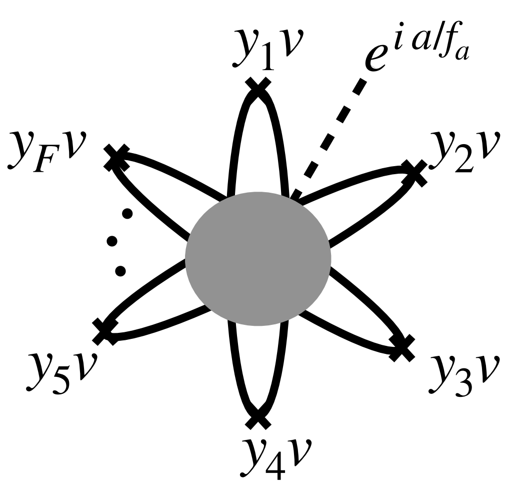
- Two novel aspects when treating high-energy instantons:

  1. Higgs is propagating degree of freedom and can close ’t Hooft operator



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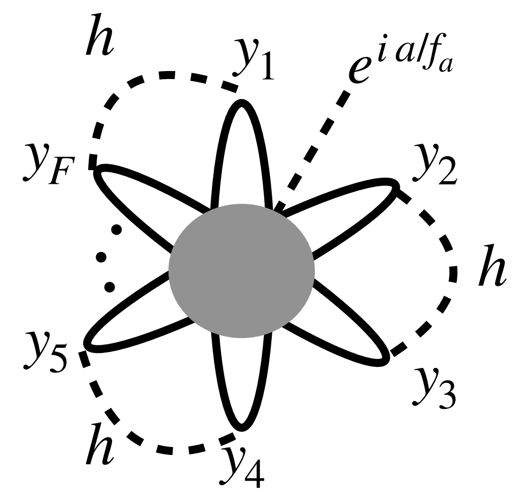

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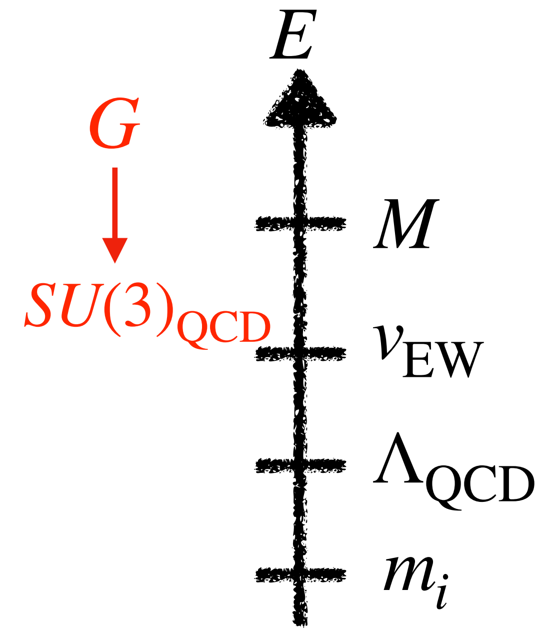

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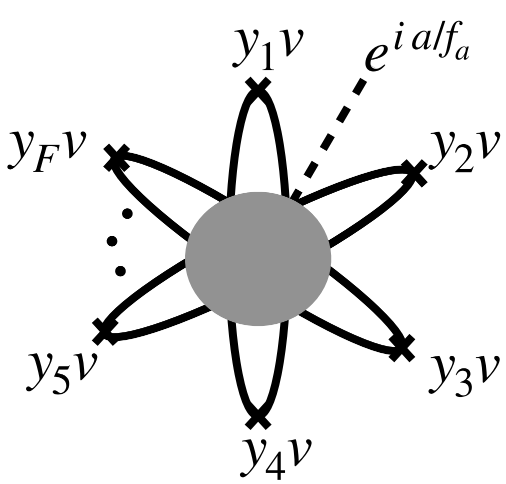

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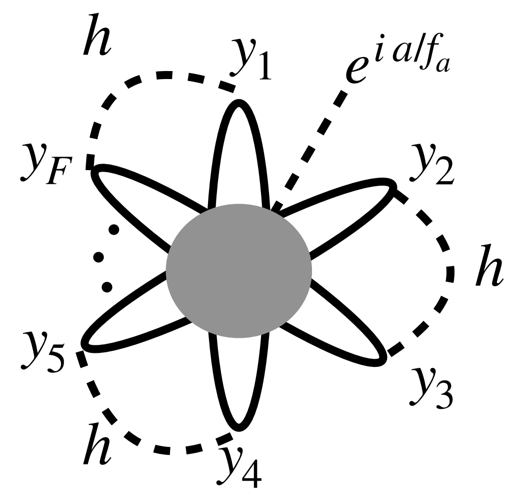

  2. Matching of G instantons to QCD instantons

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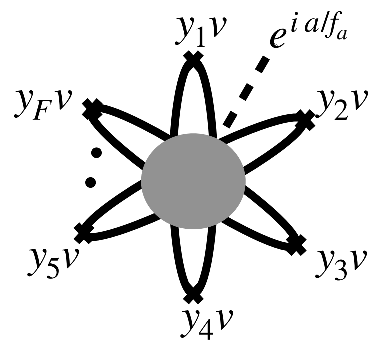

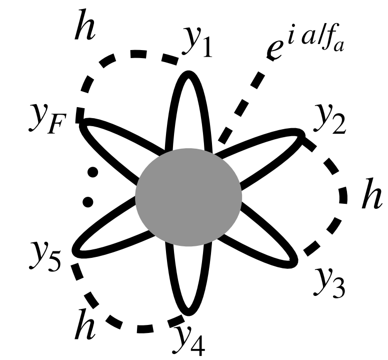

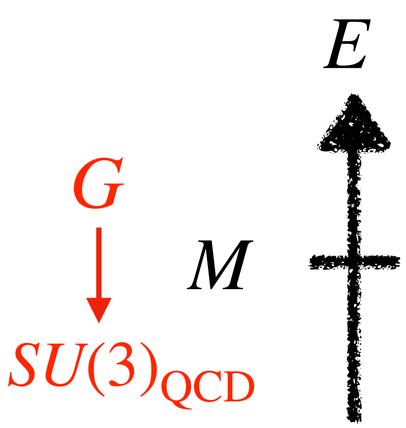

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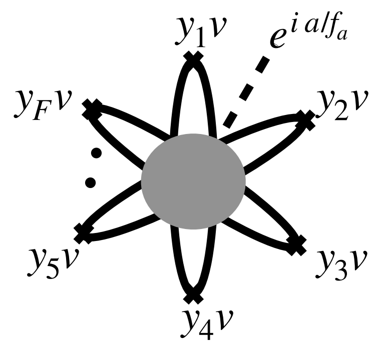
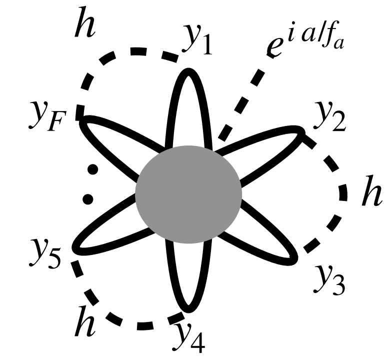
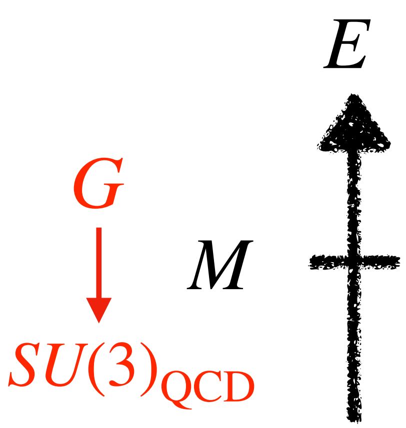




dominates for $\frac{M}{v} > 4\pi$



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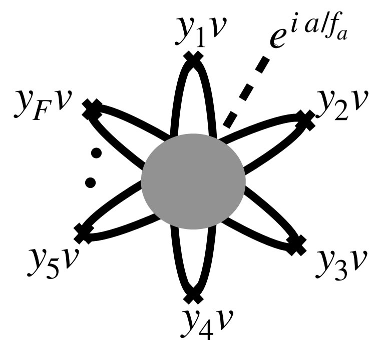

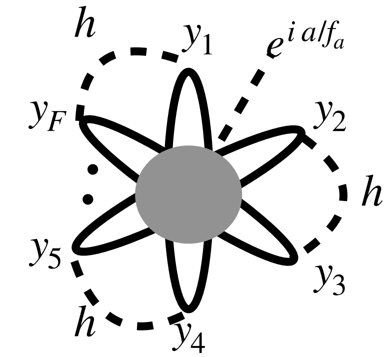

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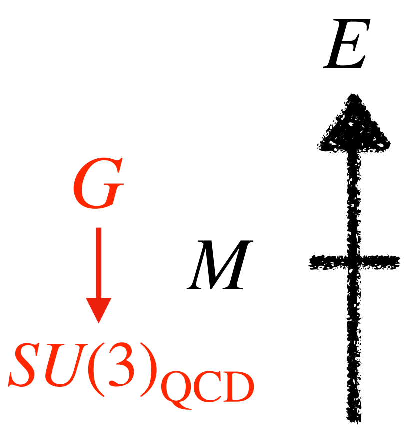

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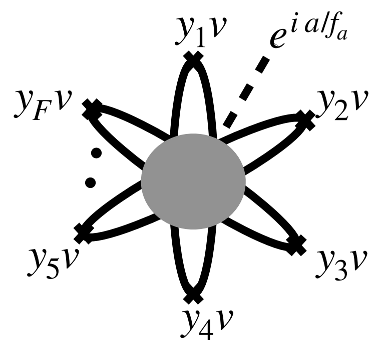

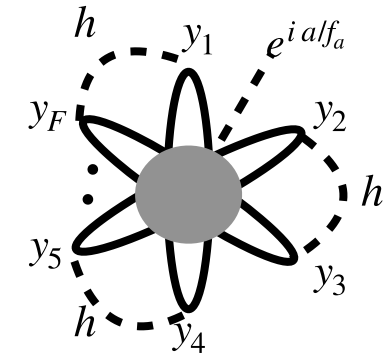

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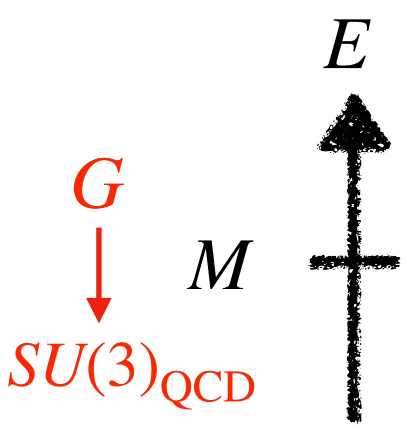

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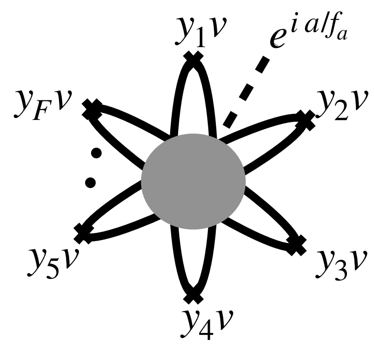
$$\boxed{\frac{m'^2_{a,M}}{m^2_{a,{\rm QCD}}} = \frac{1}{(4\pi)^F}\left(\frac{\Lambda_{\rm QCD}}{v}\right)^F \left(\frac{\Lambda_{\rm QCD}}{M}\right)^{b_{\rm QCD}-4} \ll 1}$$

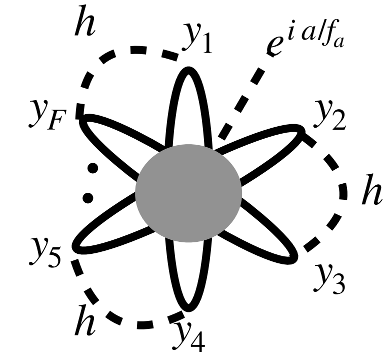
highly suppressed for $b_{\rm QCD} > 4$

$(F = 6 \Rightarrow b_{\rm QCD} = 7)$

Axion Mass in Partially Broken Gauge Groups

Csáki, MR, Shirman arXiv:1912.02197





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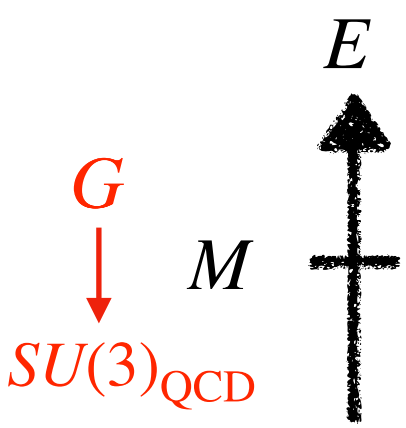

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Caveat: matching condition modified for non-trivial embedding of QCD

$$\left(\frac{\Lambda}{M}\right)^{k\, b_G} = \left(\frac{\Lambda_{\text{QCD}}}{M}\right)^{b_{\text{QCD}}}$$

k : index of embedding

Index of Embedding

Intriligator, Seiberg '95
Csáki, Murayama '98

- Intuitive interpretation: 1-instanton configuration in IR group H

$$\simeq$$

k-instanton solution in UV group G

→ some broken instantons are topologically distinct from QCD instantons and scale as $1/k$ "fractional" instantons

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- Example: $SU(N)^k \to SU(N)_D$

→ 1 instanton in $SU(N)_D$ is (1,1,…,1) instanton in $SU(N)^k$

→ e.g. (1,0,…,0) instanton of $SU(N)^k$ absent in $SU(N)_D$

UV Instantons in non-trivial Embedding

Csáki, MR, Shirman arXiv:1912.02197

- UV instanton contribution with index of embedding k $\left(\frac{\Lambda}{M}\right)^{k\, b_G} = \left(\frac{\Lambda_{\rm QCD}}{M}\right)^{b_{\rm QCD}}$

$$\frac{m'^2_{a,M}}{m^2_{a,\rm QCD}} = \frac{1}{(4\pi)^F}\left(\frac{\Lambda_{\rm QCD}}{v}\right)^F \left(\frac{\Lambda_{\rm QCD}}{M}\right)^{\frac{b_{\rm QCD}}{k}-4}$$

$(F = 6 \Rightarrow b_{\rm QCD} = 7)$

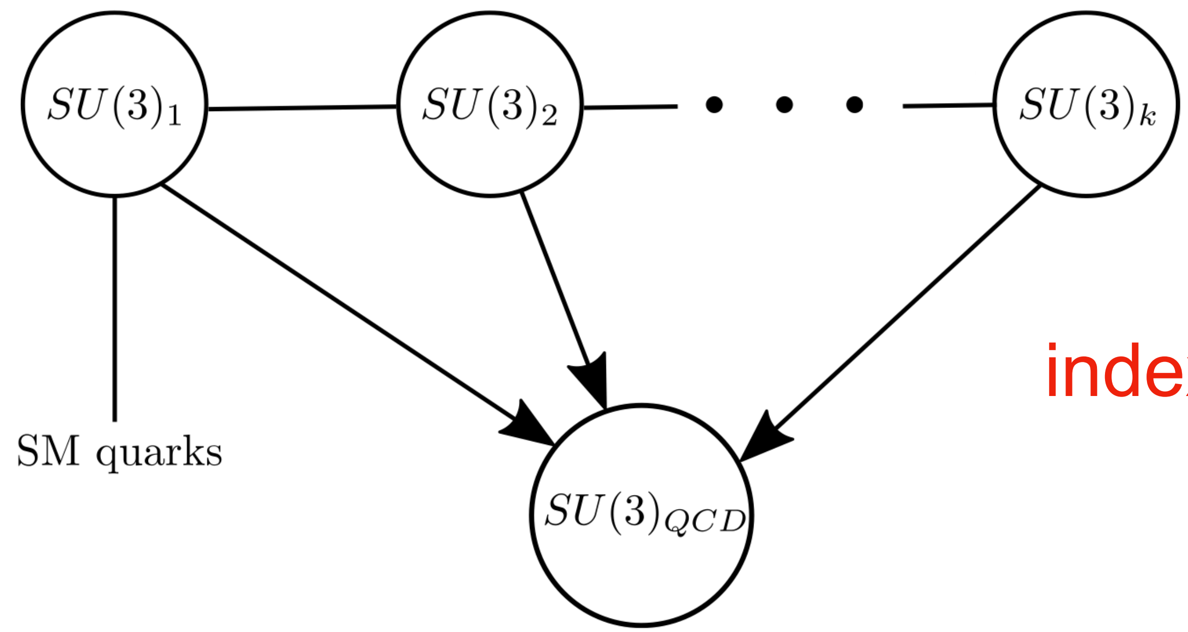
Small instantons dominate already for k=2 if M is sufficiently large!

→ **Larger enhancement with larger k**

Product Group Model

Agrawal, Howe '17

- Simple realization of product group model with non-trivial index of embedding



index of embedding: k

→ theory with k axions

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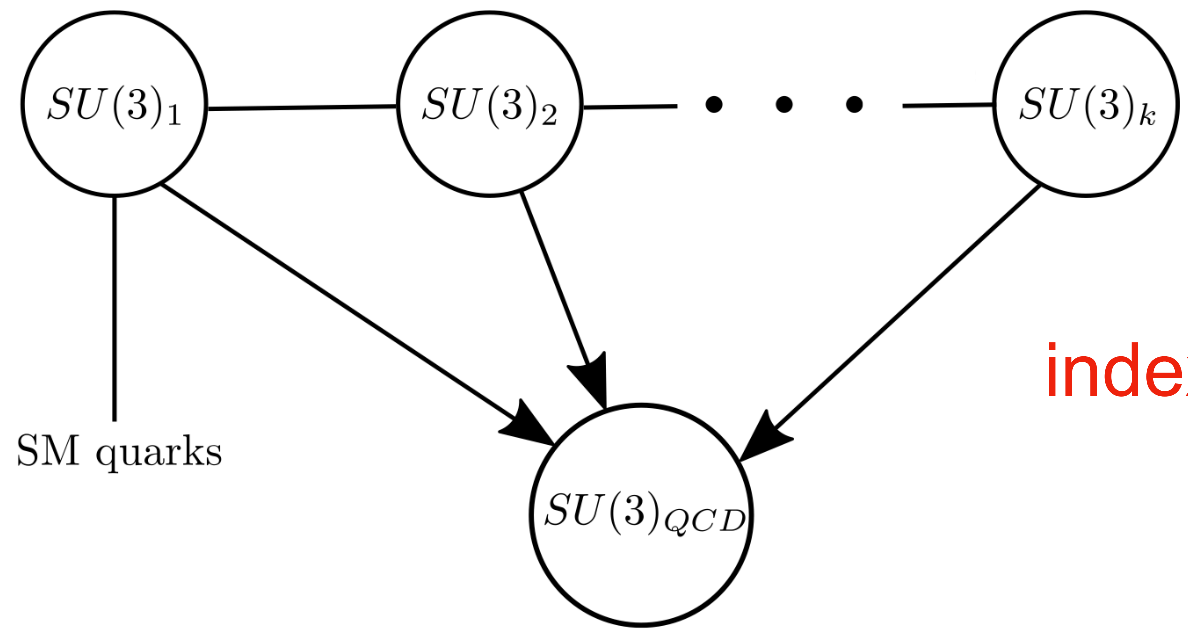


index of embedding: k

➡ theory with k axions

- We did full one-instanton calculation in constrained instanton framework

Affleck '81

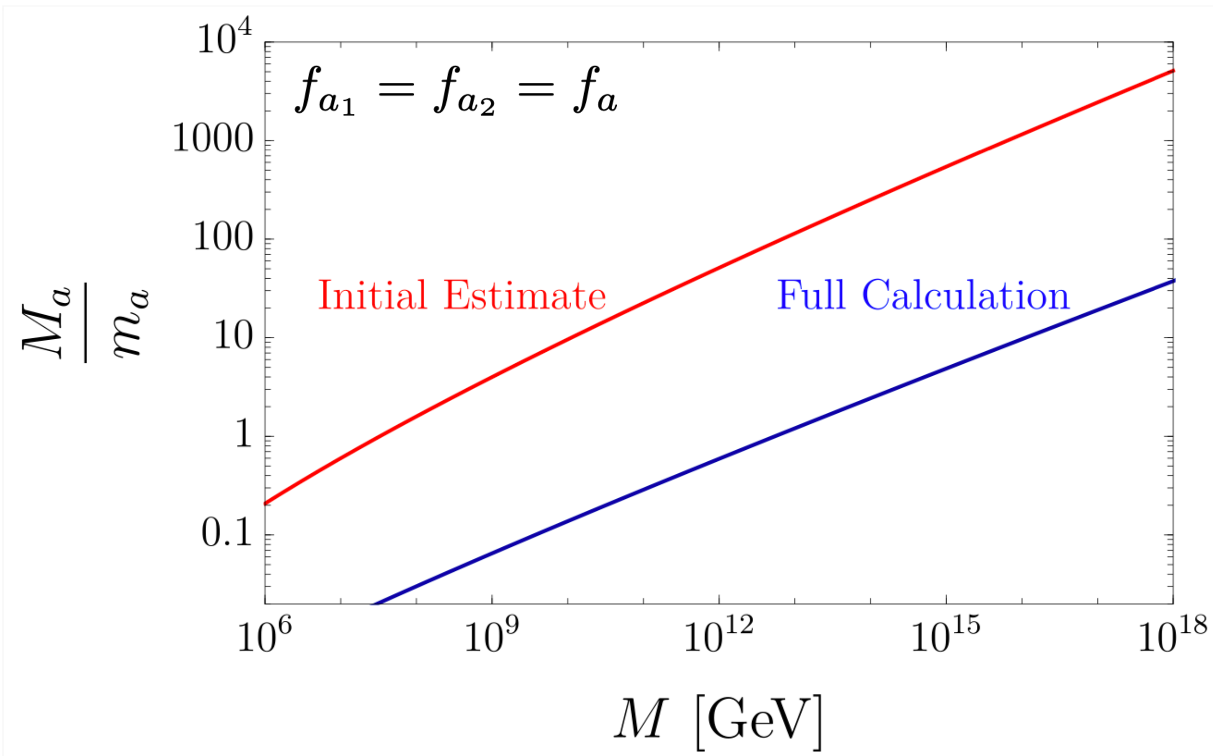
$$e^{-\frac{2\pi}{\alpha(\rho)}} \to e^{-\frac{2\pi}{\alpha(\rho)}-2\pi^2\rho^2 v_\Sigma^2}$$

Scalar vev cuts off large instantons

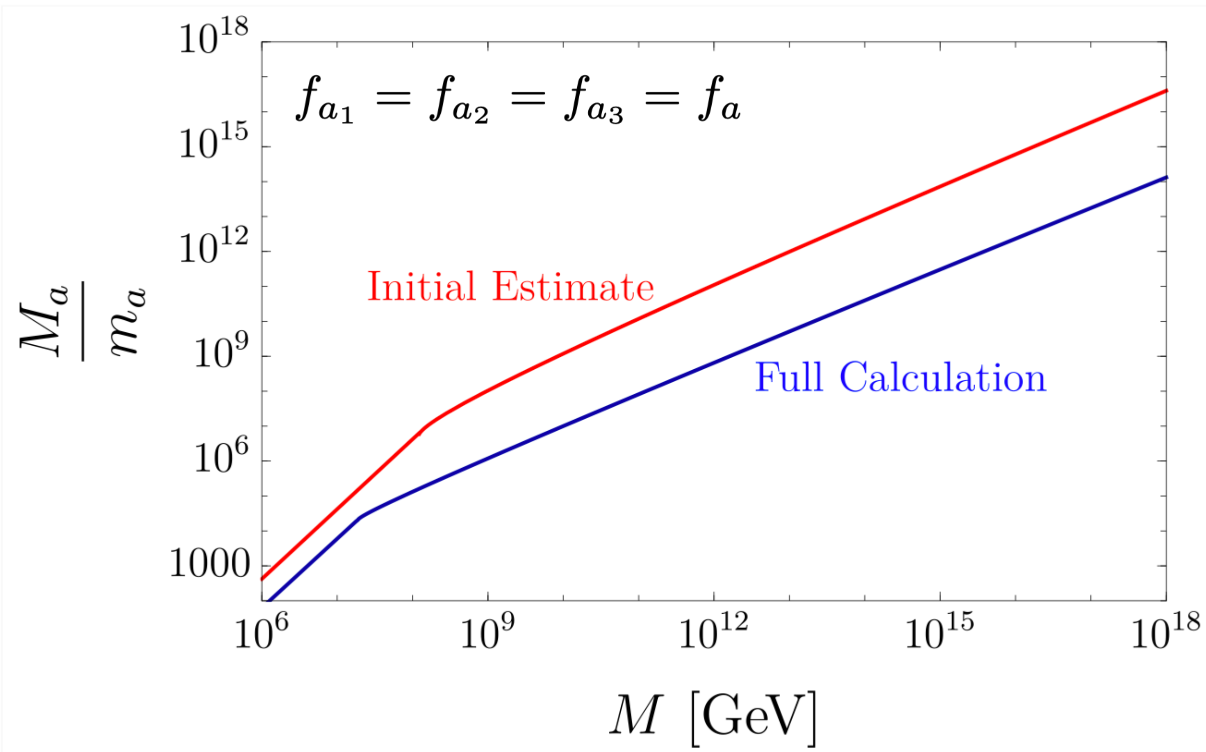
Product Group Model

Agrawal, Howe '17
Csáki, MR, Shirman '19

$k = 2$



$k = 3$



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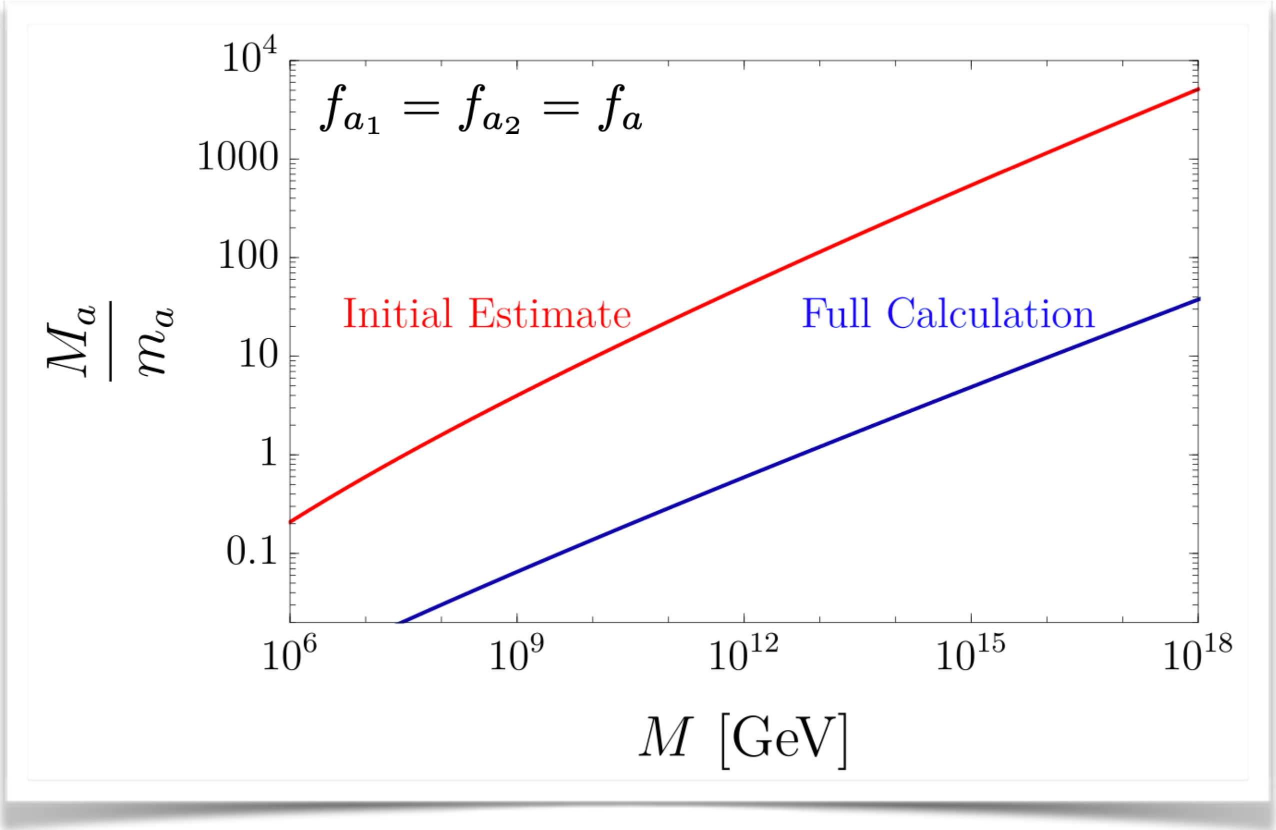

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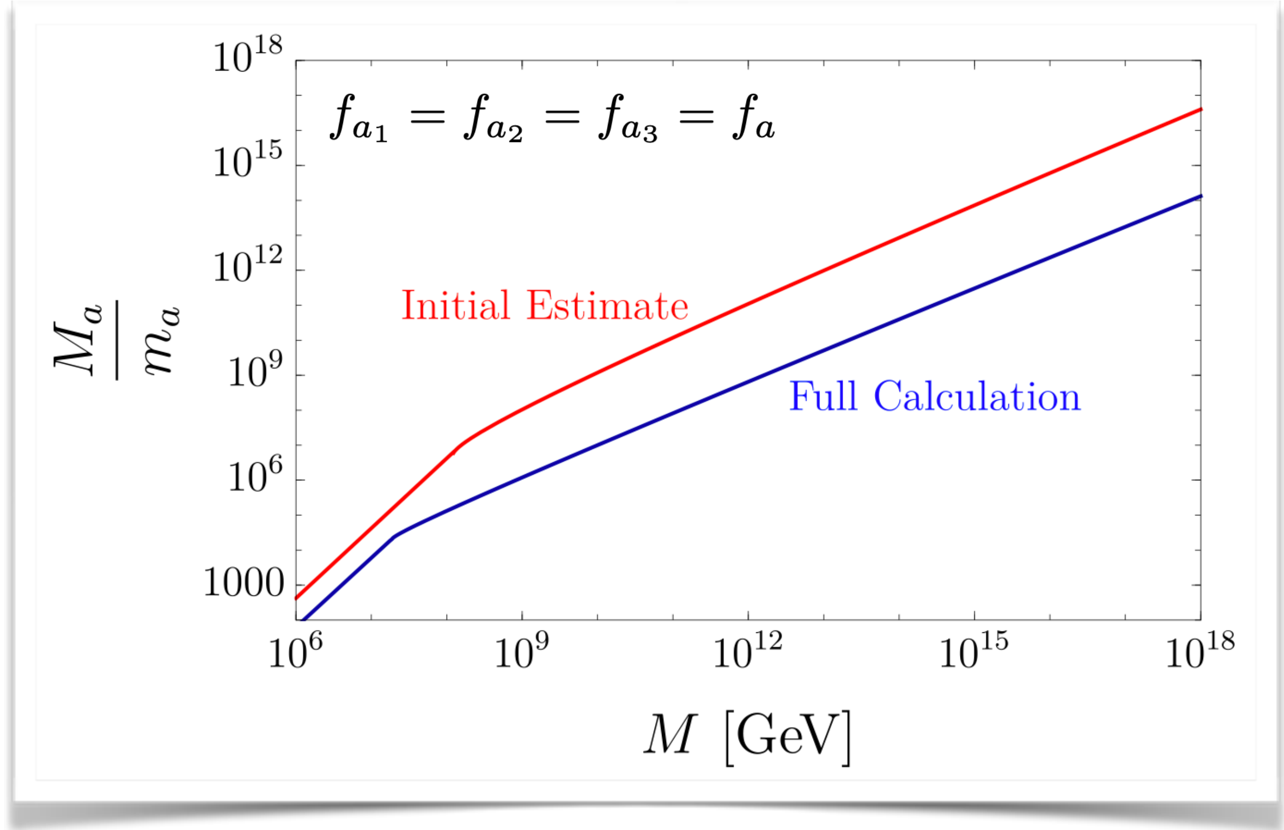

Suppression compared to Agrawal and Howe:

$$\frac{m_{a_i}^2}{\tilde{m}_{a_i}^2} \simeq 2^{-6} \cdot \left(\frac{M}{2\pi v_\Sigma}\right)^{b_i - 4}$$

Product Group Model

Agrawal, Howe ‘17
Csáki, MR, Shirman ‘19

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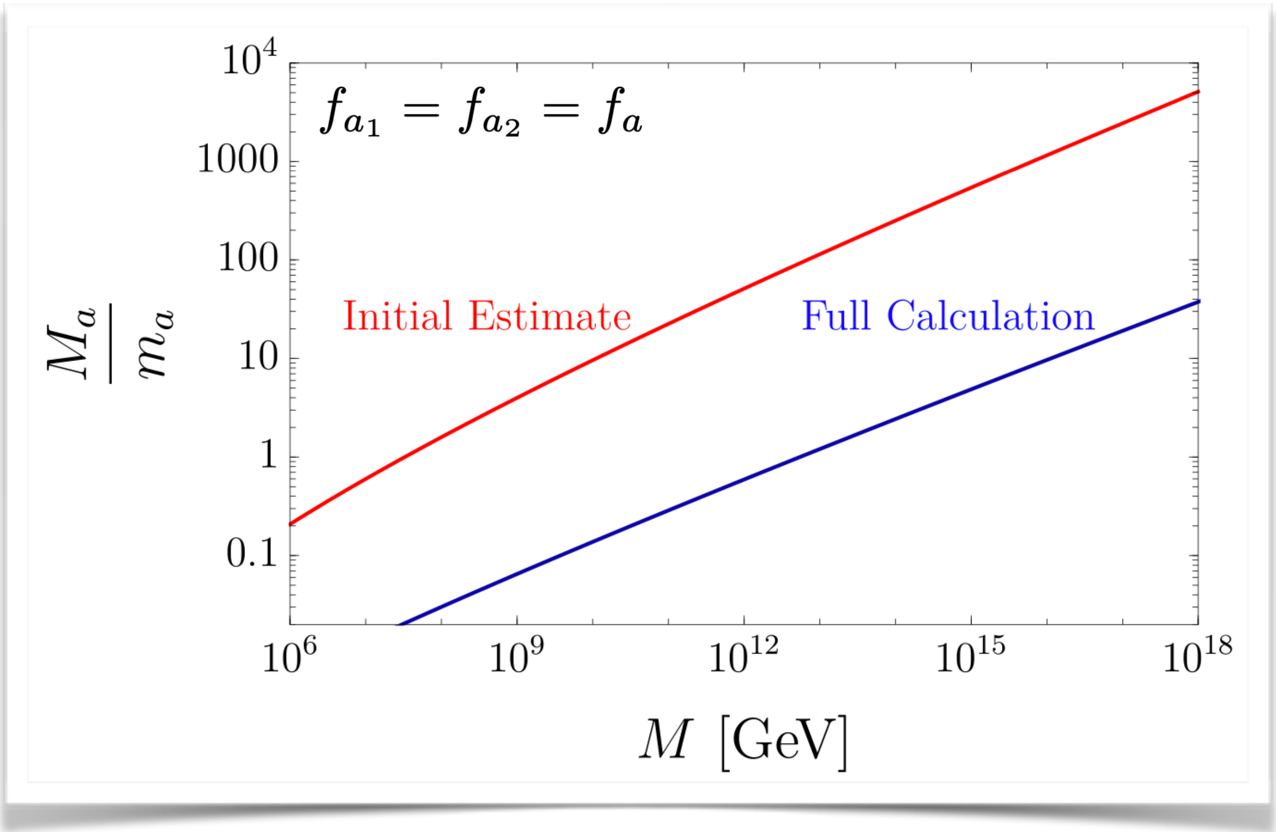

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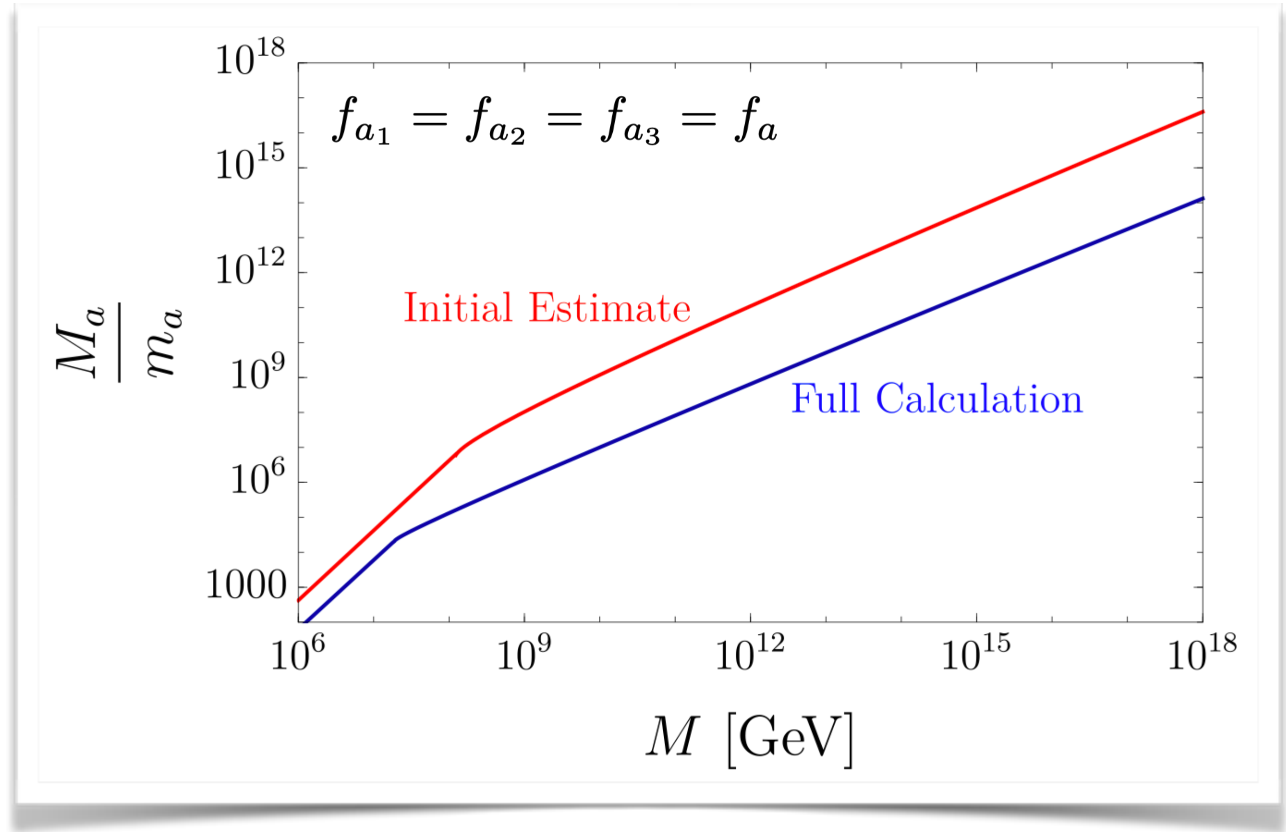

Suppression compared to Agrawal and Howe:

Missing factor of 2^{-6} in ’t Hooft’s original calculation

’t Hooft ’76 (Erratum)

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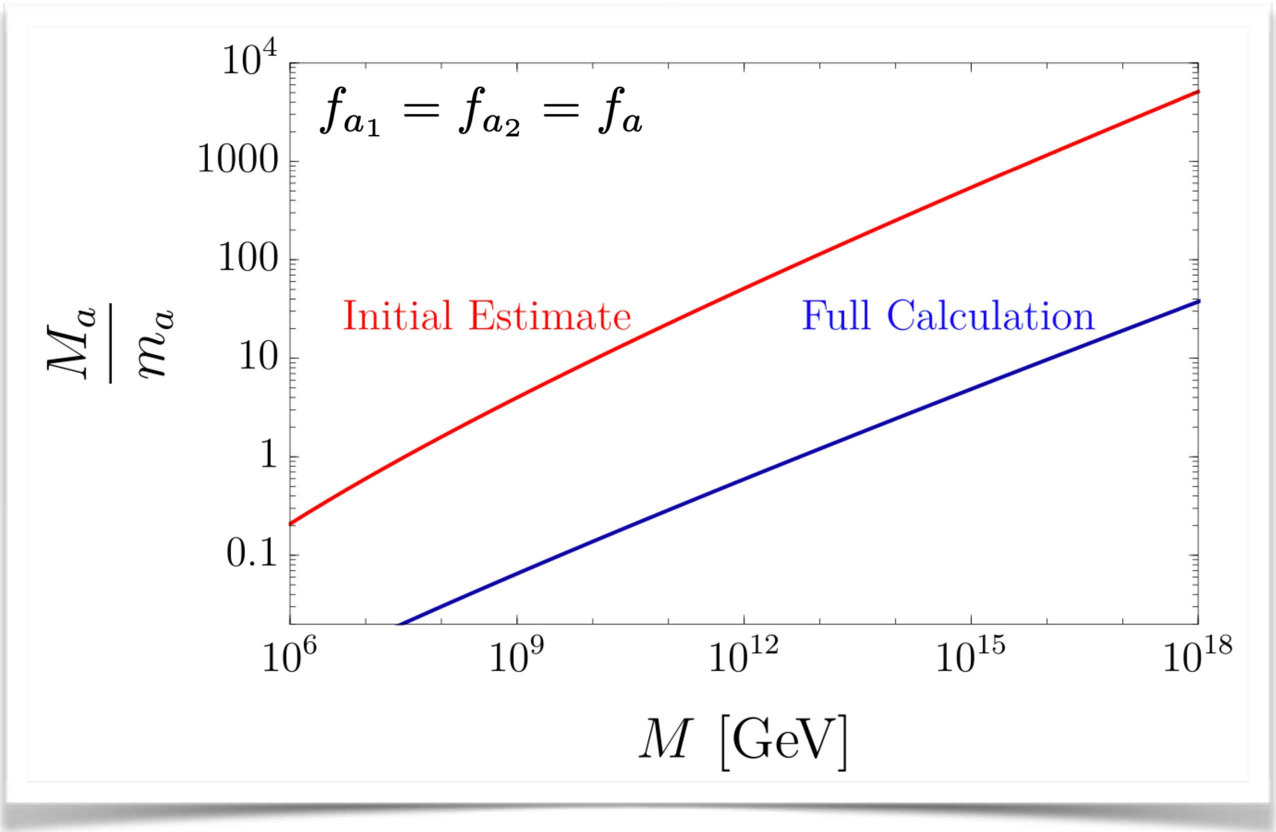

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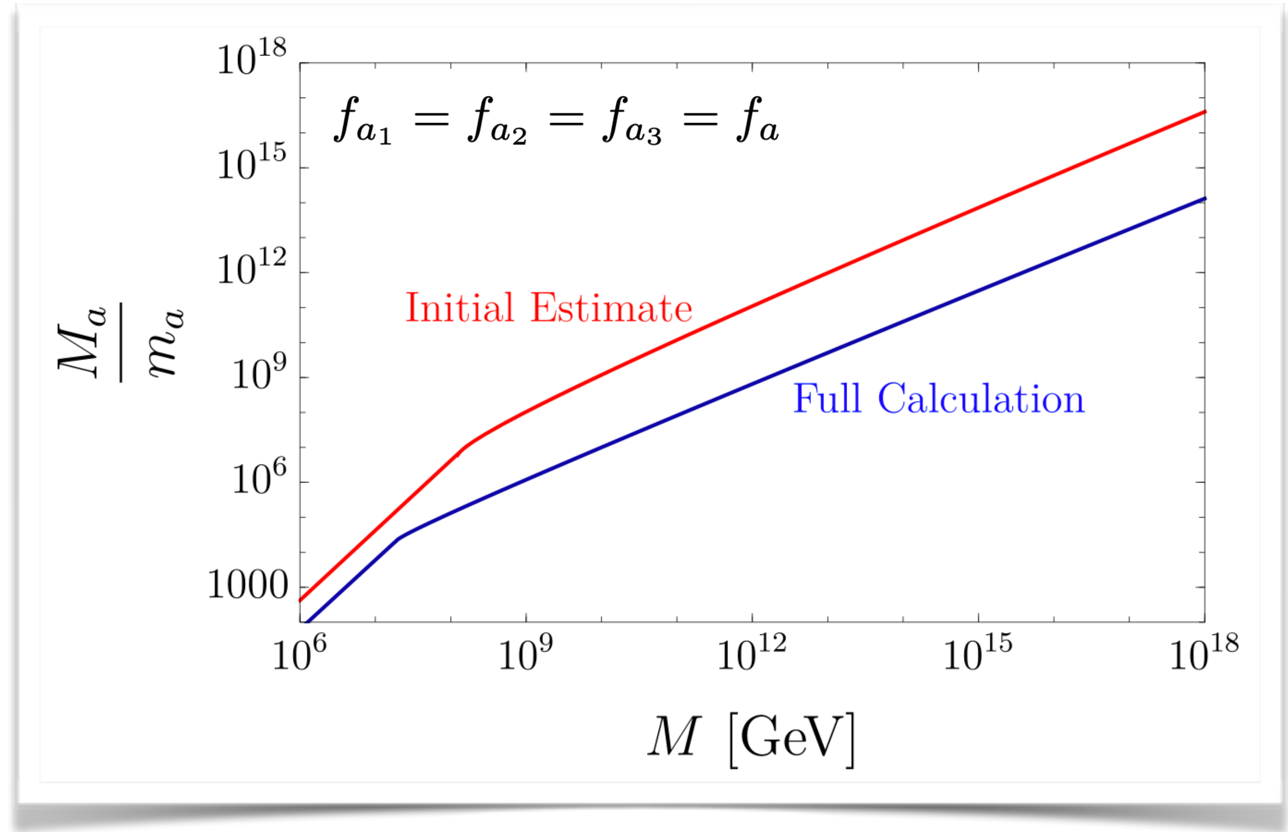

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Conversion from mass threshold $M = g_{\mathrm{eff}} v_\Sigma$ to effective cutoff in constrained instanton framework

Take Home Message

$G \to SU(3)_{\mathrm{QCD}}$ with non-trivial **index of embedding:**

1. Some broken UV instantons do not match to QCD instantons
2. “fractional” instantons are enhanced compared to QCD instantons
3. Already simple **product group** models can **significantly** enhance axion mass

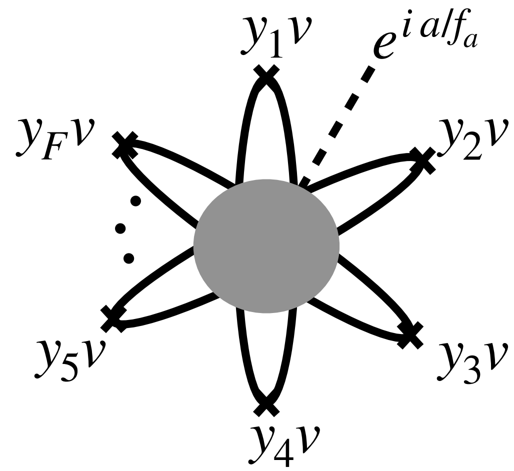
Way Forward:

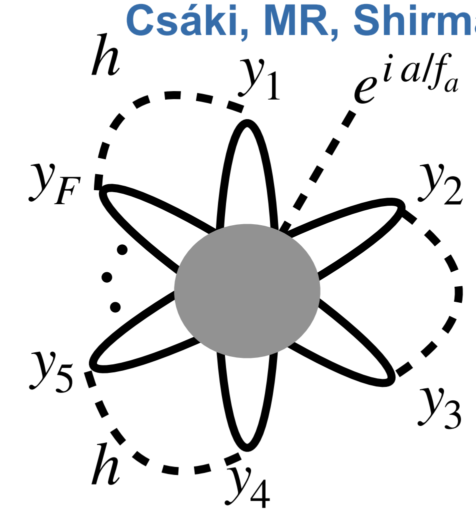
Construct model with simple UV gauge group and non-trivial index of embedding

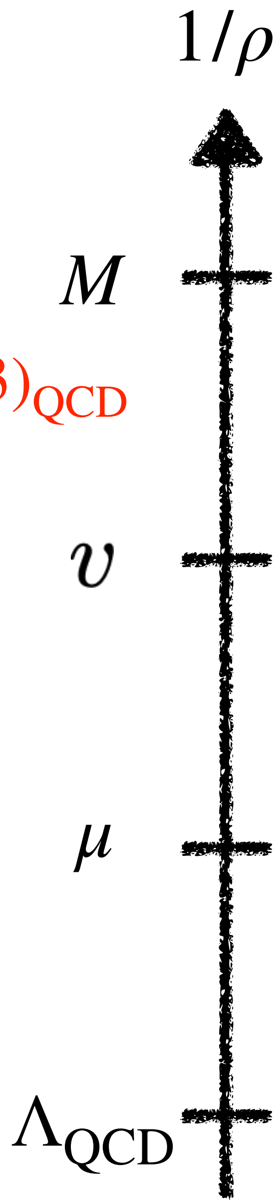
Backup

Instanton Contributions from different Scales

Csáki, MR, Shirman arXiv:1912.02197







G → $SU(3)_{\rm QCD}$

$$m_M^2 f_a^2 \sim \left(\frac{yv}{M}\right)^F \frac{\Lambda^{b_G}}{M^{b_G-4}}$$

dominates for $\frac{M}{v} > 4\pi$

$$m_M'^2 f_a^2 \sim \boxed{\left(\frac{y}{4\pi}\right)^F} \frac{\Lambda^{b_G}}{M^{b_G-4}}$$

$$m_v^2 f_a^2 \sim y^F \frac{\Lambda_{\rm QCD}^{b_{\rm QCD}}}{v^{b_{\rm QCD}-4}}$$

always subleading

$$m_v'^2 f_a^2 \sim \boxed{\left(\frac{y}{4\pi}\right)^F} \frac{\Lambda_{\rm QCD}^{b_{\rm QCD}}}{v^{b_{\rm QCD}-4}}$$

$$m_\mu^2 f_a^2 \sim \left(\frac{\prod_f m_f}{\mu^F}\right) \frac{\Lambda_{\rm QCD}^{b_{\rm QCD}}}{\mu^{b_{\rm QCD}-4}}$$

Higgs decouples

extrapolate

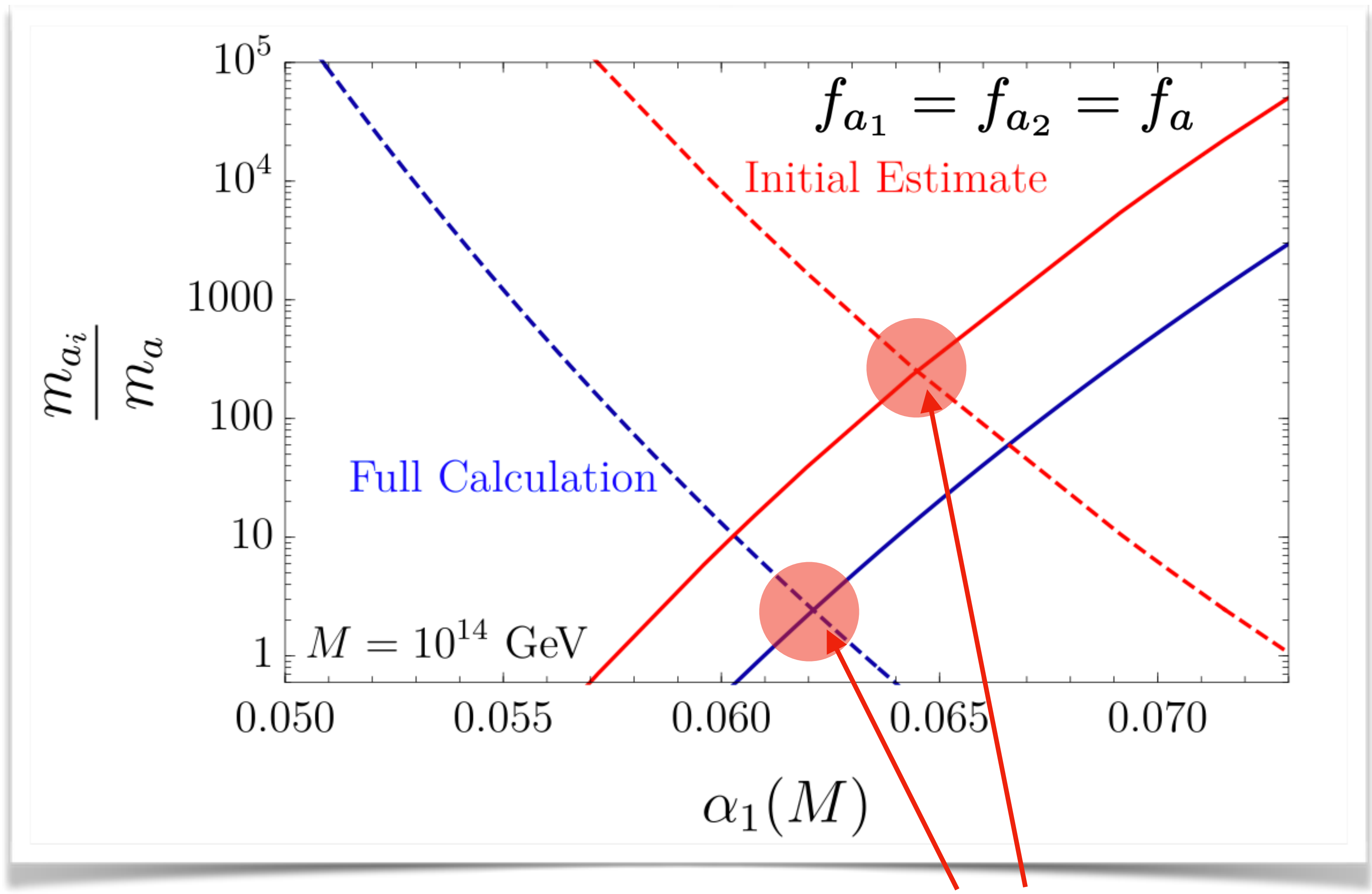
$$m_{\rm QCD}^2 f_a^2 \sim \Big(\prod_f m_f\Big)\Lambda_{\rm QCD}^{4-F}$$

Axion Mass in Agrawal-Howe Model

- Individual axion masses in $k = 2$ model

solid: $SU(3)_1$ axion

dashed: $SU(3)_2$ axion



Same contribution to both axions

Full 1-Instanton Calculation

Csáki, MR, Shirman arXiv:1912.02197

- 1-Instanton contribution to vacuum energy in completely broken $SU(N)$

→ integrate fluctuations around constrained instanton solution

Affleck '81

$$e^{-\frac{2\pi}{\alpha(\rho)}} \to e^{-\frac{2\pi}{\alpha(\rho)} - 2\pi^2\rho^2 v_\Sigma^2}$$

Scalar vev cuts off large instantons

Instanton density

$$\int \mathcal{D}A \prod_i \mathcal{D}\phi_i \, e^{-S_0} \sim \text{const.}\ e^{-i\theta} \int \frac{d^4x_0\, d\rho\, d\tilde{\mu}}{\rho^5} \left(\frac{2\pi}{\alpha}\right)^{2N} e^{-\frac{2\pi}{\alpha(\rho)} - 2\phi^2\rho^2\langle\phi\rangle^2} \prod_{f=1}^{F} \rho\, d\xi_f^{(0)} d\bar{\xi}_f^{(0)}$$

Collective coordinates

Fermion zero modes

- Fermion zero-mode integration projects out Yukawa interactions

$$\int \mathcal{D}H e^{-S_0[H]} \prod_{f=1}^{F} \rho\, d\xi_f^{(0)} d\bar{\xi}_f^{(0)} e^{i\int d^4x \sum_f y_f/\sqrt{2} H \bar{\psi}_f \psi_f} = \int \mathcal{D}H e^{-S_0[H]} \prod_{f=1}^{F} \left(\frac{i y_f \rho}{\sqrt{2}} \int d^4x \sum_f H \bar{\psi}_f^{(0)} \psi_f^{(0)} \right)$$

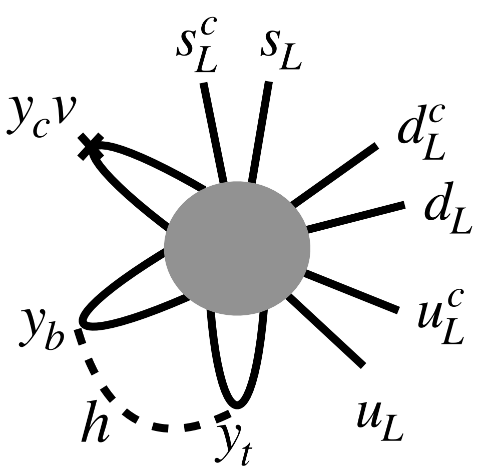
→ remaining integrals can be solved analytically

η' Mass from Small Instantons

- 't Hooft operator for F=6 QCD $$\frac{\Lambda^{b_G}}{M^{b_G+14}} \boxed{\det_{i,j}(\bar{\psi}_i \psi_j)}\, e^{i\theta} + h.c.$$

  breaks $U(1)_A$

- Low-energy QCD contains only 3 flavors → close remaining legs



$$\frac{y_t y_b y_c}{(4\pi)^2} \frac{v}{M^6} \left(\frac{\Lambda}{M}\right)^{b_G} \det_{i,j=u,d,s}(\bar{\psi}_i \psi_j)\, e^{i\theta} + h.c.$$

↓ match to QCD at M

$$\frac{y_t y_b y_c}{(4\pi)^2} \frac{v}{M^6} \left(\frac{\Lambda_{\rm QCD}}{M}\right)^{\frac{b_{\rm QCD}}{k}} \det_{i,j=u,d,s}(\bar{\psi}_i \psi_j)\, e^{i\theta} + h.c.$$

- Chiral symmetry breaking $\langle \bar{\psi}_i \psi_j \rangle \sim \Lambda_{\rm QCD}^3 \Sigma_{ij}$ with $\Sigma_{ij} \sim e^{\frac{i\eta'}{f_{\eta'}}} e^{\frac{i\pi^a T^a}{f_\pi}}$

$$\frac{y_t y_b y_c}{(4\pi)^2} \frac{v}{M} \boxed{\left(\frac{\Lambda_{\rm QCD}}{M}\right)^{\frac{b_{\rm QCD}}{k}+5}} \Lambda_{\rm QCD}^4 \det \Sigma\, e^{i\theta} + h.c.$$

suppressed for any k