# Optical Properties of Axion Backgrounds

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Based on arXiv:1911.10221 with L. Ventura (U. Aveiro) and work in progress to appear May/June

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# Outline

1. Motivating new scalars, axions and axion-like particles (ALPs)
2. Axion coupling to electromagnetism
3. Axion backgrounds
4. Optical properties: Faraday rotation, time-delay, frequency shifts, refraction
5. Summary

# Axions

We shall consider an ALP scalar field $a$

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu a \partial_\nu a - \frac{1}{2} m_a^2 a^2 - \frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}, \qquad \tilde{F}_{\mu\nu} = \frac{\varepsilon_{\rho\sigma\mu\nu}}{2} F^{\rho\sigma}$$

Main motivations:

- **Occurs in many extensions of the SM (string theory etc.)**
- **Strong CP Problem - "QCD axion"** Peccei, Quinn (1977)

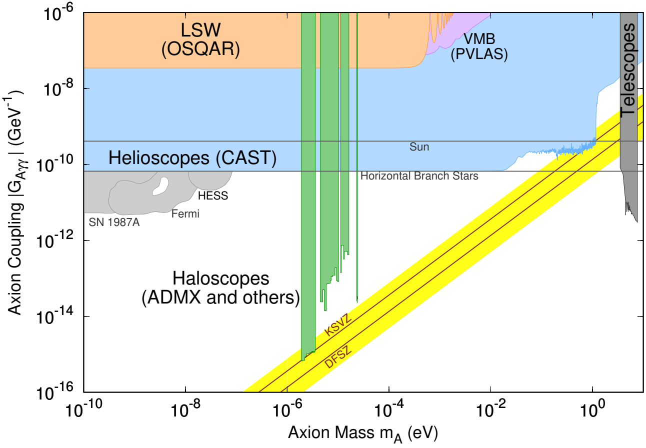
$$\mathcal{L}_\theta = \frac{\theta_{QCD}}{32\pi} \mathrm{Tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu} \qquad \theta \lesssim 10^{-10}$$

- **DM Candidate** - QCD axion or ultra light scalar DM

# Axions

Many experiments attempting to constrain the axion coupling to electromagnetism

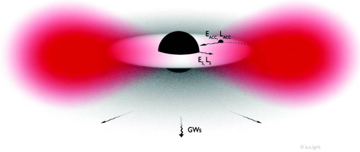
$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$




PDG

# Many possible astrophysical axion backgrounds

-**Superradiant black holes**

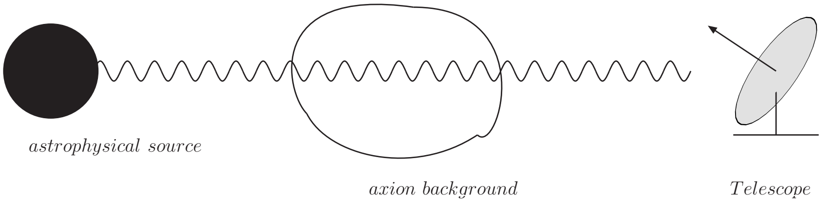



- **Axion stars**
- **Axion miniclusters**
- **DM halo**

## Question

how do photons scatter off such backgrounds

?



# Maxwell's equations with axions

$$\partial_\mu F^{\mu\nu} = g_{a\gamma\gamma}\partial_\mu a \tilde{F}^{\mu\nu} + j^\nu, \qquad \partial_{[\mu} F_{\mu\nu]} = 0$$

In terms of electric and magnetic fields:

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \rho - g_{a\gamma\gamma}\mathbf{B} \cdot \nabla a\,, \\
\nabla \times \mathbf{B} - \dot{\mathbf{E}} &= \mathbf{J} + g_{a\gamma\gamma}\dot{a}\mathbf{B} + g_{a\gamma\gamma}\nabla a \times \mathbf{E}\,, \\
\nabla \cdot \mathbf{B} &= 0 \\
\dot{\mathbf{B}} + \nabla \times \mathbf{E} &= 0\,.
\end{aligned}$$

Introduce WKB approximation: photon wavelength shorter than axion gradient scales:

$$\partial_\mu \partial_\nu a / \partial_\rho a \ll \partial_\mu \mathbf{E}/\mathbf{E}, \partial_\mu \mathbf{B}/\mathbf{B} \implies \lambda_\gamma \ll \lambda_a$$

and introduce local plane wave solutions:

$$\mathbf{E} = \mathbf{E}_0 e^{iS}, \qquad \mathbf{B} = \mathbf{B}_0 e^{iS}, \qquad \partial^\mu S = k^\mu = (\omega, \mathbf{k})$$

New ingredient - JM, Ventura 2019

**Plasma:** $\mathbf{J} = \sigma \cdot \mathbf{E}, \qquad \sigma(\omega) = \frac{i\omega_p^2}{\omega}$

e.g. interstellar medium, lab plasma etc. Putting this together:

$$\Box \mathbf{E} + \omega_{\rm p}^2 \mathbf{E} - \frac{(\nabla \mathbf{B}) \cdot \nabla a}{1 - \omega_{\rm p}^2/\omega^2} + g_{a\gamma\gamma} \left[ \dot{a}\dot{\mathbf{B}} + \nabla a \times \dot{\mathbf{E}} \right] \simeq 0,$$

$$\Box \mathbf{B} + \omega_{\rm p}^2 \mathbf{B} - g_{a\gamma\gamma} \left[ \dot{a} \nabla \times \mathbf{B} + \nabla a (\nabla \cdot \mathbf{E}) - (\nabla a \cdot \nabla) \mathbf{E} \right] \simeq 0$$

This is a matrix equation:

$$\mathbf{M}(\omega, \mathbf{k}) \cdot \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} = 0,$$

$\Longrightarrow$ Eigenvalues $D^{\pm}$ of $\mathbf{M}$ must vanish along photon rays:

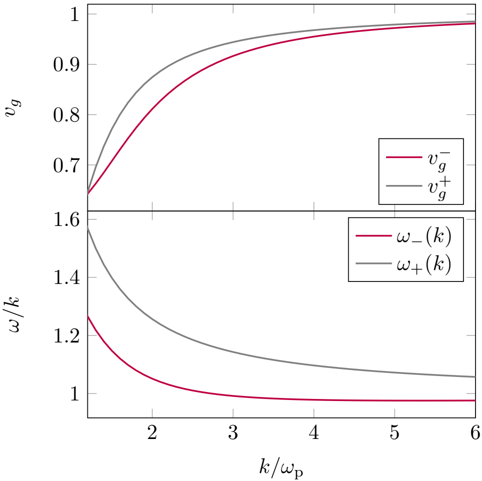
$$\begin{aligned} D^{\pm} &= k^2 - \omega_{\rm p}^2 \\ &\pm \frac{1}{[\omega^2 - \omega_{\rm p}^2]^{1/2}} \Big[ \omega^2 g_{a\gamma\gamma}^2 \left( (k \cdot \partial a)^2 - k^2 (\partial_\mu a)^2 \right) \\ &+ \omega_{\rm p}^2 g_{a\gamma\gamma}^2 \left( \dot{a}^2 k^2 - 2\dot{a}\omega (k \cdot \partial a) + (\partial_\mu a)^2 \omega^2 \right) \Big]^{1/2} \end{aligned}$$

$$\text{dispersion relation } \omega = \omega(k): \qquad \mathrm{D}^{\pm} = 0$$

## Axion backgrounds are birefringent

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} F_{\mu\nu}\tilde{F}^{\mu\nu}, \qquad \text{violates P}$$

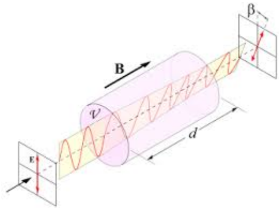
**LH** and **RH** polarisations different dispersion in axion background




where $v_g = \partial\omega/\partial k$.

Well-known application **Faraday Rotation** Harrari-Sikivie 1992

1D motion: $\omega \simeq |\mathbf{k}| \pm g_{a\gamma\gamma}\left[\dot{a} + \hat{\mathbf{k}} \cdot \nabla a\right]$




$$v_p^{\mathrm{Left}} \neq v_p^{\mathrm{Right}}$$

**Rotation Angle:** $\Delta\phi = g_{a\gamma\gamma}(a_{\mathrm{f}} - a_{\mathrm{i}})$

Constraints on existence of axon-like fields

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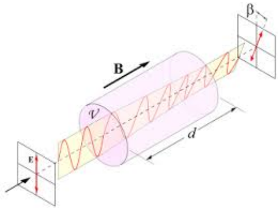


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Constraints on existence of axon-like fields

**Important:** note it depends on surface terms/axion fields at trajectory endpoints.

Deeper interpretation:

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$
$$\nabla \cdot \mathbf{D} = \rho, \qquad \nabla \times \mathbf{H} - \partial_t \mathbf{D} = \mathbf{j}$$

There is a clear constitutive relation

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} 1 & -g_{a\gamma\gamma} a \\ g_{a\gamma\gamma} a & 1 \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}$$

**Axion backgrounds $\leftrightarrow$ magneto-electric media**

$$\mathbf{D}_i = \alpha_{ij} E_j + \beta_{ij} B_j$$
$$\mathbf{H}_i = \gamma_{ij} E_j + \delta_{ij} B_j$$

Optical properties of axion fields = optical properties of space-time dependent magneto-electric medium!

**For example:**

For a stationary axion background, the photon equations of motion are given by

$$-\nabla^2 \mathbf{E} + \nabla(\nabla \cdot \mathbf{E}) + \omega^2 \mathbf{E} - g_{a\gamma\gamma}\omega \nabla a \times \mathbf{E} = 0,$$

Stationary axion background **effective dielectric tensor given by**

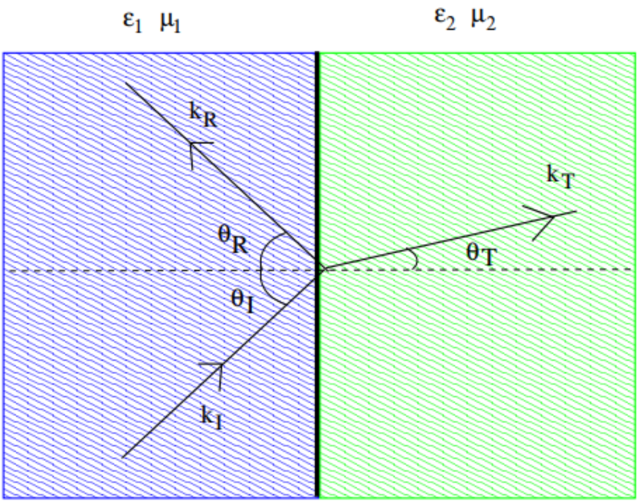
$$\varepsilon_{ij}^{\mathrm{eff}}(\omega) = \delta_{ij} - \frac{i g_{a\gamma\gamma}\varepsilon_{ijk}}{\omega}\nabla_k a,$$

Just like a magneto-optic background

$$\nabla a \leftrightarrow \mathbf{B}_{ext.}.$$

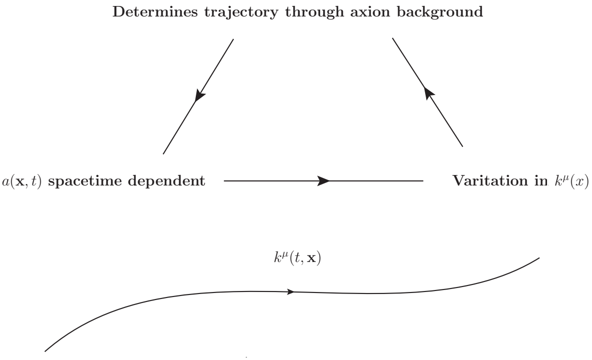
So a stationary axion background acts like a dielectric medium with magneto-optic properties!

## And we know refraction happens for dielectrics...




so by analogy, we expect refraction in axion backgrounds...

# But there are some subtleties - Blas, Caputo et al 2019



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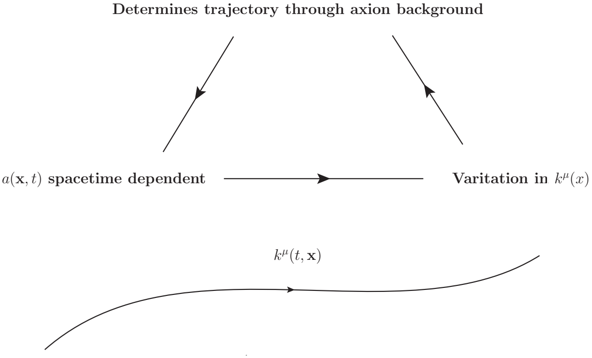



How do we deal with this consistently to track the photon's evolution?

**Answer:** Blas et al (2019) $\rightarrow$ Weinberg (1964) $\rightarrow$ Hamilton 1800s

**Hamiltonian Optics!**

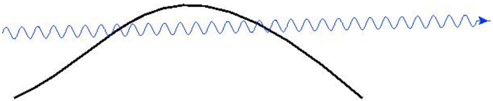
$$\frac{d\mathbf{x}}{dt} = \frac{\partial\omega}{\partial\mathbf{k}},, \qquad \frac{d\mathbf{k}}{dt} = -\frac{\partial\omega}{\partial\mathbf{x}}, \qquad \frac{d\omega}{dt} = \frac{\partial\omega}{\partial t}$$

Hamiltonian Optics:

$$\frac{df}{dt} = \{f, \omega\} + \partial_t f,$$

Where $\omega = \omega(\mathbf{x}, \mathbf{k}; t)$ is a "one-particle Hamiltonian" satisfying the mass-shell condition:

$$\omega_{\mathrm{p}} = 0: \qquad k^2 = \pm g_{a\gamma\gamma}\left[(k\cdot\partial a)^2 - (\partial\cdot a)^2 k^2\right]^{1/2}$$



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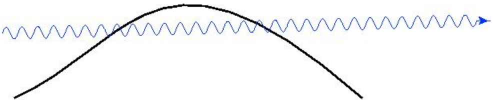


**This isn't just pedantry - it really matters...**

# Refraction

# Black Hole Superradiance

**Axion** : $g^{\mu\nu}\nabla_\mu\nabla_\nu a + m_a^2 a = 0$

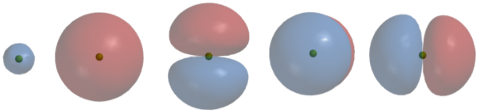
$$ds^2 = -dt^2\left(1 - \frac{2GM}{r}\right) + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Hydrogen-like solutions - e.g.Detweiler 1980

$$a = \psi(r) Y_{\ell m}(\theta, \varphi) e^{-i\omega t} \qquad - \frac{d\psi}{dr^2} + V(r)\psi = \omega^2\psi$$

$$V(r) \simeq \frac{\ell(\ell+1)}{r^2} - \frac{2GM\mu^2}{r} + \mu^2,$$ “Gravitational atom”

Gives rise to eigenvalue problem for spinning black holes

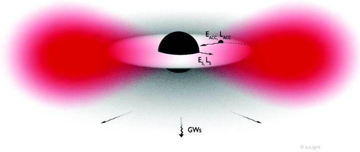


Combing the **rotation** and **confining** nature of a Kerr BH leads to an instability with discrete eigenvalues

$$a \simeq e^{-i\omega t} Y_{lm} \psi(r) \qquad \omega = \omega_R + i\omega_I \qquad a = J/M$$

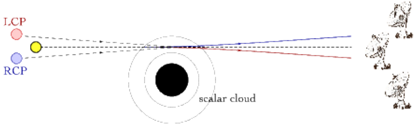
$$\omega_R = \mu - \frac{\mu}{2}\left(\frac{GM\mu}{\ell + m + 1}\right)^2, \qquad \omega_I \sim (GM\mu)^{4\ell+5}\left(\frac{am}{GM} - 4\mu GM\right)$$

Axion feeds on BH angular momentum, spinning it down.

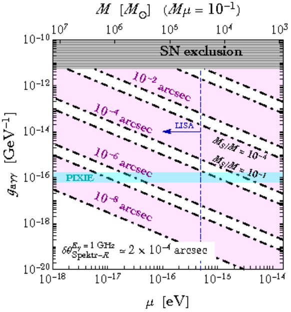


# Polarisation-dependent bending around BH

Plascencia, Urbano (2017)




**Claim:** $\mathcal{O}(g_{a\gamma\gamma})$ bending of light $\delta\theta$



## Apply carefully Hamilton's equations to same problem - Blas et al 2019

"No chiral bending of light by axion clumps"

No refraction at $\mathcal{O}(g_{a\gamma\gamma})$ in absence of plasma

**Apply carefully Hamilton's equations to same problem - Blas et al 2019**

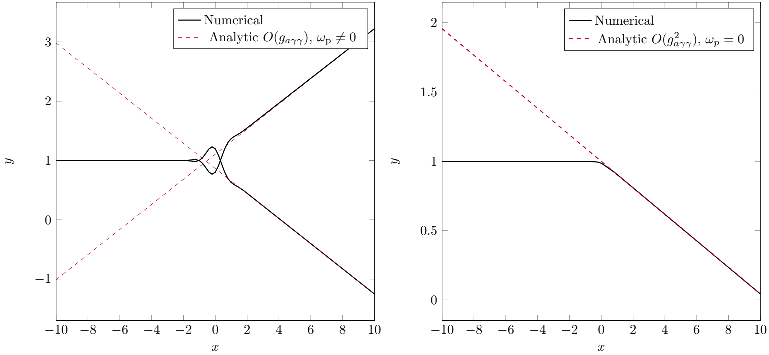
"No chiral bending of light by axion clumps"

No refraction at $\mathcal{O}(g_{a\gamma\gamma})$ in absence of plasma

**JM, Ventura 2019**

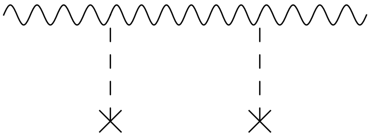
We argue that bending does occur at orders $\mathcal{O}(\omega_{\rm p}^2 g_{a\gamma\gamma})$ and $\mathcal{O}(g_{a\gamma\gamma}^2)$

$$a(t,x) = a_0 sin(m_a t) e^{-(x^2+y^2)/r_c^2}$$



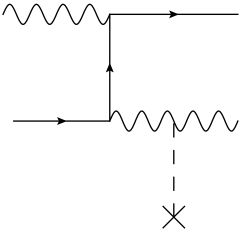
## Microphysics/Diagramatics?

$\mathcal{O}(g_{a\gamma\gamma}^2)$



no plasma

$\mathcal{O}(\omega_{\mathrm{p}}^2 g_{a\gamma\gamma})$



finite plasma

# Frequency shifts

For simplicity consider 1D motion $a = a(t, x)$

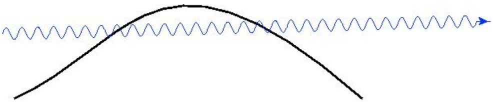
$$\mathbf{k}(t) = (k(t), 0, 0), \quad \mathbf{x}(t) = (x(t), 0, 0)$$

Simple dispersion relation:

$$\omega(k) = \left[k^2 + \omega_{\mathrm{p}}^2 \mp g_{a\gamma\gamma} k \dot{a} + \frac{g_{a\gamma\gamma}^2 a'^2}{2}\right]^{1/2} \mp \frac{g_{a\gamma\gamma} a'}{2}$$

Solve Hamilton's equations:

$$\frac{d\mathbf{x}}{dt} = \frac{\partial \omega}{\partial \mathbf{k}},, \qquad \frac{d\mathbf{k}}{dt} = -\frac{\partial \omega}{\partial \mathbf{x}}, \qquad \frac{d\omega}{dt} = \frac{\partial \omega}{\partial t}$$



# Frequency Shifts

$$\Delta\omega^{\pm}(t) = \mp\frac{g_{a\gamma\gamma}}{2}\left[\dot{a}_{\mathrm{f}} - \dot{a}_{\mathrm{i}}\right]$$ Blas et al 2019

$a(t, x) \sim \psi(\mathbf{x})\sin(m_a t)$ at emission/detection
$\implies$ frequency modulation

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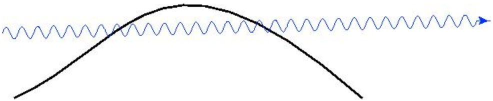
## Additional Effects

JM, Ventura 2019

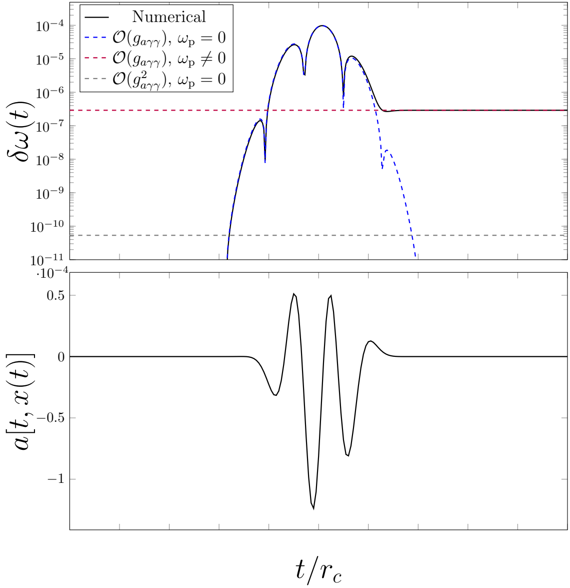
$$\Delta\omega_p = -\frac{g_{a\gamma\gamma}}{2}\int_{t_i}^{t_f} dt'\left[n_{\rm pl}\ddot{a}[t', x_0(t')] + \dot{a}'[t, x_0(t')]\right],$$

$$\Delta\omega_a = -\frac{g_{a\gamma\gamma}^2}{4k_0}\int_{t_i}^{t_f} dt'\partial_t(\partial a)^2,$$



Send a photon through an axion background:

$$a = a_0 \sin(m_a t) e^{-x^2/r_c^2}$$



## Potential to be physically important!

$$\Delta\omega_p = -\frac{g_{a\gamma\gamma}}{2}\int_{t_i}^{t_f} dt' \left[n_{\rm pl}\ddot{a}[t', x_0(t')] + \dot{a}'[t, x_0(t')]\right],$$
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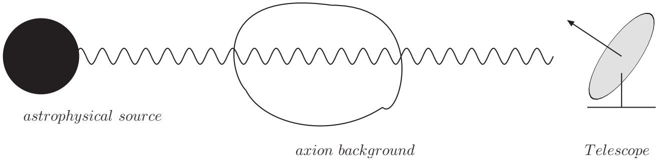
Detection for intermediate axions along line of sight!




Not just surface terms!

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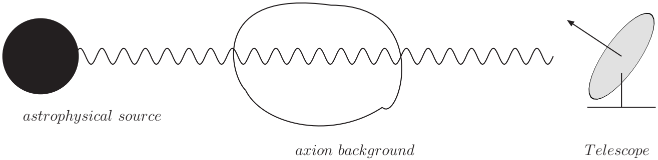
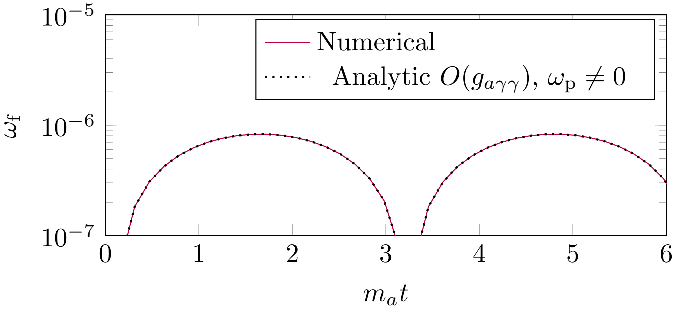


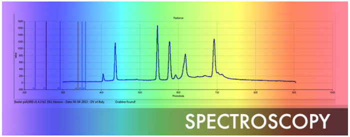

Not just surface terms!

Also running of effect with respect to frequency/momentum $k_0$

Oscillating axion background $\implies$ frequency modulations




**Future work?** precision astrophysical spectroscopy to hunt for temporal modulations?



**Time delays**

# Time-delays

$$t = \int \frac{dt'}{v_g(t')}$$

Group velocity splitting between left/right polarisations:

$$v_g^+ - v_g^- = \pm \frac{g_{a\gamma\gamma}}{4k_0} \frac{\omega_\mathrm{p}^2}{k_0^2} \left[a' - \dot{a}\right]$$

Different arrival times for left/right images!

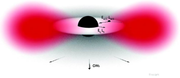
$$\Delta t_p = \mp \frac{g_{a\gamma\gamma}}{4k_0} \frac{\omega_\mathrm{p}^2}{k_0^2} \int_0^{t_\mathrm{f}} dt' \left[a' - \dot{a}\right] .$$

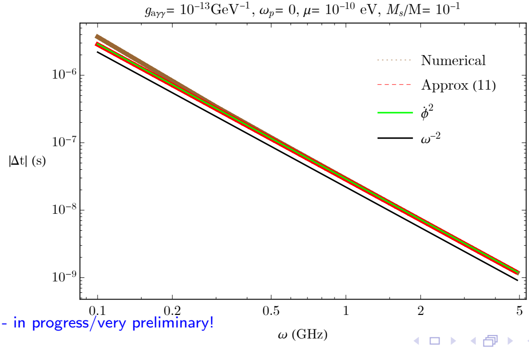
# Dispersive time delays from superradiant BH

$$t_a = -\frac{g_{a\gamma\gamma}^2}{8k_\gamma^2}\int d\ell\,\dot{a}^2$$

$$\Delta t_a(\omega_{\rm p}) = \pm\frac{g_{a\gamma\gamma}\omega_{\rm p}{}^2}{2k_\gamma^3}\int d\ell\,\dot{a}$$

$$\Delta t = \pm\frac{g_{a\gamma\gamma}^3}{8k_\gamma^3}\int d\ell\,\dot{a}^3$$



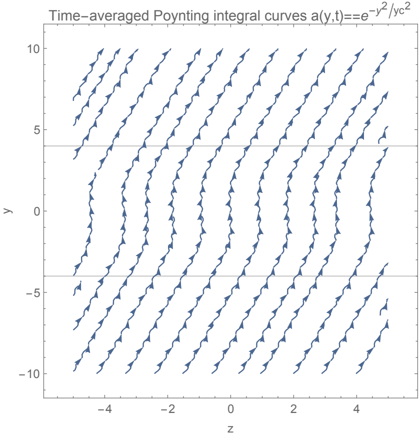



JM with L Ventura - in progress/very preliminary!

**In Progress** study effects with full solutions

$$\Box \mathbf{E} + \nabla(\nabla \cdot \mathbf{E}) + \dot{\mathbf{J}}_p + \dot{\mathbf{J}}_a = 0,$$
$$\Box \mathbf{B} - \nabla \times \mathbf{J}_a - \nabla \times \mathbf{J}_\mathrm{p} = 0.$$

where $\quad \mathbf{J}_a = g_{a\gamma\gamma}\left[\dot{a}\mathbf{B} + \nabla a \times \mathbf{E}\right], \qquad \dot{\mathbf{J}}_\mathrm{p} = \omega_\mathrm{p}^2\,\mathbf{E},$




**Refraction**

**In Progress** Use green functions

$$\mathbf{E}_{\mathrm{scat}} = g_{a\gamma\gamma} \int d^4x' G(x,x') \mathbf{E}_{\mathrm{inc}}(x')$$

Covers all cases:

- **"Geometric Optics":** $\omega_\gamma \gg m_a$ (verify existing results)
- **"Mie":** $\omega_\gamma \sim m_a$ : interesting resonant/enhanced behaviour?
- **"Raleigh":** $\omega_\gamma << m_a$

Perhaps expect non-adiabatic (history-dependent) Faraday rotation for $\omega_{\mathrm{p}} \neq 0$
$\implies$ implications for axion CMB constraints?

# Summary

(i) There are a variety of interesting effects which can happen in axion backgrounds

(ii) Plasma has interesting effects

(iii) Microphysical understanding of the power dependence on $g_{a\gamma\gamma}$ desirable - e.g. Feynman Diagrams.

(iv) Derive signatures for astrophysical setups- CMB, BHs, Halo etc...or in the lab??

Thanks for listening!

**Backup Slides**

Consider

$$\Box\phi + m_a^2\phi + \lambda(x)\phi = 0$$

Two expansions:

**Perturbation Theory:** $\lambda, \quad \lambda^2, \quad \lambda^3, \cdots$

**Gradient/WKB:** $\lambda, \quad \partial\lambda, \quad \partial^2\lambda, \quad \partial^3\lambda, \cdots$

Consider

$$\Box\phi + m_a^2\phi + \lambda(x)\phi = 0$$

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**Axion**

$$\partial_\mu F^{\mu\nu} - \chi_\mu \tilde{F}^{\mu\nu} = 0, \qquad \chi_\mu = g_{a\gamma\gamma}\partial_\mu\phi$$

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Blas, Caputo et al (2019)

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Ventura, McDonald (2019)

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