Studying the nonlinear properties of an accelerator / storage ring via tune scans

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Motivation

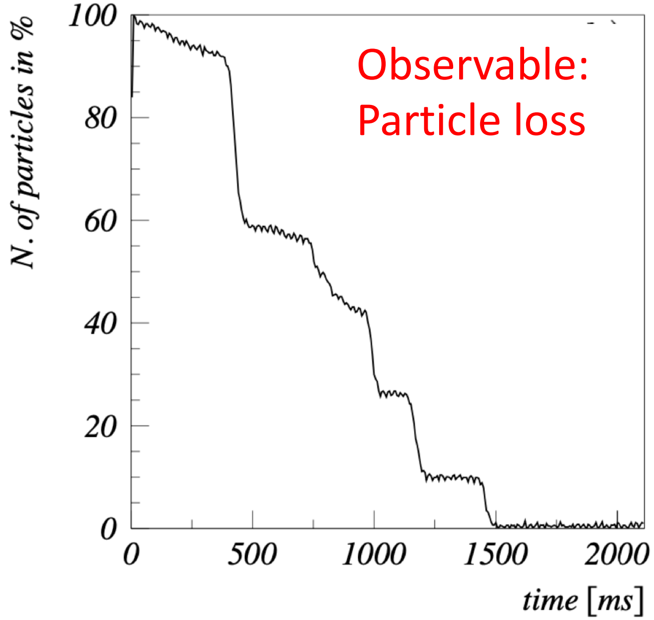
Nonlinear motion afflicts the particle dynamics in two ways

1) Excites resonances
2) Creates an amplitude dependent detuning

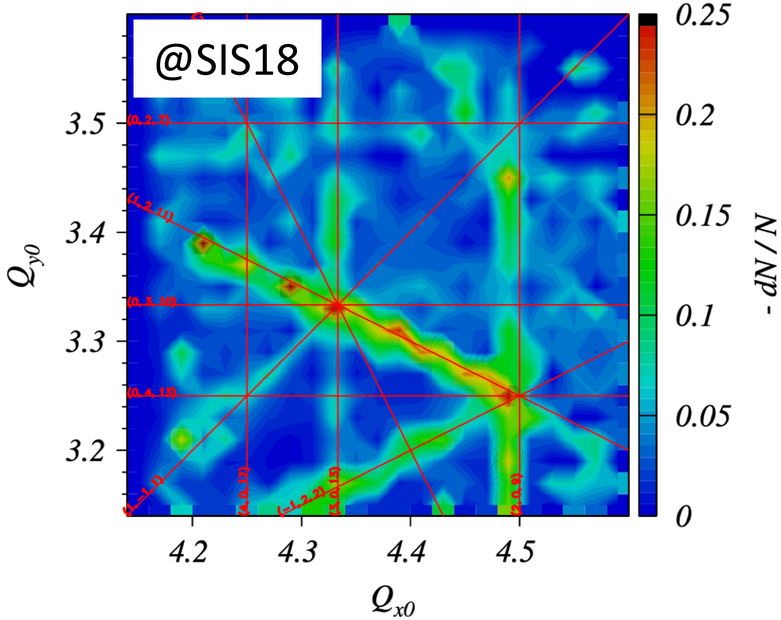
Optimization of accelerators requires a machine modeling as accurate as possible, so one can find a proper strategy to optimize the machine performances

Goal: retrieving the strength and location of an accelerator nonlinear components

Dynamical tune scan method



Outcome: resonance chart



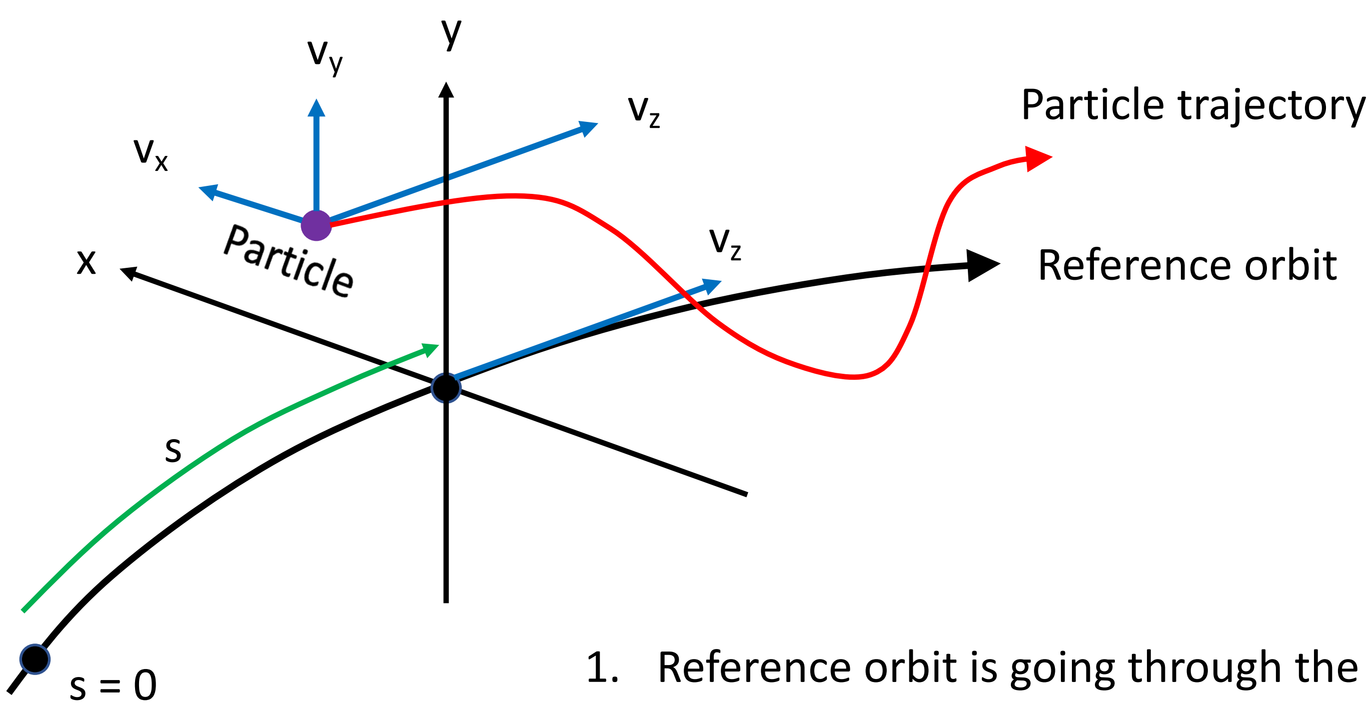
Static tune scan methods

Based on the dependence of particles tune from magnets the feed-down

WARNING: not to be confused with the amplitude dependent detuning.

Observable: particle tunes → Outcome: modeling of machine nonlinearities

Ideal Accelerator Model



1. Reference orbit is going through the magnetic center of the main magnets.

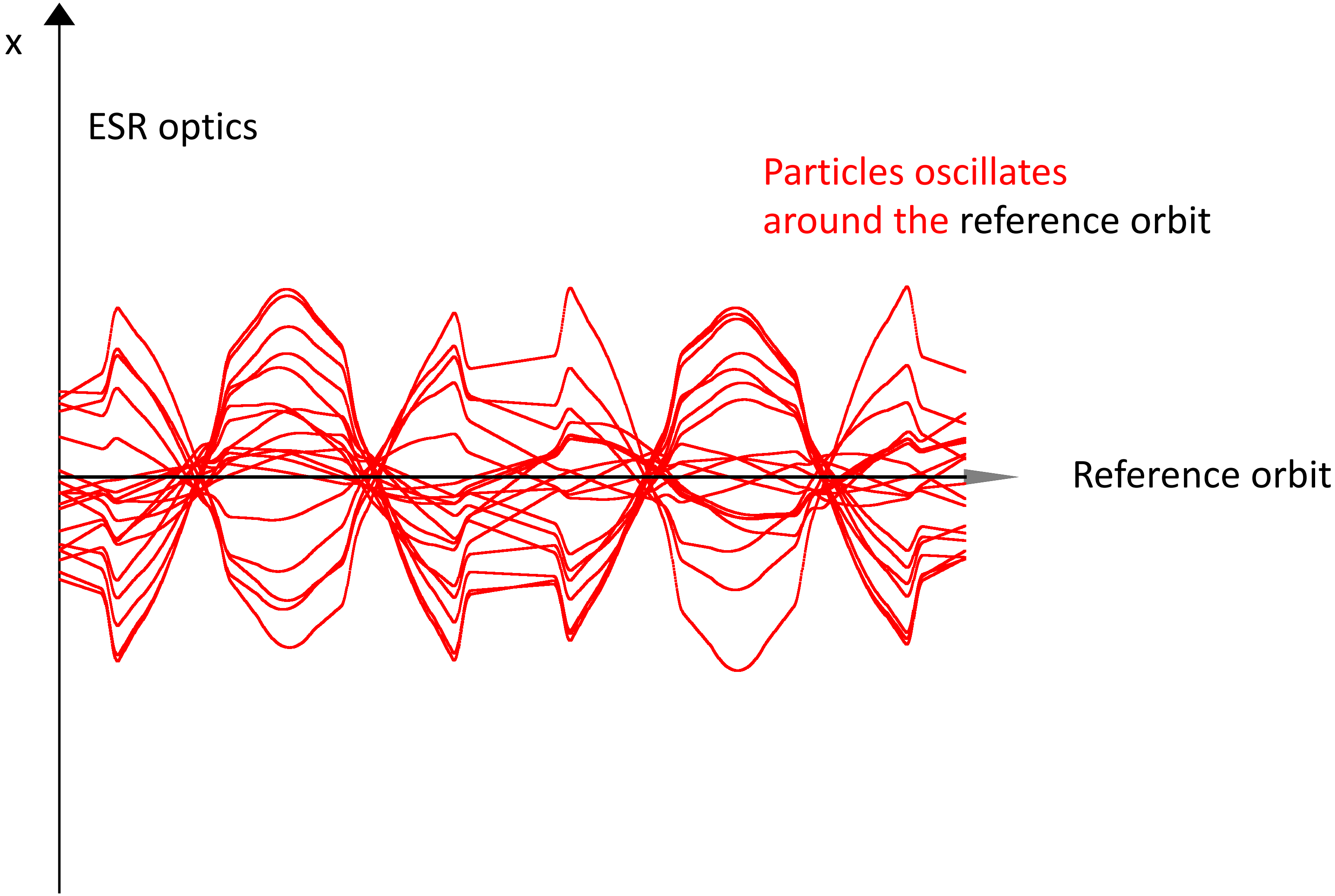
2. Particles oscillates around the reference orbit

Ideal dynamics

$$\begin{cases} \dfrac{d^2x}{ds^2} + \left(\dfrac{1}{\rho(s)^2} - k_1(s)\right)x = 0 \\ \\ \dfrac{d^2y}{ds^2} + k_1(s)y = 0 \end{cases}$$

The coordinates are measured with respect to the reference orbit

Ideal Particle Dynamics



For an off-momentum particle

Chromatic effect

$$k_x(s) \Rightarrow \frac{k_x(s)}{1+\frac{\delta p}{p_0}}$$

Dispersive term

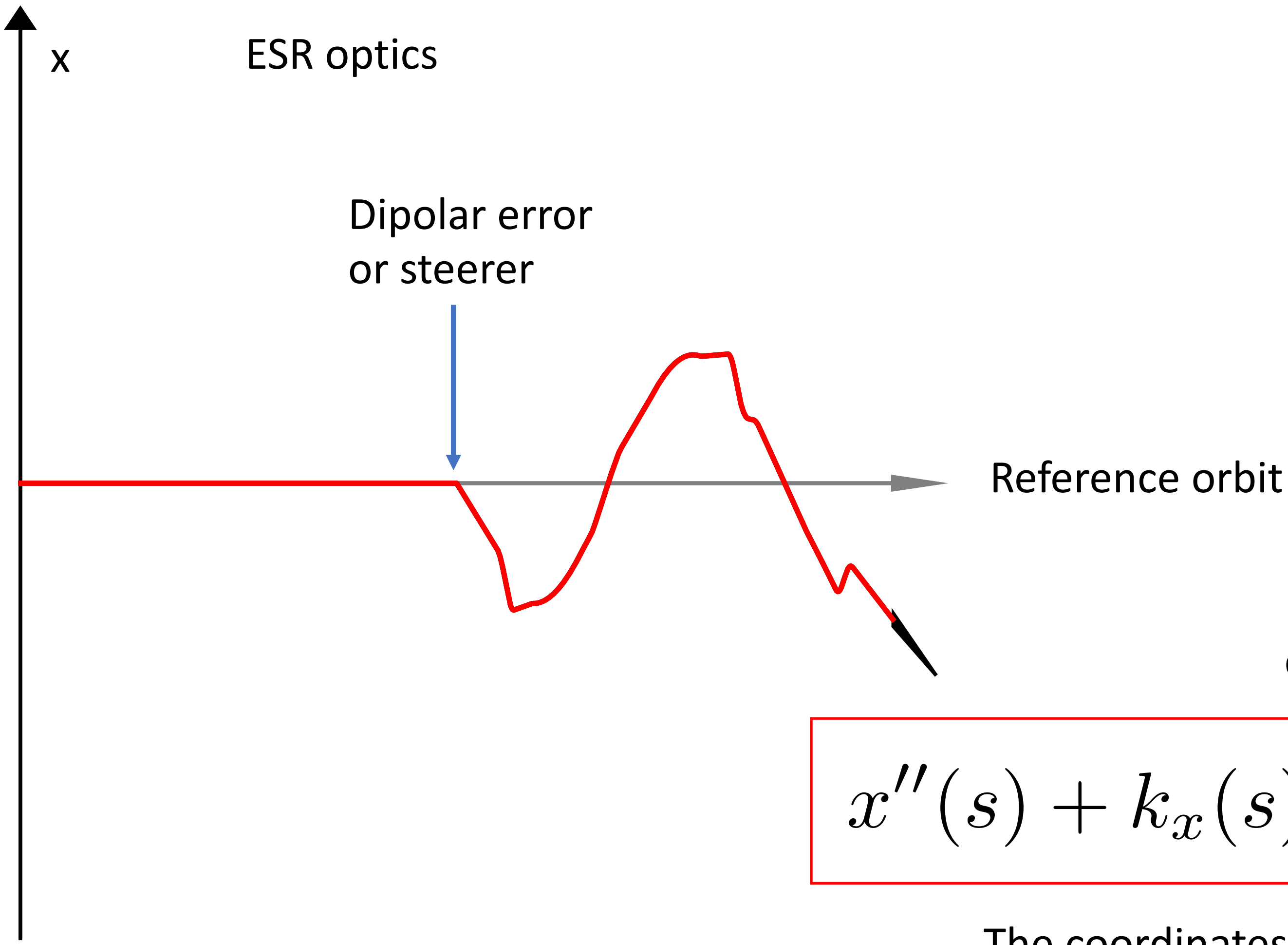
$$\frac{1}{\rho(s)}\frac{\delta p}{p_0}$$

$$x''(s) + \frac{k_x(s)}{1+\frac{\delta p}{p_0}}x = \frac{1}{\rho(s)}\frac{\delta p}{p_0}$$

$$y''(s) + \frac{k_y(s)}{1+\frac{\delta p}{p_0}}x = 0$$

The coordinates are measured with respect to the reference orbit

Including localized dipoles



Add the kick to the equation of motion

$$x''(s) + k_x(s)x = \theta_k(s)$$

The coordinates are measured with respect to the reference orbit

Including the nonlinearities

$$
\begin{cases}
\dfrac{d^2x}{ds^2} + \left(\dfrac{1}{\rho(s)^2} - k_1(s)\right)x = \underbrace{\mathrm{Re}\left[\displaystyle\sum_{n=2}^{M} \frac{k_n(s)+ij_n(s)}{n!}(x+iy)^n\right]}_{\mathcal{N}_x} \\[2em]
\dfrac{d^2y}{ds^2} + k_1(s)y = \underbrace{-\mathrm{Im}\left[\displaystyle\sum_{n=2}^{M} \frac{k_n(s)+ij_n(s)}{n!}(x+iy)^n\right]}_{\mathcal{N}_y} .
\end{cases}
$$

The coordinates are measured with respect to the reference orbit

General equations of motion

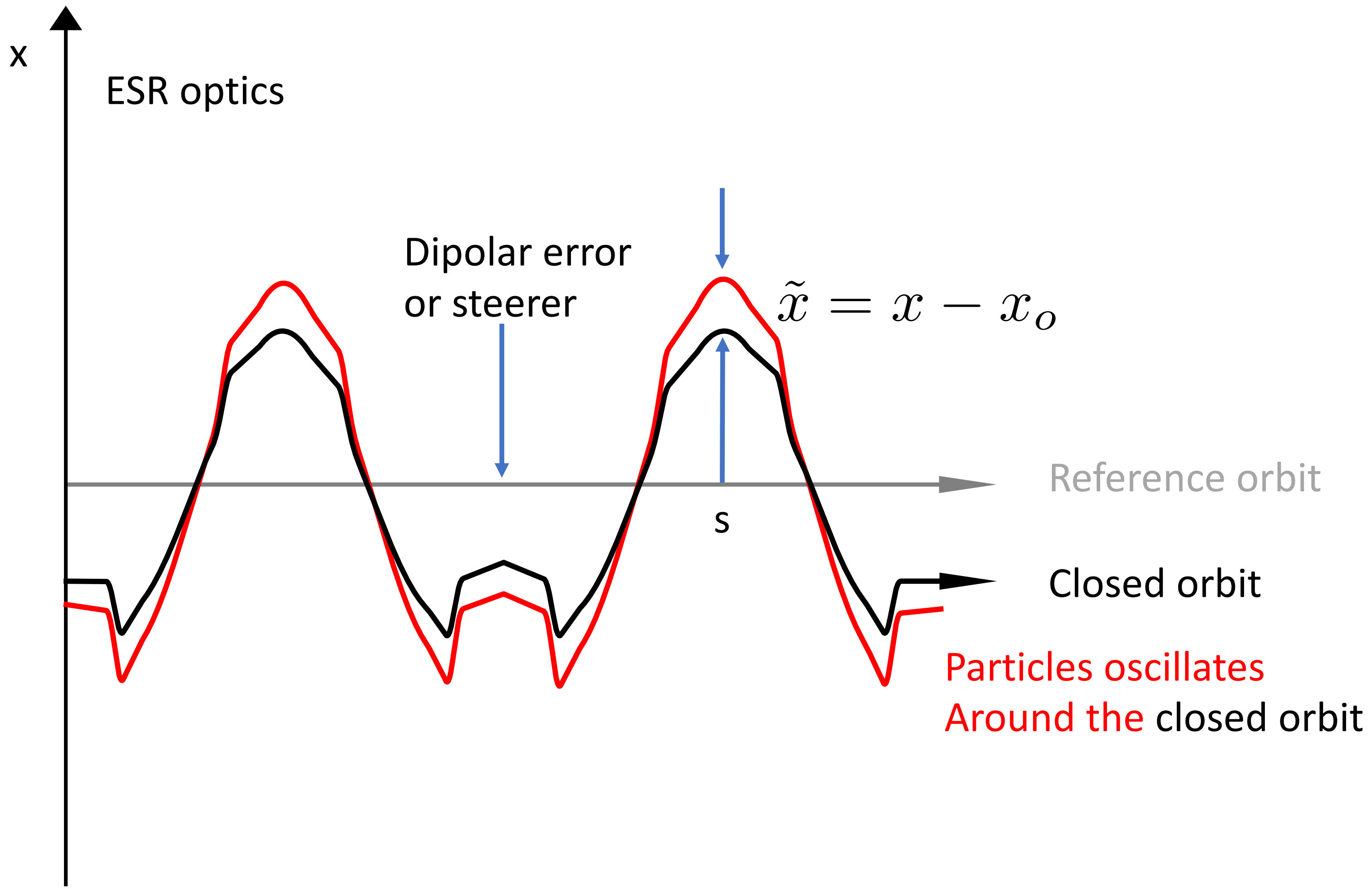
$$x'' + \frac{k_x}{1+\frac{\delta p}{p_0}} x = \frac{1}{\rho}\frac{\delta p}{p_0} + \frac{\theta_x}{1+\frac{\delta p}{p_0}} + \mathcal{N}_x(x,y),$$

$$y'' + \frac{k_y}{1+\frac{\delta p}{p_0}} y = \frac{\theta_y}{1+\frac{\delta p}{p_0}} + \mathcal{N}_y(x,y).$$

The coordinates are measured with respect to the reference orbit

On-momentum particles

Dynamics around the COD of an on-momentum beam



Dynamics of nearby particles to the closed orbit

$$\tilde{x}'' + (k_x + \tilde{k})\tilde{x} = \tilde{j}\tilde{y},$$
$$\tilde{y}'' + (k_y - \tilde{k})\tilde{y} = \tilde{j}\tilde{x},$$

$$\tilde{k} = \sum_{n\geq 1} \tilde{k}_1^{(n)}, \qquad \tilde{j} = \sum_{n\geq 1} \tilde{j}_1^{(n)}.$$

n	$\tilde{k}_1^{(n)}$	$\tilde{j}_1^{(n)}$
1	k_1	j_1
2	$k_2 x_o - j_2 y_o$	$k_2 y_o + j_2 x_o$
3	$\frac{1}{2}k_3(x_o^2 - y_o^2) - j_3 x_o y_o$	$k_3 x_o y_o + \frac{1}{2}j_3(x_o^2 - y_o^2)$

Effect of feed-down components on particle tunes

$$\Delta Q_x = \frac{1}{4\pi}\int_0^C \beta_x(s)\tilde{k}(s)ds,$$
$$\Delta Q_y = -\frac{1}{4\pi}\int_0^C \beta_y(s)\tilde{k}(s)ds,$$

The control of the COD allows, using the tunes measurement, to retrieve information on the nonlinear error

Linear coupling feed-down errors requires including the normal mode frequency. How to include this contribution has already been worked out.

Already verified experimentally in SIS18 by A. Parfenova, also tried in SPS

Off-momentum particles

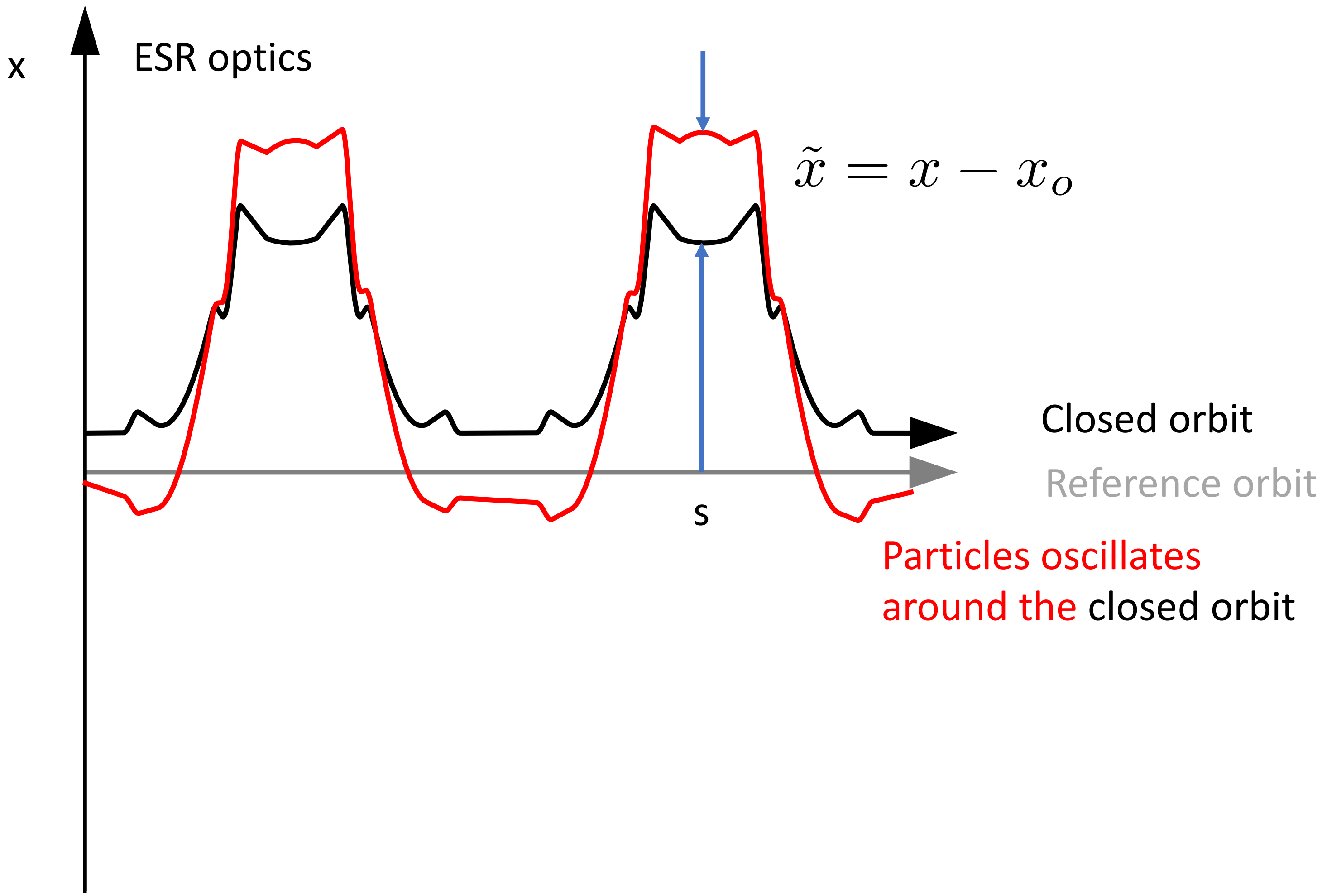
Including the effect of the dispersion

Idea: study the feed down by controlling the COD via Dispersion

The question is: how do we control the dispersion? Can we ?

These questions are the subject of this investigation

Dynamics of off-momentum particles



Particles oscillates around the closed orbit

The closed orbit

Solution of the equation

$$x_o'' + \frac{k_x}{1+\frac{\delta p}{p_0}} x_o = \frac{1}{\rho}\frac{\delta p}{p_0} + \cancel{\frac{\theta_x}{1+\frac{\delta p}{p_0}}} + \mathcal{N}_x(x_o, y_o)$$

$$y_o'' + \frac{k_y}{1+\frac{\delta p}{p_0}} y_o = \cancel{\frac{\theta_y}{1+\frac{\delta p}{p_0}}} + \mathcal{N}_y(x_o, y_o)$$

with the periodicity: $x_o(s) = x_o(s+L)$

$$y_o(s) = y_o(s+L)$$

this is not the only condition of periodicity

Tune-shift due to the dispersion closed orbit

$$Q_{x,o} = Q_x - \frac{1}{4\pi}\int_0^L \beta_x(s)k_x(s)ds\frac{\delta p}{p_0} + \frac{1}{4\pi}\int_0^L \beta_x(s)k_2(s)x_o ds - O_{\delta p}(2)$$

Chromatic contribution

Contribution due to sextupoles and the closed orbit that now is determined by the dispersion!

Very Problematic

NOTE: changing the quadrupoles will change the dispersion, hence x_o

Tune-shift due the “perturbed” dispersion closed orbit

$$\tilde{Q}_{x,o} = Q_x - \frac{1}{4\pi}\int_0^L \beta_x(s)k_x(s)ds\frac{\delta p}{p_0} +$$

$$+\frac{1}{4\pi}\int_0^L \beta_x(s)k_x(s)\delta k(s)ds + \frac{1}{4\pi}\int_0^L \beta_x(s)k_2(s)x_k(s)ds +$$

+ ....

Here x_k is the CO when the quadrupoles are perturbed

The idea is to compare two measurements of an off-momentum beam.

1) One measurement set is for a reference machine settings
2) In a second measurements the quadrupoles are perturbed so to "deform the dispersion"

$$\tilde{Q}_{x,o} - Q_{x,o} = \frac{1}{4\pi}\int_0^L \beta_x(s)k_x(s)\delta k(s)ds + \frac{1}{4\pi}\int_0^L \beta_x(s)k_2(s)\delta x_o(s)ds$$

This term is the detuning induced by the change of one quadrupole and is independent on the nonlinear errors

This term is due to the closed orbit deformation

$$\delta x_o = x_k - x_o$$

Due to the dispersion change

Due to the change of quadrupoles

It depends also from $k_2(s)$

If the change of the closed orbit amplitude depends linearly on the change of the quadrupoles there is a chance to have a method to retrieve the nonlinear errors

Can we predict $\delta x_o(s)$?

First order contribution

$$\delta x_o'' + \frac{k_x}{1+\frac{\delta p}{p_0}}\delta x_o = -\frac{k_x}{1+\frac{\delta p}{p_0}}\delta k\, x_o + O_{\delta o}(2) + O_{K_n x_{co}}(1)\delta x_o$$

$$\delta y_o'' + \frac{k_y}{1+\frac{\delta p}{p_0}}\delta y_o = 0 + O_{\delta o}(2) + O_{K_n x_{co}}(1)\delta y_o$$

It can be shown that $$\delta x_o(s) = \sum_{m=1}^{N_{quad}} \Lambda_m(s)\Delta k_m \frac{\delta p}{p_0}$$

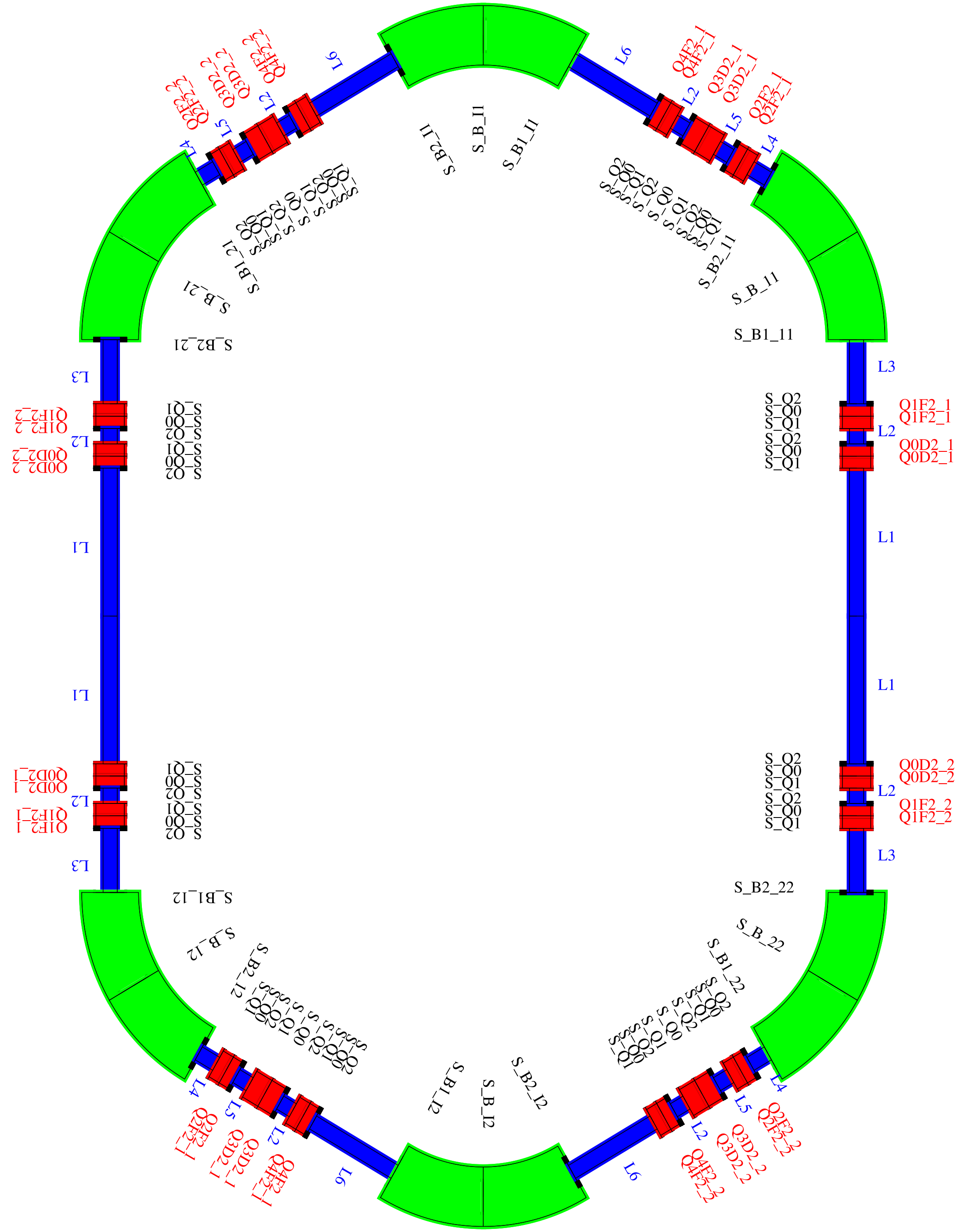
The function $\Lambda_m(s)$ is an "additional" optics function associated to the perturbation of one quadrupole (the m-th quadrupole).

TESTS

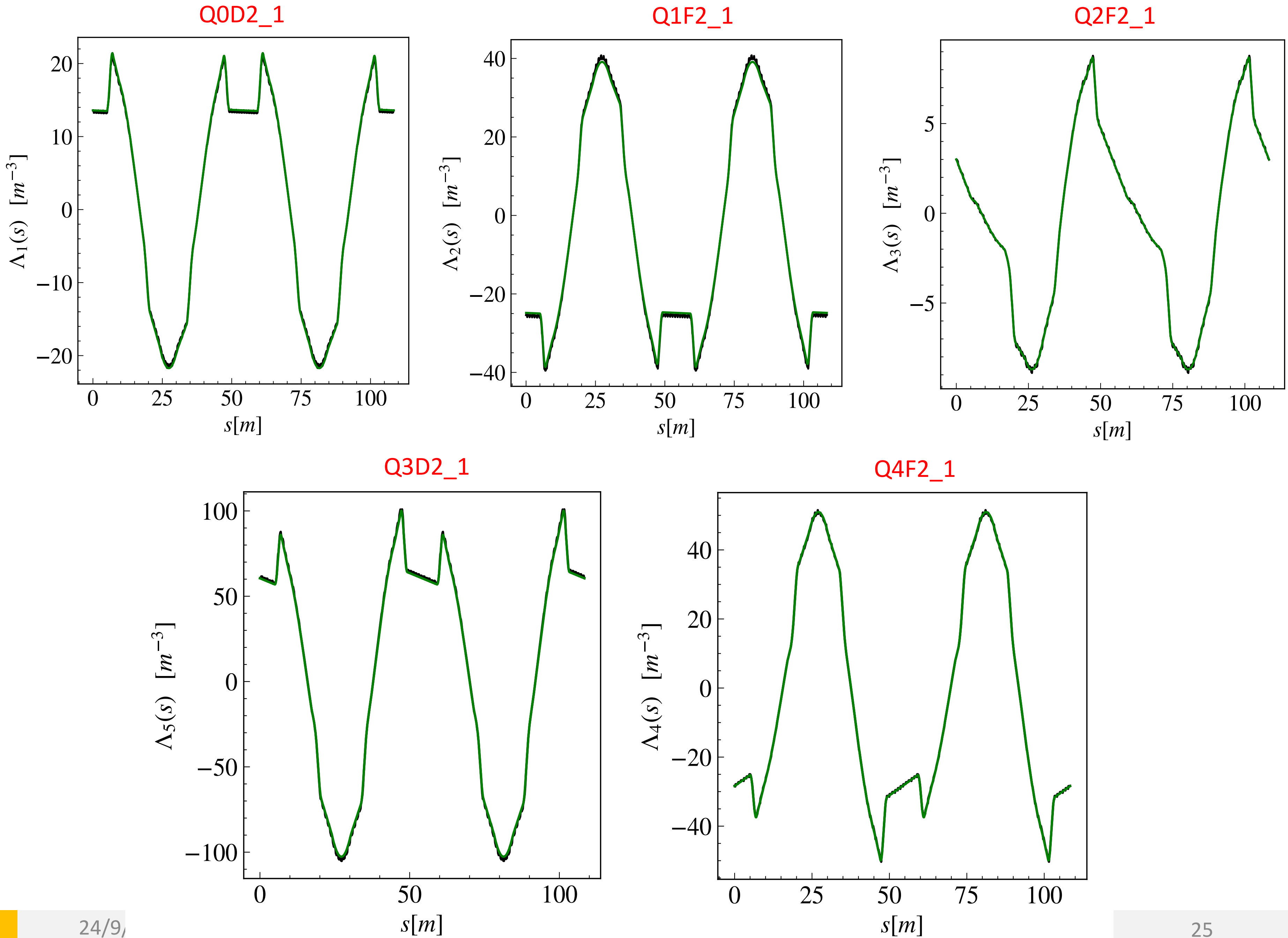
ESR

3 sextupolar errors location in each bending magnet

Quadrupoles coupled in families



Example of $\Lambda_m(s)$ functions for ESR



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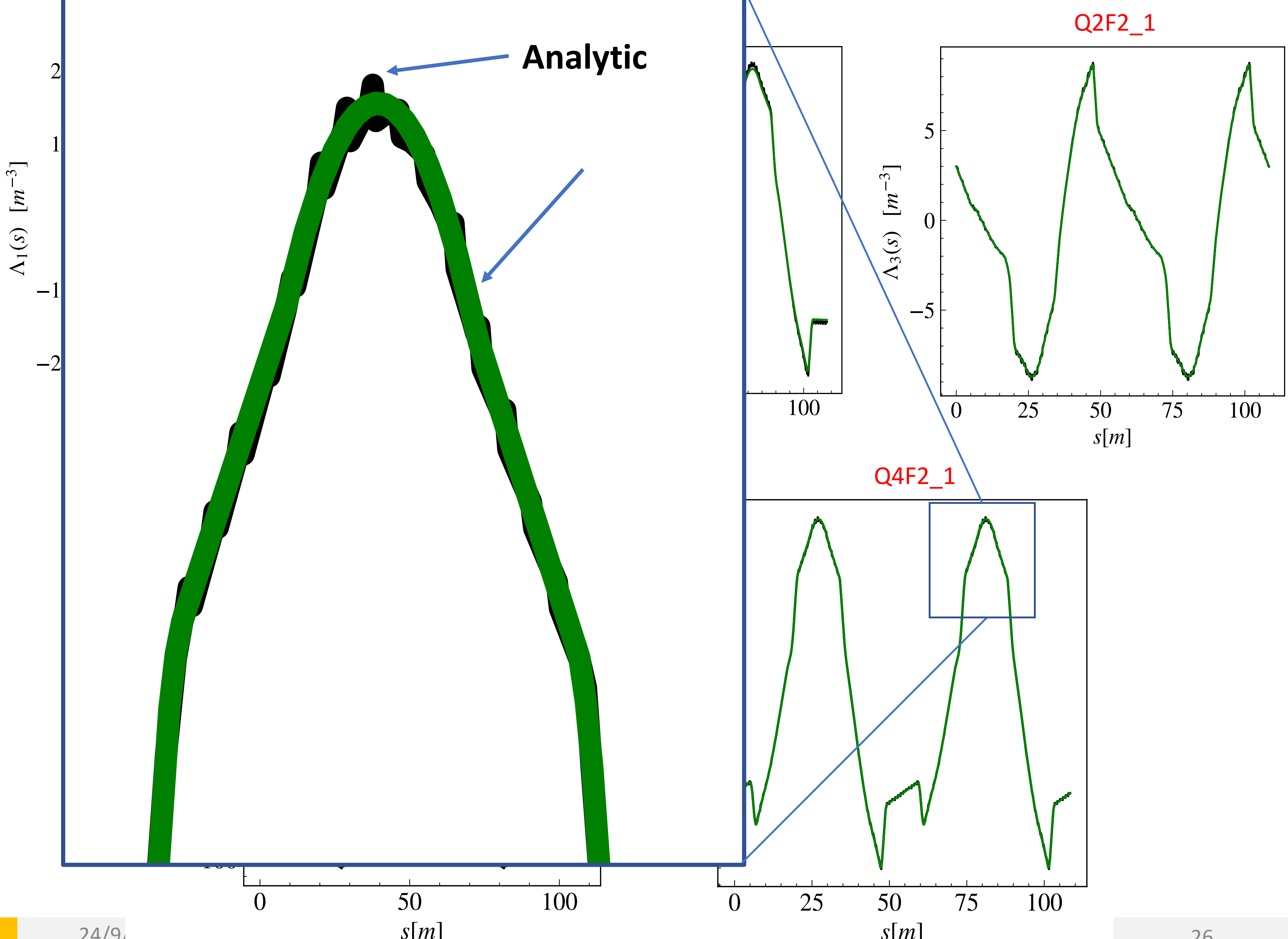


Numerical Tests

Simulation (sim)

compute the tunes via tracking: this is the simulation of the real experiment

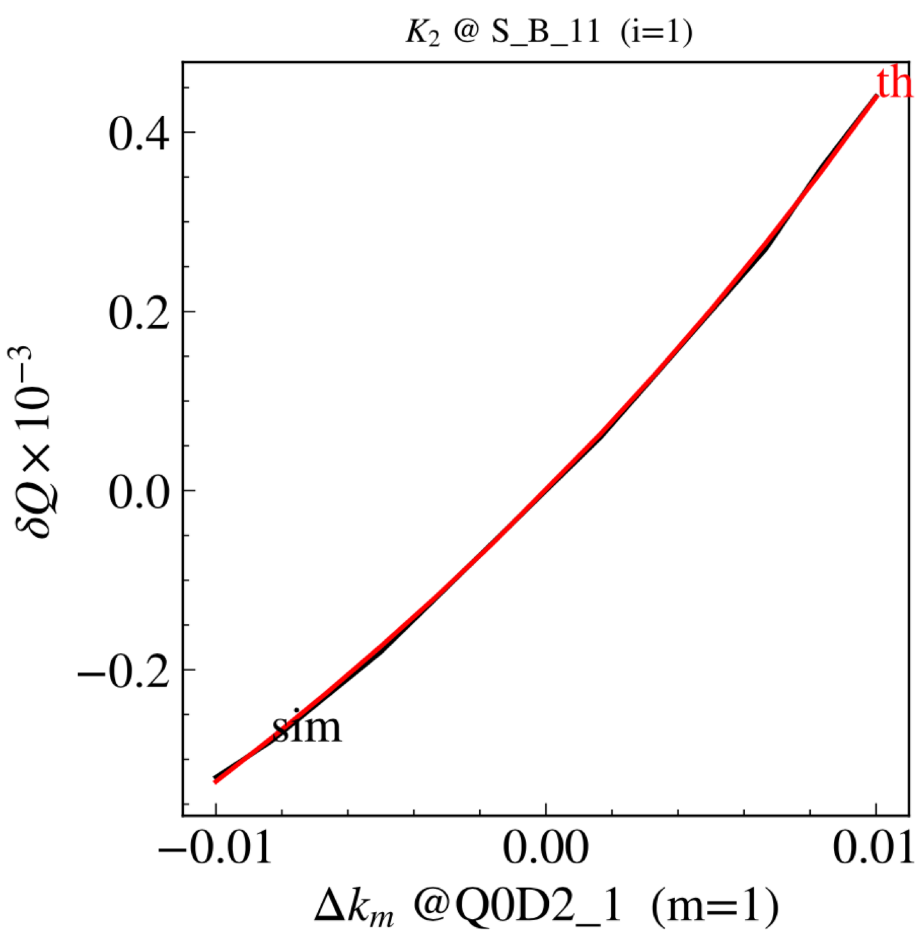
1) Compute the closed orbit
2) Set a particle near to the CO
3) Track it for 5000 turns and take coordinates at each turn
4) Make an FFT filtered properly to get high precision tune.

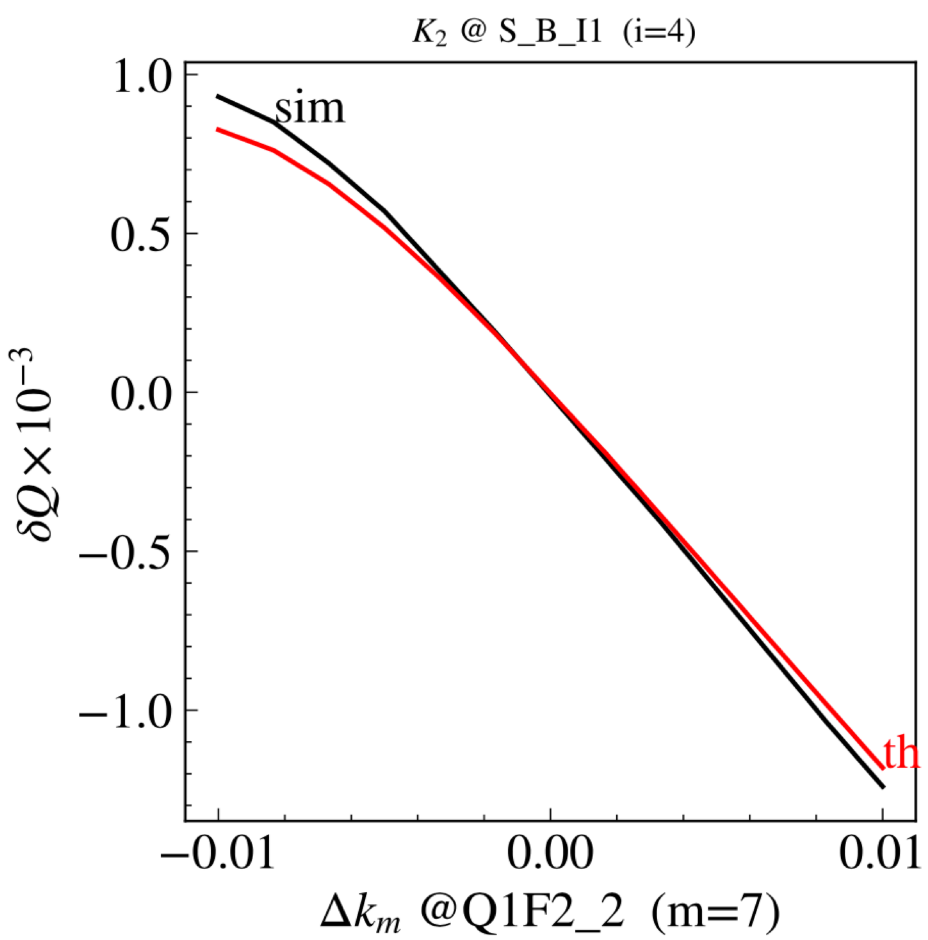
Theory (th)

compute the tunes using optical functions following the theoretical approach

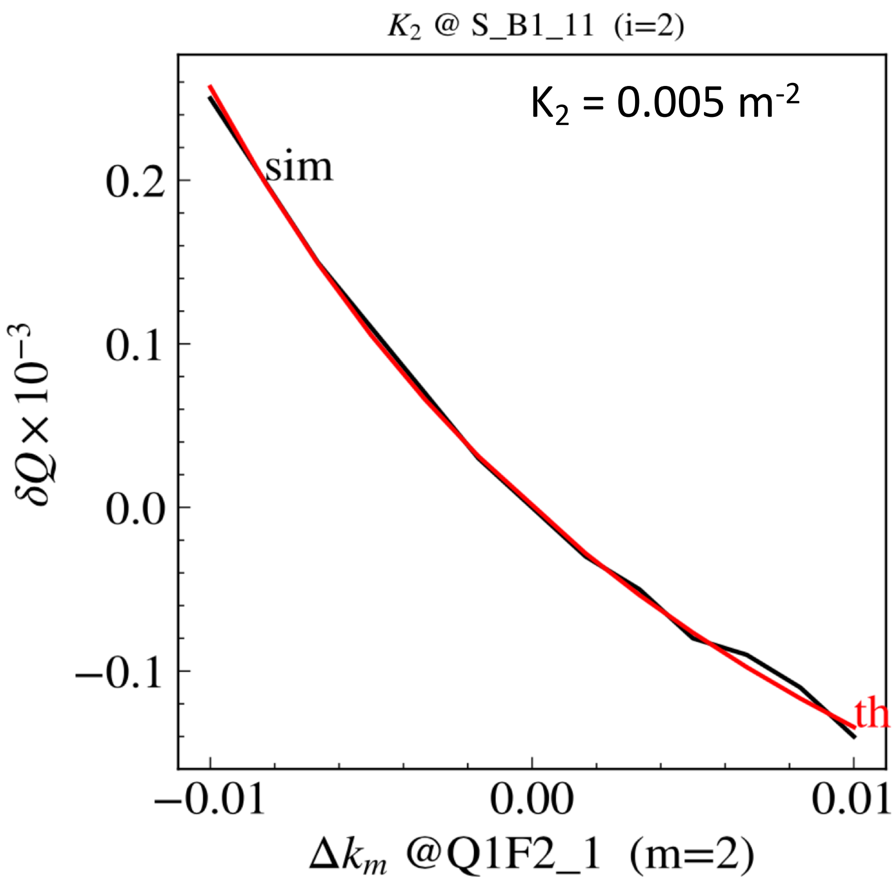
1) Uses global optics properties
2) Make use of the COD --> this requires care to details ….

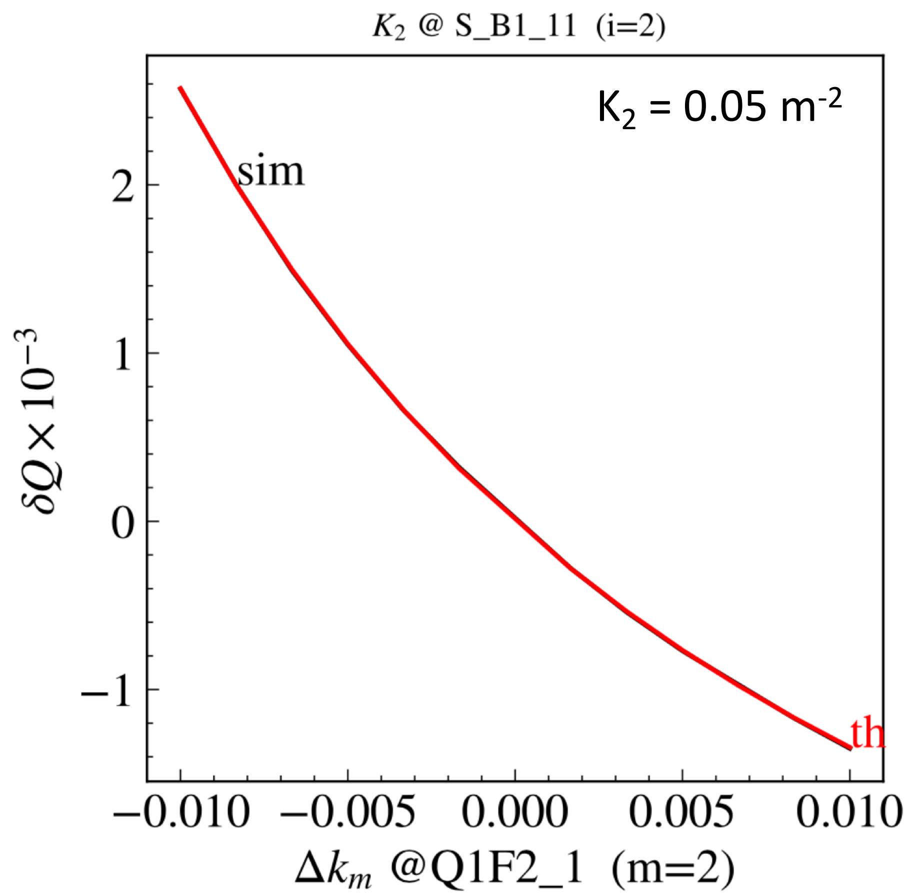
With chromaticity corrected



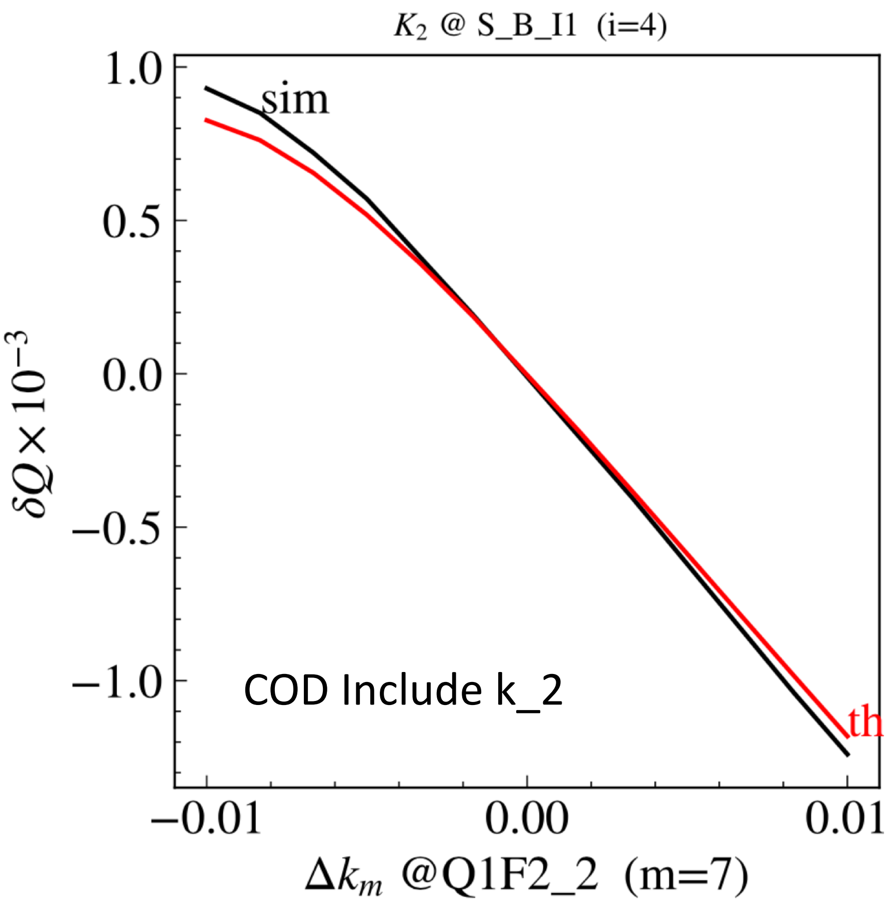


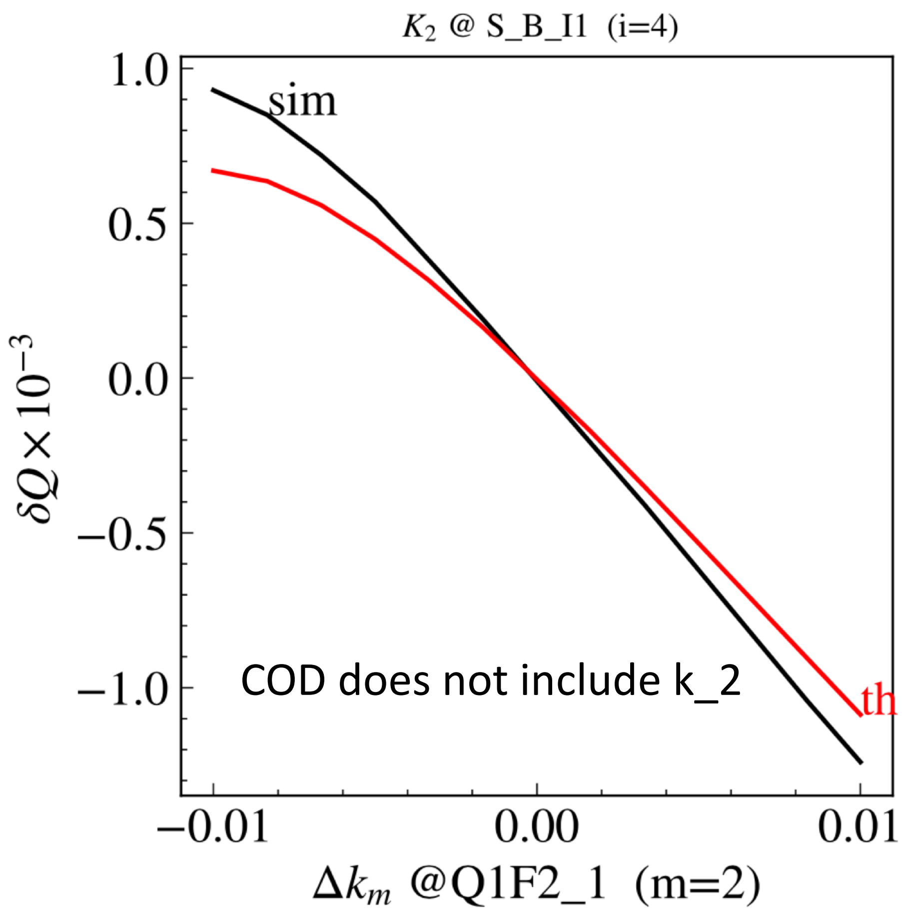
Predictivity of the nonlinear error strength





Issue: does the presence of the nonlinear error change x_k?

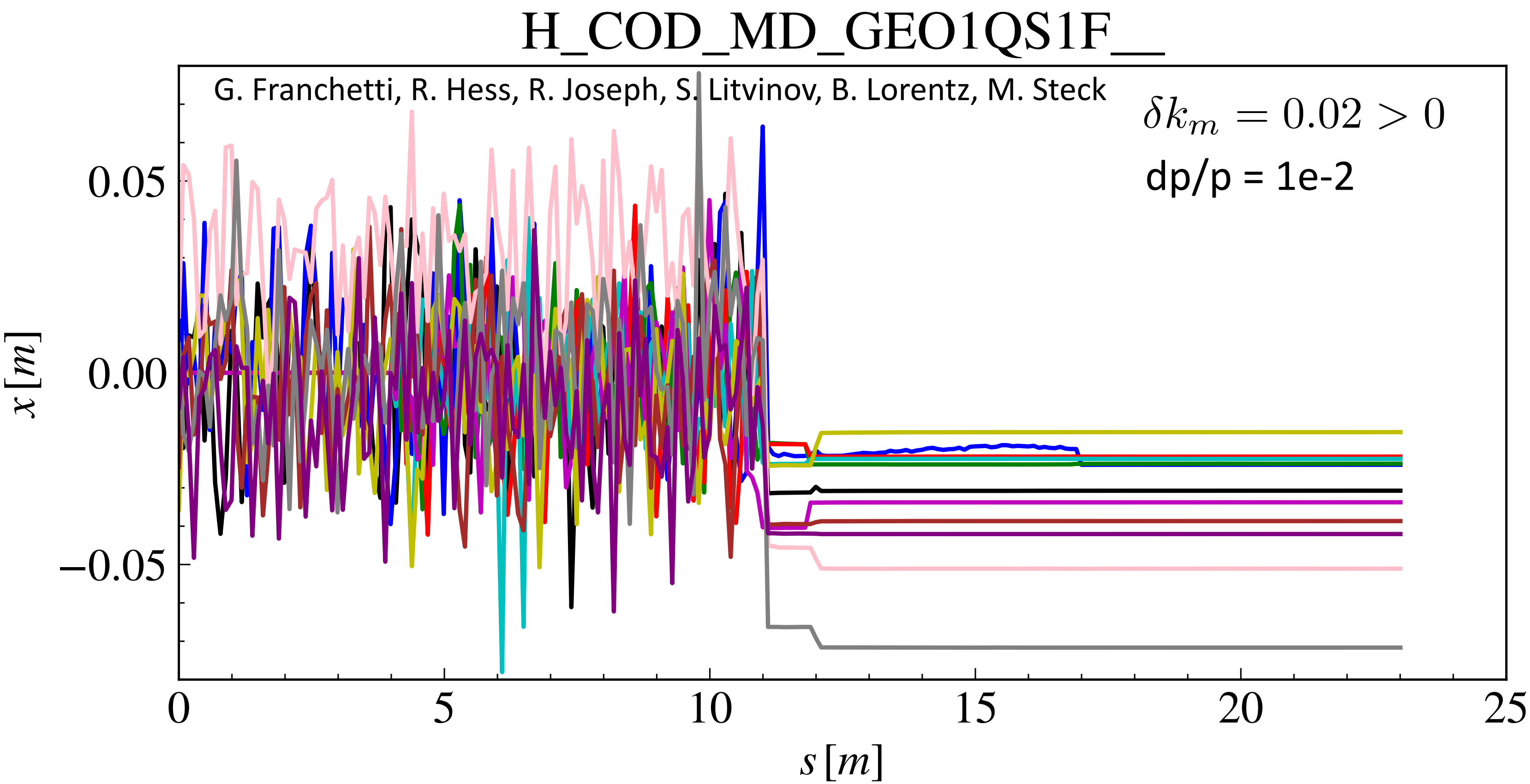




Toward experimental testing @ ESR

Last beam time an experimental campaign has been initiated in ESR

Question investigated: Is it COD induced by dp/p and dk enough to see an effect?



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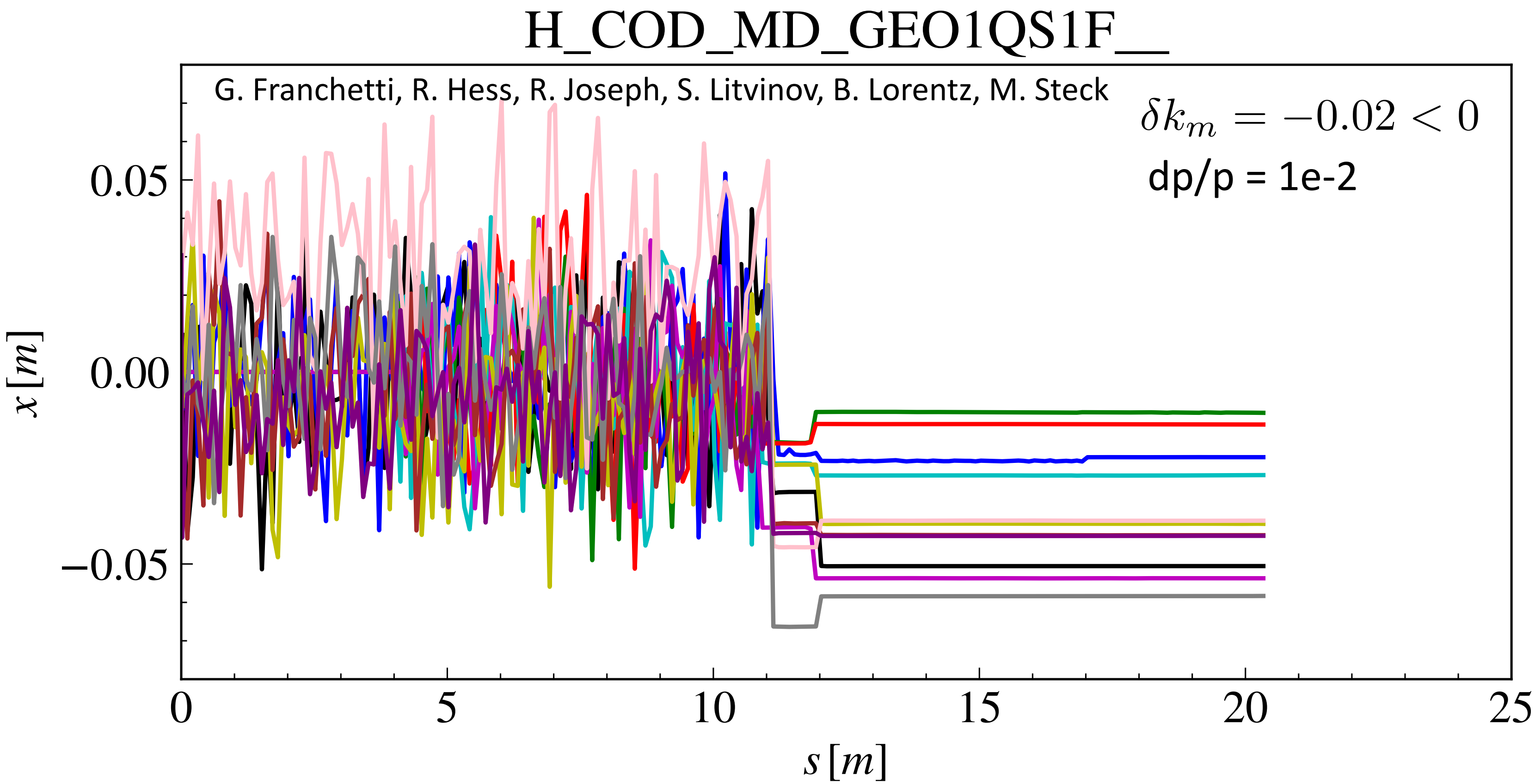

Summary and Outlook

1. For a lattice with chromaticity corrected, perturbing quads yield a detuning that compared with the unperturbed lattice contains information on the sextupolar components;
2. Simulations confirm this very well;
3. ESR has large momentum acceptance, hence allows testing this approach;
4. First experimental exploration was initiated last beam time: the big response of the CO to quad perturbation is very good and promising.

5. Next step: first benchmarking of tunes versus the ESR measurements, and finding the next experimental setup for complete experimental validation;
6. Next^2 step: disentangling the chromatic effect now included in the tunes so to retrieve the pure nonlinear contributions also when chromaticity cannot be compensated.

7. **Final goal, reach improved modeling of the ESR 2nd order nonlinearities, and later "explore the applicability" of this method to the 3rd order nonlinear components (very very hard…).**