How to suppress exponential growth

On the parametric resonance of photons in an axion background

Elisa Todarello (KIT)

Based on 2004.01669 with Ariel Arza and Thomas Schwetz

DESY Virtual Theory Forum
September 25th, 2020

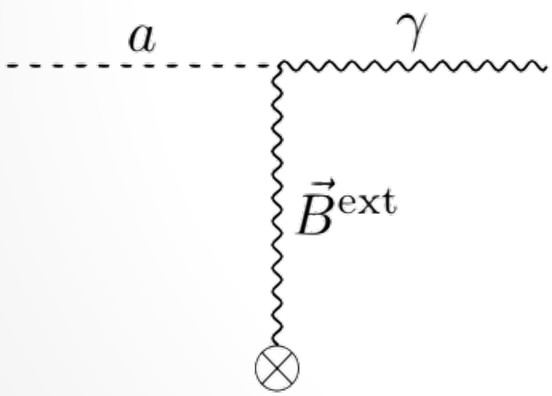
Outline

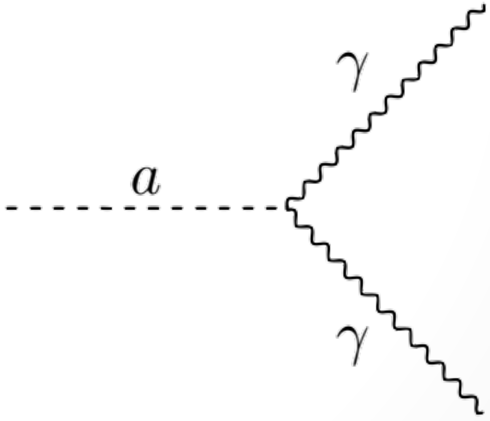
- Essentials of the axion-photon parametric resonance
- (Momentum dispersion effects)
- Gravitational redshift effects

Axion-photon interactions

- Lagrangian

$$\mathcal{L}_{a\gamma\gamma} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a a^2 + \boxed{\frac{1}{4}g_{a\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu}}$$



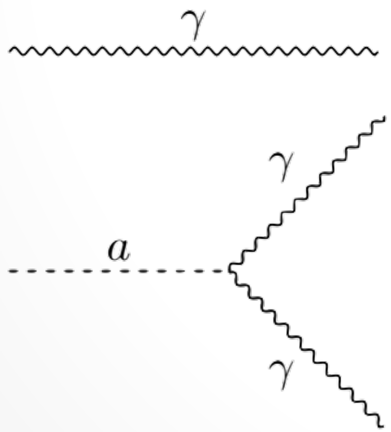


Axion-photon interactions

- Spontaneous decay rate

$$\Gamma_{a\to\gamma\gamma} = 1.1 \times 10^{-49}\ \mathrm{s}^{-1} \left(\frac{m_a}{10^{-5}\ \mathrm{eV}}\right)^5$$

- Stimulated decay



Possibility of exponential growth of the number of photons if the frequency is within a resonance band

See also talk by Oindrila GHOSH

Axion-photon interactions

Quantum photon
Carenza et al.,1911.07838

- Axion and photon as classical fields

- Neglect back-reactions on axion + non-relativistic axion field: neglect gradients

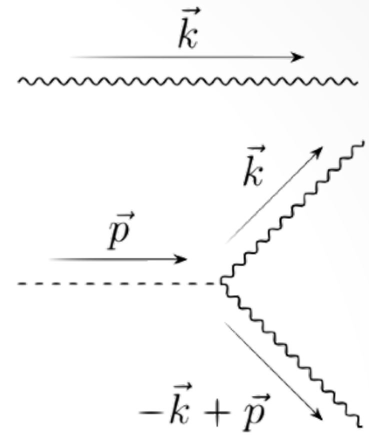
$$\boxed{\left(\partial_t^2 - \nabla^2\right)\vec{A} = -g_{a\gamma}\partial_t a \vec{\nabla} \times \vec{A}}$$

- Parametric oscillator

$$\ddot{x} + \omega_0^2 x = -\omega_0^2 h \cos(\Omega t)\, x$$

$$\omega_0 \approx \frac{\Omega}{2} \qquad \Rightarrow \qquad x \propto e^{\frac{1}{4} h\omega_0 t}$$

The monochromatic axion



$$a(t,\vec{x}) = \frac{\sqrt{2\rho_a}}{m_a}\sin\left(\omega_a t - \vec{p}\cdot\vec{x}\right)$$

- Exponential growth

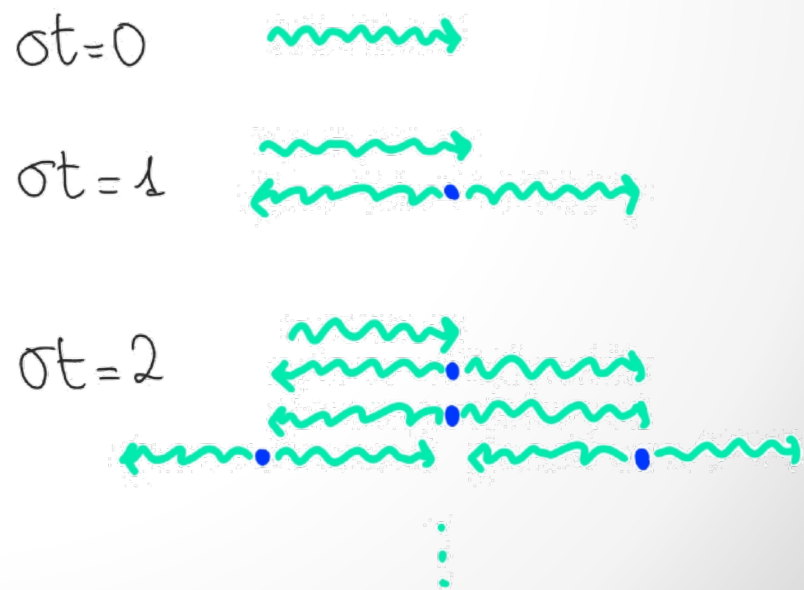
$$\sigma = \frac{g_{a\gamma}}{2}\sqrt{\frac{\rho_a}{2}}$$

$$s_{\vec{k}} = \sqrt{\sigma^2 - \epsilon_{\vec{k}}^2/4}$$

- Resonance band

$$\epsilon_{\vec{k}} = 2k - m_a - p\cos\varphi_{\vec{k}}$$

$$-2\sigma < \epsilon_{\vec{k}} < 2\sigma$$



The monochromatic axion

- Maximum growth factor in the local neighborhood

$$\sigma \simeq 6 \times 10^{-24}\,\mathrm{eV} \left(\frac{g_{a\gamma}}{10^{-11}\,\mathrm{GeV}^{-1}} \right) \left(\frac{\rho_a}{0.4\,\mathrm{GeV/cm}^3} \right)^{1/2}$$

$$\sigma^{-1} \simeq 3.5 \text{ yr} \simeq 1 \text{ pc}$$

- Center of the resonance band $\epsilon_{\vec{k}} = 0$

$$k_{\mathrm{res}} = \frac{m_a}{2}\left(1 + v_a \cos\varphi_{\vec{k}}\right)$$

- Width of the resonance band 4σ

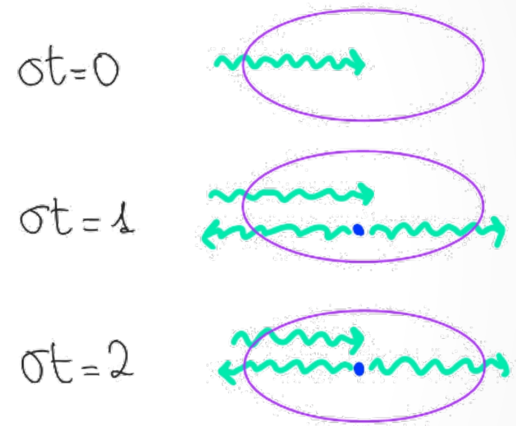
$$m_a = 10^{-5} \text{ eV} \qquad \delta v_a \lesssim 10^{-18} \qquad \delta\varphi \lesssim 10^{-15}$$

The multichromatic axion

- Axion momentum heavily affects the resonance!

- Important effects
  - Clumping
  - Gravitational redshift
  - Cosmological redshift
  - Photon plasma mass

$$\sigma R > 1$$



- References (a non exhaustive list)

  Preskill, Wise and Wilczek +Abbott and Sikivie + Dine and Fischler (1983)

  Alonso-Alvarez, Gupta, Jaeckel, and Spannowsky (1911.07885)

  Tkachev, Sov. Astron. Lett. 12 (1986)

  + Phys. Lett. B191 (1987)

  Hertzberg and Schiappacasse (1805.00430)

  Arza (1810.03722)

  Sigl, Trivedi (1907.04849)

  Carenza, Mirizzi, Sigl (1911.07838)

  Wang, Shao, and Li (2002.09144)

  Levkov, Panin, Tkachev (2004.05179)

Gravitational redshift

- Weak gravity

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)\delta_{ij}dx^i dx^j$$

- Assume external potential

$$\begin{aligned} \left(\partial_0^2 - (1+4\Phi)\nabla^2\right) A^i &= g_{a\gamma}\partial_0 a\, \epsilon^{0ijk}\partial_j A_k \\ (1-2\Phi)\partial_0^2 a - (1+2\Phi)\nabla^2 a + m_a^2 a &= 0 \end{aligned}$$

- Solve for axion at zeroth order in the eikonal approximation

$$a(t,\vec{x}\,) = (\mathrm{const}) \times e^{iS(t,\vec{x}\,)}$$

$$\partial^2 S \ll (\partial S)^2$$

- Valid if potential changes slowly over axion de Broglie wavelength

Gravitational redshift

- Momenta $k_\mu = \partial_\mu S_\gamma \qquad p_\mu = \partial_\mu S_a$

$$
\begin{aligned}
S_\gamma(z) &= \pm k_0 t \pm k_* \int_{z_*}^{z} dz' \, (1 - 2\Phi) \\
S_a(t, z) &= \pm p_0 t \pm \int_{z_*}^{z} dz' \, (1 - 2\Phi) \sqrt{p_*^2 - 2m_a^2 \Phi}
\end{aligned}
$$

- Photon k_0, axion p_0 do not redshift
- Photon momentum, axion momentum redshift differently
- Detuning is due to momentum redshift, not to energy redshift (disagree with discussion in Wang et al., 2002.09144)

Gravitational redshift

- Remember monochromatic axion

$$-2\sigma < \epsilon_{\vec{k}} < 2\sigma \qquad \epsilon_{\vec{k}} = 2k - m_a - p\cos\varphi_{\vec{k}}$$

- Axion momentum redshift moves centre of the band

$$\begin{aligned} 2k - m_a - \sqrt{p_*^2 - 2m_a^2\Phi_1} &= -2\sigma \\ 2k - m_a - \sqrt{p_*^2 - 2m_a^2\Phi_2} &= +2\sigma \end{aligned}$$

- Detuning distance

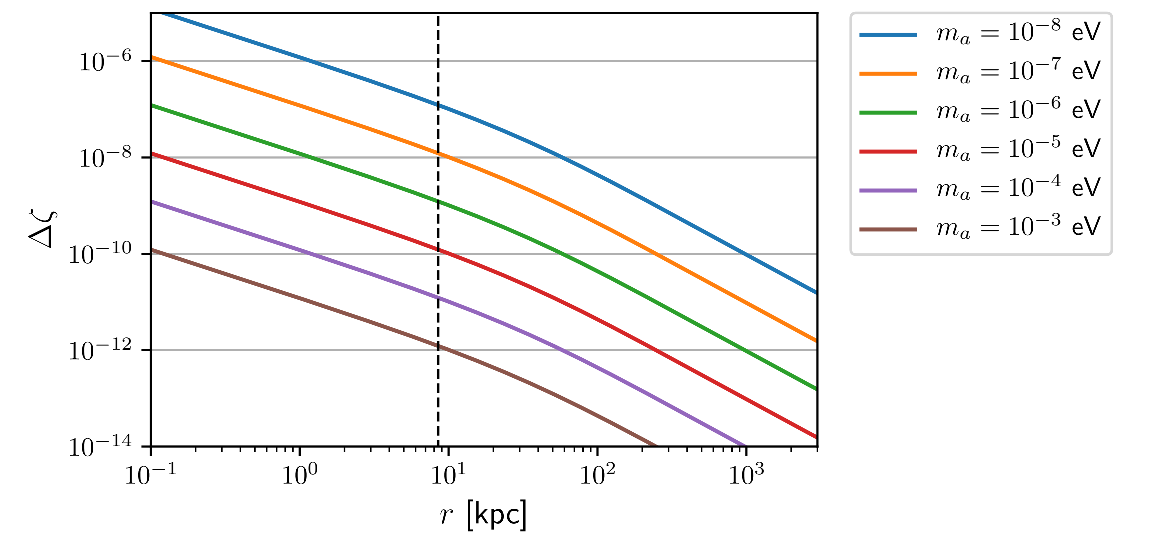
$$\Delta\zeta \equiv \sigma|z_1 - z_2| \approx \frac{4\sigma^2}{m_a^2}\frac{\sqrt{p_*^2 - 2m_a^2\Phi}}{|\partial_z\Phi|}$$

Gravitational redshift

- Local neighborhood

$$\Delta\zeta \sim 10^{-10} \left(\frac{10^{-5}\ \mathrm{eV}}{m_a}\right) \left(\frac{g_{a\gamma}}{10^{-11}\ \mathrm{GeV}^{-1}}\right)^2 \left(\frac{\rho_a}{0.4\ \mathrm{GeV/cm}^3}\right)$$

- NFW Milky Way



Conclusions

- Necessary conditions for exponential growth
- Momentum spread: $\sigma R > 1$

$$R < \frac{g_{a\gamma}^2}{8} M \sim 10^5\,\mathrm{m} \left(\frac{g_{a\gamma}}{10^{-11}\ \mathrm{GeV}^{-1}}\right)^2 \left(\frac{M}{10^{-13}\ M_\odot}\right)$$

- Gravitational redshift: $\sigma \Delta z > 1$

$$R < \left(\frac{g_{a\gamma}^4 M}{m_a^2 G_N}\right)^{1/3} \sim 10^6\,\mathrm{m} \left(\frac{g_{a\gamma}}{10^{-11}\ \mathrm{GeV}^{-1}}\right)^{4/3} \left(\frac{M}{10^{-13}\ M_\odot}\right)^{1/3} \left(\frac{10^{-5}\ \mathrm{eV}}{m_a}\right)^{2/3}$$

- Gravitational redshift: exponential growth does not happen in Milky way even in the case of a smooth axion DM component. Bounds of Sigl et al., 1907.04849 don't apply

Thank you

$\Delta\zeta = 4.3$